

Circular Cylinder Cross-Flow

530.767 CFD-Final Project

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This Project focuses on the CFD analysis of flow around circular and elliptical cylinders using a discretized Navier-Stokes (N-S) solver. This solver use 2nd order central difference scheme in space and a second order (AB2 for convection and CN2 for viscous) fractional step in time. By separate update of the face velocity ("semi-staggered" approach), we could approach zero divergence with the compact stencil for pressure. We also implemented Immersed Boundary Method (IBM) with stairs step method for inner boundary. We validated our solver based on uniform flow and channel flow, and compared our result with theordically solution.

We then simulated the flow cross circular cylinder at Re=150, checked our result against existing literature, ensuring soundness and reliability in our computational results. We also studied the influence of domain size, grid resolution, and Reynolds number on flow behavior was explored by modifying domain sizes from 8×4 to 8×2 , adjusting grid resolutions from $\Delta = 1/32$ and $\Delta = 1/16$, and analyzing results at Re = 300 and Re = 1000. Our findings indicate that for smaller domain sizes accelerate flow evolution and promote earlier vortex shedding due to enhanced fluid disturbances. Increasing the Reynolds number similarly affects flow susceptibility to disturbances. Conversely, a coarser grid resolution dampens disturbances, delaying vortex shedding but not preventing it, illustrating the fundamental role of grid resolution in fluid dynamics.

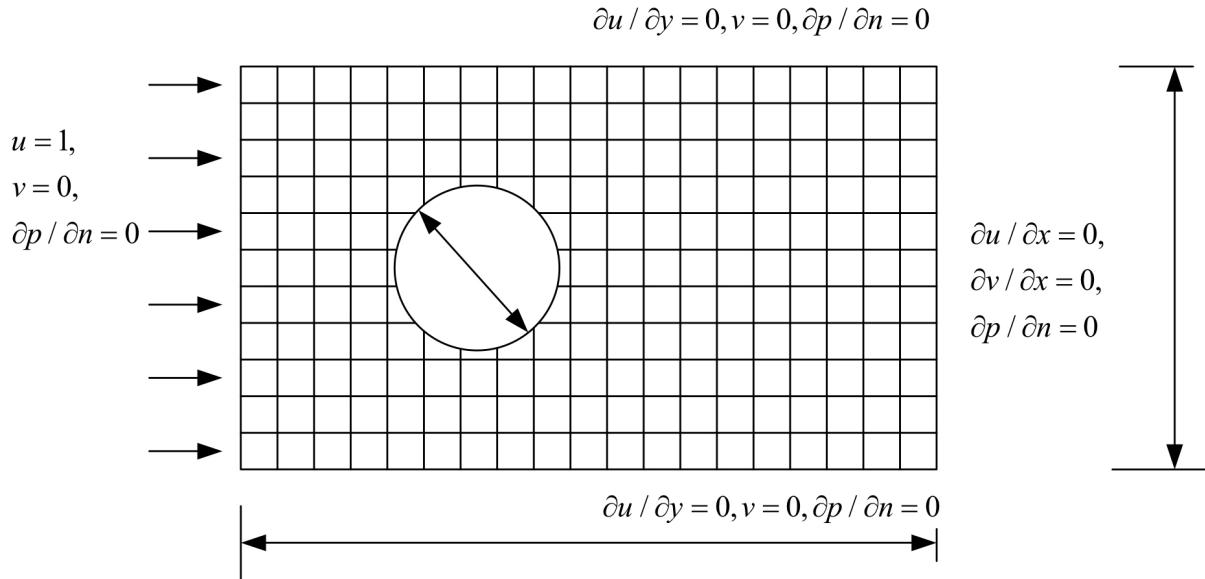
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1 Question Review



Circular cylinder in a cross-flow

Consider a circular cylinder in a cross-flow as shown above.
The flow Reynolds number based on D is $Re_D = 150$.

We will simulate this flow using:

- Incompressible Navier-Stokes equations
- A Cartesian grid
- The Stair-Step, immersed boundary method.
- You can use either staggered or co-located (cell-centered) grid. If you use a co-located arrangement, then you should employ a separate face velocity update as shown in the class.
- A fractional-step method for time-stepping
- Use 2^{nd} order central difference in space, and mixed implicit-explicit scheme in time.
- Domain size (L and H) shown in the figure is only suggestion. You can try different domain size.
- You can use either uniform or non-uniform grid. But non-uniform grid is recommended.
- At the outflow boundary, you will have to adjust the net flow in order to satisfy global mass conservation.

Tasks

- a) Generate an appropriate computational grid and perform the flow simulation with the given boundary conditions but without the cylinder. Set the initial velocity inside the domain to zero. Check to make sure that your simulation converges well and you should see a uniform flow develop in your domain. The next sanity check is to impose a no-slip boundary conditions on the top and bottom walls in which case, you should see a flow similar to the flow in the entrance of a channel.
- b) Next, perform the simulation with the circular cylinder using the Stair-Step immersed boundary method. You should get unsteady flow exhibiting vortex shedding. If you don't get vortex shedding, try to improve your numerical schemes or grid resolution.
- c) Plot, velocity, pressure, and vorticity contours at several time instances.
- d) Quantify the vortex shedding frequency and compare the value against existing published studies.
- e) Quantify the pressure drag coefficient on the cylinder and compare the value against existing published studies.
- f) Examine the effect of domain size and grid resolution on the simulation results. Key quantities to be compared are the mean pressure drag coefficient and the shedding frequency.
- g) Simulate the same flow at $Re_D = 300$ and 1000 and discuss the results in comparison to the $Re_D = 150$ simulation. Do your results at higher Reynolds number match published results?
- h) Optional Extra credit – simulate the flow around a cylinder at a 3:1 elliptic cylinder with an angle-of-attack of 30 degrees, and a Reynolds number (based on major axis) of 300. Show that your code can predict the flow and vortex shedding from this shape. Plot the coefficient of pressure drag and lift versus Reynolds number.

2 N-S Solver

2.1 Discretization of N-S Equation

The general steps solving the N-S equation is showing below:

- (I): Using the data from the current time step and the previous time step to get u^*, v^* .
Then use u^*, v^* to get U^*, V^* .
- (II): Using U^*, V^* , solving the pressure poisson equation (PPE), get p^{n+1} .
- (III): Using p^{n+1} , and u^*, v^* , get u^{n+1}, v^{n+1} .
Using p^{n+1} , and U^*, V^* , get U^{n+1}, V^{n+1} .

2.1.1 Step (I):

Using the the data we got from current time step and the previous time step get u^*, v^* .
Then use u^*, v^* get U^*, V^* .

Update u^*, v^* :

The first step is going to update u^*, v^* :

$$\frac{\vec{u}^* - \vec{u}^n}{\Delta t} = -[\frac{3}{2}\nabla \cdot (\vec{U}^n \vec{u}^n) - \frac{1}{2}\nabla \cdot (\vec{U}^{n-1} \vec{u}^{n-1})] + \frac{1}{Re} \nabla^2 \frac{\vec{u}^* + \vec{u}^n}{2} \quad (1)$$

which is:

$$\underbrace{[1 - \frac{\Delta t}{2Re} \nabla^2] \vec{u}^*}_{(1)} = -\frac{3}{2} \underbrace{[\Delta t \nabla \cdot (\vec{U}^n \vec{u}^n)]}_{(2)} + \frac{1}{2} \underbrace{[\Delta t \nabla \cdot (\vec{U}^{n-1} \vec{u}^{n-1})]}_{(3)} + \underbrace{[1 + \frac{\Delta t}{2Re} \nabla^2] \vec{u}}_{(4)} \quad (2)$$

Could say, (2), (3) are convection term, and (4) is the diffusion term of the eqaution.
Could use this equation to update u^*, v^* .

Where,

$$\begin{aligned} (1) &= \left\{ 1 - \frac{\Delta t}{2Re} \nabla^2 \right\} u^* = P^* - \left[\frac{E^* + W^* - 2P^*}{\Delta x^2} + \frac{N^* + S^* - 2P^*}{\Delta y^2} \right] \frac{\Delta t}{2Re} \\ (2) &= \frac{\Delta t}{\Delta} \left((U_e^n \frac{E^n + P^n}{2} - U_w^n \frac{W^n + P^n}{2}) + (V_n^n \frac{N^n + P^n}{2} - V_s^n \frac{S^n + P^n}{2}) \right) \\ (3) &= \frac{\Delta t}{\Delta} \left((U_e^{n-1} \frac{E^{n-1} + P^{n-1}}{2} - U_w^{n-1} \frac{W^{n-1} + P^{n-1}}{2}) + (V_n^{n-1} \frac{N^{n-1} + P^{n-1}}{2} - V_s^{n-1} \frac{S^{n-1} + P^{n-1}}{2}) \right) \\ (4) &= P^n + \frac{\Delta t}{2Re \Delta^2} (E^n + W^n + N^n + S^n - 4P^n) \end{aligned}$$

Could found in this expression, the formula to calculate u^*, v^* is same, only input u or v in to the equation:

P means (i, j) , W means $(i - 1, j)$, E means $(i + 1, j)$, S means $(i, j - 1)$, N means $(i, j + 1)$.

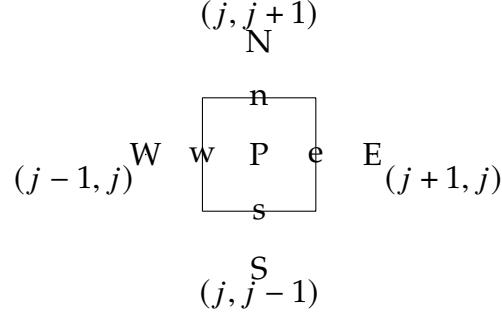


Figure 1: Illustration of a grid layout with directional points.

Put the result in, could get:

$$aW + bP + cE + eS + fN = d$$

The left hand side is the unknown characters, and the right hand side b is known from previous and current time step. Could see, this equation in our system means $Ax = d$, where A is consist of a, b, c, e, f , is a pentadiagonal matrix:

$$\begin{bmatrix} 1 & 0 & & 0 & & \\ a & b & c & & f & \\ & a & b & c & & f \\ & & a & b & c & 0 \\ 0 & & \ddots & \ddots & \ddots & \\ & e & & a & b & c \\ & e & & a & b & c \\ & 0 & & 0 & 1 & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \vdots \\ x_{n-2} \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ \vdots \\ d_{n-2} \\ d_{n-1} \\ d_{j-i} \end{bmatrix}$$

Solve this system, we could get u^* , and v^*

Update U^*, V^* :

$$U_e^* = \frac{u_P^* + u_E^*}{2}, V_n^* = \frac{v_P^* + v_N^*}{2}$$

2.1.2 Step (II):

Update p^{n+1} :

$$\begin{aligned} \nabla^2 p &= \frac{\nabla \cdot \vec{u}^*}{\Delta t} = \frac{\frac{U_e^* - U_w^*}{\Delta x} + \frac{V_n^* - V_s^*}{\Delta y}}{\Delta t} \\ \frac{P_E + P_W - 2P_P}{\Delta x^2} + \frac{P_N + P_S - 2P_P}{\Delta y^2} &= \frac{\frac{U_e^* - U_w^*}{\Delta x} + \frac{V_n^* - V_s^*}{\Delta y}}{\Delta t} \\ P_E + P_W + P_N + P_S - 4P_P &= \frac{\Delta}{\Delta t} (U_e^* - U_w^* + V_n^* - V_s^*) = RHS \end{aligned}$$

Similarly, this equation also give us a pentadiagonal system:

$$\begin{bmatrix} 1 & 0 & & 0 & & f & & \\ a & b & c & & & & f & \\ & a & b & c & & & & \\ & & a & b & c & & 0 & \\ 0 & & \ddots & \ddots & \ddots & & & \\ e & & & a & b & c & & \\ & e & & & a & b & c & \\ & & 0 & & 0 & 1 & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \vdots \\ x_{n-2} \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ \vdots \\ d_{n-2} \\ d_{n-1} \\ d_n \end{bmatrix}$$

2.1.3 Step (III):

Update u^{n+1}, v^{n+1} :

The equation to update the u^{n+1}, v^{n+1} is showing below:

$$\vec{u}^{n+1} = \vec{u}^* - \Delta t (\nabla p)_{cc}$$

where we could calculate the cell center pressure divergence $(\nabla p)_{cc}$, and:

$$\begin{cases} u^{n+1} = u^* - \Delta t \cdot \frac{P_E - P_W}{2\Delta x} \\ v^{n+1} = v^* - \Delta t \cdot \frac{P_N - P_S}{2\Delta y} \end{cases}$$

Update U^{n+1}, V^{n+1} :

The equation to update the U^{n+1}, V^{n+1} is showing below:

$$\vec{U}^{n+1} = \vec{U}^* - \Delta t (\nabla p)_{fc}$$

where we could calculate the face center pressure divergence $(\nabla p)_{fc}$, and:

$$\begin{cases} U_e^{n+1} = U_e^* - \Delta t \cdot \frac{P_E - P_P}{\Delta x} \\ V_n^{n+1} = V_n^* - \Delta t \cdot \frac{P_N - P_P}{\Delta y} \end{cases}$$

2.1.4 LineSOR implementation

In the discretization steps (I) and (II) of the Navier-Stokes equations, we obtain a pentadiagonal matrix system. However, our Tridiagonal Matrix Algorithm (TDMA) can only solve systems represented by tridiagonal matrices. To address this, we transfer the additional diagonal lines, e and f, to the right-hand side and continue iterating. This process is repeated until the results satisfy the original requirements of the pentadiagonal system.

The origional pentadiagonal system:

$$\begin{bmatrix} 1 & 0 & & 0 & & f & & \\ a & b & c & & & & f & \\ & a & b & c & & & & \\ & & a & b & c & & 0 & \\ 0 & & \ddots & \ddots & \ddots & & & \\ e & & & a & b & c & & \\ & e & & & a & b & c & \\ & & 0 & & 0 & 1 & & \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \vdots \\ u_{n-2} \\ u_{n-1} \\ u_n \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ \vdots \\ d_{n-2} \\ d_{n-1} \\ d_n \end{bmatrix}$$

For each line j, the system could be represented as:

$$[E \mid T \mid F] \begin{bmatrix} [u_{j-1}^{n+1}] \\ [u_j^{n+1}] \\ [u_{j+1}^{n+1}] \end{bmatrix} = [[d_j]]$$

Could say, E mans operating the previous line (j-1) to current line (j), and F means operating on the next line (j+1) to current line. Where $[E \mid T \mid F]$ is:

$$\left[\begin{array}{cccc|cccccccc|cccccc} 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & e_2 & 0 & \cdots & 0 & 0 & a_2 & b_2 & c_2 & \ddots & \vdots & 0 & 0 & f_2 & 0 & \cdots & 0 & 0 \\ 0 & 0 & e_3 & \cdots & 0 & 0 & 0 & a_3 & \ddots & \ddots & 0 & 0 & 0 & 0 & f_3 & \cdots & 0 & 0 \\ 0 & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & c_2 & 0 & 0 & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & e_{n-1} & 0 & 0 & \ddots & \ddots & 0 & b_{n-1} & c_{n-1} & 0 & 0 & 0 & \cdots & f_{n-1} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \end{array} \right]$$

LineSOR:

For LineSOR, we move the two diagonal line (e and f) to the right hand side:

$$[0 \mid T \mid 0] \begin{bmatrix} [u_{j-1}^{k+1}] \\ [u_j^{k+1}] \\ [u_{j+1}^{k+1}] \end{bmatrix} = [[d_j]] - [E \mid 0 \mid F] \begin{bmatrix} [u_{j-1}^k] \\ [u_j^k] \\ [u_{j+1}^k] \end{bmatrix}$$

Which is:

$$[T] \begin{bmatrix} [u_j^{k+1}] \end{bmatrix} = [[d_j]] - [E] \begin{bmatrix} [u_{j-1}^k] \end{bmatrix} - [F] \begin{bmatrix} [u_{j+1}^k] \end{bmatrix}$$

Where we could use TDMA to solve this equation. As this result is different from directly solve the equation we previously got, we need it keep iterating, until the error small enough:

$$Error = [E \mid T \mid F] \begin{bmatrix} [u_{j-1}^{k+1}] \\ [u_j^{k+1}] \\ [u_{j+1}^{k+1}] \end{bmatrix} - [[d_j]] < 1e-6$$

2.2 Immersed Boundary Method

2.2.1 Coefficient Correction

As our original Governing Equation as follows:

$$c_W f_W + c_S f_S + c_P f_P + c_N f_N + c_E f_E = RHS$$

Lets first consider the East point of our control point is solid point, which in two different kind of boundary condition is:

- Dirichlet condition: $f = f_F$
- Neumann condition: $\frac{\partial f}{\partial n} = f_p$

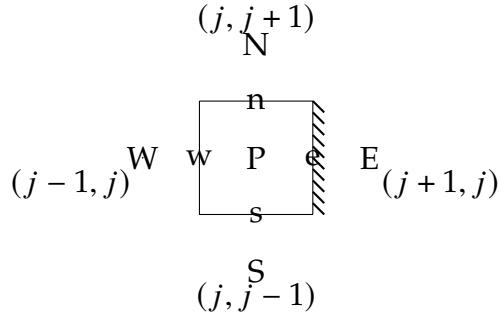


Figure 2: Illustration of a grid layout with directional points.

Dirichlet condition

For boundary at East:

$$c_W f_W + c_S f_S + c_P f_P + c_N f_N + \underbrace{c_E f_E}_{2E-f_P} = \text{RHS}$$

Where, E is the value at the east boundary. The equation then becomes:

$$\underbrace{c_W f_W + c_S f_S + c_N f_N}_{(1)} + \underbrace{(c_P - c_E) f_P}_{(2)} = \text{RHS} - 2E$$

For different boundary location:

$$(1) = iF_W \cdot c_W f_W + iF_S \cdot c_S f_S + iF_N \cdot c_N f_N + iF_E \cdot c_E f_E$$

$$(2) = [c_P - ((1 - iF_E)c_E + (1 - iF_S)c_S + (1 - iF_N)c_N(1 - iF_W)c_W)] f_P$$

Where iF is the field (array) have same shape with u , but only contains 0 (for solid point) or 1 (for fluid point) on each point, to determine whether the point is in fluid.

As the value at our inner boundary are 0, the equation then becomes:

$$(1) + (2) = \text{RHS}$$

Neumann condition

For boundary at East:

Similar, our inner boundary is 0-gradient,

$$c_W f_W + c_S f_S + c_P f_P + c_N f_N + \underbrace{c_E f_E}_{f_P} = \text{RHS}$$

Where, E is the value at the east boundary. The equation then becomes:

$$\underbrace{c_W f_W + c_S f_S + c_N f_N}_{(1)} + \underbrace{(c_P + c_E) f_P}_{(2)} = \text{RHS}$$

For different boundary location:

$$(1) = iF_W \cdot c_W f_W + iF_S \cdot c_S f_S + iF_N \cdot c_N f_N + iF_E \cdot c_E f_E$$

$$(2) = [c_P + ((1 - iF_E)c_E + (1 - iF_S)c_S + (1 - iF_N)c_N(1 - iF_W)c_W)] f_P$$

The equation then becomes:

$$(1) + (2) = RHS$$

Thus, change coefficients for N,S,E,W points is showing below::

$$c_W = iF_W \cdot c_W$$

$$c_S = iF_S \cdot c_S$$

$$c_N = iF_N \cdot c_N$$

$$c_E = iF_E \cdot c_E$$

The C_P for Dirichlet condition is showing below:

$$c_P = c_P - ((1 - iF_E)c_E + (1 - iF_S)c_S + (1 - iF_N)c_N(1 - iF_W)c_W)$$

The change coefficients for Neumann:

$$c_P = c_P + ((1 - iF_E)c_E + (1 - iF_S)c_S + (1 - iF_N)c_N(1 - iF_W)c_W)$$

2.3 Setting

2.3.1 Solver field

As in the discretization of N-S equation, we have u , v , p , and U , V fields. To handle boundary condition, we use ghost point. For convenient, we set all the fields in same size: $(N + 2) \times (N + 2)$, the u , v , p fields are:

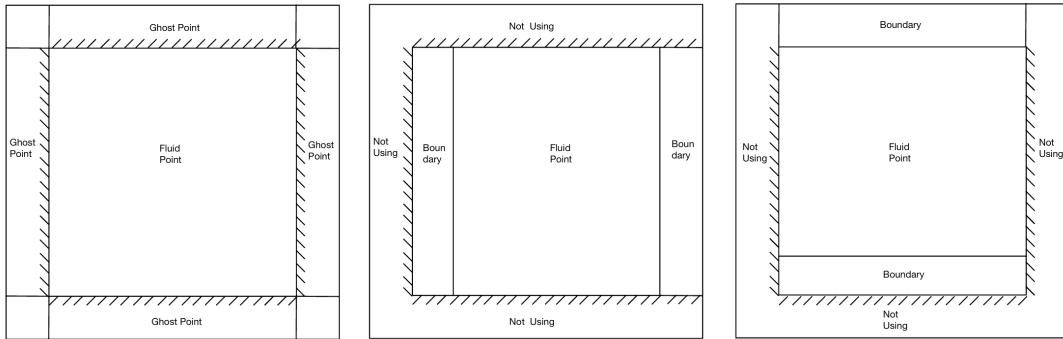


Figure 3: $u/v/p$, U , and V field

The left one is for u , v , and p field, where they are defined at cell center. It contains ghost point, which is in the solid part near to the boundary. U field is in the middle, which is 1 column, and 2 rows less than $u/v/p$ field. V field is the right one, where it is 1 row, and 2 column less than $u/v/p$ field.

2.4 Solver Architecture

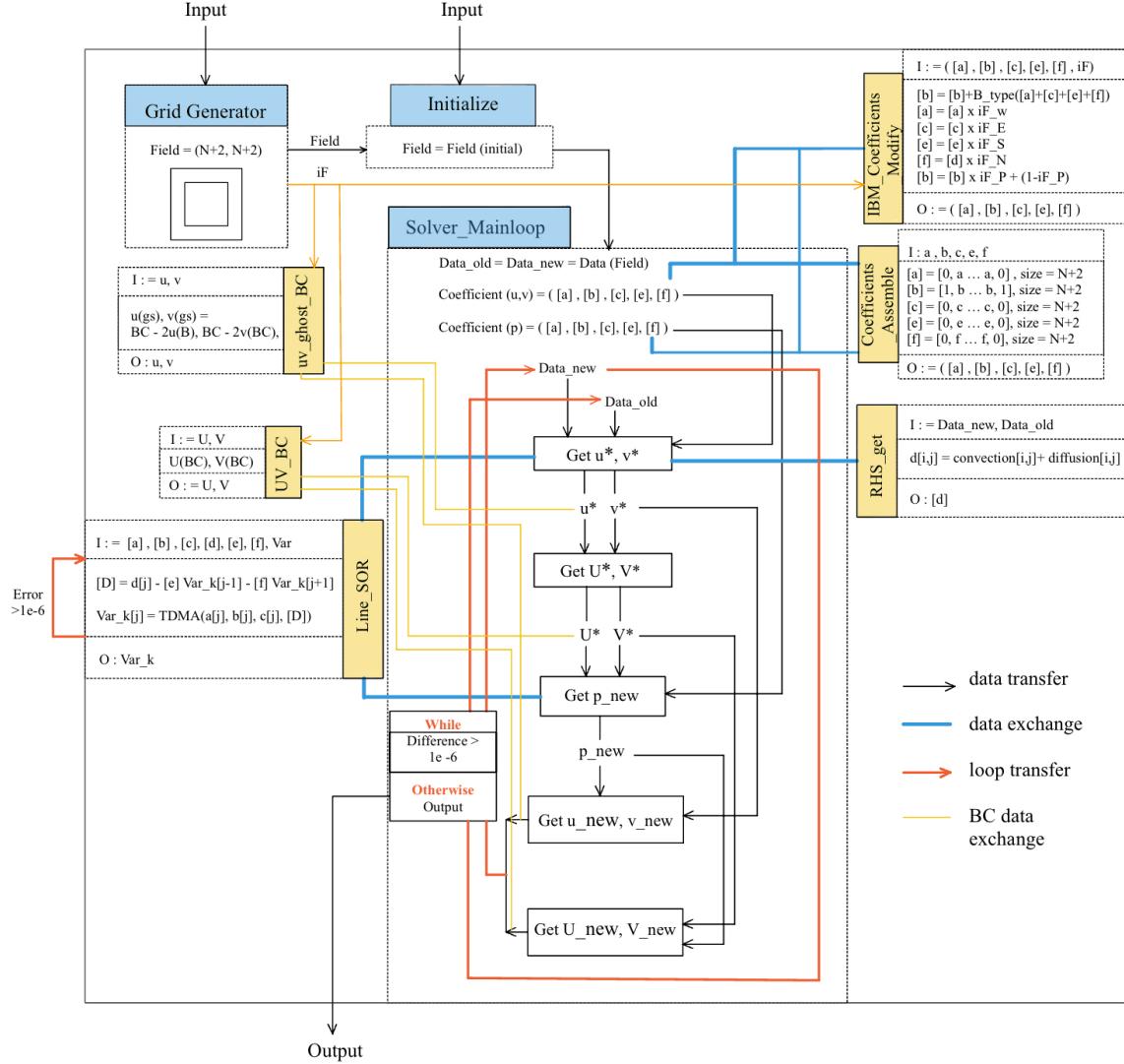


Figure 4: Solver Architecture

This diagram shows our Solver structure. By Using grid generator to generate fields for u, v, p , and U, V , transfer the data to initialize the initial value, then use the main loop to doing time iteration till the fields get to steady state, we could output the result.

- **I/O:** I means function input, and O means function output.
- **Data Transfer Lines:** The diagram uses different colored lines to indicate various types of data movement, while the lines with arrow means one-way data transfer, which means value assignment, and the lines without arrow means data exchange, where there is data input to function, but there is also data get back from function.

3 Solver Testing–Uniform and Channel Case

3.1 Uniform flow

We used the same boundary condition as the requirement shown before but without the cylinder. The simulation converges, and the uniform flow developed has been observed:

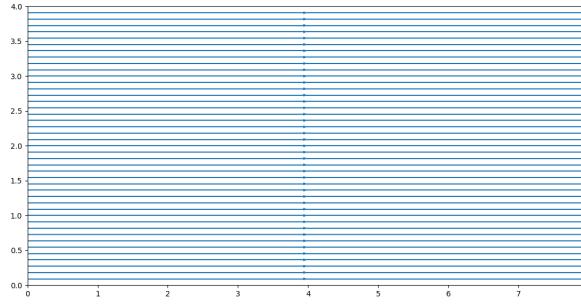


Figure 5: Streamline of the result

3.2 Channel Flow

3.2.1 Flow setting

We setup channel flow case for test our N-S solver is working well, where the setting is showing below:

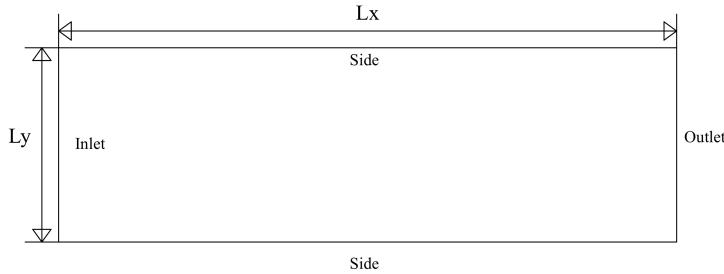


Figure 6: Domain and boundary setting for Channel flow

The Domain setting as shown above. The key parameters as follows:

- **Ly=1, Lx=10:** Width and Length of the domain
- **Inlet:** $u = 1, v = 0, \frac{\partial p}{\partial x} = 0$
- **Side:** $u = v = 0, \frac{\partial p}{\partial y} = 0$
- **Outlet:** $\frac{\partial u}{\partial x} = 0, v = 0, \frac{\partial p}{\partial x} = 0$

3.2.2 Result

The result is showing below:

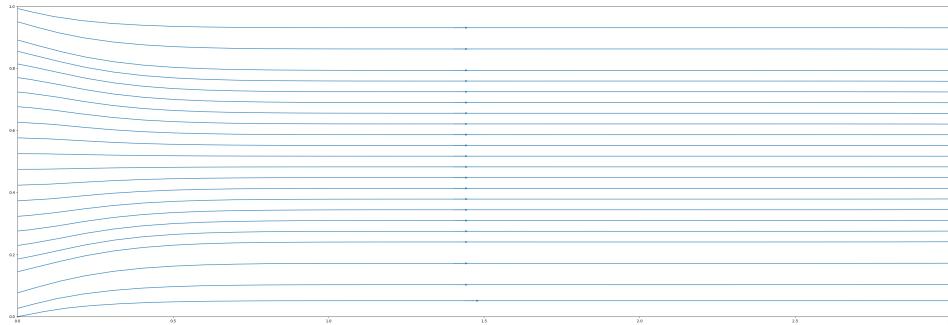


Figure 7: Streamline at $t=50s$

This diagram shows the streamline at $t=50s$, where we could see there the flow do follow our boundary condition and developed as expected.

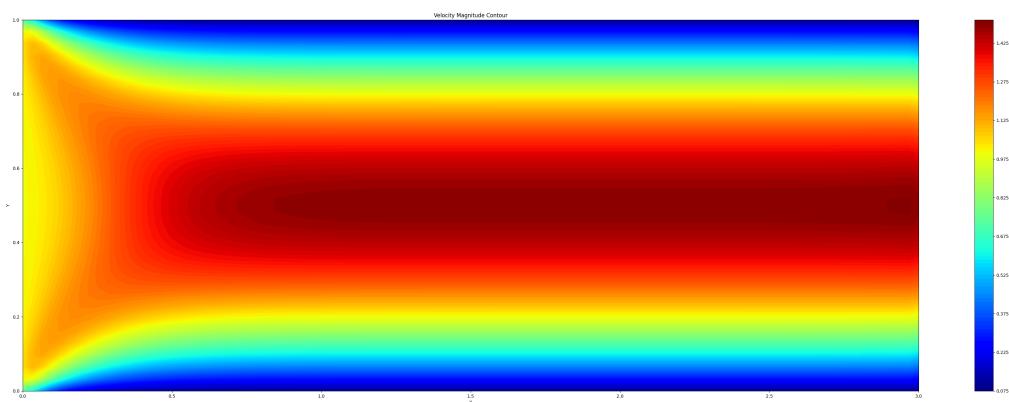


Figure 8: Velocity contour at $t=50s$

This diagram showed the velocity contour at $t=50s$, we could observe it do develop boundary layer and fully developed after certain length, and velocity have maximum value at the center of the channel.



Figure 9: Pressure contour

3.2.3 Comparison with theoretical solution

To check our result's precision, we checked our result with the theoretical solution, where the channel flow's theoretically solution is showing below:

$$u_{\text{exact}} = U \left(1 - \left(\frac{y - 0.5}{0.5} \right)^2 \right)$$

We compare this result with our numerical result, could find our numerical result is close to the exact solution:

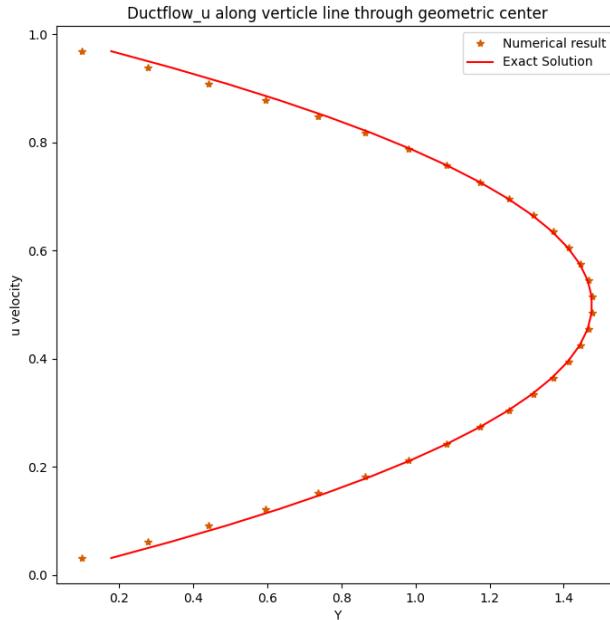


Figure 10: u comparison

4 Re=150–Circular cylinder in a cross-flow

4.1 Settings

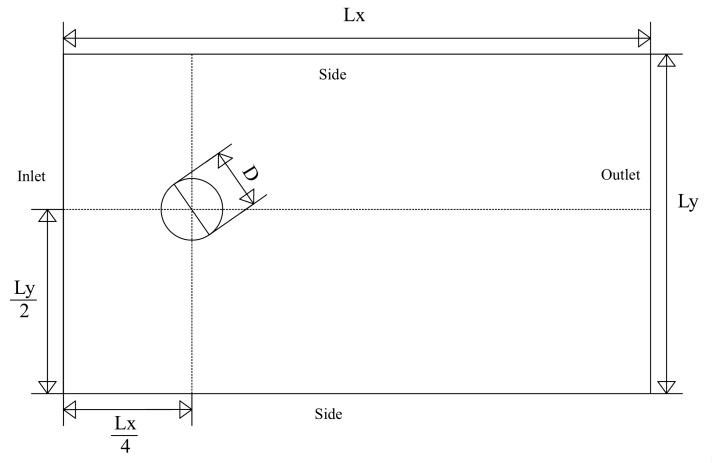


Figure 11: Domain and boundary setting for circular cylinder

The Domain setting as shown above. The key parameters as follows:

- **Ly=4, Lx=8:** Width and Length of the domain
- **Circle Center:** the location of circular cylinder center= $(\frac{L_x}{4}, \frac{L_y}{2})$
- **D=0.5:** The diameter of the circle
- **Inlet:** $u = 1, v = 0, \frac{\partial p}{\partial x} = 0$
- **Side:** $\frac{\partial u}{\partial y} = 0, \frac{\partial v}{\partial y} = 0, \frac{\partial p}{\partial y} = 0$
- **Outlet:** $\frac{\partial u}{\partial x} = 0, \frac{\partial v}{\partial x} = 0, \frac{\partial p}{\partial x} = 0$

4.2 Result among time

4.2.1 t=10 and t=50

By setting the parameters as the chapter shown before, we now exhibit the result of $t=10$ and $t=100$ as follows:

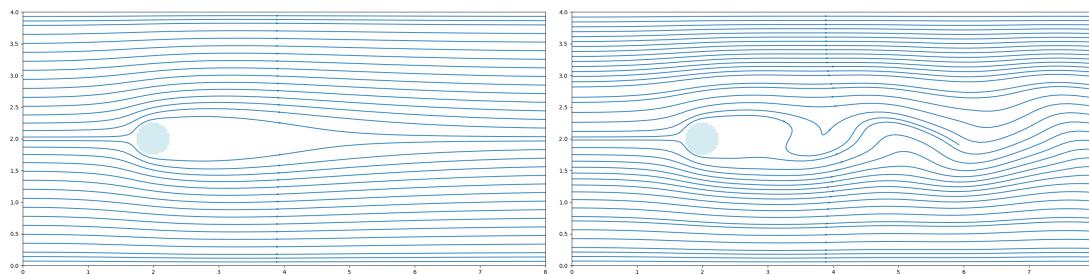


Figure 12: Streamline at $t=10$ (left), Streamline at $t=10$ $t=50s$ (right)

Based on the figures shows for $t=10$ and $t=50$, we could find at $t=10$, the flow is kind of steady, where it shows the flow have not been disturbed, and flow pattern is symmetric along y-direction.

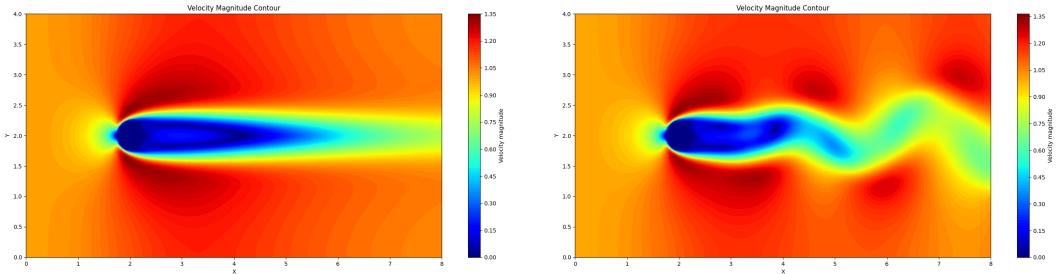


Figure 13: Velocity at $t=10$ (left), Pressure at $t=10$ $t=50s$ (right)

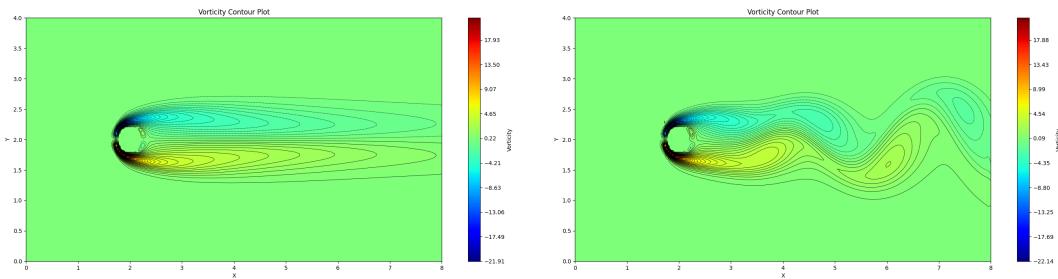


Figure 14: Vorticity at $t=10$ (left), Vorticity at $t=10$ $t=50s$ (right)

The same result shown in vorticity contour and pressure contour, where we could find the flow is not symmetric and start to show vortices.

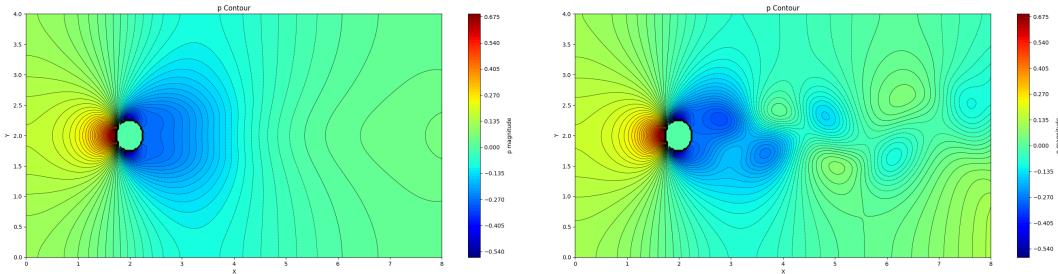


Figure 15: Pressure at $t=10$ (left), Pressure at $t=50s$ (right)

4.2.2 $t=50$ and $t=100$

By setting the parameters as the chapter shown before, we now exhibit the result of $t=50$ and $t=100$ as follows:

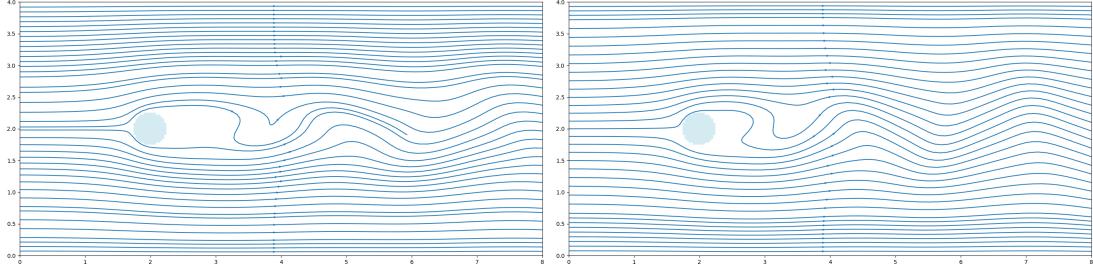


Figure 16: Streamline at $t=50$ (left), Streamline at $t=100s$ (right)

From $t=50$ to $t=100$, we could also find that the vortex is shedding from cylinder and let the flow more unsteady.

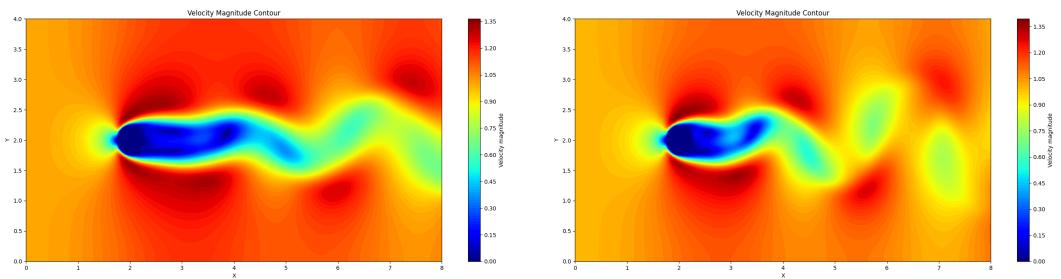


Figure 17: Velocity at $t=50$ (left), Pressure at $t=100s$ (right)

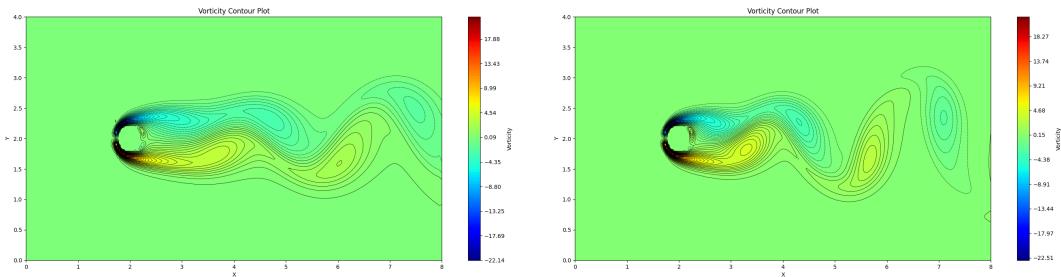


Figure 18: Vorticity at $t=50$ (left), Vorticity at $t=100s$ (right)

From these result, we could see small vortices lying up along y-center side.

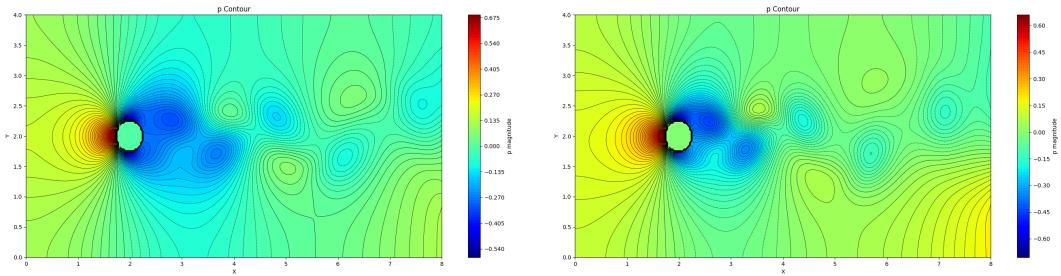


Figure 19: Pressure at $t=50$ (left), Pressure at $t=100s$ (right)

From the figure of pressure contour, we could noticed that the pressure shows its maximum at the front point of the circle, and the minimum (negative) at the back of the circle. While the vortex shedding showing up, we could find a similar pattern of the vortex, velocity and pressure contour arrangement.

4.2.3 Result Comparison of time (t=10, 50, and 100)

To get more comprehensive understanding of the vortex shedding among time, we let the result of t=10, t=50, and t=100 together as follows:

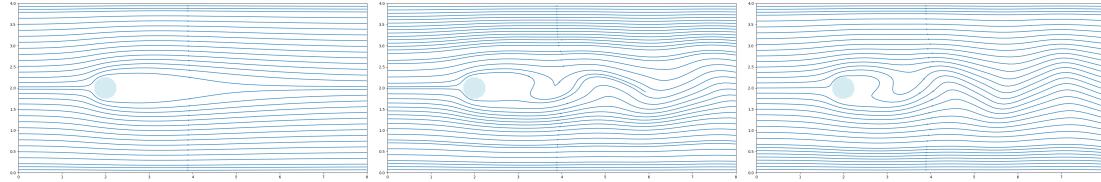


Figure 20: Streamline at t=10 (left), t=50 (middle), t=100s (right)

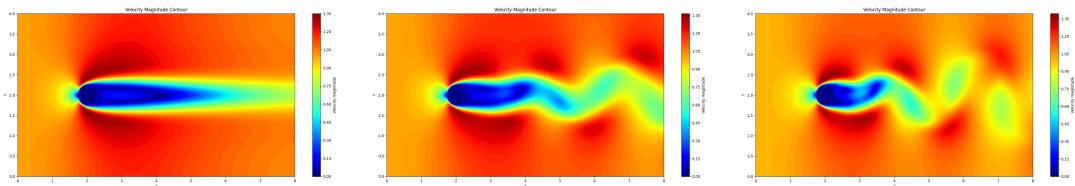


Figure 21: Velocity at t=10 (left), t=50 (middle), t=100s (right)

At t=100, we could observe the periodic behaviour typical of vortex shedding.

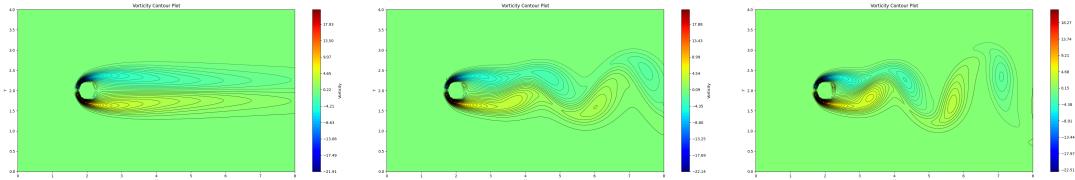


Figure 22: Vorticity at t=10 (left), t=50 (middle), t=100s (right)

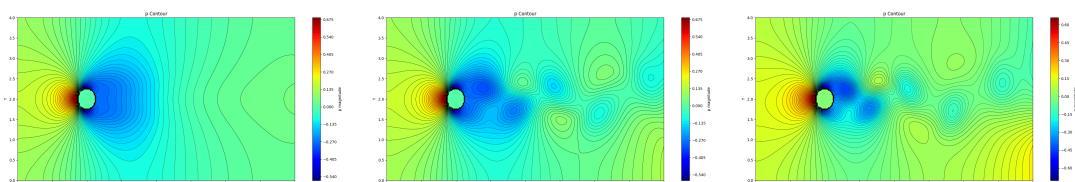


Figure 23: Pressure at t=10 (left), t=50 (middle), t=100s (right)

Here we put the result among time together. Where it exhibit clearly, that the appearance of vortex shedding is the result of revolution of vortex due to the circular cylinder.

From streamline plots, pressure disturbance, to vorticity maps, we could observe the evolution from initial flow around the cylinder to the distinct formation of a Kármán vortex street. From t=10 seconds to t=100 seconds, the pressure and vorticity maps reveal changes in the intensity and positioning of these vortices.

4.3 Re=150 Result Value and Comparison

4.3.1 Drag (c_D) and Lift (c_L) coefficient

For large time scale, we sampling 1 second once, the result is showing below:

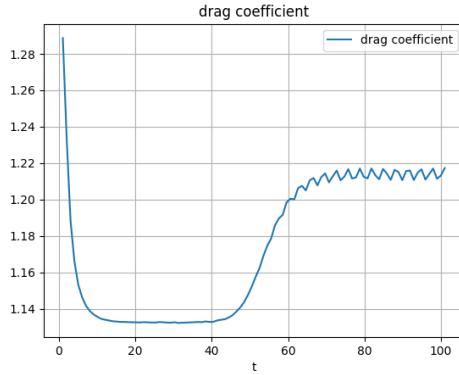


Figure 24: c_D among long time

Based on this figure, we could find the flow become steady after $t=70$. To increase our result's accuracy, we sampling around $t=100$ and $t=110$, sampling 0.01s once. The result is showing below:

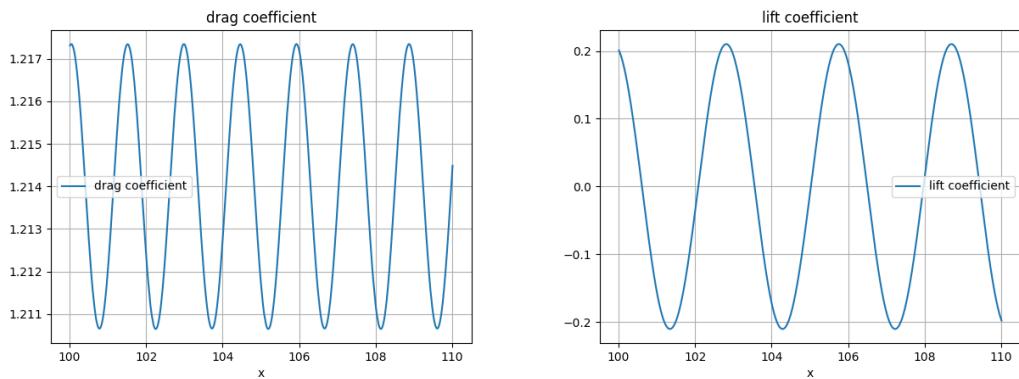


Figure 25: c_L and c_D among time

We could get the mean drag coefficient (c_D):

$$c_D = 1.212$$

Also, we could get lift coefficient (c_L):

$$c_L = 0 \pm 0.21$$

4.3.2 Vortex Shedding frequency

By sampling the result of drag and lift coefficient (c_D and c_L), and using Fast Fourier Transform (FFT), we could get the vortex shedding frequency:

$$f = 0.34$$

Based on that, we could calculate Strouhal number (St):

$$St = \frac{fD}{U} = 0.17$$

4.3.3 Result value compare with literature

According to Norberg (2003)[2], the relationship of St number with Re is shown below:

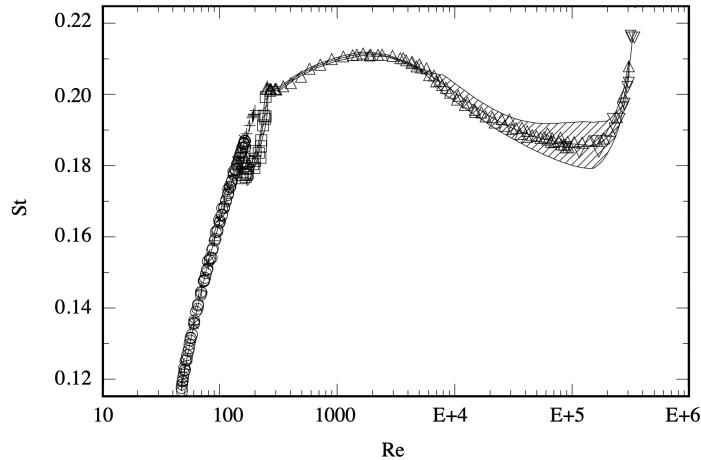


Figure 26: St number with Re (Norberg, 2003[2])

As our result shown St number is 0.166 at $Re=150$, we could say our result matched. The more detailed result is from appendix of Norberg's review (2003)[2]:

From Re in range of (47,190):

$$St = 0.2663 - \frac{1.019}{\sqrt{Re}}$$

Which shows $St = 0.183$ at $Re = 150$, which is also closed to our result.

We also compared with Qu (2013)[3], where shows $St = 0.184$, and $c_{D_p} = 1.02$. Compare with our $c_D = 1.21$, shows our result may not exactly accurate at calculating drag coefficient.

The total comparison table is showing below:

Table 1: Result comparison with Qu(2003)[3]

$Re=150$	c_{D_p}	c_L	St
Result	1.212	0 ± 0.21	0.17
Qu[3]	1.02	0.35	0.184

5 Influence of Domain Size: 8×4 VS. 8×2

5.1 Result Comparison among time

To examine the influence of grid size, we narrow our domain at y-axis, make our domain from 8×4 to 8×2 .

5.1.1 $t=50$ and $t=100$ comparison

The following figures shown below is the vorticity and velocity contour of 8×2 (upper) and 8×4 (lower):

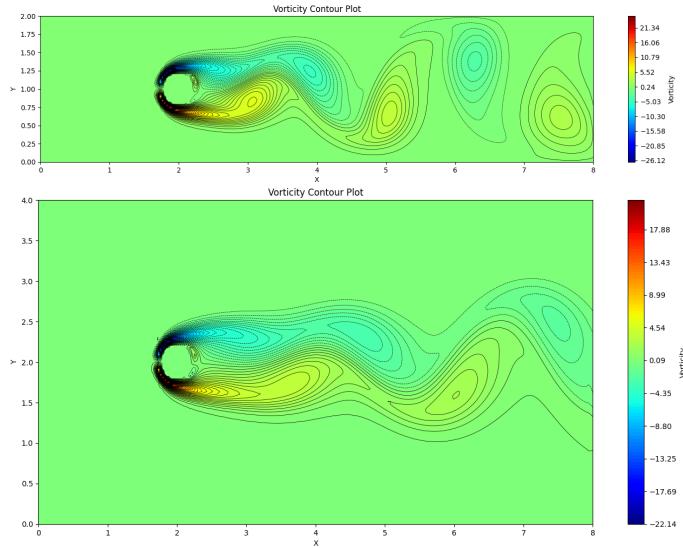


Figure 27: Vorticity at $t=50$

We could find as the domain has been narrowed, the evolution of the flow seems been accelerated, which shows vorticities shedding out of cylinder at $t=50$, domain size 8×2 , but still only starting to been disturbed $t=50$, domain size 8×4 .

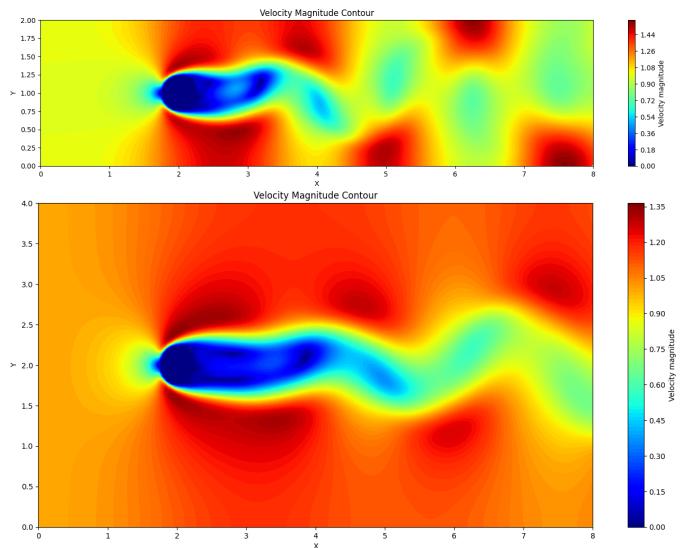


Figure 28: Velocity at $t=50$

We could find the domain 8×2 is not enough for the flow simulation, where its boundary is too close, where the vortices touched the boundary. The flow evolution been speed up, where you can see the vortex shedding.

5.2 Drag (c_D) and Lift (c_L) coefficient

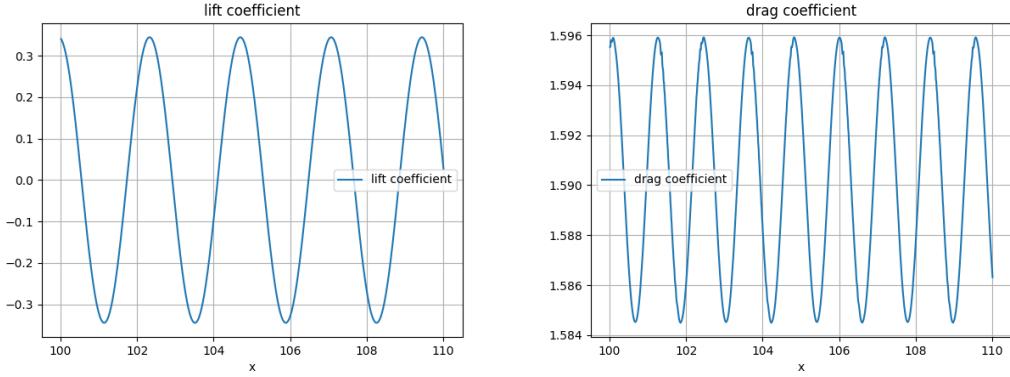


Figure 29: c_L and c_D among time

We could get the mean drag coefficient (c_D):

$$c_D|_{8 \times 2} = 1.588$$

5.2.1 Vortex Shedding frequency

By sampling the result of drag and lift coefficient (c_D and c_L), and using Fast Fourier Transform (FFT), we could get the vortex shedding frequency:

$$f|_{8 \times 2} = 0.4$$

Based on that, we could calculate Strouhal number (St):

$$St|_{8 \times 2} = \frac{fD}{U}|_{8 \times 2} = 0.2$$

5.3 Comparison of domain size 8×4 with 8×2

We could compare our narrow domain (8×2) with our result shown in the previous chapter:

Table 2: Result comparison of domain size's effect

Domain Size	c_D	c_L	St
8×4	1.212	0 ± 0.21	0.17
8×2	1.588	0 ± 0.35	0.2

We could find, after we narrowed the domain size, the result value (c_D and c_L) become larger, which may because as we narrowed the domain size, the vortex become

much easier to touch the side boundary of the domain, and been mirrored back as we use the symmetric boundary condition. This may cause the flow developed much more faster.

6 Influence of Grid Resolution: $\Delta=1/32$ VS. $\Delta=1/16$

To examine how the grid resolution could influence our result, we use the coarse grid $\Delta=1/16$.

6.1 $\Delta=1/32$ VS. $\Delta=1/16$ at different time

The result shown below is $\Delta = 1/16$ (left), and $\Delta=32$ (right). From the $\Delta=1/16$ figures, we could find this kind of grid is coarse, where we could easily observe the "stairs" at the edge of the circle.

6.1.1 $t=50$

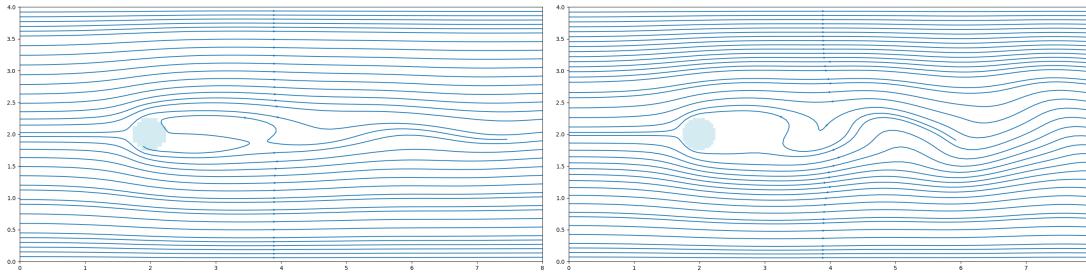


Figure 30: Streamline of $\Delta=1/16$ (left), $\Delta=1/32$ (Right)

Based on the streamline and velocity contour comparison of the $\Delta=1/16$ with $\Delta=1/32$, we could notice that the flow's development seems been slowed down at $\Delta=1/16$ grid, that the flow is still not been hugely disturbed at $t=50 \Delta=1/16$, but for $\Delta=1/32$ result at $t=50$, is already starting to show the oscillation.

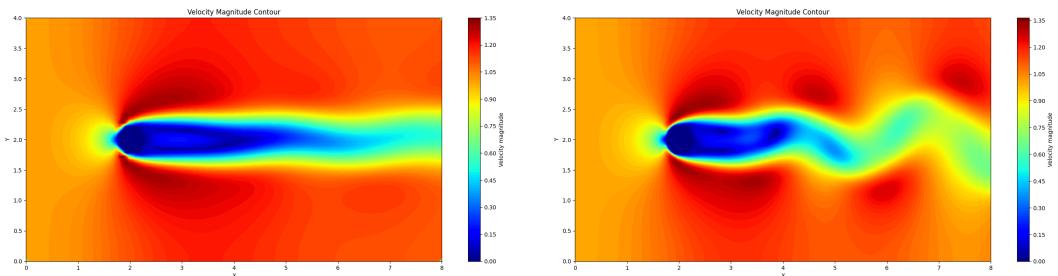


Figure 31: Velocity of $\Delta=1/16$ (left), $\Delta=1/32$ (Right)

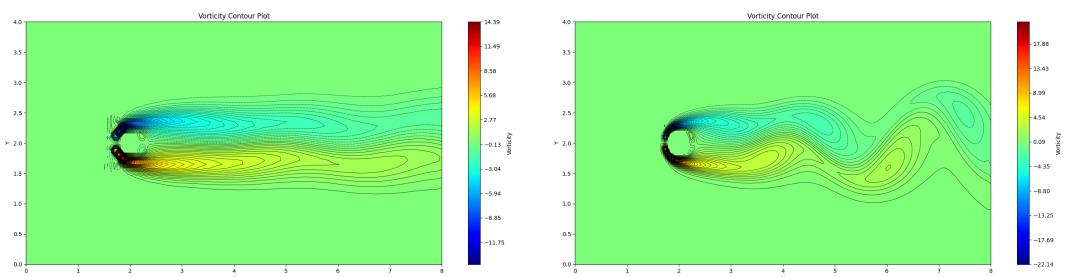


Figure 32: Vorticity of $\Delta=1/16$ (left), $\Delta=1/32$ (Right)

The similar result shown in the vorticity and pressure contour, where the flow is much un-disturbed at of $\Delta=1/16$.

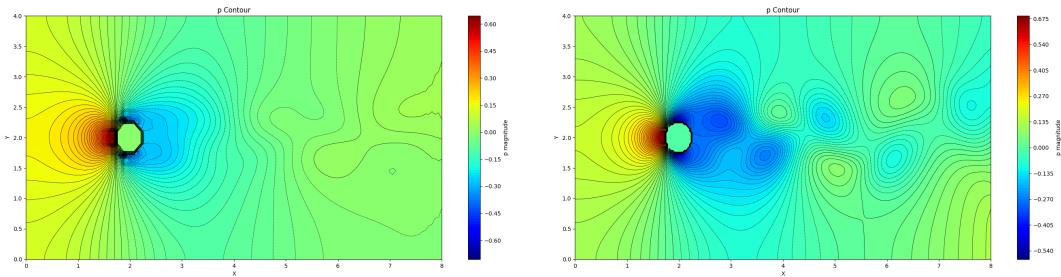


Figure 33: Pressure of $\Delta=1/16$ (left), $\Delta=1/32$ (Right)

Based on the comparison of the different grid resolution, we could find that the flow evolution seems been slowed down as the grid become coarse. This may due to the added numerical viscosity at coarse grid.

6.1.2 t=100

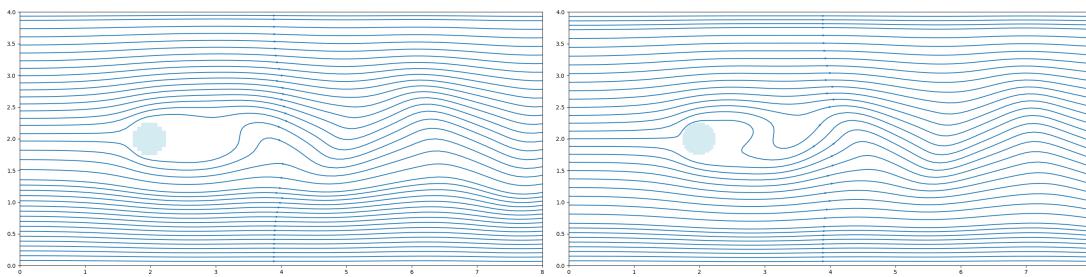


Figure 34: Streamline of $\Delta=1/16$ (left), $\Delta=1/32$ (Right)

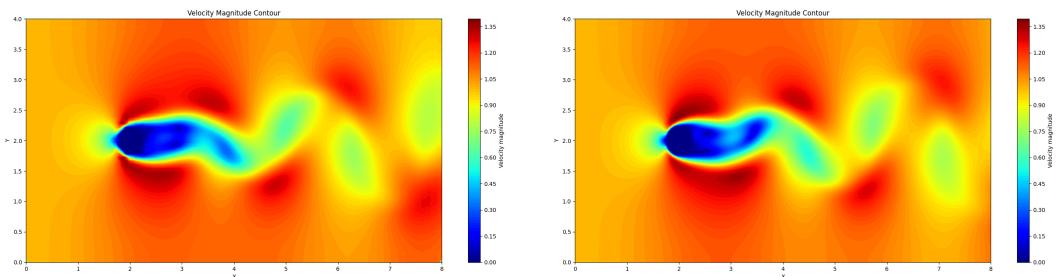


Figure 35: Velocity of $\Delta=1/16$ (left), $\Delta=1/32$ (Right)

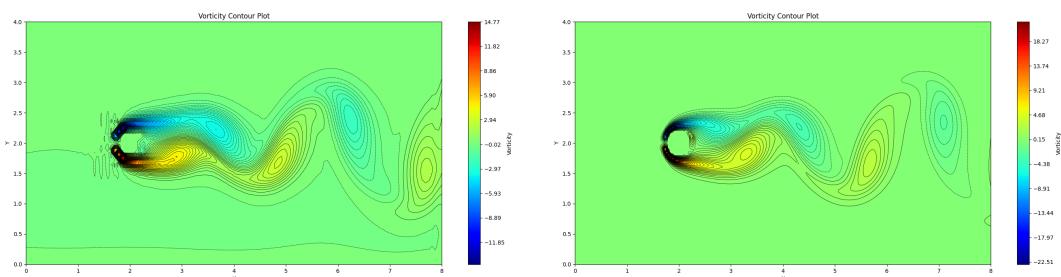


Figure 36: Vorticity of $\Delta=1/16$ (left), $\Delta=1/32$ (Right)

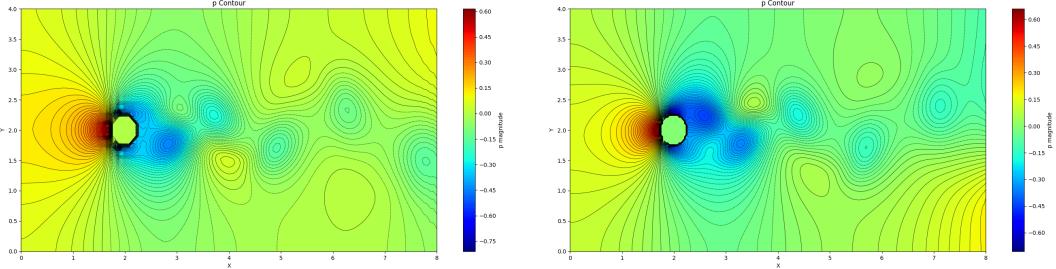


Figure 37: Pressure of $\Delta=1/16$ (left), $\Delta=1/32$ (Right)

Based on the result, we could get the conclusion: The narrow domain could accelerate the flow evolution, and the coarse grid could slow down the flow development among time due to the large numerical viscosity.

6.2 Drag (c_D) and Lift (c_L) coefficient

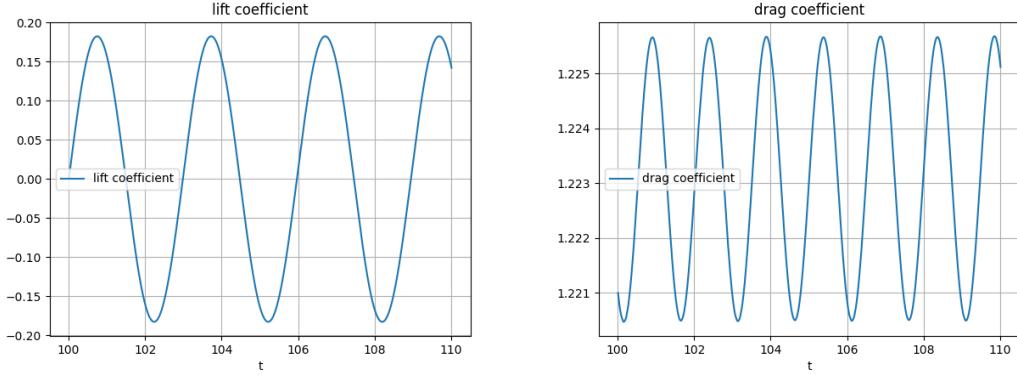


Figure 38: c_L and c_D among time

We could get the mean drag coefficient (c_D):

$$c_D = 1.22$$

6.2.1 Vortex Shedding frequency

By sampling the result of drag and lift coefficient (c_D and c_L), and using Fast Fourier Transform (FFT), we could get the vortex shedding frequency:

$$f = 0.3$$

Based on that, we could calculate Strouhal number (St):

$$St = \frac{fD}{U} = 0.15$$

6.3 Result comparison of grid resolution: 1/32 and 1/16

Table 3: Result comparison of grid resolution effect

Grid Size	c_D	c_L	St
1/32	1.212	0 ± 0.21	0.17
1/16	1.22	0 ± 0.18	0.15

We could find that as we use the coarse grid, the result (c_L) is little bit smaller. As we also could notice that the flow developed much more slower than the finer grid, which may because as we use coarse grid, we make the fluid cannot catch the number of small part's of the circle cylinder's influence of the flow, and make the numerical viscosity much higher, which could influence the result.

7 Re=300 Result

To observe Re number effect for circular cylinder cross-sectional flow, we simulate the Re=300 case, and compare with Re=150 result which is shown at the previous section. The result is showing below:

7.1 Re=300 VS. Re=150 at different time

The result of Re=300 compared with Re=150 result is showing below:

7.1.1 t=50

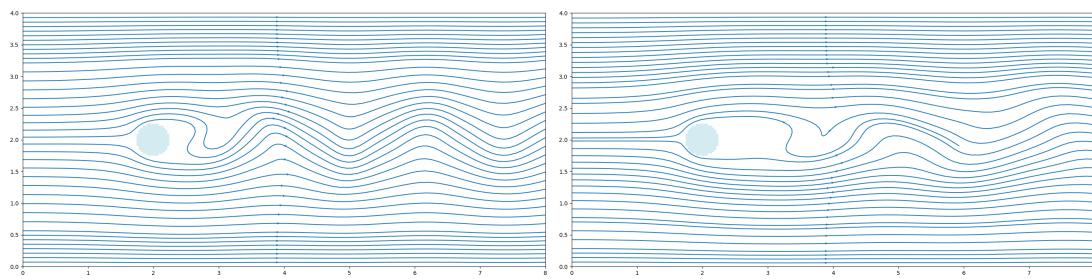


Figure 39: Streamline for Re=300 (left), Re=150 (right)

We could notice for the higher Re number, the development for the flow seems been speed up at Re=300, which could already observe vortex shedding at t=50, where for Re=150 the flow is just stating to oscillate.

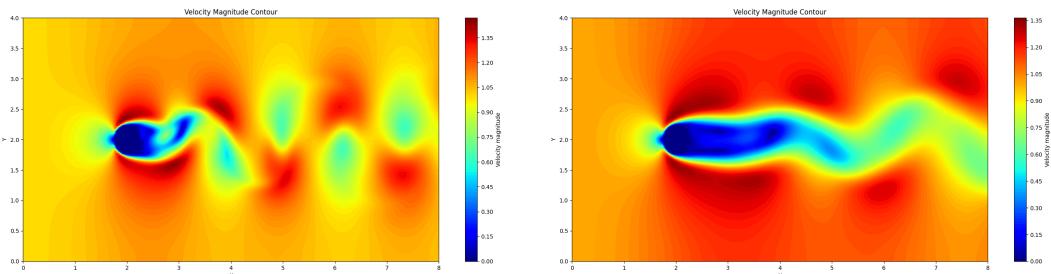


Figure 40: Velocity for Re=300 (left), Re=150 (right)

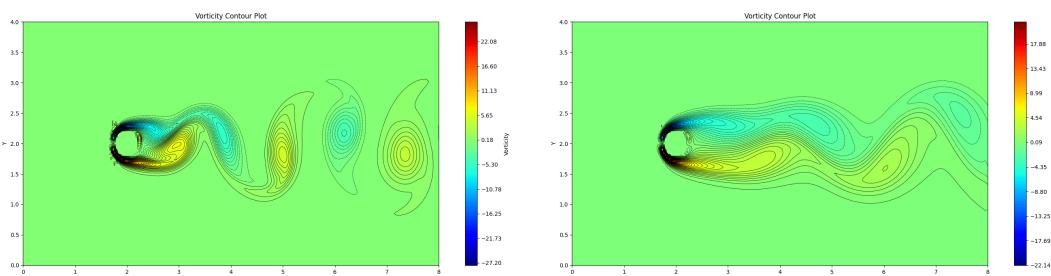


Figure 41: Vorticity for Re=300 (left), Re=150 (right)

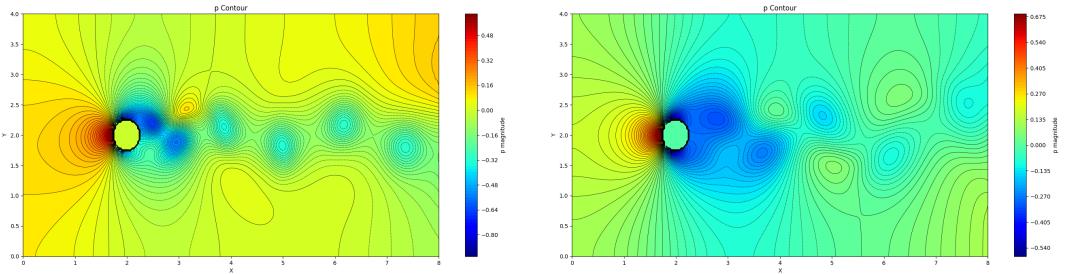


Figure 42: Pressure for $Re=300$ (left), $Re=150$ (right)

7.1.2 $t=100$

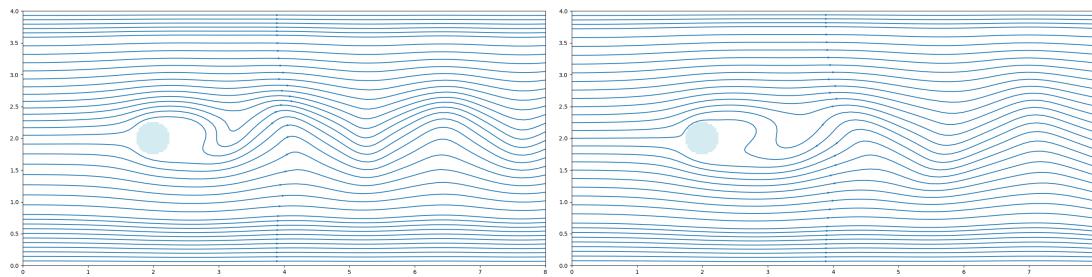


Figure 43: Streamline for $Re=300$ (left), $Re=150$ (right)

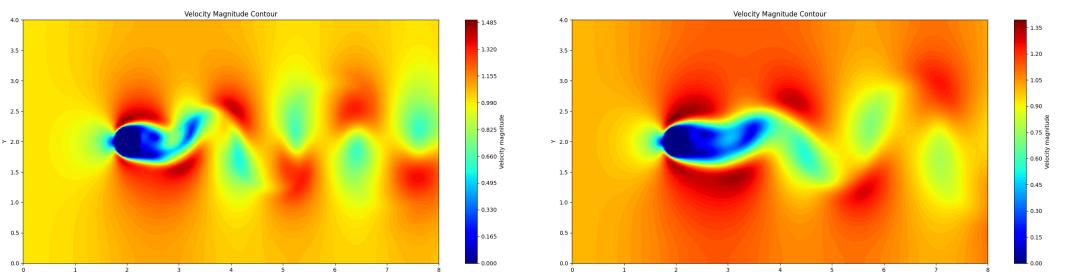


Figure 44: Velocity for $Re=300$ (left), $Re=150$ (right)

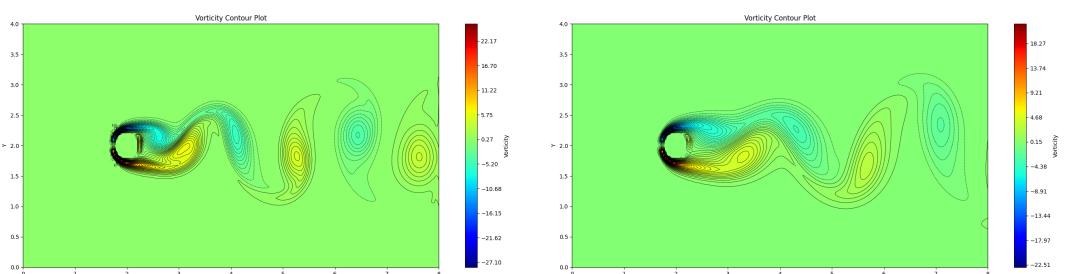


Figure 45: Vorticity for $Re=300$ (left), $Re=150$ (right)

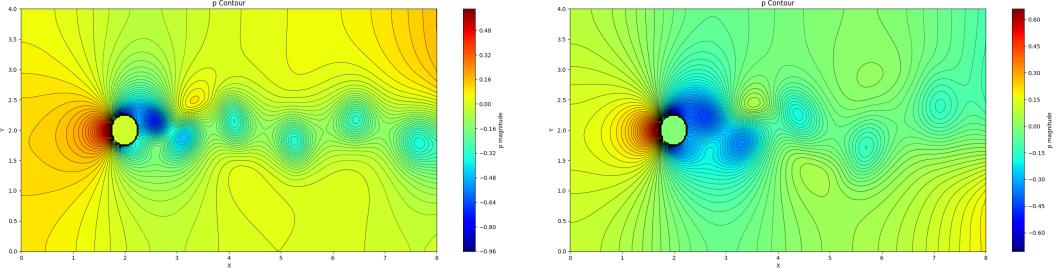


Figure 46: Pressure for $Re=300$ (left), $Re=150$ (right)

7.2 $Re=300$ Result Value and Comparison

7.2.1 Drag (c_D) and Lift (c_L) coefficient

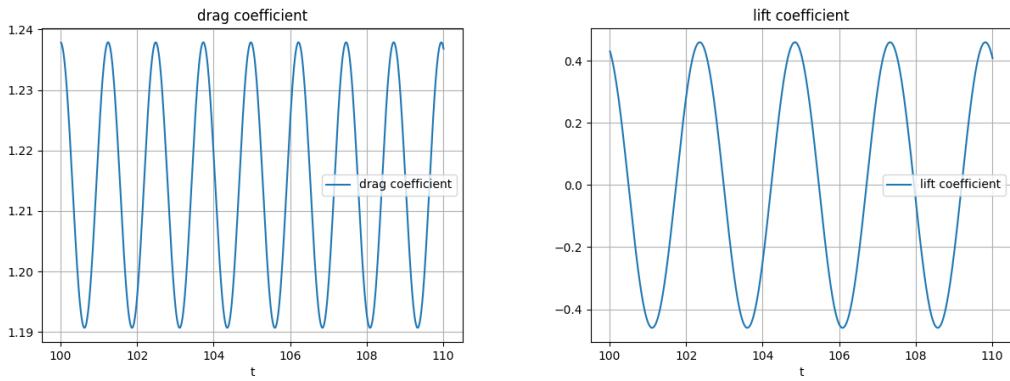


Figure 47: c_L and c_D among time

We could get the mean drag coefficient (c_D):

$$c_D = 1.21$$

Also, we could get lift coefficient (c_L):

$$c_L = 0 \pm 0.46$$

7.2.2 Vortex Shedding frequency

By sampling the result of drag and lift coefficient (c_D and c_L), and using Fast Fourier Transform (FFT), we could get the vortex shedding frequency:

$$f = 0.4$$

Based on that, we could calculate Strouhal number (St):

$$St = \frac{fD}{U} = 0.2$$

7.2.3 Result value compare with literature

According to Franke(1990)[1], the result of St at $Re = 300$: $St = 0.205$, where our result is $St = 0.2$, which is close.

Table 4: Result comparison with literature

	c_{D_p}	c_{L_p}	St
Re=300	1.21	0 ± 0.46	0.2
Franke[1]	1.11	0 ± 0.77	0.205

8 Re=1000 Result

To observe Re number effect for circular cylinder cross-sectional flow, we simulate the Re=1000 case, and compare with Re=150 result which is shown at the previous section. The result is showing below:

8.1 Re=1000 VS. Re=150 at different time

The following figures shows Re=1000 result (left), and Re=150 result (right):

8.1.1 t=50

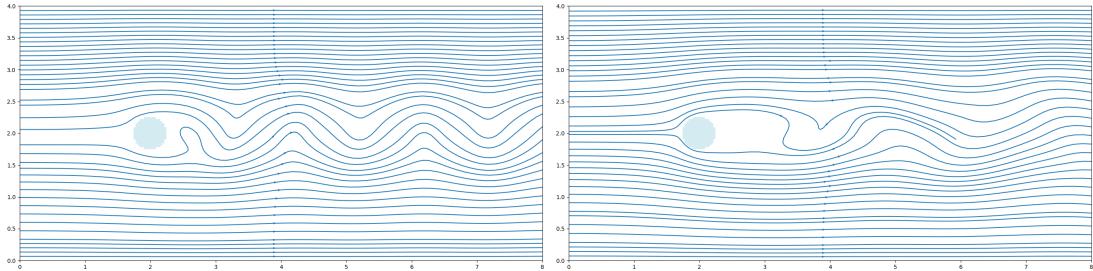


Figure 48: Streamline for Re=300 (left), Re=150 (right)

Based on the streamline and velocity contour, we could clearly observe vortex shedding from circular cylinder at Re=1000, t=50. But for Re=150, t=50, we could find the flow is only starting to been disturbed, but not shown vortex shedding yet.

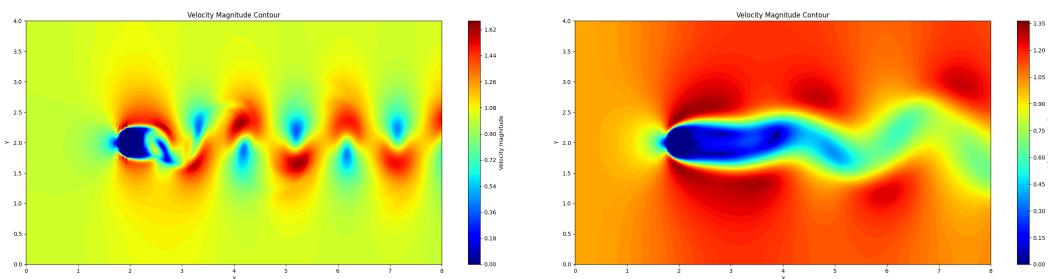


Figure 49: Velocity contour for Re=300 (left), Re=150 (right)

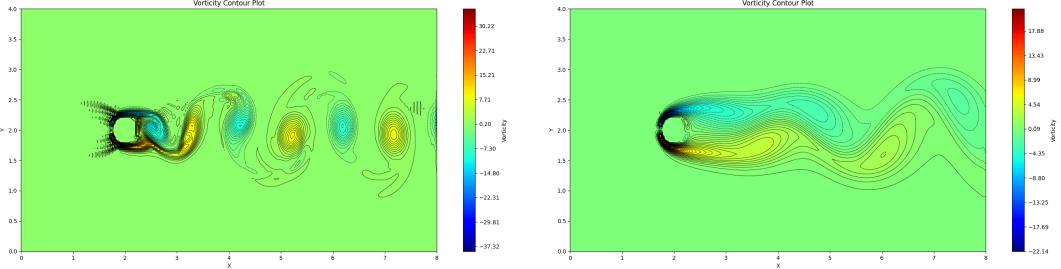


Figure 50: Vorticity for $\text{Re}=300$ (left), $\text{Re}=150$ (right)

From vorticity and pressure contour, we could get the similar observation: the vortex shedding is already shown up at $\text{Re}=1000$, but not clearly shown up at $\text{Re}=150$, $t=50$.

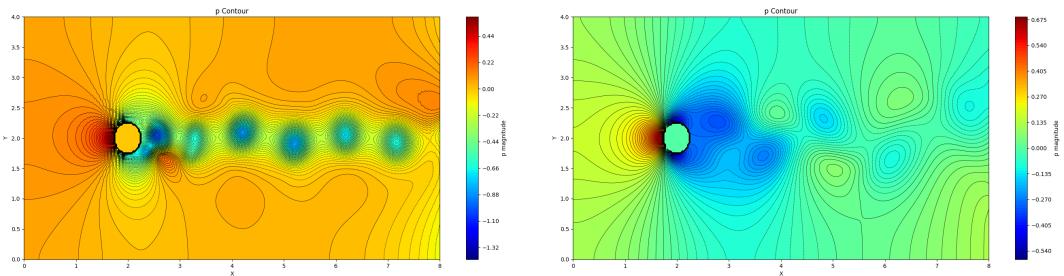


Figure 51: Pressure for $\text{Re}=300$ (left), $\text{Re}=150$ (right)

8.1.2 $t=100$

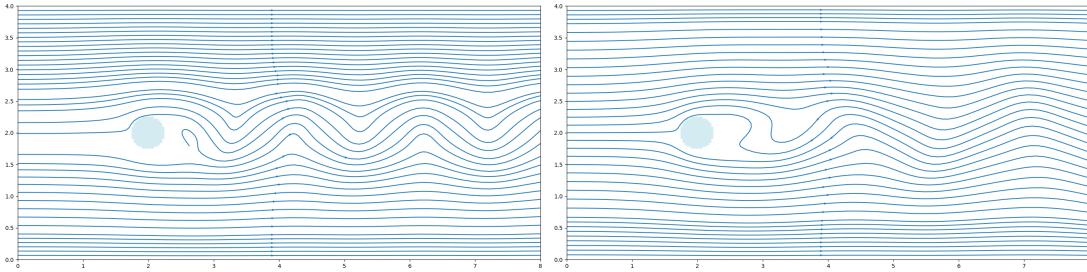


Figure 52: Streamline for $\text{Re}=300$ (left), $\text{Re}=150$ (right)

We could also notice that for $\text{Re}=1000$, the vortices kindly lying up, which neatly aligned after the circular cylinder.

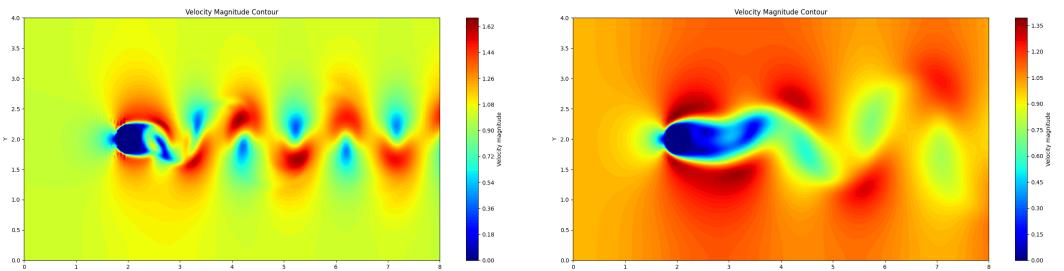


Figure 53: Velocity for $\text{Re}=300$ (left), $\text{Re}=150$ (right)

The similar result shown at vorticity and pressure contour, which we could see there vorticities and pressure extremum points arranged neatly.

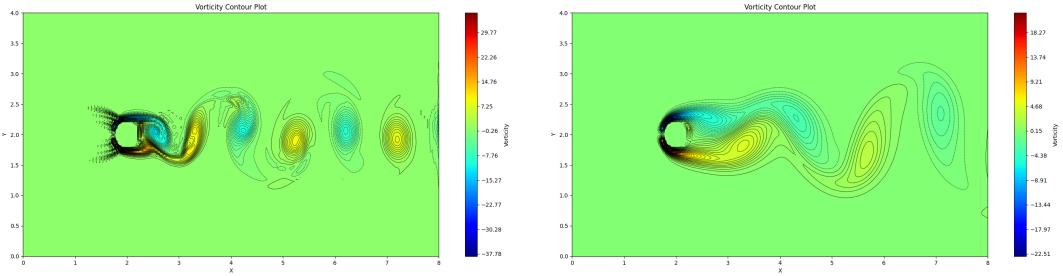


Figure 54: Vorticity for Re=300 (left), Re=150 (right)

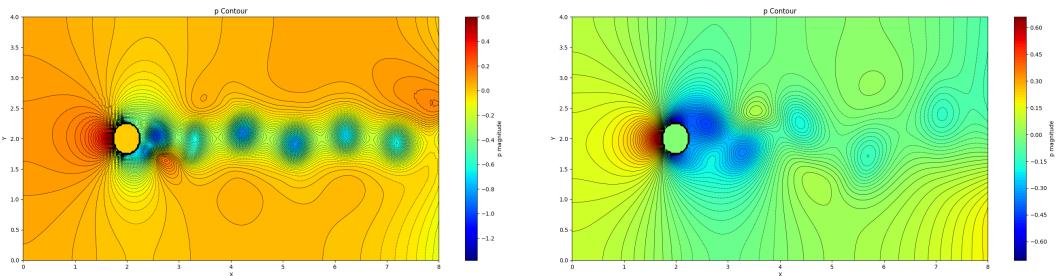


Figure 55: Pressure for Re=300 (left), Re=150 (right)

8.2 Re=1000 Result Value and Comparison

8.2.1 Drag (c_D) and Lift (c_L) coefficient

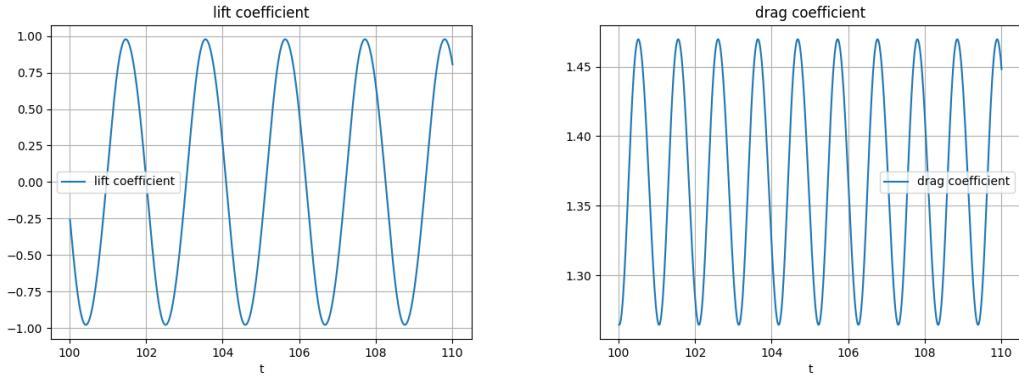


Figure 56: c_L and c_D among time

We could get the mean drag coefficient (c_D):

$$c_D = 1.36$$

Also, we could get lift coefficient (c_D):

$$c_L = 0 \pm 0.97$$

8.2.2 Vortex Sheding frequency

By sampling the result of drag and lift coefficient (c_D and c_L), and using Fast Fourier Transform (FFT), we could get the vortex shedding frequency:

$$f = 0.4$$

Based on that, we could calculate Strouhal number (St):

$$St = \frac{fD}{U} = 0.2$$

8.2.3 Result value compare with literature

According to Qu(2003)[3], the result of St at $Re = 1000$: $St = 0.2365$, where our result is $St = 0.2$, could say our result is close.

Table 5: Comparison with Qu(2003)[3] result

	c_{D_p}	c_{L_p}	St
Re=1000	1.36	0 ± 0.97	0.2
Qu[3]	1.34	0 ± 1.30	0.236

8.3 Comparison and discussion of Re number effect on result

C_D and C_L comparison

Table 6: Result comparison of Re effect

Re	c_{D_p}	c_{L_p}	St
Re=150	1.212	0 ± 0.21	0.17
Re=300	1.21	0 ± 0.46	0.2
Re=1000	1.36	0 ± 0.97	0.2

Based on the comparison, we could find for higher Re , the c_L 's maximum value will be higher, and c_D may have the same effect but not obviously in our result.

8.3.1 Comparison among time

To have a more comprehensive understanding of the Re number's effect of our result, we put $Re=1000$ (left), $Re=300$ (middle), and $Re=150$ (right)'s figures together to check how Re could influence the simulation outcome:

t=50:

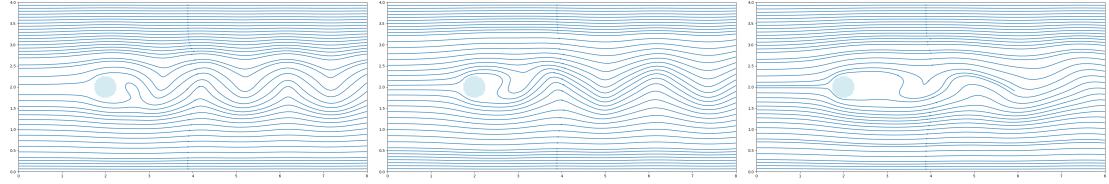


Figure 57: Streamline for $Re=1000$ (left), $Re=300$ (middle), $Re=150$ (right)

Based on the streamline and velocity contour comparison of $Re=1000$, $Re=300$, and $Re=150$, we could find that as Re become higher, the vortices evolution seems been speed up, and its become aligned up at Re become higher

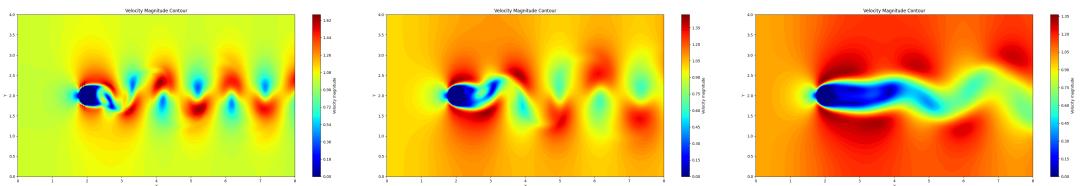


Figure 58: Velocity contour for $Re=1000$ (left), $Re=300$ (middle), $Re=150$ (right)

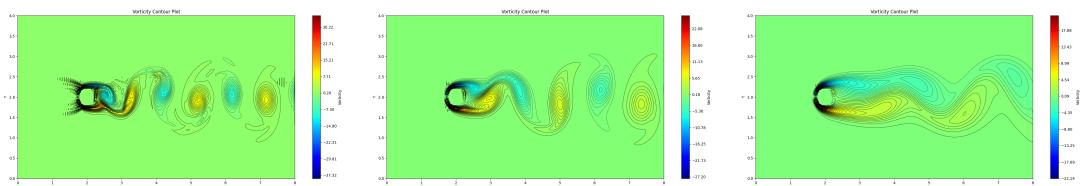


Figure 59: Vorticity for $Re=1000$ (left), $Re=300$ (middle), $Re=150$ (right)

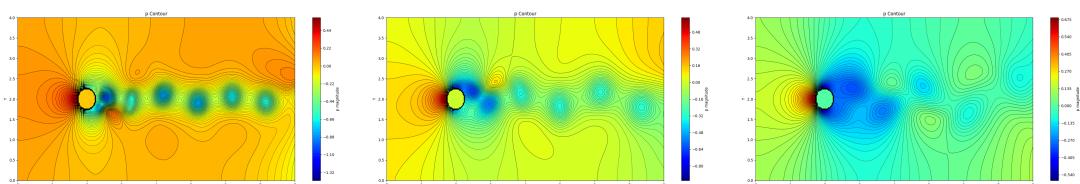


Figure 60: Pressure for $Re=1000$ (left), $Re=300$ (middle), $Re=150$ (right)

t=100:

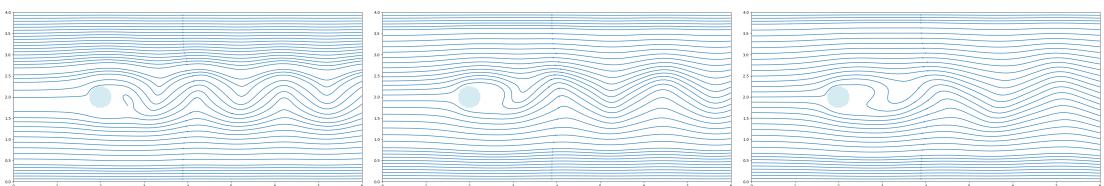


Figure 61: Streamline for $Re=1000$ (left), $Re=300$ (middle), $Re=150$ (right)

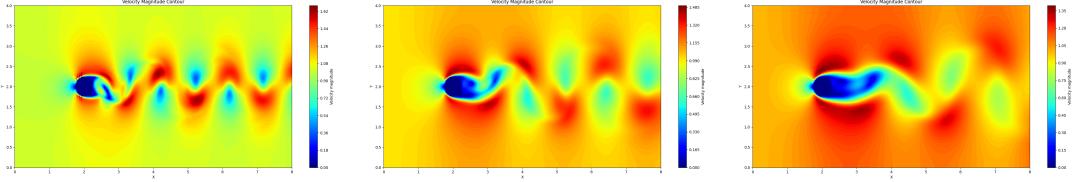


Figure 62: Velocity for Re=1000 (left), Re=300 (middle), Re=150 (right)

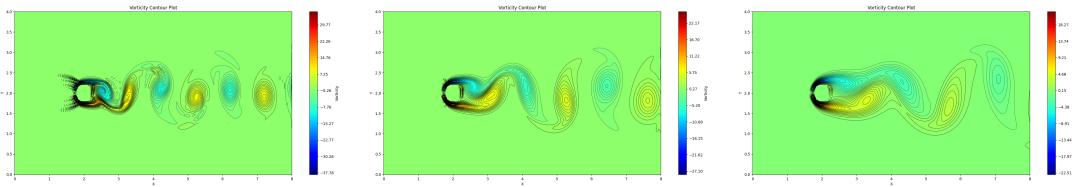


Figure 63: Vorticity for Re=1000 (left), Re=300 (middle), Re=150 (right)

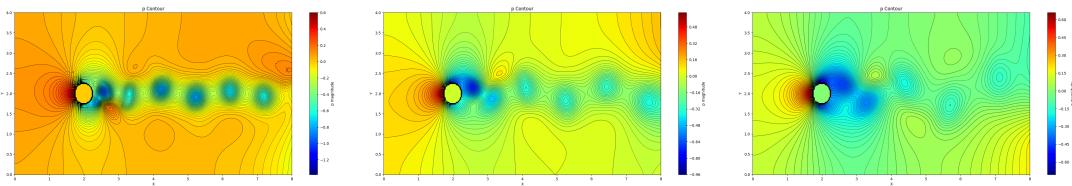


Figure 64: Pressure for Re=1000 (left), Re=300 (middle), Re=150 (right)

Based on the Re result comparison among time, we could find that as we increase Re number, the flow develop become much faster, which make the vortex shedding show up earlier.

We also could observe that as the Re number increase, the vortices begin to align neatly after the circular cylinder.

9 Re=300, Elliptic cylinder

In this section, we replaced our circular cylinder with 3:1 elliptic cylinder, and its angle of attack at 30 degrees, and Re=300.

9.1 Elliptic Setting

Based on the requirement shown above, our ellipse control equation as follows:

$$\frac{((x - x_0) \cos \theta_0 + (y - y_0) \sin \theta_0)^2}{a^2} + \frac{(-(x - x_0) \sin \theta_0 + (y - y_0) \cos \theta_0)^2}{b^2} = 1$$

Where, θ_0 is the angle of the ellipse rotation. In our case, $\theta_0 = -30$ to full fill our angle-of-attack requirement.

The domain setting is as follows:

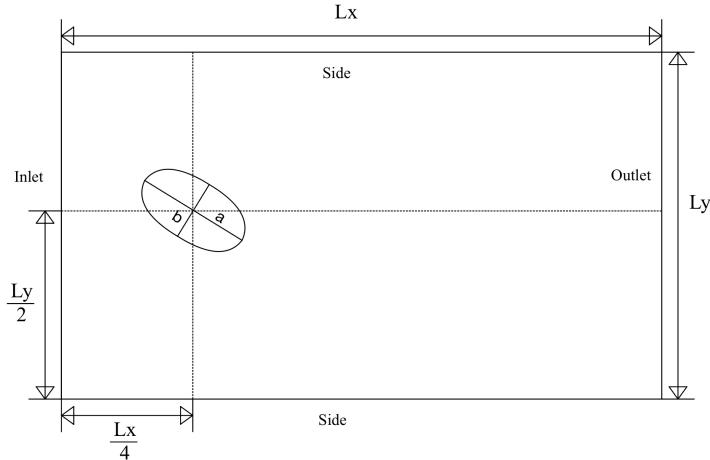


Figure 65: Domain and boundary setting for Elliptic cylinder

The Domain setting as shown above. The key parameters as follows:

- **Ly=4, Lx=8:** Width and Length of the domain
- **Ellipse Center:** the location of Elliptic cylinder center= $(\frac{Lx}{4}, \frac{Ly}{2})$
- **a=0.6, b=0.3:** The long, and short axis of the ellipse.
- **Inlet:** $u = 1, v = 0, \frac{\partial p}{\partial x} = 0$
- **Side:** $\frac{\partial u}{\partial y} = 0, \frac{\partial v}{\partial y} = 0, \frac{\partial p}{\partial y} = 0$
- **Outlet:** $\frac{\partial u}{\partial x} = 0, \frac{\partial v}{\partial x} = 0, \frac{\partial p}{\partial x} = 0$

9.2 t=50 and t=100

By setting the parameters as the chapter shown before, we now exhibit the result of t=50 and t=100 as follows:

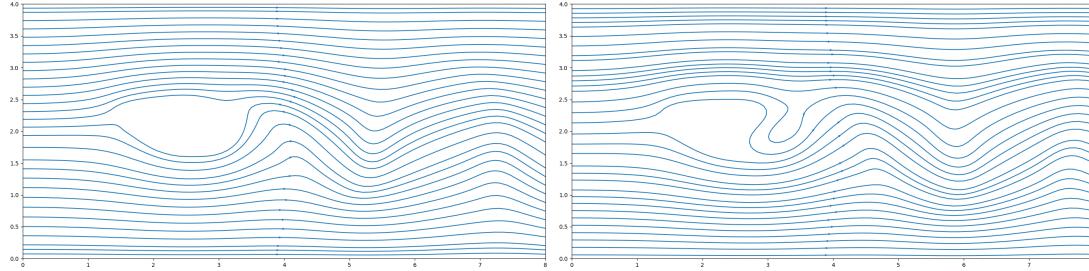


Figure 66: Streamline at t=50 (left), t=100s (right)

Based on the streamline and velocity contour, we could find the

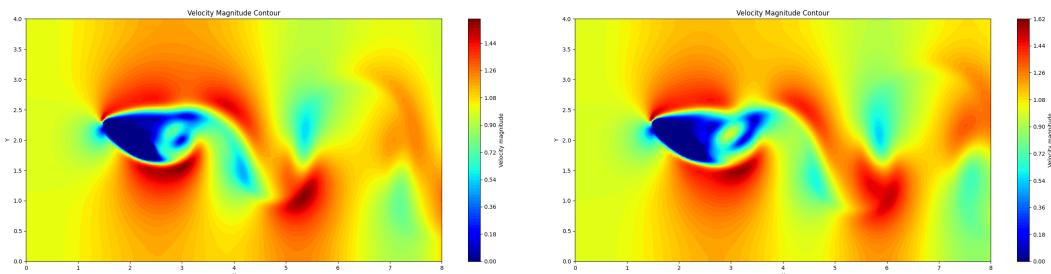


Figure 67: Velocity at t=50 (left), t=100s (right)

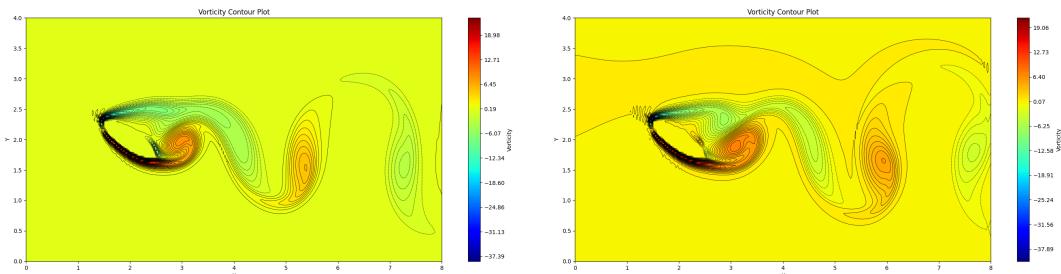


Figure 68: Vorticity at t=50 (left), t=100s (right)

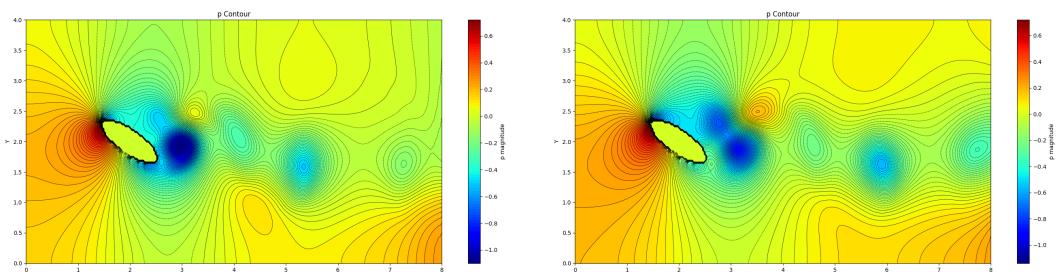


Figure 69: Pressure at t=50 (left), t=100s (right)

9.2.1 Comparison among time ($t=5, 10, 50, 100$)

As we could already observe vortex shedding showing up at $t=50$, and $t=100$, we here plot the result among more time steps:

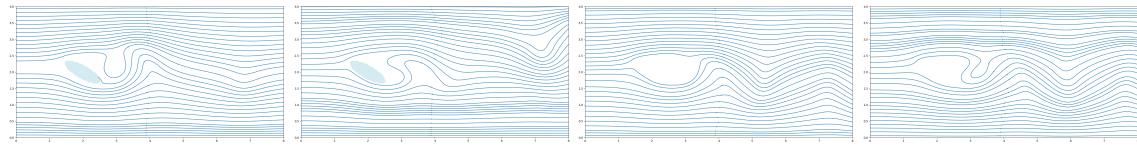


Figure 70: Streamline at $t=2$ (left), $t=100s$ (right)

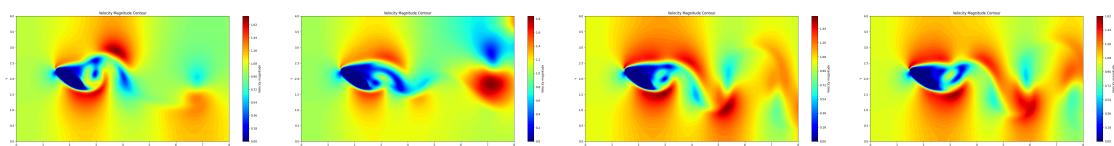


Figure 71: Velocity at $t=5$ (left), $t=10$ (middle left), $t=50$ (middle right) $t=100s$ (right)

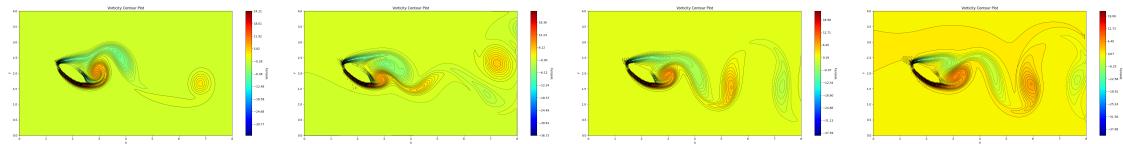


Figure 72: Vorticity at $t=5$ (left), $t=10$ (middle left), $t=50$ (middle right) $t=100s$ (right)

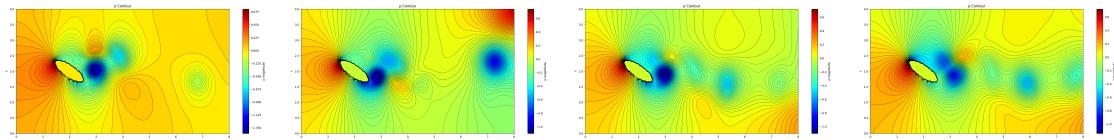


Figure 73: Pressure at $t=5$ (left), $t=10$ (middle left), $t=50$ (middle right) $t=100s$ (right)

9.3 Elliptic Result Value and Comparison

9.3.1 Drag (c_D) and Lift (c_L) coefficient

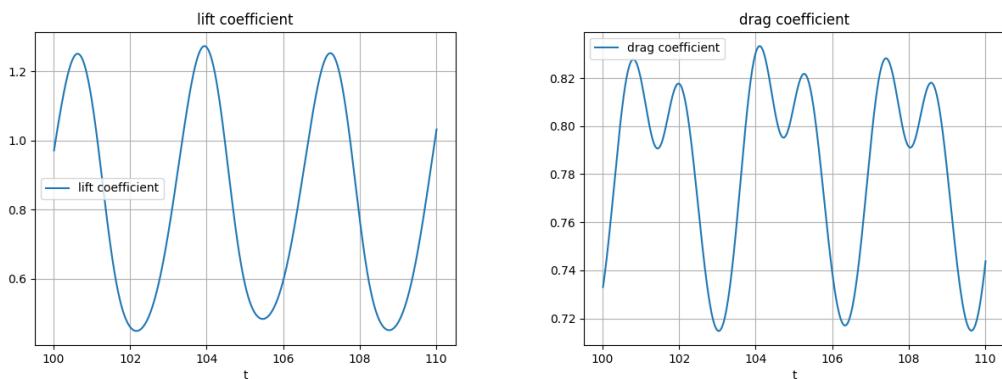


Figure 74: c_L and c_D among time

We could get the mean drag coefficient (c_L):

$$c_L = 0.81$$

Also, we could get mean lift coefficient (c_D):

$$c_D = 0.76$$

9.3.2 Vortex Shedding frequency

By sampling the result of drag and lift coefficient (c_D and c_L), and using Fast Fourier Transform (FFT), we could get the vortex shedding frequency:

$$f = 0.2$$

Based on that, we could calculate Strouhal number (St):

$$St = \frac{fD}{U} = 0.36$$

10 Discussion and Conclusion

10.1 Review of the result

In this project, we discretized N-S equation and applied to our N-S solver. The solver perform well in uniform flow and channel flow, which its result matched therodially result. We applied our solver to the circular cylinder cross sectional flow, it shows vortex shedding after several time steps.

10.1.1 Re=150, domain size= 8×4 , grid resolution $\Delta = 1/32$

We compared our result for different time scale. We could observe the vortex generating among time, and dislodged from circular cylinder. Among time, the flow been disturbed significantly.

10.1.2 Re=150, domain size= 8×2 , grid resolution $\Delta = 1/32$

In order to explore how could the domain size influence our result, we simulate the same flow in domain size 8×2 , which is **Re=150, domain size= 8×2 , grid resolution $\Delta = 1/32$** case:

For this case, we compared our result with 8×4 case, we could find that the vortex evolution been speed up, which could see the vortex shedding appear early, and been squeezed as the vortex touched the sides of the domain.

10.1.3 Re=150, domain size= 8×4 , grid resolution $\Delta = 1/16$

In order to explore how could grid resolution can influence our result, we also tried a coarse grid. We let grid size $\Delta = 1/16$, which is **Re=150, domain size= 8×4 , grid resolution $\Delta = 1/16$** case:

In this case, we could find that the flow evolution seems been slowed down, compare with $\Delta = 1/32$ grid resolution. This may due to the numerical viscosity which been added by our discretization, and as the grid become coarser, the viscosity increased, make the vortex hard to shed.

10.1.4 Re=300, domain size= 8×4 , grid resolution $\Delta = 1/32$

To discuss Re number's influence of the result of the flow, we use Re=300 and compare with Re=150 case:

In this case, we could find the vortex generated earlier, and the flow become more disturbed for Re=300, compare with Re=150 case at the same time scale. The vortex shedding appearance seems been speed up.

10.1.5 Re=1000, domain size= 8×4 , grid resolution $\Delta = 1/32$

To discuss Re number's influence of the result of the flow more, we also tried Re=1000 case

The vortex generating and shedding out of the circular cylinder seems been more speed up, and we also could notice that vortex lying up, neatly aligned after the circular cylinder.

We put $Re=1000$, $Re=300$, and $Re=150$ cases together to get a more comprehensive understanding of the vortex shedding. We could find as Re number become higher, the vortex shedding appear earlier, and the vortex aligned up more neatly.

10.1.6 Elliptic Case: $Re=300$, domain size= 8×4 , grid resolution $\Delta = 1/32$

In this case, we replaced our circular cylinder to the elliptic cylinder, and we could observe the vortex been generated at the lower right side of the cylinder, and among time, the vortex become more exhibit.

10.2 Result and Observation

	Re 150	Re 300	Re 1000	8x2 Narrow Domain	1/6 Coarse Grid	Re 150 [3]	Re 300 [1]	Re 1000 [3]
C_D	1.212	1.21	1.36	1.588	1.22	1.02	1.11	1.34
C_L	± 0.21	± 0.46	± 0.97	± 0.35	± 0.18	± 0.35	± 0.77	± 1.30
St	0.17	0.2	0.2	0.2	0.15	0.184	0.205	0.236

Table 7: Total Comparison: (Reference: Qu(2003)[3], Franke(1990)[1])

Based the observation and comparison, we could find that for small domain, the flow evolution is much faster, to make the vortex shedding show up earlier, increase Re number have the similar effect. Decrease grid resolution, in the other hand, have the opposite effect, which could slow down the vortex generation, however, it could not prevent vortex generation.

10.3 Discussion

Based the observation and comparison, we could find that for small domain, the vortex shedding occur earlier, which may because of the the flow been squeezed, and as we use the symmetric boundary condition at the side of domain ($\frac{\partial u}{\partial n}=0$), the flow touched the side of domain and been mirrored back, make the flow layering together, make the vortex show up earlier.

We found increase Re number have the similar effect, where Re number been increase, the flow become much easier to be disturbed, and make the vortex generate earlier.

Based on the comparison of coarse grid comparison, we found as we use large grid size (poor resolution), the flow evolution become slower. This may because of the grid size make the flow harder to been disturbed by the cylinder, and make the wave easier to transfer to the edge of the domain, make the flow evolution much slower.

However, coarse grid cannot prevent the vortex shedding, as we finally observed the vortex shedding after the circular cylinder.

10.4 Conclusion

In this project, we discretized the Navier-Stokes equations to build and validate an N-S solver, which was then applied to flow simulations around circular and elliptical cylinders. At a Reynolds number (Re) of 150, we compared our results with existing literature and confirmed that our results are both sound and reliable.

To analyze how domain size, grid resolution, and Reynolds number influence our numerical results, we adjusted the domain size from 8×4 to 8×2 , and modified the grid resolution from $\Delta = 1/32$ to $\Delta = 1/16$. We also explored the outcomes at $Re = 300$ and $Re = 1000$.

We observed that smaller domain sizes accelerated the flow evolution, leading to earlier vortex shedding. This phenomenon is likely due to the constrained flow being compressed, which accelerates disturbances within the fluid. Similarly, increasing the Reynolds number enhances the flow's susceptibility to disturbances, thereby promoting earlier vortex formation. These effects are consistent across different configurations, confirming the impact of domain size and Reynolds number on the observed fluid dynamics.

Conversely, reducing the grid resolution had an inhibitory effect on the rate of flow evolution. A coarser grid introduces greater numerical viscosity due to our discretization method, which damps fluid disturbances and delays the development of vortices. However, even with reduced resolution, vortex shedding cannot be entirely prevented, only delayed. This indicates that while grid resolution influences the speed of flow disturbances, it does not alter the fundamental dynamics responsible for vortex shedding.

References

- [1] R. Franke, W. Rodi, and B. Schönung. Numerical calculation of laminar vortex-shedding flow past cylinders. *Journal of Wind Engineering and Industrial Aerodynamics*, 35:237–257, 1990.
- [2] C. Norberg. Fluctuating lift on a circular cylinder: review and new measurements. *Journal of Fluids and Structures*, 17(1):57–96, 2003.
- [3] Lixia Qu, Christoffer Norberg, Lars Davidson, Shia-Hui Peng, and Fujun Wang. Quantitative numerical analysis of flow past a circular cylinder at reynolds number between 50 and 200. *Journal of Fluids and Structures*, 39:347–370, 2013.

Appendix

The Project Data and python scripts are located at: [Github link \(click this\)](#)
Our basic Solver code is showing below:

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3 plt.ioff()
4 import copy
5 import re
6
7 class mesher:
8     def __init__(self, data):
9         self.Lx = data["Lx"]
10        self.Ly = data["Ly"]
11        self.dxy = data["dx"]
12
13    def parameter_grid(self): # generate large grid
14        Field1 = grid_generator( self.Ly, self.Lx, self.dxy)
15        N_rows, N_cols,a = np.shape(Field1)
16
17        # save location y,x in field[j,i,0],
18        for j in range(0, N_rows):
19            for i in range(0,N_cols):
20                Field1[j,i,0] , Field1[j,i,1] = (j-1/2)*self.dxy, (i-1/2) *
21 self.dxy
22                # save fluid(1)/Solid(0) in field[j,i,2]
23                Field1[:, :, 2] = i_fluid_detector(Field1[:, :, 0], Field1[:, :, 1],
24 Field1[:, :, 2])
25                Field1[:, :, 3] = 1 - Field1[:, :, 2]
26        return Field1
27
28    def grid_generator( y_size, x_size, grid_size):
29        print(1.1)
30        Nx , Ny = int(x_size/grid_size), int(y_size/grid_size)
31        print(1.2)
32        print(Ny,Nx)
33        Field = np.full((Ny+2, Nx+2, 4), 0 ,dtype = 'float')
34        print(1.3)
35        return Field
36
37    def i_fluid_detector(field_y, field_x, field):
38        N_rows, N_cols = np.shape(field)
39        for j in range(0, N_rows):
40            for i in range(0, N_cols ):
41                result = 0
42                result = circle_function(field_y[j,i], field_x[j,i])
43                if result < 0:
44                    field[j,i] = 0
45                else:
46                    field[j,i] = 1
47        return field
48
49    def circle_function(y,x):
50        R= 0.25
51        x_c = data['Lx']*1/4
52        y_c = data['Ly']*1/2
53        result = (x-x_c)**2 + (y-y_c)**2 - R**2
```

```

52     return result
53
54 def IBM_uv_coeff_change(c_W, c_E, c_S, c_N, c_P, iF, iS): # iF: ifluid. iS:
55     isolid
56     c_P[1:-1,1:-1] = c_P[1:-1,1:-1] - (
57         iS[1:-1,:-2]*c_W[1:-1,1:-1] +
58         iS[1:-1,2:] * c_E[1:-1,1:-1] +
59         iS[:-2,1:-1]*c_S[1:-1,1:-1] +
60         iS[2:,1:-1]*c_N[1:-1,1:-1] )
61     c_W[1:-1,1:-1] = iF[1:-1,:-2] * c_W[1:-1,1:-1]
62     c_E[1:-1,1:-1] = iF[1:-1,2:] * c_E[1:-1,1:-1]
63     c_S[1:-1,1:-1] = iF[:-2,1:-1] * c_S[1:-1,1:-1]
64     c_N[1:-1,1:-1] = iF[2:,1:-1] * c_N[1:-1,1:-1]
65
66     c_P[1:-1,1:-1] = iF[1:-1,1:-1] * c_P[1:-1,1:-1] + iS[1:-1,1:-1]
67     c_E[1:-1,1:-1] = Solid_coeff_correct(c_E, iF)
68     c_W[1:-1,1:-1] = Solid_coeff_correct(c_W, iF)
69     c_N[1:-1,1:-1] = Solid_coeff_correct(c_N, iF)
70     c_S[1:-1,1:-1] = Solid_coeff_correct(c_S, iF)
71
72     return c_W, c_E, c_S, c_N, c_P
73
74 def IBM_p_coeff_change(c_W, c_E, c_S, c_N, c_P, iF, iS): # iF: ifluid. iS:
75     isolid
76     c_P[1:-1,1:-1] = c_P[1:-1,1:-1] + (
77         iS[1:-1,:-2]*c_W[1:-1,1:-1] +
78         iS[1:-1,2:] * c_E[1:-1,1:-1] +
79         iS[:-2,1:-1]*c_S[1:-1,1:-1] +
80         iS[2:,1:-1]*c_N[1:-1,1:-1] )
81     c_W[1:-1,1:-1] = iF[1:-1,:-2] * c_W[1:-1,1:-1]
82     c_E[1:-1,1:-1] = iF[1:-1,2:] * c_E[1:-1,1:-1]
83     c_S[1:-1,1:-1] = iF[:-2,1:-1] * c_S[1:-1,1:-1]
84     c_N[1:-1,1:-1] = iF[2:,1:-1] * c_N[1:-1,1:-1]
85
86     c_P[1:-1,1:-1] = iF[1:-1,1:-1] * c_P[1:-1,1:-1] + iS[1:-1,1:-1]
87     c_E[1:-1,1:-1] = Solid_coeff_correct(c_E, iF)
88     c_W[1:-1,1:-1] = Solid_coeff_correct(c_W, iF)
89     c_N[1:-1,1:-1] = Solid_coeff_correct(c_N, iF)
90     c_S[1:-1,1:-1] = Solid_coeff_correct(c_S, iF)
91
92     return c_W, c_E, c_S, c_N, c_P
93
94 def Solid_coeff_correct(C, iF):
95     return np.copy(C[1:-1,1:-1] * iF[1:-1,1:-1])
96
97 def TDMA(a, b, c, d): # TDMA Solver, input abcd in same length
98     # a[0], c[-1] not been used
99     do = d.copy(); ao = a.copy(); bo = b.copy(); co = c.copy()
100    N = len(d)
101    xo = np.zeros(N)
102    for rowi in range(1,N):
103        k = ao[rowi]/bo[rowi-1]
104        bo[rowi] -= co[rowi-1]*k
105        do[rowi] -= do[rowi-1]*k
106    xo[N-1] = do[N-1]/bo[N-1]
107    for rowi in range(N-2,-1,-1):
108        xo[rowi] = (do[rowi]-co[rowi]*xo[rowi+1])/bo[rowi]
109
110    return xo

```

```

108 def general_grid_generator(len_x, Nx, Ny):
109     # Creating same size of u, v, U, V
110     # where col 0, row 0, row -1, of U not used
111     # where row 0, col 0, col -1, of V not used
112     initial_value = 0.01
113     dx = len_x/Nx
114     grid = np.full((Ny+2, Nx+2), initial_value)
115     grid[0,0] = grid[-1,-1] = grid[-1,0] = grid[0,-1] = np.inf
116     u,v,U,V = data_copy(grid, grid, grid, grid)
117     p = np.copy(grid)
118     return u, v, U, V, p, dx
119
120 def data_copy(u, v, U, V):
121     return np.copy(u), np.copy(v), np.copy(U), np.copy(V)
122
123 def Ghost_BC_p(p_input): # updating ghost point for p based on BC condition
124     p = np.copy(p_input)
125     p[-1,1:-1] = p[-2,1:-1] # upper BC
126     p[0, 1:-1] = p[1,1:-1] # bottom BC
127     p[1:-1, 0] = p[1:-1,1] # left BC
128     p[1:-1, -1] = p[1:-1,-2] # right BC
129     return p
130
131 def Ghost_BC_u(u): # updating ghost point for p based on BC condition
132     u[-1,1:-1] = u[-2,1:-1] # upper BC
133     u[0, 1:-1] = u[1,1:-1] # bottom BC
134     u[1:-1, 0] = 2-u[1:-1,1] # left BC
135     u[1:-1, -1] = u[1:-1,-2] # right BC
136     u = u[:, :, :] * iF[:, :, :]
137     return u
138
139 def Ghost_BC_v(v): # updating ghost point for p based on BC condition
140     v[-1,1:-1] = -v[-2,1:-1] # upper BC
141     v[0, 1:-1] = -v[1,1:-1] # bottom BC
142     v[1:-1, 0] = -v[1:-1,1] # left BC
143     v[1:-1, -1] = v[1:-1,-2] # right BC
144     v = v[:, :, :] * iF[:, :, :]
145     return v
146
147 def UV_BC(U, V, u, iF):
148     U[1:-1,1] = 1 # left
149     V[1,:] = 0 # bottom
150     V[-1,:] = 0 # upper
151     U[1:-1,1:-1] = U[1:-1,1:-1] * iF[1:-1,1:-1]
152     U[1:-1,2:] = U[1:-1,2:] * iF[1:-1,1:-1]
153     V[1:-1,1:-1] = V[1:-1,1:-1] * iF[1:-1,1:-1]
154     V[2:,1:-1] = V[2:,1:-1] * iF[1:-1,1:-1]
155     return U,V
156
157 def coeff_assemble(rows_cols, a_e, b_e, c_e, e_e, f_e):
158     rows, cols = rows_cols[0], rows_cols[1]
159     # input elements of abcdef, get line matrix of abcdef
160     a = np.full((rows, cols), a_e) # abc in same length of Ncols of u
161     b = np.full((rows, cols), b_e)
162     c = np.full((rows, cols), c_e)
163     e = np.full((rows, cols), e_e) # ef in same length of Nrows of u
164     f = np.full((rows, cols), f_e)
165     a[:,0] = a[:, -1] = c[:,0] = c[:, -1] = 0

```

```

166 b[:,0] = b[:, -1] = 1
167 e[:, -1] = e[:, 0] = f[:, -1] = f[:, 0] = 0
168 e[0, :] = e[-1, :] = f[0, :] = f[-1, :] = 0
169 a[0, :] = a[-1, :] = c[0, :] = c[-1, :] = 0
170 return a, b, c, e, f
171
172 def RHS_uv(f, f_old, U, V, U_old, V_old, c0, r):
173     # Update Right Hand Side
174     f_P = f[1:-1, 1:-1]
175     f_W = f[1:-1, :-2]
176     f_E = f[1:-1, 2:]
177     f_S = f[:-2, 1:-1]
178     f_N = f[2:, 1:-1]
179
180     f_P_old = f_old[1:-1, 1:-1]
181     f_W_old = f_old[1:-1, :-2]
182     f_E_old = f_old[1:-1, 2:]
183     f_S_old = f_old[:-2, 1:-1]
184     f_N_old = f_old[2:, 1:-1]
185     convection_new = c0 * (U[1:-1, 2:] * (f_E + f_P) / 2 -
186                             U[1:-1, 1:-1] * (f_W + f_P) / 2 +
187                             V[2:, 1:-1] * (f_N + f_P) / 2 -
188                             V[1:-1, 1:-1] * (f_S + f_P) / 2)
189     convection_old = c0 * (U_old[1:-1, 2:] * (f_E_old + f_P_old) / 2 -
190                             U_old[1:-1, 1:-1] * (f_W_old + f_P_old) / 2 +
191                             V_old[2:, 1:-1] * (f_N_old + f_P_old) / 2 -
192                             V_old[1:-1, 1:-1] * (f_S_old + f_P_old) / 2)
193
194     diffusion = f_P + r * (f_E + f_W + f_N + f_S - 4 * f_P)
195     return -3/2 * convection_new + 1/2 * convection_old + diffusion
196
197 def Residual(u, a, b, c, d, e, f):
198     return (a[1:-1, 1:-1] * u[1:-1, :-2] +
199             b[1:-1, 1:-1] * u[1:-1, 1:-1] +
200             c[1:-1, 1:-1] * u[1:-1, 2:] +
201             e[1:-1, 1:-1] * u[:-2, 1:-1] +
202             f[1:-1, 1:-1] * u[2:, 1:-1] -
203             d[:, :])
204
205 def LineSOR_u(a, b, c, d, e, f, u):
206     # input abcdef, and u, get :D
207     # then use TDMA get new u
208     # abc is in same size of N_rows of u, ef in same size of N_cols of u
209     # ! Input d is LIKE inner field of u, need to cut each line to use !
210     u_k = np.copy(u)
211     u_k_new = np.copy(u)
212     n_rows = np.size(u, 0)
213     w = 1.3
214     D = np.zeros_like(a[1, :])
215     Res = 100
216     k = 0
217     while Res > 1e-6:
218         u_k = np.copy(u_k_new)
219         for j in range(1, n_rows - 1): # the number is based on d, which is (N-1) x (N-1)
220             D[1:-1] = d[j-1, :] - e[j, 1:-1] * u_k_new[j-1, 1:-1] - f[j, 1:-1] *
221             u_k_new[j+1, 1:-1]
222             D[0], D[-1] = 2 - u_k_new[j, 1], u_k_new[j, -2] # ghost point =

```

```

ghost point
222     u_k_new[j,:] = TDMA(a[j,:],b[j,:],c[j,:],D) # TDMA solve uk
each line
223     u_k_new[j,1:-1] = u_k[j,1:-1]*(1-w) +u_k_new[j,1:-1]*w # SOR
term
224     u_k_new = Ghost_BC_u(u_k_new)
225     Res = np.max(np.abs(Residual(u_k_new, a, b, c, d, e, f)))
# print("Res_u=")
# print(Res)
227     u = np.copy(u_k)
228     # print(Residual(u_k_new, a, b, c, d, e, f))
229     k+=1
230
231 return u_k_new, k, Res
232
233 def LineSOR_v(a,b,c,d,e,f, u):
# input abcdef, and u, get D
234 # then use TDMA get new u
# abc is in same size of N_rows of u, ef in same size of N_cols of u
# ! Input d is LIKE inner field of u, need to cut each line to use !
237 u_k = np.copy(u)
238 u_k_new = np.copy(u)
239 n_rows = np.size(u,0)
240 w = 1.3
241 D = np.zeros_like(a[1,:])
242 Res = 100
243 k = 0
244 while Res>1e-6:
245     u_k = np.copy(u_k_new)
246     for j in range(1,n_rows-1): # the number is based on d, which is (N
-1)x(N-1)
247         D[1:-1] = d[j-1,:] - e[j,1:-1]*u_k_new[j-1,1:-1] - f[j,1:-1]*
u_k_new[j+1,1:-1]
248         D[0], D[-1] = -u_k_new[j,1], u_k_new[j,-2] # ghost point =
ghost point
249     u_k_new[j,:] = TDMA(a[j,:],b[j,:],c[j,:],D) # TDMA solve uk
each line
250     u_k_new[j,1:-1] = u [j,1:-1]*(1-w) +u_k_new[j,1:-1]*w # SOR
term
251     u_k_new = Ghost_BC_v(u_k_new)
252     Res = np.max(np.abs(Residual(u_k_new, a, b, c, d, e, f)))
# print("Res_v=v")
# print(Res)
254     u = np.copy(u_k)
255     k+=1
256     # print(Residual(u_k_new, a, b, c, d, e, f))
257 return u_k_new, k, Res
258
259 def LineSOR_p(a,b,c,d,e,f, u):
# input abcdef, and u, get D
260 # then use TDMA get new u
# abc is in same size of N_rows of u, ef in same size of N_cols of u
# ! Input d is LIKE inner field of u, need to cut each line to use !
263 u = Ghost_BC_p(u)
264 u_k = np.copy(u)
265 u_k_new = np.copy(u)
266 n_rows = np.size(u,0)
267 w = 1.95
268 D = np.zeros_like(a[1,:])

```

```

272 Res = 100
273 k = 0
274 while Res>1e-6:
275     u_k = np.copy(u_k_new)
276     for j in range(1,n_rows-1): # the number is based on d, which is (N
277         D[1:-1] = d[j-1,:]- e[j,1:-1]*u_k_new[j-1,1:-1] - f[j,1:-1]*
278         u_k_new[j+1,1:-1]
279         D[0], D[-1] = u_k_new[j,1], u_k_new[j,-2] # ghost point = ghost
280         point
281         u_k_new[j,:] = TDMA(a[j,:],b[j,:],c[j,:],D) # TDMA solve uk
282         each line
283         u_k_new[j,1:-1] = u_k[j,1:-1]*(1-w) +u_k_new[j,1:-1]*w # SOR
284         term
285         u_k_new = Ghost_BC_p(u_k_new)
286         Res = np.max(np.abs(Residual(u_k_new, a, b, c, d, e, f)))/n_rows**2
287         # print("Res_p=")
288         # print(Res)
289         k+=1
290     return u_k_new, k, Res
291
292 def Solver(u, v, U, V , u_old, v_old, U_old, V_old, p, data,
293 parameter_field):
294     Re = data["Re"]; dx = data["dx"]; dt = data["dt"]; t = data["t"]; t_0 =
295     data['t_0']
296     N = 1/data['dx']
297     r = dt/(2 * Re * dx**2 ); c0 = dt/(dx)
298
299     rows_cols = np.shape(u)
300     c_W_uv , c_P_uv , c_E_uv , c_S_uv , c_N_uv  = \
301         coeff_assemble(rows_cols, -r, 1+4*r, -r, -r, -r)
302     c_W_uv , c_E_uv , c_S_uv , c_N_uv , c_P_uv = \
303         IBM_uv_coeff_change(c_W_uv, c_E_uv, c_S_uv, c_N_uv, c_P_uv, iF,
304         iS)
305
306     c_W_p , c_P_p , c_E_p , c_S_p , c_N_p  = \
307         coeff_assemble(rows_cols, 1/(dx*dx), -4/(dx*dx), 1/(dx*dx), 1/(dx
308         *dx), 1/(dx*dx))
309     c_W_p , c_E_p , c_S_p , c_N_p , c_P_p = \
310         IBM_p_coeff_change(c_W_p, c_E_p, c_S_p, c_N_p, c_P_p, iF, iS)
311
312     RHS = np.zeros_like(u)
313     U, V = UV_BC(U, V, u, iF)
314
315     u = Ghost_BC_u(u)
316     v = Ghost_BC_v(v)
317     # u, v = Ghost_BC(u,v, iF)
318
319     u_star, v_star, U_star, V_star = data_copy(u, v, U, V)
320     u_new, v_new, U_new, V_new = data_copy(u, v, U, V)
321     # u_old, v_old, U_old, V_old = data_copy(u, v, U, V)
322
323     n_max = int(t/dt)
324     Difference = 100
325     n= int( t_0/dt)
326
327     while n<n_max and Difference>1e-8 :
328         # while n<n_max:

```

```

321     U, V = UV_BC(U, V, u, iF) # Setup BC
322
323     # Step 1: get u_star, v_star, U_star, V_star
324     u = Ghost_BC_u(u)
325     v = Ghost_BC_v(v)
326     u_star = Ghost_BC_u(u_star)
327     v_star = Ghost_BC_v(v_star)
328
329     # get u_star:
330     d = RHS_uv(u, u_old, U, V, U_old, V_old, c0, r)
331     rhs= d[:, :] * iF[1:-1, 1:-1]
332     u_star, a, Resu = LineSOR_u(c_W_uv, c_P_uv, c_E_uv, rhs, c_S_uv
333 , c_N_uv, u)
334     u_star = Ghost_BC_u(u_star)
335     v_star = Ghost_BC_v(v_star)
336
337     # get v_star:
338     d = RHS_uv(v, v_old, U, V, U_old, V_old, c0, r)
339     rhs= d[:, :] * iF[1:-1, 1:-1]
340     v_star, a, Resv= LineSOR_v(c_W_uv, c_P_uv, c_E_uv, rhs, c_S_uv,
341 c_N_uv, v)
342
343     u_star = Ghost_BC_u(u_star)
344     v_star = Ghost_BC_v(v_star)
345
346     # get U_star, V_star
347     U_star[1:-1, 2:] = (u_star[1:-1, 1:-1] + u_star[1:-1, 2:]) / 2
348     V_star[2:, 1:-1] = (v_star[2:, 1:-1] + v_star[1:-1, 1:-1]) / 2
349     U_star, V_star = UV_BC(U_star, V_star, u_star, iF) # Setup BC
350
351     Diff = np.sum(U_star[1:-1, -1] - U_star[1:-1, 1])
352     U_star[1:-1, -1] -= 1 / (rows_cols[0] - 2) * Diff
353
354     # Step 2: get New Pressure P
355     d = (
356         U_star[1:-1, 2:] - U_star[1:-1, 1:-1] +
357         V_star[2:, 1:-1] - V_star[1:-1, 1:-1]
358     ) / (dt * dx)
359
360     # print(d[4, 5] - (U_star[5, 7] - U_star[5, 6] + V_star[6, 6] - V_star[5, 6])
361     / (dt * dx))
362
363     rhs= d[:, :] * iF[1:-1, 1:-1]
364     p_new, k, Resp= LineSOR_p(c_W_p, c_P_p, c_E_p, rhs, c_S_p, c_N_p
365 , p)
366
367     # print(p_new - Ghost_BC_p(p_new))
368     p_new = Ghost_BC_p(np.copy(p_new))
369
370     # Step 3: get u_new, v_new, U_new, V_new
371     # update u_new,
372     u_new[1:-1, 1:-1] = u_star[1:-1, 1:-1] - dt / dx * (iF[1:-1, 2:] * p_new
373 [1:-1, 2:] + iS[1:-1, 2:] * p_new[1:-1, 1:-1] -
374                                     iF[1:-1, :-2] * p_new
375 [1:-1, :-2] - iS[1:-1, :-2] * p_new[1:-1, 1:-1]) / 2
376     v_new[1:-1, 1:-1] = v_star[1:-1, 1:-1] - dt / dx * (iF[2:, 1:-1] * p_new
377 [2:, 1:-1] + iS[2:, 1:-1] * p_new[1:-1, 1:-1] -
378                                     iF[:: -2, 1:-1] * p_new
379 [:: -2, 1:-1] - iS[:: -2, 1:-1] * p_new[1:-1, 1:-1]) / 2

```

```

371     u_new = Ghost_BC_u(u_new); v_new = Ghost_BC_v(v_new)
372     # update U_new , V_new
373     U_new[1:-1,2:] = U_star[1:-1,2:] - dt/dx*(p_new[1:-1,2:]-p_new
374     [1:-1,1:-1])
375     V_new[2:,1:-1] = V_star[2:,1:-1] - dt/dx*(p_new[2:,1:-1]-p_new
376     [1:-1,1:-1])
377     U_new, V_new = UV_BC(U_new, V_new, u_new, iF) # Setup BC
378
379     u_old, v_old, U_old, V_old = data_copy(u, v, U, V)
380     u, v, U, V = data_copy(u_new, v_new, U_new, V_new)
381     # N = rows_cols[1]-2
382     Difference = np.sum(np.abs(u[1:-1,1:-1] - u_old[1:-1,1:-1]) +
383                         np.abs(v[1:-1,1:-1] - v_old[1:-1,1:-1]) +
384                         np.abs(p_new[1:-1,1:-1] - p[1:-1,1:-1]))
385     p = np.copy(p_new)
386     div = (U_new[1:-1, 2:] - U_new[1:-1, 1:-1]+V_new[2:, 1:-1] - V_new
387     [1:-1, 1:-1])/dx
388     div = np.linalg.norm(div)
389     p = p[:, :] * iF[:, :]
390     u = Ghost_BC_u(u)
391     v = Ghost_BC_v(v)
392     p = Ghost_BC_p(p)
393
394     n +=1
395     if n%100 == 0 or n==n_max:
396         output_save(u,v,p, U, V, u_old,v_old, U_old, V_old, data['Lx'],
397         data['Ly'] ,label=[N,Re,n*dt])
398     if n%1000 == 0:
399         draw_control(u,v,p, data['Lx'], data['Ly'], label=[N,Re,t, n*dt])
400
401     print("time=%3f, Difference=%9f, p iteration times=%d, Res_p=%f,
402 div=%2f "%((n+1)*dt,Difference,k,Resp, div))
403     output_save(u,v,p, U, V, u_old,v_old, U_old, V_old, data['Lx'],
404     data['Ly'] ,label=[N,Re,n*dt])
405     draw_control(u,v,p, data['Lx'], data['Ly'], label=[N,Re, n*dt])
406
407     return u, v, p
408
409 def output_save(u,v,p, U, V, u_old,v_old, U_old, V_old, len_x, len_y ,label):
410     N = label[0]; Re = label[1]; t = label[2]
411     with open(f'result/save/output_N{N}_Re={Re}_t={t}_{len_x}x{len_y}222.
412     txt', 'w') as f:
413         f.write(f'For grid size N= {label}:\n')
414         f.write('Array u:\n')
415         f.write(','.join(map(str, u)) + '\n\n')
416         f.write('Array v:\n')
417         f.write(','.join(map(str, v)) + '\n\n')
418         f.write('Array p:\n')
419         f.write(','.join(map(str, p)) + '\n')
420
421         f.write('Array U:\n')
422         f.write(','.join(map(str, U)) + '\n')
423         f.write('Array V:\n')
424         f.write(','.join(map(str, V)) + '\n')
425
426         f.write('Array u_old:\n')
427         f.write(','.join(map(str, u_old)) + '\n\n')

```

```

420     f.write('Array v_old:\n')
421     f.write(','.join(map(str, v_old)) + '\n\n')
422
423     f.write('Array U_old:\n')
424     f.write(','.join(map(str, U_old)) + '\n\n')
425     f.write('Array V_old:\n')
426     f.write(','.join(map(str, V_old)) + '\n\n')
427
428 def draw_control(u,v,p, Lx, Ly, label):
429     draw_streamline(u[1:-1,1:-1],v[1:-1,1:-1], Lx, Ly , label)
430     draw_velocity_contour(u[1:-1,1:-1],v[1:-1,1:-1], Lx, Ly , label)
431     draw_u_contour(u[1:-1,1:-1],v[1:-1,1:-1], Lx, Ly , label)
432     draw_p_contour(p[1:-1,1:-1],v[1:-1,1:-1], Lx, Ly , label)
433
434 def draw_streamline(u, v, len_x, len_y ,label):
435     rows, cols = np.shape(u)
436     X = np.linspace(0, len_x, cols)
437     Y = np.linspace(0, len_y, rows)
438     L = np.sqrt(len_x**2+len_y**2)
439     plt.figure(figsize=(15*len_x/L, 15*len_y/L))
440     plt.axis([0,len_x,0,len_y])
441     plt.streamplot(X, Y, u, v, density=1.5, minlength=1, arrowsize=0.5)
442     plt.savefig(f'result/stline_N{label[0]}_Re={label[1]}_t={label[2]}_{len_x}x{len_y}.png')
443     plt.close()
444
445 def draw_velocity_contour(u, v, len_x, len_y,label):
446     levels=100
447     rows, cols = np.shape(u)
448     X, Y = np.linspace(0, len_x, cols), np.linspace(0, len_y, rows)
449     velocity_magnitude = np.sqrt(u**2 + v**2)
450     L = np.sqrt(len_x**2+len_y**2)
451     plt.figure(figsize=(15*len_x/L, 15*len_y/L))
452     plt.contourf(X, Y, velocity_magnitude, levels=levels, cmap='jet')
453     plt.colorbar(label='Velocity magnitude')
454     plt.xlabel('X')
455     plt.ylabel('Y')
456     plt.title('Velocity Magnitude Contour')
457     plt.savefig(f'result/uv_N{label[0]}_Re={label[1]}_t={label[2]}_{len_x}x{len_y}.png')
458     plt.close()
459
460 def draw_u_contour(u, v, len_x, len_y ,label):
461     levels=100
462     rows, cols = np.shape(u)
463     X, Y = np.linspace(0, len_x, cols), np.linspace(0, len_y, rows)
464     L = np.sqrt(len_x**2+len_y**2)
465     plt.figure(figsize=(15*len_x/L, 15*len_y/L))
466     plt.contourf(X, Y, u, levels=levels, cmap='jet')
467     plt.colorbar(label='u magnitude')
468     plt.xlabel('X')
469     plt.ylabel('Y')
470     plt.title('u Contour')
471     plt.savefig(f'result/u_N{label[0]}_Re={label[1]}_t={label[2]}_{len_x}x{len_y}.png')
472     plt.close()
473     # plt.show()
474

```

```

475
476 def draw_p_contour(p, v, len_x, len_y ,label):
477     levels=100
478     rows, cols = np.shape(p)
479     X, Y = np.linspace(0, len_x, cols), np.linspace(0, len_y, rows)
480     L = np.sqrt(len_x**2+len_y**2)
481     plt.figure(figsize=(15*len_x/L, 15*len_y/L))
482     plt.contourf(X, Y, p, levels=levels, cmap='jet')
483     plt.colorbar(label='u magnitude')
484     plt.xlabel('X')
485     plt.ylabel('Y')
486     plt.title('u Contour')
487     plt.savefig(f'result/p_N{label[0]}_Re={label[1]}_t={label[2]}_{len_x}x{len_y}.png')
488     plt.close()
489     # plt.show()
490
491
492
493 def load_2d_arrays_with_numpy(filename):
494     try:
495         with open(filename, 'r') as file:
496             content = file.read()
497     except FileNotFoundError:
498         print("CANNOT FIND FILE!! CHECK THE PATH")
499         return None, None, None
500
501     def extract_2d_array(array_content):
502         array_data = []
503         matches = re.findall(r'\[(.*?)\]', array_content.replace('\n', ''), re.DOTALL)
504         for match in matches:
505             formatted_match = ' '.join(match.split())
506             numbers = np.fromstring(formatted_match, sep=' ')
507             array_data.append(numbers)
508         return np.array(array_data)
509
510     u_data = re.search(r'Array u:\s*((?:\[.*?\]\[, \s]*)+)', content, re.DOTALL)
511     v_data = re.search(r'Array v:\s*((?:\[.*?\]\[, \s]*)+)', content, re.DOTALL)
512     U_data = re.search(r'Array U:\s*((?:\[.*?\]\[, \s]*)+)', content, re.DOTALL)
513     V_data = re.search(r'Array V:\s*((?:\[.*?\]\[, \s]*)+)', content, re.DOTALL)
514     u_old_data = re.search(r'Array u_old:\s*((?:\[.*?\]\[, \s]*)+)', content, re.DOTALL)
515     v_old_data = re.search(r'Array v_old:\s*((?:\[.*?\]\[, \s]*)+)', content, re.DOTALL)
516     U_old_data = re.search(r'Array U_old:\s*((?:\[.*?\]\[, \s]*)+)', content, re.DOTALL)
517     V_old_data = re.search(r'Array V_old:\s*((?:\[.*?\]\[, \s]*)+)', content, re.DOTALL)
518     p_data = re.search(r'Array p:\s*((?:\[.*?\]\[, \s]*)+)', content, re.DOTALL)
519
520     if not (u_data and v_data and p_data):
521         print("CANNOT GET DATA!! CANNOT GET DATA!!")

```

```

522         return np.array([]), np.array([]), np.array([])
523
524 u = extract_2d_array(u_data.group(1))
525 v = extract_2d_array(v_data.group(1))
526 p = extract_2d_array(p_data.group(1))
527 U = extract_2d_array(U_data.group(1))
528 V = extract_2d_array(V_data.group(1))
529 u_old = extract_2d_array(u_old_data.group(1))
530 v_old = extract_2d_array(v_old_data.group(1))
531 U_old = extract_2d_array(U_old_data.group(1))
532 V_old = extract_2d_array(V_old_data.group(1))
533 return u, v, U, V, u_old, v_old, U_old, V_old, p
534
535 def Recover_result(filename):
536     u, v, U, V, u_old, v_old, U_old, V_old, p = load_2d_arrays_with_numpy(
537         filename)
538     pattern = r'output_N(\d+\.\d+)_Re=(\d+\.\d+)_t=(\d+\.\d+)_'
539     match = re.search(pattern, filename)
540     if match:
541         N = float(match.group(1))
542         Re = float(match.group(2))
543         t_0 = float(match.group(3))
544     else:
545         print("No match found")
546     Ly = 1/N * (np.size(u, 0) - 2)
547     Lx = 1/N * (np.size(u, 1) - 2)
548     dy = dx = 1/N; dt = 0.01
549     data_value = np.array([Re, Ly, Lx, dy, dx, t_0, dt])
550     keys = ["Re", "Ly", "Lx", "dy", "dx", "t_0", "dt"]
551     data_re = dict(zip(keys, data_value))
552     return data_re, u, v, U, V, u_old, v_old, U_old, V_old, p
553
554 def main():
555     N = 32; Ly = 4; Lx = 8; dy = dx = 1/N
556     Re = 150
557     t = 200; dt = 0.01
558     t_0 = 0; u_0 = 0
559     ######
560     # Recover data from existing file
561     data_re, u_0, v_0, U_0, V_0, \
562         u_old_0, v_old_0, U_old_0, V_old_0, p_0 \
563         = Recover_result(filename = f'result/save/output_N32.0_Re=150.0'
564                           '_t=73.0_8.0x4.0222.txt')
565     t_0 = data_re['t_0']
566     Re = data_re['Re']
567     Ly = data_re["Ly"]
568     Lx = data_re["Lx"]
569     dx = data_re['dx']
570     dt = data_re['dt']
571     print(Ly)
572     print('data_read:', data_re)
573     #####
574     global data
575     data_value = np.array(\
576         [Re, Ly, Lx, dy, dx, t, dt, t_0]\
577     )
578     keys = \
579         ["Re", "Ly", "Lx", "dy", "dx", "t", "dt", "t_0"]

```

```

578     data = dict(zip(keys, data_value))
579     print('data_init=', data)
580     mesh1 = mesher(data)
581     parameter_field = mesh1.parameter_grid()
582
583     global iF, iS
584     iF = parameter_field[:, :, 2]
585     iS = parameter_field[:, :, 3]
586
587     u, v, U, V, p = [np.zeros_like(parameter_field[:, :, 0]) for _ in range(5)]
588     u_old, v_old, U_old, V_old = \
589         np.zeros_like(u), np.zeros_like(v), np.zeros_like(U), np.
590         zeros_like(V)
591
592     if np.sum(u_0) != 0:
593         u, v, U, V, u_old, v_old, U_old, V_old, p = u_0, v_0, U_0, V_0,
594         u_old_0, v_old_0, U_old_0, V_old_0, p_0
595
596     u, v, p = Solver(u, v, U, V, u_old, v_old, U_old, V_old, p, data,
597     parameter_field)
598
599 if __name__ == '__main__':
600     main()

```

Listing 1: N-S Cylinder Solver