Nonlinear and Adaptive Control Systems
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Assignment 1:

Stabilize the nonlinear dynamics system using backstepping method.

$$\dot{x}_1 = x_2 + x_1^2$$
$$\dot{x}_2 = x_3 + e^{x_2} x_1$$
$$\dot{x}_3 = u + x_1^2 x_3^3$$

Q1

Define

$$z_1 = x_1 \ , \ z_2 = x_2 - \alpha_1 \ , \ z_3 = x_3 - \alpha_2$$

Step 1:

$$\dot{z}_1 = \dot{x}_1 = x_2 + x_1^2 = z_2 + \alpha_1 + x_1^2$$

In order to make z1 converge to zero, it is sufficient to let $\alpha_1 = -c_1 z_1 - x_1^2$

Thus, $\dot{z}_1 = -c_1 z_1 + z_2$

Step 2:

$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 = z_3 + \alpha_2 + e^{x_2} x_1 - \dot{\alpha}_1$$

In order to make z1 and z2 converge to zero, it is sufficient to let

$$\begin{aligned}\alpha_2 &= -z_1 - c_2 z_2 - e^{x_2} x_1 + \frac{\partial \alpha_1}{\partial x_1}\dot{x}_1 \\ &= -z_1 - c_2 z_2 - e^{x_2} x_1 + (c_1 + 2x_1)(x_2 + x_1^2)\end{aligned}$$

Thus, $\dot{z}_2 = -z_1 - c_2 z_2 + z_3$

Step 3:

$$\dot{z}_3 = \dot{x}_3 - \dot{\alpha}_2 = u + x_1^2 x_3^3 - \dot{\alpha}_2$$

In order to make z1 , z2 and z3 converge to zero, it is sufficient to let

$$\begin{aligned}u &= -z_2 - c_3 z_3 - x_1^2 x_3^3 + \dot{\alpha}_2 \\ &= -z_2 - c_3 z_3 - x_1^2 x_3^3 + \frac{\partial \alpha_2}{\partial x_1}(x_2 + x_1^2) + \frac{\partial \alpha_2}{\partial x_2}(x_3 + e^{x_2} x_1) \\ &= -z_2 - c_3 z_3 - x_1^2 x_3^3 + (-1 - c_1 c_2 - 2c_2 x_1 - e^{x_2} - 2c_1 x_1 - 2x_2 - 6x_1^2)(x_2 + x_1^2) \\ &\quad + (-c_1 - c_2 - e^{x_2} x_1 - 2x_1)(x_3 + e^{x_2} x_1)\end{aligned}$$

Thus, $\dot{z}_3 = -z_2 - c_3 z_3$

Q2

The Lyapunov function is selected as

$V = \frac{1}{2}(z_1^2 + z_2^2 + z_3^2)$,which is positive definite.

Its derivative is

$\dot{V} = z_1\dot{z}_1 + z_2\dot{z}_2 + z_3\dot{z}_3 = -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2$,which is negative definite.

Therefore, the dynamics system of z is asymptotically stable,i.e.

$\lim_{t\to\infty} z_1 = \lim_{t\to\infty} x_1 - 0 = 0$, $\lim_{t\to\infty} z_2 = \lim_{t\to\infty}(x_2 - \alpha_1) = 0$, $\lim_{t\to\infty} z_3 = \lim_{t\to\infty}(x_3 - \alpha_3) = 0$

Moreover,

$$\lim_{t\to\infty} \alpha_1 = -c_1 \lim_{t\to\infty} z_1 - \lim_{t\to\infty} x_1^2 = 0$$

$$\lim_{t\to\infty} \alpha_2 = -\lim_{t\to\infty} z_1 - c_2 \lim_{t\to\infty} z_2 - e^{x_2} \lim_{t\to\infty} x_1 - \lim_{t\to\infty}(-c_1\dot{x}_1 - 2x_1\dot{x}_1) = 0$$

The equilibrium point of the nonlinear system is $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$

Q3

In order to solve this ordinary equation with designed control input, using the matlab function ode45, i.e. the method of 4th and 5th order Runge-Kutta with variable step. If the value of x2 is sufficiently big, for example, greater than 5, the exponential term in the equation tends to be extremely large. This kind of differential equation is called stiff. Then it is hard to solve the stiff equation using numerical method.

Suppose the initial state of [x1   x2   x3] is [0   0   0]. The first candidate set of [c1   c2   c3] is selected as [1   1   1]. The simulation results are shown as below. The first subplot in figure shows how the state variables converge to zero as time. The second one shows the dynamics of parameter z. The last one shows how the Lyapunov function and its derivative changes as time.

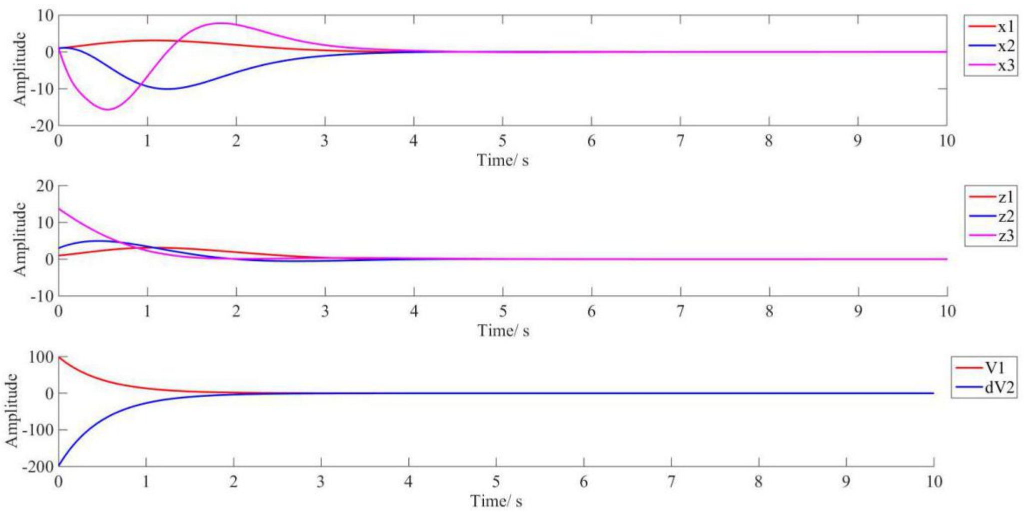

Figure 1 The dynamics of x ,z and Lyapunov Function

Change one of the c_i from 1 to 10, while the other two parameters are constant one. From figure2, it is observed that these three parameters have very similar influence on the dynamics of state variables x_i. The larger the c is, the peak of the x decreases and shifts to the left, indicating that x converges to zero faster. The effect tends to be weaker as the value of c increase. According figure3, the scaling of c_1 and c_2 have the same contribution to the input u and a little deviation occurs for case of c_3.

However, it is obvious that the c_i decide the converging speed of z_i. The figure4 illustrates this phenomenon. The details are summarized in table1. It can be inferred that the increase of c_i have positive effect on the previous z_j, as well as negative effect on the latter z_k, where $j \geq i$ and $k<i$. The effect is weaker as the value of c increase. This can be understood intuitively from the dynamics of z mentioned in Q1.

Maximum deviation & settling time of	Increase c1	Increase c2	Increase c3
x_i	Decrease	Decrease	Decrease
z_1	Decrease	Decrease	Decrease
z_2	Increase	Decrease	Decrease
z_3	Increase	Increase	Decrease

Table 1

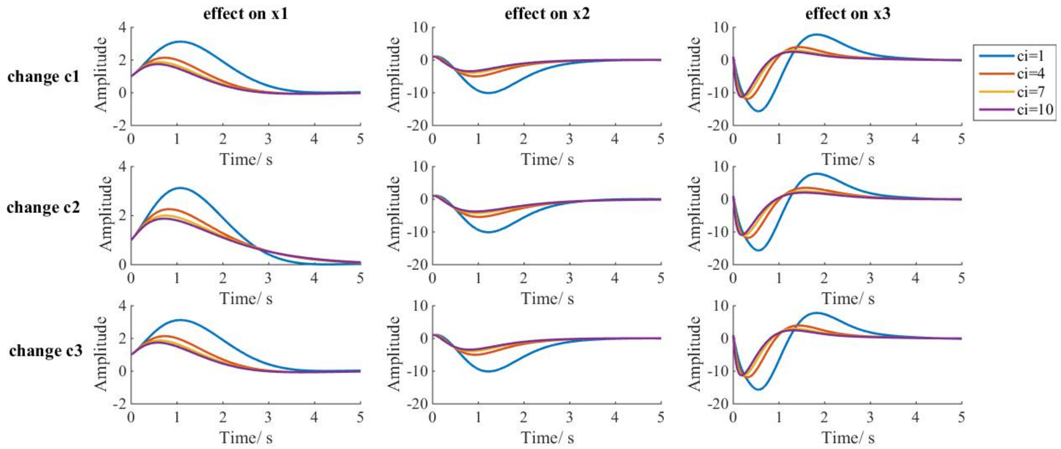

Figure 2 The effect of c on state variables x

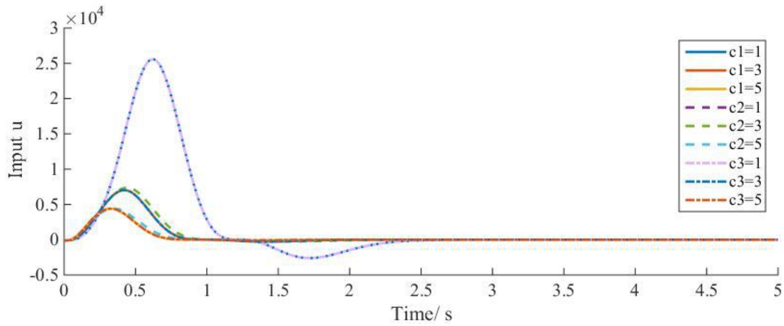

Figure 3The effect of c on input u

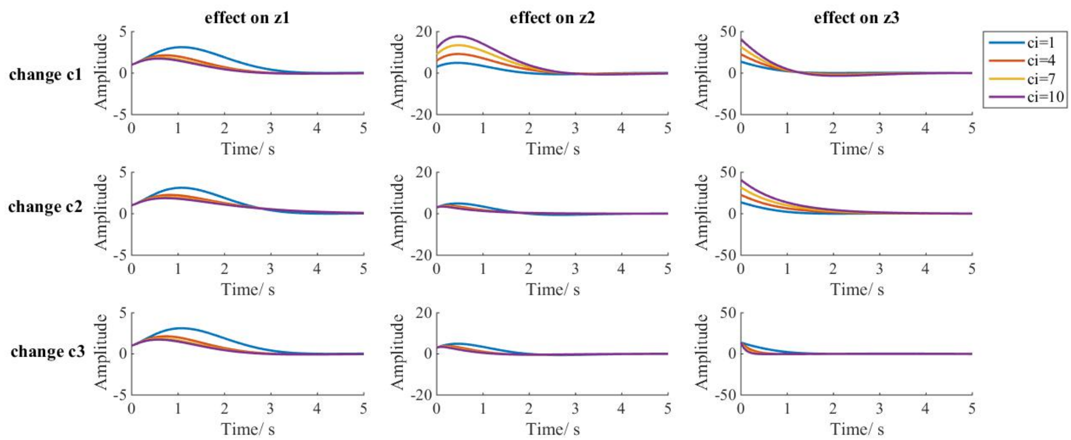

Figure 4 The effect of c on z