36

$\cancel{T\begin{bmatrix}1\\0\\0\end{bmatrix} + T\begin{bmatrix}0\\1\\0\end{bmatrix} + T\begin{bmatrix}0\\0\\1\end{bmatrix} = [\text{illegible}]}$

$T(\vec{e_1}) = \begin{bmatrix}7\\11\end{bmatrix}$, $T(\vec{e_2}) = \begin{bmatrix}6\\9\end{bmatrix}$, $T(\vec{e_3}) = \begin{bmatrix}-13\\7\end{bmatrix}$

$\therefore A = \begin{bmatrix}7 & 6 & -13\\11 & 9 & 7\end{bmatrix}$

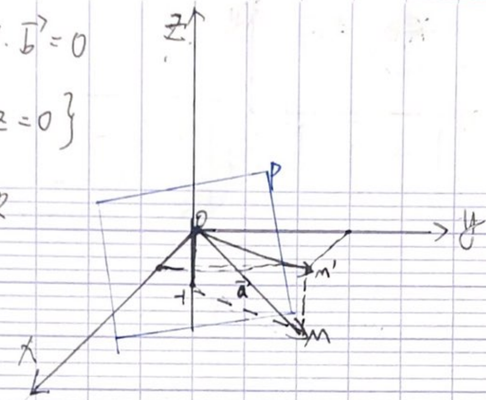
38

$\vec{a} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ , $\vec{a} \cdot \vec{b} = 0$

$\therefore \vec{a}^{\perp} = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; x + 3y - z = 0 \right\}$

$\therefore \vec{b} = \begin{pmatrix} \beta - 3\alpha \\ \alpha \\ \beta \end{pmatrix}, \; \alpha, \beta \in \mathbb{R}$

$\vec{a} \perp$ le plan P




39

$\vec{b} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ , $\vec{a_1} \cdot \vec{b} = 0$ et $\vec{a_2} \cdot \vec{b} = 0$ et $\vec{a_3} \cdot \vec{b} = 0$ , On peut obtenir

$\Rightarrow \begin{pmatrix} x_1 + x_2 + x_3 + x_4 = 0 \\ x_1 + 2x_2 + 3x_3 + 4x_4 = 0 \\ x_1 + 9x_2 + 9x_3 + 7x_4 = 0 \end{pmatrix} \rightsquigarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 4 & 0 \\ 1 & 9 & 9 & 7 & 0 \end{array}\right] \begin{array}{l} L_2 \leftarrow L_2 - L_1 \\ L_3 \leftarrow L_3 - L_1 \end{array} \Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 8 & 8 & 6 & 0 \end{array}\right]$

$\begin{array}{l} L_1 \leftarrow L_1 - L_2 \\ L_3 \leftarrow L_3 - 8L_2 \end{array} \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -1 & -2 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & -8 & -18 & 0 \end{array}\right] \begin{array}{l} L_3 \leftarrow -\frac{1}{8} \times L_3 \\ L_1 \leftarrow L_1 + L_3 \\ L_2 \leftarrow L_2 - 2L_3 \end{array} \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 1 & 0 & -\frac{3}{2} & 0 \\ 0 & 0 & 1 & \frac{9}{4} & 0 \end{array}\right]$

$\therefore \vec{b} = \begin{pmatrix} -\frac{1}{4}\alpha \\ \frac{3}{2}\alpha \\ -\frac{9}{4}\alpha \\ \alpha \end{pmatrix} \quad \alpha \in \mathbb{R}$

47

$\vec{a} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \quad \therefore \vec{u} = \frac{\vec{a}}{\|\vec{a}\|} = \frac{1}{\sqrt{2^2+1+2^2}} \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$

$\therefore \mathrm{Proj}_{\langle \vec{u} \rangle}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \left( \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{bmatrix} \right) \cdot \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{bmatrix} = \begin{bmatrix} \frac{10}{9} \\ \frac{5}{9} \\ \frac{10}{9} \end{bmatrix}$

51

$\vec{a} = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \quad \therefore \vec{u} = \frac{\vec{a}}{\|\vec{a}\|} = \frac{1}{\sqrt{4^2+3^2}} \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$\mathrm{Proj}_{\langle \vec{u} \rangle} \quad \begin{array}{l} \mathbb{R}^2 \to \mathbb{R}^2 \\ \vec{x} \to (\vec{x} \cdot \vec{u}) \cdot \vec{u} \end{array}$

$\mathrm{Proj}_{\langle \vec{u} \rangle}(\vec{e_1}) = \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \frac{1}{5} \begin{pmatrix} 4 \\ 3 \end{pmatrix} \right) \frac{1}{5} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \frac{1}{25} \begin{pmatrix} 16 \\ 12 \end{pmatrix}$

$\mathrm{Proj}_{\langle \vec{u} \rangle}(\vec{e_2}) = \left( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \frac{1}{5} \begin{pmatrix} 4 \\ 3 \end{pmatrix} \right) \frac{1}{5} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \frac{1}{25} \begin{pmatrix} 12 \\ 9 \end{pmatrix}$

$A = \frac{1}{25} \begin{pmatrix} 16 & 12 \\ 12 & 9 \end{pmatrix}$

(43) $\vec{BC} = \vec{AC} - \vec{AB}$

$\therefore \left\| \vec{BC} \right\| = \left\| (\vec{AC} - \vec{AB}) \right\| = \sqrt{\|\vec{AC}\|^2 - 2(\vec{AC} \cdot \vec{AB}) + \|\vec{AB}\|^2}$

$= \sqrt{9 - 2 \times 4 + 4} = \sqrt{5}$

$\therefore AB^2 + BC^2 = 4 + 5 = 9 = AC^2$

$\therefore$ le triangle (ABC) est rectangle.