Circular Cylinder Cross-Flow
530.767 CFD-Final Project
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This Project focuses on the CFD analysis of flow around circular and elliptical cylinders using a discretized Navier-Stokes (N-S) solver. This solver use 2^{nd} order central difference scheme in space and a second order (AB2 for convection and CN2 for viscous) fractional step in time. By separate update of the face velocity ("semi-staggered" approach), we could approach zero divergence with the compact stencil for pressure. We also implemented Immersed Boundary Method (IBM) with stairs step method for inner boundary. We validated our solver based on uniform flow and channel flow, and compared our result with theordically solution.

We then simulated the flow cross circular cylinder at Re=150, checked our result against existing literature, ensuring soundness and reliability in our computational results. We also studied the influence of domain size, grid resolution, and Reynolds number on flow behavior was explored by modifying domain sizes from 8×4 to 8×2, adjusting grid resolutions from $\Delta = 1/32$ and $\Delta = 1/16$, and analyzing results at Re = 300 and Re = 1000. Our findings indicate that for smaller domain sizes accelerate flow evolution and promote earlier vortex shedding due to enhanced fluid disturbances. Increasing the Reynolds number similarly affects flow susceptibility to disturbances. Conversely, a coarser grid resolution dampens disturbances, delaying vortex shedding but not preventing it, illustrating the fundamental role of grid resolution in fluid dynamics.

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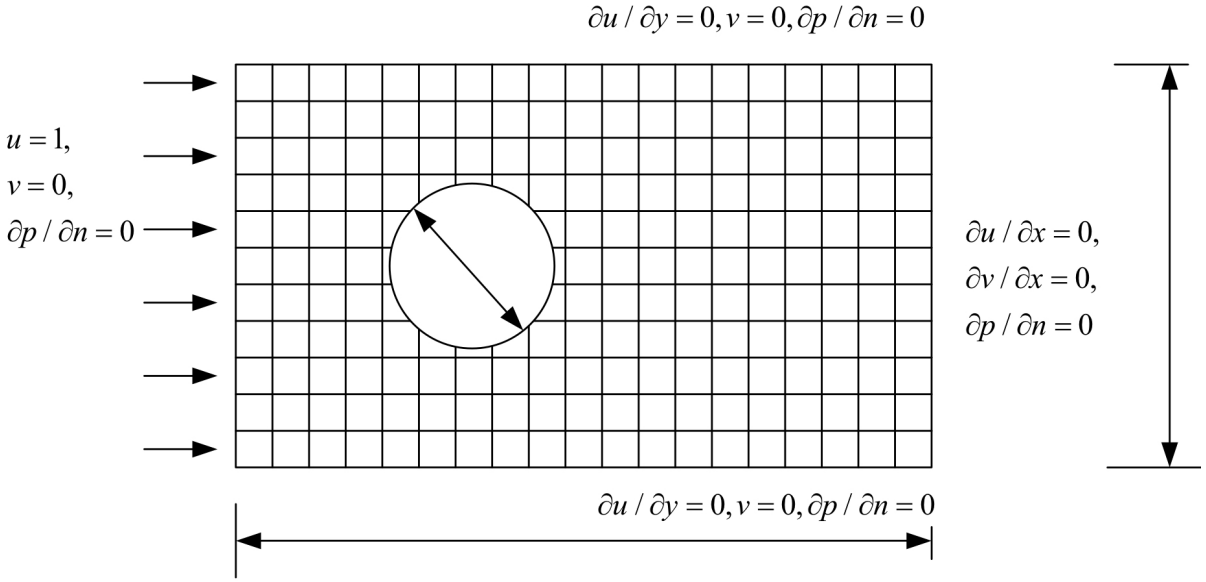
1 Question Review



Circular cylinder in a cross-flow

Consider a circular cylinder in a cross-flow as shown above.
The flow Reynolds number based on D is $Re_D = 150$.

We will simulate this flow using:

- Incompressible Navier-Stokes equations
- A Cartesian grid
- The Stair-Step, immersed boundary method.
- You can use either staggered or co-located (cell-centered) grid. If you use a co-located arrangement, then you should employ a separate face velocity update as shown in the class.
- A fractional-step method for time-stepping
- Use 2^{nd} order central difference in space, and mixed implicit-explicit scheme in time.
- Domain size (L and H) shown in the figure is only suggestion. You can try different domain size.
- You can use either uniform or non-uniform grid. But non-uniform grid is recommended.
- At the outflow boundary, you will have to adjust the net flow in order to satisfy global mass conservation.

Tasks

a) Generate an appropriate computational grid and perform the flow simulation with the given boundary conditions but without the cylinder. Set the initial velocity inside the domain to zero. Check to make sure that your simulation converges well and you should see a uniform flow develop in your domain. The next sanity check is to impose a no-slip boundary conditions on the top and bottom walls in which case, you should see a flow similar to the flow in the entrance of a channel.

b) Next, perform the simulation with the circular cylinder using the Stair-Step immersed boundary method. You should get unsteady flow exhibiting vortex shedding. If you don't get vortex shedding, try to improve your numerical schemes or grid resolution.

c) Plot, velocity, pressure, and vorticity contours at several time instances.

d) Quantify the vortex shedding frequency and compare the value against existing published studies.

e) Quantify the pressure drag coefficient on the cylinder and compare the value against existing published studies.

f) Examine the effect of domain size and grid resolution on the simulation results. Key quantities to be compared are the mean pressure drag coefficient and the shedding frequency.

g) Simulate the same flow at Re_D = 300 and 1000 and discuss the results in comparison to the Re_D = 150 simulation. Do your results at higher Reynolds number match published results?

h) Optional Extra credit – simulate the flow around a cylinder at a 3:1 elliptic cylinder with an angle-of-attack of 30 degrees, and a Reynolds number (based on major axis) of 300. Show that your code can predict the flow and vortex shedding from this shape. Plot the coefficient of pressure drag and lift versus Reynolds number.

2 N-S Solver

2.1 Discretization of N-S Equation

The general steps solving the N-S equation is showing below:

(I): Using the data from the current time step and the previous time step to get u^*, v^*. Then use u^*, v^* to get U^*, V^*.
(II): Using U^*, V^*, solving the pressure poisson equation (PPE), get p^{n+1}.
(III): Using p^{n+1}, and u^*, v^*, get u^{n+1}, v^{n+1}.
Using p^{n+1}, and U^*, V^*, get U^{n+1}, V^{n+1}.

2.1.1 Step (I):

Using the the data we got from current time step and the previous time step get u^*, v^*. Then use u^*, v^* get U^*, V^*.
Update u^*, v^*:
The first step is going to update u^*, v^*:

$$\frac{\vec{u}^* - \vec{u}^n}{\Delta t} = -[\frac{3}{2}\nabla \cdot (\vec{U}^n\vec{u}^n) - \frac{1}{2}\nabla \cdot (\vec{U}^{n-1}\vec{u}^{n-1})] + \frac{1}{Re}\nabla^2\frac{\vec{u}^* + \vec{u}^n}{2} \tag{1}$$

which is:

$$\underbrace{[1 - \frac{\Delta t}{2Re}\nabla^2]\vec{u}^*}_{(1)} = -\frac{3}{2}\underbrace{[\Delta t\nabla \cdot (\vec{U}^n\vec{u}^n)]}_{(2)} + \frac{1}{2}\underbrace{[\Delta t\nabla \cdot (\vec{U}^{n-1}\vec{u}^{n-1})]}_{(3)} + \underbrace{[1 + \frac{\Delta t}{2Re}\nabla^2]\vec{u}}_{(4)} \tag{2}$$

Could say, (2), (3) are convection term, and (4) is the diffusion term of the eqaution. Could use this equation to update u^*, v^*.
Where,

$$(1) = \left\{1 - \frac{\Delta t}{2Re}\nabla^2\right\}u^* = P^* - \left[\frac{E^* + W^* - 2P^*}{\Delta x^2} + \frac{N^* + S^* - 2P^*}{\Delta y^2}\right]\frac{\Delta t}{2Re}$$

$$(2) = \frac{\Delta t}{\Delta}\left((U_e^n\frac{E^n + P^n}{2} - U_w^n\frac{W^n + P^n}{2}) + (V_n^n\frac{N^n + P^n}{2} - V_s^n\frac{S^n + P^n}{2})\right)$$

$$(3) = \frac{\Delta t}{\Delta}\left((U_e^{n-1}\frac{E^{n-1} + P^{n-1}}{2} - U_w^{n-1}\frac{W^{n-1} + P^{n-1}}{2}) + (V_n^{n-1}\frac{N^{n-1} + P^{n-1}}{2} - V_s^{n-1}\frac{S^{n-1} + P^{n-1}}{2})\right)$$

$$(4) = P^n + \frac{\Delta t}{2Re\Delta^2}(E^n + W^n + N^n + S^n - 4P^n)$$

Could found in this expression, the formula to calculate u^*, v^* is same, only input u or v in to the equation:
P means (i, j), W means $(i-1, j)$, E means $(i+1, j)$, S means $(i, j-1)$, N means $(i, j+1)$.

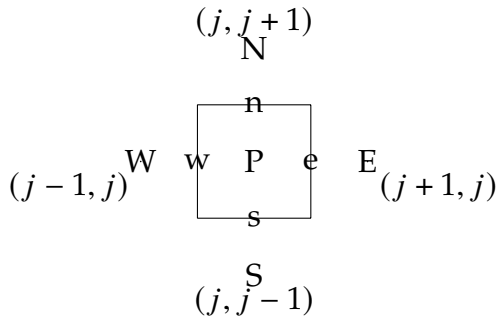


Figure 1: Illustration of a grid layout with directional points.

Put the result in, could get:

$$aW + bP + cE + eS + fN = d$$

The left hand side is the unknown characters, and the right hand side b is known from previous and current time step. Could see, this equation in our system means $Ax = d$, where A is consist of a,b,c,e,f, is a pentadiagonal matrix:

$$\begin{bmatrix} 1 & 0 & & & 0 & & & & \\ a & b & c & & & f & & & \\ & a & b & c & & & f & & \\ & & a & b & c & & & 0 & \\ 0 & & & \ddots & \ddots & \ddots & & & \\ & e & & & a & b & c & & \\ & & e & & & a & b & c & \\ & & & 0 & & & 0 & 1 & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \vdots \\ x_{n-2} \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ \vdots \\ d_{n-2} \\ d_{n-1} \\ d_{j \cdot i} \end{bmatrix}$$

Solve this system, we could get u^*, and v^*

Update U^*, V^*:

$$U_e^* = \frac{u_P^* + u_E^*}{2}, V_n^* = \frac{v_P^* + v_N^*}{2}$$

2.1.2 Step (II):

Update p^{n+1}:

$$\nabla^2 p = \frac{\nabla \cdot \vec{u}^*}{\Delta t} = \frac{\frac{U_e^* - U_w^*}{\Delta x} + \frac{V_n^* - V_s^*}{\Delta y}}{\Delta t}$$

$$\frac{P_E + P_W - 2P_P}{\Delta x^2} + \frac{P_N + P_S - 2P_P}{\Delta y^2} = \frac{\frac{U_e^* - U_w^*}{\Delta x} + \frac{V_n^* - V_s^*}{\Delta y}}{\Delta t}$$

$$P_E + P_W + P_N + P_S - 4P_P = \frac{\Delta}{\Delta t}\left(U_e^* - U_w^* + V_n^* - V_s^*\right) = RHS$$

Similarly, this equation also give us a pentadiagonal system:

$$\begin{bmatrix} 1 & 0 & & & 0 & & & \\ a & b & c & & & f & & \\ & a & b & c & & & f & \\ & & a & b & c & & & 0 \\ 0 & & & \ddots & \ddots & \ddots & & \\ & e & & & a & b & c & \\ & & e & & & a & b & c \\ & & & 0 & & & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \vdots \\ x_{n-2} \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ \vdots \\ d_{n-2} \\ d_{n-1} \\ d_n \end{bmatrix}$$

2.1.3 Step (III):

Update u^{n+1}, v^{n+1}:
The equation to update the u^{n+1}, v^{n+1} is showing below:

$$\vec{u}^{n+1} = \vec{u}^* - \Delta t(\nabla p)_{cc}$$

where we could calculate the cell center pressure divergence $(\nabla p)_{cc}$, and:

$$\begin{cases} u^{n+1} = u^* - \Delta t \cdot \frac{P_E - P_W}{2\Delta x} \\ v^{n+1} = v^* - \Delta t \cdot \frac{P_N - P_S}{2\Delta y} \end{cases}$$

Update U^{n+1}, V^{n+1}:
The equation to update the U^{n+1}, V^{n+1} is showing below:

$$\vec{U}^{n+1} = \vec{U}^* - \Delta t(\nabla p)_{fc}$$

where we could calculate the face center pressure divergence $(\nabla p)_{fc}$, and:

$$\begin{cases} U_e^{n+1} = U_e^* - \Delta t \cdot \frac{P_E - P_P}{\Delta x} \\ V_n^{n+1} = V_n^* - \Delta t \cdot \frac{P_N - P_P}{\Delta y} \end{cases}$$

2.1.4 LineSOR implementation

In the discretization steps (I) and (II) of the Navier-Stokes equations, we obtain a pentadiagonal matrix system. However, our Tridiagonal Matrix Algorithm (TDMA) can only solve systems represented by tridiagonal matrices. To address this, we transfer the additional diagonal lines, e and f, to the right-hand side and continue iterating. This process is repeated until the results satisfy the original requirements of the pentadiagonal system.

The origional pentadiagonal system:

$$\begin{bmatrix} 1 & 0 & & & 0 & & & \\ a & b & c & & & f & & \\ & a & b & c & & & f & \\ & & a & b & c & & & 0 \\ 0 & & & \ddots & \ddots & \ddots & & \\ & e & & & a & b & c & \\ & & e & & & a & b & c \\ & & & 0 & & & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \vdots \\ u_{n-2} \\ u_{n-1} \\ u_n \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ \vdots \\ d_{n-2} \\ d_{n-1} \\ d_n \end{bmatrix}$$

For each line j, the system could be represented as:

$$\left[\begin{array}{c|c|c} E & T & F \end{array}\right] \begin{bmatrix} [u_{j-1}^{n+1}] \\ [u_j^{n+1}] \\ [u_{j+1}^{n+1}] \end{bmatrix} = \left[[d_j]\right]$$

Could say, E mans operating the previous line (j-1) to current line (j), and F means operating on the next line (j+1) to current line. Where $\left[\begin{array}{c|c|c} E & T & F \end{array}\right]$ is:

$$\left[\begin{array}{cccccc|cccccc|cccccc} 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & e_2 & 0 & \cdots & 0 & 0 & a_2 & b_2 & c_2 & \ddots & \vdots & 0 & 0 & f_2 & 0 & \cdots & 0 & 0 \\ 0 & 0 & e_3 & \cdots & 0 & 0 & 0 & a_3 & \ddots & \ddots & 0 & 0 & 0 & 0 & f_3 & \cdots & 0 & 0 \\ 0 & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & c_2 & 0 & 0 & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & e_{n-1} & 0 & 0 & \ddots & \ddots & 0 & b_{n-1} & c_{n-1} & 0 & 0 & 0 & \cdots & f_{n-1} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \end{array}\right]$$

LineSOR:

For LineSOR, we move the two diagonal line (e and f) to the right hand side:

$$\left[\begin{array}{c|c|c} 0 & T & 0 \end{array}\right] \begin{bmatrix} [u_{j-1}^{k+1}] \\ [u_j^{k+1}] \\ [u_{j+1}^{k+1}] \end{bmatrix} = \left[[d_j]\right] - \left[\begin{array}{c|c|c} E & 0 & F \end{array}\right] \begin{bmatrix} [u_{j-1}^{k}] \\ [u_j^{k}] \\ [u_{j+1}^{k}] \end{bmatrix}$$

Which is:

$$[T]\left[[u_j^{k+1}]\right] = \left[[d_j]\right] - [E]\left[[u_{j-1}^{k}]\right] - [F]\left[[u_{j+1}^{k}]\right]$$

Where we could use TDMA to solve this equation. As this result is different from directly solve the equation we previously got, we need it keep iterating, until the error small enough:

$$Error = \left[\begin{array}{c|c|c} E & T & F \end{array}\right] \begin{bmatrix} [u_{j-1}^{k+1}] \\ [u_j^{k+1}] \\ [u_{j+1}^{k+1}] \end{bmatrix} - \left[[d_j]\right] < 1e-6$$

2.2 Immersed Boundary Method

2.2.1 Coefficient Correction

As our original Governing Equation as follows:

$$c_W f_W + c_S f_S + c_P f_P + c_N f_N + c_E f_E = \text{RHS}$$

Lets first consider the East point of our control point is solid point, which in two different kind of boundary condition is:

- Dirichlet condition: $f = f_F$
- Neumann condition: $\frac{\partial f}{\partial n} = f_p$

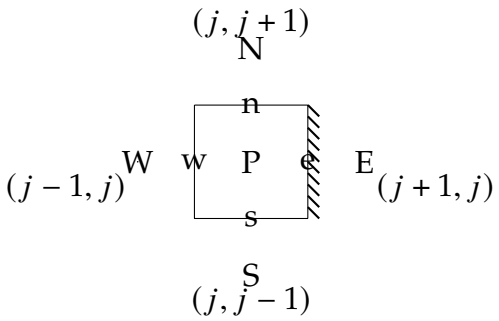


Figure 2: Illustration of a grid layout with directional points.

Dirichlet condition
For boundary at East:

$$c_W f_W + c_S f_S + c_P f_P + c_N f_N + c_E \underbrace{f_E}_{2E - f_P} = \text{RHS}$$

Where, E is the value at the east boundary. The equation then becomes:

$$\underbrace{c_W f_W + c_S f_S + c_N f_N}_{(1)} + \underbrace{(c_P - c_E) f_P}_{(2)} = \text{RHS} - 2E$$

For different boundary location:

$$(1) = iF_W \cdot c_W f_W + iF_S \cdot c_S f_S + iF_N \cdot c_N f_N + iF_E \cdot c_E f_E$$

$$(2) = \left[c_P - ((1 - iF_E)c_E + (1 - iF_S)c_S + (1 - iF_N)c_N(1 - iF_W)c_W)\right] f_P$$

Where iF is the field (array) have same shape with u, but only contains 0 (for solid point) or 1 (for fluid point) on each point, to determine whether the point is in fluid.

As the value at our inner boundary are 0, the equation then becomes:

$$(1) + (2) = RHS$$

Neumann condition
For boundary at East:
Similar, our inner boundary is 0-gradient,

$$c_W f_W + c_S f_S + c_P f_P + c_N f_N + c_E \underbrace{f_E}_{f_P} = \text{RHS}$$

Where, E is the value at the east boundary. The equation then becomes:

$$\underbrace{c_W f_W + c_S f_S + c_N f_N}_{(1)} + \underbrace{(c_P + c_E) f_P}_{(2)} = \text{RHS}$$

For different boundary location:

$$(1) = iF_W \cdot c_W f_W + iF_S \cdot c_S f_S + iF_N \cdot c_N f_N + iF_E \cdot c_E f_E$$

$$(2) = [c_P + ((1 - iF_E)c_E + (1 - iF_S)c_S + (1 - iF_N)c_N(1 - iF_W)c_W)]\, f_P$$

The equation then becomes:

$$(1) + (2) = RHS$$

Thus, change coefficients for N,S,E,W points is showing below::

$$c_W = iF_W \cdot c_W$$

$$c_S = iF_S \cdot c_S$$

$$c_N = iF_N \cdot c_N$$

$$c_E = iF_E \cdot c_E$$

The C_P for Dirichlet condition is showing below:

$$c_P = c_P - ((1 - iF_E)c_E + (1 - iF_S)c_S + (1 - iF_N)c_N(1 - iF_W)c_W)$$

The change coefficients for Neumann:

$$c_P = c_P + ((1 - iF_E)c_E + (1 - iF_S)c_S + (1 - iF_N)c_N(1 - iF_W)c_W)$$

Finally, we need to make sure for solid points, our equation could work:

$$c_P = c_P \cdot iF + iS, RHS = RHS \cdot iF$$

Where, iF is the field only contains 1 at fluid points, 0 for solid points. iS, on the contrary, only contains 1 at solid points, but 0 for fluid points.

2.3 Solver field Setting

As in the discretization of N-S equation, we have u, v, p, and U, V fields. To handle boundary condition, we use ghost point. For convenient, we set all the fields in same size: $(N + 2) \cdot (N + 2)$, the u, v, p fields are:

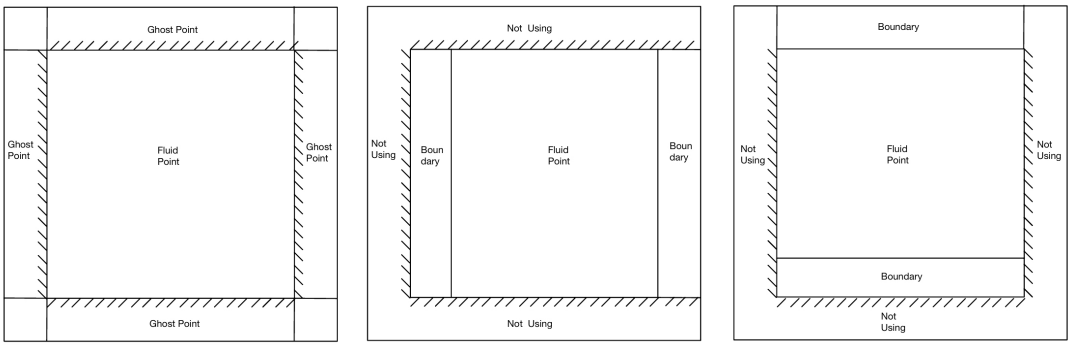


Figure 3: u/v/p, U, and V field

The left one is for u, v, and p field, where they are defined at cell center. It contains ghost point, which is in the solid part near to the boundary. U field is in the middle, which is 1 column, and 2 rows less than u/v/p field. V field is the right one, where it is 1 row, and 2 column less than u/v/p field.

2.4 Solver Architecture

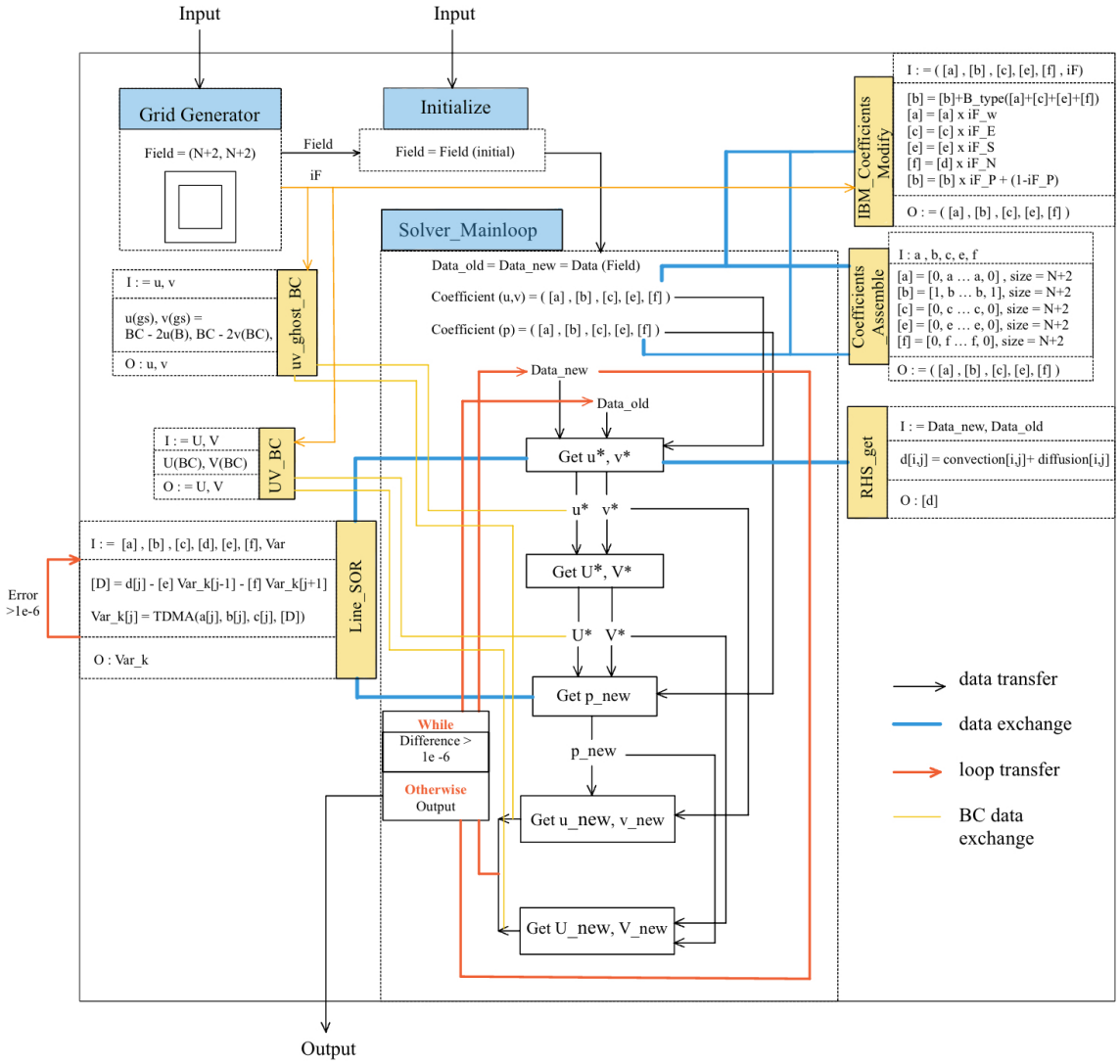


Figure 4: Solver Architecture

This diagram shows our Solver structure. By Using grid generator to generate fields for u, v, p, and U, V, transfer the data to initialize the initial value, then use the main loop to doing time iteration till the fields get to steady state, we could output the result.

- **I/O**: I means function input, and O means function output.
- **Data Transfer Lines**: The diagram uses different colored lines to indicate various types of data movement, while the lines with arrow means one-way data transfer, which means value assignment, and the lines without arrow means data exchange, where there is data input to function, but there is also data get back from function.

3 Solver Testing–Uniform and Channel Case

3.1 Uniform flow

We used the same boundary condition as the requirement shown before but without the cylinder. The simulation converges, and the uniform flow developed has been observed:

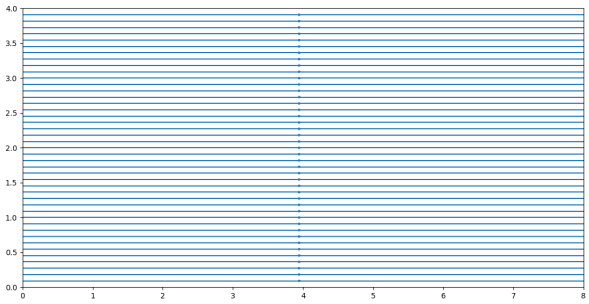


Figure 5: Streamline of the result

3.2 Channel Flow

3.2.1 Flow setting

We setup channel flow case for test our N-S solver is working well, where the setting is showing below:

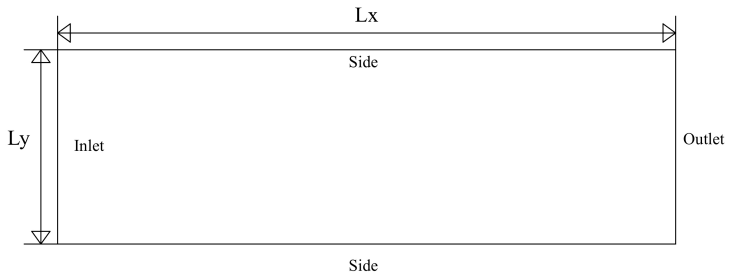


Figure 6: Domain and boundary setting for Channel flow

The Domain setting as shown above. The key parameters as follows:

- **Ly=1, Lx=10**: Width and Length of the domain
- **Inlet**: $u = 1, v = 0, \frac{\partial p}{\partial x} = 0$
- **Side**: $u = v = 0, \frac{\partial p}{\partial y} = 0$
- **Outlet**: $\frac{\partial u}{\partial x} = 0, v = 0, \frac{\partial p}{\partial x} = 0$

3.2.2 Result

The result is showing below:

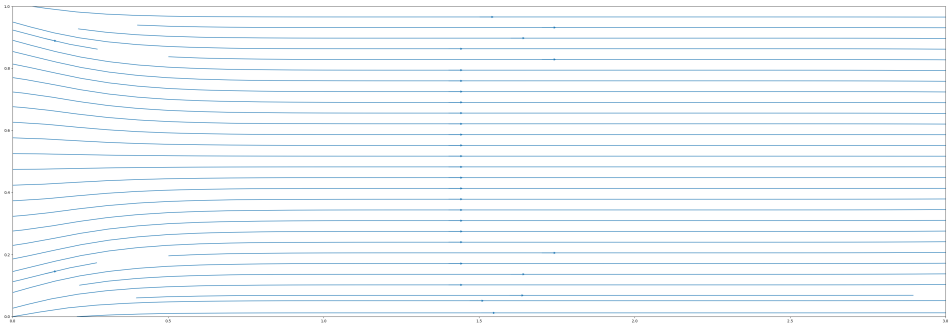


Figure 7: Streamline at t=50s

This diagram shows the streamline at t=50s, where we could see there the flow do follow our boundary condition and developed as expected.

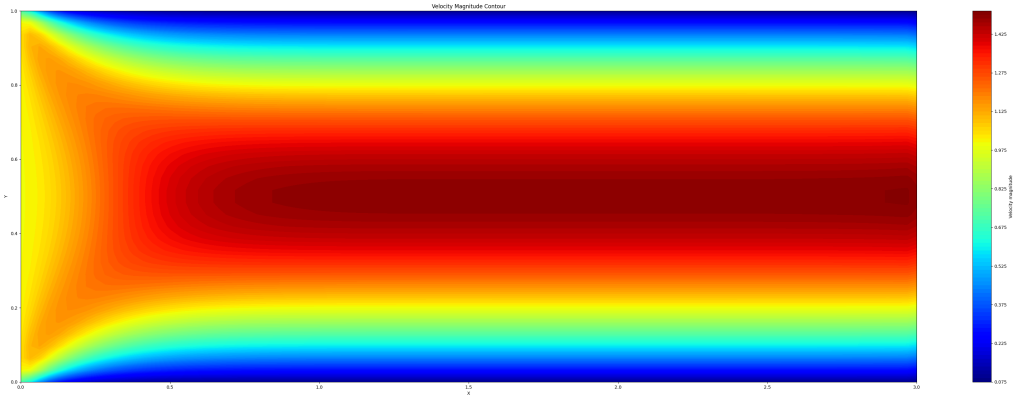


Figure 8: Velocity contour at t=50s

This diagram showed the velocity contour at t=50s, we could observe it do develop boundary layer and fully developed after certain length, and velocity have maximum value at the center of the channel.

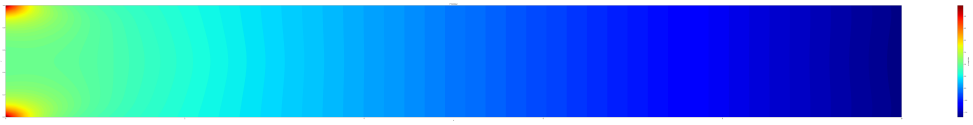


Figure 9: Pressure contour

3.2.3 Comparison with theoretical solution

To check our result's precision, we checked our result with the theoretical solution, where the channel flow's theoretically solution is showing below:

$$u_{\text{exact}} = U\left(1 - \left(\frac{y - 0.5}{0.5}\right)^2\right)$$

We compare this result with our numerical result, could find our numerical result is close to the exact solution:

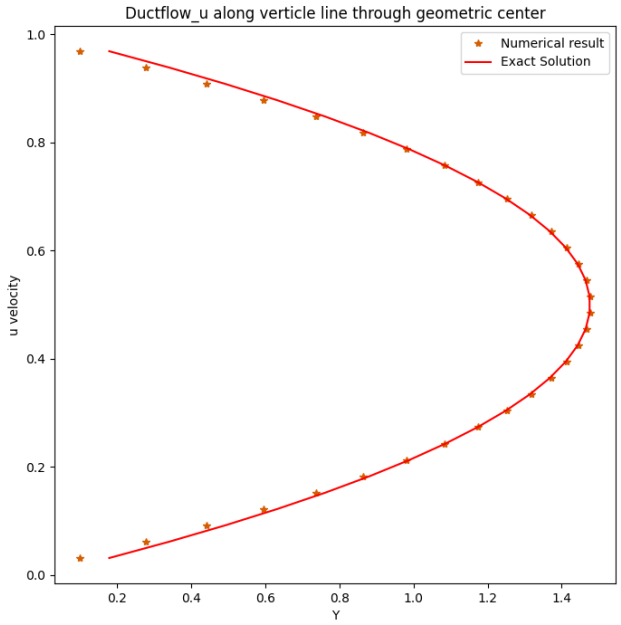


Figure 10: u comparison

4 Re=150–Circular cylinder in a cross-flow

4.1 Settings

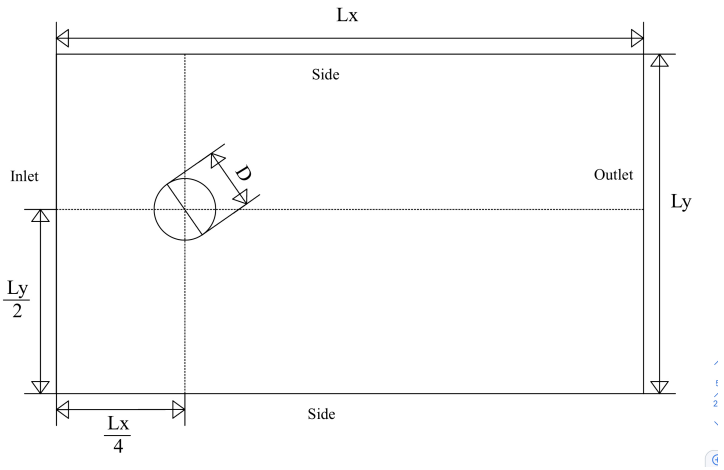


Figure 11: Domain and boundary setting for circular cylinder

The Domain setting as shown above. The key parameters as follows:

- **Ly=4, Lx=8**: Width and Length of the domain
- **Circle Center**: the location of circular cylinder center=$(\frac{Lx}{4}, \frac{Ly}{2})$
- **D=0.5**: The diameter of the circle
- **Inlet**: $u = 1, v = 0, \frac{\partial p}{\partial x} = 0$
- **Side**: $\frac{\partial u}{\partial y} = 0, \frac{\partial v}{\partial y} = 0, \frac{\partial p}{\partial y} = 0$
- **Outlet**: $\frac{\partial u}{\partial x} = 0, \frac{\partial v}{\partial x} = 0, \frac{\partial p}{\partial x} = 0$

4.2 Result among time

4.2.1 t=10 and t=50

By setting the parameters as the chapter shown before, we now exhibit the result of t=10 and t=100 as follows:

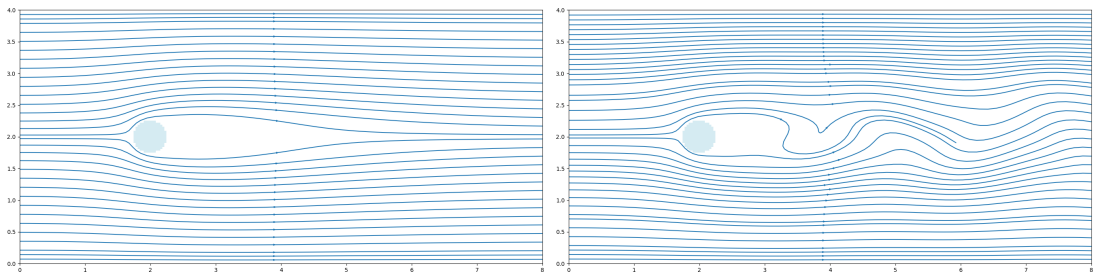


Figure 12: Streamline at t=10 (left), Streamline at t=10 t=50s (right)

Based on the figures shows for t=10 and t=50, we could find at t=10, the flow is kind of steady, where is shows the flow have not been disturbed, and flow pattern is symmetric along y-direction.

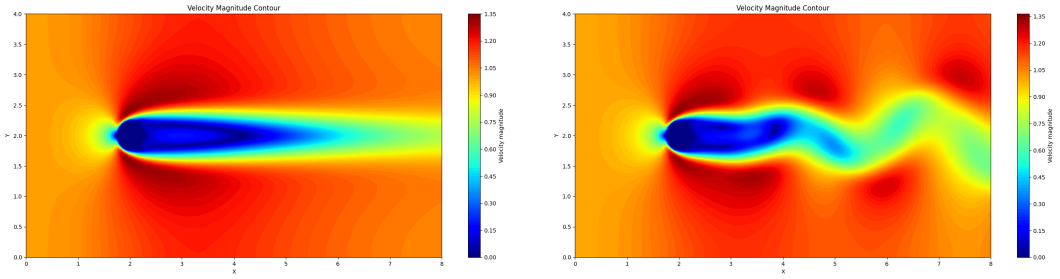


Figure 13: Velocity at t=10 (left), Pressure at t=10 t=50s (right)

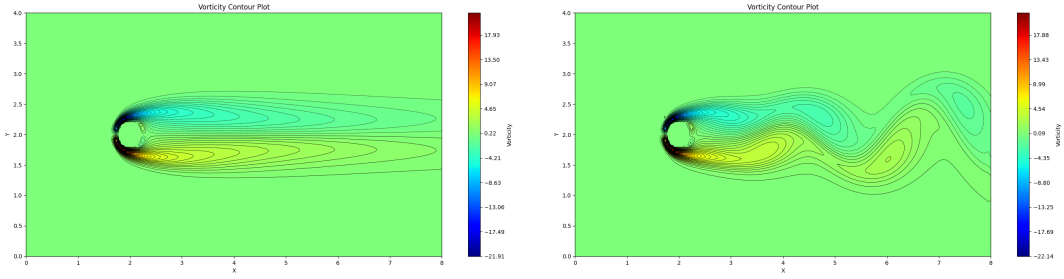


Figure 14: Vorticity at t=10 (left), Vorticity at t=10 t=50s (right)

The same result shown in vorticity contour and pressure contour, where we could find the flow is not symmetric and start to show voticies.

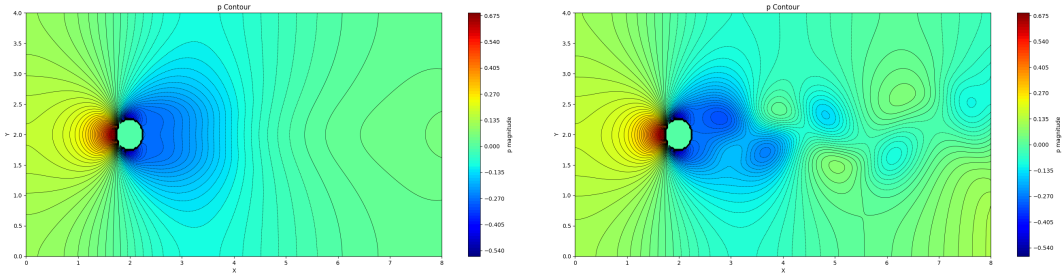


Figure 15: Pressure at t=10 (left), Pressure at t=50s (right)

4.2.2 t=50 and t=100

By setting the parameters as the chapter shown before, we now exhibit the result of t=50 and t=100 as follows:

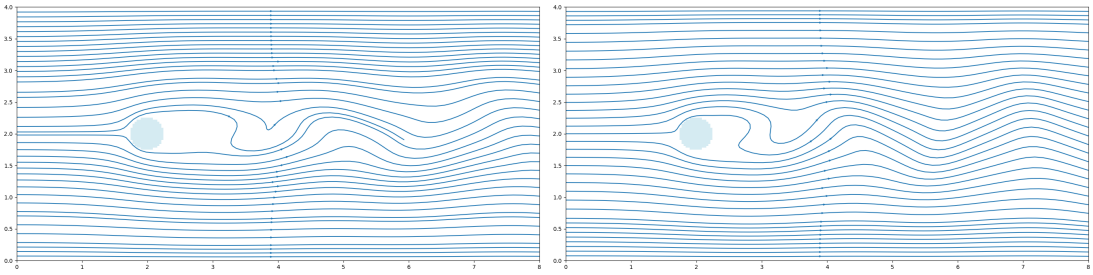


Figure 16: Streamline at t=50 (left), Streamline at t=10 t=100s (right)

Form t=50 to t=100, we could also find that the vortex is shedding from cylinder and let the flow more unsteady.

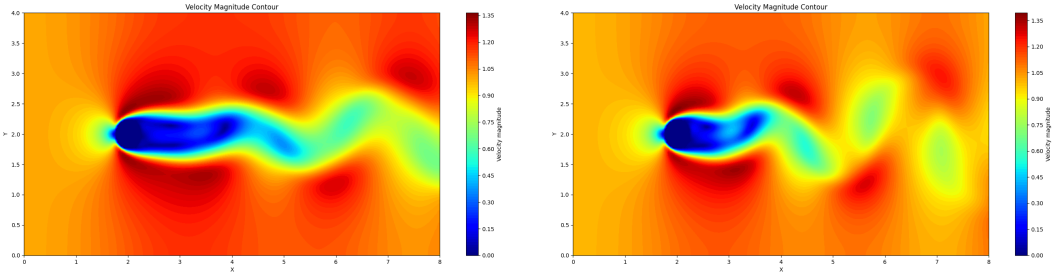


Figure 17: Velocity at t=50 (left), Pressure at t=10 t=100s (right)

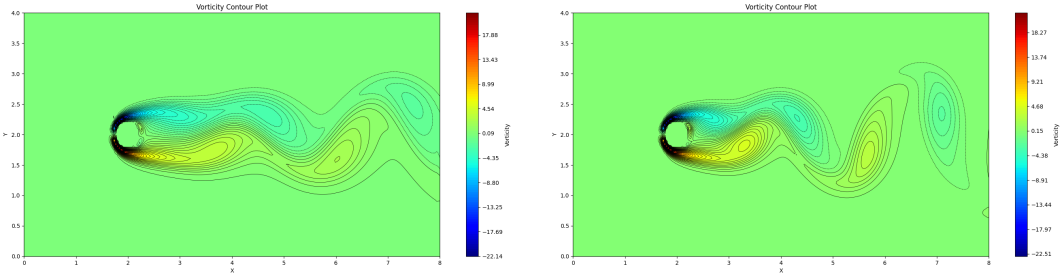


Figure 18: Vorticity at t=50 (left), Vorticity at t=100s (right)

From these result, we could see small vortices lying up along y-center side.

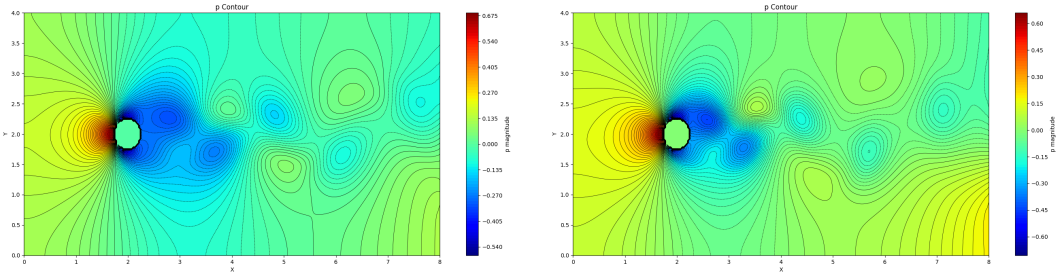


Figure 19: Pressure at t=50 (left), Pressure at t=100s (right)

From the figure of pressure contour, we could noticed that the pressure shows its maximum at the front point of the circle, and the minimum (negative) at the back of the circle. While the vortex shedding showing up, we could find a similar pattern of the vortex, velocity and pressure contour arrangement.

4.2.3 Result Comparison of time (t=10, 50, and 100)

To get more comprehensive understanding of the vortex shedding among time, we let the result of t=10, t=50, and t=100 together as follows:

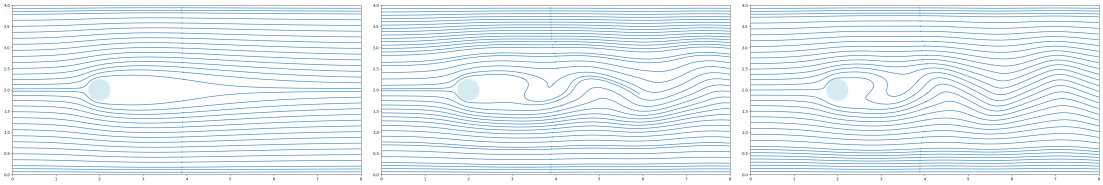


Figure 20: Streamline at t=10 (left), t=50 (middle), t=100s (right)

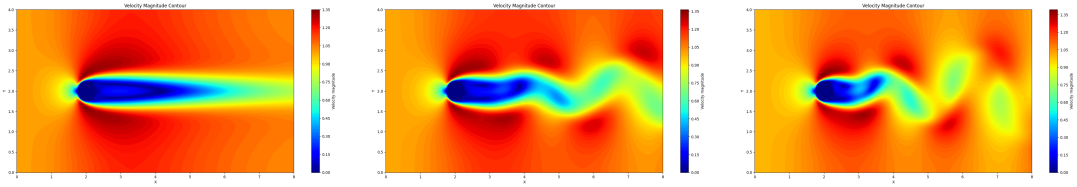


Figure 21: Velocity at t=10 (left), t=50 (middle), t=100s (right)

At t=100, we could observe the periodic behaviour typical of vortex shedding.

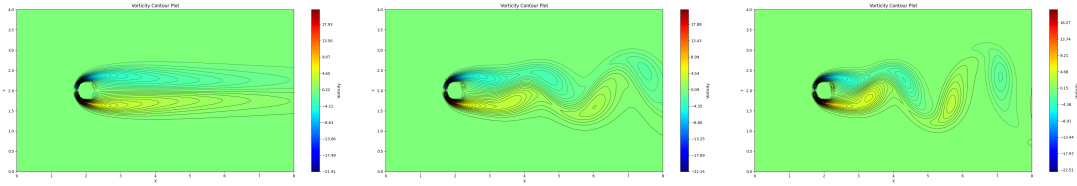


Figure 22: Vorticity at t=10 (left), t=50 (middle), t=100s (right)

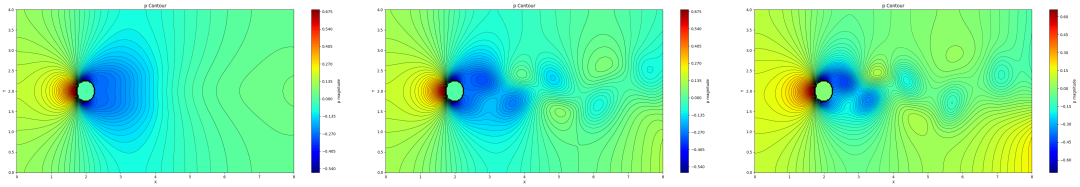


Figure 23: Pressure at t=10 (left), t=50 (middle), t=100s (right)

Here we put the result among time together. Where it exhibit clearly, that the appearance of vortex shedding is the result of revolution of vortex due to the circular cylinder.

From streamline plots, pressure disturbance, to vorticity maps, we could observe the evolution from initial flow around the cylinder to the distinct formation of a Kármán vortex street. From t=10 seconds to t=100 seconds, the pressure and vorticity maps reveal changes in the intensity and positioning of these vortices.

4.3 Re=150 Result Value and Comparison

4.3.1 Drag (c_D) and Lift (c_L) coefficient)

For large time scale, we sampling 1 second once, the result is showing below:

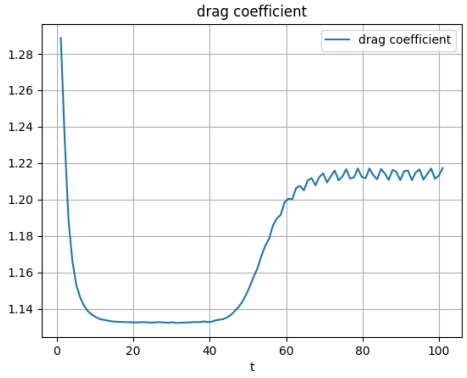


Figure 24: c_D among long time

Based on this figure, we could find the flow become steady after t=70. To increase our result's accuracy, we sampling around t=100 and t=110, sampling 0.01s once. The result is showing below:

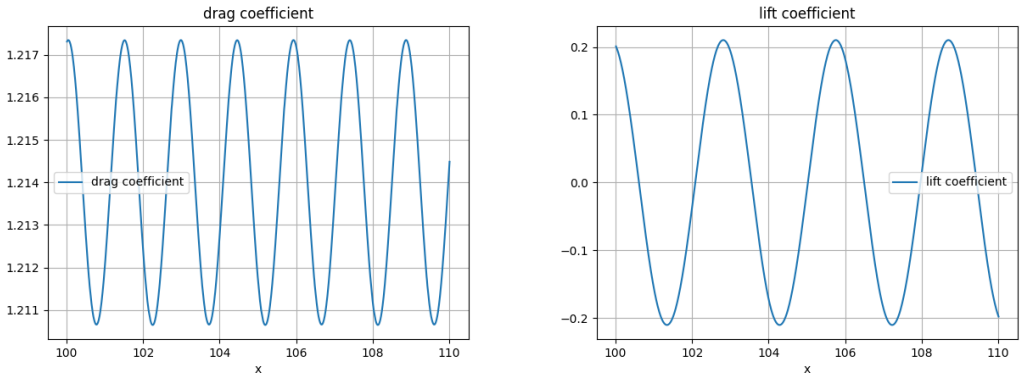


Figure 25: c_L and c_D among time

We could get the mean drag coefficient (c_D):

$$c_D = 1.212$$

Also, we could get lift coefficient (c_D):

$$c_L = 0 \pm 0.21$$

4.3.2 Vortex Shedding frequency

By sampling the result of drag and lift coefficient (c_D and c_L), and using Fast Fourier Transform (FFT), we could get the vortex shedding frequency:

$$f = 0.34$$

Based on that, we could calculate Strouhal number (St):

$$St = \frac{fD}{U} = 0.17$$

4.3.3 Result value compare with literature

According to Norberg (2003)[2], the relationship of St number with Re is shown below:

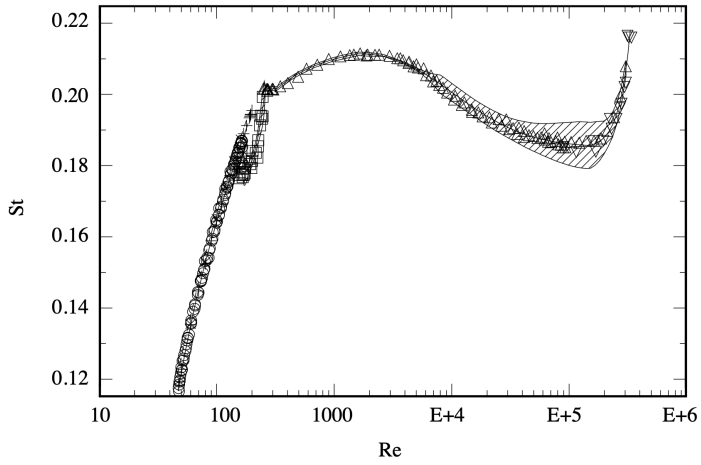


Figure 26: St number with Re (Norberg, 2003[2])

As our result shown St number is 0.166 at Re=150, we could say our result matched. The more detailed result is from appendix of Norberg's review (2003)[2]:

From Re in range of (47,190):

$$St = 0.2663 - \frac{1.019}{\sqrt{Re}}$$

Which shows St = 0.183 at Re = 150, which is also closed to our result.

We also compared with Qu (2013)[3], where shows St = 0.184, and c_{Dp} = 1.02. Compare with our c_D = 1.21, shows our result may not exactly accurate at calculating drag coefficient.

The total comparison table is showing below:

Table 1: Result comparison with Qu(2003)[3]

Re=150	c_{D_p}	c_L	St
Result	1.212	0 ± 0.21	0.17
Qu[3]	1.02	0.35	0.184

5 Influence of Domain Size: 8×4 VS. 8×2

5.1 Result Comparison among time

To examine the influence of grid size, we narrow our domain at y-axis, make our domain from $8 \times 4$to 8×2.

5.1.1 t=50 and t=100 comparison

The following figures shown below is the vorticity and velocity contour of 8×2(upper) and 8×4(lower):

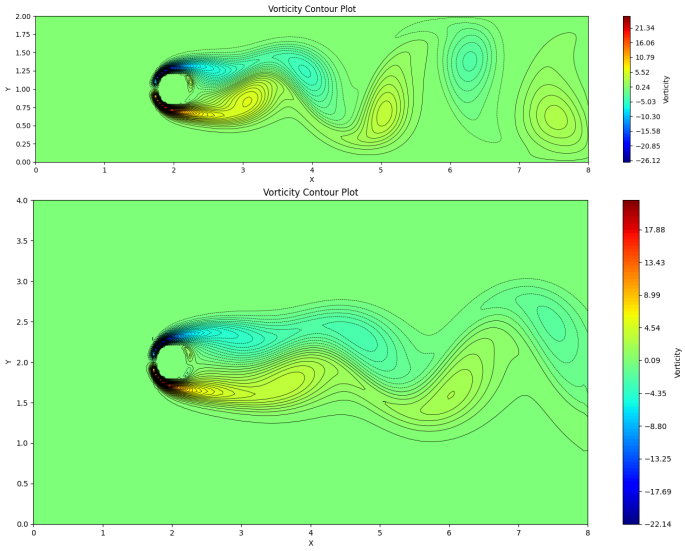


Figure 27: Vorticity at t=50

We could find as the domain has been narrowed, the evolution of the flow seems been accelerated, which shows voticies shedding out of cylinder at t=50, domain size 8×2, but still only starting to been disturbed t=50, domain size 8×4.

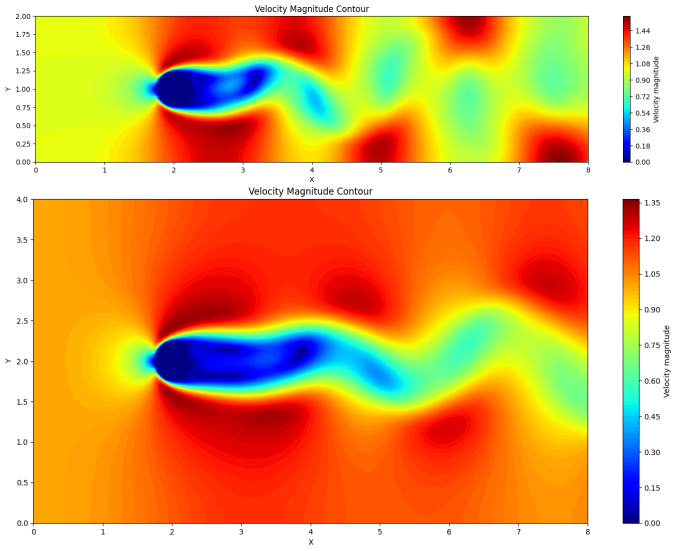


Figure 28: Velocity at t=50

We could find the domain 8×2 is not enough for the flow simulation, where its boundary is too close, where the vortices touched the boundary. The flow evolution been speed up, where you can see the vortex shedding.

5.2 Drag (c_D) and Lift (c_L) coefficient)

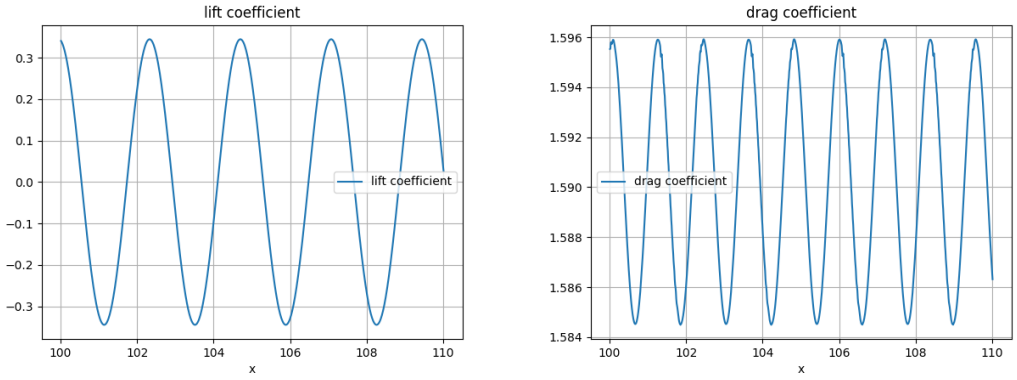


Figure 29: c_L and c_D among time

We could get the mean drag coefficient (c_D):

$$c_D|_{8\times2} = 1.588$$

5.2.1 Vortex Shedding frequency

By sampling the result of drag and lift coefficient (c_D and c_L), and using Fast Fourier Transform (FFT), we could get the vortex shedding frequency:

$$f|_{8\times2} = 0.4$$

Based on that, we could calculate Strouhal number (St):

$$St|_{8\times2} = \frac{fD}{U}|_{8\times2} = 0.2$$

5.3 Comparison of domain size 8×4 with 8×2

We could compare our narrow domain (8×2) with our result shown in the previous chapter:

Table 2: Result comparison of domain size's effect

Domain Size	c_D	c_L	St
8×4	1.212	0 ± 0.21	0.17
8×2	1.588	0 ± 0.35	0.2

We could find, after we narrowed the domain size, the result value (c_D and c_L) become larger, which may because as we narrowed the domain size, the vortex become

much easier to touch the side boundary of the domain, and been mirrored back as we use the symmetric boundary condition. This may cause the flow developed much more faster.

6 Influence of Grid Resolution: Δ=1/32 VS. Δ=1/16

To examine how the grid resolution could influence our result, we use the coarse grid Δ=1/16.

6.1 Δ=1/32 VS. Δ=1/16 at different time

The result shown below is Δ =1/16 (left), and Δ=32 (right). Form the Δ=1/16 figures, we could find this kind of grid is coarse, where we could easily observe the “stairs" at the edge of the circle.

6.1.1 t=50

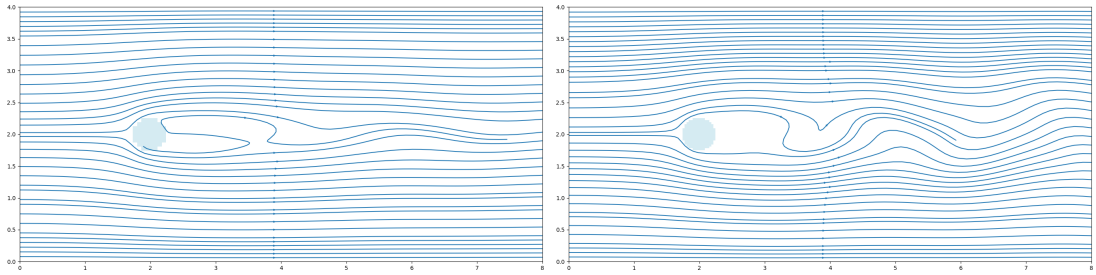


Figure 30: Streamline of Δ=1/16 (left), Δ=1/32 (Right)

Based on the streamline and velocity contour comparison of the Δ=1/16 with Δ=1/32, we could notice that the flow’s development seems been slowed down at Δ=1/16 grid, that the flow is still not been hugely disturbed at t=50 Δ=1/16, but for Δ=1/32 result at t=50, is already starting to show the oscillation.

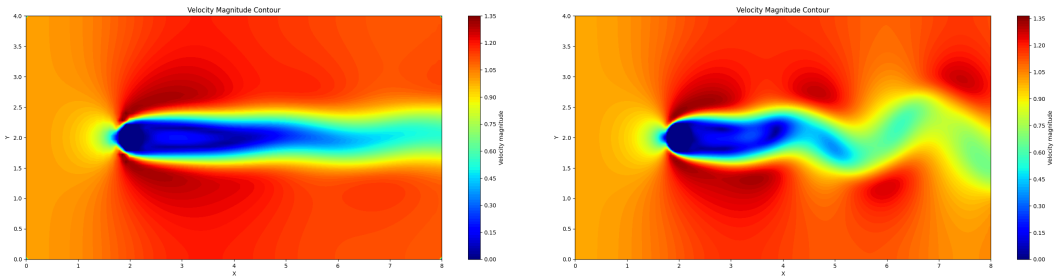


Figure 31: Velocity of Δ=1/16 (left), Δ=1/32 (Right)

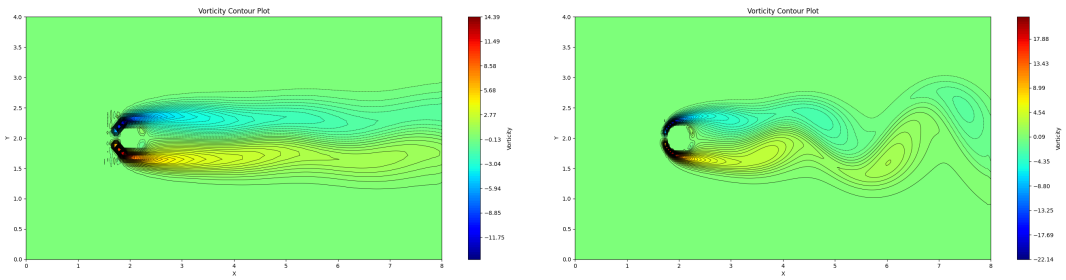


Figure 32: Vorticity of Δ=1/16 (left), Δ=1/32 (Right)

The similar result showned in the vorticity and pressure contour, where the flow is much un-disturbed at of Δ=1/16.

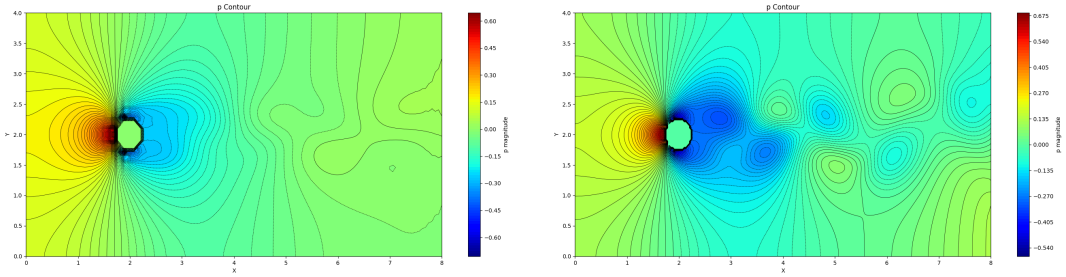


Figure 33: Pressure of Δ=1/16 (left), Δ=1/32 (Right)

Based on the comparison of the different grid resolution, we could find that the flow evolution seems been slowed down as the grid become coarse. This may due to the added numerical viscosity at coarse grid.

6.1.2 t=100

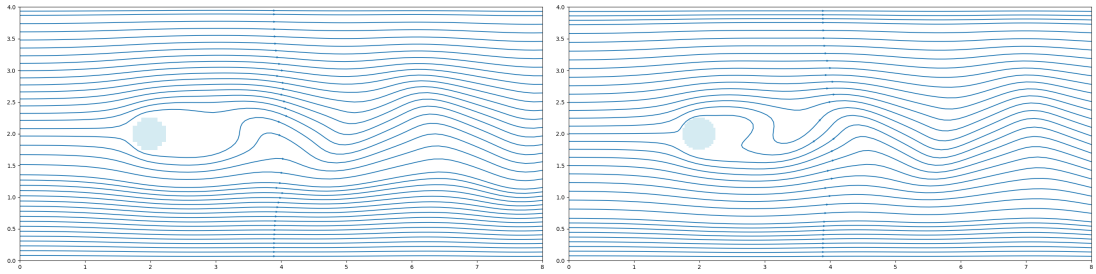


Figure 34: Streamline of Δ=1/16 (left), Δ=1/32 (Right)

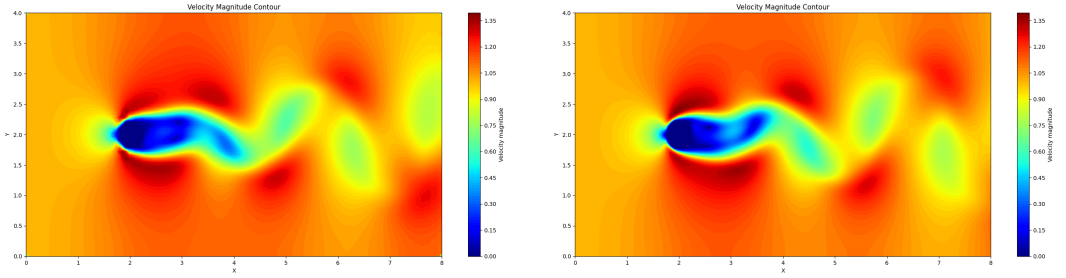


Figure 35: Velocity of Δ=1/16 (left), Δ=1/32 (Right)

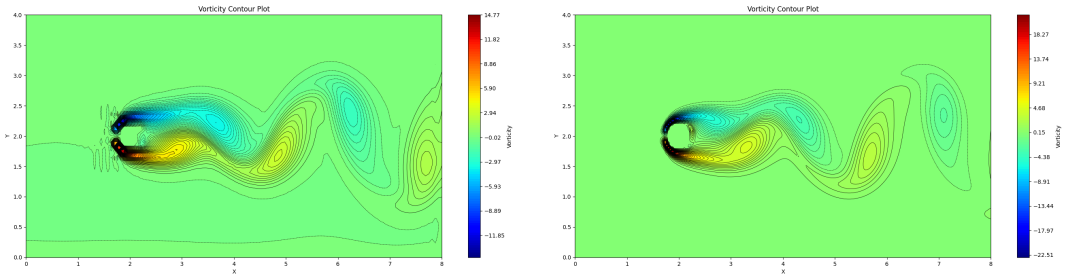


Figure 36: Vorticity of Δ=1/16 (left), Δ=1/32 (Right)

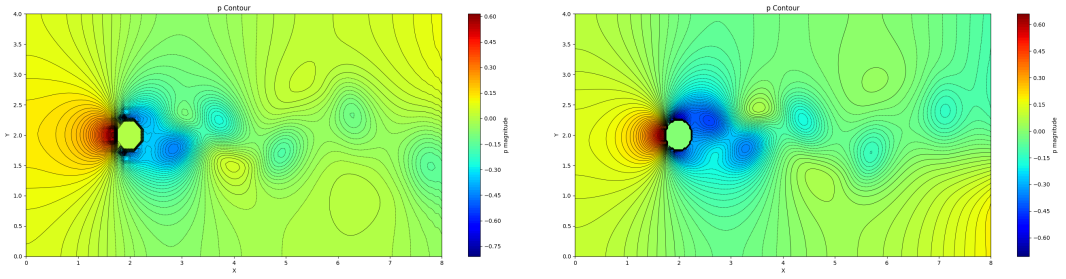


Figure 37: Pressure of Δ=1/16 (left), Δ=1/32 (Right)

Based on the result, we could get the conclusion: The narrow domain could accelerate the flow evolution, and the coarse grid could slow down the flow development among time due to the large numerical viscosity.

6.2 Drag (c_D) and Lift (c_L) coefficient)

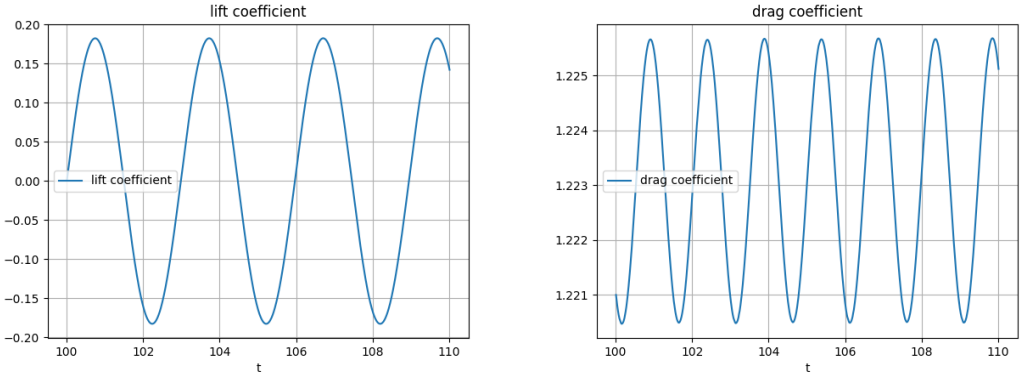


Figure 38: c_L and c_D among time

We could get the mean drag coefficient (c_D):

$$c_D = 1.22$$

6.2.1 Vortex Shedding frequency

By sampling the result of drag and lift coefficient (c_D and c_L), and using Fast Fourier Transform (FFT), we could get the vortex shedding frequency:

$$f = 0.3$$

Based on that, we could calculate Strouhal number (St):

$$St = \frac{fD}{U} = 0.15$$

6.3 Result comparison of grid resolution: 1/32 and 1/16

Table 3: Result comparison of grid resolution effect

Grid Size	c_D	c_L	St
1/32	1.212	0 ± 0.21	0.17
1/16	1.22	0 ± 0.18	0.15

We could find that as we use the coarse grid, the result (c_L) is little bit smaller. As we also could notice that the flow developed much more slower than the finer grid, which may because as we use coarse grid, we make the fluid cannot catch the number of small part's of the circle cylinder's influence of the flow, and make the numerical viscosity much higher, which could influence the result.

7 Re=300 Result

To observe Re number effect for circular cylinder cross-sectional flow, we simulate the Re=300 case, and compare with Re=150 result which is shown at the previous section. The result is showing below:

7.1 Re=300 VS. Re=150 at different time

The result of Re=300 compared with Re=150 result is showing below:

7.1.1 t=50

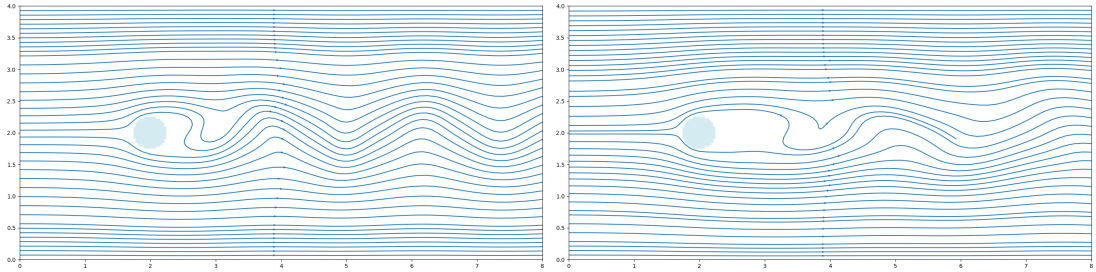


Figure 39: Streamline for Re=300 (left), Re=150 (right)

We could notice for the higher Re number, the development for the flow seems been speed up at Re=300, which could already observe vortex shedding at t=50, where for Re=150 the flow is just stating to oscillate.

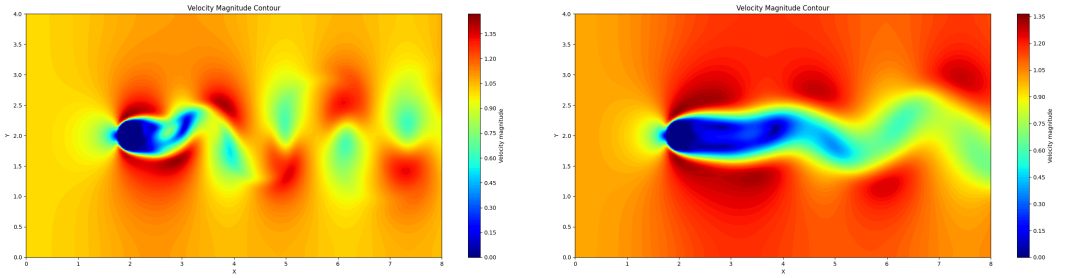


Figure 40: Velocity for Re=300 (left), Re=150 (right)

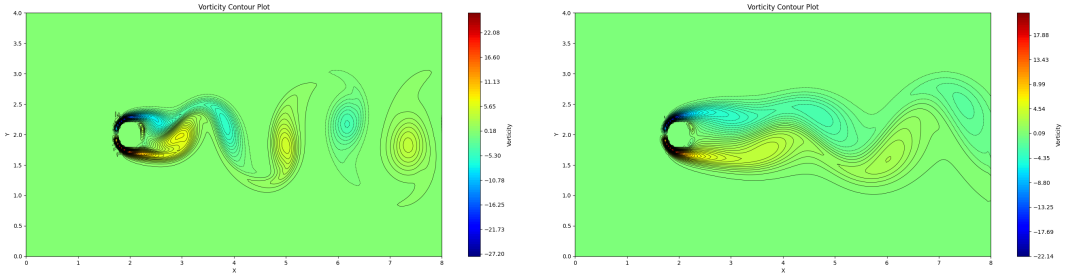


Figure 41: Vorticity for Re=300 (left), Re=150 (right)

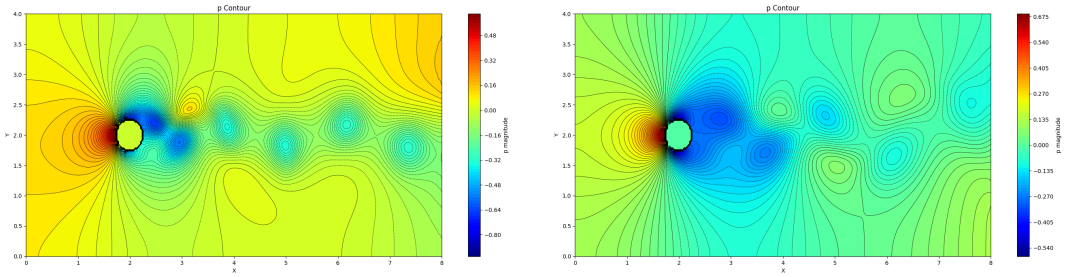


Figure 42: Pressure for Re=300 (left), Re=150 (right)

7.1.2 t=100

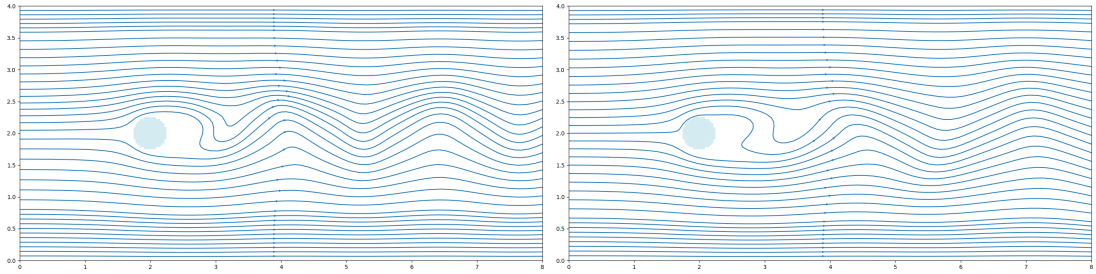


Figure 43: Streamline for Re=300 (left), Re=150 (right)

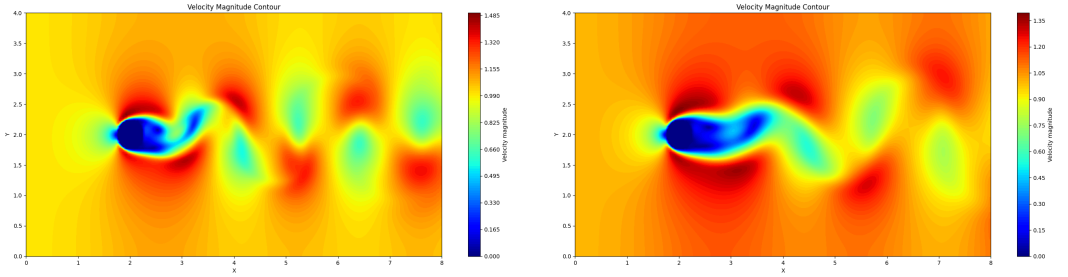


Figure 44: Pressure for Re=300 (left), Re=150 (right)

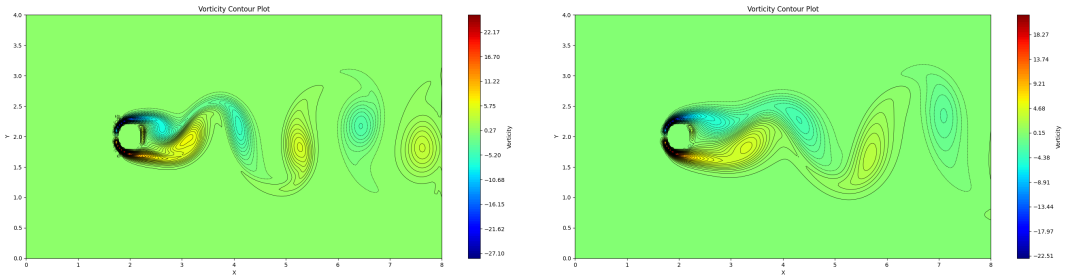


Figure 45: Vorticity for Re=300 (left), Re=150 (right)

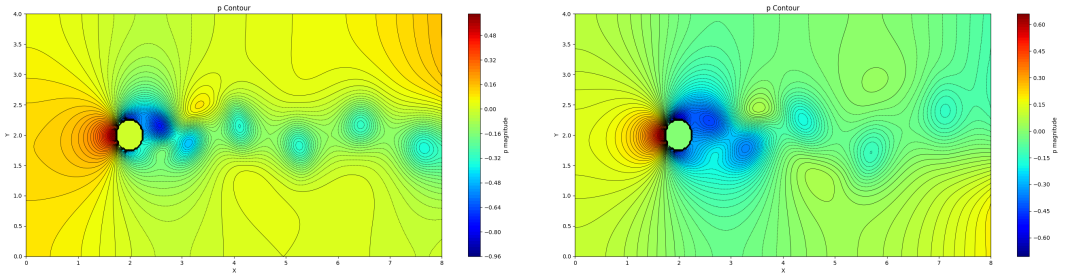


Figure 46: Pressure for Re=300 (left), Re=150 (right)

7.2 Re=300 Result Value and Comparison

7.2.1 Drag (c_D) and Lift (c_L) coefficient

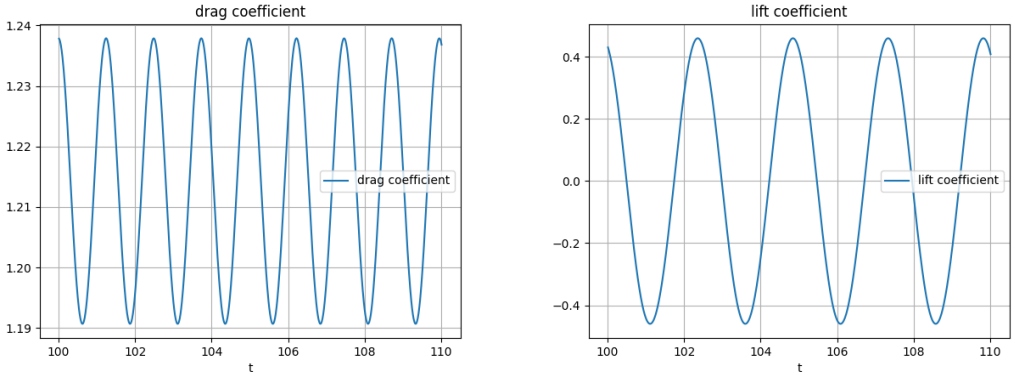


Figure 47: c_L and c_D among time

We could get the mean drag coefficient (c_D):

$$c_D = 1.21$$

Also, we could get lift coefficient (c_D):

$$c_L = 0 \pm 0.46$$

7.2.2 Vortex Shedding frequency

By sampling the result of drag and lift coefficient (c_D and c_L), and using Fast Fourier Transform (FFT), we could get the vortex shedding frequency:

$$f = 0.4$$

Based on that, we could calculate Strouhal number (St):

$$St = \frac{fD}{U} = 0.2$$

7.2.3 Result value compare with literature

According to Franke(1990)[1], the result of St at $Re = 300$: $St = 0.205$, where our result is $St = 0.2$, which is close.

Table 4: Result comparison with literature

	c_{D_p}	c_{L_p}	St
Re=300	1.21	0 ± 0.46	0.2
Franke[1]	1.11	0 ± 0.77	0.205

8 Re=1000 Result

To observe Re number effect for circular cylinder cross-sectional flow, we simulate the Re=1000 case, and compare with Re=150 result which is shown at the previous section. The result is showing below:

8.1 Re=1000 VS. Re=150 at different time

The following figures shows Re=1000 result (left), and Re=150 result (right):

8.1.1 t=50

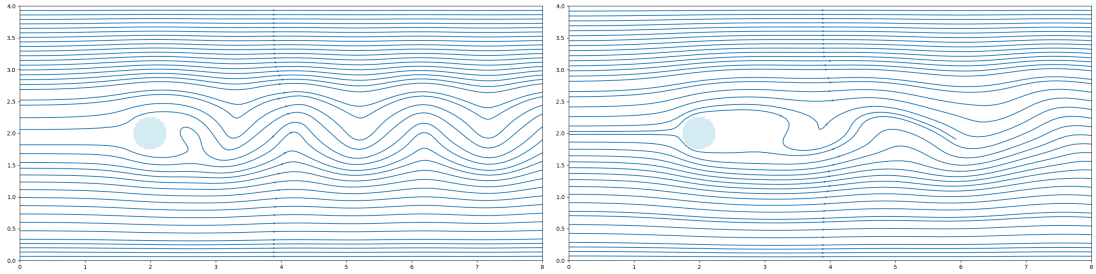


Figure 48: Streamline for Re=300 (left), Re=150 (right)

Based on the streamline and velocity contour, we could clearly observe vortex shedding from circular cylinder at Re=1000, t=50. But for Re=150, t=50, we could find the flow is only starting to been disturbed, but not shown vortex shedding yet.

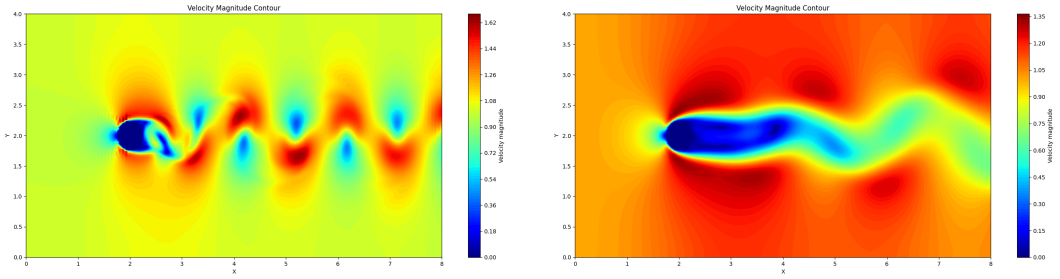


Figure 49: Velocity contour for Re=300 (left), Re=150 (right)

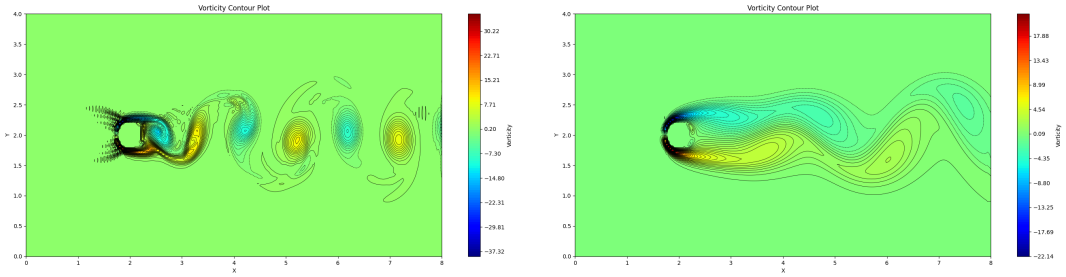


Figure 50: Vorticity for Re=300 (left), Re=150 (right)

From vorticity and pressure contour, we could get the similar observation: the vortex shedding is already shown up at Re=1000, but not clearly shown up at Re=150, t=50.

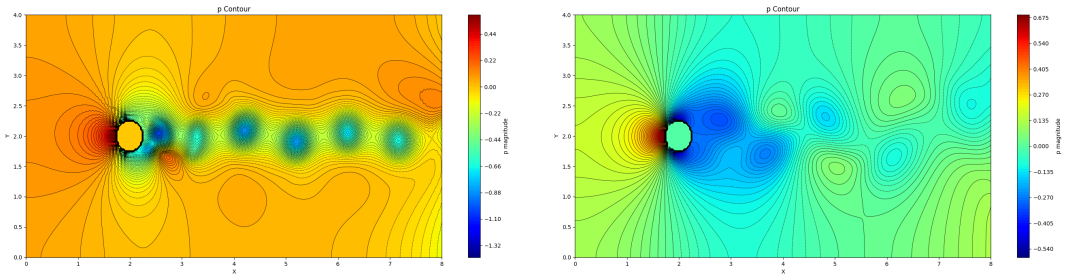


Figure 51: Pressure for Re=300 (left), Re=150 (right)

8.1.2 t=100

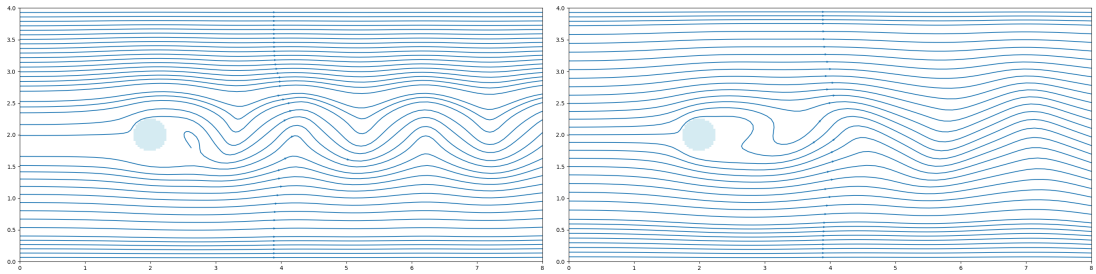


Figure 52: Streamline for Re=300 (left), Re=150 (right)

We could also notice that for Re=1000, the vortices kindly lying up, which neatly aligned after the circular cylinder.

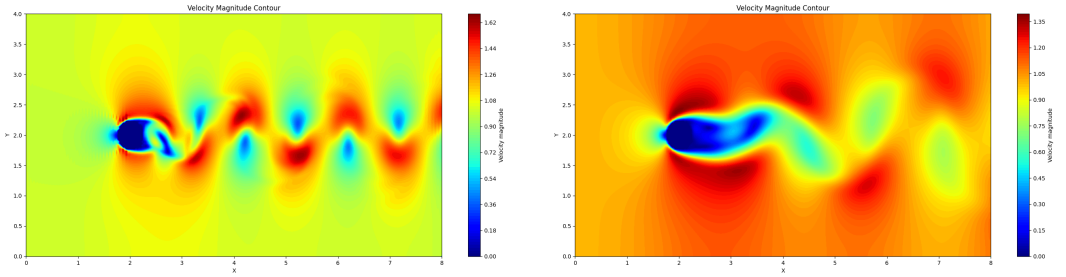


Figure 53: Velocity for Re=300 (left), Re=150 (right)

The similar result shown at vorticity and pressure contour, which we could see there voticies and pressure extremum points arranged neatly.

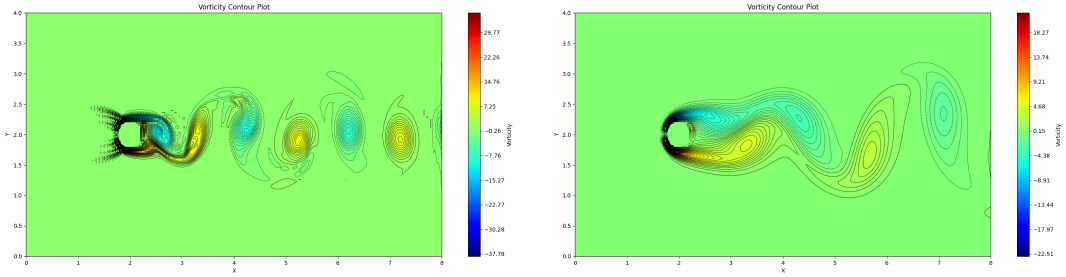


Figure 54: Vorticity for Re=300 (left), Re=150 (right)

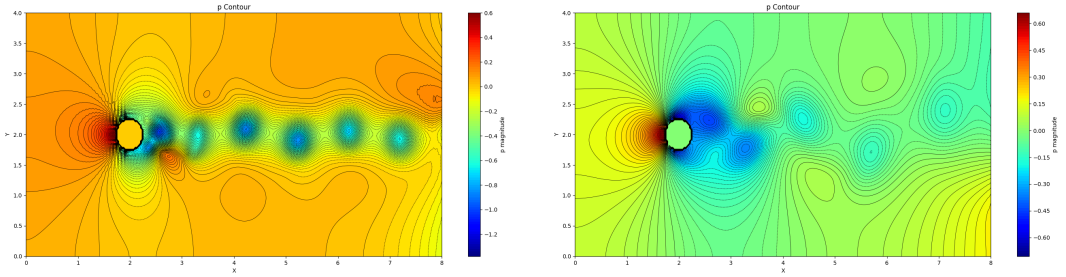


Figure 55: Pressure for Re=300 (left), Re=150 (right)

8.2 Re=1000 Result Value and Comparison

8.2.1 Drag (c_D) and Lift (c_L) coefficient

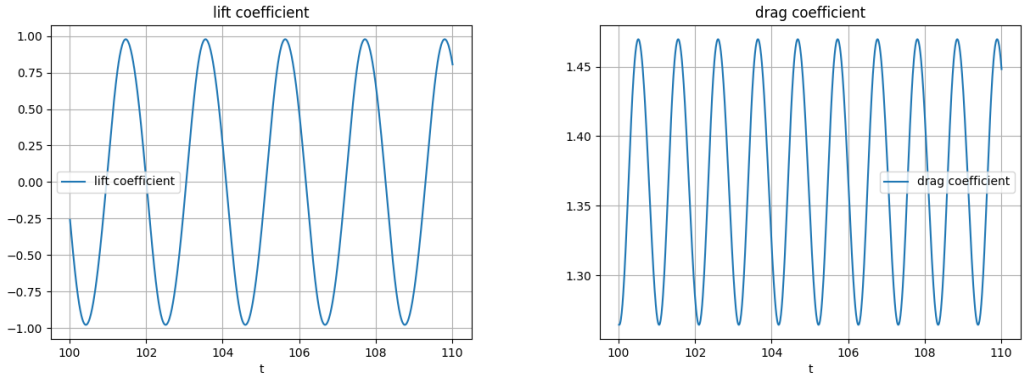


Figure 56: c_L and c_D among time

We could get the mean drag coefficient (c_D):

$$c_D = 1.36$$

Also, we could get lift coefficient (c_D):

$$c_L = 0 \pm 0.97$$

8.2.2 Vortex Shedding frequency

By sampling the result of drag and lift coefficient (c_D and c_L), and using Fast Fourier Transform (FFT), we could get the vortex shedding frequency:

$$f = 0.4$$

Based on that, we could calculate Strouhal number (St):

$$St = \frac{fD}{U} = 0.2$$

8.2.3 Result value compare with literature

According to Qu(2003)[3], the result of St at $Re = 1000$: $St = 0.2365$, where our result is $St = 0.2$, could say our result is close.

Table 5: Comparison with Qu(2003)[3] result

	c_{D_p}	c_{L_p}	St
Re=1000	1.36	0 ± 0.97	0.2
Qu[3]	1.34	0 ± 1.30	0.236

8.3 Comparison and discussion of Re number effect on result

8.3.1 C_D and C_L comparison

Table 6: Result comparison of Re effect

Re	c_{D_p}	c_{L_p}	St
Re=150	1.212	0 ± 0.21	0.17
Re=300	1.21	0 ± 0.46	0.2
Re=1000	1.36	0 ± 0.97	0.2

Based on the comparison, we could find for higher Re, the c_L's maximum value will be higher, and c_D may have the same effect but not obviously in our result.

8.3.2 Comparison among time

To have a more comprehensive understanding of the Re number's effect of our result, we put Re=1000 (left), Re=300 (middle), and Re=150 (right)'s figures together to check how Re could influence the simulation outcome:

t=50:

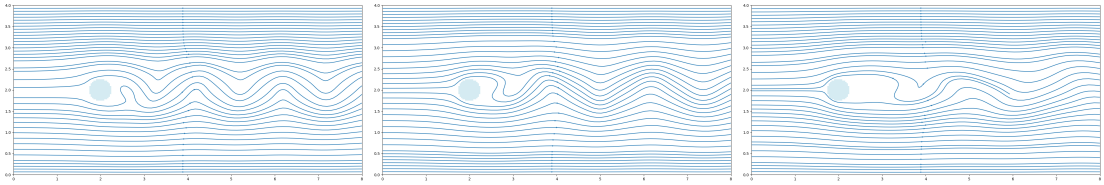


Figure 57: Streamline for Re=1000 (left), Re=300 (middle), Re=150 (right)

Based on the streamline and velocity contour comparison of Re=1000, Re=300, and Re=150, we could find that as Re become higher, the vortices evolution seems been speed up, and its become aligned up at Re become higher

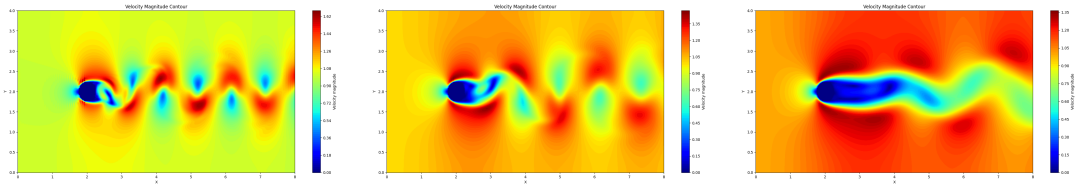


Figure 58: Velocity contour for Re=1000 (left), Re=300 (middle), Re=150 (right)

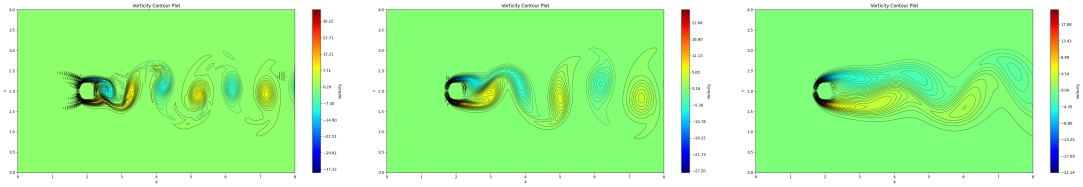


Figure 59: Vorticity for Re=1000 (left), Re=300 (middle), Re=150 (right)

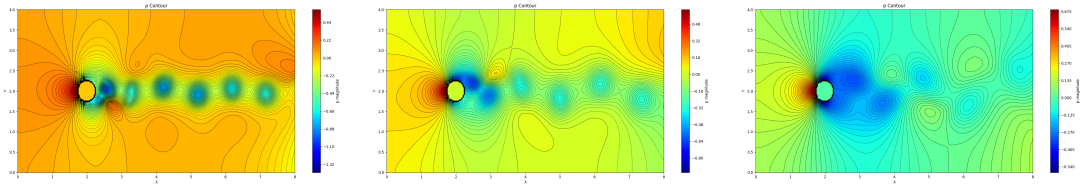


Figure 60: Pressure for Re=1000 (left), Re=300 (middle), Re=150 (right)

t=100:

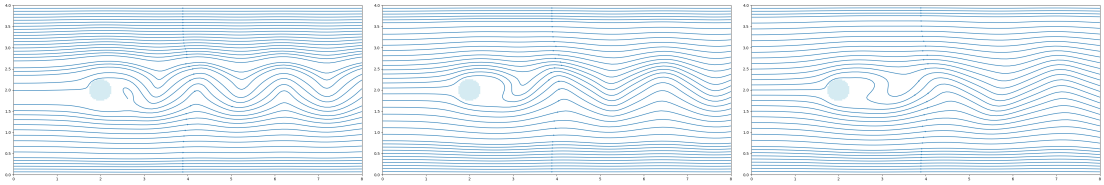


Figure 61: Streamline for Re=1000 (left), Re=300 (middle), Re=150 (right)

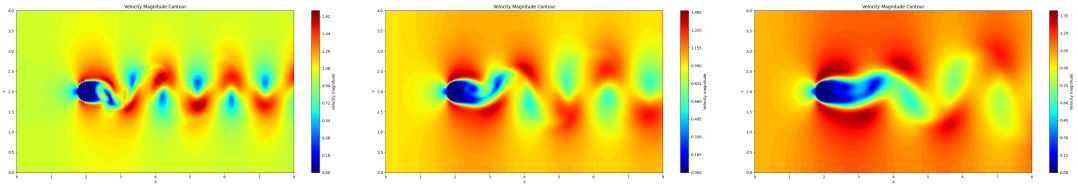


Figure 62: Velocity for Re=1000 (left), Re=300 (middle), Re=150 (right)

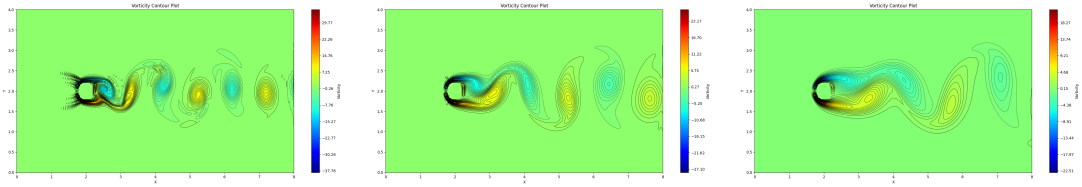


Figure 63: Vorticity for Re=1000 (left), Re=300 (middle), Re=150 (right)

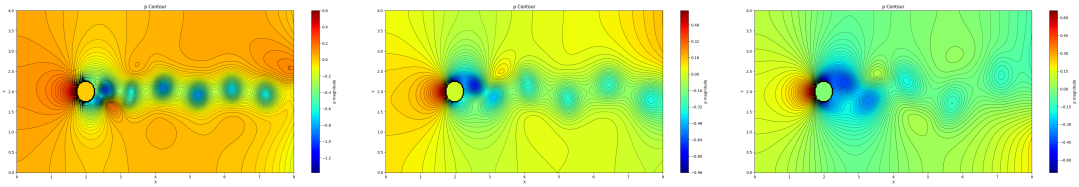


Figure 64: Pressure for Re=1000 (left), Re=300 (middle), Re=150 (right)

Based on the Re result comparison among time, we could find that as we increase Re number, the flow develop become much faster, which make the vortex shedding show up earlier.

We also could observe that as the Re number increase, the vortices begin to align neatly after the circular cylinder.

9 Re=300, Elliptic cylinder

In this section, we replaced our circular cylinder with 3:1 elliptic cylinder, and its angle of attack at 30 degrees, and Re=300.

9.1 Elliptic Setting

Based on the requirement shown above, our ellipse control equation as follows:

$$\frac{((x-x_0)\cos\theta_0+(y-y_0)\sin\theta_0)^2}{a^2}+\frac{(-(x-x_0)\sin\theta_0+(y-y_0)\cos\theta_0)^2}{b^2}=1$$

Where, θ_0 is the angle of the ellipse rotation. In our case, $\theta_0 = -30$ to full fill our angle-of-attack requirement.

The domain setting is as follows:

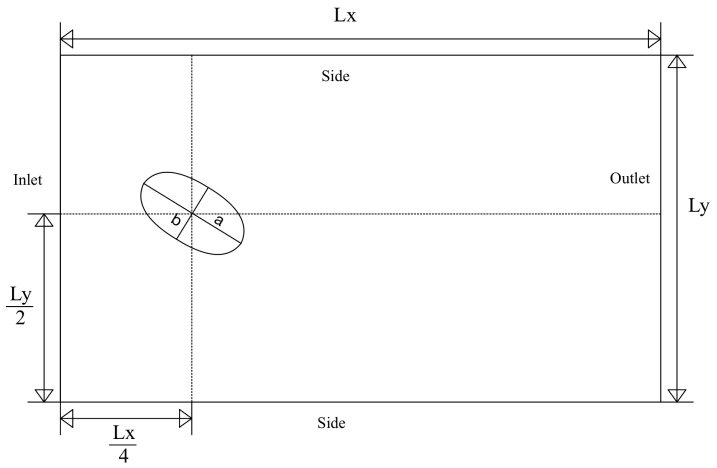


Figure 65: Domain and boundary setting for Elliptic cylinder

The Domain setting as shown above. The key parameters as follows:

- **Ly=4, Lx=8**: Width and Length of the domain
- **Ellipse Center**: the location of Elliptic cylinder center=$(\frac{Lx}{4}, \frac{Ly}{2})$
- **a=0.6, b=0.3**: The long, and short axis of the ellipse.
- **Inlet**: $u = 1, v = 0, \frac{\partial p}{\partial x} = 0$
- **Side**: $\frac{\partial u}{\partial y} = 0, \frac{\partial v}{\partial y} = 0, \frac{\partial p}{\partial y} = 0$
- **Outlet**: $\frac{\partial u}{\partial x} = 0, \frac{\partial v}{\partial x} = 0, \frac{\partial p}{\partial x} = 0$

9.2 t=50 and t=100

By setting the parameters as the chapter shown before, we now exhibit the result of t=50 and t=100 as follows:

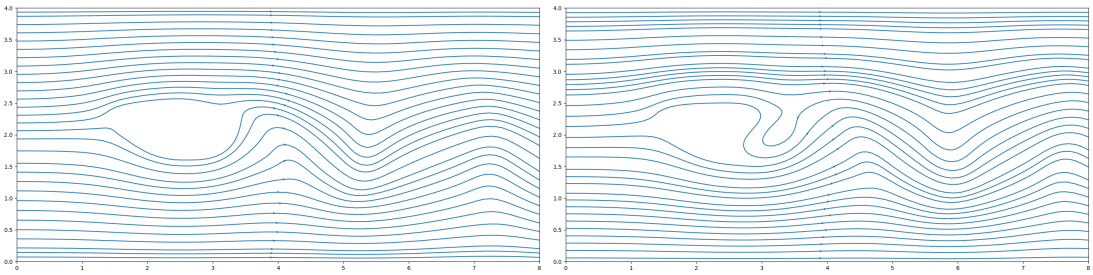


Figure 66: Streamline at t=50 (left), t=100s (right)

Based on streamline and velocity, we could see there is a vortex generated at the east south point of the ellipse, and the cortex generated at this point then shed off from ellipse. We could observe the vortices shedding is already shown up at t=50.

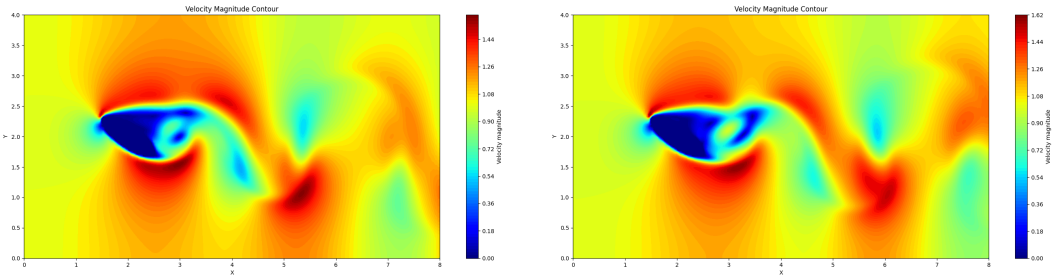


Figure 67: Velocity at t=50 (left), t=100s (right)

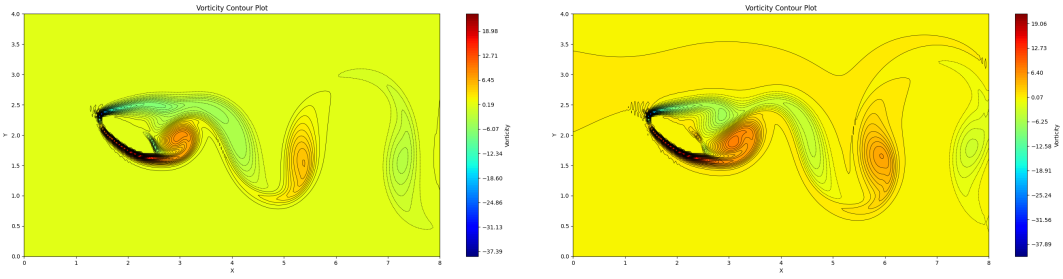


Figure 68: Vorticity at t=50 (left), t=100s (right)

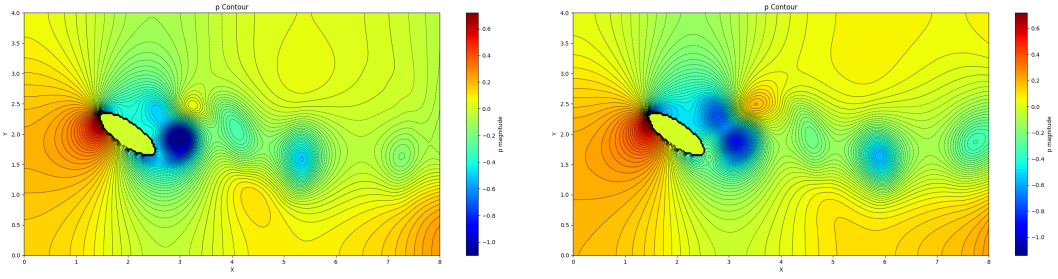


Figure 69: Pressure at t=50 (left), t=100s (right)

9.2.1 Comparison among time (t=5, 10, 50, 100)

As we could already observe vortex shedding showing up at t=50, and t=100, we here plot the result among more time steps:

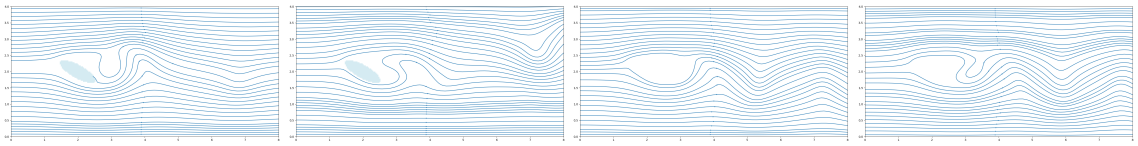


Figure 70: Streamline at t=5 (left), t=10 (middle left), t=50 (middle right) t=100s (right)

Based on the time comparison for more time scale figures, we could see how the vortex generated and shed off from ellipse. Based on the pressure contour, we could see there is main pressure low point shown at the ease south side of the ellipse.

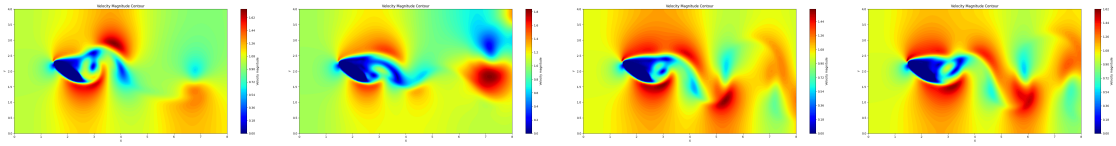


Figure 71: Velocity at t=5 (left), t=10 (middle left), t=50 (middle right) t=100s (right)

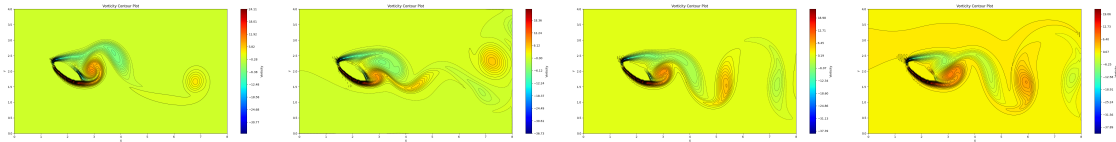


Figure 72: Vorticity at t=5 (left), t=10 (middle left), t=50 (middle right) t=100s (right)

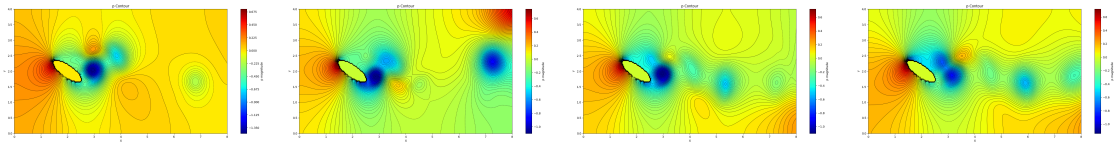


Figure 73: Pressure at t=5 (left), t=10 (middle left), t=50 (middle right) t=100s (right)

At t=50 and t=100, we could see the vortex aligned up after the ellipse.

9.3 Elliptic Result Value and Comparison

9.3.1 Drag (c_D) and Lift (c_L) coefficient

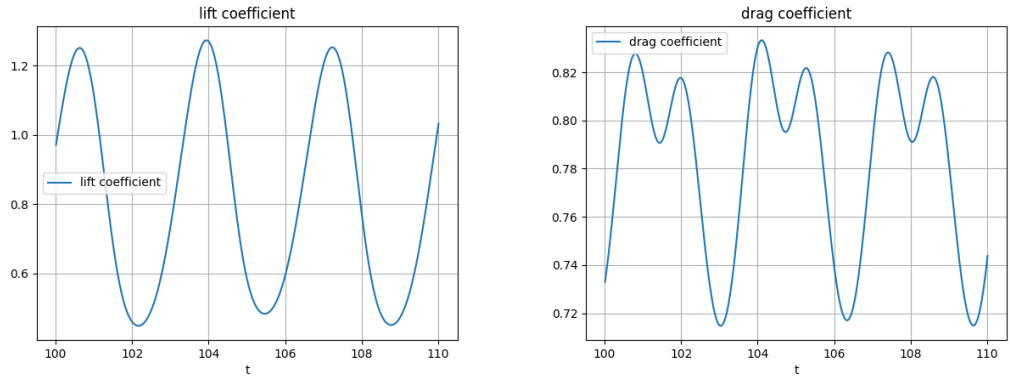


Figure 74: c_L and c_D among time

We could get the mean drag coefficient (c_L):

$$c_L = 0.81$$

Also, we could get mean lift coefficient (c_D):

$$c_D = 0.76$$

9.3.2 Vortex Shedding frequency

By sampling the result of drag and lift coefficient (c_D and c_L), and using Fast Fourier Transform (FFT), we could get the vortex shedding frequency:

$$f = 0.2$$

Based on that, we could calculate Strouhal number (St):

$$St = \frac{fD}{U} = 0.36$$

10 Discussion and Conclusion

10.1 Review of the result

In this project, we discritized N-S equation and applied to our N-S solver. The solver perform well in uniform flow and channel flow, which its result matched therodially result. We applied our solver to the circular cylinder cross sectional flow, it shows vortex shedding after several time steps.

10.1.1 Re=150, domain size= 8×4, grid resolution $\Delta = 1/32$

We compared our result for different time scale. We could observe the vortex generating among time, and dislodged from circular cylinder. Among time, the flow been disturbed significantly.

10.1.2 Re=150, domain size= 8×2, grid resolution $\Delta = 1/32$

In order to explore how could the domain size influence our result, we simulate the same flow in domain size 8×2, which is **Re=150, domain size=** 8×2**, grid resolution** $\Delta = 1/32$ case:

For this case, we compared our result with 8×4 case, we could find that the vortex evolution been speed up, which could see the vortex shedding appear early, and been squeezed as the vortex touched the sides of the domain.

10.1.3 Re=150, domain size= 8×4, grid resolution $\Delta = 1/16$

In order to explore how could grid resolution can influence our result, we also tried a coarse grid. We let grid size $\Delta = 1/16$, which is **Re=150, domain size=** 8×4**, grid resolution** $\Delta = 1/16$ case:

In this case, we could find that the flow evolution seems been slowed down, compare with $\Delta = 1/32$ grid resolution. This may due to the numerical viscosity which been added by our discretization, and as the grid become coarser, the viscosity increased, make the vortex hard to shed.

10.1.4 Re=300, domain size= 8×4, grid resolution $\Delta = 1/32$

To discuss Re number's influence of the result of the flow, we use Re=300 and compare with Re=150 case:

In this case, we could find the vortex generated earlier, and the flow become more disturbed for Re=300, compare with Re=150 case at the same time scale. The vortex shedding appearance seems been speed up.

10.1.5 Re=1000, domain size= 8×4, grid resolution $\Delta = 1/32$

To discuss Re number's influence of the result of the flow more, we also tried Re=1000 case

The vortex generating and shedding out of the circular cylinder seems been more speed up, and we also could notice that vortex lying up, neatly aligned after the circular cylinder.

We put Re=1000, Re=300, and Re=150 cases together to get a more comprehensive understanding of the vortex shedding. We could find as Re number become higher, the vortex shedding appear earlier, and the vortex aligned up more neatly.

10.1.6 Elliptic Case: Re=300, domain size= 8×4, grid resolution $\Delta = 1/32$

In this case, we replaced our circular cylinder to the elliptic cylinder, and we could observe the vortex been generated at the lower right side of the cylinder, and among time, the vortex become more exhibit.

10.2 Result and Observation

	Re 150	**Re 300**	**Re 1000**	**8x2 Narrow Domain**	**1/6 Coarse Grid**	Re 150 [3]	Re 300 [1]	Re 1000 [3]
C_D	1.212	1.21	1.36	1.588	1.22	1.02	1.11	1.34
C_L	±0.21	±0.46	±0.97	±0.35	±0.18	±0.35	±0.77	±1.30
St	0.17	0.2	0.2	0.2	0.15	0.184	0.205	0.236

Table 7: Total Comparison: (Reference: Qu(2003)[3], Franke(1990)[1])

Based the observation and comparison, we could find that for small domain, the flow evolution is much faster, to make the vortex shedding show up earlier, increase Re number have the similar effect. Decrease grid resolution, in the other hand, have the opposite effect, which could slow down the vortex generation, however, it could not prevent vortex generation.

10.3 Discussion

Based the observation and comparison, we could find that for small domain, the vortex shedding occur earlier, which may because of the the flow been squeezed, and as we use the symmetric boundary condition at the side of domain ($frac\partial\partial n$=0), the flow touched the side of domain and been mirrored back, make the flow layering together, make the vortex show up earlier.

We found increase Re number have the similar effect, where Re number been increase, the flow become much easier to be disturbed, and make the vortex generate earlier.

Based on the comparison of coarse grid comparison, we found as we use large grid size (poor resolution), the flow evolution become slower. This may because of the grid size make the flow harder to been disturbed by the cylinder, and make the wave easier to transfer to the edge of the domain, make the flow evolution much slower.

However, coarse grid cannot prevent the vortex shedding, as we finally observed the vortex shedding after the circular cylinder.

10.4 Conclusion

In this project, we discretized the Navier-Stokes equations to build and validate an N-S solver, which was then applied to flow simulations around circular and elliptical cylinders. At a Reynolds number (Re) of 150, we compared our results with existing literature and confirmed that our results are both sound and reliable.

To analyze how domain size, grid resolution, and Reynolds number influence our numerical results, we adjusted the domain size from 8×4 to 8×2, and modified the grid resolution from $\Delta = 1/32$ to $\Delta = 1/16$. We also explored the outcomes at Re = 300 and Re = 1000.

We observed that smaller domain sizes accelerated the flow evolution, leading to earlier vortex shedding. This phenomenon is likely due to the constrained flow being compressed, which accelerates disturbances within the fluid. Similarly, increasing the Reynolds number enhances the flow's susceptibility to disturbances, thereby promoting earlier vortex formation. These effects are consistent across different configurations, confirming the impact of domain size and Reynolds number on the observed fluid dynamics.

Conversely, reducing the grid resolution had an inhibitory effect on the rate of flow evolution. A coarser grid introduces greater numerical viscosity due to our discretization method, which damps fluid disturbances and delays the development of vortices. However, even with reduced resolution, vortex shedding cannot be entirely prevented, only delayed. This indicates that while grid resolution influences the speed of flow disturbances, it does not alter the fundamental dynamics responsible for vortex shedding.

References

[1] R. Franke, W. Rodi, and B. Schönung. Numerical calculation of laminar vortex-shedding flow past cylinders. *Journal of Wind Engineering and Industrial Aerodynamics*, 35:237–257, 1990.

[2] C. Norberg. Fluctuating lift on a circular cylinder: review and new measurements. *Journal of Fluids and Structures*, 17(1):57–96, 2003.

[3] Lixia Qu, Christoffer Norberg, Lars Davidson, Shia-Hui Peng, and Fujun Wang. Quantitative numerical analysis of flow past a circular cylinder at reynolds number between 50 and 200. *Journal of Fluids and Structures*, 39:347–370, 2013.

Appendix

The Project Data and python scripts are located at: Github link (click this)
Our basic Solver code is showing below:

```
import numpy as np
import matplotlib.pyplot as plt
plt.ioff()
import copy
import re

class mesher:
    def __init__(self, data):
        self.Lx = data["Lx"]
        self.Ly = data["Ly"]
        self.dxy = data["dx"]

    def parameter_grid(self): # generate large grid
        Field1 = grid_generator( self.Ly,  self.Lx, self.dxy)
        N_rows, N_cols,a = np.shape(Field1)

        # save location y,x in field[j,i,0],
        for j in range(0, N_rows):
            for i in range(0,N_cols):
                Field1[j,i,0] , Field1[j,i,1] = (j-1/2)*self.dxy, (i-1/2)*self.dxy
        # save fluid(1)/Solid(0) in field[j,i,2]
        Field1[:,:,2] = i_fluid_detector(Field1[:,:,0], Field1[:,:,1], Field1[:,:,2])
        Field1[:,:,3] = 1 - Field1[:,:,2]
        return Field1

def grid_generator( y_size,  x_size, grid_size):
        print(1.1)
        Nx , Ny = int(x_size/grid_size), int(y_size/grid_size)
        print(1.2)
        print(Ny,Nx)
        Field = np.full((Ny+2, Nx+2, 4), 0 ,dtype = 'float')
        print(1.3)
        return Field

def i_fluid_detector(field_y, field_x, field):
    N_rows, N_cols = np.shape(field)
    for j in range(0, N_rows):
        for i in range(0, N_cols ):
            result = 0
            result = circle_function(field_y[j,i], field_x[j,i])
            if result < 0:
                field[j,i] = 0
            else:
                field[j,i] = 1
    return field

def circle_function(y,x):
    R= 0.25
    x_c = data['Lx']*1/4
    y_c = data['Ly']*1/2
    result = (x-x_c)**2 + (y-y_c)**2 - R**2
```

```
    return result

def IBM_uv_coeff_change(c_W, c_E, c_S, c_N, c_P, iF, iS): # iF: ifluid. iS: isolid
        c_P[1:-1,1:-1] = c_P[1:-1,1:-1] -(
            iS[1:-1,:-2]*c_W[1:-1,1:-1] +
            iS[1:-1,2:]*c_E[1:-1,1:-1] +
            iS[:-2,1:-1]*c_S[1:-1,1:-1] +
            iS[2:,1:-1]*c_N[1:-1,1:-1] )
        c_W[1:-1,1:-1] = iF[1:-1,:-2] * c_W[1:-1,1:-1]
        c_E[1:-1,1:-1] = iF[1:-1,2:] * c_E[1:-1,1:-1]
        c_S[1:-1,1:-1] = iF[:-2,1:-1] * c_S[1:-1,1:-1]
        c_N[1:-1,1:-1] = iF[2:,1:-1] * c_N[1:-1,1:-1]

        c_P[1:-1,1:-1] = iF[1:-1,1:-1] * c_P[1:-1,1:-1] + iS[1:-1,1:-1]
        c_E[1:-1,1:-1] = Solid__coeff_correct(c_E, iF)
        c_W[1:-1,1:-1] = Solid__coeff_correct(c_W, iF)
        c_N[1:-1,1:-1] = Solid__coeff_correct(c_N, iF)
        c_S[1:-1,1:-1] = Solid__coeff_correct(c_S, iF)
        return c_W, c_E, c_S, c_N, c_P


def IBM_p_coeff_change(c_W, c_E, c_S, c_N, c_P, iF, iS): # iF: ifluid. iS: isolid
        c_P[1:-1,1:-1] = c_P[1:-1,1:-1] +(
            iS[1:-1,:-2]*c_W[1:-1,1:-1] +
            iS[1:-1,2:]*c_E[1:-1,1:-1] +
            iS[:-2,1:-1]*c_S[1:-1,1:-1] +
            iS[2:,1:-1]*c_N[1:-1,1:-1] )
        c_W[1:-1,1:-1] = iF[1:-1,:-2] * c_W[1:-1,1:-1]
        c_E[1:-1,1:-1] = iF[1:-1,2:] * c_E[1:-1,1:-1]
        c_S[1:-1,1:-1] = iF[:-2,1:-1] * c_S[1:-1,1:-1]
        c_N[1:-1,1:-1] = iF[2:,1:-1] * c_N[1:-1,1:-1]

        c_P[1:-1,1:-1] = iF[1:-1,1:-1] * c_P[1:-1,1:-1] + iS[1:-1,1:-1]
        c_E[1:-1,1:-1] = Solid__coeff_correct(c_E, iF)
        c_W[1:-1,1:-1] = Solid__coeff_correct(c_W, iF)
        c_N[1:-1,1:-1] = Solid__coeff_correct(c_N, iF)
        c_S[1:-1,1:-1] = Solid__coeff_correct(c_S, iF)
        return c_W, c_E, c_S, c_N, c_P

def Solid__coeff_correct(C, iF):
        return np.copy(C[1:-1,1:-1] * iF[1:-1,1:-1])

def TDMA(a, b, c, d):  # TDMA Solver, input abcd in same length
    # a[0], c[-1] not been used
    do = d.copy(); ao = a.copy(); bo = b.copy(); co = c.copy()
    N = len(d)
    xo = np.zeros(N)
    for rowi in range(1,N):
        k = ao[rowi]/bo[rowi-1]
        bo[rowi] -= co[rowi-1]*k
        do[rowi] -= do[rowi-1]*k
    xo[N-1] = do[N-1]/bo[N-1]
    for rowi in range(N-2,-1,-1):
        xo[rowi] = (do[rowi]-co[rowi]*xo[rowi+1])/bo[rowi]
    return xo

```

```
def general_grid_generator(len_x, Nx, Ny):
    # Creating same size of u, v, U, V
    # where col 0, row 0, row -1, of U not used
    # where row 0, col 0, col -1, of V not used
    initial_Value = 0.01
    dx = len_x/Nx
    grid = np.full((Ny+2, Nx+2), initial_Value)
    grid[0,0] = grid[-1,-1] = grid[-1,0] = grid[0,-1] = np.inf
    u,v,U,V = data_copy(grid, grid, grid, grid)
    p = np.copy(grid)
    return u, v, U, V, p, dx

def data_copy(u, v, U, V):
    return np.copy(u), np.copy(v), np.copy(U), np.copy(V)

def Ghost_BC_p(p_input): # updating ghost point for p based on BC condition
    p = np.copy(p_input)
    p[-1,1:-1] =   p[-2,1:-1] # upper BC
    p[0, 1:-1] =   p[1,1:-1] # bottom BC
    p[1:-1, 0] =   p[1:-1,1] # left BC
    p[1:-1, -1] =  p[1:-1,-2] # right BC
    return p

def Ghost_BC_u(u): # updating ghost point for p based on BC condition
    u[-1,1:-1] =   u[-2,1:-1] # upper BC
    u[0, 1:-1] =   u[1,1:-1] # bottom BC
    u[1:-1, 0] =   2-u[1:-1,1] # left BC
    u[1:-1, -1] =  u[1:-1,-2] # right BC
    u = u[:,:]*iF[:,:]
    return u

def Ghost_BC_v(v): # updating ghost point for p based on BC condition
    v[-1,1:-1] =   -v[-2,1:-1] # upper BC
    v[0, 1:-1] =   -v[1,1:-1] # bottom BC
    v[1:-1, 0] =   -v[1:-1,1] # left BC
    v[1:-1, -1] =   v[1:-1,-2] # right BC
    v = v[:,:]*iF[:,:]
    return v

def UV_BC(U, V, u, iF):
    U[1:-1,1] = 1 # left
    V[1,:] = 0 # bottom
    V[-1,:] = 0 # upper
    U[1:-1,1:-1] = U[1:-1,1:-1] * iF[1:-1,1:-1]
    U[1:-1,2:]   = U[1:-1,2:]   * iF[1:-1,1:-1]
    V[1:-1,1:-1] = V[1:-1,1:-1] * iF[1:-1,1:-1]
    V[2:,1:-1]   = V[2:,1:-1]   * iF[1:-1,1:-1]
    return U,V

def coeff_assemble(rows_cols, a_e, b_e, c_e, e_e, f_e):
    rows, cols = rows_cols[0], rows_cols[1]
    # input elements of abcdef, get line matrix of abcdef
    a = np.full((rows, cols), a_e ) # abc in same length of Ncols of u
    b = np.full((rows, cols), b_e )
    c = np.full((rows, cols), c_e )
    e = np.full((rows, cols), e_e ) # ef in same length of Nrows of u
    f = np.full((rows, cols), f_e )
    a[:,0]= a[:,-1] = c[:,0] = c[:,-1] = 0
```

```
    b[:,0] = b[:,-1] = 1
    e[:,-1] =e[:,0] = f[:,-1]= f[:,0]=0
    e[0,:] = e[-1,:] = f[0,:] = f[-1,:] = 0
    a[0,:] = a[-1,:] = c[0,:] = c[-1,:] = 0
    return a,b,c, e,f

def RHS_uv(f, f_old, U, V, U_old, V_old, c0, r):
    # Update Right Hand Side
    f_P = f[1:-1,1:-1]
    f_W = f[1:-1,:-2]
    f_E = f[1:-1,2:]
    f_S = f[:-2,1:-1]
    f_N = f[2:,1:-1]

    f_P_old = f_old[1:-1,1:-1]
    f_W_old = f_old[1:-1,:-2]
    f_E_old = f_old[1:-1,2:]
    f_S_old = f_old[:-2,1:-1]
    f_N_old = f_old[2:,1:-1]
    convection_new = c0*( U[1:-1,2:] * (f_E + f_P)/2 -
                          U[1:-1,1:-1] * (f_W + f_P)/2 +
                          V[2:,1:-1]  * (f_N + f_P)/2 -
                          V[1:-1,1:-1] * (f_S + f_P)/2 )
    convection_old = c0*( U_old[1:-1,2:] * (f_E_old + f_P_old)/2 -
                          U_old[1:-1,1:-1] * (f_W_old + f_P_old)/2 +
                          V_old[2:,1:-1]  * (f_N_old + f_P_old)/2 -
                          V_old[1:-1,1:-1] * (f_S_old + f_P_old)/2 )

    diffusion = f_P + r*(f_E + f_W + f_N + f_S -4*f_P)
    return -3/2*convection_new + 1/2*convection_old + diffusion

def Residual(u, a, b, c, d, e, f):
    return (a[1:-1,1:-1] * u[1:-1,:-2] +
                      b[1:-1,1:-1] * u[1:-1,1:-1] +
                      c[1:-1,1:-1] * u[1:-1,2:] +
                      e[1:-1,1:-1] * u[:-2,1:-1] +
                      f[1:-1,1:-1] * u[2:,1:-1] -
                      d[:,:])

def LineSOR_u(a,b,c,d,e,f, u):
    # input abcdef, and u, get :D
    # then use TDMA get new u
    # abc is in same size of N_rows of u, ef in same size of N_cols of u
    # ! Input d is LIKE inner field of u, need to cut each line to use !
    u_k = np.copy(u)
    u_k_new = np.copy(u)
    n_rows = np.size(u,0)
    w = 1.3
    D = np.zeros_like(a[1,:])
    Res = 100
    k = 0
    while Res>1e-6:
        u_k = np.copy(u_k_new)
        for j in range(1,n_rows-1): # the number is based on d, which is (N
    -1)x(N-1)
            D[1:-1] = d[j-1,:] - e[j,1:-1]*u_k_new[j-1,1:-1] - f[j,1:-1]*
    u_k_new[j+1,1:-1]
            D[0], D[-1] = 2-u_k_new[j,1], u_k_new[j,-2] # ghost point =
```

```
    ghost point
            u_k_new[j,:] = TDMA(a[j,:],b[j,:],c[j,:],D) # TDMA solve uk
    each line
            u_k_new[j,1:-1] = u_k[j,1:-1]*(1-w) +u_k_new[j,1:-1]*w # SOR
    term
        u_k_new = Ghost_BC_u(u_k_new)
        Res = np.max(np.abs(Residual(u_k_new, a, b, c, d, e, f)))
        # print("Res_u=")
        # print(Res)
        u = np.copy(u_k)
        # print(Residual(u_k_new, a, b, c, d, e, f))
        k+=1
    return u_k_new, k, Res

def LineSOR_v(a,b,c,d,e,f, u):
    # input abcdef, and u, get D
    # then use TDMA get new u
    # abc is in same size of N_rows of u, ef in same size of N_cols of u
    # ! Input d is LIKE inner field of u, need to cut each line to use !
    u_k = np.copy(u)
    u_k_new = np.copy(u)
    n_rows = np.size(u,0)
    w = 1.3
    D = np.zeros_like(a[1,:])
    Res = 100
    k = 0
    while Res>1e-6:
        u_k = np.copy(u_k_new)
        for j in range(1,n_rows-1): # the number is based on d, which is (N
    -1)x(N-1)
            D[1:-1] = d[j-1,:] - e[j,1:-1]*u_k_new[j-1,1:-1] - f[j,1:-1]*
    u_k_new[j+1,1:-1]
            D[0], D[-1] = -u_k_new[j,1], u_k_new[j,-2] # ghost point =
    ghost point
            u_k_new[j,:] = TDMA(a[j,:],b[j,:],c[j,:],D) # TDMA solve uk
    each line
            u_k_new[j,1:-1] = u [j,1:-1]*(1-w) +u_k_new[j,1:-1]*w # SOR
    term
        u_k_new = Ghost_BC_v(u_k_new)
        Res = np.max(np.abs(Residual(u_k_new, a, b, c, d, e, f)))
        # print("Res_v=v")
        # print(Res)
        u = np.copy(u_k)
        k+=1
        # print(Residual(u_k_new, a, b, c, d, e, f))
    return u_k_new, k, Res

def LineSOR_p(a,b,c,d,e,f, u):
    # input abcdef, and u, get D
    # then use TDMA get new u
    # abc is in same size of N_rows of u, ef in same size of N_cols of u
    # ! Input d is LIKE inner field of u, need to cut each line to use !
    u = Ghost_BC_p(u)
    u_k = np.copy(u)
    u_k_new = np.copy(u)
    n_rows = np.size(u,0)
    w = 1.95
    D = np.zeros_like(a[1,:])
```

```
    Res = 100
    k = 0
    while Res>1e-6:
        u_k = np.copy(u_k_new)
        for j in range(1,n_rows-1): # the number is based on d, which is (N-1)x(N-1)
            D[1:-1] = d[j-1,:] - e[j,1:-1]*u_k_new[j-1,1:-1] - f[j,1:-1]*u_k_new[j+1,1:-1]
            D[0], D[-1] = u_k_new[j,1], u_k_new[j,-2] # ghost point = ghost point
            u_k_new[j,:] = TDMA(a[j,:],b[j,:],c[j,:],D) # TDMA solve uk each line
            u_k_new[j,1:-1] = u_k[j,1:-1]*(1-w) +u_k_new[j,1:-1]*w # SOR term
        u_k_new = Ghost_BC_p(u_k_new)
        Res = np.max(np.abs(Residual(u_k_new, a, b, c, d, e, f)))/n_rows**2
        # print("Res_p=")
        # print(Res)
        k+=1
    return u_k_new, k, Res

def Solver(u, v, U, V , u_old, v_old, U_old, V_old,  p, data, parameter_field):
    Re = data["Re"]; dx = data["dx"]; dt = data["dt"]; t = data["t"]; t_0 = data['t_0']
    N = 1/data['dx']
    r = dt/(2 * Re * dx**2 ); c0 = dt/(dx)

    rows_cols = np.shape(u)
    c_W_uv , c_P_uv , c_E_uv , c_S_uv , c_N_uv  =\
          coeff_assemble(rows_cols, -r, 1+4*r, -r, -r, -r)
    c_W_uv , c_E_uv , c_S_uv , c_N_uv , c_P_uv = \
          IBM_uv_coeff_change(c_W_uv, c_E_uv, c_S_uv, c_N_uv, c_P_uv, iF, iS)

    c_W_p , c_P_p , c_E_p , c_S_p , c_N_p  = \
          coeff_assemble(rows_cols, 1/(dx*dx), -4/(dx*dx), 1/(dx*dx), 1/(dx*dx), 1/(dx*dx))
    c_W_p , c_E_p , c_S_p , c_N_p , c_P_p = \
          IBM_p_coeff_change(c_W_p, c_E_p, c_S_p, c_N_p, c_P_p, iF, iS)

    RHS = np.zeros_like(u)
    U, V = UV_BC(U, V, u, iF)

    u = Ghost_BC_u(u)
    v = Ghost_BC_v(v)
    # u, v = Ghost_BC(u,v, iF)

    u_star, v_star, U_star, V_star = data_copy(u, v, U, V)
    u_new, v_new, U_new, V_new = data_copy(u, v, U, V)
    # u_old, v_old, U_old, V_old = data_copy(u, v, U, V)

    n_max = int(t/dt)
    Difference = 100
    n= int( t_0/dt)

    while n<n_max and Difference>1e-8 :
    # while n<n_max:
```

```
        U, V = UV_BC(U, V, u, iF) # Setup BC

        # Step 1: get u_star, v_star, U_star, V_star
        u = Ghost_BC_u(u)
        v = Ghost_BC_v(v)
        u_star = Ghost_BC_u(u_star)
        v_star = Ghost_BC_v(v_star)

        # get u_star:
        d = RHS_uv(u, u_old, U, V, U_old, V_old, c0, r)
        rhs= d[:,:]*iF[1:-1,1:-1]
        u_star ,a, Resu = LineSOR_u(c_W_uv , c_P_uv , c_E_uv, rhs , c_S_uv , c_N_uv, u)
        u_star = Ghost_BC_u(u_star)
        v_star = Ghost_BC_v(v_star)

        # get v_star:
        d = RHS_uv(v, v_old, U, V, U_old, V_old, c0, r)
        rhs= d[:,:]*iF[1:-1,1:-1]
        v_star ,a, Resv= LineSOR_v(c_W_uv , c_P_uv , c_E_uv, rhs , c_S_uv , c_N_uv, v)

        u_star = Ghost_BC_u(u_star)
        v_star = Ghost_BC_v(v_star)

        # get U_star, V_star
        U_star[1:-1,2:] =  (u_star[1:-1,1:-1] + u_star[1:-1,2:])/2
        V_star[2:,1:-1] =  (v_star[2:,1:-1] + v_star[1:-1,1:-1])/2
        U_star, V_star = UV_BC(U_star, V_star, u_star, iF) # Setup BC

        Diff = np.sum(U_star[1:-1, -1]-U_star[1:-1,1])
        U_star[1:-1, -1] -= 1/(rows_cols[0]-2) * Diff

        # Step 2: get New Pressure P
        d = (
            U_star[1:-1, 2:] - U_star[1:-1, 1:-1] +
            V_star[2:, 1:-1] - V_star[1:-1, 1:-1]
        )/(dt*dx)

        # print(d[4,5]-(U_star[5,7]-U_star[5,6]+V_star[6,6] - V_star[5,6])/(dt*dx))

        rhs= d[:,:]*iF[1:-1,1:-1]
        p_new,k, Resp= LineSOR_p(c_W_p , c_P_p , c_E_p, rhs , c_S_p , c_N_p , p)
        # print(p_new-Ghost_BC_p(p_new))
        p_new = Ghost_BC_p(np.copy(p_new))

        # Step 3: get u_new, v_new, U_new , V_new
        # update u_new,
        u_new[1:-1,1:-1] = u_star[1:-1, 1:-1] - dt/dx*(iF[1:-1,2:]*p_new[1:-1,2:] + iS[1:-1,2:]*p_new[1:-1,1:-1] -
                                                       iF[1:-1,:-2]*p_new[1:-1,:-2] - iS[1:-1,:-2]*p_new[1:-1,1:-1])/2
        v_new[1:-1,1:-1] = v_star[1:-1, 1:-1] - dt/dx*(iF[2:,1:-1]*p_new[2:,1:-1] + iS[2:,1:-1]*p_new[1:-1,1:-1] -
                                                       iF[:-2,1:-1]*p_new[:-2,1:-1] - iS[:-2,1:-1]*p_new[1:-1,1:-1])/2
```

```
371
372         u_new = Ghost_BC_u(u_new); v_new = Ghost_BC_v(v_new)
373         # update U_new , V_new
374         U_new[1:-1,2:] = U_star[1:-1,2:] - dt/dx*(p_new[1:-1,2:]-p_new
    [1:-1,1:-1])
375         V_new[2:,1:-1] = V_star[2:,1:-1] - dt/dx*(p_new[2:,1:-1]-p_new
    [1:-1,1:-1])
376         U_new, V_new = UV_BC(U_new, V_new, u_new, iF) # Setup BC
377
378         u_old, v_old, U_old, V_old = data_copy(u, v, U, V)
379         u, v, U, V = data_copy(u_new, v_new, U_new, V_new)
380         # N = rows_cols[1]-2
381         Difference = np.sum(np.abs(u[1:-1,1:-1] - u_old[1:-1,1:-1]) +
382                             np.abs(v[1:-1,1:-1] - v_old[1:-1,1:-1]) +
383                             np.abs(p_new[1:-1,1:-1] - p[1:-1,1:-1]) )
384         p = np.copy(p_new)
385         div = (U_new[1:-1, 2:] - U_new[1:-1, 1:-1]+V_new[2:, 1:-1] - V_new
    [1:-1, 1:-1])/dx
386         div = np.linalg.norm(div)
387         p = p[:,:]*iF[:,:]
388         u = Ghost_BC_u(u)
389         v = Ghost_BC_v(v)
390         p = Ghost_BC_p(p)
391
392         n +=1
393         if n%100 == 0 or n==n_max:
394             output_save(u,v,p, U, V, u_old,v_old, U_old, V_old, data['Lx'],
     data['Ly'] ,label=[N,Re,n*dt])
395         if n%1000 == 0:
396             draw_control(u,v,p, data['Lx'], data['Ly'], label=[N,Re,t, n*dt
    ])
397         print("time=%.3f, Difference=%.9f, p iteration times=%d, Res_p=%f,
    div=%.2f "%((n+1)*dt,Difference,k,Resp, div))
398     output_save(u,v,p, U, V, u_old,v_old, U_old, V_old, data['Lx'], data['
    Ly'] ,label=[N,Re,n*dt])
399     draw_control(u,v,p, data['Lx'], data['Ly'], label=[N,Re, n*dt])
400     return u, v, p
401
402 def output_save(u,v,p, U, V, u_old,v_old, U_old, V_old, len_x, len_y ,label
    ):
403     N = label[0]; Re = label[1]; t = label[2]
404     with open(f'result/save/output_N{N}_Re={Re}_t={t}_{len_x}x{len_y}222.
    txt', 'w') as f:
405         f.write(f'For grid size N= {label}:\n')
406         f.write('Array u:\n')
407         f.write(','.join(map(str, u)) + '\n\n')
408         f.write('Array v:\n')
409         f.write(','.join(map(str, v)) + '\n\n')
410         f.write('Array p:\n')
411         f.write(','.join(map(str, p)) + '\n')
412
413         f.write('Array U:\n')
414         f.write(','.join(map(str, U)) + '\n')
415         f.write('Array V:\n')
416         f.write(','.join(map(str, V)) + '\n')
417
418         f.write('Array u_old:\n')
419         f.write(','.join(map(str, u_old)) + '\n\n')
```

```
        f.write('Array v_old:\n')
        f.write(','.join(map(str, v_old)) + '\n\n')

        f.write('Array U_old:\n')
        f.write(','.join(map(str, U_old)) + '\n\n')
        f.write('Array V_old:\n')
        f.write(','.join(map(str, V_old)) + '\n\n')

def draw_control(u,v,p, Lx, Ly, label):
    draw_streamline(u[1:-1,1:-1],v[1:-1,1:-1], Lx, Ly , label)
    draw_velocity_contour(u[1:-1,1:-1],v[1:-1,1:-1], Lx, Ly , label)
    draw_u_contour(u[1:-1,1:-1],v[1:-1,1:-1], Lx, Ly , label)
    draw_p_contour(p[1:-1,1:-1],v[1:-1,1:-1], Lx, Ly , label)

def draw_streamline(u, v, len_x, len_y ,label):
    rows, cols = np.shape(u)
    X = np.linspace(0, len_x, cols)
    Y = np.linspace(0, len_y, rows)
    L = np.sqrt(len_x**2+len_y**2)
    plt.figure(figsize=(15*len_x/L, 15*len_y/L))
    plt.axis([0,len_x,0,len_y])
    plt.streamplot(X, Y, u, v,  density=1.5, minlength=1, arrowsize=0.5)
    plt.savefig(f'result/stline_N{label[0]}_Re={label[1]}_t={label[2]}_{
   len_x}x{len_y}.png')
    plt.close()

def draw_velocity_contour(u, v, len_x, len_y,label):
    levels=100
    rows, cols = np.shape(u)
    X, Y = np.linspace(0, len_x, cols), np.linspace(0, len_y, rows)
    velocity_magnitude = np.sqrt(u**2 + v**2)
    L = np.sqrt(len_x**2+len_y**2)
    plt.figure(figsize=(15*len_x/L, 15*len_y/L))
    plt.contourf(X, Y, velocity_magnitude, levels=levels, cmap='jet')
    plt.colorbar(label='Velocity magnitude')
    plt.xlabel('X')
    plt.ylabel('Y')
    plt.title('Velocity Magnitude Contour')
    plt.savefig(f'result/uv_N{label[0]}_Re={label[1]}_t={label[2]}_{len_x}x
   {len_y}.png')
    plt.close()

def draw_u_contour(u, v, len_x, len_y ,label):
    levels=100
    rows, cols = np.shape(u)
    X, Y = np.linspace(0, len_x, cols), np.linspace(0, len_y, rows)
    L = np.sqrt(len_x**2+len_y**2)
    plt.figure(figsize=(15*len_x/L, 15*len_y/L))
    plt.contourf(X, Y, u, levels=levels, cmap='jet')
    plt.colorbar(label='u magnitude')
    plt.xlabel('X')
    plt.ylabel('Y')
    plt.title('u Contour')
    plt.savefig(f'result/u_N{label[0]}_Re={label[1]}_t={label[2]}_{len_x}x{
   len_y}.png')
    plt.close()
    # plt.show()

```

```
475
476 def draw_p_contour(p, v, len_x, len_y ,label):
477     levels=100
478     rows, cols = np.shape(p)
479     X, Y = np.linspace(0, len_x, cols), np.linspace(0, len_y, rows)
480     L = np.sqrt(len_x**2+len_y**2)
481     plt.figure(figsize=(15*len_x/L, 15*len_y/L))
482     plt.contourf(X, Y, p, levels=levels, cmap='jet')
483     plt.colorbar(label='u magnitude')
484     plt.xlabel('X')
485     plt.ylabel('Y')
486     plt.title('u Contour')
487     plt.savefig(f'result/p_N{label[0]}_Re={label[1]}_t={label[2]}_{len_x}x{
    len_y}.png')
488     plt.close()
489     # plt.show()
490
491
492
493 def load_2d_arrays_with_numpy(filename):
494     try:
495         with open(filename, 'r') as file:
496             content = file.read()
497     except FileNotFoundError:
498         print("CANNOT FIND FILE!! CHECK THE PATH")
499         return None, None, None
500
501     def extract_2d_array(array_content):
502         array_data = []
503         matches = re.findall(r'\[(.*?)]', array_content.replace('\n', ''),
    re.DOTALL)
504         for match in matches:
505             formatted_match = ' '.join(match.split())
506             numbers = np.fromstring(formatted_match, sep=' ')
507             array_data.append(numbers)
508         return np.array(array_data)
509
510     u_data = re.search(r'Array u:\s*((?:\[\s*.*?\s*\][,\s]*)+)', content,
    re.DOTALL)
511     v_data = re.search(r'Array v:\s*((?:\[\s*.*?\s*\][,\s]*)+)', content,
    re.DOTALL)
512     U_data = re.search(r'Array U:\s*((?:\[\s*.*?\s*\][,\s]*)+)', content,
    re.DOTALL)
513     V_data = re.search(r'Array V:\s*((?:\[\s*.*?\s*\][,\s]*)+)', content,
    re.DOTALL)
514     u_old_data = re.search(r'Array u_old:\s*((?:\[\s*.*?\s*\][,\s]*)+)',
    content, re.DOTALL)
515     v_old_data = re.search(r'Array v_old:\s*((?:\[\s*.*?\s*\][,\s]*)+)',
    content, re.DOTALL)
516     U_old_data = re.search(r'Array U_old:\s*((?:\[\s*.*?\s*\][,\s]*)+)',
    content, re.DOTALL)
517     V_old_data = re.search(r'Array V_old:\s*((?:\[\s*.*?\s*\][,\s]*)+)',
    content, re.DOTALL)
518     p_data = re.search(r'Array p:\s*((?:\[\s*.*?\s*\][,\s]*)+)', content,
    re.DOTALL)
519
520     if not (u_data and v_data and p_data):
521         print("CANNOT GET DATA!! CANNOT GET DATA!!")
```

```
        return np.array([]), np.array([]), np.array([])

    u = extract_2d_array(u_data.group(1))
    v = extract_2d_array(v_data.group(1))
    p = extract_2d_array(p_data.group(1))
    U = extract_2d_array(U_data.group(1))
    V = extract_2d_array(V_data.group(1))
    u_old = extract_2d_array(u_old_data.group(1))
    v_old = extract_2d_array(v_old_data.group(1))
    U_old = extract_2d_array(U_old_data.group(1))
    V_old = extract_2d_array(V_old_data.group(1))
    return u, v, U, V , u_old, v_old, U_old, V_old,  p

def Recover_result(filename):
    u, v, U, V , u_old, v_old, U_old, V_old,  p = load_2d_arrays_with_numpy
    (filename)
    pattern = r'output_N(\d+\.\d+)_Re=(\d+\.\d+)_t=(\d+\.\d+)_'
    match = re.search(pattern, filename)
    if match:
        N = float(match.group(1))
        Re = float(match.group(2))
        t_0 = float(match.group(3))
    else:
        print("No match found")
    Ly = 1/N* (np.size(u,0)-2)
    Lx = 1/N* (np.size(u,1)-2)
    dy = dx = 1/N; dt = 0.01
    data_value = np.array([Re, Ly, Lx, dy, dx ,t_0 ,dt])
    keys = ["Re", "Ly", "Lx", "dy", "dx" ,"t_0" ,"dt"]
    data_re = dict(zip(keys, data_value))
    return data_re, u, v, U, V , u_old, v_old, U_old, V_old,  p

def main():
    N = 32; Ly = 4; Lx = 8; dy = dx = 1/N
    Re = 150
    t = 200; dt = 0.01
    t_0 = 0; u_0 = 0
    ##########################################
    # Recover data from existing file
    data_re, u_0, v_0, U_0, V_0 , \
        u_old_0, v_old_0, U_old_0, V_old_0,  p_0 \
            = Recover_result(filename = f'result/save/output_N32.0_Re=150.0
    _t=73.0_8.0x4.0222.txt')
    t_0 = data_re['t_0']
    Re = data_re['Re']
    Ly = data_re["Ly"]
    Lx = data_re["Lx"]
    dx = data_re['dx']
    dt = data_re['dt']
    print(Ly)
    print('data_read:',data_re)
    ##########################################
    global data
    data_value = np.array(\
        [ Re , Ly , Lx , dy , dx , t , dt , t_0 ]\
            )
    keys = \
        ["Re","Ly","Lx","dy","dx","t","dt","t_0"]
```

```
    data = dict(zip(keys, data_value))
    print('data_init=',data)
    mesh1 =  mesher(data)
    parameter_field = mesh1.parameter_grid()

    global iF, iS
    iF = parameter_field[:,:,2]
    iS = parameter_field[:,:,3]

    u, v, U, V ,p = [np.zeros_like(parameter_field[:,:,0]) for _ in range
    (5)]
    u_old, v_old, U_old, V_old =\
          np.zeros_like(u), np.zeros_like(v), np.zeros_like(U), np.
    zeros_like(V)

    if np.sum(u_0) !=0:
        u, v, U, V , u_old, v_old, U_old, V_old,  p =u_0, v_0, U_0, V_0 ,
    u_old_0, v_old_0, U_old_0, V_old_0,  p_0

    u,v, p =Solver(u, v, U, V , u_old, v_old, U_old, V_old,  p, data,
    parameter_field)
if __name__ == '__main__':
    main()
```

Listing 1: N-S Cylinder Solver