Homework No.3 for Math 3121

Due Time: Oct 22, 11pm.

Problem 1. Let $\sigma \in S_8$ be the element

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 4 & 3 & 2 & 1 & 5 & 8 & 7 \end{pmatrix}.$$

(1) Compute σ^2. (2). Decompose σ as a product of disjoint cycles. (3). Compute the order of σ. (4). Compute σ^{-1}.

Problem 2. Let $\sigma \in S_8$ be of the form

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 6 & 3 & 2 & a & b & 1 & 7 \end{pmatrix}.$$

Suppose σ is an **odd** permutation,

(1). Find a and b. (2). Decompose σ as a product of disjoint cycles.

(3). Compute the order of σ. (4). Decompose σ^{-1} as a product of disjoint cycles. (5). Comupte σ^{2020}.

Problem 3. Give an example of a subgroup in S_4 that has order 6.

Problem 4. If H is a subgroup of S_n ($n \geq 2$), and H is not contained in A_n, prove that $2|H \cap A_n| = |H|$.

Problem 5. Let G be a finite group, H_1 and H_2 are subgroups of G with $|H_1| = 24$ and $|H_2| = 25$. Prove that $H_1 \cap H_2 = \{e\}$.