

Literature review

Household Energy Use Modeling– A summary on the methodology

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Part I

Econometric Models

Chapter 1

Econometric Models

1.1 ECM and SEM

[NaiefEltony1995]

This paper tests two models for energy demand projection on Kuwait data and forecast the future energy demand in Kuwait. The long-run and short-run price and income elasticities were estimated. The root mean square percent error (RMSPE) and mean percent error (MPE) measured the deviation of the estimates from the observations.

1.1.1 Method 1: ECM

Cointegration and Error-Correction Model (ECM) takes into account the time-series characteristics of the data. It combines cointegration techniques and an error-correction model and has the advantages of

- Being easy to distinguish between the short- and long- run response.
- Estimating the speed of adjustment toward the long-run values.

Three-step projection

1. Examine the time-series effect.

To test if the time-series have a unit root – whether it is first-difference, second-difference, or n-difference stationary series.

A time-series process is considered *stationary* if the **mean and variance are constant over time** and if **the auto-correlation between values at two points depend only on the distance and not on the time period**.

Augmented Dickey-Fuller (ADF) test Running a regression for each series considered, with first-difference of the variables as the independent variable (δX_t , left-hand-side, LHS), and its first-lagged level (X_{t-1}) and the lagged first-differences (δX_{t-1}) as independent variables (RHS)

$$\Delta X_t = d_0 + d_1 \times X_{t-1} + \sum_{i=1}^m d_2 \times \Delta X_{t-1} + e_t \quad \text{for } m = 2, 4, \quad (1.1)$$

where e_t is the stationary random error that is normally distributed.

H_0 : the not-differenced form of the series is nonstationary (the series, in itself, is nonstationary.) Reject H_0 if d_1 is statistically significant and larger than the critical values reported in literature.

Conventional Dickey-Fuller (DF) test Based on Eq. (1.1) when the RHS summation is deleted:

$$\Delta X_t = d_0 + d_1 \times X_{t-1} + e_t \quad \text{for } m = 2, 4, \quad (1.2)$$

H_0 : the not-differenced form of the series is nonstationary (the series, in itself, is nonstationary.)

Reject H_0 if d_1 is statistically significant and larger than the critical values reported in literature.

2. Investigate the cointegration between variables.

If the variables are found to be nonstationary, the second step would be carried out.

Cointegration means the variables possess a long-run relationship. If each one of the variables are nonstationary but a linear combination of them is stationary, the variables could be considered cointegrated.

And if they are found to be cointegrated, the long-run elasticities would be estimated from the cointegration regression.

The following cointegrating regression is estimated because there are more than two variables:

$$\ln(E_t) = F_0 + F_1 \times \ln(P_t) + F_2 \times \ln(Y_t) + U_t, \quad (1.3)$$

where E_t is the per-capita energy consumption, P_t is the real price of energy, Y_t is the real per-capita income. U_t is the residual (normal distribution).

U_t should subject to a DF test. If the null hypothesis H_0 is accepted, the variables are cointegrated, and F_1 and F_2 are the **long-run price and income elasticities** respectively.

3. Estimate the short-run elasticities and the speed of adjustment from an ECM.

If the variables are cointegrated, the following ECM is estimated:

$$\Delta \ln(E_t) = J_0 + \sum_{i=0}^n J_{1i} \times \Delta \ln P_{t-i} + \sum_{i=0}^m J_{2i} \times \ln(Y_{t-i}) + \sum_{i=0}^s J_{3i} \Delta \ln(E_{t-i}) + J_4 \times U_{t-1} + Z_t, \quad (1.4)$$

where the lag-order n , m , and s are chosen to make Z_t white noise, and with U_{t-1} given by Eq. (1.3).

Coefficient J_{1i} is the **short-run price elasticities**, J_{2i} is the **short-run income elasticities**, and J_{4i} represents the speed of adjustment toward the long-run equilibrium.

Implementation

T-statistics from the conventional DF test and the ADF test are given to examine the stationarity and cointegration of variables (income, price, and energy demand). The cointegration Durbin-Watson (CRDW) test for the residuals in Eq. (1.2) and Eq. (1.3) are performed to reveal the cointegration. The parameter estimates can be found in Fig. 1.1 and Fig. 1.2.

In Fig. 1.1 and Fig. 1.2, the elasticities of energy demand showed that energy demand was price inelastic in both short- and long- run but income elastic in the long- run. The long-term effects of income and price changes were greater than the short run, coinciding the assumption of the slow adjustment of firms' and households' energy-using stocks in the short run.

1.1.2 Method 2: SEM

A **Simultaneous Equation Model (SEM)** assumed that the **desired** energy demand per capita (E_t^*) in year t depends on the real price of energy (P_t) and real per capita GDP (Y_t) in the form of a log-linear function. The planned energy demand could be written as

$$\ln(E_t^*) = A_0 + A_1 \times \ln(P_t) + A_2 \times \ln(Y_t), \quad (1.5)$$

Table 2

COINTEGRATING ENERGY DEMAND: DEPENDENT VARIABLE $\ln E$

Regressor	Parameter Estimate		T-Statistic
Constant	10.0520		5.91
$\ln P$	-0.2337		-5.82
$\ln Y$	1.2135		3.02
R^2	0.8080		
CRDW ^a	1.2653		
DF ^b	-4.2803		
ADF(1) ^c	-4.1725		
Residual correlogram			
Lag 1 to 4	0.0477	-0.2833 -0.1423	-0.0154

^aCointegrated Durbin-Watson.^bDickey-Fuller.^cAugmented Dickey-Fuller, first difference.

Figure 1.1: Cointegrating energy demand from Eq. (1.2). The parameter estimates for $\ln(P)$ and $\ln(Y)$ are the **long-run price- and income- elasticities**

But the **actual** energy demand per capita (E_t) is not necessarily equal to the desired level due to technological rigidity and the inertia in endusers' decision making. The actual demand E_t is thusly assumed to adjust towards the E_t^* with a lag such that it is a function of the current year's economic variables and the energy consumption in the previous year ($t - 1$), as indicated by Eq. (1.6).

$$\ln(E_t) = \delta \times (A_0 + A_1 \times \ln(P_t) + A_2 \times \ln(Y_t)) + (1 - \delta) \times \ln(E_{t-1}), \quad (1.6)$$

where $\delta(\in [0, 1])$ and $(1 - \delta)$ are the weighting coefficient between the current economic variables and the inertia from the previous year. A_1 and A_2 are the price- and income- elasticities.

Y_t is an endogenous variable which depends upon the level of energy consumption (E_t), whose output is altered by the technology and the structure of the economy. The structural and technological characteristics are measured by *the percentage of output by the nonoil sectors* S_t . The relation is depicted as:

$$\ln(Y_t) = C_0 + C_1 \times \ln(E_t) + C_2 \times \ln(S_t), \quad (1.7)$$

Eq. (1.6) and Eq. (1.7) are simultaneously and endogenously determined using a two-stage least square method.

Implementation

The cointegration tests performed at the log level of the variables indicated nonstationarity but cointegration among the log-variables. The authors therefore used the undifferenced forms of the variables to preserve the information about long-run relationships. The statistical results of the SEM are shown in Fig. 1.3.

Table 3

**ERROR-CORRECTING MODEL OF ENERGY DEMAND:
DEPENDENT VARIABLE $\Delta \ln E_t$**

Regressor	Parameter Estimate	T-statistic	Elasticities	
			Short Run	Long Run ^c
Constant	3.355	8.609		
$\Delta \ln P_t$	-0.1208	-2.768		
$\Delta \ln Y_t$	0.4825	7.559		
E_{t-1}	0.9669	6.424		
U_{t-1}	0.6336	3.690		
R^2	0.9475			
SEE ^a	0.1761			
DW ^b	1.9534			
P			-0.12	-0.23
Y			0.48	1.21

^aStandard error of the estimation.

^bDurbin-Watson.

^cObtained from cointegrating regression.

Figure 1.2: ECM of energy demand (Eq. (1.3)). The parameter estimates for $\Delta \ln(P)$ and $\Delta \ln(Y)$ are the short-run price- and income- elasticities.

$$\begin{aligned}
 \ln E_t &= 8.5396 - 0.2091 \ln P_t + 0.5455 \ln Y_t + 0.5185 \ln E_{t-1} \\
 \text{T-statistics} & \quad (7.45) \quad (-2.0) \quad (4.49) \quad (1.89) \\
 R^2 &= 0.6984 \quad \text{SEE} = 0.4655 \\
 \\
 \ln Y_t &= 11.279 + 0.6439 \ln E_t - 0.2975 \ln S_t + 0.3299 D7480 . \\
 & \quad (18.4) \quad (3.81) \quad (-1.84) \quad (2.44) \\
 R^2 &= 0.6180 \quad \text{SEE} = 0.3484
 \end{aligned}$$

Figure 1.3: Statistical results of the SEM (Eq. (1.6) and Eq. (1.7)). The short-run price- and income- elasticities are -0.2091 and 0.5455 respectively. The long-run price elasticity = $-0.2091/(1 - 0.5185) = -0.4343$. The long-run income elasticity = $0.5455/(1 - 0.5185) = 1.1329$.

Part II

Demand Systems

Chapter 2

Demand Systems

2.1 Quadratic Expenditure System(QES)

2.1.1 Methodology: QES

QES exploits the full potential of Engel curve flexibility and can be estimated with a relatively small number of free parameters.

Indirect Utility Function Each theoretically plausible quadratic expenditure system is generated by the indirect utility function Eq. (2.1)

$$\Psi(P, \mu) = -\frac{g(P)}{\mu - f(P)} - \frac{\alpha(P)}{g(P)}, \quad (2.1)$$

where $P^T = (p_1 p_2 \dots p_n)$ is the vector of prices for n commodity groups and μ denotes total expenditure.

Demand Functions Quadratic in Expenditure

[Howe1979] defined a system of demand functions quadratic in expenditure, by the indirect utility function, of the form

$$h_i(P, \mu) = \frac{1}{g^2}(\partial\alpha - \frac{\partial g}{g}\alpha)\mu^2 + [\frac{\partial g}{g} - \frac{2f}{g^2}(\partial\alpha - \frac{\partial g}{g}\alpha)]\mu + \frac{f^2}{g^2}(\partial\alpha - \frac{\partial g}{g}\alpha) - \frac{\partial g}{g}f + \partial f, \quad (2.2)$$

where f , g , and α are functions homogeneous of degree one, equivalently

$$h_i(P, \mu) = \frac{1}{g^2}(\partial\alpha - \frac{\partial g}{g}\alpha)(\mu - f)^2 + \frac{\partial g}{g}(\mu - f) + \partial f. \quad (2.3)$$

A *homogeneous function* is a function of several variables such that, if all its arguments are multiplied by a scalar, then its value is multiplied by some power of this scalar, called *the degree of homogeneity*, or simply the degree; that is, if k is an integer, a function f of n variables is homogeneous of degree k if

$$f(sx_1, \dots, sx_n) = s^k f(x_1, \dots, x_n) \quad (2.4)$$

for every x_1, \dots, x_n , and $s \neq 0$.

The system for implementation

The system easy for implementation is characterized by the indirect utility function Eq. (2.5)

$$\Psi(P, \mu) = -\frac{\prod_i^n P_i^{a_i}}{\mu - \sum_i^n p_i \hat{b}_i} - \frac{\sum_i^n p_i c_i}{\prod_i^n p_i^{a_i}}, \quad (2.5)$$

and the expenditure functions Eq. (2.6)

$$p_i q_i(P, \mu) = p_i \hat{b}_i + a_i \left(\mu - \sum_j p_j \hat{b}_j \right) + (c_i p_i - \sum_j p_j c_j) \prod_j p_j^{2a_j} \left(\mu - \sum_j p_j \hat{b}_j \right)^2. \quad (2.6)$$

under the following constraints.

$$g(P) = \prod_i p_i^{a_i} \quad (2.7)$$

$$f(P) = \sum_i p_i \hat{b}_i \quad (2.8)$$

$$\alpha(P) = \sum_i p_i c_i \quad (2.9)$$

$$\sum_i a_i = 1 \quad (2.10)$$

The system has $3n - 1$ free parameters, for each of the commodity exists a parameter set $\{a_i, \hat{b}_i, c_i\}$.

2.1.2 Examples

Estimation in Germany

[SCHULTE2017512]

This paper applied a quadratic expenditure system to project the **cross- and own- price elasticiteis** and **income- (expenditure) elasticities of residential energy demand (heating and electricity)** in Germany using the official expenditure data from 1993 to 2008.

Definition

They distinguished 10 commodities, as shown in Fig. 2.1, and estimated their mean expenditure, own-price and cross-price elasticities Fig. 2.2 at the national average level. Six household type (Fig. 2.3) and the income quartiles were distinguished to examine the income (expenditure) (Fig. 2.4) and price elasticities (Fig. 2.5) of energy demand in different households.

Expenditure elasticity

At the expenditure elasticities of electricity, heating increased with income level (ranging from 0.253 to 0.495 for electricity, from 0.279 to 0.452 for heating) and from single households to coupled households.

Price elasticity

At the national average level, the own-price elasticities of electricity, heating are -0.4310 and -0.5008, indicating a price-inelastic residential energy demand in Germany. Higher income households showed greater elasticity in both electricity and heating demand (around 3 times in the top quartile than in the bottom). Elasticity also grew from single households to coupled households.

Stone-Lewbel Cross Section Prices

They reflect the fact that the composition of consumed commodity groups differs among households, making the perceived price of the commodity groups different among households. It is based on a theory of household specific price indices under the *weakly separable* demand assumption.// In the case of Cobb-Douglas *within group utility functions*, the price indices could be constructed as

$$u_i(q_i, s) = g_i \prod_h q_{ih}^{w_{ih}(s)}, \quad \sum_h w_{ih} = 1, \quad (2.11)$$

where i is the commodity group, h is the composite in commodity group i , s denotes a demographic characteristics vector, g_i is the scaling factor for the commodity group, q_{ih} is the consumed quantity and w_{ih} is the (within group) budget share of good h in group i .

In this case, the household specific price index $p_i(s)$ is derived as

$$p_i(s) = \frac{1}{g_i} \prod_i \left(\frac{\hat{p}_{ih}}{w_{ih}} \right)^{w_{ih}}, \quad (2.12)$$

whereby \hat{p}_{ih} denotes prices for good h of commodity group i and the scaling factor g_i represents

Table 1
Definition of commodity groups.

No.	Code	Comprised goods
1	ELECTRICITY	electricity
2	HEATING	gas, oil, solid fuels, district heating
3	TRANSPORT	car fuel, public transport
4	FOOD	food, food away from home, (alcoholic) beverages, tobacco
5	CLOTHES	clothes, shoes, shoe repair
6	HOUSING	rent, rent equivalent for homeowners, maintenance and repair
7	HEALTH	health care, personal hygiene, care of the elderly/disabled
8	MOBILITY	private transport (except for car fuel), communication
9	EDUCATION	education, entertainment, child daycare
10	OTHERS	furniture, household appliances, jewellery, vacation trips, financial services, other services

Figure 2.1: Commodity definition.

Table 3

Mean expenditure, own-price and cross-price elasticities for all commodity g expenditure and prices distributions. Standard errors in parentheses are deri

	1	2	3	4
μ	0.398 8 (0.003 6)	0.405 5 0.006 4)	0.636 9 (0.004 9)	0.658 3 (0.002 7)
p_1	-0.431 0 (0.005 9)	-0.004 8 (0.000 2)	-0.009 5 (0.000 1)	-0.008 7 (0.000 1)
p_2	-0.008 0 (0.000 2)	-0.500 8 (0.005 4)	-0.015 4 (0.000 3)	-0.013 5 (0.000 2)
p_3	-0.008 6 (0.000 2)	-0.007 3 (0.000 4)	-0.572 6 (0.005 9)	-0.014 6 (0.000 3)
p_4	-0.021 4 (0.000 9)	-0.014 8 (0.001 7)	-0.050 2 (0.001 2)	-0.725 9 (0.004 9)

Figure 2.2: Mean expenditure, own-price and cross-price elasticities (electricity, heating, transport, food) at the national average level.

Table 2

Definition of household types and composition of data set.

Code	S0	S1	C0	C1	C2	C3
Type	<i>Single without children</i>	<i>Single with one child</i>	<i>Couple without children</i>	<i>Couple with one child</i>	<i>Couple with two children</i>	<i>Couple with three children</i>
Share	25.8%	2.8%	39.3%	11.6%	15.8%	4.7%

Age of children: from 0 up to and including 17 years.

Figure 2.3: Household type.

Table 4

Expenditure elasticities for energy goods and food for different household types at different total expenditure levels: Predicted values at the means of the 2008 household type specific total expenditure quartiles and at respective price means. Standard errors in parentheses are derived with the delta method.

Good		S0	S1	C0	C1	C2	C3
1	μ_{0-25}	0.260 (0.004)	0.253 (0.004)	0.281 (0.003)	0.291 (0.003)	0.298 (0.004)	0.281 (0.004)
	μ_{25-50}	0.333 (0.004)	0.302 (0.004)	0.353 (0.004)	0.355 (0.004)	0.356 (0.004)	0.342 (0.004)
	μ_{50-75}	0.391 (0.004)	0.348 (0.005)	0.407 (0.004)	0.405 (0.004)	0.403 (0.004)	0.389 (0.005)
	μ_{75-100}	0.485 (0.004)	0.437 (0.005)	0.495 (0.007)	0.477 (0.006)	0.471 (0.007)	0.462 (0.008)
2	μ_{0-25}	0.279 (0.003)	0.293 (0.004)	0.311 (0.003)	0.353 (0.005)	0.362 (0.005)	0.367 (0.006)
	μ_{25-50}	0.343 (0.003)	0.338 (0.005)	0.364 (0.005)	0.401 (0.006)	0.408 (0.007)	0.405 (0.009)
	μ_{50-75}	0.387 (0.004)	0.378 (0.005)	0.398 (0.008)	0.431 (0.009)	0.436 (0.010)	0.432 (0.012)
	μ_{75-100}	0.452 (0.007)	0.445 (0.008)	0.419 (0.017)	0.447 (0.017)	0.434 (0.019)	0.407 (0.024)
3	μ_{0-25}	0.447 (0.005)	0.485 (0.009)	0.482 (0.005)	0.425 (0.005)	0.484 (0.005)	0.515 (0.008)
	μ_{25-50}	0.533 (0.005)	0.556 (0.009)	0.584 (0.005)	0.522 (0.005)	0.588 (0.006)	0.613 (0.008)
	μ_{50-75}	0.601 (0.005)	0.618 (0.009)	0.668 (0.006)	0.592 (0.006)	0.652 (0.007)	0.687 (0.009)
	μ_{75-100}	0.723 (0.005)	0.736 (0.009)	0.807 (0.009)	0.721 (0.008)	0.773 (0.010)	0.830 (0.012)
4	μ_{0-25}	0.610 (0.005)	0.570 (0.006)	0.540 (0.004)	0.546 (0.004)	0.576 (0.004)	0.581 (0.005)
	μ_{25-50}	0.667 (0.004)	0.625 (0.006)	0.595 (0.003)	0.605 (0.003)	0.634 (0.003)	0.649 (0.004)
	μ_{50-75}	0.705 (0.003)	0.660 (0.005)	0.625 (0.003)	0.642 (0.003)	0.662 (0.003)	0.688 (0.004)
	μ_{75-100}	0.749 (0.003)	0.736 (0.004)	0.656 (0.006)	0.677 (0.005)	0.693 (0.005)	0.720 (0.005)

1: ELECTRICITY, 2: HEATING, 3: TRANSPORT, 4: FOOD.

Figure 2.4: Income (expenditure) elasticities of electricity, heating, transport, and food demand in different households.

Table 5

Price elasticities for different household types: Predicted values at the means of the household type specific total expenditure quartiles and respective prices. Standard errors in parentheses are derived with the delta method.

Good		S0	S1	C0	C1	C2	C3
1	μ_{0-25}	-0.179 (0.004)	-0.174 (0.007)	-0.234 (0.004)	-0.227 (0.005)	-0.238 (0.005)	-0.215 (0.006)
	μ_{25-50}	-0.282 (0.005)	-0.244 (0.007)	-0.353 (0.006)	-0.341 (0.006)	-0.351 (0.006)	-0.324 (0.007)
	μ_{50-75}	-0.376 (0.006)	-0.319 (0.008)	-0.467 (0.007)	-0.440 (0.007)	-0.449 (0.007)	-0.430 (0.008)
	μ_{75-100}	-0.566 (0.008)	-0.501 (0.010)	-0.724 (0.011)	-0.657 (0.010)	-0.665 (0.010)	-0.676 (0.012)
	μ_{0-25}	-0.205 (0.003)	-0.215 (0.008)	-0.281 (0.004)	-0.302 (0.006)	-0.320 (0.006)	-0.311 (0.009)
	μ_{25-50}	-0.313 (0.004)	-0.294 (0.009)	-0.413 (0.005)	-0.439 (0.007)	-0.463 (0.007)	-0.451 (0.010)
	μ_{50-75}	-0.411 (0.005)	-0.378 (0.009)	-0.542 (0.006)	-0.559 (0.009)	-0.587 (0.008)	-0.592 (0.012)
	μ_{75-100}	-0.616 (0.008)	-0.584 (0.012)	-0.845 (0.013)	-0.829 (0.014)	-0.861 (0.014)	-0.921 (0.020)
3	μ_{0-25}	-0.295 (0.005)	-0.316 (0.012)	-0.367 (0.006)	-0.308 (0.006)	-0.350 (0.006)	-0.352 (0.010)
	μ_{25-50}	-0.416 (0.006)	-0.412 (0.012)	-0.506 (0.006)	-0.433 (0.007)	-0.485 (0.007)	-0.488 (0.010)
	μ_{50-75}	-0.515 (0.006)	-0.502 (0.012)	-0.628 (0.007)	-0.533 (0.007)	-0.585 (0.008)	-0.605 (0.011)
	μ_{75-100}	-0.700 (0.007)	-0.693 (0.013)	-0.862 (0.010)	-0.731 (0.009)	-0.786 (0.010)	-0.842 (0.013)
	μ_{0-25}	-0.471 (0.006)	-0.446 (0.012)	-0.495 (0.005)	-0.476 (0.006)	-0.497 (0.005)	-0.480 (0.008)
	μ_{25-50}	-0.602 (0.006)	-0.543 (0.011)	-0.628 (0.005)	-0.609 (0.006)	-0.629 (0.006)	-0.615 (0.008)
	μ_{50-75}	-0.707 (0.006)	-0.630 (0.011)	-0.746 (0.006)	-0.715 (0.007)	-0.731 (0.006)	-0.731 (0.008)
	μ_{75-100}	-0.902 (0.007)	-0.831 (0.011)	-1.010 (0.009)	-0.934 (0.009)	-0.946 (0.009)	-0.975 (0.011)

1: ELECTRICITY, 2: HEATING, 3: TRANSPORT, 4: FOOD.

Figure 2.5: Price elasticities of electricity, heating, transport, and food demand in different households