Nonlinear and Adaptive Control Systems
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Assignment 2:

A plant is described by a second-order model

$$y(s)=\frac{b_1 s+b_2}{s^2+a_1 s+a_2}u(s)$$

The plant is expected to follow the system

$$y_m(s)=\frac{s+3}{s^2+3s+2}r$$

Q1

Suppose the parameters of the plant are known,

$$y(s)=\frac{2s+2}{s^2+s-1}u(s)=2\frac{s+1}{s^2+s-1}u(s)$$

Transform y in the form of

$$y(s)=\frac{1}{s^2+3s+2}\left[2(s+1)u(s)+\left(s^2+3s+2-s^2-s+1\right)y(s)\right]$$
$$=2\frac{s+3}{s^2+3s+2}\left(\frac{s+1}{s+3}u(s)+\frac{2s+3}{2(s+3)}y(s)\right)$$
$$=2\frac{s+3}{s^2+3s+2}\left(u(s)-\frac{2}{s+3}u(s)-\frac{3}{2(s+3)}y(s)+y(s)\right)$$

Thus, the error is

$$e_1=y-y_m=2\frac{s+3}{s^2+3s+2}\left(u(s)-\frac{2}{s+3}u(s)-\frac{3}{2(s+3)}y(s)+y(s)-\frac{1}{2}r(s)\right)$$

In order to eliminate error, the input u can be selected as

$$u(s)=\frac{2}{s+3}u(s)+\frac{3}{2(s+3)}y(s)-y(s)+\frac{1}{2}r(s)$$

Figure 1 illustrates the simulation result of the above system with reference signal as sin(t). It is obvious that the plant track the reference system perfectly.

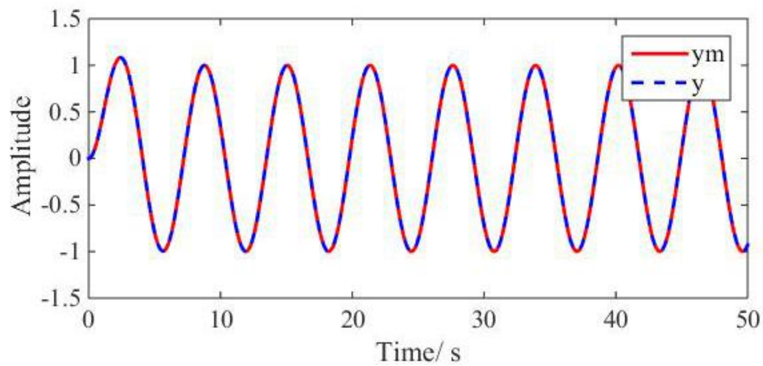

Figure 1 The tracking performance when r=sin(t) and the parameters are known

Q2

If the parameters are unknown, design the input u as

$$u(s)=\frac{\theta_1}{s+3}u(s)+\frac{\theta_2}{s+3}y(s)+\theta_3 y(s)+\theta_4 r(s)$$
$$=\boldsymbol{\theta}^T\boldsymbol{\omega}$$

where

$$\boldsymbol{\omega} = \begin{bmatrix} \frac{1}{s+3}u & \frac{1}{s+3}y & y & r \end{bmatrix}^T$$

$$\boldsymbol{\theta}^T = [\theta_1 \quad \theta_2 \quad \theta_3 \quad \theta_4]$$

The adaptive law of the parameter θ should be

$$\dot{\hat{\boldsymbol{\theta}}} = -sign(k_p)\boldsymbol{\Gamma} e_1 \boldsymbol{\omega}$$

where Γ is the adaptive gain ,which is a positive definite matrix.
Moreover, we can infer

$$\dot{\tilde{\boldsymbol{\theta}}} = \dot{\boldsymbol{\theta}} - \dot{\hat{\boldsymbol{\theta}}} = -\dot{\hat{\boldsymbol{\theta}}}$$

where theta hat is the estimated value of θ and theta tilde is the deviation between the estimated and the true value.
For the reason that the stability analysis do not need the value of the target system, to be general, the known parameters are not substituted in the latter proving.
The output error can be computed as

$$e_1 = k_p \frac{Z_m}{R_m}\left(\hat{\theta}^T \omega - \theta^T \omega\right) = k_m \frac{Z_m}{R_m}\left(-\frac{k_p}{k_m}\tilde{\theta}^T \omega\right)$$

For this assignment, The system is strictly minimum phase. Its relative degree is 1, so the Nyquist plot lies entirely in the right half plane. The poles are negative, namely -1 and -2, so that the system is strictly stable. Thus the target system is SPR (strictly positive real).
Transform the transfer function into state space form,

$$\dot{e} = A_m e + b_m$$
$$e_1 = c_m^T e$$

For this assignment,

$$A_m = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}, b_m = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, c_m^T = [1 \quad 3]$$

According to the Kalman-Yakubovic Lemma, since the is strictly positive real, there exist a pair of positive matrix P and Q such that

$$A_m^T P + P A_m = -Q$$
$$P b_m = c_m$$

For this assignment, the appropriate value of the pair can be selected as

$$P = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 1 \end{bmatrix}, Q = \begin{bmatrix} 2 & 1.5 \\ 1.5 & 2 \end{bmatrix}$$

which are both symmetric and their eigenvalues are positive, thus are positive real.
It is noticeable that e_1 is derivable and if e is square integrable and derivable , so e_1 is.
Design the Lyapunov function as

$$V=\frac{1}{2}e^{T}Pe+\frac{1}{2}\left|\frac{k_p}{k_m}\right|\tilde{\theta}^{T}\Gamma^{-1}\tilde{\theta}=\frac{1}{2}e^{T}Pe+\frac{1}{2}\left|k_p\right|\tilde{\theta}^{T}\Gamma^{-1}\tilde{\theta}$$

The derivative of the Lyapunov function can be computed as

$$\dot{V}=-\frac{1}{2}e^{T}Qe+e^{T}Pb_m\left(-k_p\tilde{\boldsymbol{\theta}}^{T}\boldsymbol{\omega}\right)+\left|\frac{k_p}{k_m}\right|\tilde{\boldsymbol{\theta}}^{T}\boldsymbol{\Gamma}^{-1}\dot{\tilde{\boldsymbol{\theta}}}$$

$$=-\frac{1}{2}e^{T}Qe$$

which is negative semi-definite. Therefore the e and θ are bounded. And e is also square integrable. From the previous discussion , it is known that the e_1 is also square integrable and derivable ,i.e. $e_1\in L_2\cap L_\infty, \dot{e}_1\in L_\infty$
Therefore from the Barbalat's lemma, we can conclude that

$$\lim_{t\to\infty}e_1(t)=0$$

Q3

Simulating with the initial value of $\hat{\theta}=[1.5, 2, -1.5, 0]$, which has no effect on the general discussion. The gain matrix for case one are selected as *I*. The second gain matrix is selected as *10·I*.

Figure2 illustrates the tracking performance and tracking error when r=sin(t). In the starting period, the tracking performance is poor, then after few steps, the tracking error converges to zero. This phenomenon is consistent with the above proving, which indicates the e_1 should converge to zero.

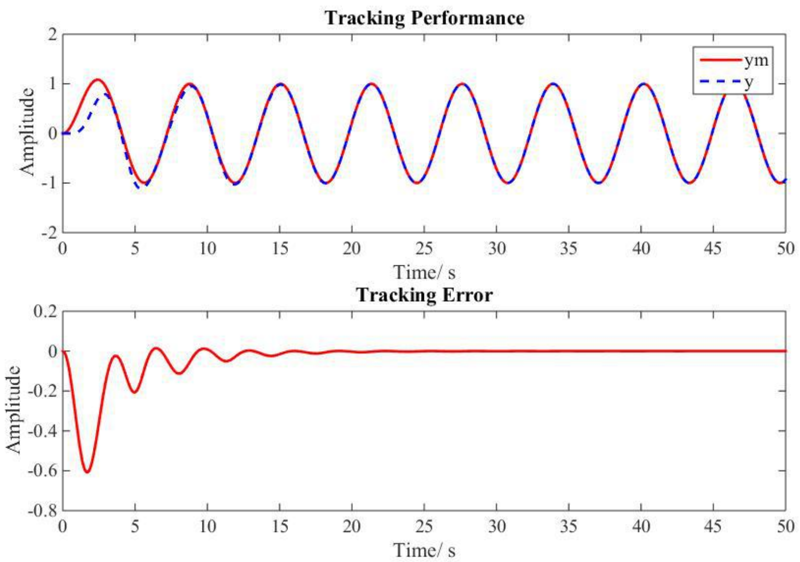

Figure 2 The tracking performance and tracking error when r=sin(t)

Figure3 illustrates the tracking performance and tracking error when r=sin(t)+sin(3t). The convergence takes a longer time. In order to make a clear illustration of the error plot, a good set of initial value and gain matrix is selected, which is mentioned before.

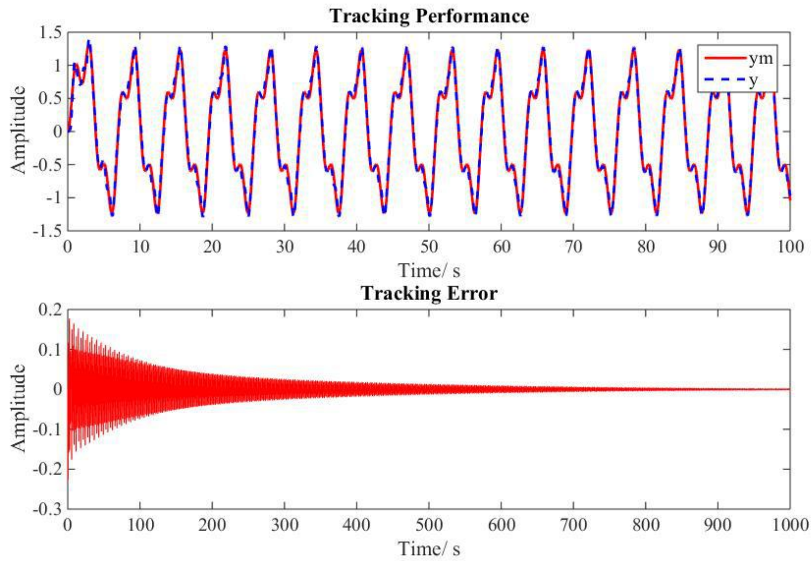

Figure 3 The tracking performance and tracking error when r=sin(t)+sin(3t)

If we want the parameters θ converge to the real value, the input signal should be sufficient rich of order 2n[1]. For this second order system, the input should be at least order of 4. Moreover, the $u = \sum_{1}^{n} \sin(\omega_i t)$ is PE of order 2n. It implies the θ hat will converge to θ when the second reference signal, r=sin(t)+sin(3t) is imposed. The first reference signal should just be bounded. The figure4 illustrates the above conclusion, in the first plot, the θ tilde converges to nonzero value and in the second plot it finally converges to zero.

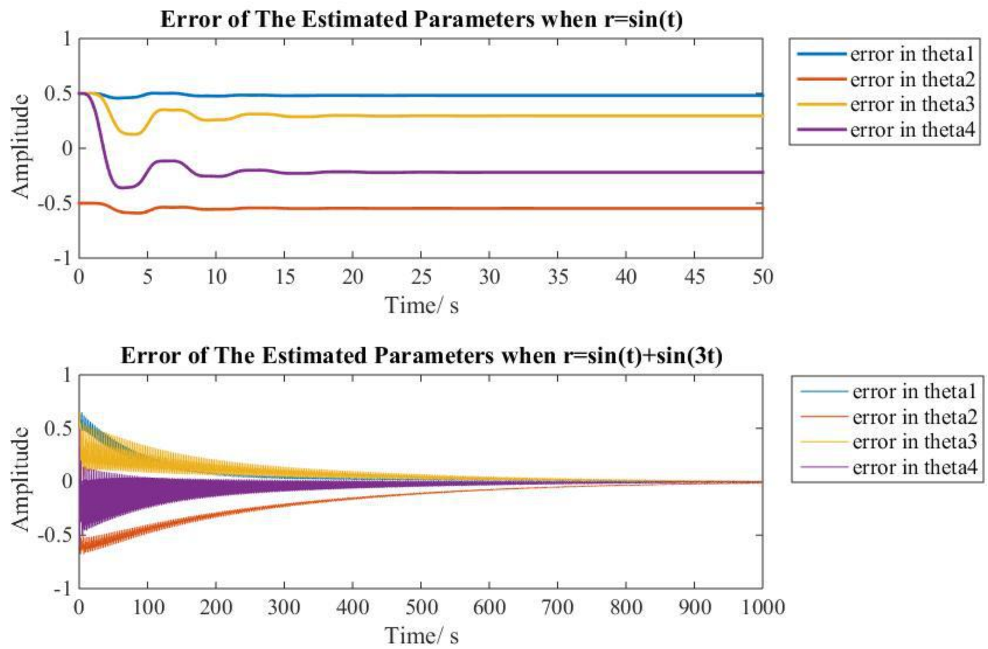

Figure 4 The error between estimated value and real value of θ

Reference: [1] Ioannou, P. A., & Sun, J. (1995). Robust adaptive control. Prentice-Hall, Inc.