$S5$:

(52) $\vec{v} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$, $\vec{u} = \frac{1}{\|\vec{v}\|} \cdot \vec{v} = \frac{1}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$T(x) = Sym_L(\vec{x}) = 2(x \cdot \vec{u}) \cdot \vec{u} - \vec{x}$

$T(\vec{e_1}) = 2 \cdot \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \frac{1}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix} \right) \cdot \frac{1}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{25}\begin{pmatrix} 32 \\ 24 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{7}{25} \\ \frac{24}{25} \end{pmatrix}$

$T(\vec{e_2}) = 2 \cdot \left(\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \frac{1}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix} \right) \cdot \frac{1}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{25}\begin{pmatrix} 24 \\ 18 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{24}{25} \\ -\frac{7}{25} \end{pmatrix}$

$\therefore A = \begin{pmatrix} \frac{7}{25} & \frac{24}{25} \\ \frac{24}{25} & -\frac{7}{25} \end{pmatrix}$

(57) $\vec{v} = \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$, $\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{4^2+3^2+3^2}}\begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix} = \frac{1}{\sqrt{34}}\begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$

$T(x) = Sym_V(\vec{x}) = \vec{x} - 2(\vec{x} \cdot \vec{u}) \cdot \vec{u}$

$T(\vec{e_1}) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - 2\left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cdot \frac{1}{\sqrt{34}}\begin{pmatrix} 4 \\ 3 \\ 3 \end{pmatrix} \right) \cdot \frac{1}{\sqrt{34}}\begin{pmatrix} 4 \\ 3 \\ 3 \end{pmatrix} = \frac{1}{17}\begin{pmatrix} 1 \\ -12 \\ -12 \end{pmatrix}$

$T(\vec{e_2}) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - 2\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \frac{1}{\sqrt{34}}\begin{pmatrix} 4 \\ 3 \\ 3 \end{pmatrix} \right) \cdot \frac{1}{\sqrt{34}}\begin{pmatrix} 4 \\ 3 \\ 3 \end{pmatrix} = \frac{1}{17}\begin{pmatrix} -12 \\ 8 \\ -9 \end{pmatrix}$

$T(\vec{e_3}) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - 2\left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot \frac{1}{\sqrt{34}}\begin{pmatrix} 4 \\ 3 \\ 3 \end{pmatrix} \right) - \frac{1}{\sqrt{34}}\begin{pmatrix} 4 \\ 3 \\ 3 \end{pmatrix} = \frac{1}{17}\begin{pmatrix} -12 \\ -9 \\ 8 \end{pmatrix}$

$A = \frac{1}{17}\begin{pmatrix} 1 & -12 & -12 \\ -12 & 8 & -9 \\ -12 & -9 & 8 \end{pmatrix}$

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B : homothétie

C : projection

E : transvection

A : symétrie

D : rotation

61

1) $\vec{n} = \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$

2) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sqrt{2}\alpha - \beta \\ \alpha + 0\beta \\ 0\alpha + \beta \end{pmatrix} = \alpha \underbrace{\begin{pmatrix} \sqrt{2} \\ 1 \\ 0 \end{pmatrix}}_{\vec{w_1}} + \beta \underbrace{\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}}_{\vec{w_2}}$

$\vec{v} = \alpha \vec{w_1} + \beta \vec{w_2} \quad (\alpha, \beta \in \mathbb{R})$

3) $\vec{u} = \frac{1}{\|\vec{n}\|} \vec{n} = \frac{1}{\sqrt{1+2+1}} \cdot \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$

Thm : $\mathrm{Sym}_V(\vec{x}) = \vec{x} - 2(\vec{x} \cdot \vec{u}) \cdot \vec{u}$

$T(\vec{e_1}) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - 2\left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}\right) \cdot \frac{1}{2}\begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{2} \\ -1 \end{pmatrix}$

$T(\vec{e_2}) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - 2\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}\right) \cdot \frac{1}{2}\begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \sqrt{2} \\ 0 \\ \sqrt{2} \end{pmatrix}$

$T(\vec{e_3}) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - 2\left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}\right) \cdot \frac{1}{2}\begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ \sqrt{2} \\ 1 \end{pmatrix}$

$A = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{2} & -1 \\ \sqrt{2} & 0 & \sqrt{2} \\ -1 & \sqrt{2} & 1 \end{pmatrix}$

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1) $\vec{n} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$

$\vec{u} = \dfrac{\vec{n}}{\|\vec{n}\|} = \dfrac{1}{\sqrt{3^2+1+1}} \cdot \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = \dfrac{1}{\sqrt{11}} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$

Tun: $\text{proj}_P(\vec{x}) = \vec{x} - (\vec{x} \cdot \vec{u}) \cdot \vec{u}$

$T(\vec{e_1}) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - \left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cdot \dfrac{1}{\sqrt{11}} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \right) \cdot \dfrac{1}{\sqrt{11}} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = \dfrac{1}{11} \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix}$

$T(\vec{e_2}) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - \left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \dfrac{1}{\sqrt{11}} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \right) \cdot \dfrac{1}{\sqrt{11}} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = \dfrac{1}{11} \begin{pmatrix} -3 \\ 10 \\ 1 \end{pmatrix}$

$T(\vec{e_3}) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - \left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot \dfrac{1}{\sqrt{11}} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \right) \cdot \dfrac{1}{\sqrt{11}} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = \dfrac{1}{11} \begin{pmatrix} 3 \\ 1 \\ 10 \end{pmatrix}$

$A = \dfrac{1}{11} \begin{pmatrix} 2 & -3 & 3 \\ -3 & 10 & 1 \\ 3 & 1 & 10 \end{pmatrix}$

2) $\text{proj}_P(\vec{v}) = A \cdot \vec{v} = \dfrac{1}{11} \begin{pmatrix} 2 & -3 & 3 \\ -3 & 10 & 1 \\ 3 & 1 & 10 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \dfrac{1}{11} \begin{pmatrix} 8 \\ -12 \\ 12 \end{pmatrix} = \begin{pmatrix} \frac{8}{11} \\ -\frac{12}{11} \\ \frac{12}{11} \end{pmatrix}$