中山大学理工学院 2013 学年第 3 学期期末

信号与系统（A 卷 ）答案

12 年级微电子（2+2）专业 姓名：____________ 学号：_________ 教师姓名：陈晖 考试成绩：____________

ANSWER ALL THE PROBLEMS ON THIS SHEET!

1. **(Total 24 pts)** The input $x(t)$ and the output $y(t)$ of a stable casual continuous-time LTI system are related by the differential equation

$$\frac{d^2y(t)}{dt^2}+3\frac{dy(t)}{dt}+2y(t)=x(t)$$

(a) **(3 pts)** Write the spectral domain equation for the system.

$(j\omega)^2Y(j\omega)+3j\omega Y(j\omega)+2Y(j\omega)=X(j\omega)$

(b) **(3 pts)** List the transfer function $H(j\omega)$

$$H(j\omega)=\frac{Y(j\omega)}{X(j\omega)}=\frac{1}{(j\omega+1)(j\omega+2)}=\frac{1}{j\omega+1}-\frac{1}{j\omega+2}$$

(c) **(5 pts)** Find the impulse response $h(t)$ for this system

$h(t)=e^{-t}u(t)-e^{-2t}u(t)$

(d) **(5 pts)** If the system input is $x(t)=e^{-2t}u(t)$, calculate the system output response $y(t)$.

$$x(t)=e^{-2t}u(t)\to X(j\omega)=\frac{1}{j\omega+2}$$

$$Y(j\omega)=X(j\omega)H(j\omega)=\frac{1}{(j\omega+1)(j\omega+2)}-\frac{1}{(j\omega+2)^2}$$

$$y(t)=e^{-t}u(t)-e^{-2t}u(t)-te^{-2t}u(t)$$

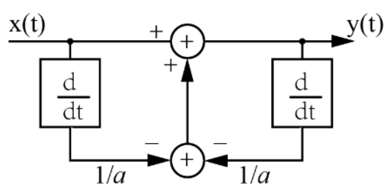
(e) **(3 pts)** If the system is cascaded with another filter with

$$H_2(j\omega)=\frac{a-j\omega}{a+j\omega}$$

Write the steady state response of the whole system $H(j\omega)H_2(j\omega)$ upon a unit step function (which means DC response)

$$|H(0)H_2(0)|=\frac{1}{2}$$

(f) **(5 pts)** Draw the block diagram of the $H_2(j\omega)$ system



2. **(Total 24 pts)** A discrete-time system has input x[n] and output y[n]. The Fourier transforms of these signals are related by the equation

$$Y(e^{j\omega})=\frac{1}{1-\frac{1}{3}e^{-j\omega}}X(e^{j\omega})$$

(a) **(4 pts)** Determine the difference equation for this system.

$$y[n]-\frac{1}{3}y[n-1]=x[n]$$

(b) **(5 pts)** Determine the output y[n] upon an input $x[n]=(1/2)^n u[n]$.

$$y[n]=3\left(\frac{1}{2}\right)^n u[n]-2\left(\frac{1}{3}\right)^n u[n]$$

(c) **(4 pts)** Determine the output y[n] upon an input x[n]=cos(πn/8) + sin(πn/4).

$$y[n]=1.42\cos(\pi n/8-0.182)+1.25\sin(\pi n/4-0.299)$$

(d) **(3 pts)** Is this system linear? Justify your answer

Yes, it is linear. Since when x[n]=a x1[n]+b x2[n], y[n]=ay1[n]+by2[n]

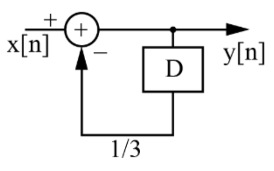
(e) **(2 pts)** If the system is a filter, what kind of filter it is?

Low pass filter

(f) **(3 pts)** What is the phase of this filter at its maximum transfer?

It's at maximum transfer when w=0, and the phase is also zero

(g) **(3 pts)** Draw the block diagram for this system



3. **(Total 16 pts)** Consider a continuous-time LTI system for which the input x(t) and the output y(t) are related by the differential equation

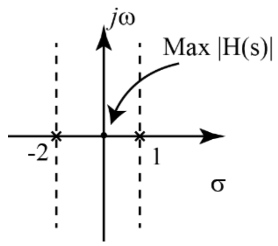
$$\frac{d^2y(t)}{dt^2}+\frac{dy(t)}{dt}-2y(t)=x(t)$$

and Let X(s) and Y(s) denote Laplace transform of x(t) and y(t) respectively, and let H(s) denote the transfer function of h(t), the system's unit impulse response.

(a) **(4 pts)** Determine $H(s)$ as a ratio of two polynomials in s.

$$H(s)=\frac{1}{s^2+s-2}=\frac{1/3}{s-1}-\frac{1/3}{s+2}$$

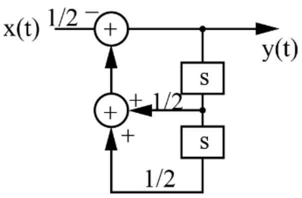
(b) **(3 pts)** Sketch the pole-zero pattern of $H(s)$ and mark the position for the maximum $|H(s)|$



(c) **(5 pts)** Determine $h(t)$ for the case that the system is stable (ROC are all on the right side of poles)

$$h(t)=\frac{1}{3}(e^{t}u(t)-e^{-2t}u(t))$$

(d) **(4 pts)** Draw the block diagram of the system



4. **(Total 14 pts)** Answer following short questions in brief sentences:

(a) **(3 pts)** In the problem 1-3 above, what are the spectral domain tools you use?

Continuous Time Fourier Transform (CTFT)

Discrete Time Fourier Transform (DTFT)

Laplace transform

(b) **(3 pts)** What are the bases of spectral space used in the above spectral domain analysis?

$e^{j\omega}$, $e^{j\omega}$, e^{s}

(c) **(3pts)** How to use the spectral bases above to find the spectral components of a temporal signal? Answer with analysis equations and give brief explanation.

Do the integration of signal and the conjugate of the baseover the time domain

(d) **(2 pts)** What is the first principles to link the temporal and spectral domain analysis

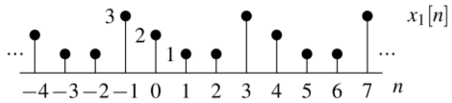
Convolution of one domain correspond to multiplication of the other,

(e) **(2 pts)** Can the methods above be used in nonlinear systems?

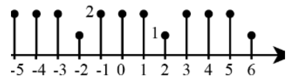
No.

5. **(Total 23 pts)** Sketch the signals following the instruction:

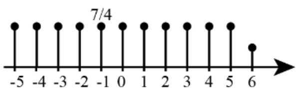
A periodic signal $x_1[n]$ is represented as graph below,



(a) **(3 pts)** Even part of the signal



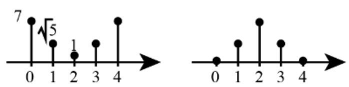
(b) **(3 pts)** Draw the DC of the signal $x_1[n]$



(c) **(5 pts)** Calculate $x_2[n]=e^{-jk(2\pi/4)n}$ and fill the table below

		n				$\sum_{n=0}^{3}x_1[n]x_2[n]$
		0	1	2	3	
k	0	1	1	1	1	7
	1	1	-j	-1	j	2j+1
	2	1	-1	1	-1	-1
	3	1	j	-1	-j	1-2j
	4	1	1	1	1	7

(d) **(5 pts)** Draw the DFT of $x_1[n]$ with amplitude and phase



(e) **(3 pts)** Explain the relation between (b) and (d)

DC corresponds to the frequency component of 0

(f) **(4 pts)** Draw convolution of $x_1[n]$ with $x_3[n]=\delta[n+1]+\delta[n-1]$

END OF EXAM PROBLEMS