Self-Capacitance of Single-Layer Inductors With Separation Between Conductor Turns

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***Abstract*—This letter presents the technique for estimating the self-capacitance of single-layer air-core solenoid inductors with separation between the insulated turns. In single-layer air-core inductors, the self-capacitance is due to the conductor turn-to-turn capacitances. The analytical framework to determine the turn-to-turn capacitances of single-layer air-core inductors with uniformly and nonuniformly separated conductor turns is established. The influence of the wire insulation coating is taken into consideration. A representative design example of a single-layer air-core inductor is presented and its self-capacitance and self-resonant frequency are predicted. The presented analytical approach was tested by experimental measurements on the designed inductor. The derived analytical expressions are useful for designing air-core inductors for high frequency (HF) and very HF applications such as electromagnetic interference/electromagnetic compatibility filters and radio and TV transmitters.**

***Index Terms*—Inductors, EMI filters, parasitic capacitance, self-capacitance, inductor model, air-core inductor, solenoid inductor, transmitters.**

I. Introduction

AIR-CORE inductors are becoming attractive in a wide variety of applications, including high-frequency (HF) switched-mode power supplies, electromagnetic interference (EMI)/electromagnetic compatibility (EMC) filters, aerospace and communication systems, healthcare industry, radio and TV broadcasting, and wireless electric charging. The bandwidth of these inductors is limited by its self-capacitance [1]–[11]. Therefore, there is a need to accurately predict the self-capacitance so that the maximum capabilities of the inductor can be exploited for over a large range of frequencies.

The turn-to-turn capacitance is caused by the insulated separation between the adjacent conductor turns. These capacitances in series form the self-capacitance. The separation between the conductor turns is due to the wire insulation coating of a finite thickness or the air space between the conductor turns or both. Many works in the literature have addressed the methods to estimate the self-capacitance in inductors using different approaches [1]–[6]. This letter extends the earlier work presented in [1] by taking into account the separation between the insulated conductor turns. The expression for self-capacitance is derived in terms of wire insulation thickness and the air separation between the adjacent insulated conductor turns. It is also shown that a single-layer inductor with a wider separation produces a smaller self-capacitance yielding a wide bandwidth.

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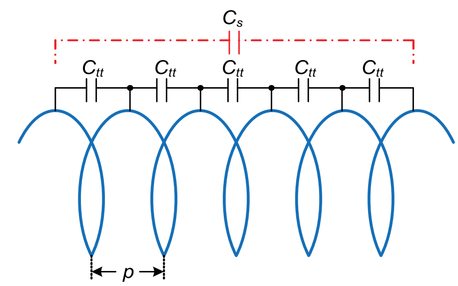

Fig. 1. Representation of an air-core inductor showing its turn-to-turn capacitance C_{tt} and the total self-capacitance C_s caused by uniform separation of turns.

The analysis assumes the following.

1) The self-capacitance is linear and frequency- and voltage invariant. Its reactance is infinite at dc and does not affect the low-frequency performance of the inductor.
2) An insulated cylindrical structure supports the inductor winding. The winding-to-core capacitance is ignored and the relative permeability of the core is $\mu_{\rm rc} = 1$.
3) The analysis is performed up to the first resonant frequency.

II. Turn-to-Turn and Self-Capacitances

Fig. 1 shows a physical representation of a single-layer air-core inductor. The winding pitch p is the distance between the center-points for adjacent conductor turns. It is assumed that the inductor is wound on a cylindrical core made of insulated material such as polyvinyl chloride (PVC). The turn-to-turn capacitance $C_{\rm tt}$ between the conductor turns is due to the air space between the turns and the insulated wire coating. The lumped self-capacitance is C_s. The length of the core is l_c and the diameter of the core is D_c. It is the objective of this letter to derive closed-form analytical expressions for $C_{\rm tt}$ and C_s for inductors with uniform and nonuniform pitch lengths.

A. *Expression for Inductor With Fixed Pitch Length*

An air-core inductor with a uniform pitch between the conductor turns is shown in Fig. 1. Fig. 2 shows two adjacent round conductor turns of the single-layer air-core solenoid

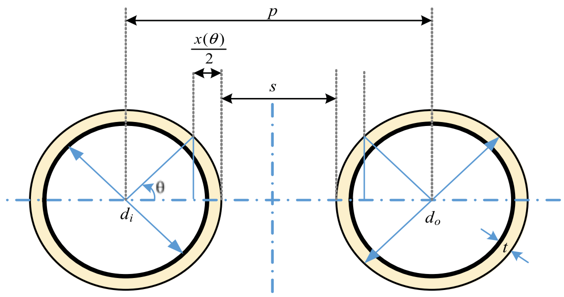

Fig. 2. Two adjacent round turns of single-layer solenoid inductor with turn separation to determine the turn-to-turn capacitance C_{tt}.

inductor with a finite turn separation required to determine the turn-to-turn capacitance C_{tt}. In the figure, d_i and d_o are the inner and outer diameters of the round conductor, $r_i = d_i/2$ and $r_o = d_o/2$ are the inner and outer radii of the round conductor, p is the winding pitch, t is the thickness of the wire insulation coating, s is the fixed air space between the outer surfaces of the conductors along the horizontal axis, and θ is the angle produced by the elementary capacitance from the horizontal plan to a point on the conductor outer surface. The distance $x(\theta)$ is the path of the electric field on the horizontal axis projected by θ. For example, $x(\theta) = 0$ for $\theta = 0$, $x(\theta) = r_o/2$ for $\theta = \pi/4$, and $x(\theta) = r_o$ for $\theta = \pi/2$.

In general, the elementary capacitance can be expressed as

$$dC = \frac{\epsilon r}{dr} d\theta dl. \tag{1}$$

In (1), $\epsilon = \epsilon_0 \epsilon_r$, where $\epsilon_0 = 8.854 \times 10^{-12}$ F/m is the absolute permittivity of free space and ϵ_r is the relative permittivity of the insulator. Each turn is made of a conductor of finite length l. The elementary capacitance of the insulation coating due to the elementary surface subtended by θ for the radius r ranging from r_i to r_o and the length l ranging from zero to the mean turn length l_T for one turn is

$$dC_c = \epsilon d\theta \int_0^{l_T} dl \int_{r_i}^{r_o} \frac{r}{dr} = \frac{\epsilon l_T}{\ln \frac{r_o}{r_i}} d\theta. \tag{2}$$

Therefore, the elementary capacitance due to the wire insulation coatings of the two adjacent turns connected in series is

$$dC_{\mathrm{ttc}} = \frac{dC_c}{2} = \frac{\epsilon l_T}{2 \ln \frac{r_o}{r_i}} d\theta = \frac{\epsilon l_T}{2 \ln \frac{d_o}{d_i}} d\theta. \tag{3}$$

From Fig. 2, the distance between two adjacent turns is $s = p - d_o$. The total air gap between the two adjacent turns is the sum of the distance between the outer surfaces s and the length $x(\theta)$ given by

$$g(\theta) \approx x(\theta) + s = x(\theta) + p - d_o. \tag{4}$$

From the geometrical considerations related to the area subtended by θ, one obtains

$$\cos\theta = \frac{\frac{d_o}{2} - \frac{x(\theta)}{2}}{\frac{d_o}{2}} = 1 - \frac{x(\theta)}{d_o} \tag{5}$$

yielding

$$x(\theta) = d_o(1 - \cos\theta). \tag{6}$$

Substituting for $x(\theta)$ in (6) into (4) gives

$$g(\theta) = d_o(1 - \cos\theta) + p - d_o = d_o \left(\frac{p}{d_o} - \cos\theta \right). \tag{7}$$

The elementary surface dS of the wire including the insulation coating has the form of an elementary ring of length l_T is

$$dS = \frac{d_o l_T}{2} d\theta. \tag{8}$$

Therefore, the elementary capacitance due to the air gap between the two turns with separation s is

$$dC_g = \frac{\epsilon_o dS}{g(\theta)} = \frac{\epsilon_o l_T}{2\left(\frac{p}{d_o} - \cos\theta\right)} d\theta. \tag{9}$$

The series combination of the elementary capacitances of the air gap between the two adjacent turns and the insulation coating of the two adjacent turns is given by

$$\begin{aligned} dC_{\mathrm{tt}}(\theta) &= \frac{dC_{\mathrm{ttc}} dC_g}{dC_{\mathrm{ttc}} + dC_g} \\ &= \frac{\epsilon_o l_T}{2\left(\frac{1}{\epsilon_r} \ln \frac{d_o}{d_i} + \frac{p}{d_o} - \cos\theta\right)} d\theta. \end{aligned} \tag{10}$$

Therefore, the turn-to-turn capacitance is obtained by integrating (10) between the limits $\pi/2$ to $-\pi/2$ to yield

$$C_{\mathrm{tt}} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dC_{\mathrm{tt}}(\theta) d\theta = 2 \int_0^{\frac{\pi}{2}} dC_{\mathrm{tt}}(\theta) d\theta. \tag{11}$$

The following equation is useful to solve the integral given in (11):

$$\int \frac{dx}{a - \cos x} = \frac{2}{\sqrt{a^2 - 1}} \tan^{-1} \left[\sqrt{\frac{a+1}{a-1}} \tan\left(\frac{x}{2}\right) \right]. \tag{12}$$

Using (12), the turn-to-turn capacitance in (11) becomes

$$\begin{aligned} C_{\mathrm{tt}} &= \frac{2\epsilon_o l_T}{\sqrt{\left(\frac{1}{\epsilon_r} \ln \frac{d_o}{d_i} + \frac{p}{d_o}\right)^2 - 1}} \\ &\quad \times \tan^{-1} \left(\sqrt{1 + \frac{2}{\frac{1}{\epsilon_r} \ln \frac{d_o}{d_i} + \frac{p}{d_o} - 1}} \right). \end{aligned} \tag{13}$$

In terms of the insulation coating thickness t, the turn-to-turn capacitance is

$$\begin{aligned} C_{\mathrm{tt}} &= \frac{2\epsilon_o l_T}{\sqrt{\left[\frac{1}{\epsilon_r} \ln \left(1 + \frac{2t}{d_i}\right) + \frac{p}{d_o}\right]^2 - 1}} \\ &\quad \times \tan^{-1} \left[\sqrt{1 + \frac{2}{\frac{1}{\epsilon_r} \ln \left(1 + \frac{2t}{d_i}\right) + \frac{p}{d_o} - 1}} \right]. \end{aligned} \tag{14}$$

For a core diameter D_c and wire outer diameter d_o, the effective coil diameter of one turn is $D_t = D_c + d_o$. Therefore, the mean length per turn of the conductor over a core with circular cross section is $l_T = \pi D_t$ to yield the turn-to-turn capacitance

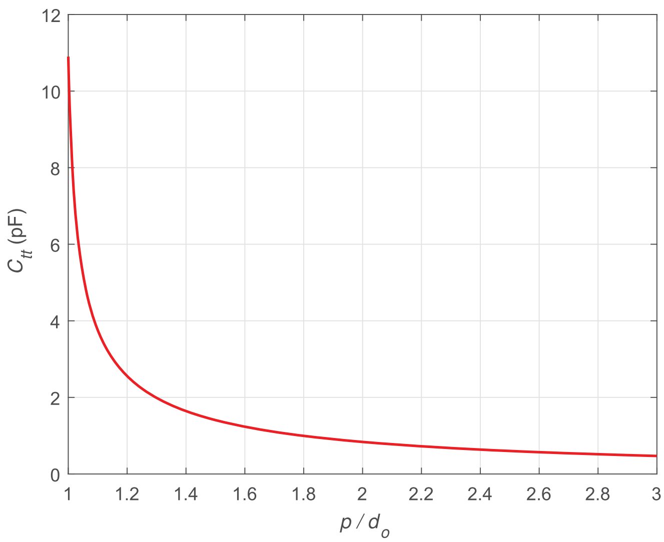

Fig. 3. Turn-turn capacitance $C_{\rm tt}$ of the single-layer air-core inductor as a function of the normalized pitch p/d_o.

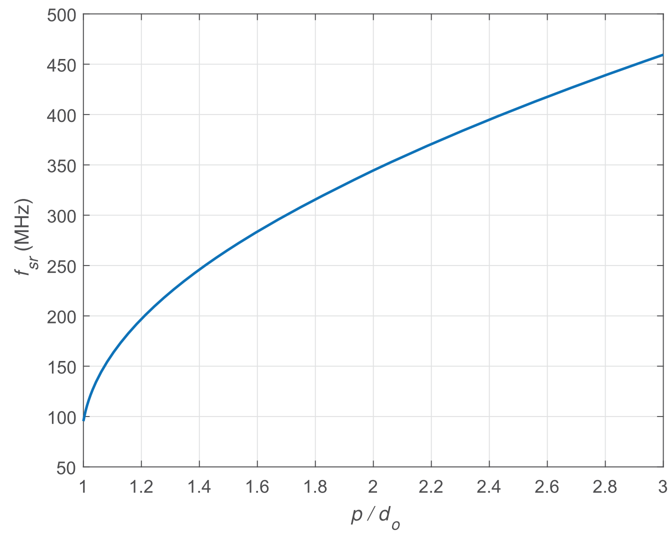

Fig. 4. Self-resonant frequency $f_{\rm sr}$ of the single-layer air-core inductor as a function of the normalized pitch p/d_o.

as

$$C_{\rm tt} = \frac{2\epsilon_o \pi D_t}{\sqrt{\left(\frac{1}{\epsilon_r}\ln\frac{d_o}{d_i} + \frac{p}{d_o}\right)^2 - 1}} \times \tan^{-1}\left(\sqrt{1 + \frac{2}{\frac{1}{\epsilon_r}\ln\frac{d_o}{d_i} + \frac{p}{d_o} - 1}}\right). \quad (15)$$

For illustrative purposes, consider an example inductor with the following data: $d_o = 1.22$ mm, $d_i = 1.15$ mm, $t = (d_o - d_i)/2 = 0.035$ mm, $D_c = 2.4$ cm, $D_t = D_c + d_o = 25.22$ mm, and $\epsilon_r = 3.3$. The number of turns is $N = 33$. Using (15), the turn-turn capacitance as a function of the pitch normalized with respect to the outer diameter p/d_o is shown in Fig. 3. The capacitance reduces with increase in the winding pitch indicating that a large pitch length is desirable. One may express the turn-to-turn capacitance in terms of the separation length $s = p - d_o$ as

$$C_{\rm tt} = \frac{2\epsilon_o \pi D_t}{\sqrt{\left(1 + \frac{1}{\epsilon_r}\ln\frac{d_o}{d_i} + \frac{s}{d_o}\right)^2 - 1}} \times \tan^{-1}\left(\sqrt{1 + \frac{2}{\frac{1}{\epsilon_r}\ln\frac{d_o}{d_i} + \frac{s}{d_o}}}\right). \quad (16)$$

The network of turn-to-turn capacitances are in series [5] for the single-layer air-core solenoid inductors. Therefore, the inductor self-capacitance is

$$C_s = \frac{C_{\rm tt}}{N-1}. \quad (17)$$

The self-resonant frequency $f_{\rm sr}$ for the inductor with inductance L is

$$f_{\rm sr} = \frac{1}{2\pi\sqrt{LC_s}}. \quad (18)$$

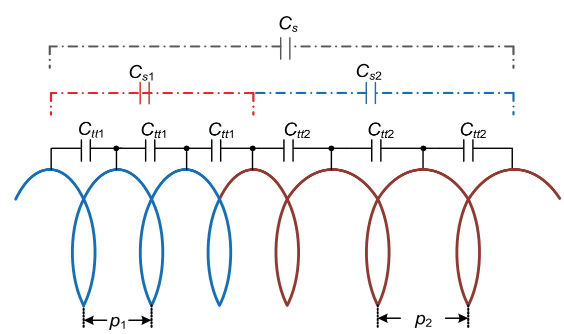

Fig. 5. Representation of an air-core inductor showing its turn-to-turn capacitances $C_{\rm tt1}$, $C_{\rm tt2}$ and the local self-capacitances C_{s1}, C_{s2}, and total self-capacitance C_s caused by different lengths of separation between insulated conductor turns.

Fig. 4 shows the self-resonant frequency as a function of the normalized winding pitch. The self-resonant frequency increases with the pitch, whereas the self-capacitance reduces in a manner similar to Fig. 3. Therefore, an inductor with large pitch can produce a high bandwidth.

B. *Expression for Inductor With Multiple Pitch Lengths*

One can exploit the advantage of the series-connected turn-to-turn capacitances in single-layer air-core inductors to boost the bandwidth. For an air-core inductor with segments having different pitch lengths as shown in Fig. 5, the turn-to-turn capacitances are correspondingly different for each segment. Consider an inductor with uniform core and wire diameters but different pitch lengths. The pitch lengths and number of turns are p_1, N_1 in segment 1 and p_2, N_2 in segment 2 such that $p_1 > p_2$. The total number of conductor turns is $N = N_1 + N_2$. In accordance with (15) and Fig. 3, the turn-to-turn capacitances $C_{\rm tt1}$ and $C_{\rm tt2}$ in segments 1 and 2, respectively, obey the relationship $C_{\rm tt1} < C_{\rm tt2}$. Consequently, the self-capacitances of each segment are $C_{s1} = C_{\rm tt1}/(N_1 - 1)$ and $C_{s2} = C_{\rm tt2}/(N_2 - 1)$

producing the total self-capacitance

$$C_s = \frac{1}{\frac{1}{C_{s1}} + \frac{1}{C_{s2}}} \quad (19)$$

and for an inductor with k different pitch lengths, the total self-capacitance is

$$C_s = \frac{1}{\frac{1}{C_{s1}} + \frac{1}{C_{s2}} + \frac{1}{C_{s3}} \cdots\cdots \frac{1}{C_{sk}}}. \quad (20)$$

Therefore, a single-layer inductor with different pitch lengths can yield a lower effective self-capacitance compared to that of an inductor with a single pitch length. This could be a useful feature especially for HF and very HF applications.

III. Experimental Validation

The analytical results given by the proposed method were compared with those measured for several inductors. The specifications of the air-core solenoid inductor are as follows. The diameter of the PVC bobbin is $D_c = 2.4$ cm, therefore the radius and area of the core are $r_c = D_c/2 = 1.2$ cm and $A_c = 4.52\,\text{cm}^2$, respectively. The length of the core is $l_c = 6.5$ cm. The number of copper conductor turns was $N = 33$ and the chosen wire was AWG17 with inner diameter $d_i = 1.15$ mm and outer diameter $d_o = 1.22$ mm. With the Nagaoka's coefficient K [11], [12], the calculated inductance is

$$L = \frac{\mu_0 A_c N^2}{l_c K} = \frac{\mu_0 A_c N^2}{l_c \left(1 + 0.9\frac{D_c}{2l_c}\right)} = 8.16\,\mu\text{H}. \quad (21)$$

The turn-to-turn and self-capacitances computed using (15) and (17) are $C_{\rm tt} = 1.14$ pF and $C_s = 35.74$ fF. The estimated self-resonant frequency using (18) is 294.55 MHz.

An inductor with the above specifications was design, built, and tested to validate the analytical results. The pitch length was

$$p = \frac{l_c}{N-1} = 2.013\,\text{mm} \quad (22)$$

to give $p/d_o = 1.665$. The inductor was measured with an HP4194A impedance/gain-phase analyzer. The inductance measured at a frequency of 200 Hz was $8.69\,\mu$H. The self-capacitance was measured as $C_s = 37.55$ fF. Hence, the self-resonant frequency using the measured inductance and self-capacitance can be found as $f_{\rm sr} = 278.74$ MHz. The predicted and measured results were in good agreement.

IV. Conclusion

An analytical approach for predicting the self-capacitance of single-layer air-core solenoid inductors as a function of the separation between adjacent insulated conductor turns has been presented. The expression for the turn-to-turn capacitance and the self-capacitance have been derived in terms of the wire insulation thickness t, the winding pitch p, and the separation s. The self-capacitance decreases with increase in the separation between the turns. The reduction in the self-capacitance is high at smaller pitch lengths and low at large pitch lengths. The self-resonant frequency increases with pitch length. As a result, air-core inductors with pitch $p >> d_o$ can yield a higher bandwidth than those at $p \leq d_o$, where d_o is the outer diameter of the conductor. It has also been shown that the bandwidth can be boosted for inductors with nonuniform winding pitch as they produce significantly lower self-capacitance. Experimental results for an air-core inductor have been presented. The bandwidth in the order of 250 MHz was obtained. The self-capacitance in single-layer air-core inductors is very small as the turn-to-turn capacitances are in series resulting in a large bandwidth.

The contributions of this letter are as follows.

1) A new and improved method to compute the self-capacitance of single-layer air-core solenoid inductors is given.
2) The relationship between the self-capacitance and the self-resonant frequency on the winding pitch is discussed.
3) The results of this letter are useful for estimating the turn-to-turn capacitance of inductors used in HF applications such as EMI/EMC filters and transmitters in radio and TV broadcasting stations.

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