Q.1)

- True
- False
- True

Q.2)

1. Where f(n) = 3n3 + n2 + n and g(n) = n3

=>C1 n3 ≤ $3n^3 + n^2 + n$ ≤ C2 n^3 ➔ (÷$\boldsymbol{n3}$)

=> C1 ≤ 3 +$\frac{1}{n}$ +$\frac{1}{n^2}$ ≤ C2 ➔ sub n = 1

=>C1 ≤ 3 + $\mathbf{1}$ + $\mathbf{1}$ ≤ C2

=>C1 ≤ 5 ≤ C2

Satisfies the condition that C1 and C2 > 0

2.

Where f(n) = $2^{n+1}$and g(n) = 2^n

➔ C1 2^n≤ 2^{n+1}≤ C2 2^n ➔(÷$\boldsymbol{2n}$)

➔ C1 ≤ $\mathbf{2}$ ≤ C2 ➔ sub n = 1

➔ C1 ≤ 2 ≤ C2

Satisfies the condition that C1 and C2 > 0

الموضوع: التاريخ: / /

$g(n) = \ln(n)$, $f(n) = \log(n) + \log(\log(n))$

$\Rightarrow$ Using definition Method:-

$\therefore C_1 \ln(n) \leq \log(n) + \log(\log(n)) \leq C_2 \ln n$

for $\ln(n) = \log_e n$

$\therefore C_1 \log_e n \leq \log(n) + \log(\log(n)) \leq C_2 \log_e n$

~~for log~~

$\Rightarrow C_1 \log_e(n) \leq \frac{\log_e(n)}{\log_e(10)} + \log(\log(n)) \leq C_2 \log_e(n)$

$[\div \log_e(n)]$

$C_1 \leq \frac{1}{\log_e(10)} + \cancel{\frac{\log(\log(n))}{\log_e(n)}}^{\text{zero}} \leq C_2$

if $n = 10$

$\therefore C_1 \leq \frac{1}{\log_e(10)} \leq C_2$

$C_1 \leq 0.43 \leq C_2$

3.

Q.3)

1. Answer…

   f(n) =n^3, g(n) = n^2

   Using the limit test

   $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ = $\lim_{n\to\infty} \frac{n^3}{n^2}$ = $\lim_{n\to\infty} n$ = ∞

   ∴ Proved f(n) = Ω (g(n))

2. Answer…

   f(n) = log(n), g(n) = $(\log n)^2$ Using the limit test $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ $\lim_{n\to\infty} \frac{\log(\mathrm{n})}{(\log n)^2}$ =

   $\lim_{n\to\infty} \frac{1}{\log(\mathrm{n})}$ = $\mathbf{1}/\infty$ = $\mathbf{0}$

   ∴ Proved f(n) = O(g(n))

Q.4)

Answer…

Let the running time be $f(n)$. If $f(n) = \theta\ (g(n))$, then $0 < c1g(n) \leq f(n) \leq c2g(n)$ where c1 and c2 are positive constants. Since $c1g(n) \leq f(n)$, therefore $f(n) = \Omega\ (g(n))$. Moreover, since $f(n) \leq c2g(n)$, then $f(n) = O(g(n))$. Proving the other direction, if $f(n) = O(g(n)$, and $f(n) = \Omega\ (g(n))$, this implies that there are two positive constants c1 and c2 such that $f(n) \leq c2g(n)$ and $c1g(n) \leq f(n)$. Therefore, $c1g(n) \leq f(n) \leq c2g(n)$ satisfying the definition of the θ notation. Hence, $f(n) = \theta\ (g(n)$.

Q.5)

Answer…

Suppose not.

Let $f(n) \in o(g(n)) \cap \omega(g(n))$ Now $f(n) = \omega(g(n))$ if and only if $g(n) = o(f(n))$ and $f(n) = o(g(n))$ by assumption. By transitivity property, $f(n) = o(f(n))$ i.e. for all constants $c > 0$, $f(n) < cf(n)$. Choose $c < 1$

and we have the desired contradiction from the asymptotic non-negativity of f(n).