Stevens Institute of Technology

SYS-611 Homework #2

Due Sep. 21 2022

Submit the following using the online submission system: 1) Cover sheet with name, date, and collaborators, 2) Written responses in PDF format, 3) All work (e.g. .xlsx or .py files).

2.1 Discrete Process Generator [10 points]

Consider the random variable X to be the *maximum value* of two six-sided dice.

(a) 2 PTS Enumerate the sample space (*hint: there are 36 cases*) and, for each one, identify the random variable value X.

(b) 1 PT Write the probability mass function (PMF) $p(x)$ and cumulative distribution function (CDF) $F(x)$ as a table for $1 \leq x \leq 6$.

(c) 1 PT What is the population mean (μ_x), also known as expected value ($E[X]$)?

(d) 1 PT Using a bar chart, plot the PMF $p(x)$ for $1 \leq x \leq 6$. Label the axes.

(e) 1 PT Using a line chart, plot the CDF $F(x)$ for $1 \leq x \leq 6$. Label the axes.

(f) 4 PTS Using the inverse transform method, develop a discrete process generator for X and generate $n = 1000$ samples $x_1, x_2, \ldots, x_{1000}$. Report the following:

   (i) Plot a histogram of the samples

   (ii) Sample mean ($\bar{x}$)

   (iii) Sample standard deviation (s_x)

   (iv) 95% confidence interval for the population mean

2.2 Continuous Process Generator [9 points]

Consider the random variable Y to be the time (measured in minutes) to drink a cup of coffee. Assume Y is distributed as a ramp-up distribution with the following probability density function (PDF):

$$f(y) = \begin{cases} (9-y)/18 & 3 \leq y \leq 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) 1 PT What is the population mean (μ_y), also known as expected value ($E[Y]$)?

(b) 1 PT Using a line chart, plot the PDF $f(y)$ for $0 \leq y \leq 10$. Label the axes.

(c) 2 PTS Either using calculus or geometry, derive an equation for the CDF $F(y)$.

(d) 1 PT Using a line chart, plot the CDF $F(y)$ for $0 \leq y \leq 10$. Label the axes.

(e) 4 PTS Using the inverse transform method, develop a continuous process generator for Y and generate $n = 1000$ samples $y_1, y_2, \ldots, y_{1000}$. Report the following:

   (i) Plot a histogram of the samples using appropriately-sized bins
   (ii) Sample mean ($\bar{y}$)
   (iii) Sample standard deviation (s_y)
   (iv) 95% confidence interval for population mean

2.3 Arrivals at Café Java [6 points]

The manager at Café Java wants to build a simulation model to improve operations. The first step is to simulate customer arrivals. A baseline model assumes customers arrive with an inter-arrival time Z (time between customers) that follows an Exponential distribution with rate parameter $\lambda = 2$ customers per minute with PDF and CDF equations:

$$f(z) = \lambda e^{-\lambda \cdot z} \qquad F(z) = 1 - e^{-\lambda \cdot z}$$

(a) 2 PTS Using the inverse transform method, develop a continuous process generator for Z and generate $n = 1000$ samples of inter-arrival periods $z_1, z_2, \ldots, z_{1000}$.

List the first 10 samples $z_1, \ldots, z_{10}$ and plot a histogram of all 1000 z_i samples.

(b) 1 PT Compute the arrival times for each of the 1000 customers $t_1, t_2, \ldots, t_{1000}$. The arrival time is the cumulative sum of inter-arrival times for all prior customers:

$$t_1 = z_1, \quad t_2 = z_1 + z_2, \quad \ldots, \quad t_n = \sum_{i=1}^{n} z_i = t_{n-1} + z_n$$

List the first 10 samples $t_1, \ldots, t_{10}$.

(c) 2 PTS Count the number of customers arriving in each 1-minute interval for the first 300 minutes $k_1, k_2, \ldots, k_{300}$. For example, k_1 counts the number of customers with $0.0 \leq t_i < 1.0$ and k_{300} counts the number of customers with $299.0 \leq t_i < 300.0$.

List the first 5 samples $k_1, \ldots, k_5$ and plot a histogram of all 300 k_i samples.

(d) 1 PT The random variable K_j represents the number of customers arriving in the jth minute. What probability distribution does K_j follow? (hint: which distribution counts the frequency of exponentially-distributed events in unit time intervals?)