Homework No.2 for Math 3121

Deadline: Oct 9, 11pm.   Late submissions will **not** be accepted.

Problem 1. Determine whether the given subset of $\mathbb{C}^*$ is a subgroup of $(\mathbb{C}^*, \cdot)$ (just answer "yes" or "no", no reasons needed.)
(1) The set of positive real numbers   (2). The set of positive integers
(3) The set $\{z \in \mathbb{C}^* \,|\, z^{2020} = 1\}$.
(4) The set $\{1, -1, 2, \frac{1}{2}\}$.
(5) The set $\{3^n \,|\, n \in \mathbb{Z}\}$.

Problem 2. Determine whether the given subset of $GL(3, \mathbb{R})$ is a subgroup of $GL(3, \mathbb{R})$ (just answer "yes" or "no", no reasons needed.)
(1) The set of 3×3 upper triangular matrices with all diagonal entries 1.
(2) The set of invertible 3×3 matrices with all entries non-negative.

Problem 3. Compute the order of elements (just give the answers, no details needed):
(1). $-1, -i, \frac{1}{2} + \frac{\sqrt{3}}{2}i, 3$ in $G = \mathbb{C}^*$,
(2). $5, 14, 8$ in $\mathbb{Z}_{12}$.
(3). $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, where $\theta = \frac{\pi}{1010}$.

Problem 4.(just give the answers, no details needed). (1) Give an example of a cyclic group with 101 elements. (2) Give an example of an infinite cyclic group. (3) Give an example of a finite group that is NOT cyclic.

Problem 5. Let $\sigma, \tau \in S_8$ be the elements

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 4 & 3 & 2 & 1 & 5 & 8 & 7 \end{pmatrix}, \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 1 & 6 & 2 & 3 & 5 & 7 & 8 \end{pmatrix}$$

(1) Compute $\sigma\tau$ and $\tau\sigma$. (2). Compute σ^{-1}.

Problem 6. Let G be a finite group. Suppose H is a non-empty subset of G that is closed under the binary operation of G, prove that H is a subgroup.