DIGITAL CONTROL COURSEWORK

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Q1.

The transfer function of the system is

$$G(s)=\frac{Y(s)}{U(s)}=\frac{1}{s^2+5s+6}=\frac{1}{(s+2)(s+3)}$$

The open-loop performance can be estimated from the transfer function. The poles of the system are -2 and -3, indicating that the system is stable. The open loop gain is 1/6. After calculation, the natural frequency is 2.5. The damping ratio is 1.02, which is larger than 1. Thus, there will be no overshoot. The rising time is approximately 1.41 s.

The simulated open-loop step response of the system is showed as below.

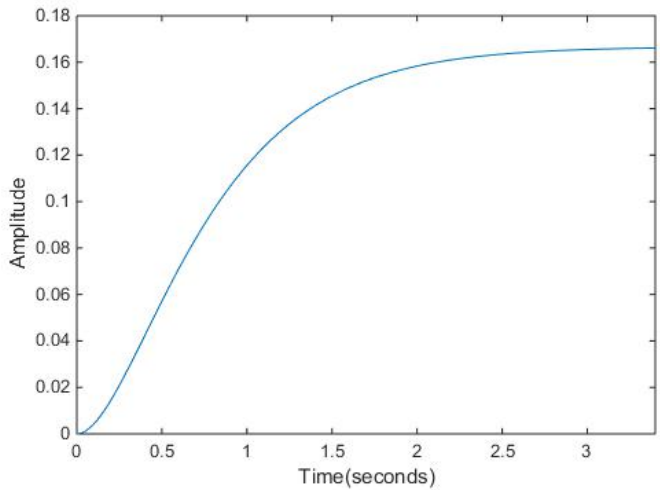

Figure 1 step response for Q1

Q2.

The PD controller can be designed as

$$C(s)=k_p+sk_d$$

The closed loop transfer function is

$$H(s)=\frac{G(s)C(s)}{1+G(s)C(s)}=\frac{k_p+k_d s}{s^2+(k_d+5)s+6+k_p}$$

To place the poles, the parameter can be computed as

$$k_p+6=s_1s_2=(-4+5j)(-4-5j)$$
$$k_d+5=-(s_1+s_2)=-(-4+5j-4-5j)$$

Thus, the PD controller is

$$C(s)=35+3s$$

Substitute the parameters in the closed loop transfer function,

$$H(s)=\frac{3s+35}{s^2+8s+41}$$

From the transfer function, the real parts of the poles are positive. Thus the system is stable. The steady state gain is 0.8537 . The The damping ratio is 0.6247 ,indicating that the system is under damped.

The simulated closed-loop step response of the system is showed as below. The rise time can be measured as 0.246 s. The settling time is 0.84s. The overshoot is 10.01%.

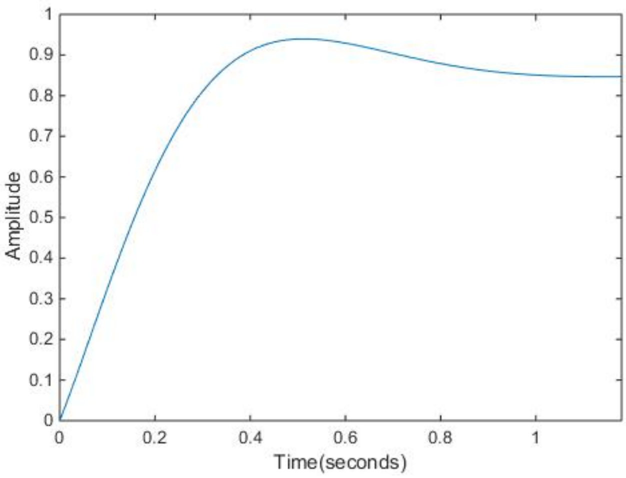

Figure 2 step response for Q2

Q3.

According to the shannon sampling theorem, the sampling rate should be at least 2 times (always 10 times) of ,for simplicity, natural frequency. Thus the sampling time should be less than 1.25 s. Thus in these two case, the aliasing may not occurred.

(a) The discrete transfer function of the system is

$$G_{d1}(z)=\frac{0.1156z+0.02134}{z^2-0.1851z+0.006738}$$

Figure 3 shows the open-loop step response of this system. If we concerned much about the transient process, this sampling time of 1 second may be a little bit larger.

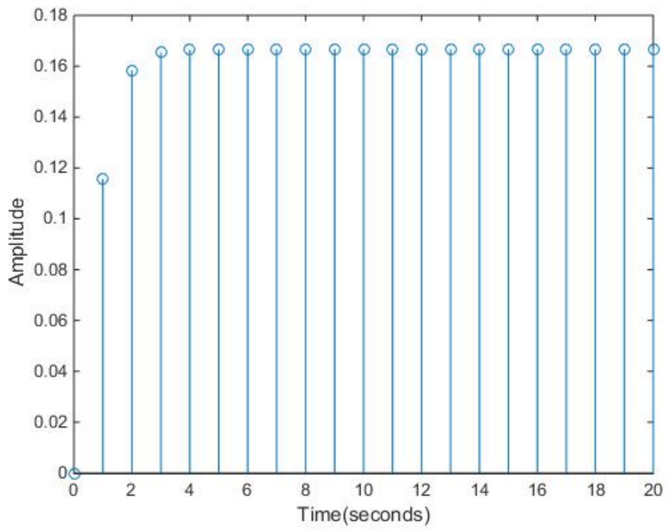

Figure 3 step response for Q3(a)

(b) The discrete transfer function of the system is

$$G_{d2} = \frac{0.004241z + 0.00359}{z^2 - 1.56z + 0.6065}$$

Figure 4 shows the open-loop step response of this system. Compared to figure 3, due to the larger sampling rate, the transit behaviour is better captured. However, from the transfer function, it is noted that the small sampling time may results some computation effort, due to the small value of the parameters.

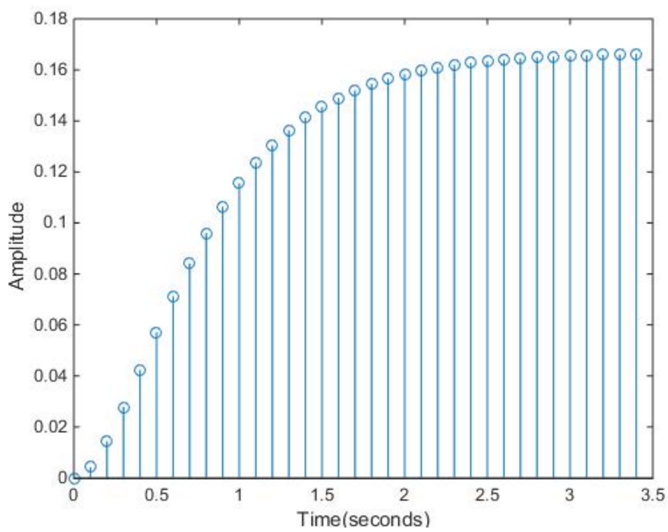

Figure 4 step response for Q3(b)

Q4.

The PD controller can be designed as

$$C_d = k_p + k_d \frac{1 - z^{-1}}{T}$$

(a) The characteristic equation of poles is

$$d_1(z) = z^3 + [0.1156(k_p + k_d) - 0.1851]z^2 + (0.006738 + 0.02134k_p - 0.0943kd)z - 0.02134kd$$

In order to place two of the poles at the desired location,

$$d_1(z_1) = d(e^{-4+5j}) = 0$$
$$d_1(z_2) = d(e^{-4-5j}) = 0$$

The kp and kd can be computed by solving these two equations.
Thus, the PD controller is

$$C_{d1} = -0.1936 + 0.0028 \cdot (1 - z^{-1})$$

The overall closed loop transfer function is

$$H_{d1}(z) = \frac{-0.02206 \cdot z^2 - 0.004396 \cdot z - 0.00006}{z^3 - 0.2072 \cdot z^2 + 0.002342z - 0.0006}$$

By letting z=1, the steady state value is -0.0344.

Figure 5 shows the closed-loop step response of the system. Due to the negative k_p , the step response is negative.

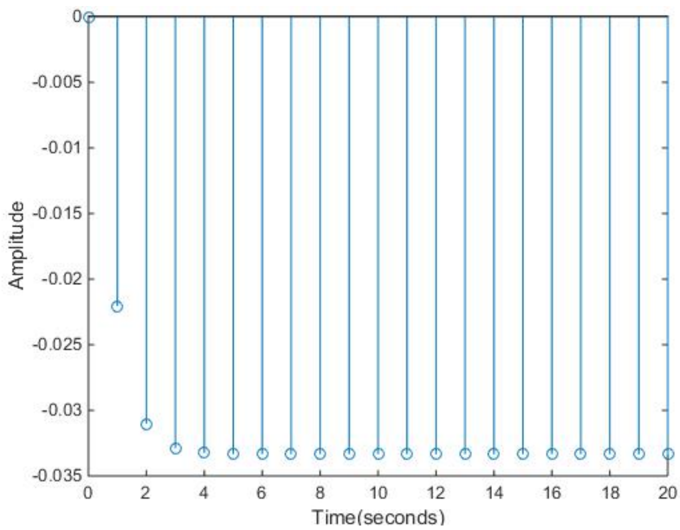

Figure 5 closed-loop step response for Q4(a)

The poles and zeros for this closed-loop transfer function are showed as below. All the three poles are in the unit circle, which means the system is stable. The real parts of poles are small so that the response is slow. Compared to the continuous case, there is one additional zero and one additional pole in the transfer function. These undesired zeros and pole lead to the negative gain. The desired poles are very close to the zero near the origin. Thus the system's behaviour is dominated by the other pole.Thus there are almost no oscillation in the step response.

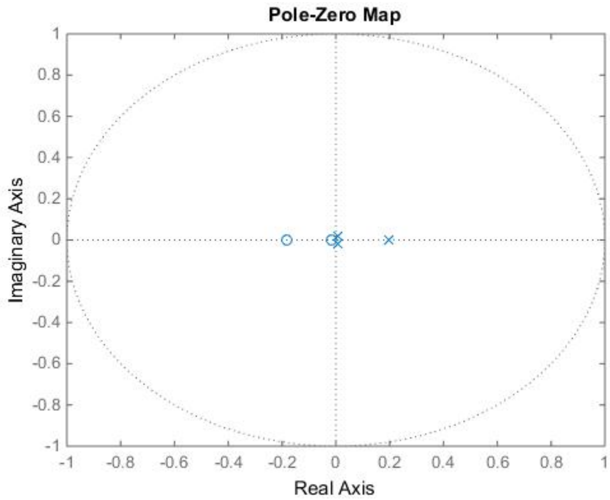

Figure 6 pole-zero map for Q4(a)

(b) The characteristic equation of poles is

$$d_2(z) = z^3 + [0.004241(k_p + k_d) - 1.56]z^2 + (0.6065 + 0.00359k_p - 0.00065kd)z - 0.00359kd$$

In order to place two of the poles at the desired location,

$$d_2(z_1) = d(e^{-0.4+0.5j}) = 0$$
$$d_2(z_2) = d(e^{-0.4-0.5j}) = 0$$

The kp and kd can be computed by solving these two equations.
Thus, the PD controller is

$$C_{d2} = 22.4833 + 23.5559 \cdot (1 - z^{-1})$$

Compared to the previous case, the kp is larger. Thus the response is much faster.
The overall transfer function is

$$H_{d2} = \frac{0.1952 \cdot z^2 + 0.06537 \cdot z - 0.08456}{z^3 - 1.364 \cdot z^2 + 0.6719 \cdot z - 0.08456}$$

The steady state gain is 0.7881.
The closed-loop step response of the system is

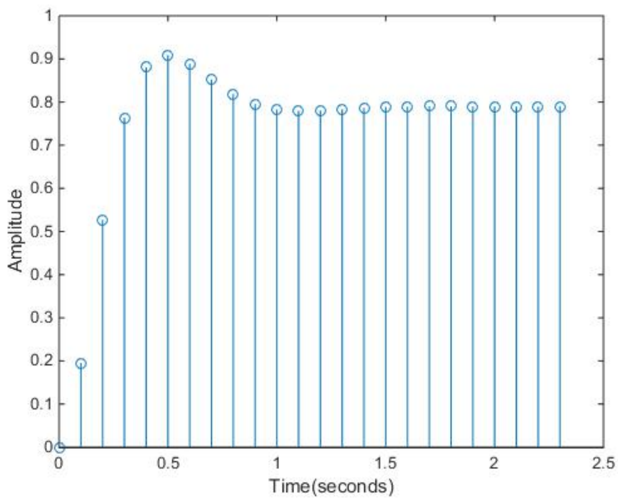

Figure 7 closed-loop step response for Q4(b)

The poles and zeros for this closed-loop transfer function are showed as below. The situation of desired poles is much better than the previous case. Thus, the system's behaviour is similar to the continuous case. Compared these two cases with different sampling time, it can be inferred that more accurate the discrete system, more effective PD controller can take effect.

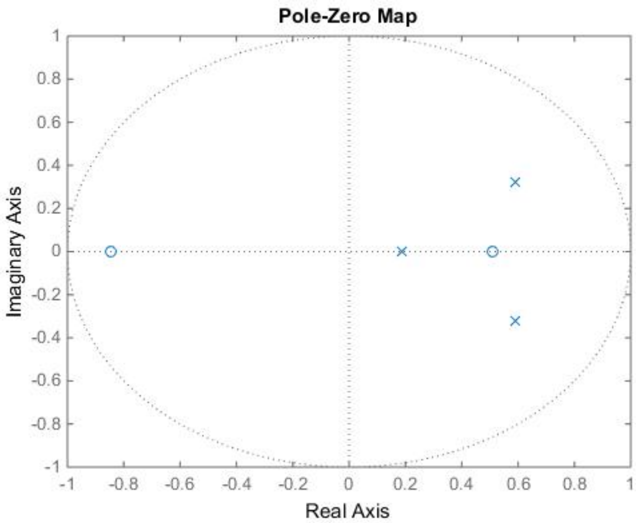

Figure 8 pole-zero map for Q4(b)