EEEN60108 CONTROL FUNDAMENTALS

A REPORT

ON

THE DEVELOPMENT OF A MODEL AND CONTROLLER OF A VERTICAL TAKE-OFF AND LANDING (VTOL) SYSTEM USING LABVIEW AND SIMULINK

Yi Zhou

STUDENT ID: 99767145

1. SUMMARY

This report is mainly on the identification and control of a vertical take-off and landing(VTOL) experiment device using Labview. A Proportional Integral Derivative (PID) controller is designed to enable the beam stable at the desired position. The basic tuning criterion of PID is summarized.

2. INTRODUCTION

The experiment is conducted in a VTOL ,which stands for 'vertical take-off and landing', trainer developed by Labview. The Labview GUI is used to set input signal, measure the necessary signal and implement the controller.

The device mainly consists of two parts: the DC electric motor and mechanical part. A DC electric motor is used as the power source of the mechanical system. A rotary beam is considered as the model of VTOL aircraft. A fan and an adjustable counterweight is mounted at two ends of the beam respectively. The pitch angle position of the beam can be acquired by a rotary encoder.

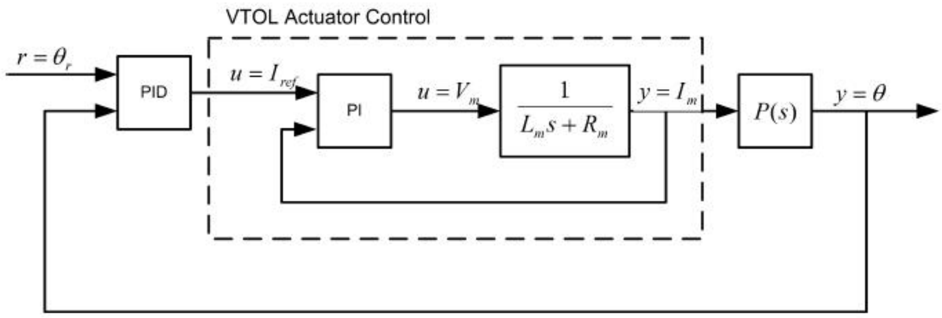


Figure 2.1 The Cascade Control

The cascade control is implemented in the VTOL trainer as figure 2.1. The system is broken into two sub systems: the inner loop to control the current and outer loop to control the pitch angle position. In the inner loop, a PI controller is used. In the outer loop, the modified PID controller, which can avoid derivative kick, is used to regulate the pitch angle position.

3. RESULTS AND DISCUSSION

3.1 Current Control

3.1.1 The Motor Dynamics

The mathematical model of the DC motor is

$$u_m = R_m i_m + L_m \frac{di_m}{dt} \tag{3.1}$$

Where u_m is the voltage , i_m is the current, R_m is the resistance and L_m is the inductance.

When the system reaches its steady state, the derivative term equals zero. Using the measured data, the linear model between the current and voltage in the steady state can be described as

$$U = 2.2173 \times I + 0.7238 \tag{3.2}$$

Regardless the constant term, the approximate steady state gain equals 2.2173, which can be considered as the resistance of the circuit.

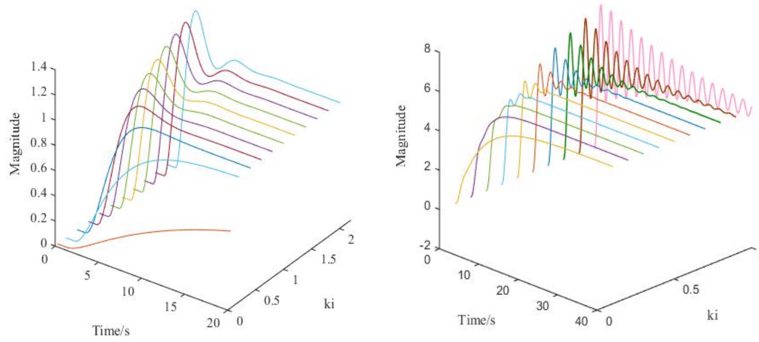
Apparently, due to the inductance in the circuit, the circuit should be considered as a dynamical system. It is observed that the transient response of current lags behind the the step change of voltage signal. In order to eliminate the effect of the inductance, PI controller is implemented. The kp is set to 0.25 and ki is set to 10. As a result, the current can track the reference signal very well.

3.1.2 Tuning of the Current Controller

To investigate the effect of integral controller, set the kp=2.35 , ki=0 and kp=0 ,ki=10 separately. Without the integral term, there always exists a steady state error. In other words, the integral controller can provide the necessary input to the system to compensate the steady state error. In aspect of the rise time, the proportional controller with a small kp have almost the same effect of a integral controller with relative large ki.Therefore ,the proportional controller is more effective as a source of growth than the integral controller.

According to the measured data in Appendix, as the ki parameter increases from 0.5 to 1, the settling time shorten from 4 to 1. It is reasonable because the integral term can be considered as a less effective proportional term to some extent. However, if a much larger pi is used, the settling time should increase. The figure shows the actual effect of different values of ki on first and second system.

An intuitive interpretation of this phenomenon is given below. Suppose a system with closed loop integral controller tracks a step signal. When the response hit the reference line, the integral term should be equals to its desired value, which properly eliminates the steady state error. If there is no overshoot, which means the damping ratio of overall system is over 1, a larger ki can accelerate the response and shorten the settling time. However, if a overshoot occurs, the integral term should deviate from its desired value, thus causes a new error. Then, the feedback takes effect and the output is subject to oscillate around the set point. Thus increase the settling time. A simplified mathematical proof of this deduction is shown in Appendix.



(a) First Order System (b) Second Order System

Figure 3.1 The Effect of Different Magnitude of KI for Integral Controller

3.1.3 Flight Dynamics

The steady state relationship between current and pitch angle position of the beam is investigated in this section. The physical model of this VTOL device can be described by the equation

$$K_t I_m + m_2 g l_2 \cos\theta - m_1 g l_1 \cos\theta - \frac{1}{2} m_h g l_h \cos\theta = 0 \tag{3.3}$$

The current is proportional to the cosine of pitch angle, which means the system is nonlinear. In order to linearize this model, the range of angles should be restricted near the horizontal position (-20 deg to 20 deg is suggested) where the cosine terms can be considered as constant one.

According to the measured data, the steady-state gain between the current (as input) and the pitch angle position in radius is 0.6.

This system also shows dynamical behavior when set a step change of current. Compared with the inner loop dynamics, the response of this mechanical system is much slower. Therefore, the dynamics of inner loop can be ignored when analyzing the response of pitch angle position.

3.2 Modeling

Set the kp and ki for inner loop to 0 and 50 respectively. To obtain the open loop transfer function of current and pitch angle, firstly, stabilize the beam to horizontal position. Then, the response of a step change in the current set point from 1.08 A to 1.38 A is measured.

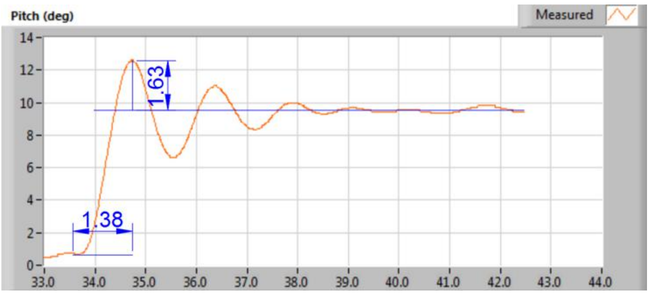


Figure 3.2 The Measured Response

From the plot above, the gain G, natural frequency ω_n and damping ratio ζ can be computed using the magnitude of input and output signal ,peak overshoot value M_p and its time t_p. It can be measured from the figure above that the M_p is 1.63 and t_p is 1.38 s. Notice that the pitch angle should convert to radius from degree when solving the gain.

$$\begin{aligned} M_p &= e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \times 100\% \\ t_p &= \frac{\pi}{\omega_n \sqrt{1-\xi^2}} \end{aligned} \tag{3.4}$$

Computation results of the the gain G, natural frequency ω_n and damping ratio ζ are 0.55, 2.40, 0.32 respectively. Then the transfer function is constructed as

$$G(s) = \frac{G}{s^2 + 2\zeta\omega_n s + \omega_n{}^2} = \frac{3.168}{s^2 + 1.54s + 5.76} \tag{3.5}$$

The labview itself can also provide the system identification tool. A periodic square signal is used as input. Set the amplitude equals 0.2A ,and frequency equals 0.2Hz. Then run the software for at least 20 seconds, the estimated transfer function is obtained as

$$G(s) = \frac{0.537899}{0.164349s^2 + 0.288779s + 1} = \frac{3.27}{s^2 + 1.76s + 6.08} \tag{3.6}$$

The two poles of the model are -0.8786 + 2.3049i and -0.8786 - 2.3049i which all located in the left half plane. Thus the plant model is stable. The gain, natural frequency and damping ratio are 0.54, 2.47, 0.36 respectively. Since the damping ratio is less than 1, the system is called under damping and tends to oscillate around the set point.

To verify the estimated transfer function, re-run the previous experiment to get the measured data as well as the theoretical curve. The frequency and gain match well, but the damping ratio of the real system is obviously smaller than the simulated one. The reason maybe the sampling time for one period is too short. The disturbances also can contributes to the deviation.

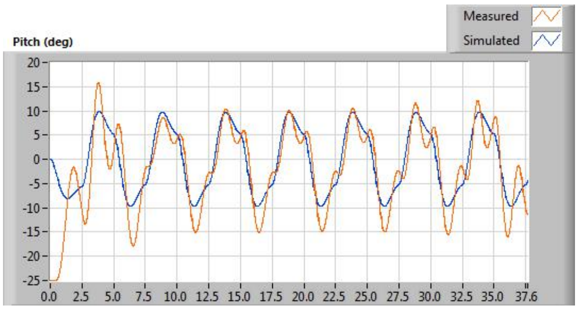


Figure 3.3 The Difference Between the Measured Data and Simulated Results

3.3 Flight Control

3.3.1 PD Controller Steady-State Analysis

Firstly, the system should be stabilized at the horizontal position. In the current control section, set kp_c=0 and ki_c=50. In the position control parameter section, set kp=1, ki=2, kd=1. After the system is stable at zero degree set point, change the PID parameters to 1, 0, 1 respectively. Then apply a step change which amplitude is 10 in the set point. The response of the system is recorded. The measured steady state error of the pitch angle position is

$$SS\_Error = (\frac{10-3.3}{10}) \times 100\% = 67\% \tag{3.7}$$

Using the model computed in the previous section, the theoretical solution of steady state error is obtained

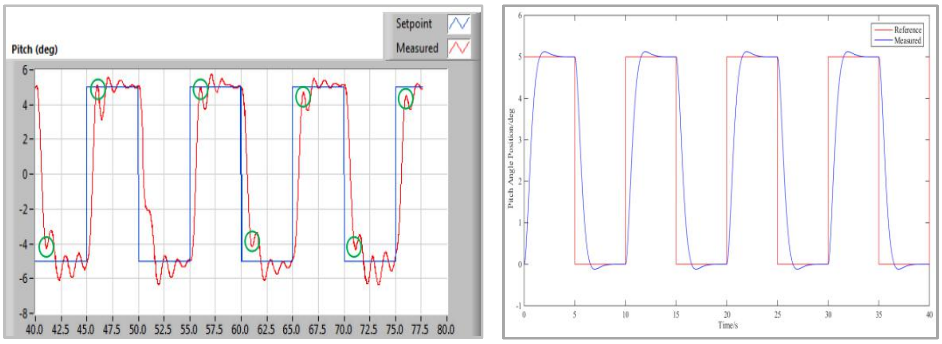
$$\begin{aligned} SS\_Error &= \left(1 - \frac{SS\_Gain \times Value\_of\_Setpoint}{Value\_of\_Setpoint}\right) \times 100\% \\ &= 1 - \lim_{s \to 0} \frac{kp \cdot G(s)}{(kp + s \cdot kd) \cdot G(s) + 1} = 1 - \frac{1 \times 0.54}{1 \times 0.54 + 1} = 65\% \end{aligned} \tag{3.8}$$

As the result shows, the theoretical value of steady state error is very closed to the measured value.

3.3.2 Tuning of the PID Controller

A PID controller which is described in part 2 is implemented to this system.As far as we concerned, the rising time is the most important factor. Because the frequency of the input is relatively high, the response should be fast enough to track the signal. Thus we first decide a larger kp parameter. However, if kp is too large, the response tends to oscillate especially for this periodic input. The integral term is mainly used for compensating the steady state error. Derivative term can make the curve smooth. It acts like a barrier to the change of the response so it may increase the rising time. After several attempts, the final result we determined is kp=1.7, ki=3, kd=1. The performance of the system is measured as: rising time is 0.92 s,overshoot is 6%, steady state error is zero.

Compared with the simulation results by Simulink, the results is similar in most of the performance, namely rising time is 0.88 s, overshoot is 4% and steady state error is zero. However,the measured results is much more oscillatory which is consistent with the previous results in section 3.2.



(a) The Measured Data in Labview (b) The Simulated Results in Simulink

Figure 3.4 The Response of Step Change

3.3.2 Some Observation

When tuning the PID controller, I prefer to start with a zero kd parameter. However, the output signal of this system oscillates and becomes unstable when using a P controller. According to Appendix , It is proved that a second order system can not be destabilized by proportional control. However, for this periodic square input, if the settling time is too long to stabilize the response, the system cannot get stable in one period and tends to become oscillatory. If the overshoot is too large, the nonlinear behavior is also non-negligible.

There is an odd phenomenon which is hard to understand in the figure 3.4 (a). It is noticeable that those peak of the response is under the reference signal. At those specific peak points, the derivative term is zero while the proportional and integral terms are obviously both positive. Given a positive input. It behaves like the non-minimum phase system. This may owe to some disturbance with time delay or the uncertainty of the system like some nonlinear terms. It would help to understand if the process signal can be measured.

When setting the derivative term a comparative large number, the smooth curve cannot be acquired. The response oscillate around the set point in a small amplitude. It can be seen that the derivative term is very sensitive to noise.

4. CONCLUSIONS

The relationship of voltage to current and current to pitch angle position both can be simplified as a linear model. The transfer function obtained by system identification agrees well with measured one in aspect of the gain and natural frequency. However, the damping ratio have some deviation. The basic criterion of trail and error method for tuning PID is summarized. It can be seen that these three parameters can have both positive and negative effects on the response. The design of a PID controller should depends on the realistic requirements of the system.

5. REFERENCES

[1] The University of Manchester. Laboratory Manual on Developing a Model and Controller of a VTOL system using LabVIEW and Matlab/Simulink

[2] Quanser. Student Workbook QNET VTOL for NI ELVIS

[3] Lehigh University. ME389_AERO01_VTOL_Guideline

6.APPENDIX

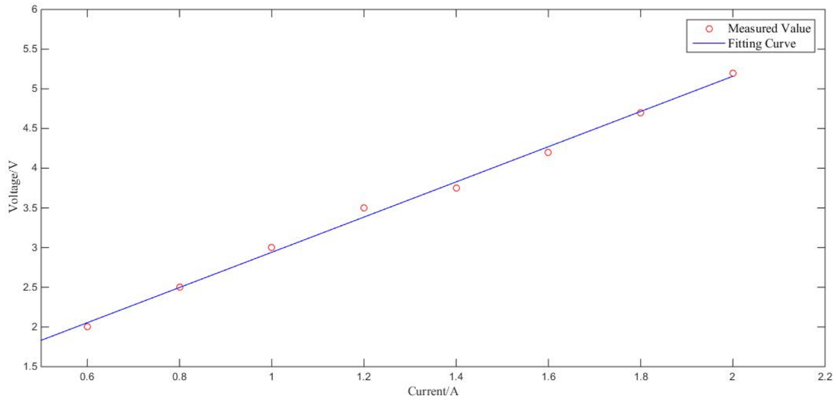


Figure 6.1 The Linear Model of Voltage and Current

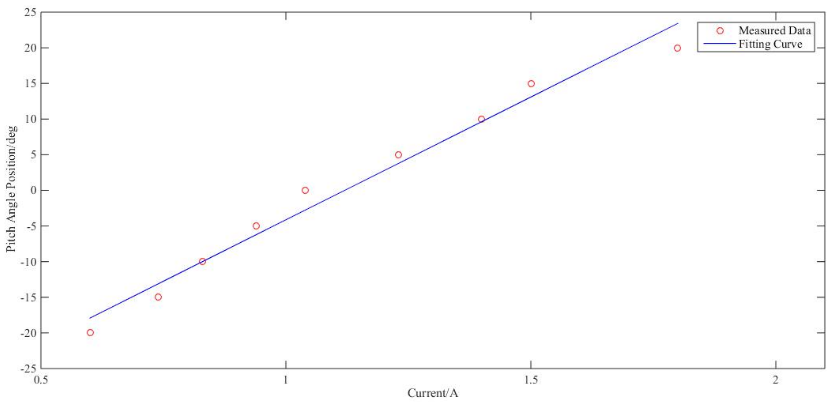


Figure 6.2 The Linear Model of Current and Pitch Angle Position

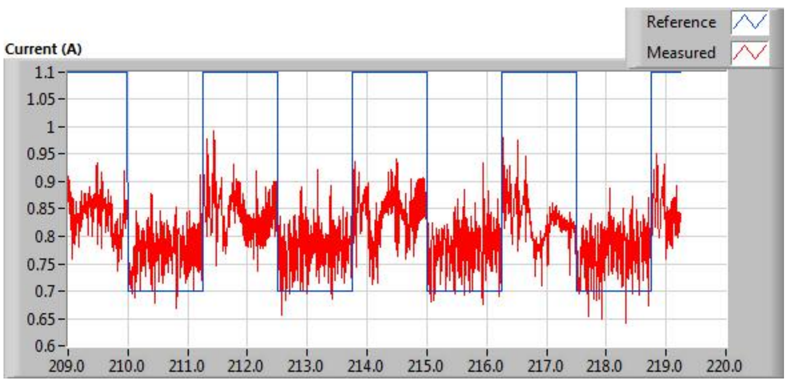


Figure 6.3 KP=0.25,KI=0 for Inner Loop

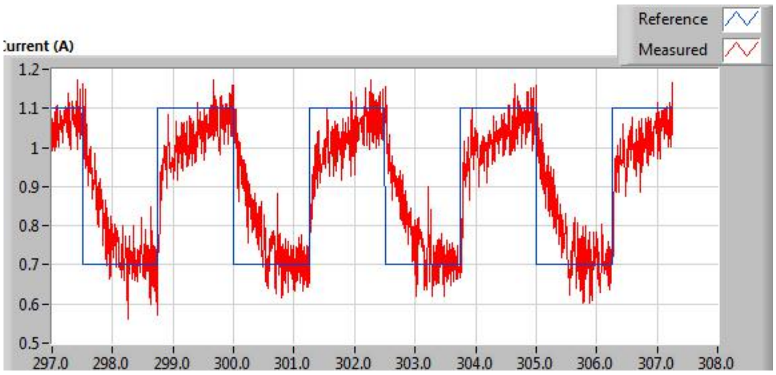


Figure 6.4 KP=0,KI=0 for Inner Loop

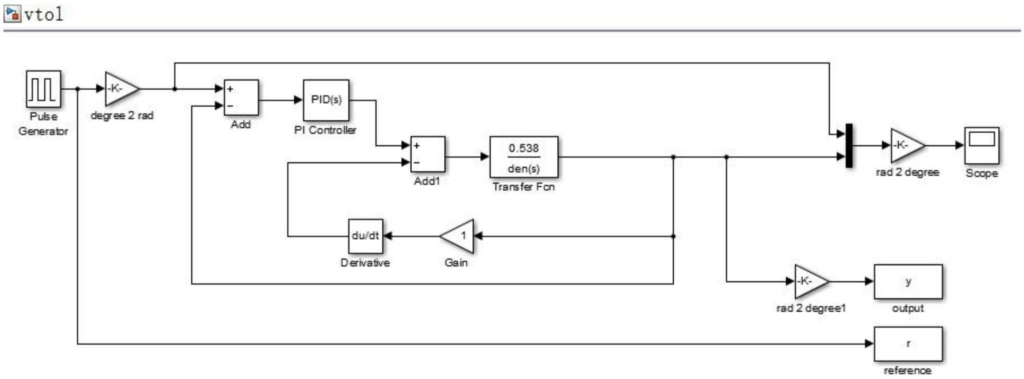


Figure 6.5 The Simulink Model of the PID Control

Proof 1

The transfer function of a first order system with integral feedback control is

$$\frac{k_i b}{s^2 + as + k_i b}$$

The natural frequency and damping ratio are

$$\omega_n = \sqrt{k_i b}$$

$$\zeta = \frac{a}{2\sqrt{k_i b}}$$

For ζ less than 1

$$\frac{e^{-\zeta w_n t}}{\sqrt{1-\xi^2}} = 0.05$$

Thus if the ki increase, the settling time may increase.

Proof 2

Suppose the transfer function of a stable system is

$$\frac{c}{s^2 + as + b}$$

Where

$s_1 + s_2 = -a < 0$
$s_1 s_2 = b > 0$

If implemented with a P controller, the new transfer function is

$$\frac{c}{s^2 + as + b + kp \cdot c}$$

Where

$s_1 + s_2 = -a < 0$
$s_1 s_2 = b + kp \cdot c > b > 0$

Thus, the new poles the system should still be negative. The system is always stable no matter what the value of kp is.