

**Problem Chosen**

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## **Power Allocation Planning in Time Trials**

### **Summary**

Solo cyclists can generate varying amounts of power at varying times, which is referred to as a power curve. The purpose of this paper is to demonstrate how to distribute power for various types of cyclists on various sections of the track in order to assist them in finishing the race faster.

We developed the OmPD model (omni power duration model) to define the power curves for both types of riders. The OmPD model describes the maximum average power that a rider can produce at various times. We collected the mean maximal power values at various power profile time points for time trial experts and sprinters, and then used the  $CP_{3-hyd}$  model to fit the critical parameters describing the power profile  $W$  (work above critical power), and  $CP$  (critical power), with  $R^2 = 0.99$ , and plotted the rider's power profile.

To determine how cyclists' power is distributed across different tracks, we developed a ramp power output model and an anaerobic energy consumption model. Through the use of Lagrange multipliers, it was determined that riding the entire course at a constant speed was the fastest way to complete the race. The optimal power allocation strategy was derived by simplifying the three courses into some combinations of typical stages and combining them with the optimization model. By comparing our power allocation method to the actual finish time, it was determined that our method could assist athletes in achieving better results.

We modeled the power output under the influence of wind, taking into account the effects of wind speed and direction on cyclists. We discovered that in a 6m/s windy environment, the rider's speed can be increased by 31.86 percent compared to when there is no wind; however, in a 6m/s windy environment, the rider's speed will be decreased by 27.35 percent. As a result, we believe that wind speed and direction have a significant effect on riding.

To determine the model's sensitivity to rider deviation from the target power, we simulated rider deviation from the target power using manufactured random errors. Using the men's time trial specialists' performance on the Tokyo Olympic track as a benchmark, their average riding power on each section was allowed to deviate by 18w from the optimal riding power, resulting in an average finish time 12.15 seconds slower than the fastest finish time. The smallest possible variation in speed occurs on downhill sections, at 0.272m/s, followed by flat ground, and the greatest variation occurs on uphill sections, at up to 0.483m/s.

For the team time trial model extension, CFD calculations were used to average out the resistance experienced by the athletes in each position, which corresponds to 58.4 percent of the resistance experienced by an individual riding at the same speed. Additionally, we determined the magnitude of the power that the rider in each position in the team should use. We also considered the possibility of riders dropping out during the race and determined that avoiding dropouts as much as possible aided in completing the race as quickly as possible.

We discovered that increasing  $cp$  and  $W$  is more beneficial for the race, and that  $cp$  has a significantly greater influence on the model than  $W$ . As a result, it is critical to obtain precise  $CP$  for the model.

Finally, we created a rider's guide for team sport directors to assist them in achieving a more balanced power distribution throughout the race.

**Keywords:** power curve, individual time trial, ompd model, power output model, anaerobic exertion model, air resistance

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# 1 Introduction

## 1.1 Problem Background

Cycle sport is a competitive physical activity that has been popular throughout the world for more than a century. The issue that has generated the most worry in competition is how to ride faster. The performance of a rider is determined by a variety of elements, including the event and the course. Additionally, there are numerous types of riders, making it difficult for anyone to succeed. Nowadays, technological advancements have enabled precise recording and efficient output data collection in the field, enabling applied scientific analysis for this sport to become a reality. Since the 1980s, when the first power meter for cycling was invented, power has become the primary metric for cycling performance. The power curve, which displays the maximum amount of power an average rider can generate over time, becomes an important analytical tool.

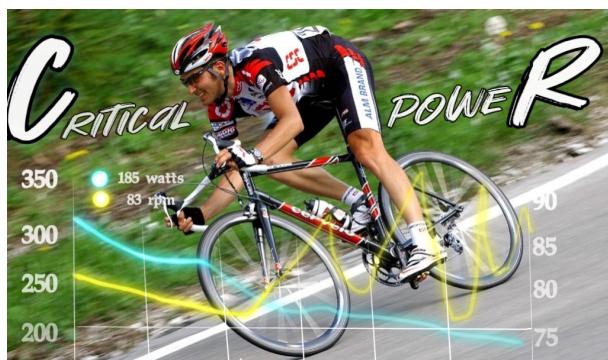


Figure 1: Critical Power in Bicycle Trial

## 1.2 Restatement of the Problem

To reduce the time required to travel a given distance, we must determine the optimal way to apply power to a specific rider while he or she traverses a time trial course, taking into account the rider's capability and power curve. The cyclist has a maximum amount of energy that can be expended across the route, as well as cumulative limits from previous aggressiveness and exceeding power curve limits. When combined with the specified constraints, the restatement of the problem is as follows:

- Define the power profile of time trial specialist and a different type rider, also consider the gender of the rider.
- Apply the model to various time trial courses for each power profile defined above, including at least one listed below:
  - 2021 Olympic Time Trial course in Tokyo
  - 2021 UCI World Championship time trial course in Flanders

- One from your own design includes at least four sharp turns and at least one nontrivial road grade, the end of the course should be near its start.
- Determine the weather impact including wind directions and strength, measure how the results could be affected by weather and environment's small differences.
- The rider may miss the power target, they need some idea of the possible range of expected split times at key parts, need to measure how sensitive the results of deviations from the target power.
- Extend the model to optimal power for team trial of six riders team, while the time is determined of the fourth rider crosses the finish line.

### 1.3 Literature Review

In the realm of power profiling, such approaches have been utilized. The most fundamental of these is average output values, which indicate that the system is beginning to comprehend the requirements but has not fully utilized its performance potential and hence cannot give peak performance. The following approach describes the power output by time at a certain intensity (binning), which can represent the range of intensities, but the range chosen is usually problematic, and also provides no insight into the duration of individual attempts. This challenge can be solved by using physiological thresholds to define bins. It does provide a broader insight into individual athletes, but it is unable to amalgamate data from different athletes, as the representation is made up of psychological responses that do not represent the same absolute power output.

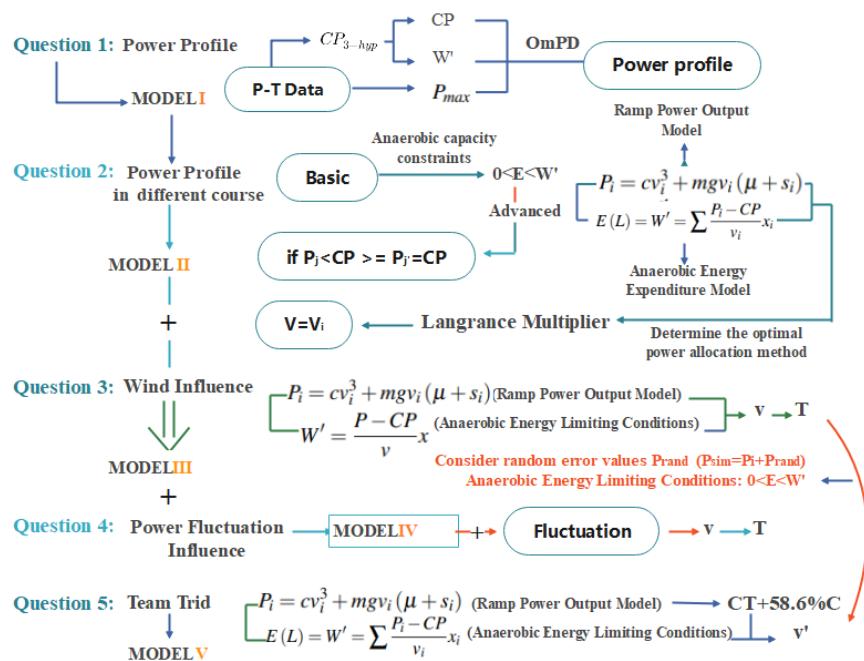
To address this constraint, exposure variation analysis (EVA) uses a two-bin system in a conventional manner to represent the intensity and duration of individual efforts. It is "very potent" in demonstrating the pacing strategy and stochastic nature of power output, and is particularly effective in describing situations in which numerous short submaximal sprints are alternated with intervals of rest.

However, because all of these approaches have failed to adequately describe power outputs for individual efforts (van Erp and Sanders 2020; Puchowicz et al. 2020; Quod et al. 2010; Vogt et al. 2007b), the mean maximal power output (MMP) is used to do so (van Erp and Sanders 2020; Puchowicz et al. 2020; Quod et al. 2010; Vogt et al. 2007b). However, the MMP profile is a consequence of a variety of biological and technical parameters, making it difficult to establish relationships between them. As a result, the researchers analyze MMP data using the PD profiles model. There are three types of critical power models: the two-parameter Critical Power (CP) model, the three-parameter Critical Power (CP) model (also known as the  $CP_{3-hyp}$  Estimate model), and the OmPD model, which is a combination of the two. This is detailed in our model.

## 1.4 Our Approach

To begin, the power curves of four different types of cyclists were plotted using the OmPD model, and the overall trend of the curves indicated that output power declined significantly at initially and then gradually over time. The power curve can be used to express the athlete's maximal output power at a given moment. Following that, a ramp power output model and an anaerobic capacity consumption model were constructed, and the Lagrange multiplier approach was used to find the ideal output power distribution for different stages of the three courses.

The power output model was then constructed under the influence of wind, and it was discovered that wind direction and speed had a significant effect on riding. Following that, the speed distribution range and the effect of divergence from the goal power on the stage completion time were calculated using random simulation. Finally, the successful model was expanded to the team time trial, with the conclusion that preventing dropouts would result in a speedier finish.



## 2 Assumptions and Explanations

- **Assumption:** Variations in a cyclist's power output due to acceleration and deceleration can be ignored.  
→ **Explanation:** The power out amount using for acceleration during the competition is extremely small compared with the power output using for keep movement situation and climb.
- **Assumption:** Ignore the acceleration process and calculate the sudden change of

speed.

→ **Explanation:** The acceleration process is very short and the speed does not change much.

- **Assumption:** The cyclist always maintains the same aerodynamic position.  
→ **Explanation:** Riders maintain as low a wind resistance position as possible at all times during the race in order to maintain maximum efficiency.
- **Assumption:** There is no wind speed influence in Model I and Model III(Model III will consider the wind speed influence).  
→ **Explanation:** Air resistance will be taken into account in all processes, but under normal circumstances low wind speed will not affect the rider greatly.

### 3 Model Preparation

#### 3.1 Notatoins

Notation	Name	Explanation
$CP$	Critical Power	
$W'$	Consumption Energy	The total work above critical power
$E$	Residual Anaerobic Energy	Energy the rider can comsume
$P_{max}, P_{peak}$	Peak Power	The max of the power output
$P, PO$	Power Output	Rider's power output to keep moving
$t$	Time from the beginning of the trial	Rourse current time
$T$	Finish Time	Time to finish the course
$CP_{TTF}$	Time to task failure at critical power	Time that could matain critical power
$A$	Fixed constnt	
$c$	Drag coefficient	Air resistance coefficient
$c_T$	Air resistance correction factor	Team resistance coefficient
$v$	Velocity	Rider's speed
$\mu$	Friction coefficient	Ground resistance
$s$	Uphill grade of the road	Slope
$x$	Moving distance	Displacement coordinate
$L$	Total course length	The length of the course

Table 1: Notatons mentioned in this paper and their explanation

#### 3.2 Data Collection

We obtain the power-time data of each kind of riders from Manuel and Pedro's research for 4 kinds of riders: male-time trial specialist, female-time trial specialist, male-sprinter, and female-sprintermean.

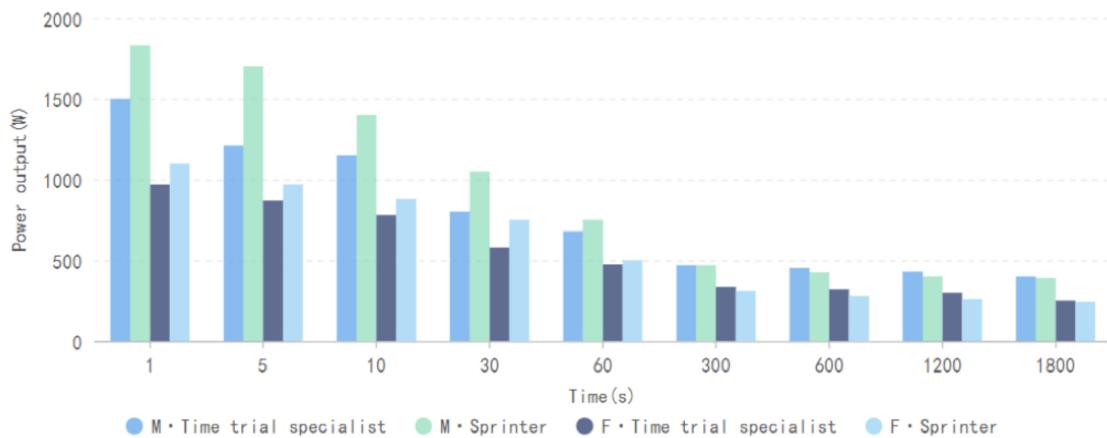


Figure 2: Maximal power values attained at the different power profile time points.

## 4 Model I: Combined Power Profile Model for Flat Road

Our purpose with model I is to establish the power profile for various types of riders. To do so, we used the best rider's power-time statistics, divided riders into two categories: time trial specialists and sprinters, and then further divided them by gender. So far, four types of riders have been identified: male time trial specialists, female time trial specialists, male sprinters, and female sprinters. We acquire the  $CP$  and  $W'$  values for each type of rider using this data. We then fit the  $CP_3 - hyp$  estimate model to the  $P_{max}$  data from the power-time data to generate the power curve.

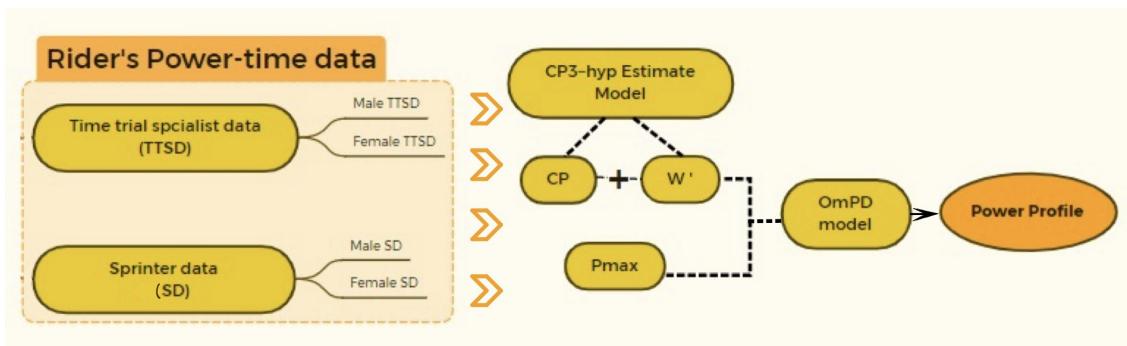


Figure 3: The syllabus of Model I, while TTSD means Time trial specialist data, and SD means Sprinter data

As the beginning of modeling, we obtain the power-time data of each kind of riders from Manuel and Pedro's research.

### 4.1 Obtain $CP$ and $W'$ by $CP_3 - hyp$ Estimate model

In cycling, critical power ( $CP$ ) and work above  $CP$  ( $W'$ ) can be estimated through linear and nonlinear models. Mattioni (2017) found the 3-parameter hyperbolic model

$(CP_{3-hyp})$  is the most accurate model through the exponential ( $CP_{exp}$ ), 3-parameter hyperbolic ( $CP_{3-hyp}$ ), 2-parameter hyperbolic ( $CP_{2-hyp}$ ), linear ( $CP_{linear}$ ), and linear 1/time ( $CP_{1/time}$ ) models, using different combinations of TTE (time-to-exhaustion) trials of different durations. The formula as follows:

$$CP_{3-hyp}: t = \frac{W'}{PO - CP} + \frac{W'}{CP - P_{max}} \quad (1)$$

While  $PO$  determines power output (could be considered as  $P$ ),  $P_{max}$  is the maximal instantaneous power (in watts),  $t$  is the cycling time. Import the 4 kinds of rider's power data into this model, we can obtain the  $CP$  and  $W'$  for each rider.

Type of the Rider	Critical Power (CP)	Comsumption Work ( $W'$ )
male-time trial specialist	410.4	24315.5
female-time trial specialist	282.4	22087.3
male-sprinter	374.5	31093.8
female-sprinter	240.2	24370.8

Table 2:  $CP$  and  $W'$  for different types of rider

However, the  $CP_{3-hyp}$  model does not perform well in long-term forecasts (more than 40 minutes). The  $CP_{3-hyp}$  model assumes that athletes can maintain their movement indefinitely while exercising at CP power, which is obviously not the case in reality. As a result, we resort to the OmPD model for power curve prediction, as it is more accurate for long time estimation.

## 4.2 Using the OmPD Model to Define Rider's Power Profile

The omni-domain power-duration (OmPD) model could describe the entire domain of maximal mean power (MMP) data from cyclists (Michael et al. 2020), it integrates basic models for short and long periods, extended the CP model for longer durations. The equation of the OmPD model as follows:

$$P_{(t)} = \frac{W'}{t} \times \left(1 - e^{-t \times \frac{P_{max}-CP}{W'}}\right) + CP; \quad t \leq CP_{TTF} \quad (2)$$

$$P_{(t)} = \frac{W'}{t} \times \left(1 - e^{-t \times \frac{P_{max}-CP}{W'}}\right) + CP - A \times \ln\left(\frac{t}{CP_{TTF}}\right); \quad t > CP_{TTF} \quad (3)$$

In this equation,  $P_{(t)}$  represent the power output,  $W'$  represent the total work above the line of critical power,  $CP$  is critical power,  $t$  represent the time in seconds,  $CP_{TTF}$  represent the time to task failure at critical power,  $A=10$ , represents a fixed constant for the decline in power output over time,  $\ln$  is natural logarithm to the base of e (2.718)

According to Poole et al. in 2016, athletes cannot keep moving forever at  $CP$  power and exercise at  $CP$  power can only be sustained for 20-40 min at most. therefore, we take  $CP_{TTF}=30$  min.

Based on the  $CP_{3-hyp}$  Estimate model and the OmPD model, we fitted the three parameters of 4 kinds of riders—the peak power  $P_{peak}$  from rider's power-time data, the  $CP$  and  $W'$  obtained by  $CP_{3-hyp}$  Estimate mode—in to the OmPD model, and obtain the power profile of the different types of riders.

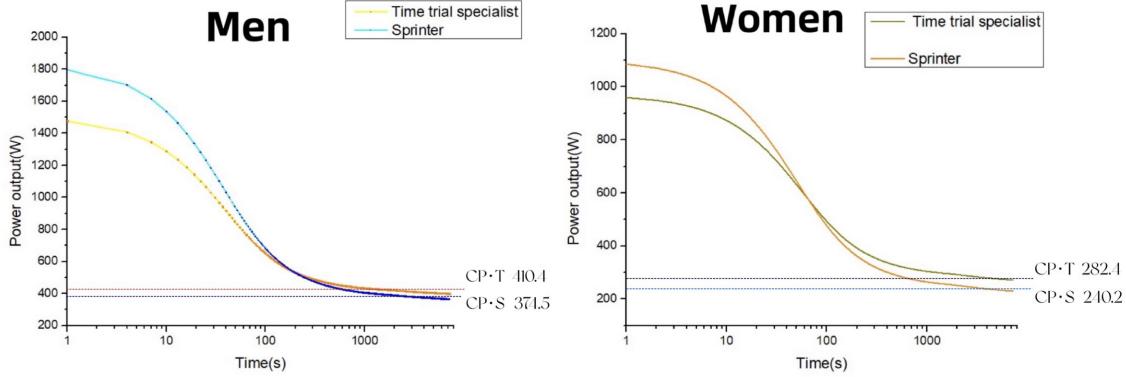


Figure 4: The four types rider's curve profile

## 5 Model II: All-Terrain Power Model

### 5.1 Basic Terrain Power Output Model

The cyclist's power output  $P$  could be measured in following formula:

$$P_i = cv_i^3 + mgv_i(\mu + s_i) \quad (4)$$

where  $c$  is the drag coefficient (in  $\text{kg m}^{-1}$ ),  $v$  is velocity (in  $\text{m s}^{-1}$ ),  $m$  is the mass (in kg) of the cyclist (and bicycle),  $\mu$  is the friction coefficient,  $s$  is the uphill grade of the road (sine of the road angle), and  $g = 9.8 \text{ m s}^{-2}$ .

Typical values for the drag and friction coefficients for cyclist on an aero bicycle are  $c = 0.17 \text{ kg m}^{-1}$  and  $\mu = 0.003$  (Kyle, 2003)

According to the work above the critical power is the factor eventually consume the rider's total energy, we use  $P - CP$  to measure the work—which expressed as consumption of anaerobic energ—in a constant grade of terrian. Consider the distant the cyclist need to travel is easy to obtain, use the following equation to change the independent variable from time to distant.

$$E = \int_0^t (P - CP) dt = \int_0^x \frac{(P - CP)}{v} dx \quad (5)$$

In 2016, Poole and his team demonstrated that it is optimum to maintain the same pace during this portion when riding on the same slope. Thus, throughout the competition, the following equation can be used to determine the total work performed above the critical power, which is also the same as the rider's energy consumption in perfect conditions.

$$E(L) = W' = \sum \frac{P_i - CP}{v_i} x_i \quad (6)$$

The overall distance the cyclist must travel is calculated by adding the length of movement required for each segment, and the total time is calculated by multiplying the distance of each segment by its duration.

$$\sum x_i = L \quad T = \sum \frac{x_i}{v_i} \quad (7)$$

Our objective is to minimize the time  $T$  required to complete the route while maintaining a fixed  $W^{prime}$  that cannot exceed the rider's entire energy. Using lagrange multipliers to minimize  $T$ : If  $E$  and  $T$  are functions of the rider's velocity  $v$ , their gradients must be parallel at extreme values of  $T$  and  $E$ .

$$\nabla E = \lambda \nabla T \quad (8)$$

Using the derivatives of  $v_i$ , we can find the velocity in different terrian need to be same to obtain the minimum total time.

$$\nabla E = \frac{d}{dv_i} \left( \frac{P_i - CP}{v_i} \right) x_i = x_i \frac{d}{dv_i} \left( cv_i^2 + mg(\mu + s_i) - \frac{CP}{v_i} \right) = \lambda \nabla T \quad (9)$$

Eliminating  $\lambda$ , the solution is the equality for each  $v_i$ . Which means we need the constant velocity in the whole competition, include get through the uphill, downhill and flat road, and rely on these equations, this optimal speed can be solved.

## 5.2 Advanced Power Output Model

However, there is a circumstance in which the rider exhausts his energy on the uphill stretch, causing the remaining energy at the top to become negative, while the power production on the downhill section is negative, replenishing the total energy to zero. This is because we place constraints on the outcomes but do not monitor the process. The rider cannot exhaust his energy during the actual competition, and the remaining energy cannot be less than zero. As a result, the following constraints must be added:

$$0 \leq E(x) \leq W' \quad 0 \leq x \leq L \quad (10)$$

These limitations indicate that riders cannot deplete their energy during the competition by performing work to the point where their remaining energy is less than zero, nor can they perform negative work in other processes to compensate for their energy depletion.

To guarantee that the model fits these limits, we continuously subtract downhill from back to front, ensuring that work consumption is always positive, until we reach the point where uphill power can exceed CP.

### 5.3 Applying the Model To the Actual Course

After developing the all-terrain power distribution model, we wanted to validate it on a range of real-world tracks. The question requests that we submit realistic applications for three courses: the 2021 Olympic Time Trial in Tokyo, the 2021 UCI World Championship time trial in Belgium, and one course of our own design.

As a result, we analyzed the terrain of the Tokyo track, the Belgian track, and the track we designed based on the Italian Alpine region track, and used our all-terrain power output model to determine the optimal finish times for four different types of runners on the three tracks, as well as the speed and power distribution by position.

The picture below depicts the three tracks' geographical locations on the globe, with a pretty even distribution of diameters and little variation in climatic conditions.



Figure 5: Race route map and track location

### 5.3.1 Toyko Course

To begin, using the competition's terrain map, simplification and abstraction can be obtained, with the slope of each ramp calculated using the height topographical map as follows:

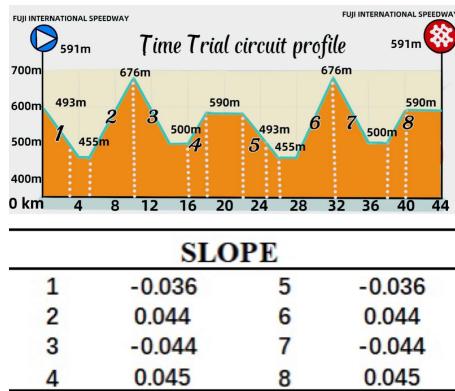


Figure 6: terrain map with slope

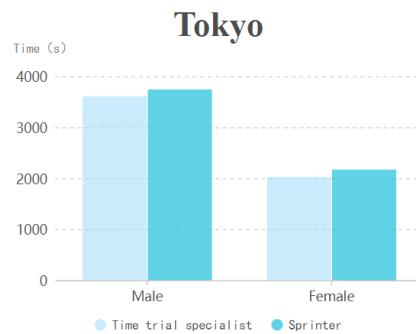


Figure 7: Finish times of each type of runner on the Tokyo track

As illustrated in the picture, the model was applied to the finish timings of each type of runner on the Tokyo Olympic track. Male results were in the 3600s, while female results were in the 2000s. Regardless of gender, time trial professionals achieved somewhat better performance than sprinters. In the actual race, the men's time trial time was 55 minutes and 04 seconds, while the women's time trial time was 30 minutes and 13 seconds, which is consistent with our model projections.

The figure above depicts the speed and power distributions on the Tokyo circuit. According to the speed distribution chart, riders travel faster downhill, at roughly 17m/s for men. When travelling uphill, the speed is approximately 8m/s for both males and females, and approximately 13m/s for males and 11m/s for females on flat roads. Conclusions that can be drawn include the following:

- **Conclusion 1:** In terms of speed, downhill is the fastest, followed by flat road and uphill is the slowest

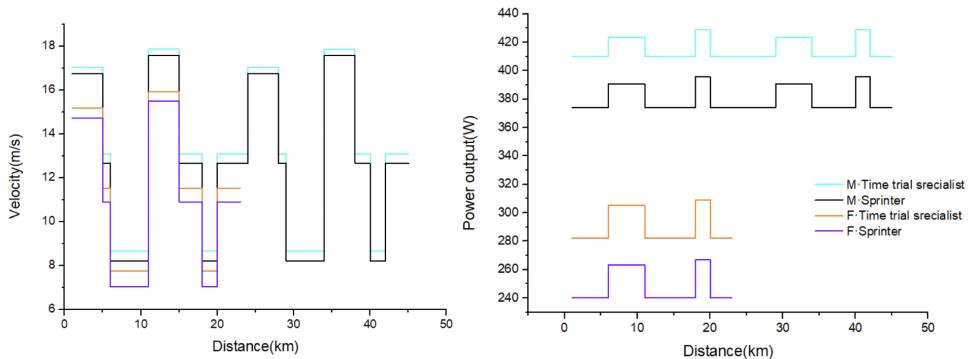


Figure 8: Speed distribution and power distribution of each type of driver in different sections

- **Conclusion 2:** Compared with women, men are 17m/s and women are 15m/s on downhill, and both are 8 on uphill, with men slightly faster.
- **Conclusion 3:** For both men and women, time trial specialists are slightly faster than sprinters.
- **Conclusion 4:** Both males and females will go uphill with maximum power output to ensure speed. The rest of the road has been CP output to save energy.

### 5.3.2 Belgium Course

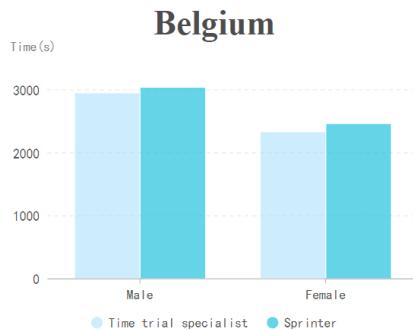


Figure 9: Different types rider finish time

The graph above illustrates the finishing times of four different sorts of athletes on the Belgian track. As you can see, the male competitors ended in the mid-3000s, while the female athletes finished in the mid-2400s. Men finished in 47 minutes and 47 seconds, while women finished in 36 minutes and 05 seconds. When compared to the preceding computation, the model fits well.

The relatively flat surface of the Brussels course results in horizontal and visibly stratified speed and power distribution curves.

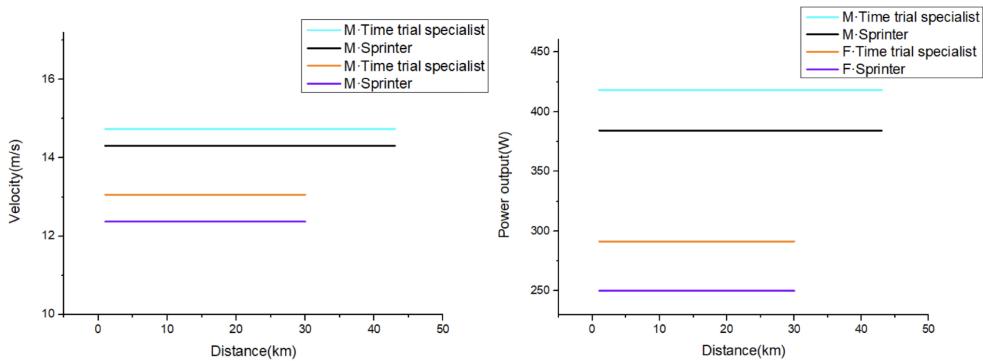


Figure 10: Speed distribution and power distribution of each type of driver in different sections

As can be observed from the speed distribution graph, males travel at a substantially faster rate than females, at approximately 14.5 m/s. Additionally, time trial specialists have much faster speeds than sprinters within the same gender.

The power diagram depicts a similar state, and the path can be thought of as being flat.

- **Conclusion 1:** Because of the horizontal road so according to the same speed
- **Conclusion 2:** Overall, male speed is faster than female, and time trial expert speed is greater than sprinters' speed
- **Conclusion 3:** Constant power output on horizontal roads is the most optimal strategy

### 5.3.3 Italy Course

A sketch of the Italian track terrain is shown below:

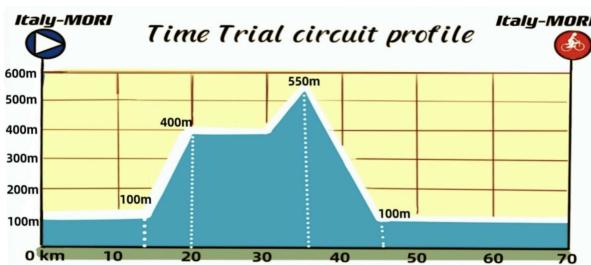


Figure 11: terrain map

The image above depicts the Italian track's finish time (we designed the track ourselves with reference to the Italian mountain track). The track features a significant climb, a minor uphill, and a significant downhill. The graph below illustrates the fluctuation in speed and power with position on the Italian circuit. On the large uphill portion, the

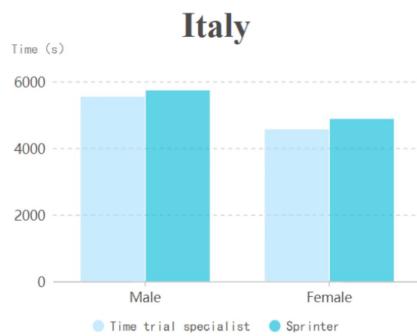


Figure 12: Different types rider finish time

output power is substantially higher and the speed decreases significantly, although the power remains stable on the other sections.

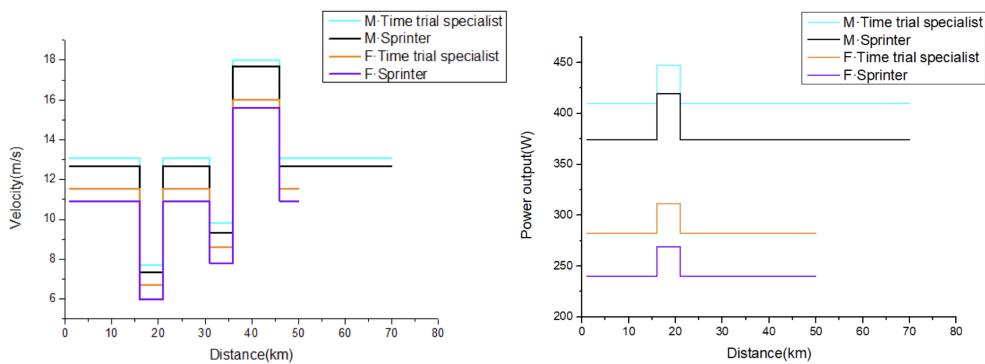


Figure 13: Speed distribution and power distribution of each type of driver in different sections

As illustrated in the graphic, the speed and power distributions of each type of player vary depending to the track's topography; in the major uphill, power production exceeds the CP and speed is lowest, while power remains steady at the CP in other sections. The speed is greatest on the downhill, and the speed of the horizontal part behind is proportional to the rise in speed on the downhill.

## 6 Model III: Wind-Performance Model

To determine the effect of different wind speeds on riders' performance, we calculated the rider's speed on flat tracks for various wind speeds using the output power equation with wind speed (equation(11)) and the "work consumption calculation" (bb), and then calculated the rider's finish time for each wind speed. Wind speed is proven to have a bigger effect on the rider's finishing time.

Considering the effect of wind speed on the rider, the output power equation of model 2 (equation(4)) is modified to output power equation with wind speed, where  $w$  is the wind speed, and removed  $s$ :

$$P = cv(v+w)^2 + mgv\mu \quad (11)$$

Additionally, modify the work consumption equation from equation (6) as follows:

$$W' = \frac{P - CP}{v}x \quad (12)$$

Combining these two equations yields the corresponding race speed for a certain rider when the wind speed is entered. Due to the fact that this model is employed on flat terrain, the finish time can be calculated by dividing the total distance traveled by the race speed  $T = x/v$ . As a result, a relationship between wind speed and finish time can be formed, yielding the following results:

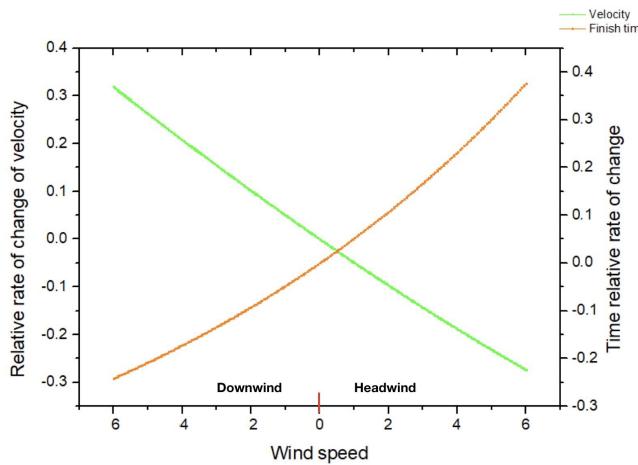


Figure 14: The relationship between wind speed, rider's velocity and finish time

As can be seen from this graph, wind speed has a significant effect on the rider's finish time. When a tailwind of 6m/s is present, the speed can be enhanced by 31.86 percent and the time saved by 24.16 percent. On the other hand, a 6m/s headwind reduces the driver's speed by 27.35 percent. Requires an additional 37.64 percent of time.

## 7 Model IV: Power Fluctuation Model

While the ideal power distribution may be calculated mathematically, there are numerous disruptions in nature, and riders are certain to fall short of entirely aligning with the optimal power curve during actual movement. As a result, we generate a random fluctuation for each type of road section using the ideal power distribution determined in the second question and examine the change in speed and the change in length of finish time for each road section.

---

**Algorithm 1:** Stochastic simulation of power deviation

---

```

1 special treatment of the first line;
2 for  $j = 1 \rightarrow 1000$  do
3   Clear energy accumulation value  $W_t$ ;
4   for  $i = 1 \rightarrow 14$  do
5     if  $W_t < W'$  then
6        $P_{sim,i} = P_i + P_{rand}$  ;
7       //  $P_i$  generates random value in  $\pm (P_m - CP)$ 
8       based on  $P_i$  solve  $v_i$ ;
9       if  $P_{sim,i} > CP$  then
10      W will consume energy according to the anaerobic energy
11      consumption model E;
12      if  $W_t + e > W'$  then
13        Energy consumption E up to  $W' - W_t$  ;
14        Solving for  $v_i$  according to the anaerobic energy
15        expenditure model;
16        Solving for  $P_i$  according to the power output model;
17      end
18    else
19      Energy consumption E=0;
20    end
21  else
22    The output power satisfies  $P_{sim,i} = P_i$  and the energy consumption
23    is 0;
24    Calculate the time used for the ith segment  $t_i$  and record the speed
25     $v_{j,i}$ ;
26  end
27 Calculate the time  $T_j$  for the jth simulation;
28 end Calculate the average, maximum, and minimum values of  $T_j$ ;

```

---

Figure 15: The relationship between wind speed, rider's velocity and finish time

### 7.1 Simulation of Power Fluctuations

We found the ideal power distribution in the second question. To replicate power variation in the actual world, we utilize a Matlab program to generate random fluctuations in the power of each section, because the player's maximum power variation value is less than  $P_{max} - CP$  and the fluctuations' range meets the following relation:

$$|\varepsilon| \leq |P_{max} - CP| \quad (13)$$

Take the male time trial specialist's race in Tokyo for example, where his  $P_{max} = 428(W)$ ,  $CP = 410(W)$ , then his variation range is  $\pm 18$  (W). The rider's power range still has to meet the constraints

## 7.2 Rider's $v$ and $T$ Under the Influence of Power Fluctuations

By applying the perturbation, we can acquire the power distribution of each segment under the perturbation, from which we can determine the maximum and minimum values of each part's speed, as well as the average speed.

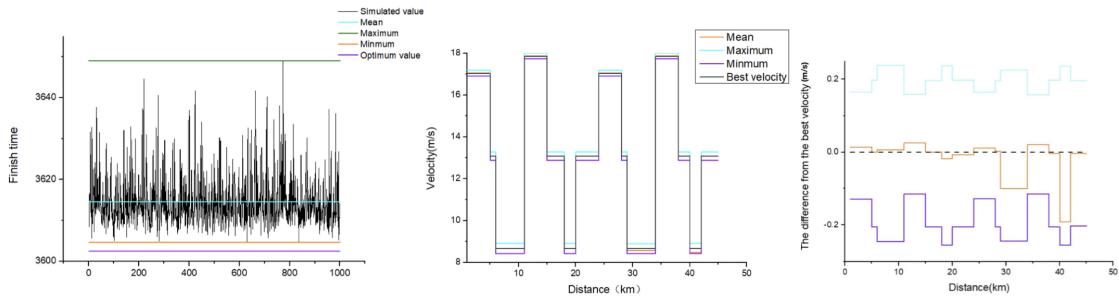


Figure 16: Velocity and power output under the influence of power fluctuations

The best completion time was found to be 12.148 seconds quicker than the average, 46.6 seconds faster than the longest, and 2.222 seconds slower than the shortest. Due to the fact that the longest and shortest finish times are caused by random errors, the model optimization outcomes are superior. Simultaneously, we discovered that the downhill segment had the least conceivable range of speed variation, 0.272m/s, followed by the flat sector, and the uphill section had the biggest possible range of speed variation, up to 0.483m/s.

## 8 Model V: Optimized Team Trial Model

### 8.1 Model Overview

The tactic in team time trial (TTT) is for team members to alternate taking the lead to confront the slipstream (also known as taking a pull), while others recuperate behind the leader. Previous research indicated that resistance could be reduced by up to 50%. (Martin et al. 1998). The preceding approach estimated the air resistance-constrained single-player speed.

This model takes the single-player motion scenario on a flat track, corrects it using hydrodynamic analysis to obtain the team's air resistance coefficient, calculates the

team's motion speed in this instance, and calculates the appropriate riding power for each position player on the team. Additionally, we assess the situation of players that miss and drop out.

## 8.2 Basic Formula and resistance coefficient

The basic power output equation and the work consumption equation are as follows in models I and II:

$$P = cv^3 + mg\mu v \quad W' = \frac{P - CP}{v}x \quad (14)$$

where  $c$  is the drag coefficient,  $\mu$  is the friction coefficient. By using these two equations, we can solve for the rider's speed.

## 8.3 Determination of Drag Coefficient $c$

In a team time trial, competitors frequently advance by taking turns breaking the wind, ensuring that each athlete spends equal time in each position on the team over the course of the race. Thus, the athlete's resistance can be considered to be the average of the resistance encountered in each position.

In 2018, Bert's team presented the aerodynamic analysis using CFD (Computational Fluid Dynamics) simulation with 3D RANS equations and  $k-\epsilon$  models, studied the air resistance for the group from 2 up to 9 cyclists, and found in the 6 member's team trial situation, the air resistance can be reduced to 58.6% of the single person condition (Bert et al. 2018)

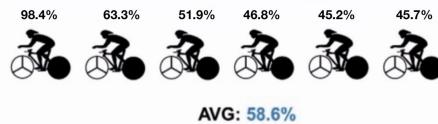


Figure 17: Drag of every rider in pacelines of 6 riders, as a percentage of the drag of an isolated rider, and the average drag percentage for the whole pacelin

Therefore, we are able to solve the corresponding velocity  $v$  in the present model by applying the equation using  $c_T$  instead of  $c$

$$c_T = c \times 58.6\% \quad (15)$$

Where,  $c_T$  is the air resistance correction factor for a team of six. Therefore, it is possible to get the team's race speed:  $v' = 15.6483m/s = 56.3339km/h$

In this case, we can obtain the riding power of each position member of the team.



Figure 18: riding power of each position member

## 8.4 Players falling out of line

In a real-world team race, participants will drop out of the team owing to blunders. As a result, we examined the effect of players leaving out at various positions on the team's finish time. The energy used by mistakes increases in direct proportion to the distance traveled, whereas the earlier the fallout occurs, the less energy is contributed to the team. Thus, a relationship between the dropped team's position and the team's finish time can be established.

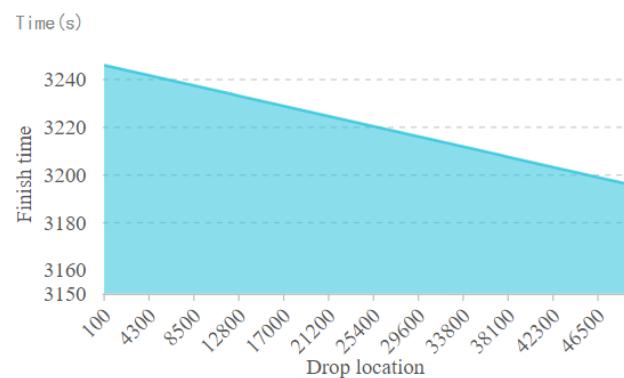


Figure 19: PD curve influenced by CP

To finish the race as soon as possible, you need to try to avoid dropping out.

## 9 Model Test

### 9.1 Sensitivity Analysis

There are two methods for determining the effectiveness of a runner's race: the runner's power distribution curve and the finish time. The PD curve is influenced by two indicators:  $CP$  (critical power) and  $W'$  (anaerobic capacity). We experimented with tiny tweaks to each of these three metrics to observe how they affected the power curve and finish time.

#### 9.1.1 Sensitivity Analysis of PD curves

To begin, we conducted a sensitivity analysis of the runners' PD curves using the men's time trial expert as an example. We modified one of the CP indicators by 5% to generate three distinct but comparable PD curves:

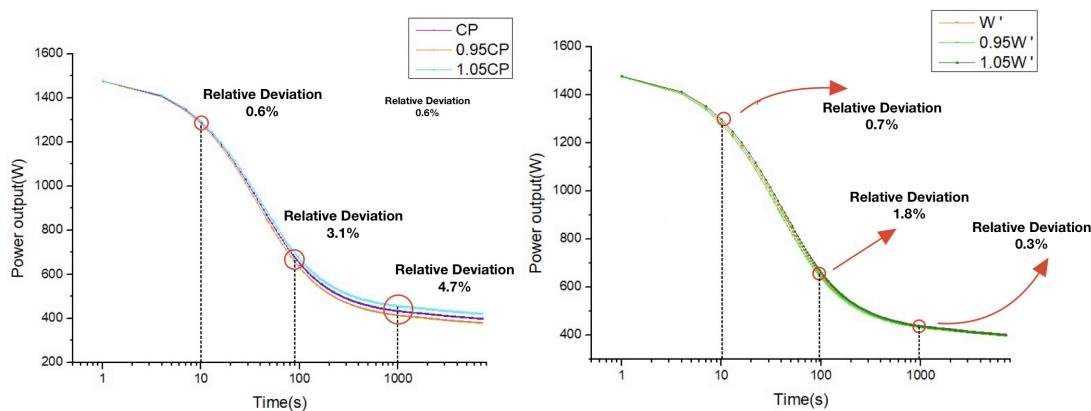


Figure 20: PD curve influenced by CP and PD curve influenced by  $W'$

It can be found that the shape of the PD curve hardly changes, and it can be seen that the smallest change is within 1% before the first period (10s), around 3.1% in the middle period, and the largest change in the second period, but stable at around 5%

Similarly, we analyzed in the same way the PD curve as influenced by the anaerobic capacity ( $W'$ ). It can be noticed that the curve is least affected (0.7%) in the first period (before the tenth second), in the middle section, it is affected by about 2%, and in the latter section, the power curve is affected by another decrease (1000s, 0.3%). The influence of  $W'$  on the curve is small compared to CP

#### 9.1.2 Sensitivity Analysis of Finish Time

In order to analyze the effect of CP and  $W'$  on the finish time, we still take the men's time trial experts as an example and analyze the corresponding change in the finish time

by making the CP and  $W'$  value change up or down independent by 5% of the original value.

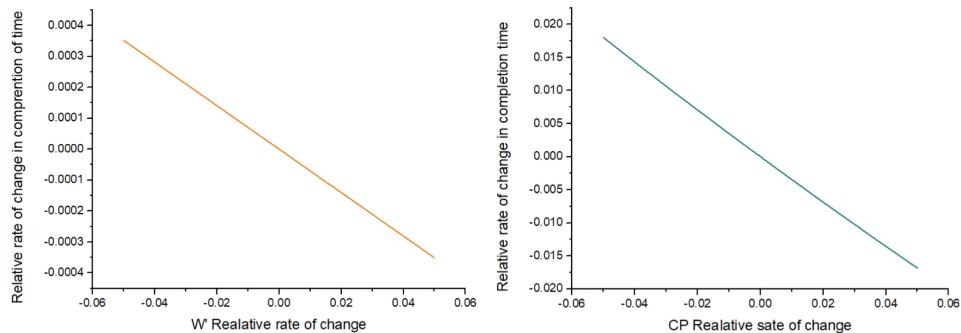


Figure 21: Finish time influenced by CP

It can be seen that the larger the CP or  $W'$ , the shorter the finish time, and the relationship between the two is approximately linear. The change of CP and time is in the same order of magnitude. And it is concluded that the finish time is more influenced by the CP.

## 10 Model Strengths

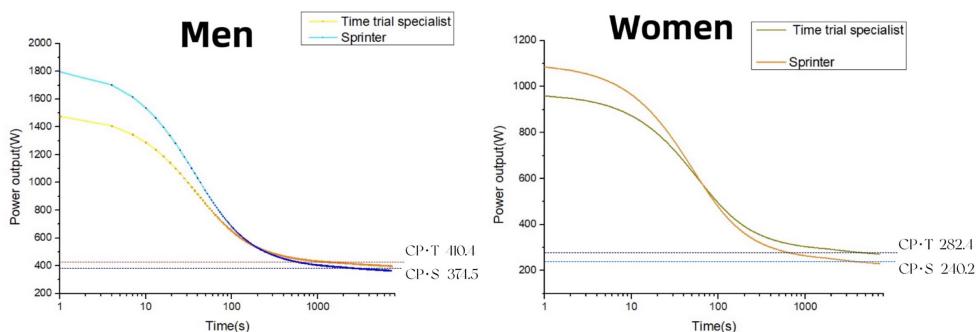
- **Predicting long times.** The model can predict the race power, speed factor, and finish time distribution for up to one hour.
- **Accurate prediction data.** The model simulates rider performance that is similar to that of a real competitor.
- **Consistent with physiological basis.** The model is based on a large number of previous studies on the physiological state of the rider and the corresponding theory, which can better match the actual race conditions of the rider.

## 11 Rider's Guide: Developing speed in a race involves meticulous planning.

To win a race, your team requires not only extensive training and professional equipment, but also sophisticated power planning.

I'm curious whether you've ever struggled to choose riders for a time trial. Have you ever had a cyclist on your team ride too aggressively in the early stages of a race and then fade away? Our model may be able to assist you! To begin, if your team wishes to compete in time trials, you should select as many time trial specialists as feasible. A time trial specialist's power curve suggests that they excel at long, long distance sports such as time trials. They have superior endurance, can ride at a higher power level for longer periods of time, and while they are not as explosive as sprinters, they can perform better in lengthy races than sprinters.

Second, your team members should attempt to keep a steady speed throughout the race. Mathematically, it is faster to bike at an even speed to the finish line. Therefore, team members must be exercised to maximize force on the uphill and conserve energy on the downhill and flat to avoid overexertion and tiredness.



Wind speed and direction can have a significant effect on riding and can seriously disrupt the race, so ensure that the team is familiar with the weather conditions and has measures in place to cope with them prior to the race. Train your team members, if possible, to ride with less departure from the planned power. Power deviation also has an effect on the outcome, and the ability to ride within the planned power range is critical to achieving a successful result.



If your team is racing in a team time trial, bear in mind that ensuring that no team member falls off is critical to achieving a good performance, and training your team

mates to stay off as long as possible can help you win! Finally, I wish your squad the best of luck in the upcoming races.

Team 2220770

## References

- [1] Leo, P., Spragg, J., Podlogar, T., Lawley, J. S., Mujika, I. (2021). Power profiling and the power-duration relationship in cycling: a narrative review. European journal of applied physiology, 1-16.
- [2] Maturana, F. M., Fontana, F. Y., Pogliaghi, S., Passfield, L., Murias, J. M. (2018). Critical power: how different protocols and models affect its determination. Journal of Science and Medicine in Sport, 21(7), 742-747.
- [3] Puchowicz, M. J., Baker, J., Clarke, D. C. (2020). Development and field validation of an omni-domain power-duration model. Journal of Sports Sciences, 38(7), 801-813.
- [4] Gordon, S. (2005). Optimising distribution of power during a cycling time trial. Sports Engineering, 8(2), 81-90.
- [5] Manuel Mateo-March, Teun van Erp, Xabier Muriel, Pedro L. Valenzuela, Mikel Zabala, Robert P. Lamberts, Alejandro Lucia, David Barranco-Gil y Jesús G. Pallarés. The record power profile in professional female cyclists: normative values obtained from a large database. Int J Sports Physiol Perform. 2021. Online ahead of print. <https://doi.org/10.1123/ijspp.2021-0372>
- [6] Pedro L. Valenzuela, Xabier Muriel Teun van Erp, Manuel Mateo-March, Alex Gandía, Mikel Zabala, Robert P. Lamberts, Alejandro Lucia, David Barranco-Gil y Jesús G. Pallarés. The record power profile in professional male cyclists: normative values obtained from a large database. Int J Sports Physiol Perform. 2021. Online ahead of print. <https://doi.org/10.1123/ijspp.2021-0263>
- [7] Blocken, B., Topalar, Y., van Druenen, T., Andrianne, T. (2018). Aerodynamic drag in cycling team time trials. Journal of Wind Engineering and Industrial Aerodynamics, 182, 128-145.
- [8] Martin, J.C., Milliken, D.L, Cobb, J.E., McFadden, K.L., Coggan, A.R., 1998. Validation of a mathematical model for road cycling power. J. Appl. Biomech. 14, 276-291.