CC blanc du 12/3/2021 IE-4 ZHILE ZHANG

Exo 1 :

a) $\vec{w} = [\vec{v_1}\ \vec{v_2}\ \vec{v_3}] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $m \in \mathbb{R}$

$\Leftrightarrow \begin{pmatrix} 0 \\ m \\ 1 \end{pmatrix} = x_1 \begin{pmatrix} -1 \\ 0 \\ m \end{pmatrix} + x_2 \begin{pmatrix} -1 \\ m \\ 1 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ m \\ 0 \end{pmatrix}$

$\Leftrightarrow$ $-x_1 - x_2 + 0x_3 = 0$
$0x_1 + m \cdot x_2 + m \cdot x_3 = m$
$m \cdot x_1 + x_2 + 0x_3 = 1$

$\rightsquigarrow \left[\begin{array}{ccc|c} -1 & -1 & 0 & 0 \\ 0 & m & m & m \\ m & 1 & 0 & 1 \end{array}\right]$

$L_1 \leftarrow (-1) L_1$ $\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & m & m & m \\ m & 1 & 0 & 1 \end{array}\right]$ Si $m=0$: $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]$, $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ \lambda \end{pmatrix}$ $\lambda \in \mathbb{R}$

Si $m \neq 0$: $\left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ m & 1 & 0 & 1 \end{array}\right]$ $L_3 \leftarrow L_3 - mL_1$ $\left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1-m & 0 & 1 \end{array}\right]$ $L_1 \leftarrow L_1 - L_2$, $L_3 \leftarrow L_3 - (1-m) L_2$ $\rightarrow$

$\left[\begin{array}{cccc} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & m-1 & m \end{array}\right]$ Si $m \neq 0$ et $m = 1$: $\left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right]$ Pas de solution

Si $m \neq 0$ et $m \neq 1$: $\left[\begin{array}{cccc} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & \frac{m}{m-1} \end{array}\right]$ $L_1 \leftarrow L_1 + L_3$, $L_2 \leftarrow L_2 - L_3$ $\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{m}{m-1} - 1 \\ 0 & 1 & 0 & 1 - \frac{m}{m-1} \\ 0 & 0 & 1 & \frac{m}{m-1} \end{array}\right]$

Conclusion : Quand $m = 0$ ou ($m \neq 1$ et $m \neq 0$) $\vec{w}$ est combinaison linéaire des $\vec{v_1}, \vec{v_2}, \vec{v_3}$.

b) Quand $m = 0$, $\vec{w} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\vec{v_1} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$, $\vec{v_3} = \vec{0}$,

$\vec{w} = (\vec{v_1}\ \vec{v_2}) \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

Quand $m \neq 0$ et $m \neq 1$, $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{m}{m-1} - 1 \\ 1 - \frac{m}{m-1} \\ \frac{m}{m-1} \end{pmatrix}$ $m \in \mathbb{R}$

$\therefore \vec{w} = [\vec{v_1}\ \vec{v_2}\ \vec{v_3}] \begin{bmatrix} \frac{m}{m-1} - 1 \\ 1 - \frac{m}{m-1} \\ \frac{m}{m-1} \end{bmatrix}$ $m \in \mathbb{R}$

Exo2 :

1) $\vec{v} = \begin{pmatrix}0\\2\\-3\end{pmatrix}$, $\vec{v}$ est le vecteur orthogonal à ce plan

$\therefore \vec{u} = \frac{1}{\|\vec{v}\|}\vec{v} = \frac{1}{\sqrt{2^2+3^2}}\begin{pmatrix}0\\2\\-3\end{pmatrix} = \frac{1}{\sqrt{13}}\begin{pmatrix}0\\2\\-3\end{pmatrix}$

2) $T_{(m)} : \mathbb{R}^3 \to \mathbb{R}^3$

$\vec{x} \to \vec{x} - (x \cdot \vec{u}) \cdot \vec{u}$

$T(\vec{e_1}) = \begin{pmatrix}1\\0\\0\end{pmatrix} - \left(\begin{pmatrix}1\\0\\0\end{pmatrix} \cdot \frac{1}{\sqrt{13}}\begin{pmatrix}0\\2\\-3\end{pmatrix}\right) \cdot \frac{1}{\sqrt{13}}\begin{pmatrix}0\\2\\-3\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix} = \frac{1}{13}\begin{pmatrix}13\\0\\0\end{pmatrix}$

$T(\vec{e_2}) = \begin{pmatrix}0\\1\\0\end{pmatrix} - \left(\begin{pmatrix}0\\1\\0\end{pmatrix} \cdot \frac{1}{\sqrt{13}}\begin{pmatrix}0\\2\\-3\end{pmatrix}\right) \cdot \frac{1}{\sqrt{13}}\begin{pmatrix}0\\2\\-3\end{pmatrix} = \frac{1}{13}\begin{pmatrix}0\\9\\6\end{pmatrix}$

$T(\vec{e_3}) = \begin{pmatrix}0\\0\\1\end{pmatrix} - \left(\begin{pmatrix}0\\0\\1\end{pmatrix} \cdot \frac{1}{\sqrt{13}}\begin{pmatrix}0\\2\\-3\end{pmatrix}\right) \cdot \frac{1}{\sqrt{13}}\begin{pmatrix}0\\2\\-3\end{pmatrix} = \frac{1}{13}\begin{pmatrix}0\\6\\4\end{pmatrix}$

$\therefore A = \frac{1}{13}\begin{pmatrix}13 & 0 & 0\\0 & 9 & 6\\0 & 6 & 4\end{pmatrix}$

3) $T(\vec{v}) = A \cdot \vec{v} = \frac{1}{13}\begin{pmatrix}13 & 0 & 0\\0 & 9 & 6\\0 & 6 & 4\end{pmatrix} \cdot \begin{pmatrix}-1\\1\\2\end{pmatrix} = \begin{pmatrix}-1\\ \frac{21}{13}\\ \frac{14}{13}\end{pmatrix}$

4) $T(\vec{x}) = A \cdot \vec{x} = \frac{1}{13}\begin{pmatrix}13 & 0 & 0\\0 & 9 & 6\\0 & 6 & 4\end{pmatrix} \cdot \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}2\\3\\2\end{pmatrix}$

$\Leftrightarrow \begin{pmatrix}x_1\\ \frac{9}{13}x_2 + \frac{6}{13}x_3\\ \frac{6}{13}x_2 + \frac{4}{13}x_3\end{pmatrix} = \begin{pmatrix}2\\3\\2\end{pmatrix} \sim \left(\begin{array}{ccc|c}1 & 0 & 0 & 2\\0 & 9 & 6 & 39\\0 & 6 & 4 & 26\end{array}\right)$ $L_2 \leftarrow L_2 \times \frac{1}{9}$, $L_3 \leftarrow L_3 - 6L_2$

$\to \begin{bmatrix}1 & 0 & 0 & 2\\0 & 1 & \frac{2}{3} & \frac{13}{3}\\0 & 0 & 0 & 0\end{bmatrix}$ $\therefore \vec{x} = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}2\\ \frac{13}{3} - \frac{2}{3}\alpha\\ \alpha\end{pmatrix}$ $\alpha \in \mathbb{R}$