Homework No.4 for Math 3121

Deadline: Nov. 12, 11:00pm

Problem 1. Determine if the following maps are homomorphisms of groups (No reasons needed).

(1). $\Phi : \mathbb{R}^* \to \mathbb{R}^*, \quad \Phi(a) = 2020a$

(2). $\Phi : \mathbb{R}^* \to \mathbb{R}^*, \quad \Phi(a) = a^{2020}$

(3). $\Phi : \mathbb{R} \to \mathbb{R}, \quad \Phi(a) = 2020a$

(4). $\Phi : \mathbb{R} \to \mathbb{R}, \quad \Phi(a) = a^2$

(5). $\Phi : GL(n, \mathbb{R}) \to \mathbb{R}^*, \quad \Phi(A) = det(A)^7$.

(6). $\Phi : \mathbb{R} \to \mathbb{R}^*, \quad \Phi(a) = 10^a$.

(7). $\Phi : \mathbb{R}^* \to \mathbb{R}, \quad \Phi(a) = 10^a$.

(8). $\Phi : \mathbb{R}^* \to \mathbb{R}, \quad \Phi(a) = \log_{10}(a^2)$.

(9). $\Phi : GL(2, \mathbb{R}) \to \mathbb{R}, \quad \Phi(A) = \log |det(A)|$.

(10). $\Phi : S_3 \to S_4$,

$$\Phi(\sigma) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ i & j & k & 4 \end{pmatrix}, \quad \text{where } \sigma = \begin{pmatrix} 1 & 2 & 3 \\ i & j & k \end{pmatrix}.$$

Problem 2. (no reasons needed). (1) Find a homomorphism $\Phi : \mathbb{C}^* \to \mathbb{C}^*$ such that $Ker(\Phi) = U_{10}$. (2) Find a homomorphism $\Phi : \mathbb{C} \to \mathbb{R}^*$ such that $\Phi(i) = 2$.

Problem 3. Let A and B be groups. Find an isomorphism $\Phi : A \times B \to B \times A$.

Problem 4. Let G and G' be finite groups, suppose that $|G|$ and $|G'|$ are relatively prime, prove that a homomorphism $\phi : G \to G'$ must be trivial, i.e., $\phi(a) = e'$ for all $a \in G$,

Problem 5. Determine if each of the groups below is isomorphic to $\mathbb{Z}$ (no reasons needed)

(1). $\{3^n \,|\, n \in \mathbb{Z}\}$, the operation is the multiplication.

(2). $\mathbb{Q}$, the operation is the addition.

(3). $2020\mathbb{Z}$, the operation is the addition.

(4). $U = \{a \in \mathbb{C} \,|\, |a| = 1\}$, the operation is the multiplication.

(5). $(\log 10)\mathbb{Z}$, the operation is the addition.

Problem 6. Let $G = GL(2, \mathbb{R})$, determine if each of the following subgroups of G is normal (no reasons needed)

(1). $H = \{\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} \,|\, a, d \in \mathbb{R}^*\}$

(2). $H = \{\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \,|\, a \in \mathbb{R}^*\}$

(3). $H = \{A \in G \,|\, \det A = 1\}$

(4). $H = \{A \in G \,|\, A \text{ is upper triangular}\}$.

(5). $H = \{A \in G \,|\, \det A > 0\}$.