Motor Speed and Position Control
Solution Sheet
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MSc

Exercise 1:

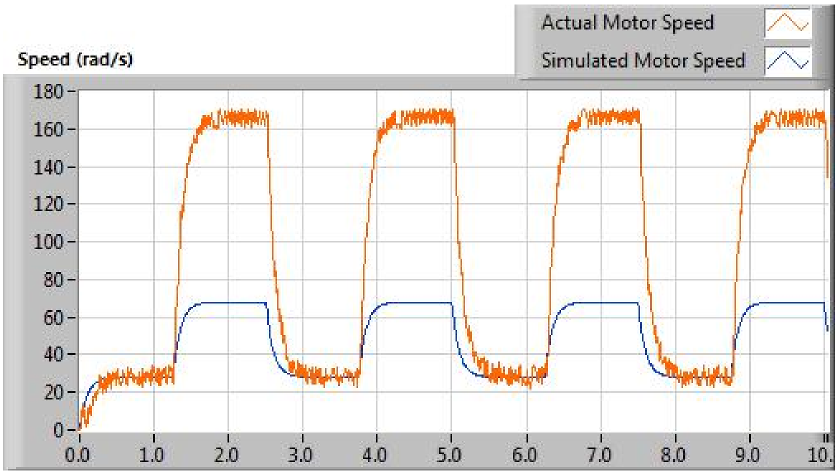

Figure 1 Speed Response

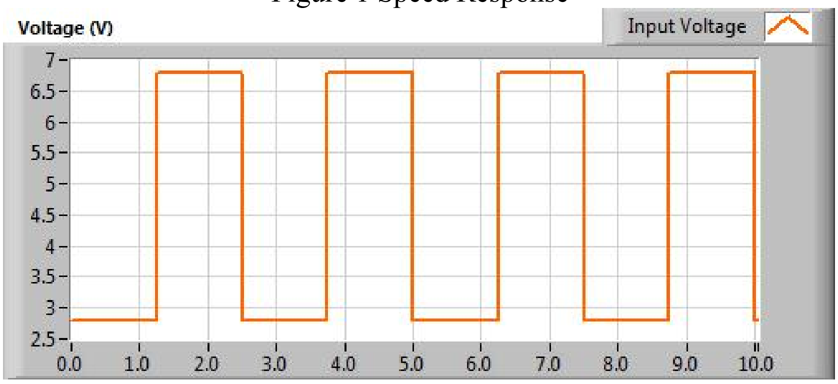

Figure 2 Input Voltage

Exercise 2:

Gain:

K=34.7346;

Time Constant:

T=0.1236;

Transfer Function:

$$G(s) = \frac{34.7}{0.12s + 1}$$

<u>Exercise 3:</u>

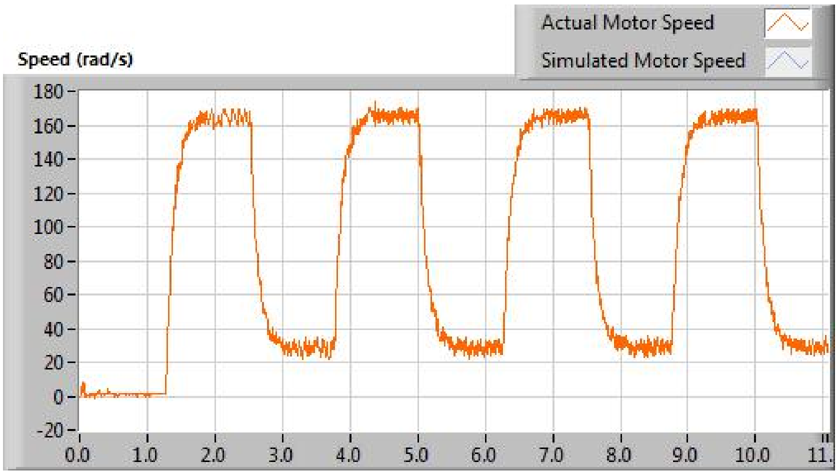

Figure 3 Actual Speed

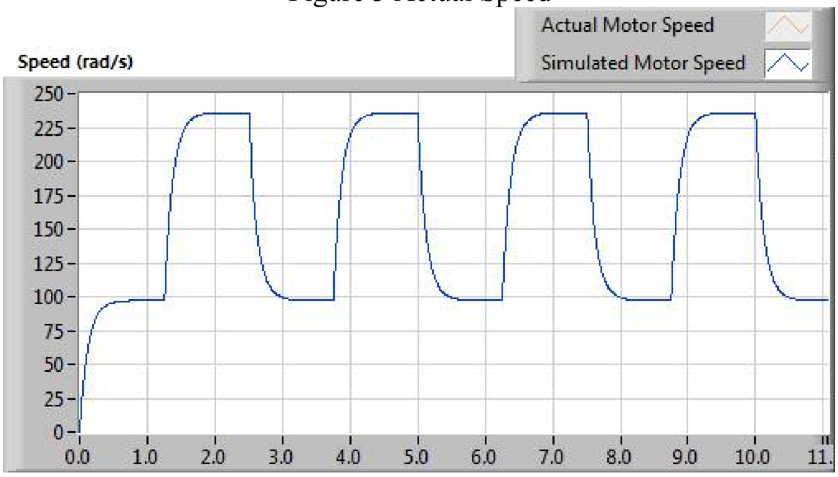

Figure 4 Simulated Speed

Problem:

The transfer function is derived from the square wave section. However, in the first step section, the system behaves differently from what the transfer function predicts. In figure 1, at starting stage, when given a step change of voltage from 0 to 2.8V, the steady motor speed is about 28 rad/s. In figure 2, the motor even has no response during the steady change. From the estimated transfer function, there is approximately a loss of 2V in voltage. Thus the speed deviates from the estimated value for about 70 rad/s. This lead to a constant difference between the actual speed and simulated speed during the latter periodic section. Besides this, the actual response has some small oscillation compared to the smooth curve of simulated one.

Analysis:

The first reason is the system may not simply be a first order system. It can be a higher order system or even a nonlinear system. The time delay is obvious in figure 3. Thus, the estimated transfer function is not applied at the starting stage.

The second reason is the friction. There are two types of friction: the micro mode and marco mode. Thus the friction is both position-variant and velocity-variant on different conditions. The model could be very complex due to these frictions.

<u>Exercise 4:</u>

If use a pure proportional controller, it is simply a first order system. Increasing kp speeds up the response. However, when kp equals 0.5, the response is strongly oscillatory and gradually falls to zero speed. The voltage signal is very noisy which oscillate between the maximum and minimum limited voltage in a fast frequency. It can be seen that the average value of voltage is almost zero due to the limitation. Thus the motor speed may fall. The ki can reduce the steady state error. However, the response of integral process is slow. When kp equals 0 and ki equals 0.05, the response is too slow to track the reference in one period. Thus the system tends to become unstable if ki is too large.When using the PI controller, if kp equals a small value like 0.05 and ki equals 1, the overshoot occurs. It can be deduced that the proportional reaction is too slow to diminish the accumulated error of integral term in this case.

Exercise 5:

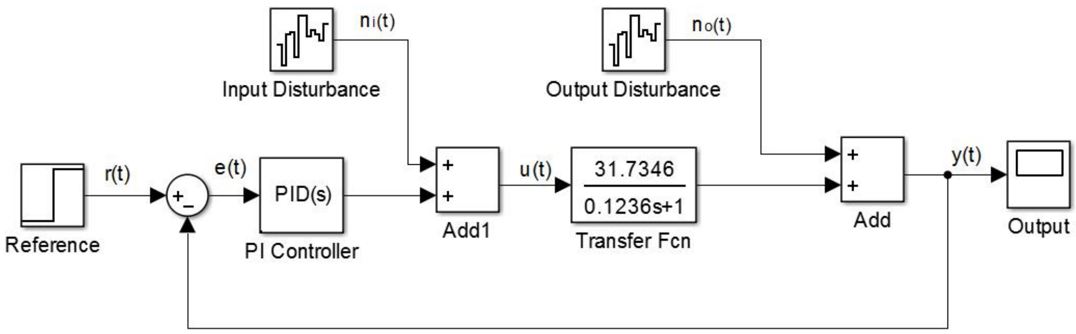

Figure 5 Block Diagram

From figure 5, the closed loop equation can be derived as

$$Y(s)=\frac{C(s)\cdot G(s)}{1+C(s)\cdot G(s)}R(s)+\frac{G(s)}{1+C(s)\cdot G(s)}N_i(s)+\frac{1}{1+C(s)\cdot G(s)}N_o(s)$$

The closed loop transfer function of the reference signal is:

$$G_c(s)=\frac{Y(s)}{R(s)}=\frac{C(s)\cdot G(s)}{1+C(s)\cdot G(s)}=\frac{\frac{k_p s+k_i}{s}\cdot\frac{34.7}{0.12s+1}}{1+\frac{k_p s+k_i}{s}\cdot\frac{34.7}{0.12s+1}}=\frac{34.7k_p s+34.7k_i}{0.12s^2+(34.7k_p+1)s+34.7k_i}$$

The closed loop transfer functions of input and output noise are

$$\frac{Y(s)}{N_i(s)}=\frac{G(s)}{1+C(s)\cdot G(s)}=\frac{34.7s}{0.12s^2+(34.7k_p+1)s+34.7k_i}$$

$$\frac{Y(s)}{N_o(s)}=\frac{1}{1+C(s)\cdot G(s)}=\frac{0.12s^2+s}{0.12s^2+(34.7k_p+1)s+34.7k_i}$$

The input disturbance could be the uncertainty in electric circuit .
The output disturbance could be the friction of the mechanical system.

Exercise 6:

According to the previous calculation, the transfer function is:

$$G_c(s)=\frac{Y(s)}{R(s)}=\frac{34.7k_p s+34.7k_i}{0.12s^2+(34.7k_p+1)s+34.7k_i}$$

Thus, in order to make the natural frequency and damping ratio 16.0 and 0.75 respectively, the kp and ki should be:

$$k_p = 0.12 \cdot \frac{\omega_n^2}{34.7} = 0.057$$

$$k_i = \frac{0.12 \times 2\omega_n \zeta - 1}{34.7} = 0.91$$

For a canonical second order system, the overshoot Mp and settling time ts can be computed as

$$M_p = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi} = 2.8\%$$

$$t_s = \frac{4.6}{\zeta \omega_n} = 0.38\, s$$

Exercise 7:

From figure 6, the overshoot is approximately 20% at 2s, which is much higher than estimation. The settling time is about 0.3 s which may not be very accurate due to the noise. It is obvious that the transfer function has one zero. The zero has effect on the transient response of the system. In fact, any in-consistency between the real and estimated model can lead to the different transient response. However, luckily, the steady state value can be controlled by the feedback system.

Figure 6 Realistic Response

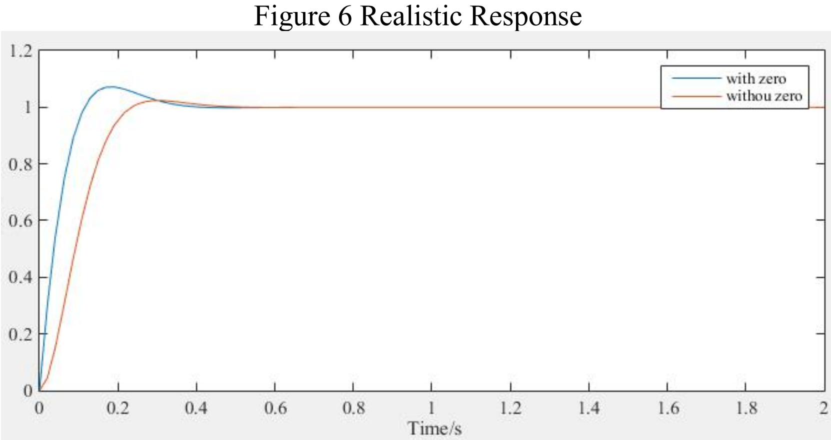

Figure 7 The effect of the zero

Exercise 8:

By setting the bsp to zero, the overshoot reduces from 30% to 22%.

The advantages:

The main advantage is the minimizing of overshoot. If bsp equals zero, the proportional term disappears. Thus, the overshoot is minimized while the rising time increase a little. The overshoot will get larger with increasing bsp. Furthermore, it is more flexible to tune the controller via setpoint weighting in order to satisfy different design compromise[1]. This kind of system which has two different signal paths for the setpoint and output is called two-degree of freedom system[1].

For multivariable system, the derivative kick can be avoided through this method. Actually, the derivative kick is probably not a bad thing in SISO system. However, in multivariable loop, the large change of controller output will act as a large load disturbance to the other loops[2]. By setting the bsp to zero, the proportional controller will act directly on the output signal rather than the error, which eliminates the proportional kick.

Reference:

[1] Åström, Karl Johan, and T. Hägglund. "PID controllers: Theory, Design and Tuning." Instrument Society of America Research Triangle Park Nc (1995).
[2] Chien, I Lung, H. P. Huang, and J. C. Yang. "A Simple Multiloop Tuning Method for PID Controllers with No Proportional Kick." Ind.eng.chem.res 38.4(1999):1456-1468.

Exercise 9:

We start from kp=0.5,ki=0,kd=0. Increasing kp, the overshoot is larger and the settling time increases. The rising time increase a little which is not obvious. By setting kp=10, the response become unstable. The steady state error is always existed.

Then we add the integral term when kp equals 2. Increasing ki should leads to the elimination of steady state error. Actually, the steady state error still exists in the positive position and even over the reference value in the negative position when ki equals 1. When the ki is larger than 2.5, both sides have a steady state error over the reference. Indeed, the integral terms reduce the steady state error, however, it seems a little excessive. The reason will be analyzed in the exe 13.

Finally, we study the effect of derivative term when kp and ki equal to 2 and 5 respectively. When kd ranges from 0.1 to 0.5, the response has no large oscillation any more. However if keep on increasing the kd, the response no longer tracks the reference. The voltage signal becomes very noisy. It can be concluded that the derivative term is very sensitive to noise.

Exercise 10:

The position can be computed through the integration of velocity. Thus, the transfer function is:

$$G_p(s) = \frac{1}{s} \cdot \frac{34.7}{0.12s+1} = \frac{34.7}{0.12s^2 + s}$$

Exercise 11:

Implemented with a PD controller, the close loop transfer function is:

$$G(s) = \frac{(k_p + sk_d)\cdot G_P(s)}{1 + (k_p + sk_d)\cdot G_P(s)} = \frac{34.7k_d s + 34.7k_p}{0.12s^2 + (34.7k_d + 1)s + 34.7k_p}$$

If the damping ratio is 0.6 and natural frequency is 25 rad/s, kp and kd should be:

$$k_p = 0.12 \cdot \frac{\omega_n^2}{34.7} = 2.161$$

$$k_d = \frac{0.12 \times 2\omega_n \zeta - 1}{34.7} = 0.075$$

The overshoot can be computed as:

$$M_p = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi} = 9.48\%$$

Exercise 12:

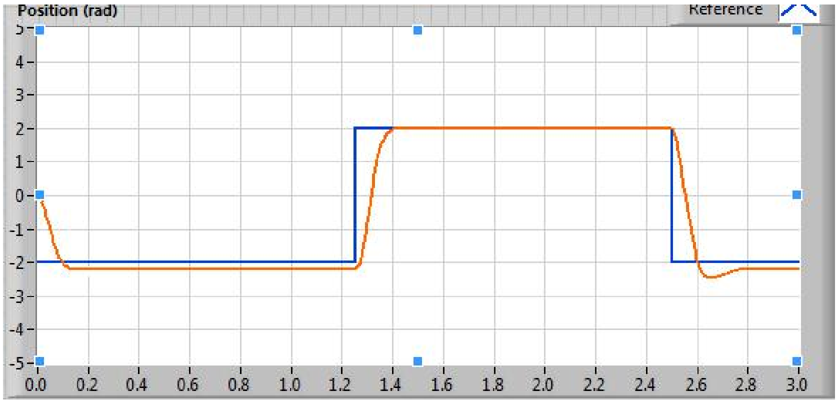

Figure 8 The response of position

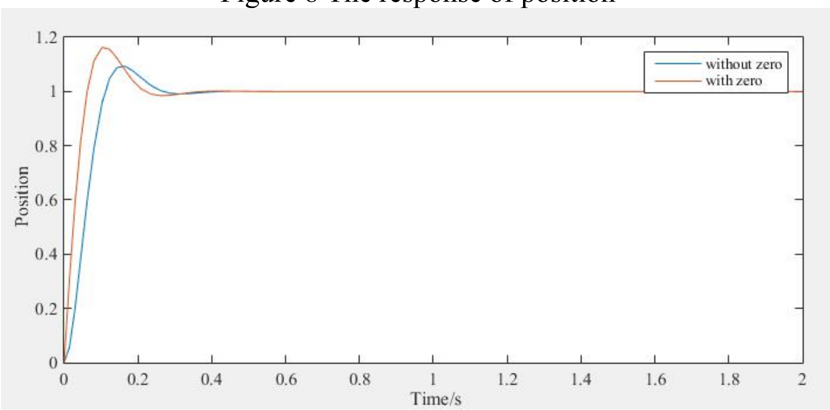

Figure 9 The effect of zero

From the figure the overshoot is almost zero in the upper position and 12.5% in the lower position.
From the results from Simulink, the existing zero increase the overshoot slightly and decrease the rising time. It can be easily observed that the response of the positive and negative position are different.This phenomenon also occurs in previous results. According to figure 7, this non-symmetric tracking error is mainly due to the nonlinear effect of the micro-dynamic friction[3]. Furthermore, this non-symmetry can be eliminated by a large ki, which will be thoroughly discussed in exe 13.

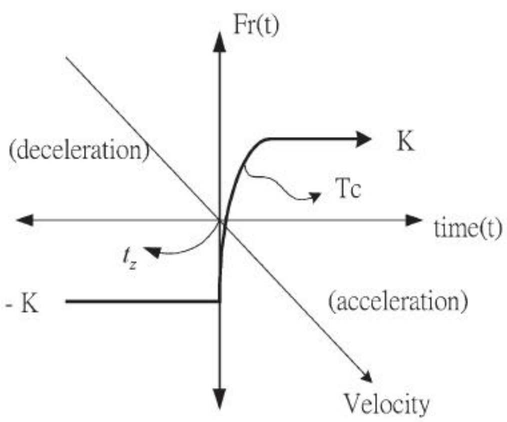

Figure 10 Simplified un-symmetric friction model[1]

Reference:

[3] Chen, J. S., et al. "Friction characterization and compensation of a linear-motor rolling-guide stage." International Journal of Machine Tools & Manufacture 43.9(2003):905-915.
[4] Tsurata, K., et al. "Genetic algorithm (GA) based modeling of nonlinear behavior of friction of a rolling ball guide way." International Workshop on Advanced Motion Control, 2000. Proceedings IEEE, 2000:181-186.

Exercise 13:

In the ideal model, if implemented with a integral controller, the value of “temporal steady state”,which means the flat section of the response, will never be over or under the reference. However, the practical lab results show a different fact. The non-symmetric tracking error discussed in exe 12 also occurs in this case.

It can be deduced that there exists something act like a barrier. In the previous exercises, the existence of friction is introduced. The phenomenon here can be simply explained as” the output torque is not large enough to overcome the barrier of friction”. Thus, the position will be stuck in the position where under the reference. This effect can be eliminate by setting a larger ki value. The integral term can accumulate the error to overcome the friction. Thus, the response come back a few seconds after the flat stage, again reach the position over the reference.