

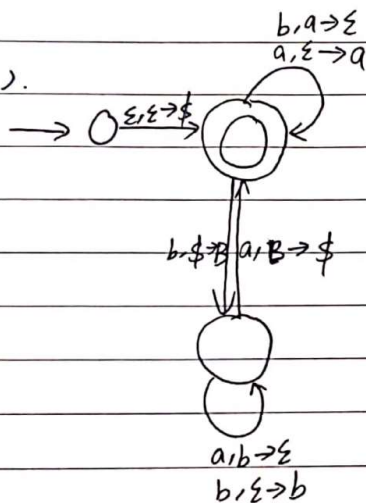
1. (a).

 $S \rightarrow aAa / bBb / a$ $A \rightarrow a / aQAQ$ $B \rightarrow b / aBA$ $Q \rightarrow alb$

(b)

 $S \rightarrow AB / BA$ $A \rightarrow a / aA$ $B \rightarrow b / aBa$

2(a).



3(a) Yes

Because R is regular language, $R \circ R$ will be regular language, too. (As Regular languages means 'are recognized by some NFA'. The concatenation of R, R must be regular since their NFA can be concatenated. i.e. link each accepting state of R 's NFA to the start state of ~~and the~~ the other R 's NFA with ϵ). Then $(R \circ R \circ R)$ must be RL for the same reason. (concatenate three NFA to recognize $R \circ R \circ R$).

(b) No

we can prove it by pumping lemma.

Let $w = a^p b^p$ so $s = a^p b^p a^p b^p a^p b^p \in \text{triple}(R)$

Since $|s| \geq p$, let $s = xyz$, $xy = a^p \Rightarrow |xy| \leq p$ and $|y| \geq 1$

Then $xyyz = a^{p+k} b^p a^p b^p a^p b^p \notin \text{triple}(R)$

Therefore, it is not necessarily Regular language.

3(c)

Assume $\text{snip}(L)$ is Regular Language

Then let $\text{tmp}(L) = \{xz \mid x \in L, y \in L\}$

$\text{tmp}(L) \circ \text{snip}(L)$ should be regular, too.

$$\text{tmp}(L) \circ \text{snip}(L) \Leftrightarrow \{xz \mid xyz \in L, |x|=|y|=|z|, x \in L, z \in L\}$$

Let $L = a^*$: then $|x|=|y|=|z| \Rightarrow x=z, |x|=|y|$

$$\text{Then } \text{tmp}(L) \circ \text{snip}(L) \Leftrightarrow \{xz \mid xyz \in L, |x|=|y|, x \in L, z \in L\}$$

$$\Leftrightarrow \{xx \mid xyx \in L, |x|=|y|, x \in L, y \in L\}$$

In this case, we know $\text{tmp}(L) \circ \text{snip}(L)$ is not regular language.

So, $\text{snip}(L)$ is not necessarily regular language