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MAST20018 - Exam Solutions 2017

Q1 a)

$x_1 :=$ # of brownies eaten
$x_2 :=$ # of scoops of choc ice cream eaten
$x_3 :=$ # of bottles of cola drunk daily ✓✓
$x_4 :=$ # of pieces of cheesecake eaten

$$\min \quad z = 50x_1 + 20x_2 + 30x_3 + 80x_4 \quad ✓$$

s.t

$$400x_1 + 200x_2 + 150x_3 + 500x_4 \geq 500 \quad ✓$$
$$3x_1 + 2x_2 \geq 6 \quad ✓$$
$$2x_1 + 2x_2 + 4x_3 + 4x_4 \geq 10 \quad ✓$$
$$2x_1 + 4x_2 + x_3 + 5x_4 \geq 8 \quad ✓$$

$$x_i \geq 0 \quad \forall i \quad ✓$$

$\boxed{8}$

b) i) Activity levels are not completely continuous. ✓
ii) The objective function contribution from each activity is not completely proportional. ie., can you buy 20% of a brownie for 10c? ✓

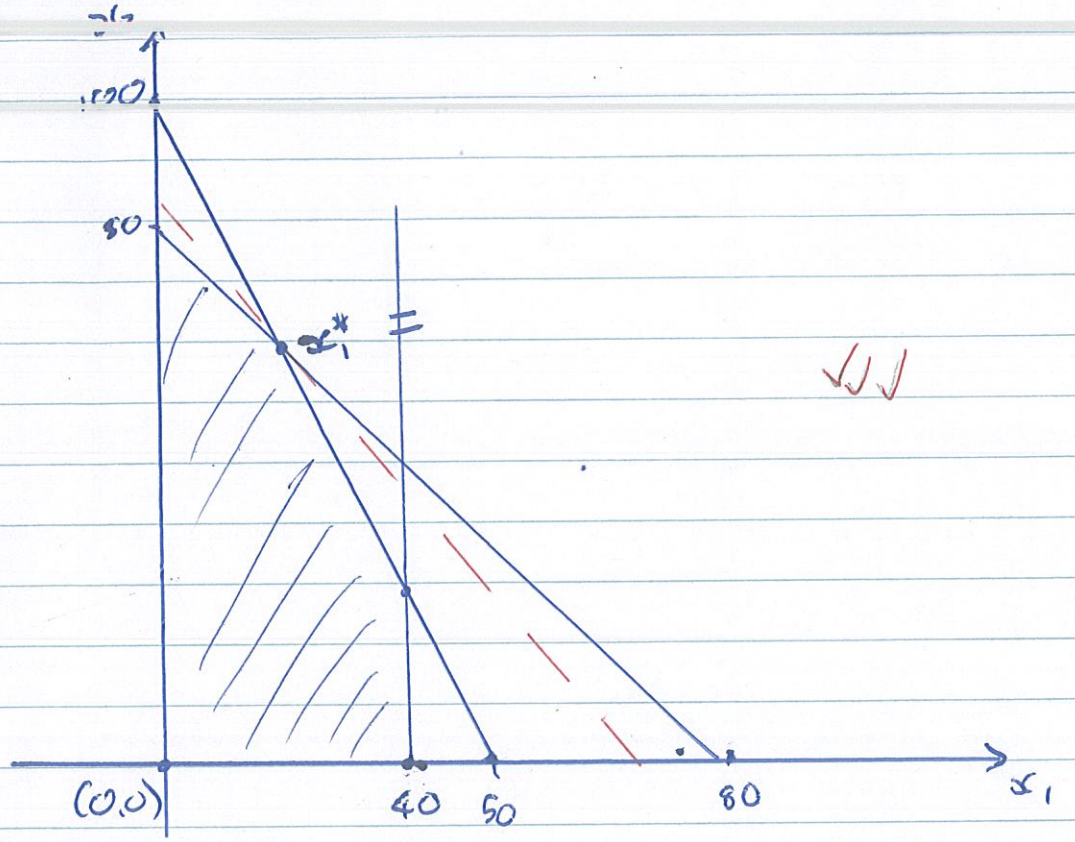
iii) Activities are not 100% independent (they may intefere through digestion, for eg) ✓

iv). There would be natural variation in eg the Calories in each item. The exact nutritional values would have some uncertainty. ✓

$\boxed{4}$

Q2).

a)



b).

If $k=0$ then $Z=0$ and all feasible points are optimal.

If $k \neq 0$ then

$$x_2 = -\frac{3}{2}x_1 + \frac{Z}{2k}.$$

If $k>0$ then we wish to minimize the y-intercept

$\therefore$ $(0,0)$ is optimal. $Z=0$

If $k<0$ the we wish to maximise the y-intercept.

$\therefore$ move the gradient to the point x_1^*

which is the intersection of

$2x_1 + x_2 = 100$ and $x_1 + x_2 = 80$.

$\therefore$ $x_1 = 20$ and $x_2 = 60$.

$z = 3k(20) + 2k(60)$

$= 60k + 120k = 180k$.

$\boxed{7}$

Q3)

a) max $-z = -2x_1 - 3x_2$ (or min)

$\frac{1}{2}x_1 + \frac{1}{4}x_2 + s_1 = 4$ ✓

$x_1 + 3x_2 - t_1 + y_1 = 36$ ✓

$x_1 + x_2 + y_2 = 10$ ✓

$x_1, x_2, s_1, t_1, y_1, y_2 \geq 0.$ ✓ $\boxed{4}$

b) min $w = y_1 + y_2$

BV	x_1	x_2	s_1	t_1	y_1	y_2	RHS
s_1	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	4
y_1	1	3	0	-1	1	0	36
y_2	1	1	0	0	0	1	10
w	0	0	0	0	-1	-1	0
→ w	2	4	0	-1	0	0	46

✓✓

BV	x_1	x_2	s_1	t_1	y_1	y_2	RHS
s_1	1/4	0	1	0	0	-1/4	$\frac{3}{2}$
y_1	-2	0	0	-1	1	-3	6
x_2	1	1	0	0	0	1	10
w	-2	0	0	-1	0	-4	6.

✓✓ $\boxed{4}$

c) The LP is infeasible since the phase 1 table has not eliminated the artificial variables. ✓✓ $\boxed{2}$

Q4).

Find canonical form:

$$\max z = x_1 + 4x_2$$

s.t

$$x_1 + 2x_2 + S_1 = 6$$
$$2x_1 + x_2 + S_2 = 8$$
$$x_1, x_2, S_1, S_2 \geq 0.$$

$\therefore$

$$A_B = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

$$A_B | I = \left[\begin{array}{cc|cc} 2 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right]$$

using Gauss-Jordan

$$A_B^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

$$\therefore \quad A_B^{-1} \cdot b = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}. \quad \text{RHS.}$$

The column for x_1 in the optimal tableau

is $A_B^{-1} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \end{bmatrix}.$

the column for s_1 in the optimal tableau is

$$A_B^{-1} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$$

$C_B = [4, 0]$ //

$\therefore C_B \cdot A_B^{-1} = [4, 0] \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix} = [2 \ 0]$

$\therefore$ coeff of x_1 in z-row is

$$[2 \ 0] \begin{bmatrix} 1 \\ 2 \end{bmatrix} - c_1 = 1$$ ✓

Coeff of S_1 in z-row is

$$[2 \ 0] \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 0 = 2$$ ✓

Z at the RHS is $C_B \cdot A_B^{-1} \cdot b = [2, 0] \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

$= 12.$ //

$\therefore$

BV	x_1	x_2	s_1	s_2	RHS
x_2	1/2	1	1/2	0	3
s_2	3/2	0	−1/2	1	5
Z	1	0	2	0	12

✓

$\boxed{15}$

Q5.

check feasibility in primal

$2 \cdot \frac{5}{26} + \frac{5}{2} + 3 \cdot \frac{27}{26}$

$= \frac{10 + 65 + 81}{26} = \frac{156}{26} = 6 \quad \therefore \; s_1 = 0$

$5 \cdot \frac{5}{26} + 4 \cdot \frac{5}{2} + \frac{27}{26} = 12 \quad \therefore \quad s_2 = 0.$

$2 \cdot \left(\frac{5}{2}\right) = 5 \quad \therefore \quad s_3 = 0$

Dual $\quad \min \; 6y_1 + 12y_2 + 5y_3$

$2y_1 + 5y_2 \geq 9$
$y_1 + 4y_2 + 2y_3 \geq 14$
$3y_1 + y_2 \geq 7$

$\boxed{4}$

$y_1, y_2, y_3 \geq 0$

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$x_1 > 0$, $x_2 > 0$, $x_3 > 0$.

$\therefore$ $t_1 = 0$ $t_2 = 0$ $t_3 = 0$.

$\therefore$ $2y_1 + 5y_2 = 9$
$3y_1 + y_2 = 7$

$-13y_1 = -26$
$y_1 = 2$
$y_2 = 1$

$y_1 + 4y_2 + 2y_3 = 14$

$\therefore$ $2 + \cancel{8}\ 4 + 2y_3 = 14$
$y_3 = \cancel{5/2}\ 4$

$w = 6y_1 + 12y_2 + 5y_3$

$= 6(2) + 12(1) + 5 \cdot \cancel{5/2}\ 4$
$= 12 + 12 + \frac{20}{2} = \cancel{36\frac{1}{2}}\ 44$

$\boxed{6}$

$z = 9(5/26) + 14(5/12) + 7(\frac{27}{26})$

$= 44$

$\therefore$ $x^* = (5/26, 5/2, 27/26)$ is optimal.

Q6.).

BV	x_1	x_2	x_3	x_4	RHS
x_2	1	1	1	0	4
x_4	1	0	-1	1	1
z	$1-3\alpha$	4α	4	0	16 ✓

2

	BV	x_1	x_2	x_3	x_4	RHS
$\Rightarrow$	x_2	1	1	1	0	4 ✓
	x_4	1	0	-1	1	1
	z	$1-7\alpha$	0	$4-4\alpha$	0	$16-16\alpha$

which is optimal iff $1 - 7\alpha \geq 0 \quad \alpha \leq 1/7$

and $4 - 4\alpha \geq 0 \quad \alpha \leq 1$ $\boxed{2}$

(i) iff $\alpha \leq 1/7$. $\therefore \alpha \in [0; 1/7]$

(ii)

BV	x_1	x_2	x_3	x_4	RHS
x_2	0	1	2	-1	3
x_1	1	0	-1	1	1
Z	0	0	$5-13\alpha$	$7\alpha-1$	$15-9\alpha$

$\boxed{2}$

which is optimal iff $5 - 13\alpha \geq 0 \quad \alpha \leq 5/13$

$7\alpha - 1 \geq 0 \quad \alpha \geq 1/7$.

$\therefore \alpha \in [1/7; 5/13]$ $\boxed{2}$

$\Rightarrow \alpha \in [1/7; 1/3]$

(iii) If $\alpha = 1/7$ then $x_2 = 4$ and $x_4 = 1$ is optimal, with $x_1 = x_3 = 0$

ie, the point $(0,4)$ is optimal in the original program

Also, $x_2 = 3$, $x_1 = 1$ and $x_3 = x_4 = 0$ is optimal. Ie $(1,3)$ is optimal in the original program.

$\therefore$ all points $\lambda(0,4) + (1-\lambda)(1,3)$ are optimal, where $\lambda \in [0,1]$.

$\boxed{2}$

$= (1-\lambda, \lambda+3)$