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A Report on

Generalised Predictive Control for DC Motor.

Introduction

Generalised predictive control(GPC) is a simple kind of model predictive control (MPC). It is widely used in industry because of its simple theory and good anticipating performance. The aim of this report is to investigate on how to design and debug GPC for the DC motor using CARIMA model and FSR model.

Task 1

Firstly, the sampling time is determined. Because this script simply use the step signal as input, the response is not so sensitive to sampling time. However, if the response change in a high frequency, the sampling time have to satisfy the Nyquist sampling theorem.

Using the pseudo random binary signal (PRBS) as input signal, the response is got as figure(1) illustrates. The next task is to identify the model for this motor using the linear least squares method.

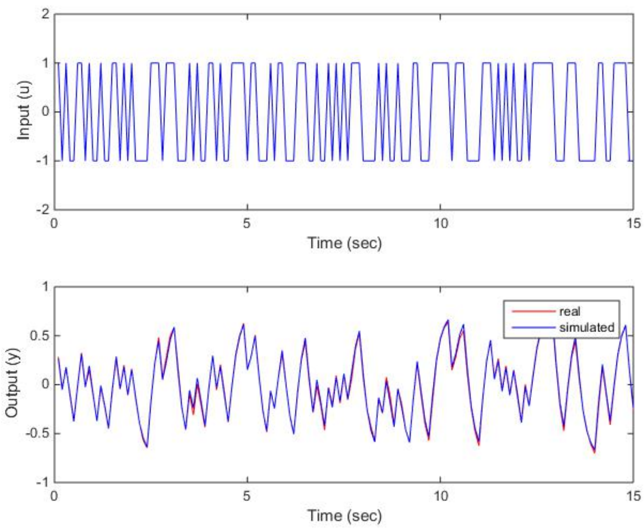

Figure 1 The input and output of the system

Linear least squares method is widely used in many disciplines. In order to find the most accurate solution x for Ax=b，

$$
\begin{aligned}
x &= \operatorname{argmin}\|Ax-b\|^2 \\
&= \operatorname{argmin}(Ax-b)^T(Ax-b) \\
&= \operatorname{argmin}\left(x^T A^T Ax - 2b^T Ax\right)
\end{aligned}
$$

The function is quadratic and the A^TA matrix is positive definite. Therefore, there exists a global minimum for this function. The x can be obtained by making the derivation of the equation equals zero. Thus,

$$x = \left(A^T A\right)^{-1} A^T b$$
$$= A \setminus b \text{ in Matlab}$$

Due to the disturbance and noise in the system, if use too high order, it tends to overfit the curve. It can be seen in the figure(2) that the minimum squares error (mse) grows when the relative order is high. In reality, the higher order also means higher computation cost which is not desired. Thus, the accurate signal with relative low order is preferred.Moreover, the feedback can easily correct the small error of the model. Therefore, the order of (3,1) is selected as an appropriate one for this system.

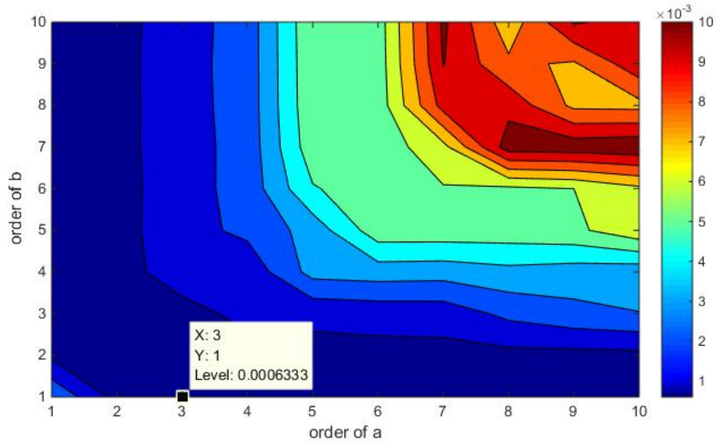

Figure 2 The mse of different system orders

Using the step signal to verify the computed model, as figure 3 shows, the real and simulated responses are very close. The accuracy of identification is satisfying.

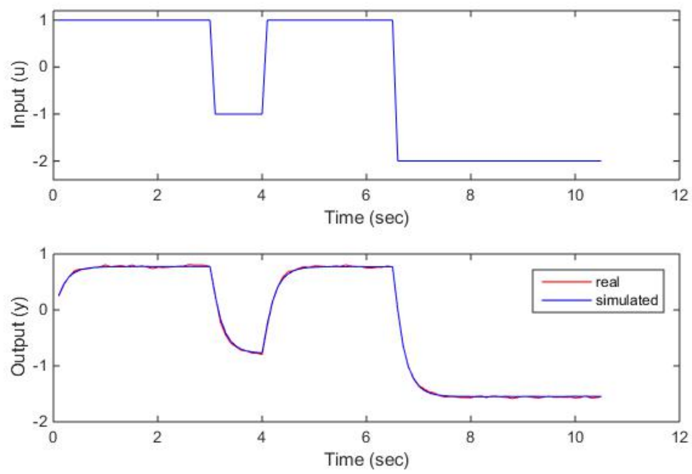

Figure 3 The result of system identification

Task 2
2.1 Realization of GPC

A function called `Matrix_Carima` is programmed to calculate the H, P, Q matrix.

```
[H,P,Q] = Matrix_Carima(Parameter);
```

Where "Parameter" is a structure array contains the previous two system orders and the selected horizon. Then, another function called `Controllaw` is programmed to imply the GPC control law to the system.

```
[u_past,y_past]=Controllaw(r,H,P,Q,lamda,Parameter);
```

Where ''r'' is the reference signal and ''lamda'' is the weighted coefficient in the cost function. Two returned arrays are saved as input and output respectively.

2.2 The effect of horizon

For this step response, the all dynamic response occur in one sampling time, then a long steady period follow. In other words, the system is slow. Thus the horizon need not to be very large. From the figure 5, When the horizon equals 1, the anticipation is not very obvious. When horizon increases, the response which is ahead of the change of signal can be clearly seen from the figure. Considering the computation effort, the horizon is set to 3.

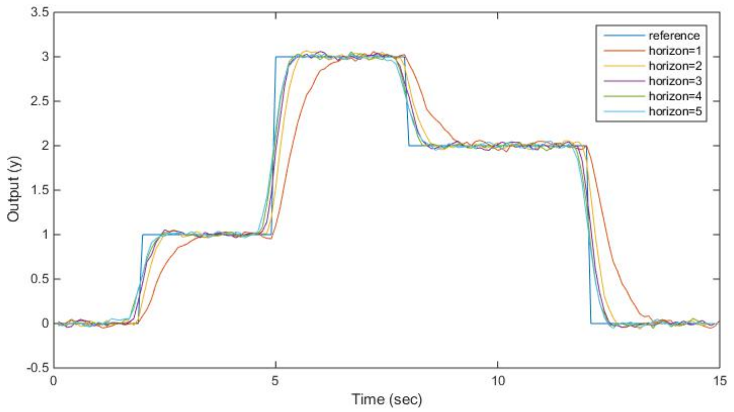

Figure 4 The effect of horizon using CARIMA model

2.3 The effect of lamda

Remind the cost function of GPC using transfer function model :

$$J=\left(\underset{\rightarrow k+1}{\mathrm{r}}-\underset{\rightarrow k+1}{y}\right)^{T}\left(\underset{\rightarrow k+1}{\mathrm{r}}-\underset{\rightarrow k+1}{y}\right)+\lambda\,\underset{\rightarrow k}{\Delta\mathrm{u}}^{T}\,\underset{\rightarrow k}{\Delta\mathrm{u}}$$

This function consists of two terms. The first term enable the system track reference. Due to the non-linear behaviour and disturbance in the system, there always exists a small fluctuation of the input signal in steady state. This may leads to fatigue in mechanical system as well as undesired fluctuation of input energy. Thus the second term in the cost function is used to regulate the input signal to eliminate this phenomenon. The parameter lamda is the weighted coefficient second term, the rise time is very short while the overshoot may occur. Considering the input signal, the abrupt change and the fluctuation in steady state are both not desired. If implemented with the second term, the performance of input becomes acceptable for the reality. However, too large lamda may slow the response and increase the tracking error. Thus, different reference, the appropriate value of lamda is set to 1.2 for this model .

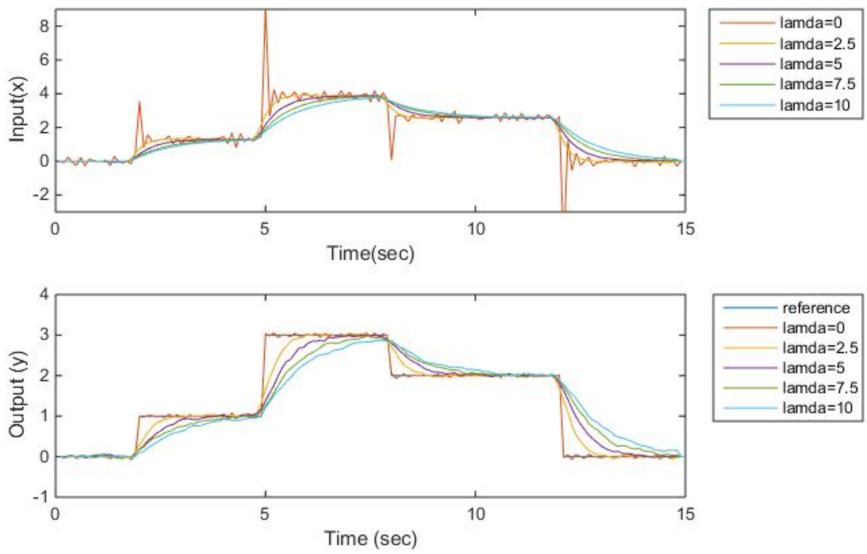

Figure 5 The effect of lamda using FSR model

2.4 The final result

Finally the model is tested with a new step reference. From figure 7, the simulated response is just what the GPC method expects. The input signal is smooth. The output response anticipates the future change of reference. There is almost no overshoot. The prediction is unbiased, which means there is no steady state error.

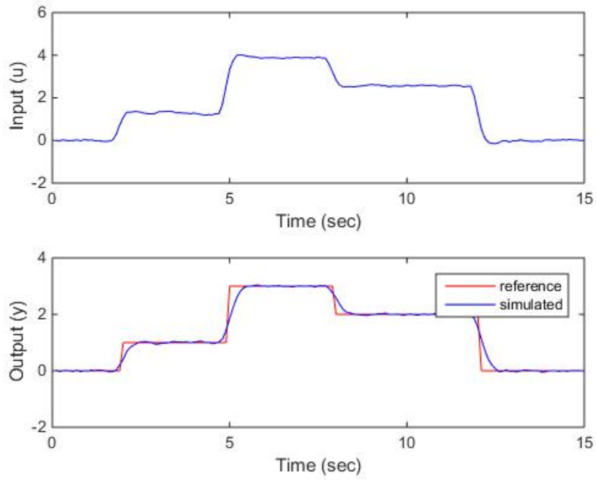

Figure 6 The simulated result using CARIMA model

Task 3

Three methods are used to formulate the step response model. The first one is directly read the ‘‘a’’ array from the step response of the original system. The second one is identify the ‘‘h’’ array from the FIR. Then translate ‘‘h’’ to a according to the simple relationship between them. Third one is directly get the ‘‘h’’ array from the CARIMA model. Then do the same process as the previous one. It should be noticed that the division of two polynomials may results a residue. However, the simulated results in figure 7 show all these three methods can get an accurate enough model.

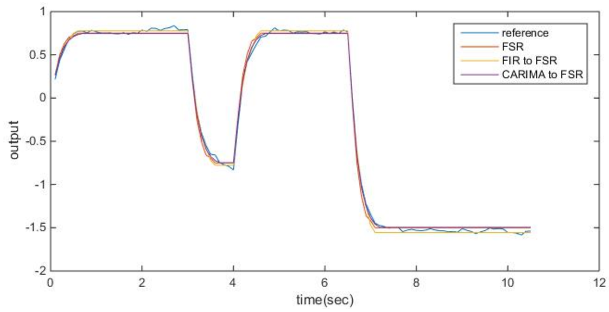

Figure 7 The output of FSR/FIR to FSR/CARIMA to FSR

The figure 8 below illustrates the effect of order on the mse in this three case. The general shapes are very similar. If using step or impulse signal, the input signal would not change in steady state. Thus, too higher order is meaningless. The order of m,h,and (a,b) is chosen as 6, 5, (1,11) respectively. The mse of the third method which derive from CARIMA is much larger than the other two when using the same order. The result of the first method is sensitive to the disturbance when collecting the system output as parameter a . Due to the limitation of length ,only the model obtained from the first method is used in the following section.

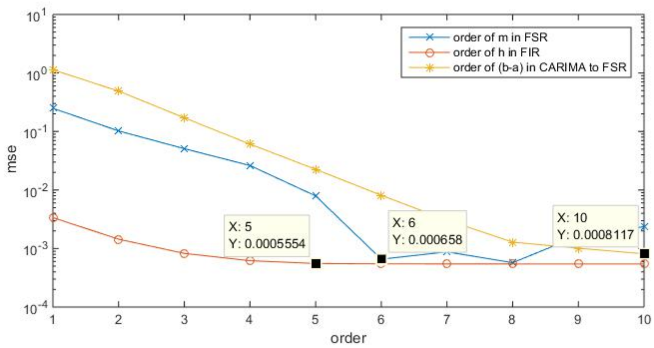

Figure 8 The mse of FSR/FIR to FSR/CARIMA to FSR

Task 4

4.1 Realization of GPC

The structure of H, P, Q matrix is different from CRIMA model, so another function is programmed.

```
[H,P,Q]=Matrix_Step(Parameter,a);
```

Where “a” is the selected order.

The control law is identical to the previous one.

```
[u_past,y_past]=Controllaw(r,H,P,Q,lamda,Parameter);
```

4.2 The effect of horizon

As figure 9 illustrates, the effect of horizon is very similar to that of CARIMA model in section 3.2. However, the horizon “p” of FSR model must be smaller than the order “a” because of the structure of P matrix. In fact, just as what is explained in section 3, those “h” with orders higher than the chosen appropriate value should be all zeros. It means that the model cannot use the data over its anticipating ability. Moreover, it is meaningless to use a extremely large order to enable a larger horizon. It has no help with accuracy and tends to overfit.

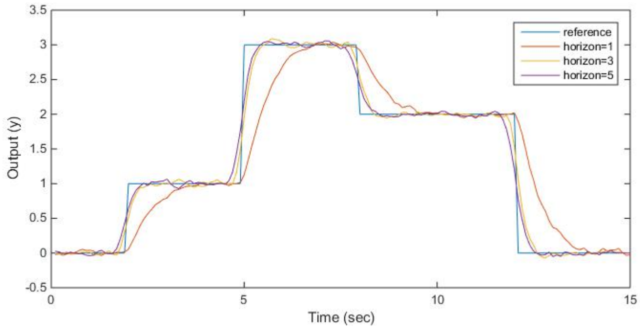

Figure 9 The effect of horizon using FSR model

4.3 The effect of lamda

The effect of lamda is also very similar to that of CARIMA model. In this case, when lamda equals zero, the oscillation of the input is very severe and results an overshoot of the input or even get unstable in some attempts. The improvement after using the lamda term is obvious. The appropriate value of lamda is set to 1 for this model.

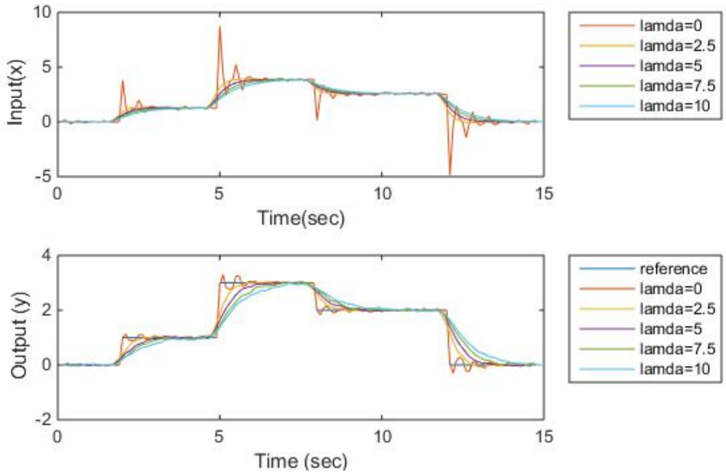

Figure 10 The effect of lamda using FSR model

4.4 The final result

Finally, the model is tested with a new reference. As figure 11 shows, The input curve is smooth. The output response can anticipate the future change of reference. There is almost no overshoot. The prediction is unbiased, which means there is no steady state error.

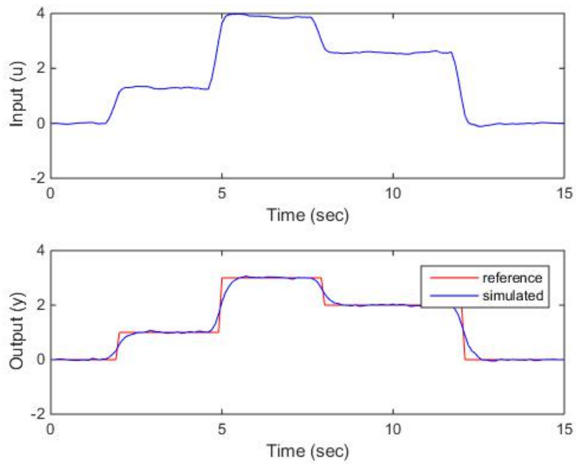

Figure 11 The simulated result using FSR model