

An improved simulated annealing–linear programming hybrid algorithm applied to the optimal coordination of directional overcurrent relays

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ARTICLE INFO

Keywords:

Directional overcurrent relay
Power system protection
Overcurrent protection
Linear programming
Simulated annealing

ABSTRACT

The coordination of directional overcurrent relays (DOCRs) is a constrained and nonlinear optimization problem which consists in finding suitable plug and time dial settings so that the relay operational times are minimized, keeping selectivity and sensitivity characteristics. Recently, several efforts have been devoted to automate the coordination of DOCRs. This paper proposes a hybrid technique entitled simulated annealing–linear programming (SA-LP) to achieve the optimal coordination of DOCRs. Five test-systems (IEEE-3, IEEE-6, IEEE-8, IEEE-15 and IEEE-30 bus) are used to verify the effectiveness of the proposed technique. Results obtained with the SA-LP are confronted against other optimization techniques reported in specialized literature, under identical conditions. The proposed approach presented good quality solutions, low computational processing times and great convergence towards the optimum solution, presenting an advantage over adaptive coordination tendency by enhancing monitoring, communication capabilities and grid control.

1. Introduction

The power system is susceptible to abnormalities that must be eliminated to avoid permanent equipment damage or power system instability. Protective relays detect and isolate the faulty section of the electrical system under the requirements of sensitivity, selectivity, reliability, and speed [1]. Directional overcurrent relays (DOCRs) are widely used as an economical means to protect sub-transmission and distribution systems [2]. These types of protective relays act based on the current levels, and it has two main adjustable parameters: plug settings (PSs) and time dial settings (TDSs). The TDS adjusts time-delay before a relay operates whenever a sensed fault current reaches a value greater than or equal to the pick-up current. The PS defines the pick-up current that flows through the relay, and its value is often expressed as multiple of the nominal current of the current transformer. These parameters are used to specify a particular time–current characteristic of a DOCR.

Relay coordination is an important aspect of protection system design, which consists of selecting suitable settings such that their fundamental protective function is accomplished with sufficient coordination margins, without excessive time delays [3]. Mathematically, the DOCRs coordination problem can be formulated as a mixed-integer nonlinear programming (MINLP) where the PSs and TDSs are treated as discrete and continuous variables, respectively [4].

An idea of robust coordination is presented by Costa et al. [5], which analyzes a classical coordination model but considering different scenarios where system data are constantly changing and selectivity is no longer guaranteed. Recently, Yazdaninejadi et al. [6] shows a different kind of coordination for meshed distribution networks, previously developed by Zeineldin et al. [7], where dual-setting relays are equipped with two inverse time-current characteristics whose parameters depend on the fault direction, adding an extra level of protection.

In an adaptive protection scheme, it is necessary to compute the DOCRs parameters at each topological change [2,8–10]. All the computational effort is held on an external computer, located in a control room, which receives the input data (system topology, relay characteristics and short-circuit levels), runs the coordination algorithm and remotely upload the parameters on the relays. A practical implementation of adaptive protection is reported in [11]. Modern power systems with distributed generation (DG) impact on protective relaying [12], especially in radial systems where coordination is performed with non-directional units [13,14], requiring the implementation of a meshed network formulation [15,16].

The coordination of DOCRs is often formulated as a nonlinear optimization problem. However, a linear formulation can be used when TDSs are unknown and PSs are pre-determined or vice-versa. Thus, classical optimization techniques such as linear programming (LP) were

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used to solve the coordination problem [1,17–19]. Nevertheless, a better solution can be obtained when the PSs are treated as variables in the optimization process.

When the problem is treated nonlinearly, meta-heuristic optimization algorithms are usually used to solve the coordination problem with less computational time compared to conventional methods, especially when the problem has non-convex characteristics [20–22]. Techniques such as evolutionary algorithms [23,24], modified particle swarm optimization (MPSO) [25,26], genetic algorithm (GA) [27], non-dominated sorting GA-II (NSGA-II) [28], differential evolution algorithm (DE) [3,29–31], seeker optimization algorithm (SOA) [32], cuckoo search algorithm (CSA) [33] and particle swarm optimization (PSO) [25], biogeography-based optimization (BBO) [4]. Authors from [34,35] references, solve coordination problem using a hybrid whale optimization algorithm (HWOA) and firefly algorithm (FA) that mimics the flashing behavior of fireflies.

New and more sophisticated algorithms, such as hybrid algorithms, are used to improve the convergence and performance of meta-heuristics. Generally, hybrid algorithms divide the original optimization problem into subproblems, where each subproblem is solved with a specific algorithm. Regarding the DOCRs coordination problem, several hybrid algorithms have been proposed in the specialized literature: a combination of GA with LP (GA-LP) [27], GA with nonlinear programming technique (GA-NLP) [20], PSO with LP (PSO-LP) [21], CSA with firefly algorithm (CSA-FFA) [22], BBO algorithm with LP (BBO-LP) [4], and ant colony optimization with LP (ACO-LP) [36].

Traditional optimization algorithms that require a convex problem characteristic are not suitable for coordination, a non-convex problem. A convex problem guarantees a global optimal. Depending on the problem size, complexity may affect the search with a high risk of being trapped in a local optimal. Amraee [32] uses standard branch-and-bound (B&B) to solve the coordination problem, also shows the advantage of the meta-heuristic SOA regarding B&B. In [9], the coordination problem was solved using binary integer programming (BIP) with CPLEX as a solver, which uses dynamic programming to improve its convergence. Damchi et al. [37] changes formulation by discretizing in small steps the PSs. The author implements B&B considering at each branch there is a linear and convex sub-problem. Srinivas et al. [38] introduces a linearized formulation of DOCR coordination problem, named convexified linear program (CLP) using McCormick envelopes. A sequential tightening algorithm reduces the relative error gap in each iteration updating variable limits.

This paper presents a novel hybrid approach to coordinate DOCRs in meshed systems entitled simulated annealing–linear programming (SA-LP), assuming smart grid application to bring fast data processing directly applicable to online adaptive coordination. Recent protection schemes published in the literature applied convex optimization techniques and problem relaxation to coordinate DOCRs, resulting in low relay operation times, but generating miss-coordination [37,38,34,35]. Against this background, the main contributions of this work are threefold: (a) a strong approach for coordinating overcurrent relays, with five well-known test cases to validate the proposed technique; (b) features fast data processing directly applicable to online adaptive coordination, which is uppermost in the smart-grid structure; (c) a fair comparison between the proposed algorithm and several others techniques published in the specialized literature, where the SA-LP found the best solutions reported and, in some cases, even better solutions, without miss-coordination.

This paper is organized as follows. Section 2 presents the formulation of DOCRs coordination problem. Section 3 shows the proposed technique for solving the coordination problem. In Section 4, the proposed algorithm is tested in five widely used test systems, and the results are confronted with others published in the specialized literature. Finally, Section 5 presents the main conclusions of this work.

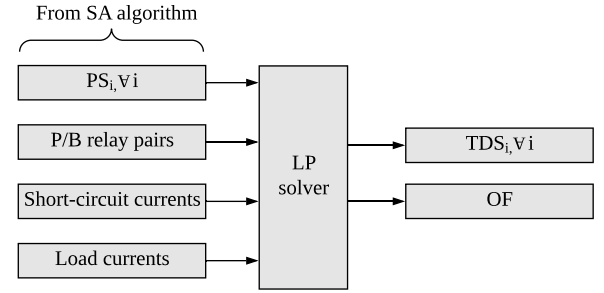


Fig. 1. The SA-LP algorithm.

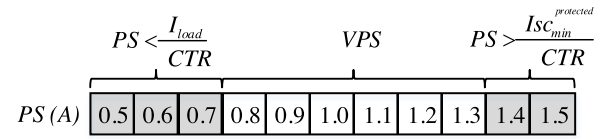


Fig. 2. Example of valid plug settings (VPS) in an arbitrary relay. The gray boxes indicate the PSs that violate restrictions (5) and (6).

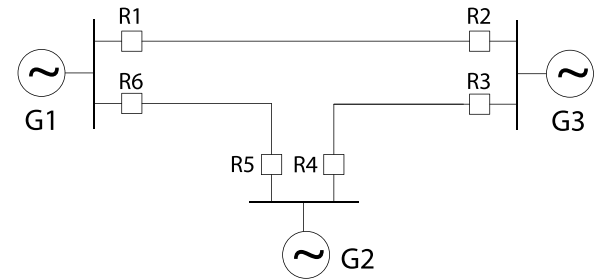


Fig. 3. Single-line diagram of the IEEE 3-bus system.

Table 1
Optimal PSs and TDSs of the IEEE 3-bus system.

Relay	PS (A)	TDS	Relay	PS (A)	TDS
R ₁	2.5	0.1067	R ₄	2.5	0.1000
R ₂	2.0	0.1083	R ₅	2.5	0.1000
R ₃	3.0	0.1000	R ₆	1.5	0.1119
Time ^a (s)	0.0362				
OF (s)	1.5987				

^a Average of 100 runs.

Table 2
Operational times between P/B protection for the IEEE 3-bus system.

Primary	Backup	ΔT (s)	ΔT^a (s)
R ₁	R ₅	0.9618	0.2205
R ₂	R ₄	0.2860	0.2028
R ₃	R ₁	0.2623	0.2000
R ₄	R ₆	0.2958	0.2000
R ₅	R ₃	0.3433	0.2022
R ₆	R ₂	0.9784	–

^a Related to transient configuration.

Table 3
Comparison of the results for the IEEE 3-bus system.

	PSO [26]	SOA [32]	BBO-LP [4]	SA-LP
OF (s)	1.9258	1.599	1.5987	1.5987

Table 4
Top 10 feasible and optimal solutions from exhaustive LP solver for IEEE 3-bus system.

N. sol.	TDS						PS (A)						OF (s)
	R_1	R_2	R_3	R_4	R_5	R_6	R_1	R_2	R_3	R_4	R_5	R_6	
1	0.1067	0.1083	0.1000	0.1000	0.1000	0.1119	2.5	2.0	3.0	2.5	2.5	1.5	1.5987
2	0.1067	0.1000	0.1000	0.1000	0.1000	0.1119	2.5	2.5	3.0	2.5	2.5	1.5	1.5990
3	0.1067	0.1084	0.1000	0.1000	0.1000	0.1000	2.5	2.0	3.0	2.5	2.5	2.0	1.5998
4	0.1067	0.1000	0.1000	0.1000	0.1000	0.1000	2.5	2.5	3.0	2.5	2.5	2.0	1.6000
5	0.1000	0.1083	0.1000	0.1000	0.1000	0.1119	3.0	2.0	3.0	2.5	2.5	1.5	1.6017
6	0.1000	0.1000	0.1000	0.1000	0.1000	0.1119	3.0	2.5	3.0	2.5	2.5	1.5	1.6020
7	0.1000	0.1084	0.1000	0.1000	0.1000	0.1000	3.0	2.0	3.0	2.5	2.5	2.0	1.6028
8	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	3.0	2.5	3.0	2.5	2.5	2.0	1.6030
9	0.1216	0.1083	0.1000	0.1000	0.1000	0.1119	2.0	2.0	3.0	2.5	2.5	1.5	1.6119
10	0.1067	0.1084	0.1000	0.1141	0.1000	0.1000	2.5	2.0	3.0	2.0	2.5	2.0	1.6120

2. Problem formulation

The main objective of the coordination problem is to calculate the TDSs and PSs that minimize the relay operational times. Relays located closer to the fault should have priority of operation, and backup relays should operate only if a primary relay fails.

2.1. Relay operational characteristics

Directional overcurrent relay (DOCRs) operates when its input current exceeds a predetermined value. The inverse definite minimum time (IDMT) DOCRs has its operational time inversely proportional to the input current. Furthermore, the directional element distinguish the direction of the fault current and it is sensitized only to a specific fault direction. Thus, it is possible to achieve selectivity in meshed systems. Otherwise, it is practically impossible to obtain selectivity on meshed systems using only non-directional overcurrent relays. The equation that describes the time-current characteristics of International Electrotechnical Commission (IEC) IDMT relay is [39]:

$$T_{i,j} = \frac{TDS_i \cdot A_i}{(I_{scj}/PS_i)^{N_i} - 1}, \quad (1)$$

where $T_{i,j}$ is the operation time of the relay i (R_i) for a fault located at j . TDS_i and PS_i are the TDS and PS of R_i , respectively. I_{scj} is the level of short-circuit seen by R_i for a fault located at j . A_i and N_i are constants related to the IEC standardized curve types of R_i . For a standard inverse curve type, these constants are 0.14 and 0.02, respectively.

2.2. Coordination problem

When the coordination problem is formulated as an optimization problem, the objective is to find the TDS and PS for all the DOCRs that minimize their operational times, without losing selectivity. The coordination problem has non-convex characteristics [20–22], so heuristic-based optimization techniques are more appropriate to solve it in a reasonable time. When TDS or PS are considered as discrete variables, the problem is formulated as mixed-integer nonlinear programming, which is even harder to solve.

2.3. Objective function

The objective is to minimize the weighted sum of all relay operational times when they act as primary protection. One of the main consequences of a faulty system is the built-up current, so the DOCRs should act as fast as possible (without violating any constraint) to reduce the mechanical and thermal stress of the electrical grid. The objective function (OF) is expressed as:

$$\min \sum_j \sum_{i=1}^m T_{i,j}, \quad (2)$$

where m is the number of DOCRs.

2.4. Constraints

The OF presented in (2) is subjected to three sets of constraints as follows:

(1) *Selectivity criteria*: Selectivity minimizes the power outage by prioritizing the action of relays near the location of faults by adding a propositional delay between operation of primary and backup (P/B) relays, known as coordination time interval (CTI). Backup relays should not act unless the primary (main) relays fail to take appropriate action. The selectivity criteria are formulated as a set of inequality constraints:

$$T_{k,j} - T_{i,j} \geq CTI, \quad (3)$$

where $T_{k,j}$ is the operational time of the k th backup protection of the i th relay for a fault at j . CTI is the coordination time interval (in seconds) to ensure the selectivity, and it depends on types of relay (electromechanical or microprocessor-based), speed of circuit breakers, and other system parameters. Typically, it is used a CTI of 0.3–0.4 and 0.1–0.2 s, for electromechanical and microprocessor-based relays, respectively [26].

(2) *Bounds of relay operational times*: The following constraints are related to the bounds of minimum and maximum operational times allowed. Although relays should operate as fast as possible, they required a minimum amount of time to operate. However, if the DOCRs takes long to operate, irreversible equipment damage and instability on the power system might occur. This set of constraints is shown as:

$$T_i^{\min} \leq T_{i,j} \leq T_i^{\max}, \quad (4)$$

where T_i^{\min} is the minimum operational time required for R_i when it acts as primary protection. This time varies with the relay manufacturer, and it is usually adopted 0.05 [3] or 0.2 s [20]. T_i^{\max} is the maximum operational time required for R_i when it acts as primary protection. This value varies with the equipment thermal limits. In [3], it is considered 1 s for T_i^{\max} .

(3) *Bounds on relay settings*: The DOCRs must allow the normal operation of the system. Thus, the PS should be greater than the maximum expected load through the relays, even in a light overload situation. The PS should be smaller than the minimum fault current seen by the respective relay. Otherwise, the relay is not sensitive for that fault. In addition, PS and TDS must be within the range available for each relay. These bounds are the constraints shown in (6) and (7). The minimum and maximum available TDS usually vary from 0.025 to 0.1 and 1.1 to 1.2, respectively [3,20]. Thus,

$$\frac{OLF \cdot I_i^{\text{load}}}{CTR_i} < PS_i < \frac{I_{sc_i}^{\min}}{CTR_i}, \quad (5)$$

$$PS_i^{\min} \leq PS_i \leq PS_i^{\max}, \quad (6)$$

$$TDS_i^{\min} \leq TDS_i \leq TDS_i^{\max}, \quad (7)$$

where OLF is the overload factor considered to estimate the maximum

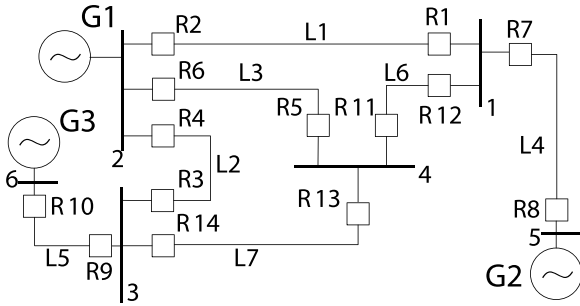


Fig. 4. Single-line diagram of the IEEE 6-bus system.

Table 5
Optimal PS and TDS of the IEEE 6-bus system.

Relay	PS (A)	TDS	Relay	PS (A)	TDS
R ₁	1.5000	0.1014	R ₈	1.2500	0.0500
R ₂	1.5000	0.1863	R ₉	1.2500	0.0500
R ₃	1.2776	0.0946	R ₁₀	1.3968	0.0500
R ₄	1.5000	0.1006	R ₁₁	1.5000	0.0650
R ₅	1.2500	0.0500	R ₁₂	1.5000	0.0506
R ₆	1.3808	0.0500	R ₁₃	1.4329	0.0500
R ₇	1.2500	0.0500	R ₁₄	1.5000	0.0708
Time ^a (s)	0.4079				
OF (s)	10.1512				

^a Average of 100 runs.

Table 6
Operational times between P/B relays for the IEEE 6-bus system.

Primary	Backup	ΔT^a (s)	Primary	Backup	ΔT^b (s)
R ₁	R ₈	0.2907	R ₁	R ₈	0.8105
R ₁	R ₁₁	4.0356	R ₂	R ₃	0.2000
R ₃	R ₁₀	0.2000	R ₄	R ₁	0.2000
R ₃	R ₁₃	0.2791	R ₅	R ₁₂	1.1119
R ₄	R ₁	0.4524	R ₆	R ₃	0.9252
R ₅	R ₁₂	0.2261	R ₇	R ₂	0.2000
R ₅	R ₁₄	0.5202	R ₇	R ₁₁	0.2370
R ₆	R ₃	0.5784	R ₉	R ₄	0.2000
R ₇	R ₂	0.2380	R ₉	R ₁₃	0.2000
R ₇	R ₁₁	0.2000	R ₁₁	R ₆	0.2000
R ₉	R ₄	0.2151	R ₁₁	R ₁₄	0.2000
R ₉	R ₁₃	0.2613	R ₁₂	R ₂	6.2261
R ₁₁	R ₆	2.3807	R ₁₂	R ₈	0.5312
R ₁₁	R ₁₄	0.3964	R ₁₃	R ₁₂	0.2000
R ₁₂	R ₂	0.7218	R ₁₄	R ₄	3.4340
R ₁₂	R ₈	0.2603	R ₁₄	R ₁₀	0.6447
R ₁₃	R ₆	0.5009			
R ₁₃	R ₁₂	0.4372			
R ₁₄	R ₄	0.2800			
R ₁₄	R ₁₀	0.2683			

^a Related to near-end faults.

^b Related to far-end faults.

load through the electric system. I_i^{load} is the maximum expected load through R_i . CTR_i is the current transformer ratio of R_i . I_{sc}^{min} is the minimum fault current seen by the R_i when it acts as primary or backup protection. PS_i^{min} and PS_i^{max} are the minimum and maximum PS_i values available, respectively. TDS_i^{min} and TDS_i^{max} are the minimum and maximum TDS available for R_i , respectively.

3. Simulated annealing-linear programming algorithm

3.1. Introduction

The proposed algorithm named as SA-LP is a hybrid technique that combines the meta-heuristic simulated annealing (SA) and linear

Table 7
Comparison of the results for the six-bus system.

	GA [42]	BIP [9]	MDE5 [3]	OCDE2 [30]	ADE [29]	SA-LP
OF (s)	10.7345	10.5380	10.3514	10.3286	10.2664	10.1512

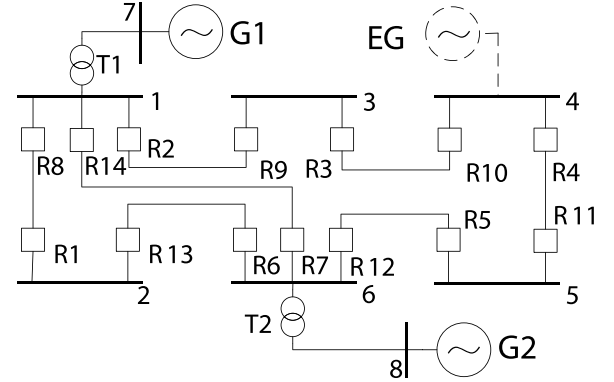


Fig. 5. Single-line diagram of the IEEE 8-bus system.

Table 8
Optimal PS and TDS of the IEEE 8-bus system.

Relay	PS (A)	TDS	Relay	PS (A)	TDS
R ₁	2.00	0.1132	R ₈	2.50	0.1697
R ₂	2.50	0.2602	R ₉	2.50	0.1473
R ₃	2.50	0.2251	R ₁₀	2.50	0.1759
R ₄	2.50	0.1603	R ₁₁	2.50	0.1869
R ₅	2.50	0.1000	R ₁₂	2.50	0.2664
R ₆	2.50	0.1731	R ₁₃	2.00	0.1138
R ₇	2.50	0.2428	R ₁₄	2.50	0.2459
Time ^a (s)	0.0602				
OF (s)	8.4271				

^a Average of 100 runs.

Table 9
Operational times between P/B relays for the IEEE 8-bus system.

Primary	Backup	ΔT (s)	Primary	Backup	ΔT (s)
R ₁	R ₆	0.3000	R ₈	R ₇	0.5768
R ₂	R ₁	0.3000	R ₈	R ₉	0.4533
R ₂	R ₇	0.3000	R ₉	R ₁₀	0.3000
R ₃	R ₂	0.3000	R ₁₀	R ₁₁	0.3000
R ₄	R ₃	0.3000	R ₁₁	R ₁₂	0.3000
R ₅	R ₄	0.3000	R ₁₂	R ₁₃	0.3000
R ₆	R ₅	0.4964	R ₁₂	R ₁₄	0.3000
R ₆	R ₁₄	0.5871	R ₁₃	R ₈	0.3000
R ₇	R ₅	0.3620	R ₁₄	R ₁	0.4235
R ₇	R ₁₃	0.4527	R ₁₄	R ₉	0.3000

programming (LP) to achieve the coordination of DOCRs. The SA algorithm is a probabilistic optimization technique that has an analogy with the physical annealing of solids [40]. The ability to escape local optimal is held by the probability of accepting solutions of inferior quality during the optimization process. In SA-LP, the SA algorithm treats the PSs as decision variables, while the optimization of TDSs is handled by an LP solver, as shown in Fig. 1. The following explains the step-by-step of the implemented algorithm.

3.2. Step-by-step process

Step 1: Create an initial solution by randomly selecting values of $PS_i \in VPS_i: \forall i$, where VPS_i is a set of PS_i values that does not violate the restrictions (5) and (6). An example is shown in Fig. 2.

Table 10
Comparison of the results for the IEEE 8-bus system.

	MPSO [25]	GA [27]	GA-LP [27]	BBO-LP [4]	BIP [9]	SOA [32]	SA-LP
OF (s)	17.33	11.001	10.949	8.7556	8.6944	8.4271	8.4271

Table 11
Top 10 feasible & optimal solutions from exhaustive from limited LP solver for the IEEE 8-bus system.

N. sol.		1	2	3	4	5	6	7	8	9	10
TDS	R_1	0.1132	0.1132	0.1132	0.1132	0.1132	0.1132	0.1132	0.1132	0.1132	0.1132
	R_2	0.2602	0.2602	0.2602	0.2602	0.2602	0.2602	0.2602	0.2602	0.2602	0.2602
	R_3	0.2251	0.2251	0.2251	0.2251	0.2251	0.2251	0.2251	0.2251	0.2251	0.2251
	R_4	0.1603	0.1603	0.1603	0.1603	0.1603	0.1603	0.1603	0.1603	0.1603	0.1603
	R_5	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
	R_6	0.1731	0.1731	0.1965	0.1965	0.1731	0.1731	0.1731	0.2268	0.1731	0.1965
	R_7	0.2428	0.2428	0.2428	0.2428	0.2428	0.2428	0.2783	0.2428	0.2783	0.2428
	R_8	0.1697	0.1700	0.1697	0.1700	0.1937	0.1940	0.1697	0.1697	0.1700	0.1937
	R_9	0.1473	0.1473	0.1473	0.1473	0.1473	0.1473	0.1473	0.1473	0.1473	0.1473
	R_{10}	0.1759	0.1759	0.1759	0.1759	0.1759	0.1759	0.1759	0.1759	0.1759	0.1759
	R_{11}	0.1869	0.1869	0.1869	0.1869	0.1869	0.1869	0.1869	0.1869	0.1869	0.1869
	R_{12}	0.2664	0.2664	0.2664	0.2664	0.2664	0.2664	0.2664	0.2664	0.2664	0.2664
	R_{13}	0.1138	0.1000	0.1138	0.1000	0.1138	0.1000	0.1138	0.1138	0.1000	0.1138
	R_{14}	0.2459	0.2459	0.2459	0.2459	0.2459	0.2459	0.2459	0.2459	0.2459	0.2459
PS (A)	R_1	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
	R_2	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
	R_3	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
	R_4	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
	R_5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
	R_6	2.5	2.5	2.0	2.0	2.5	2.5	2.5	1.5	2.5	2.0
	R_7	2.5	2.5	2.5	2.5	2.5	2.5	2.0	2.5	2.0	2.5
	R_8	2.5	2.5	2.5	2.5	2.0	2.0	2.5	2.5	2.5	2.0
	R_9	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
	R_{10}	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
	R_{11}	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
	R_{12}	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
	R_{13}	2.0	2.5	2.0	2.5	2.0	2.5	2.0	2.0	2.5	2.0
	R_{14}	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
OF (s)		8.4271	8.4292	8.4440	8.4461	8.4466	8.4487	8.4608	8.4619	8.4629	8.4635

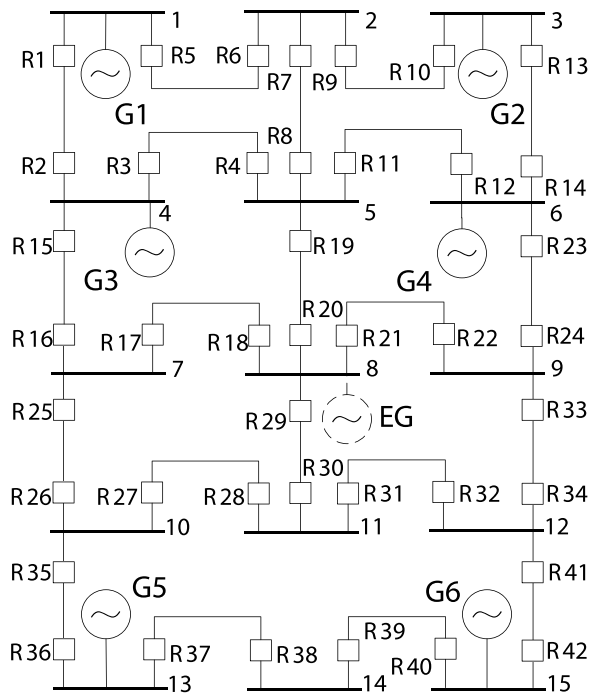


Fig. 6. Single-line diagram of the IEEE 15-bus system.

Table 12
Optimal PS and TDS of the IEEE 15-bus system.

Relay	PS (A)	TDS	Relay	PS (A)	TDS
R_1	1.50	0.1000	R_{22}	1.50	0.1092
R_2	1.00	0.1008	R_{23}	1.00	0.1094
R_3	2.00	0.1047	R_{24}	1.50	0.1000
R_4	1.00	0.1153	R_{25}	2.00	0.1027
R_5	2.00	0.1086	R_{26}	1.50	0.1120
R_6	2.00	0.1072	R_{27}	2.00	0.1041
R_7	2.00	0.1055	R_{28}	2.50	0.1051
R_8	1.50	0.1068	R_{29}	1.50	0.1042
R_9	2.00	0.1062	R_{30}	2.00	0.1011
R_{10}	1.50	0.1122	R_{31}	2.00	0.1000
R_{11}	1.50	0.1000	R_{32}	1.50	0.1055
R_{12}	1.50	0.1000	R_{33}	2.50	0.1003
R_{13}	2.00	0.1073	R_{34}	2.50	0.1073
R_{14}	1.00	0.1114	R_{35}	2.00	0.1031
R_{15}	1.00	0.1035	R_{36}	2.00	0.1000
R_{16}	1.50	0.1000	R_{37}	2.50	0.1030
R_{17}	2.00	0.1000	R_{38}	2.50	0.1062
R_{18}	1.00	0.1051	R_{39}	2.50	0.1025
R_{19}	2.00	0.1016	R_{40}	2.50	0.1043
R_{20}	1.50	0.1000	R_{41}	2.50	0.1041
R_{21}	0.50	0.1658	R_{42}	1.50	0.1043
Time ^a (s)	0.5050				
OF (s)	12.2149				

^a Average of 100 runs.

Table 13

Operational times between P/B relays for the IEEE 15-bus system.

Primary	Backup	ΔT (s)	Primary	Backup	ΔT (s)	Primary	Backup	ΔT (s)
R_1	R_6	0.2000	R_{15}	R_4	0.2000	R_{27}	R_{36}	0.2208
R_2	R_4	0.2040	R_{16}	R_{18}	0.2402	R_{28}	R_{29}	0.2000
R_2	R_{16}	0.2547	R_{16}	R_{26}	0.2060	R_{28}	R_{32}	0.2000
R_3	R_1	0.2616	R_{17}	R_{15}	0.2342	R_{29}	R_{17}	0.2752
R_3	R_{16}	0.2034	R_{17}	R_{26}	0.2000	R_{29}	R_{19}	0.2336
R_4	R_7	0.2049	R_{18}	R_{19}	0.2641	R_{29}	R_{22}	0.2000
R_4	R_{12}	0.2224	R_{18}	R_{22}	0.2305	R_{30}	R_{27}	0.2000
R_4	R_{20}	0.2510	R_{18}	R_{30}	0.2641	R_{30}	R_{32}	0.2482
R_5	R_2	0.2000	R_{19}	R_3	0.2090	R_{31}	R_{27}	0.2146
R_6	R_8	0.2000	R_{19}	R_7	0.2000	R_{31}	R_{29}	0.2628
R_6	R_{10}	0.2029	R_{19}	R_{12}	0.2176	R_{32}	R_{33}	0.2163
R_7	R_5	0.2000	R_{20}	R_{17}	0.2775	R_{32}	R_{42}	0.2469
R_7	R_{10}	0.2000	R_{20}	R_{22}	0.2024	R_{33}	R_{21}	0.2000
R_8	R_3	0.2000	R_{20}	R_{30}	0.2359	R_{33}	R_{23}	0.2000
R_8	R_{12}	0.2086	R_{21}	R_{17}	0.2416	R_{34}	R_{31}	0.2272
R_8	R_{20}	0.2372	R_{21}	R_{19}	0.2000	R_{34}	R_{42}	0.2000
R_9	R_5	0.2188	R_{21}	R_{30}	0.2000	R_{35}	R_{25}	0.2095
R_9	R_8	0.2159	R_{22}	R_{23}	0.2707	R_{35}	R_{28}	0.2000
R_{10}	R_{14}	0.2000	R_{22}	R_{34}	0.2015	R_{36}	R_{38}	0.2000
R_{11}	R_3	0.2095	R_{23}	R_{11}	0.2432	R_{37}	R_{35}	0.2000
R_{11}	R_7	0.2005	R_{23}	R_{13}	0.3770	R_{38}	R_{40}	0.2000
R_{11}	R_{20}	0.2467	R_{24}	R_{21}	12.6703	R_{39}	R_{37}	0.2000
R_{12}	R_{13}	0.2000	R_{24}	R_{34}	0.2000	R_{40}	R_{41}	0.2000
R_{12}	R_{24}	0.2040	R_{25}	R_{15}	0.2000	R_{41}	R_{31}	0.2578
R_{13}	R_9	0.2000	R_{25}	R_{18}	0.2000	R_{41}	R_{33}	0.2000
R_{14}	R_{11}	0.2332	R_{26}	R_{28}	0.2261	R_{42}	R_{39}	0.2000
R_{14}	R_{24}	0.2259	R_{26}	R_{36}	0.2564			
R_{15}	R_1	0.3090	R_{27}	R_{25}	0.2000			

Table 14

Comparison of the results for the IEEE 15-bus system.

	GA-NLP [33]	CSA [33]	B&B [32]	PSO-LP [43]	SOA [32]	SA-LP
OF (s)	19.5843	19.5521	15.335	15.002	12.227	12.2149

Step 2: Set the temperature (T_k) as initial temperature (T_0) as shown in (8).

$$T_k = T_0 = \frac{\mu}{-\ln(\phi)} f(S_0), \quad (8)$$

where ϕ is the probability of S_j such that $f(S_j) > f(S_0)$ to be accepted when $f(S_j)$ is μ times worse than $f(S_0)$ and T_k is the temperature at k th iteration.

Step 3: Create a list of relays ordered in descending order regarding their operational times for faults within the primary protection zone (T_{ij}).

Step 4: Establish a search direction for the intensification procedure on R_i as shown in Pseudocode 1. Consider that S_0 is the current solution, S_j is the neighbor solution, $f(S_0)$ is the objective function of S_0 and $f(S_j)$ is the objective function of S_j . The direction of the search can be held on higher or lower values of PS_i .

Pseudocode 1. Search direction for the intensification procedure.

- 1: Create a set of S_j with the adjacent neighborhood of $PS_i \in S_0$;
- 2: $\text{iff}(S_j) < f(S_0)$ or $\{f(S_j) > f(S_0) \text{ and } e^{\left(\frac{f(S_0)-f(S_j)}{T}\right)} > \text{random}[0, 1]\}$ then
- 3: The search is towards the neighbor S_j with the best OF;
- 4: $S_0 \leftarrow S_j$;
- 5: **else**
- 6: Go to Step 7.
- 7: **end if**

Step 5: Perform the intensification procedure on PS_i to improve $f(S_0)$, as shown in Pseudocode 2.

Pseudocode 2. Intensification procedure.

- 1: Perform an uphill or a downhill moves, regarding the search direction, on the neighborhood of $PS_i \in S_0$.
- 2: $\text{iff}(S_j) < f(S_0)$ or $\{f(S_j) > f(S_0) \text{ and } e^{\left(\frac{f(S_0)-f(S_j)}{T}\right)} > \text{random}[0, 1]\}$
- 3: $S_0 \leftarrow S_j$;
- 4: **else**
- 5: Ends the intensification procedure on PS_i .
- 6: **end if**

Step 6: If a new solution is found, update the temperature T_{k+1} with a geometric decay such as:

$$T_{k+1} = \alpha \cdot T_k, \quad (9)$$

where α is the decay factor. Otherwise, to go step 7.

Step 7: Increase the counter i if there are still relays on the list to be analyzed. Otherwise, go to step 8.

Step 8: If the incumbent solution is not updated in ξ iterations, the optimization process is terminated. Otherwise, there still room for improvement and the process restarts in Step 3.

3.3. Communication failure scenario

During a condition of communication failure, relays are unable to update their parameters. In this case, the algorithm considers the last known settings of the incommunicable relays as constants during the optimization process. Thus, the algorithm can solve the coordination problem by finding a feasible but sub-optimal solution.

3.4. Solution space size

For a given set of PSs, it is possible to compute the optimal values of TDSs because the problem becomes convex. Thus, the solution space size (SSS) is calculated as

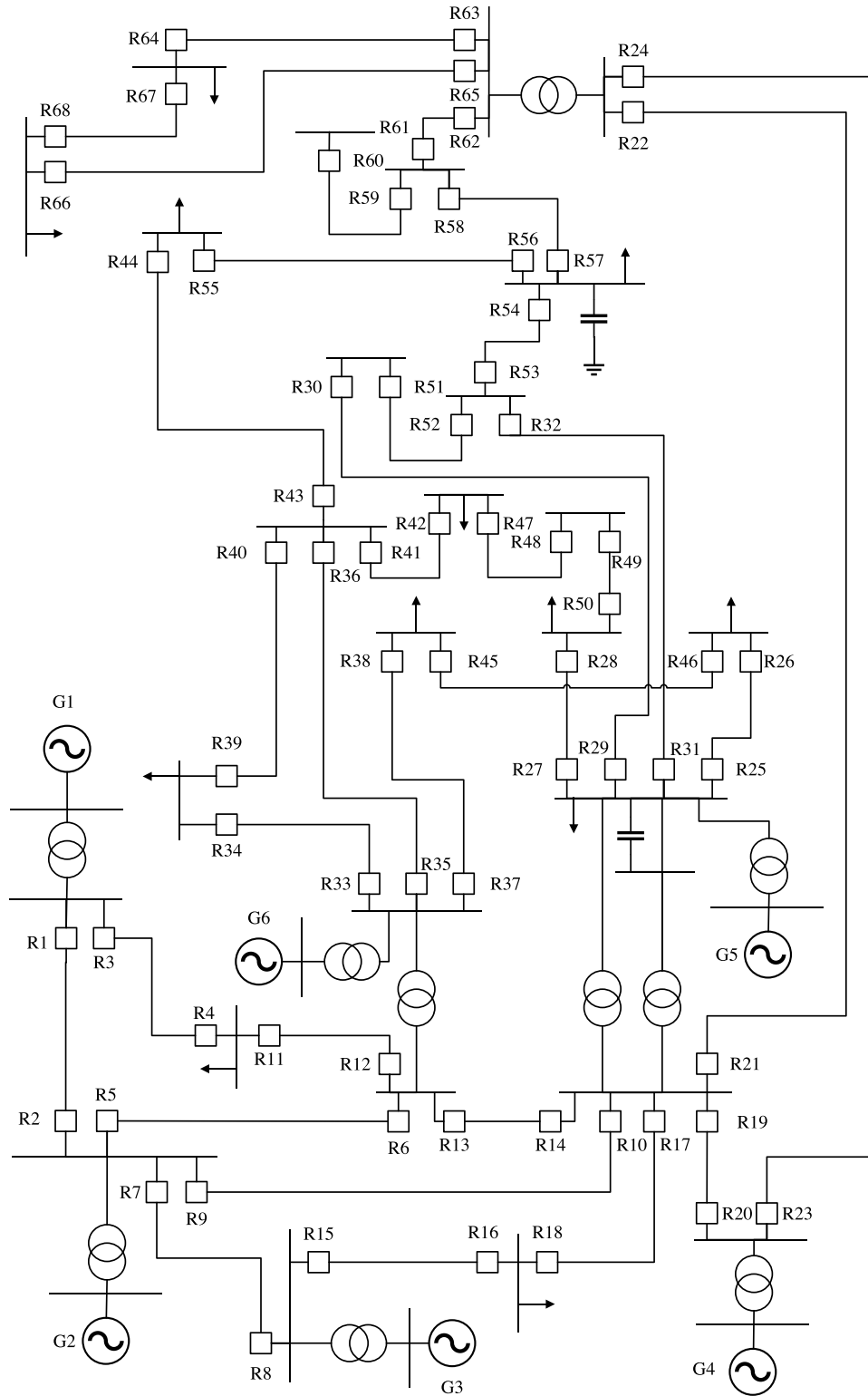


Fig. 7. Single-line diagram of the IEEE 30-bus system.

Table 15
Convergence results for the IEEE 30-bus, during 100 runs.

Min. OF (s)	Mean OF (s)	Max. OF (s)	Std. (s)
22.3936	22.8003	23.2796	0.2044

$$SSS = \prod_{i=1}^m PS_i \subset VPS_i, \quad (10)$$

where m is the number of DOCRs.

4. Simulation results and discussions

This section presents the main results obtained with the SA-LP. Five

Table 16
Optimal PS and TDS of the IEEE 30-bus system.

Relay	PS (A)	TDS	Relay	PS (A)	TDS	Relay	PS (A)	TDS
R ₁	5.0	0.1562	R ₂₄	1.5	0.1000	R ₄₇	5.0	0.1570
R ₂	5.0	0.1181	R ₂₅	5.0	0.2725	R ₄₈	5.0	0.1522
R ₃	5.0	0.1285	R ₂₆	5.0	0.1667	R ₄₉	5.0	0.1285
R ₄	2.5	0.1017	R ₂₇	5.0	0.2670	R ₅₀	5.0	0.2052
R ₅	5.0	0.1080	R ₂₈	5.0	0.1054	R ₅₁	5.0	0.2343
R ₆	1.5	0.1052	R ₂₉	5.0	0.2916	R ₅₂	5.0	0.1423
R ₇	5.0	0.1057	R ₃₀	2.5	0.1000	R ₅₃	5.0	0.1963
R ₈	2.0	0.1000	R ₃₁	5.0	0.1870	R ₅₄	5.0	0.1266
R ₉	5.0	0.1038	R ₃₂	2.0	0.1000	R ₅₅	5.0	0.1323
R ₁₀	2.0	0.1000	R ₃₃	5.0	0.1550	R ₅₆	5.0	0.1535
R ₁₁	5.0	0.1062	R ₃₄	1.5	0.1000	R ₅₇	5.0	0.1516
R ₁₂	5.0	0.1380	R ₃₅	5.0	0.2416	R ₅₈	5.0	0.1091
R ₁₃	5.0	0.1371	R ₃₆	5.0	0.1091	R ₅₉	1.5	0.1000
R ₁₄	5.0	0.1467	R ₃₇	5.0	0.2143	R ₆₀	2.5	0.1000
R ₁₅	3.5	0.1055	R ₃₈	5.0	0.1453	R ₆₁	5.0	0.1201
R ₁₆	3.5	0.1021	R ₃₉	4.5	0.1002	R ₆₂	1.5	0.2480
R ₁₇	5.0	0.1408	R ₄₀	5.0	0.1192	R ₆₃	5.0	0.1216
R ₁₈	3.0	0.1000	R ₄₁	5.0	0.2062	R ₆₄	1.5	0.1000
R ₁₉	5.0	0.1337	R ₄₂	5.0	0.1184	R ₆₅	4.0	0.1265
R ₂₀	3.0	0.1000	R ₄₃	5.0	0.1884	R ₆₆	1.5	0.1000
R ₂₁	5.0	0.1253	R ₄₄	5.0	0.1269	R ₆₇	3.0	0.1000
R ₂₂	1.5	0.1000	R ₄₅	5.0	0.1607	R ₆₈	4.0	0.1000
R ₂₃	2.0	0.1057	R ₄₆	5.0	0.1861			
Time ^a (s)			0.9799					
OF (s)			22.3936					

^a Average of 100 runs.

different test cases were used to test the effectiveness of the proposed technique. The coordination is carried out by phase relays, but a similar procedure can be performed for ground relays. The proposed algorithm is carried out using MATLAB R2015a running on a Windows 10, 64 bits platform, with a Core i5 3.5 GHz PC with 8 GB of ram. LP problems are solved using IBM ILOG CPLEX Optimization Studio v.12.5. The parameters of the SA are $\mu = 10\%$, $\phi = 5\%$, $\alpha = 60\%$, and $\xi = 20$. To evaluate its convergence, the proposed algorithm is executed 100 times.

The obtained results are confronted with other optimization algorithms reported in the specialized literature that uses the same: (1) fault levels, types and locations seen by DOCRs; (2) P/B DOCRs pairs; (3) minimum and maximum DOCRs operational times and CTI; (4) PS step size; (5) continuous TDS, therefore its representation is truncated after the fourth digit after the dot; (6) bounds on the minimum and maximum TDS and PS; (7) DOCRs IDMT curve type (IEC standard inverse); (8) system topology.

4.1. Case I: IEEE 3-bus system

In case I, the proposed algorithm is applied to the IEEE 3-bus system presented in Fig. 3. It has three buses, three lines, three generators, and six DOCRs. Detailed data of this system are presented in [32]. The coordination is held for three-phase faults in the midpoint of each line. Moreover, there are 12 selectivity constraints: 6 for steady and 6 transient configurations. The PS varies from 1.5 to 5.0 A, in uniform steps of 0.5 A. The TDS is a continuous variable with a range from 0.1 to 1.1. The adopted value of CTI is 0.2 s.

The coordination results are shown in Table 1. The operational times between P/B relays, ΔT , are shown in 2. The system is coordinated as the minimum ΔT is the CTI (0.2 s) itself. The minimum DOCR operational times are 0.2496 and 0.2444 s for normal and transient configurations, respectively. A comparison with other optimization algorithms is presented in Table 3. The proposed technique is able to find the same result (OF of 1.5987 s) as the BBO-LP [4], demanding less computational time. Results also show that the proposed algorithm has superior performance than PSO [26], SOA [32], and BBO-LP [4]. The SA-LP converged to the optimum solution in all 100 runs.

4.1.1. IEEE 3-bus: exhaustive search

The SSS for the three-bus system is 1.2×10^5 , which is relatively low. Thus, is held an exhaustive search to find the global optimal solution. Results for the best feasible solutions are shown in Table 4. The best configuration found is equal to the proposed algorithm.

4.2. Case II: IEEE 6-bus system

In this case, the coordination is held for the IEEE 6-bus system shown in Fig. 4. This system has six buses, seven lines, three generators, and fourteen DOCRs. Both close-in and far-end three-phase faults are considered in its main topology. Detailed system data are available in [3].

For a fair comparison, it is considered the same OF presented in [3]. There are 48 selectivity constraints related to both close-in and far-end faults, but 10 of these are relaxed [41]. TDS varies continuously from 0.05 to 1.10. PS varies from 1.25 to 1.50 A, with steps of 100 μ A. A CTI of 0.2 s is considered. In this problem, T_{min} is 0.05 s and T_{max} is 1.00 s.

Table 5 shows the coordination results for the IEEE 6-bus system. A comparison between the proposed algorithm and others published in the literature is shown in Table 7. Regarding the OF, the proposed algorithm outperforms the GA [42], BIP [9], modified DE 5 (MDE5) [3], adaptive DE algorithm (ADE) [30] and opposition based chaotic DE 2 (OCDE2) [29]. The difference of operational times (ΔT) between P/B relays, regarding the CTI, are shown in Table 6. The smallest ΔT found is 0.2000 s, which is the considered CTI. Also, the lowest relay operational time is 0.1863 s, which is greater than T_{min} (0.05 s). Also, SA-LP converged to the optimum solution in all runs.

4.3. Case III: IEEE 8-bus system

The following system is shown in Fig. 5. This network has eight buses, seven lines, two generators, and fourteen DOCRs. EG is an external grid modeled with a short-circuit capacity of 400 MVA. Detailed data of this system are presented in [32]. There are 20 selectivity constraints related to near-end three-phase faults. The available PS are 0.5, 0.6, 0.8, 1.0, 1.5, 2.0 and 2.5 A. The TDS can vary continuously from 0.1 to 1.1. A CTI of 0.3 s is considered.

The optimal settings for the IEEE 8-bus system are shown in Table 8. The ΔT between P/B relays are shown in Table 8. The lowest value of ΔT between P/B relays found was 0.3000 s, which is the corresponding CTI of this system. The lowest relay operational time found was 0.4076 s. A comparison of the proposed algorithm against other optimization techniques is shown in Tables 9 and 10. The SA-LP was able to find the same results as the SOA [32], which has the lowest OF than other quoted results. Also, SA-LP converged to the optimum solution in all 100 runs.

4.3.1. IEEE 8-bus: exhaustive search

The SSS for the IEEE 8-bus system is 4.8×10^6 , a not so large value. Thus, an exhaustive search was performed. Results for the 10 best solutions are shown in Table 11. The best solution found is the same as the proposed algorithm.

4.4. Case IV: IEEE 15-bus system

Case IV refers to the system presented in Fig. 6. This network has 15 buses, 21 lines, 6 generators, and 42 DOCRs. EG is an external grid modeled with a short-circuit capacity of 200 MVA. Detailed data of this system are presented in [32]. This problem has 82 selectivity constraints related to near-end three-phase faults. The minimum and maximum values of PS are 0.5 and 2.5 A, in uniform steps of 0.5 A, respectively. TDSs are continuous variables ranging from 0.1 to 1.1. The CTI for this system is 0.2 s.

The SSS for the IEEE 15-bus system is 4.5×10^{28} . The output of the proposed technique is shown in Table 12. The ΔT between P/B relays are shown in Table 13. The minimum operational time of DOCRs is

Table 17
Operational times between P/B relays for the IEEE 30-bus system.

Primary	Backup	ΔT (s)	Primary	Backup	ΔT (s)	Primary	Backup	ΔT (s)
R ₁	R ₄	0.3000	R ₁₉	R ₂₂	–	R ₄₁	R ₃₅	0.3000
R ₂	R ₆	0.3000	R ₂₀	R ₂₄	0.3096	R ₄₁	R ₃₉	0.3000
R ₂	R ₈	0.3809	R ₂₁	R ₉	0.3488	R ₄₁	R ₄₄	0.3000
R ₂	R ₁₀	0.3807	R ₂₁	R ₁₃	0.3417	R ₄₂	R ₄₈	0.3000
R ₃	R ₂	0.3000	R ₂₁	R ₁₈	0.3577	R ₄₃	R ₃₅	0.3426
R ₄	R ₁₂	0.3000	R ₂₁	R ₂₀	0.3955	R ₄₃	R ₃₉	0.3426
R ₅	R ₁	0.3000	R ₂₂	R ₂₃	0.3000	R ₄₃	R ₄₂	0.3000
R ₅	R ₈	0.3858	R ₂₃	R ₁₉	0.3000	R ₄₄	R ₅₆	0.3000
R ₅	R ₁₀	0.3856	R ₂₄	R ₂₁	0.3000	R ₄₅	R ₃₇	0.3000
R ₆	R ₁₁	0.4700	R ₂₅	R ₂₈	0.3411	R ₄₆	R ₂₅	0.3000
R ₆	R ₁₄	0.4746	R ₂₅	R ₃₀	0.3502	R ₄₇	R ₄₁	0.3000
R ₇	R ₁	0.3025	R ₂₅	R ₃₂	0.4041	R ₄₈	R ₅₀	0.3000
R ₇	R ₆	0.3074	R ₂₆	R ₄₅	0.3000	R ₄₉	R ₄₇	0.3000
R ₇	R ₁₀	0.3881	R ₂₇	R ₂₆	0.3413	R ₅₀	R ₂₇	0.3000
R ₈	R ₁₆	0.3000	R ₂₇	R ₃₀	0.3504	R ₅₁	R ₂₉	0.3000
R ₉	R ₁	0.3103	R ₂₇	R ₃₂	0.4042	R ₅₂	R ₃₁	0.4299
R ₉	R ₆	0.3152	R ₂₈	R ₄₉	0.3000	R ₅₂	R ₅₄	0.3000
R ₉	R ₈	0.3961	R ₂₉	R ₂₆	0.3000	R ₅₃	R ₃₁	0.3000
R ₁₀	R ₁₃	0.4797	R ₂₉	R ₂₈	0.3000	R ₅₃	R ₅₁	0.3000
R ₁₀	R ₁₈	0.4957	R ₂₉	R ₃₂	0.3629	R ₅₄	R ₅₅	0.3726
R ₁₀	R ₂₀	0.5336	R ₃₀	R ₅₂	0.3000	R ₅₄	R ₅₈	0.3734
R ₁₀	R ₂₂	–	R ₃₁	R ₂₆	0.5192	R ₅₅	R ₄₃	0.3000
R ₁₁	R ₃	0.3000	R ₃₁	R ₂₈	0.5192	R ₅₆	R ₅₃	0.3000
R ₁₂	R ₅	0.3000	R ₃₁	R ₃₀	0.5283	R ₅₆	R ₅₈	0.3000
R ₁₂	R ₁₄	0.3000	R ₃₂	R ₅₁	0.5789	R ₅₇	R ₅₃	0.3008
R ₁₃	R ₅	0.3046	R ₃₂	R ₅₄	0.4491	R ₅₇	R ₅₅	0.3000
R ₁₃	R ₁₁	0.3000	R ₃₃	R ₃₆	0.4239	R ₅₈	R ₆₂	0.3000
R ₁₄	R ₉	0.3000	R ₃₃	R ₃₈	0.4794	R ₅₉	R ₅₇	0.4481
R ₁₄	R ₁₈	0.3089	R ₃₄	R ₄₀	0.3000	R ₅₉	R ₆₂	0.4159
R ₁₄	R ₂₀	0.3467	R ₃₅	R ₃₄	0.4451	R ₆₁	R ₅₇	0.3000
R ₁₄	R ₂₂	–	R ₃₅	R ₃₈	0.3000	R ₆₂	R ₆₄	–
R ₁₅	R ₇	0.3000	R ₃₆	R ₃₉	0.5350	R ₆₂	R ₆₆	–
R ₁₆	R ₁₇	0.3000	R ₃₆	R ₄₂	0.4924	R ₆₃	R ₆₁	0.3000
R ₁₇	R ₉	0.3071	R ₃₆	R ₄₄	0.5350	R ₆₃	R ₆₆	–
R ₁₇	R ₁₃	0.3000	R ₃₇	R ₃₄	0.5006	R ₆₄	R ₆₈	0.6686
R ₁₇	R ₂₀	0.3538	R ₃₇	R ₃₆	0.3000	R ₆₅	R ₆₁	0.3122
R ₁₇	R ₂₂	–	R ₃₈	R ₄₆	0.3000	R ₆₅	R ₆₄	–
R ₁₈	R ₁₅	0.3000	R ₃₉	R ₃₃	0.3000	R ₆₆	R ₆₇	0.3184
R ₁₉	R ₉	0.3307	R ₄₀	R ₃₅	0.5041	R ₆₇	R ₆₃	0.3000
R ₁₉	R ₁₃	0.3236	R ₄₀	R ₄₂	0.4615	R ₆₈	R ₆₅	0.3000
R ₁₉	R ₁₈	0.3396	R ₄₀	R ₄₄	0.5041			

Table 18
Number of linear programming problems solved during 100 runs of the SA-LP.

	Minimum	Mean	Maximum	SSS
Case I	56	85.56	127	1.2×10^5
Case II	255	836.49	1891	4.2×10^{46}
Case III	96	113.71	148	4.8×10^6
Case IV	444	558.07	720	4.5×10^{28}
Case V	431	568.70	861	2.7×10^{58}

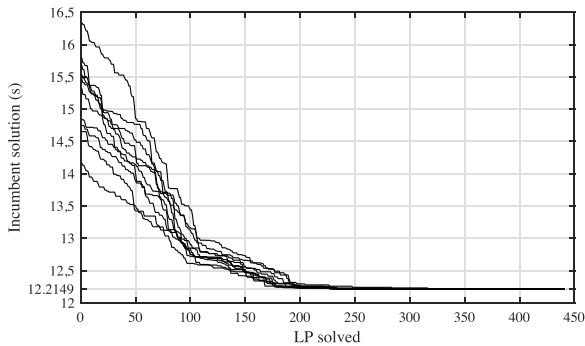


Fig. 8. Convergence of the proposed algorithm for the case IV, for 10 different starting points.

0.2176 s, as seen in Table 12. The selectivity is guaranteed because the lowest value of ΔT corresponds to the CTI itself (0.2 s). Regarding the OF, Table 14 held a comparison of the SA-LP against GA-NLP [33], CSA [33], B&B [32], PSO-LP [43] and SOA [32]. Among the quoted algorithms, the proposed technique has the lowest OF (12.2149 s). Also, SA-LP converged to the optimum solution in all runs.

4.5. Case V: IEEE 30-bus system

The last case refers to the system presented in Fig. 7. This network has 36 buses, 37 lines, 6 generators, 10 transformers, 2 fixed capacitors and 68 DOCRs. Detailed data of this system are presented in [44]. The problem has 122 selectivity restrictions. PSs can vary from 1.5 to 5 A, in uniform steps of 0.5 A. TDSs are treated as continuous variables within range of 0.1–1.0. It is considered a CTI of 0.3 s. The problem has 122 selectivity restrictions, 8 of which were relaxed because the short-circuit currents seen by the backup protections are smaller than the minimum pickup currents.

The results of the SA-LP for the IEEE 30-bus system are shown in Table 16. The SSS for this system is 2.7×10^{58} . The ΔT between P/B relays are shown in Table 17. The selectivity is guaranteed because the lowest value of ΔT is the CTI itself (0.3 s). The convergence results of the proposed technique are shown in Table 15.

4.6. Performance analysis

The SA-LP solves several LP problems during its iterative process. Approximately, 75% of the computational time demanded by the proposed algorithm is spent solving LP problems. The efficiency of the proposed algorithm was evaluated based on the number of LP solved. Thus, the minimum, mean and maximum quantity of LPs solved during the 100 runs of the SA-LP and SSS are shown in Table 18. Fig. 8 shows the convergence of the SA-LP for case IV, considering 10 different starting points.

4.7. Discussion

The SA-LP showed excellent results when compared to other algorithms reported in the specialized literature, under identical conditions. In cases II and IV, the proposed technique was able to find even better results. After 100 runs, for all cases, the proposed technique showed a low deviation on the OF. This great convergence is one of the big advantages of SA-LP, which indicates the robustness and reliability of the proposed technique.

Exhaustive methods to find an optimal solution was applied only to IEEE-3 and IEEE-8 bus systems for validation of the performance of the implemented methodology. For all other systems, computational budget and memory allocation limit the possibility of an entire comparison.

The algorithm proved to be computationally efficient, a critical requirement for online adaptive protection schemes, by evaluating only a fraction of the solution space as shown in Table 18. The average computational time required to obtain the final solution was less than one second for all test-systems used in this work.

In case II, the SA-LP treated the continuous PSs as discrete variables with a resolution of 100 μ A. In this scenario, a trade-off analysis between performance and precision (decimals after the dot) must be held as the SSS exponentially increases with the number of available PSs.

Regarding the OF, the SA-LP performed equal or better than SOA, as seen in Tables 3, 10 and 14. [32] provides the execution times of the SOA for the IEEE-8 and IEEE-15 bus systems. For the former, the execution times were 50.45 s (SOA) and 0.0602 s (SA-LP). For the latter, the execution times were 406.3 s (SOA) and 0.5050 s (SA-LP). Although the simulation was held on different machines, the ratios of execution times between SOA and SA-LP are roughly 800:1, a number too significant to be attributed to the processing power alone.

Case V has eight P/B relays (R_{10}/R_{22} , R_{14}/R_{22} , R_{17}/R_{22} , R_{19}/R_{22} , R_{62}/R_{64} , R_{62}/R_{66} , R_{63}/R_{66} and R_{65}/R_{64}) without backup protection, as seen in Table 17. Those P/B relays do not participate on the optimization process because they violate the right side of the inequality presented on (5).

5. Conclusions

The DOCRs coordination in meshed networks is solved with the new hybrid approach: SA-LP. It showed to be suitable for online adaptive protection schemes due to its great computational performance and convergence. Results show the effectiveness of the SA-LP in comparison to other algorithms, such as the ADE, BBO-LP, CSA, GA, GA-LP, GANLP, MDE5, MPSO, OCDE2, PSO, PSO-LP, BIP and SOA. The proposed approach converged to the best solutions reported in the specialized literature and, in some cases, to even better solutions.

The SA-LP showed a low deviation on the OF, which makes it reliable and robust. The proposed approach handled the coordination problem as a nonlinear mixed-integer optimization problem considering discrete PSs and continuous TDSs as decision variables.

New technologies like dual-setting relays have a future in smart grid applications where communication is growing up on protective devices but nowadays coordination based on both zones of protection, bus, and line, could be inexpensive and valid for actual systems. The most

important application of this work is the intelligent protective relays integration with smart grid technologies. If the coordination is applied as fast as shown in this paper, then a computer could process information almost instantly, making all required calculations in site. Thus, relays are loaded with robust coordinated data, being able to adapt to system changes.

Conflict of interest

None declared.

Acknowledgement

The authors thanks CAPES/CNPq for financial support.

References

- [1] A.J. Urdaneta, R. Nadira, L.G.P. Jimenez, Optimal coordination of directional overcurrent relays in interconnected power systems, *IEEE Trans. Power Deliv.* 3 (3) (1988) 903–911, <https://doi.org/10.1109/61.193867>.
- [2] M.Y. Shih, A. Conde Enríquez, L.M. Torres Treviño, On-line coordination of directional overcurrent relays: performance evaluation among optimization algorithms, *Electr. Power Syst. Res.* 110 (2014) 122–132, <https://doi.org/10.1016/j.epr.2014.01.013>.
- [3] R. Thangaraj, M. Pant, K. Deep, Optimal coordination of over-current relays using modified differential evolution algorithms, *Eng. Appl. Artif. Intell.* 23 (5) (2010) 820–829.
- [4] F.A. Albasri, A.R. Alroomi, J.H. Talaq, Optimal coordination of directional overcurrent relays using biogeography-based optimization algorithms, *IEEE Trans. Power Deliv.* 30 (4) (2015) 1810–1820, <https://doi.org/10.1109/TPWRD.2015.2406114>.
- [5] M.H. Costa, R.R. Saldanha, M.G. Ravetti, E.G. Carrano, Robust coordination of directional overcurrent relays using a matheuristic algorithm, *IET Gen. Transm. Distrib.* 11 (2) (2017) 464–474, <https://doi.org/10.1049/iet-gtd.2016.1010>.
- [6] A. Yazdani, D. Nazarpour, S. Golshannavaz, Dual-setting directional overcurrent relays: an optimal coordination in multiple source meshed distribution networks, *Int. J. Electr. Power Energy Syst.* 86 (2017) 163–176, <https://doi.org/10.1016/j.ijepes.2016.10.004>.
- [7] H.H. Zeineldin, H.M. Sharaf, D.K. Ibrahim, E.E.D.A. El-Zahab, Optimal protection coordination for meshed distribution systems with dg using dual setting directional over-current relays, *IEEE Trans. Smart Grid* 6 (1) (2015) 115–123, <https://doi.org/10.1109/TSG.2014.2357813>.
- [8] C.R. Chen, C.H. Lee, Adaptive overcurrent relay coordination for off-peak loading in interconnected power system, *Int. J. Electr. Power Energy Syst.* 63 (2014) 140–144, <https://doi.org/10.1016/j.ijepes.2014.05.068>.
- [9] R. Corrêa, G. Cardoso, O.C. de Araújo, L. Mariotto, Online coordination of directional overcurrent relays using binary integer programming, *Electr. Power Syst. Res.* 127 (2015) 118–125.
- [10] M. Shih, C. Castillo Salazar, A. Conde Enríquez, Adaptive directional overcurrent relay coordination using ant colony optimisation, *IET Gen. Transm. Distrib.* 9 (14) (2015) 2040–2049, <https://doi.org/10.1049/iet-gtd.2015.0394>.
- [11] E.C. Piescorovsky, N.N. Schulz, Comparison of non-real-time and real-time simulators with relays in-the-loop for adaptive overcurrent protection, *Electr. Power Syst. Res.* 143 (2017) 657–668, <https://doi.org/10.1016/j.epr.2016.10.049>.
- [12] S.P. George, S. Ashok, M.N. Bandyopadhyay, Impact of distributed generation on protective relays, 2013 Int. Conf. Renew. Energy Sustain. Energy (2013) 157–161, <https://doi.org/10.1109/ICRESE.2013.6927806>.
- [13] Optimal overcurrent relay coordination: a review, *Proc. Eng.* 53 (2013) 332–336, <https://doi.org/10.1016/j.proeng.2013.02.043>.
- [14] C.-R. Chen, C.-H. Lee, C.-J. Chang, Optimal overcurrent relay coordination in power distribution system using a new approach, *Int. J. Electr. Power Energy Syst.* 45 (1) (2013) 217–222, <https://doi.org/10.1016/j.ijepes.2012.08.057>.
- [15] H. Yang, F. Wen, G. Ledwich, Optimal coordination of overcurrent relays in distribution systems with distributed generators based on differential evolution algorithm, *Int. Trans. Electr. Energy Syst.* 23 (1) (2013) 1–12, <https://doi.org/10.1002/etep.635>.
- [16] A. Srivastava, J.M. Tripathi, S.R. Mohanty, B. Panda, Optimal over-current relay coordination with distributed generation using hybrid particle swarm optimization-gravitational search algorithm, *Electr. Power Comp. Syst.* 44 (5) (2016) 506–517, <https://doi.org/10.1080/15325008.2015.1117539>.
- [17] R. Ramaswami, M.J. Damborg, S.S. Venkata, Coordination of directional overcurrent relays in transmission systems—a subsystem approach, *IEEE Trans. Power Deliv.* 5 (1) (1990) 64–71, <https://doi.org/10.1109/61.107257>.
- [18] A.J. Urdaneta, H. Restrepo, S. Márquez, J. Sánchez, Coordination of directional overcurrent relay timing using linear programming, *IEEE Trans. Power Deliv.* 11 (1) (1996) 122–128, <https://doi.org/10.1109/61.484008>.
- [19] B. Chattopadhyay, M. Sachdev, T. Sidhu, An on-line relay coordination algorithm for adaptive protection using linear programming technique, *IEEE Trans. Power Deliv.* 11 (1) (1996) 165–173, <https://doi.org/10.1109/61.484013>.
- [20] P.P. Bedekar, S. Bhide, Optimum coordination of directional overcurrent relays

- using the hybrid GA-NLP approach, *IEEE Trans. Power Deliv.* 26 (1) (2011) 109–119, <https://doi.org/10.1109/TPWRD.2010.2080289>.
- [21] V.A. Papaspiliotopoulos, T.S. Kurashvili, G.N. Korres, Optimal coordination of directional overcurrent relays for distribution systems with distributed generation based on a hybrid PSO-LP algorithm, 9th Mediterranean Conference on Power Generation, Transmission Distribution and Energy Conversion (MedPower'14), Athens, Greece, 2014, pp. 1–6, <https://doi.org/10.1049/cp.2014.1697>.
- [22] V.N. Rajput, K.S. Pandya, K. Joshi, Optimal coordination of directional overcurrent relays using hybrid CSA-FFA method, 2015 12th International Conference on Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology (ECTI-CON), vol. 119 (2015) 1–6, <https://doi.org/10.1109/ECTICon.2015.7207044>.
- [23] C. So, Application of genetic algorithm for overcurrent relay coordination, 6th International Conference on Developments in Power Systems Protection, vol. 1997 (1997) 66–69, <https://doi.org/10.1049/cp.19970030>.
- [24] J.A. Sueiro, E. Diaz-Dorado, E. Míguez, J. Cidrás, Coordination of directional overcurrent relay using evolutionary algorithm and linear programming, *Int. J. Electr. Power Energy Syst.* 42 (1) (2012) 299–305.
- [25] H.H. Zeineldin, E.F. El-Saadany, M.M.a. Salama, Optimal coordination of overcurrent relays using a modified particle swarm optimization, *Electr. Power Syst. Res.* 76 (11) (2006) 988–995, <https://doi.org/10.1016/j.epsr.2005.12.001>.
- [26] M.M. Mansour, S.F. Mekhamer, N. El-Kharbawe, A modified particle swarm optimizer for the coordination of directional overcurrent relays, *IEEE Trans. Power Deliv.* 22 (3) (2007) 1400–1410, <https://doi.org/10.1109/TPWRD.2007.899259>.
- [27] A.S. Noghabi, J. Sadeh, H.R. Mashhadi, Considering different network topologies in optimal overcurrent relay coordination using a hybrid GA, *IEEE Trans. Power Deliv.* 24 (4) (2009) 1857–1863, <https://doi.org/10.1109/TPWRD.2009.2029057>.
- [28] Z. Moravej, F. Adelnia, F. Abbasi, Optimal coordination of directional overcurrent relays using NSGA-II, *Electr. Power Syst. Res.* 119 (2015) 228–236, <https://doi.org/10.1016/j.epsr.2014.09.010>.
- [29] S.S. Dash, J. Moirangthem, R. Ramaswami, K.R. K, Adaptive differential evolution algorithm for solving non-linear coordination problem of directional overcurrent relays, *IET Gen. Transm. Distrib.* 7 (4) (2013) 329–336, <https://doi.org/10.1049/iet-gtd.2012.0110>.
- [30] T.R. Chelliah, R. Thangaraj, S. Allamsetty, M. Pant, Coordination of directional overcurrent relays using opposition based chaotic differential evolution algorithm, *Int. J. Electr. Power Energy Syst.* 55 (2014) 341–350, <https://doi.org/10.1016/j.ijepes.2013.09.032>.
- [31] M. Yen, A. Conde, T.-y. Hsiao, L. Martín, T. Trevi, Enhanced differential evolution algorithm for coordination of directional overcurrent relays, *Electr. Power Syst. Res.* 143 (2017) 365–375, <https://doi.org/10.1016/j.epsr.2016.09.011>.
- [32] T. Amraee, Coordination of directional overcurrent relays using seeker algorithm, *IEEE Trans. Power Deliv.* 27 (3) (2012) 1415–1422, <https://doi.org/10.1109/TPWRD.2012.2190107>.
- [33] G. Darji, M. Patel, V. Rajput, K. Pandya, A tuned cuckoo search algorithm for optimal coordination of directional overcurrent relays, 2015 International Conference on Power and Advanced Control Engineering (ICPACE), no. 1 (2015) 162–167, <https://doi.org/10.1109/ICPACE.2015.7274936>.
- [34] T. Khurshaid, A. Wadood, S. Gholami Farkoush, J. Yu, C. Kim, S. Rhee, An improved optimal solution for the directional overcurrent relays coordination using hybridized whale optimization algorithm in complex power systems, *IEEE Access* 7 (2019) 90418–90435, <https://doi.org/10.1109/ACCESS.2019.2925822>.
- [35] T. Khurshaid, A. Wadood, S. Gholami Farkoush, C. Kim, J. Yu, S. Rhee, Improved firefly algorithm for the optimal coordination of directional overcurrent relays, *IEEE Access* 7 (2019) 78503–78514, <https://doi.org/10.1109/ACCESS.2019.2922426>.
- [36] A.E. Labrador Rivas, L.A. Gallego Pareja, T. Abrão, Coordination of distance and directional overcurrent relays using an extended continuous domain ACO algorithm and an hybrid ACO algorithm, *Electr. Power Syst. Res.* 170 (December 2018) (2019) 259–272, <https://doi.org/10.1016/j.epsr.2019.01.032>.
- [37] Y. Damchi, M. Dolatabadi, H.R. Mashhadi, J. Sadeh, Milp approach for optimal coordination of directional overcurrent relays in interconnected power systems, *Electr. Power Syst. Res.* 158 (2018) 267–274, <https://doi.org/10.1016/j.epsr.2018.01.015>.
- [38] S.T.P. Srinivas, P.P. Verma, K.S. Swarup, n, *IEEE Trans. Power Deliv.* 34 (2) (2019) 769–772, <https://doi.org/10.1109/TPWRD.2019.2892606>.
- [39] IEC, IEC 60255-151 – Measuring Relays and Protection Equipment – Part 151: Functional Requirements for Over/Under Current Protection, (2009).
- [40] D. Henderson, S.H. Jacobson, A.W. Johnson, The Theory and Practice of Simulated Annealing, Springer US, Boston, MA, 2003, pp. 287–319, https://doi.org/10.1007/0-306-48056-5_10.
- [41] D. Birla, R.P. Maheshwari, H.O. Gupta, K. Deep, M. Thakur, Application of random search technique in directional overcurrent relay coordination, *Int. J. Emerg. Electr. Power Syst.* 7 (1) (2006), <https://doi.org/10.2202/1553-779X.1271>.
- [42] R.A. Swief, A.Y. Abdelaziz, A. Nagy, Optimal strategy for Over Current relay coordination using genetic algorithm, 2014 International Conference on Engineering and Technology (ICET) (2014) 1–5, <https://doi.org/10.1109/ICEngTechnol.2014.7016786>.
- [43] H. Yang, F. Wen, G. Ledwich, Optimal coordination of overcurrent relays in distribution systems with distributed generators based on differential evolution algorithm, *Int. Trans. Electr. Energy Syst.* 23 (October 2011) (2013) 1–12, <https://doi.org/10.1002/etep.635>.
- [44] M. Shahidehpour, Y. Wang, Information system for control centers, Communication and Control in Electric Power Systems, John Wiley & Sons, Inc., Hoboken, NJ, USA, 2005, pp. 101–134, <https://doi.org/10.1002/0471462926.ch3>.