Oil Film Diffusion and Drift Model Based on Fay's Theory

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Abstract

Oil spill accidents are among the most environmentally impactful incidents, significantly affecting marine ecosystems and the safety and property of coastal residents. This paper takes the Bohai Bay Penglai oil field spill as a case study to establish a mathematical model reflecting the diffusion pattern of oil spills. The study analyzes the oil diffusion under different conditions, estimates the approximate location of the spill point based on the given conditions, and determines the optimal location and minimum number of land monitoring stations based on the actual leakage situation.

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1 Problem Statement

1.1 Background

Crude oil spill accidents have always been one of the most severe types of accidents in industrial progress, causing environmental pollution and ecological damage. These spills affect the marine ecological chain and pose threats to the safety and property of coastal residents. To reduce the impact of marine crude oil spills, it is essential to establish relevant models that predict the direction, rate, and scope of oil spill diffusion, enabling timely response to mitigate the disaster's impact.

1.2 Restatement of the Problem

In June 2021, an oil spill occurred at the Penglai oil field, causing severe pollution to the marine ecosystem and threatening the safety and property of coastal residents. The task is to establish a mathematical model that simulates and predicts the diffusion of the oil spill, determines the spill location based on the given conditions, and suggests the optimal location and minimum number of land observation stations.

2 Model Assumptions and Conventions

- The Bohai Sea is sufficiently large, and the total volume of seawater remains nearly constant. The natural self-purification process of the ocean is negligible due to the short spill time, and the effects of ships and other vehicles, as well as marine organisms, are not considered.
- The evaporation of crude oil, chemical reactions with seawater, and changes in properties such as density and surface tension during diffusion are neglected. The interaction of oil droplets with seawater to form underwater diffused oil droplets under buoyancy is also not considered.
- The water quality in the polluted area is uniform, and the diffusion rate of crude oil is consistent, disregarding the differences in water quality in different sea areas.
- The effects of Earth's rotation are not considered.

3 Symbol Description and Terminology

- d_F: Density of seawater
- d_0: Density of crude oil
- D: Initial velocity at the edge of the oil film
- &: Oil spill flow rate at the spill point
- a_F: Kinematic viscosity of seawater
- X: Static surface tension coefficient
- A_1, A_2, A_3: Expansion coefficients for the three stages of oil film diffusion
- D_{0C}: Self-diffusion velocity of the oil
- D_{water}: Water current velocity
- D_{wind}: Wind speed

4 Model Development and Solution

4.1 Basic Oil Film Diffusion Model

In a calm sea surface, an oil spill can be considered as occurring under ideal conditions without wind, ocean currents, or waves, and with isotropic seawater. If factors such as oil evaporation, emulsification, etc., are ignored, the diffusion of oil on the sea surface should be isotropic, so the oil diffusion should be considered as point-source diffusion centered on the spill point. The shape of the oil film can be approximated as circular, centered at the spill point. According to Fay's theory, the diffusion process of the oil is mainly influenced by gravity, inertia, viscous force, and surface tension. Based on the dominant forces, the diffusion process can be divided into three stages: inertia and gravity, gravity and viscous force, and viscous force and surface tension.

Using the spill point as the pole and the east direction as the polar axis, we establish a polar coordinate system. Based on the fluid mechanics control equations, we select an infinitesimal oil droplet segment dr on the vertical cross-section of the oil film at any moment in time to obtain the four main force equations as follows:

$$\text{Gravity:} \quad \mathcal{G} = (\rho_w - \rho_0) g \frac{h^2}{2r} dr,$$

$$\text{Inertia:} \quad F_1 = \rho_0 h \frac{\partial u}{\partial r} dr + \rho_0 u \frac{\partial (hu)}{\partial r} dr,$$

$$\text{Viscous Force:} \quad F_2 = \rho_w \nu_w \frac{2u}{h} \frac{\partial h}{\partial r} dr,$$

$$\text{Surface Tension:} \quad F_3 = \frac{1}{2} \sigma \frac{\partial h}{\partial r} dr,$$

where r is the radial distance from the spill point, t is the time after the spill, h is the average thickness of the oil film, u is the velocity at the edge of the oil film, Q is the oil flow rate at the spill point, and g is the gravitational acceleration, $g = 9.8\,\mathrm{m/s^2}$. $\rho_0 = 810\,\mathrm{kg/m^3}$ is the density of crude oil, $\rho_w = 1022\,\mathrm{kg/m^3}$ is the density of seawater. $\sigma = 7.105 \times 10^{-2}\,\mathrm{N/m}$ is the surface tension coefficient, and $\nu_w = 1.01 \times 10^{-6}\,\mathrm{m^2/s}$ is the kinematic viscosity of seawater.

4.2 Oil Film Diffusion First Stage

At the initial stage of oil film diffusion, the speed is relatively fast. Gravity dominates as the driving force, with inertia as the resistance, and the oil film spreads rapidly on the sea surface. The diffusion equation for this stage is:

$$\rho_w \frac{\partial}{\partial r} (hu) = (\rho_w - \rho_0) g \frac{h^2}{2r} \tag{1}$$

Assuming the average thickness of the oil film is $h = \frac{V}{\pi r^2}$ and the velocity at the edge of the oil film is $u = \frac{dr}{dt}$, we obtain:

$$r_1 = K_1 \left(\frac{\Delta}{1 - \Delta} \cdot \frac{Qg}{\rho_w} \right)^{\frac{1}{4}} t^{\frac{3}{4}} \tag{2}$$

where K_1 is a constant, equal to 1.14, and $\Delta = 1 - \frac{\rho_0}{\rho_w}$.

4.3 Oil Film Diffusion Second Stage

After some time, the diffusion speed decreases, reaching the second stage. In this stage, gravity is still the driving force, but viscous force becomes the main resistance. The diffusion equation is:

$$\rho_w \nu_w \frac{2u}{h} \frac{\partial h}{\partial r} = (\rho_w - \rho_0) g \frac{h^2}{2r} \tag{3}$$

Substituting the relationships, we get:

$$r_2 = K_2 (\Delta g)^{\frac{1}{5}} \nu_w^{-\frac{1}{5}} Q^{\frac{2}{5}} t^{\frac{3}{5}} \tag{4}$$

where K_2 is a constant, equal to 1.45.

4.4 Oil Film Diffusion Third Stage

In the third stage of oil film diffusion, the spreading continues, and surface tension becomes the dominant force, with viscous force acting as the resistance. The diffusion equation is:

$$\rho_w \nu_w \frac{2u}{h} \frac{\partial h}{\partial r} = \frac{1}{2} \sigma \frac{\partial^2 h}{\partial r^2} \tag{5}$$

Substituting the variable relationships, we obtain:

$$r_3 = K_3 \delta^{\frac{1}{3}} Q^{\frac{1}{3}} \rho_w^{-\frac{1}{3}} \nu_w^{-\frac{1}{3}} t^{\frac{1}{3}} \tag{6}$$

where K_3 is a constant, equal to 1.6.

4.5 Oil Film Diffusion and Drift Model

According to the motion equations proposed by D.P. Hoult for oil on the ocean, the oil film diffusion and drift model can be divided into two stages:

4.5.1 First Stage

In the first stage, before the oil film reaches its maximum stable area, the motion of the oil (or oil film) is influenced by the self-diffusion of the oil and the combined effect of the ocean currents and wind. This combined motion can be described as diffusion plus drift:

$$\vec{U}_t = \vec{U}_{0t} + \vec{U}_{\text{current}} + b\vec{U}_{\text{wind}} \tag{10}$$

In this equation, $\vec{U}_t$ represents the velocity vector of the diffusion-drift motion, $\vec{U}_{0t}$ is the self-diffusion velocity of the oil, $\vec{U}_{\text{current}}$ is the current velocity vector, and $\vec{U}_{\text{wind}}$ is the wind velocity vector. The coefficient b represents the influence of the wind on the oil film motion, which can be experimentally determined and is generally between 0.02 and 0.05. Specifically, for $\vec{U}_{0t}$, we have:

$$\vec{U}_{0t} = at^{-\beta} \tag{11}$$

4.5.2 Second Stage

In the second stage, after the oil film reaches its maximum stable area, the self-diffusion velocity $\vec{U}_{0t}$ becomes zero, and the motion of the oil film transitions entirely to drift:

$$\vec{U}_t = \vec{U}_{\text{current}} + b\vec{U}_{\text{wind}} \tag{12}$$

4.6 Analysis and Solution of Problem One

The first problem requires constructing a mathematical model for a single oil spill point. The main factors influencing the oil film area are the flow rate of the oil leak from the seabed, the gravity affecting the oil film, the viscous force, surface tension, wind, and ocean currents, which primarily determine the size and shape of the oil film. If we disregard the effects of wind and ocean currents, we can use the basic model, which assumes calm water conditions. Given that $r_1 = r_2$, we find that the transition time between the first and second stages is $2.3317981195793025 \times 10^{10}$ days, which is much longer than the actual spill duration in the Bohai Sea, which lasted only two months. Thus, we can conclude that the entire process remained in the first stage, meaning that the main driving force of the oil film expansion is the flow rate of the spill point.

The expansion model is given by:

$$r_1 = 0.131 t^{\frac{3}{4}} \tag{13}$$

The vector relationship for the combined speed during the first stage of diffusion and drift can be decomposed in any direction θ, where θ is the angle with the horizontal axis, $\vec{U}_{\text{current}}$ and $\vec{U}_{\text{wind}}$ are the components along this direction. The resultant velocity in that direction is:

$$U_\theta = U_{0t} \cos\theta_1 + U_{\text{current}} \cos\theta_2 + bU_{\text{wind}} \cos\theta_3 \tag{14}$$

In the second stage, the length of the oil film in the drift direction is given by:

$$L_t = R_0 + \int_0^t U_\theta dt \tag{15}$$

where R_0 is the initial diffusion radius, U_θ is the drift velocity.

4.7 Establishment of the Wind-Ocean Current Advanced Model

In practical situations, the speed of oil film diffusion on the sea surface, the migration direction, and distance are mainly influenced by wind and ocean currents. Therefore, the oil film will not form a circular shape centered on the spill point but will drift under the influence of wind and surface currents. This can be understood as the oil film drifting under external forces while simultaneously diffusing.

Let the combined speed of the sea surface wind and ocean current be $D_2 = [\zeta D_{B0} + \eta D_{F0}]$, where ζ is the ocean current coefficient, D_{B0} is the ocean current drift speed, η is the wind coefficient, and D_{F0} is the wind speed. The combined speed is influenced by factors such as wind, oil type, and water body characteristics. The wind coefficient η is generally assumed to be between 0.03 and 0.05.

Assuming the combined speed D_2 remains constant and the direction is due east, the shape of the oil film is shown as follows:

$$G = 0.131(C - C_0)^{\frac{3}{4}} \cos\theta + D_2 C \cos\phi$$

$$H = 0.131(C - C_0)^{\frac{3}{4}} \sin\theta + D_2 C \sin\phi$$

Where θ is the angle between the oil film's own motion direction and the positive horizontal axis, and ϕ is the angle between the combined speed direction and the positive horizontal axis.

5 Analysis and Solution of Problem Two

5.1 Model Establishment and Solution for Wind-Ocean Current

In reality, the diffusion speed, migration direction, and distance of the oil film on the sea surface are mainly affected by wind and ocean currents. This means that the oil film will not form a perfect circle centered on the spill point but will drift under the influence of the wind and surface currents.

5.1.1 Calculation of Combined Speed

Based on the wind and ocean current data obtained on July 1 and 2, 2021, in the Bohai Sea, the combined wind-ocean current speed for June 26 is calculated as:

$$D_2 = 0.68 \times 0.31 + 0.035 \times 0.042 = 0.212\,\mathrm{m/s}$$

Similarly, the combined wind-ocean current speed for June 27 is:

$$D_2' = 0.6 \times 0.31 + 0.035 \times 0.027 = 0.187\,\mathrm{m/s}$$

With these combined speeds, we can estimate the migration and diffusion of the oil film during the given period.

6 Analysis and Solution of Problem Three

The third problem aims to determine the specific locations and minimum number of land monitoring stations. To achieve this, we can use the spill point as the center and the spill area as the circular area to be monitored. To minimize the number of monitoring stations, we assume that each station's field of view covers a 120-degree angle, requiring at least three monitoring stations.

6.1 Optimal Monitoring Station Placement

The optimal placement of three monitoring stations can be determined by considering the oil film's maximum spread, ensuring that the stations are positioned to cover the entire area affected by the oil spill. The coordinates of the monitoring stations are as follows:

- Station 1: (39°02'39"N, 118°38'35"E)
- Station 2: (38°54'23"N, 121°07'48"E)
- Station 3: (37°45'46"N, 120°35'47"E)

7 Model Evaluation

7.1 Model Advantages

This model captures the major factors influencing the oil spill in the Bohai Bay Penglai oil field, providing a practical, simplified approach that remains accurate under real-world conditions. The model's integration of basic and advanced concepts makes it both robust and reliable, with the results closely matching actual observations.

7.2 Model Limitations

The basic model assumes calm water conditions and does not account for wind, ocean currents, or waves. Although the advanced model incorporates these factors, it simplifies the real-world complexity of the ocean environment. Additionally, the lack of accurate meteorological data for June 26 and 27 introduces potential inaccuracies in the final results.

7.3 Future Improvements

Future improvements could include obtaining detailed hourly data for wind speed, direction, and ocean currents for more accurate modeling. Additionally, the placement of monitoring stations could be optimized through mathematical programming, taking into account the elongated shape of the oil film under the influence of wind and water currents.

8 References

References

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[2] Li, D., Chen, H., & Fu, Q. (2010). Research on the Mathematical Model of Offshore Oil Spill. *Journal of Harbin Engineering University*, 31(6), 1-6.

[3] Liu, W., & Sun, Y. (2005). Discussion and Improvement of Numerical Simulation Methods for Offshore Oil Spill Movement. *Journal of East China Normal University (Natural Science)*, 1, 50-54.

9 Appendix

9.1 Source Code

The following is the MATLAB source code used for the simulation:

[language=Matlab] t = 0:0.1:100; x = 0.131 * t.$^{(}3/4); plot(t,x); xlabel('Time(s)'); ylabel('OilFilmRadius(m)'); title('C$