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Optimal coordination of over-current relays using modified differential evolution algorithms

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ABSTRACT

Optimization of directional over-current relay (DOCR) settings is an important problem in electrical engineering. The optimization model of the problem turns out to be non-linear and highly constrained in which two settings namely time dial setting (*TDS*) and plug setting (*PS*) of each relay are considered as decision variables; the sum of the operating times of all the primary relays, which are expected to operate in order to clear the faults of their corresponding zones, is considered as an objective function. In the present study, three models are considered namely IEEE 3-bus model, IEEE 4-bus model and IEEE 6-bus model. To solve the problem, we have applied five newly developed versions of differential evolution (DE) called modified DE versions (MDE1, MDE2, MDE3, MDE4, and MDE5). The results are compared with the classical DE algorithm and with five more algorithms available in the literature; the numerical results show that the modified DE algorithms outperforms or perform at par with the other algorithms.

1. Introduction

Electrical power system operates at various voltage levels from 415 V to 400 kV or even more. This system can be divided into three parts: generation, transmission and utilization (load). Among these three parts, transmission of power, which is carried out by the electrical conductors is called transmission lines placed in open. Such lines undergo abnormalities more frequently than other parts in their life time due to various reasons like faults (which create over-current) over-load, over-voltage, under-frequency etc. One well-known source for occurrence of over-voltage in such lines is lightning. These abnormalities cause interruption of the supply and may damage the equipments connected to the system, arising the need for protection. Over-current relay is the most commonly used protection scheme in the power system to protect the system from various faults.

Directional over-current relays (DOCRs) are good technical and economic alternative for the protection of interconnected sub-transmission systems and secondary protection of transmission systems (Urdaneta et al., 1997). These relays are provided in electrical power systems to isolate only the fault lines in the event of the faults in the system. Relay is a logical element and issues a trip signal to the circuit breaker if a fault occurs within the relay jurisdiction and is placed at both ends of each line. Their coordination is an important aspect of the protection system design. Relay coordination problem is to determine the sequence of relay operations for each possible fault location so that faulted section is isolated, with sufficient coordination margins and without excessive time delays. This sequence selection is a function of power network topology, relay characteristics, and protection philosophy (Birla et al., 2006).

The DOCR protection scheme consists of two types of settings namely current, referred to as 'Plug Setting' or *PS*, and 'Time Dial Setting' or *TDS*, which must be calculated. With the optimization of these settings (main objective of this paper), an efficient coordination of relays can be achieved and the faulty transmission line may be isolated, thereby maintaining a continuity of supply to healthy sections of the power systems.

In the present study, the above stated problem of coordinating each DOCR with one another in electrical power system, modelled as a non-linear constrained optimization problem, is solved using the basic differential evolution (DE) and its improved and modified versions (MDE) for IEEE 3-bus, 4-bus and 6-bus systems. The two settings (*PS* and *TDS*) of each relay are taken as decision variables. Sum of the operating times of all the primary relays, which are expected to operate in order to clear the faults of their corresponding zones, is considered as an objective function and the constraints are bound on all decision variables, complexly interrelated times of the various relays (called selectivity

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constraints) and restrictions on each term of the objective function.

The remaining of the paper is organized as follows: in Section 2, we give a brief literature review of the techniques used for coordinating directional over-current relays. In Section 3, we discuss the problem formulation. The basic or the classical DE is given in Section 4 and the modified MDE versions are discussed in Section 5. Experimental settings and numerical results are discussed in Section 6 and finally the conclusions based on the present study are drawn in Section 7.

2. Literature review

Before the application of optimization theory in these problems, trial and error approach was used but it has a well-known drawback of slow convergence rate as a result of large number of iterations needed to reach a suitable relay setting. To overcome the disadvantage of trial and error method, many authors assumed the value of DOCR settings based on expert's experience and solved them in a linear environment (Irving and Elrafie, 1993; Chattopadhyay et al., 1996; Urdaneta et al., 1996; Urdaneta et al., 2001). However, it was observed that linear approach cannot ensure correct settings of the relays (Laway and Gupta, 1993) as it did not consider all possible operating conditions of the power system. Urdaneta et al. (1988) was the first to report the application of optimization theory in the coordination of DOCR. A detailed literature survey on this problem has been performed by (Birla et al., 2005). They have classified the previous works on DOCR coordination into three categories: curve fitting technique, graph theoretical technique and optimization technique.

Sparse dual revised simplex method of linear programming has been used in (Irving and Elrafie, 1993) to optimize the *TDS* settings for assumed non-linear *PS* settings. Some linear programming techniques applied in DOCR coordination problem include Chattopadhyay et al. (1996); Urdaneta et al. (1996); Braga and Saraiva (1996); Abyaneh and Keyhani (1995); and Abdelaziz et al. (2002).

Laway and Gupta (1993) applied simplex and Rosenbrock–Hillclimb methods to optimize the *TDS* and the *PS* settings, respectively, in a similar way, as used by Urdaneta et al. (1988). The optimization of DOCR settings using artificial intelligence (AI) and nature inspired algorithms (NIA) has received considerable attention recently. Some of the NIA algorithms like evolutionary programming (So and Li, 2000a), genetic algorithm (GA) (So et al., 1997; Razavi et al., 2008; Thakur, 2007), modified evolutionary programming (So and Li, 2000b, 2004), and particle swarm optimization (Mansour and Mekhamer, 2007; Zeineldin et al., 2006; Bansal and Deep, 2008) have been applied successfully for solving this problem. Self organizing migrating algorithm (SOMA) and its hybridization with GA have been applied by (Dipti, 2007). Some of the AI methods like fuzzy logic (Abyane et al., 1997) and expert systems (Brown and Tyle, 1986; Lee et al., 1989; Hong et al., 1991; Jianping and Trecat, 1996) have also been applied to this problem. Birla et al (2006) and Deep et al (2006) used random search technique (RST2) to solve the relay coordination problem for IEEE 6-bus model and IEEE 3-bus, 4-bus models, respectively.

Although DE is a robust and a popular optimization tool for solving complex optimization problems, as far as authors know no research paper is available on the implementation of DE for optimization of DOCR settings. In this paper an effort has been made to apply DE and its modified versions on the above mentioned problem of DOCR settings and the results are compared with other contemporary algorithms.

3. Problem formulation

An important characteristic of some types of protection in an electrical circuit is their capacity to determine the direction of the flow of power. Because of this feature they inhibit opening of the associated switch when the fault current flows in the direction opposite to the setting of the relays. Directional relays can tackle this situation when relays face fault currents in both directions because they operate only when fault current flows in specified tripping direction. Hence, directional over-current relays are used extensively for the protection of feeders having infeed from both the ends (e.g. loop systems and parallel feeders).

A DOCR consists of two units: (i) an instantaneous unit and (ii) a time-delay unit.

The instantaneous unit operates with no intentional time-delay when current is above a predefined threshold value, known as the instantaneous current setting. Time-delay unit is used for current, which is below the instantaneous current setting but exceeds the normal flow due to a fault. This unit operates at the occurrence of a fault with an intentional time-delay. Two settings are associated with the time-delay unit, which are as under

- time dial setting (*TDS*)
- plug setting (*PS*) (e.g. tap setting)

The *TDS* adjusts time-delay before a relay operates whenever the fault current reaches a value equal to or greater than the pick-up current. Tap setting is a value that defines the pick-up current of the relay, and currents are expressed as multiple of this. These settings essentially specify the particular time–current characteristics from the family of available curves and the multiple of tap setting to be used to find the relay operating time for a given current flowing through the relay. "Threshold" or "Pick-up current" is the minimum current for which the relay operates and is determined by selecting one of the plug setting taps available on the relay.

3.1. General model of the problem

The mathematical model of the problem followed in this paper is same as Thakur (2007)but DE and its modified versions are used to solve the given problem instead of GA.

The operating time (T) of a DOCR is a non-linear function of the relay settings (time dial settings (*TDS*) and plug settings (*PS*) and the fault current (I) seen by the relay. Thus, relay operating time equation for a DOCR is given by

$$T = \frac{\alpha * TDS}{\left(\frac{1}{PS * CT_{pri\_rating}}\right)^{\beta} - \gamma} \tag{1}$$

Only *TDS* and *PS* are unknown variables in above equation. These are the "decision variables" of the problem. Throughout this paper, the symbol "$*$" represents scalar multiplication. α, β and γ are the constants representing the behaviour of characteristic in a mathematical way, in which operating time of the DOCR varies and are given as 0.14, 0.02 and 1.0, respectively as per [IEEE std. (1997)]. Value of CTpri_rating depends upon the number of turns in the equipment current transformer (CT). CT is used to reduce the level of the current so that relay can withstand it. With each relay one "current transformer" is used and thus, CTpri_rating is known in the problem. Value of I (Fault current passing through the relay) is also known, as it is a system dependent parameter and continuously measured by measuring instruments. Number of constraints for systems of bigger sizes depends upon the number of lines in the system. Details of the number of lines in few larger systems are given in Table 1. In practice, in electrical

Table 1
The complexity of the DOCR problem as the bus size increases.

	IEEE 3-bus	IEEE 4-bus	IEEE 6-bus
No. of lines	3	4	7
No. of DOCRs (relays)	6	8	14
No. of decision variables	12	16	28
No. of selectivity constraints	8	9	38
Constraints imposing restrictions on each term of objective function	24	32	104

engineering, power systems may be of even bigger sizes and there are other types of relays also besides DOCRs. Coordinating DOCRs with other types of relays generates even larger number of constraints as shown in Table 1. It is evident from Table 1 that simultaneous optimization of both the settings (*TDS* and *PS*) of each DOCR of the system forms a complex problem.

3.2. Objective function and constraints of the problem

The optimal coordination problem of DOCRs using optimization technique consists of minimizing an objective function (performance function) subject to certain coordination criteria and limits on problem variables. The relay, which is supposed to operate first to clear the fault, is called the primary relay. A fault close to relay is known as the close-in fault for the relay and a fault at the other end of the line is known as a far-bus fault for the relay. Conventionally, objective function in coordination studies is constituted as the summation of operating times of all primary relays, responding to clear all close-in and far-bus faults. The objective function is defined as follows:

$$Minimize\ OBJ = \sum_{i=1}^{N_{cl}} T^i_{pri\_cl\_in} + \sum_{j=1}^{N_{far}} T^j_{pri\_far\_bus} \quad (2)$$

where, N_{cl} is number of relays responding for close-in fault. N_{far} is number of relays responding for far-bus fault. $T_{pri\_cl\_in}$ is primary relay operating time for close-in fault. $T_{pri\_far\_bus}$ is primary relay operating time for far-bus fault.

The constraints are as follows:

(1) Bounds on variables *TDS*s
$TDS^i_{\min} \le TDS^i \le TDS^i_{\max}$, where, i varies from 1 to N_{cl}.
$TDS^i_{\min}$ is the lower limit and $TDS^i_{\max}$ is the upper limit of TDS^i. These limits are 0.05 and 1.1, respectively.
(2) Bounds on variables *PS*s
$PS^i_{\min} \le PS^i \le PS^i_{\max}$, where, i varies from 1 to N_{cl}.
$PS^i_{\min}$ is the lower limit and $PS^i_{\max}$ is the upper limit of PS^i. These are 1.25 and 1.50, respectively.
(3) Limits on primary operation times
This constraint imposes constraint on each term of objective function to lie between 0.05 and 1.0.
(4) Selectivity constraints for all relay pairs

$$T_{backup} - T_{primary} - CTI \ge 0$$

T_{backup} is the operating time of backup relay, $T_{primary}$ is operating time of primary relay and *CTI* is coordinating time interval.

3.3. Model 1–the IEEE 3-bus model

For the coordination problem of IEEE 3-bus model, value of each of N_{cl} and N_{far} is 6 (equal to number of relays or twice the lines). Accordingly, there are 12 decision variables (two for each relay) in this problem i.e. TDS^1–TDS^6 and PS^1–PS^6. The 3-bus system can be visualized as shown in Fig. 1.

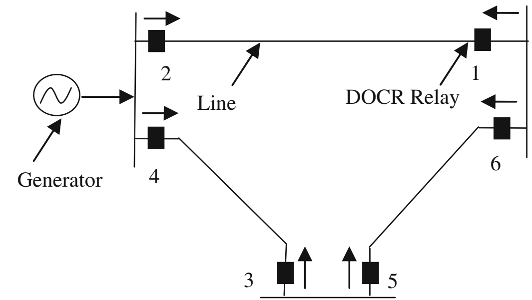

Fig. 1. A typical IEEE 3-bus DOCR coordination problem model.

Objective function (OBJ) to be minimized as given by

$$OBJ = \sum_{i=1}^{6} T^i_{pri\_cl\_in} + \sum_{j=1}^{6} T^j_{pri\_far\_bus} \quad (3)$$

where

$$T^i_{pri\_cl\_in} = \frac{0.14 * TDS^i}{\left(\frac{a^i}{PS^i * b^i}\right)^{0.02} - 1} \quad (4)$$

$$T^i_{pri\_far\_bus} = \frac{0.14 * TDS^j}{\left(\frac{c^i}{PS^j * d^i}\right)^{0.02} - 1} \quad (5)$$

The values of constants a^i, b^i, c^i and d^i are given in the Table 2.
Constraints for the model
Bounds on variables *TDS*s
$TDS^i_{\min} \le TDS^i \le TDS^i_{\max}$, where, i varies from 1 to 6 (N_{cl})
Bounds on variables *PS*s:
$PS^i_{\min} \le PS^i \le PS^i_{\max}$, where, i varies from 1 to 6 (N_{cl})
Limits on primary operation times:
This constraint imposes constraint on each term of objective function to lie between 0.05 and 1.0.
Selectivity constraints are

$$T^i_{backup} - T^i_{primary} - CTI \ge 0 \quad (6)$$

T_{backup} is the operating time of backup relay and $T_{primary}$ is the operating time of primary relay. Value of coordinating time interval (*CTI*) is 0.3. Here

$$T^i_{backup} = \frac{0.14 * TDS^p}{\left(\frac{e^i}{PS^p * f^i}\right)^{0.02} - 1} \quad (7)$$

$$T^i_{primary} = \frac{0.14 * TDS^q}{\left(\frac{g^i}{PS^q * h^i}\right)^{0.02} - 1} \quad (8)$$

The values of constants p, q, e^i, f^i, g^i and h^i are given in Table 3.

3.4. Model 2—The IEEE 4-bus model

For coordination problem of IEEE 4-bus model, value of each of N_{cl} and N_{far} is *8* (equal to the number of relays or twice the lines). Accordingly, there are 16 decision variables (two for each relay) in this problem i.e. TDS^1–TDS^8 and PS^1–PS^8. The 4-bus system can be visualized as shown in Fig. 2. The value of *CTI* for this model is 0.3. The number of selectivity constraints is 9.

The objective function and constraints for this model will be of same form as in the case of Model-I problem (with N_{cl}=8) described in the section of problem formulation. The values of constants a^i, b^i, c^i, d^i and e^i, f^i, g^i, h^i for Model-II are given in Tables 4 and 5, respectively.

Table 2
Values of constants a^i, b^i, c^i and d^i for Model 1.

$T^i_{pri\_cl\_in}$			$T^i_{pri\_far\_bus}$		
TDS^i	a^i	b^i	TDS^j	c^i	d^i
TDS^1	9.46	2.06	TDS^2	100.63	2.06
TDS^2	26.91	2.06	TDS^1	14.08	2.06
TDS^3	8.81	2.23	TDS^4	136.23	2.23
TDS^4	37.68	2.23	TDS^3	12.07	2.23
TDS^5	17.93	0.8	TDS^6	19.2	0.8
TDS^6	14.35	0.8	TDS^5	25.9	0.8

Table 3
Values of constants e^i, f^i, g^i and h^i for Model 1.

	T^i_{backup}			$T^i_{primary}$	
p	e^i	f^i	q	g^i	h^i
5	14.08	0.8	1	14.08	2.06
6	12.07	0.8	3	12.07	2.23
4	25.9	2.23	5	25.9	0.8
2	14.35	0.8	6	14.35	2.06
5	9.46	0.8	1	9.46	2.06
6	8.81	0.8	3	8.81	2.23
2	19.2	2.06	6	19.2	0.8
4	17.93	2.23	5	17.93	0.8

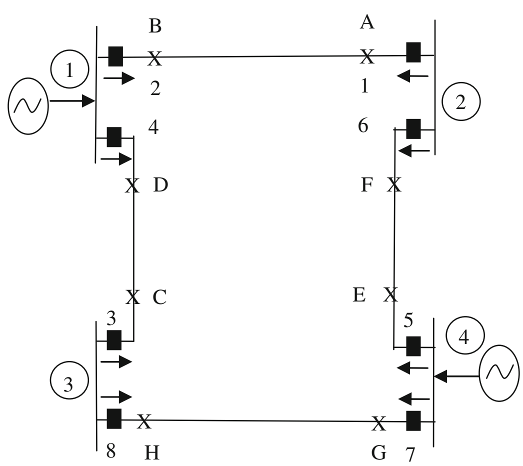

Fig. 2. A typical IEEE 4-bus DOCR coordination problem model.

Table 4
Values of constants a^i, b^i, c^i and d^i for Model 2.

$T^i_{pri\_cl\_in}$			$T^i_{pri\_far\_bus}$		
TDS^i	a^i	b^i	TDS^j	c^i	d^i
TDS^1	20.32	0.48	TDS^2	23.75	0.48
TDS^2	88.85	0.48	TDS^1	12.48	0.48
TDS^3	13.61	1.1789	TDS^4	31.92	1.1789
TDS^4	116.81	1.1789	TDS^3	10.38	1.1789
TDS^5	116.7	1.5259	TDS^6	12.07	1.5259
TDS^6	16.67	1.5259	TDS^5	31.92	1.5259
TDS^7	71.7	1.2018	TDS^8	11	1.2018
TDS^8	19.27	1.2018	TDS^7	18.91	1.2018

Table 5
Values of constants e^i, f^i, g^i and h^i for Model 2.

	T^i_{backup}			$T^i_{primary}$	
p	e^i	f^i	q	g^i	h^i
5	20.32	1.5259	1	20.32	0.48
5	12.48	1.5259	1	12.48	0.48
7	13.61	1.2018	3	13.61	1.1789
7	10.38	1.2018	3	10.38	1.1789
1	1.16	0.48	4	116.81	1.1789
2	12.07	0.48	6	12.07	1.1789
2	16.67	0.48	6	16.67	1.5259
4	11	1.1789	8	11	1.2018
4	19.27	1.1789	8	19.27	1.2018

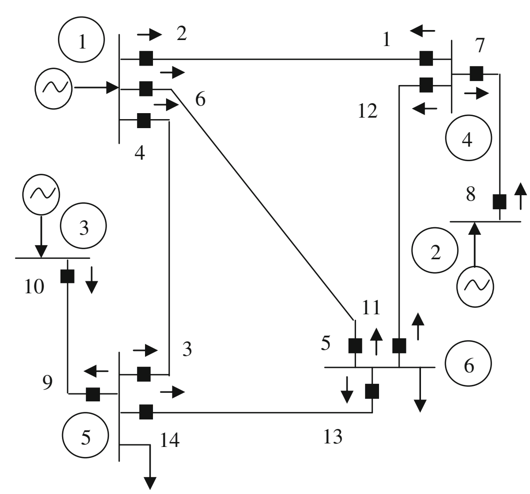

Fig. 3. A typical IEEE 6-bus DOCR coordination problem model.

3.5. Model 3–The IEEE 6-bus model

The next coordination problem is IEEE 6-bus model, value of each of N_{cl} and N_{far} is *14* (equal to number of relays or twice the lines). Accordingly, there are 28 decision variables (two for each relay) in this problem i.e. TDS^1–TDS^{14} and PS^1–PS^{14}. The 6-bus system can be visualized as shown in Fig. 3. The value of *CTI* for this model is 0.2. For the nominal state of the sample 6-bus model, 48 selectivity constraints are generated corresponding to all the possible near-end and far-end faults sensed by all the relays of the system. Based on the observation of Birla et al (2006a), 10 constraints are relaxed.

The objective function and constraints for this model will be of same form as in the case of Model-I problem with N_{cl}=14. The values of constants a^i, b^i, c^i, d^i and e^i, f^i, g^i, h^i for Model-III are given in Tables 6 and 7, respectively.

4. Basic differential evolution algorithm

Differential evolution is an evolutionary algorithm (EA) proposed by Storn and Price (1995). DE is similar to other EAs particularly genetic algorithms (GA) (Goldberg, 1986) in the sense that it uses the same evolutionary operators like selection recombination and mutation like that of GA. However, it is the application of these operators that make DE different from GA; while, in GA crossover plays a significant role; it is the mutation operator, which affects the working of DE (Karaboga and Okdem, 2004). The working of basic DE may be described as follows:

For a D-dimensional search space, each target vector $x_{i,g}$, a mutant vector $V_{i,g+1}=(v_{1,i,g+1},\ldots,v_{D,i,g+1})$ is generated by

$$v_{i,g+1}=x_{r_1,g}+F*(x_{r_2,g}-x_{r_3,g}) \tag{9}$$

Table 6
Values of constants a^i, b^i, c^i and d^i for Model 3.

$T^i_{pri\_cl\_in}$			$T^i_{pri\_far\_bus}$		
TDS^i	a^i	b^i	TDS^j	c^i	d^i
TDS^1	2.5311	0.2585	$TDS2$	5.9495	0.2585
TDS^2	2.7376	0.2585	$TDS1$	5.3752	0.2585
TDS^3	2.9723	0.4863	$TDS4$	6.6641	0.4863
TDS^4	4.1477	0.4863	$TDS3$	4.5897	0.4863
TDS^5	1.9545	0.7138	$TDS6$	6.2345	0.7138
TDS^6	2.7678	0.7138	$TDS5$	4.2573	0.7138
TDS^7	3.8423	1.746	$TDS8$	6.3694	1.746
TDS^8	5.618	1.746	$TDS7$	4.1783	1.746
TDS^9	4.6538	1.0424	$TDS10$	3.87	1.0424
TDS^{10}	3.5261	1.0424	$TDS9$	5.2696	1.0424
TDS^{11}	2.584	0.7729	$TDS12$	6.1144	0.7729
TDS^{12}	3.8006	0.7729	$TDS11$	3.9005	0.7729
TDS^{13}	2.4143	0.5879	$TDS14$	2.9011	0.5879
TDS^{14}	5.3541	0.5879	$TDS13$	4.335	0.5879

Table 7
Values of constants e^i, f^i, g^i and h^i for Model 3.

T^i_{backup}			$T^i_{primary}$		
p	e^i	f^i	q	g^i	h^i
8	4.0909	1.746	1	5.3752	0.2585
11	1.2886	0.7729	1	5.3752	0.2585
8	2.9323	1.746	1	2.5311	0.2585
3	0.6213	0.4863	2	2.7376	0.2585
3	1.6658	0.4863	2	5.9495	0.2585
10	0.0923	1.0424	3	4.5897	0.4863
10	2.561	1.0424	3	2.9723	0.4863
13	1.4995	0.5879	3	4.5897	0.4863
1	0.8869	0.2585	4	4.1477	0.4863
1	1.5243	0.2585	4	6.6641	0.4863
12	2.5444	0.7729	5	4.2573	0.7138
12	1.4549	0.7729	5	1.9545	0.7138
14	1.7142	0.5879	5	4.2573	0.7138
3	1.4658	0.4863	6	6.2345	0.7138
3	1.1231	0.2585	6	6.2345	0.7138
11	2.1436	0.7729	7	4.1783	1.746
2	2.0355	0.2585	7	4.1783	1.746
11	1.9712	0.7729	7	3.8423	1.746
2	1.8718	0.2585	7	3.8423	1.746
13	1.8321	0.5879	9	5.2696	1.0424
4	3.4386	0.4863	9	5.2696	1.0424
13	1.618	0.5879	9	4.6538	1.0424
4	3.0368	0.4863	9	4.6538	1.0424
14	2.0871	0.5879	11	3.9005	0.7729
6	1.8138	0.7138	11	3.9005	0.7729
14	1.4744	0.5879	11	2.584	0.7729
6	1.1099	0.7138	11	2.584	0.7729
8	3.3286	1.746	12	3.8006	0.7729
2	0.4734	0.2585	12	3.8006	0.7729
8	4.5736	1.746	12	6.1144	0.7729
2	1.5432	0.2585	12	6.1144	0.7729
12	2.7269	0.7729	13	4.335	0.5879
6	1.6085	0.7138	13	4.335	0.5879
12	1.836	0.7729	13	2.4143	0.5879
10	2.026	1.0424	14	2.9011	0.5879
4	0.8757	0.4863	14	2.9011	0.5879
10	2.7784	1.0424	14	5.3541	0.5879
4	2.5823	0.4863	14	5.3541	0.5879

where $r_1,r_2,r_3 \in \{1, 2, \ldots, NP\}$ are randomly chosen integers, must be different from each other and also different from the running index i. $F(>0)$ is a scaling factor, which controls the amplification of differential evolutions $(x_{r_2,g}-x_{r_3,g})$. In order to increase the diversity of the perturbed parameter vectors, crossover is introduced (Storn and Price, 1997). The parent vector is mixed with the mutated vector to produce a trial vector $U_{i,g+1}=(u_{1,i,g+1},\ldots, u_{D,i,g+1})$,

$$u_{ji,g+1} = \begin{cases} v_{ji,g+1} & \text{if} \quad (rand_j \le CR) \quad (j=j_{rand}) \\ x_{ji,g} & \text{if} \quad (rand_j > CR) \quad (j \ne j_{rand}) \end{cases} \tag{10}$$

where $j=1, 2,\ldots, D$; $rand_j \in [0, 1]$; CR is the crossover constant takes values in the range [0, 1] and $j_{rand} \in (1,2,\ldots,D)$ is the randomly chosen index.

Selection is the step to choose the vector between the target vector and the trial vector with the aim of creating an individual for the next generation.

Computational steps of classical DE are given as

Step 1	*Initialize DE parameters*
Step 2	*Randomly initialize the positions of all particles*
Step 3	*Evaluate the fitness function values of all particles (X_i) in the population*
Step 3	*Set g=1*
Step 4	*// Mutation* *Generate mutant vectors $V_{i,g+1}$ corresponding to each target vector $X_{i,g}$ using Eq.* (9)
Step 5	*// Crossover* *Generate trial vector $U_{i,g+1}$ for each particle using Eq.* (10)
Step 6	*//Selection* *Update particles position*
Step 7	*Set g=g+1*
Step 8	*If (Stopping criteria is reached) then go to step 9* *Else go to step 4*
Step 9	*Print the global best particle and the corresponding fitness function value*

5. Modified differential evolution algorithms

DE has emerged as one of the most popular technique for solving engineering design problems (Rogalsky et al., 1999; Joshi and Sanderson, 1999; Omran et al., 2005; Das et al., 2008). However, it has been observed that the performance of DE is sometimes not up to the expectations. Like most of the population based stochastic search techniques DE also suffers from the drawbacks like premature convergence and stagnation of population (Lampinen and Zelinka, 2000). Several attempts have been made in the literature to improve its performance (Omran et al., 2005a; Brest et al., 2006; Rahnamayan et al., 2008; Teo, 2006). In continuation with the efforts to improve the working of DE in terms of convergence rate as well as solution quality, in this paper we propose five modified versions of DE viz. MDE1, MDE2, MDE3, MDE4, and MDE5 to solve IEEE 3-bus, IEEE 4-bus and IEEE 6-bus model problems. Here, we would like to mention that a part of this work namely MDE1 algorithm is already published in conference proceedings (Pant et al., 2009a), where we used it for solving unconstrained test problems. Encouraged by its performance, in this paper we developed other modified versions and applied them to the DOCR problem considered. These algorithms differ from the basic DE algorithm in the phase of generating the mutant vector. The following sections briefly describe the modified versions of DE.

5.1. MDE algorithms

The MDE algorithms differ from the classical DE only in the mutation phase in a two-fold manner. These schemes make use the absolute weighted difference between the two vector points

Table 8
Optimal design variables, objective function values (*F*) and number of function of evaluations (NFE) of IEEE 3-bus Model by DE and modified DE algorithms.

	DE	MDE1	MDE2	MDE3	MDE4	MDE5
TS^1	0.05	0.05	0.05	0.05	0.05	0.05
TS^2	0.2194	0.2178	0.1979	0.1988	0.1976	0.1976
TS^3	0.05	0.05	0.05	0.05	0.05	0.05
TS^4	0.2135	0.2090	0.2094	0.2090	0.2090	0.2090
TS^5	0.19498	0.1812	0.1847	0.1812	0.1812	0.1812
TS^6	0.1953	0.1807	0.1827	0.1807	0.1806	0.1806
PS^1	1.25	1.25	1.25	1.25	1.25	1.25
PS^2	1.25	1.25	1.4999	1.4849	1.4999	1.5
PS^3	1.2500	1.25	1.25	1.25	1.25	1.25
PS^4	1.4605	1.4999	1.4999	1.4999	1.4999	1.5
PS^5	1.25	1.5	1.4318	1.4998	1.4999	1.5
PS^6	1.25	1.4999	1.4619	1.4999	1.4999	1.5
F	4.8422	4.8070	4.7873	4.7822	4.7806	4.7806
NFE	78360	72350	73350	97550	69270	38250

Table 9
Optimal design variables, objective function values (F) and number of function of evaluations (NFE) of IEEE 4-bus Model by DE and modified DE algorithms.

	DE	MDE1	MDE2	MDE3	MDE4	MDE5
TS^1	0.05	0.05	0.05	0.05	0.05	0.05
TS^2	0.2248	0.2121	0.2123	0.2121	0.2121	0.2121
TS^3	0.05	0.0500	0.05	0.05	0.05	0.05
TS^4	0.1515	0.1515	0.1515	0.1515	0.1515	0.1515
TS^5	0.1264	0.1264	0.1264	0.1264	0.1262	0.1264
TS^6	0.05	0.05	0.05	0.05	0.05	0.0500
TS^7	0.1337	0.1338	0.1371	0.1338	0.1337	0.1337
TS^8	0.0500	0.05	0.05	0.05	0.05	0.0500
PS^1	1.2734	1.2733	1.2733	1.2733	1.25	1.2734
PS^2	1.25	1.4998	1.4959	1.5	1.5	1.4999
PS^3	1.2500	1.2500	1.2500	1.25	1.25	1.2500
PS^4	1.4997	1.4996	1.4997	1.4995	1.5	1.4999
PS^5	1.4997	1.5	1.5	1.4997	1.5	1.5
PS^6	1.25	1.2500	1.25	1.25	1.25	1.2500
PS^7	1.5	1.4997	1.4274	1.4995	1.4998	1.5
PS^8	1.25	1.2500	1.25	1.25	1.25	1.25
F	3.6774	3.6694	3.6734	3.6692	3.6674	3.6694
NFE	95400	43400	67200	99700	55100	35330

in place of the usual vector difference as in classical DE and secondly, in MDE schemes amplification factor, *F* (of the usual DE), is replaced by *L*, a random variable following Laplace distribution.

The proposed mutation schemes are defined as follows:

5.1.1. MDE1 scheme

$$v_{i,g+1} = x_{r_1,g} + \mathrm{L}*|x_{r_1,g} - x_{r_2,g}| \tag{11}$$

In this scheme only two individuals are used to generate new mutant vector.

5.1.2. MDE2 scheme

$$v_{i,g+1} = x_{best,g} + \mathrm{L}*|x_{r_1,g} - x_{r_2,g}| \tag{12}$$

In MDE2 scheme, the base vector is the one having the best fitness function value; whereas the other two individuals are randomly selected.

5.1.3. MDE3 scheme

$$v'_{i,g+1} = x_{r_1,g} + \mathrm{L}*|x_{r_1,g} - x_{r_2,g}| \tag{13}$$

$$v''_{i,g+1} = x_{r_2,g} + \mathrm{L}*|x_{r_1,g} - x_{r_2,g}| \tag{14}$$

if $(f(v'_{i,g+1}) < f(v''_{i,g+1}))$ then $v_{i,g+1} = v'_{i,g+1}$
else $v_{i,g+1} = v''_{i,g+1}$

In MDE3, two offspring are generated as in Eqs. (13) and (14) using two individuals and the offspring with better fitness function value is selected as the mutant vector.

5.1.4. MDE4 scheme

If $(\mathrm{U}(0,1) > 0.2)$ then

$$v_{i,g+1} = x_{r_1,g} + \mathrm{L}*|x_{r_1,g} - x_{r_2,g}| \tag{15}$$

else

$$v_{i,g+1} = x_{r_1,g} + F*(x_{r_2,g} - x_{r_3,g}) \tag{16}$$

In MDE4 scheme, mutant vector using Eq. (11) and the basic mutant vector equation are applied probabilistically using a predefined value. A random variable following normal distribution (U(0,1)) is generated. If it is greater than 0.2 then MDE1 scheme is applied otherwise Eq. (9) is applied.

5.1.5. MDE5 scheme

$$v_{i,g+1} = x_{r_1,g} + \mathrm{L}*|x_{best,g} - x_{r_2,g}| \tag{17}$$

Finally, in MDE5 scheme absolute weighted difference between the individual having the best fitness function value and a randomly chosen individual is taken.

In Eqs. 11–17, the symbols have their usual meaning as defined in the previous section.

Table 10
Optimal design variables, objective function values (*F*) and number of function of evaluations (NFE) of IEEE 6-bus Model by DE and modified DE algorithms.

	DE	MDE1	MDE2	MDE3	MDE4	MDE5
TS^1	0.1173	0.1171	0.1149	0.1034	0.1144	0.1024
TS^2	0.2082	0.1866	0.2037	0.1863	0.1864	0.1863
TS^3	0.0997	0.0965	0.0982	0.0961	0.0947	0.0946
TS^4	0.1125	0.1119	0.10367	0.1125	0.1006	0.1067
TS^5	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500
TS^6	0.0580	0.0500	0.0500	0.0500	0.0500	0.0500
TS^7	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500
TS^8	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500
TS^9	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500
TS^{10}	0.0719	0.0706	0.0575	0.0703	0.0701	0.0563
TS^{11}	0.0649	0.0649	0.0667	0.0649	0.0649	0.0650
TS^{12}	0.0617	0.0617	0.0566	0.0509	0.0509	0.0553
TS^{13}	0.0500	0.0500	0.0635	0.0500	0.0500	0.0500
TS^{14}	0.0856	0.0860	0.0859	0.0857	0.0709	0.0709
PS^1	1.2505	1.2515	1.2635	1.4995	1.2602	1.4991
PS^2	1.2500	1.4959	1.2993	1.4999	1.4987	1.4999
PS^3	1.2512	1.2525	1.2622	1.2575	1.2761	1.2771
PS^4	1.2515	1.2632	1.4322	1.2508	1.4992	1.3650
PS^5	1.2500	1.2500	1.2500	1.2500	1.2500	1.2500
PS^6	1.2500	1.3822	1.3885	1.3810	1.3814	1.3818
PS^7	1.2500	1.2500	1.2508	1.2500	1.2500	1.2500
PS^8	1.2500	1.2501	1.2500	1.2500	1.2505	1.2500
PS^9	1.2502	1.2500	1.2514	1.2500	1.2500	1.2500
PS^{10}	1.2502	1.2501	1.4970	1.2521	1.2500	1.4996
PS^{11}	1.4998	1.4999	1.4759	1.4998	1.4999	1.4998
PS^{12}	1.2575	1.2529	1.4700	1.4997	1.5000	1.3931
PS^{13}	1.4805	1.4664	1.2728	1.4647	1.4615	1.4613
PS^{14}	1.2557	1.2500	1.2624	1.2540	1.4979	1.4974
F	10.6272	10.5067	10.6238	10.4370	10.3812	10.3514
NFE	212190	72960	18180	101580	100860	106200

Table 11
Comparison results of IEEE 3-bus, 4-bus and 6-bus models: interms of objective function vaues.

Algorithm	IEEE 3-bus model	IEEE 4-bus model	IEEE 6-bus model
DE	4.8421	3.6774	10.6272
MDE1	4.8069	3.6694	10.5067
MDE2	4.7872	3.6734	10.6238
MDE3	4.7822	3.6692	10.4370
MDE4	**4.7806**	3.6674	10.3812
MDE5	4.7806	3.6694	**10.3514**
RST2	4.8354	3.7050	10.6192
GA	5.0761	3.8587	13.7996
SOMA	8.0101	3.7892	26.1495
SOMGA	4.7898	3.6745	10.3578
LX-POL	4.8265	**3.5749**	10.6028
LX-PM	4.8286	3.5830	10.6219

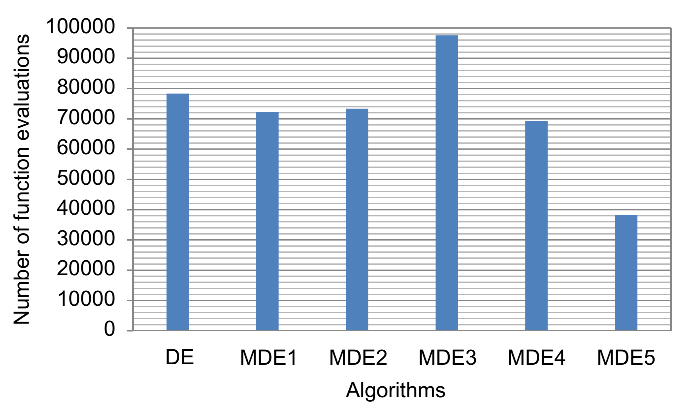

Fig. 5. Comparison of DE and modified DE algorithms in term of number of function evaluations: IEEE 3-bus model.

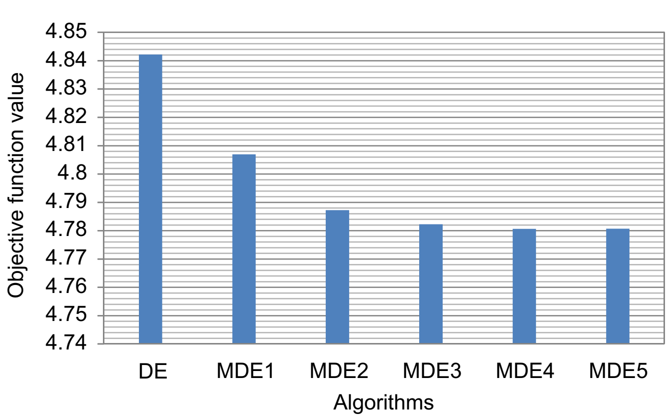

Fig. 4. Comparison of DE and modified DE algorithms in term of objective function values: IEEE 3-bus model.

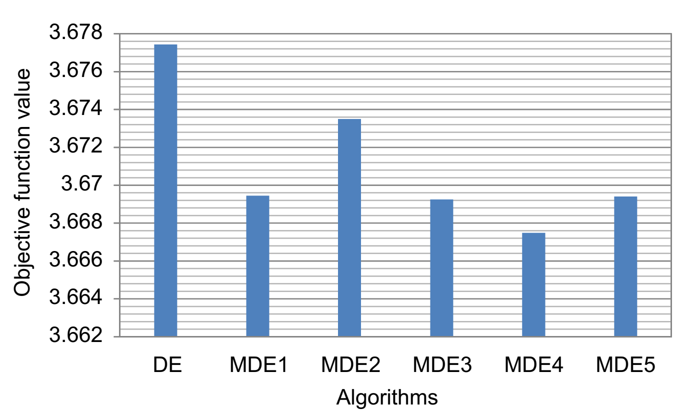

Fig. 6. Comparison of DE and modified DE algorithms in term of objective function values: IEEE 4-bus model.

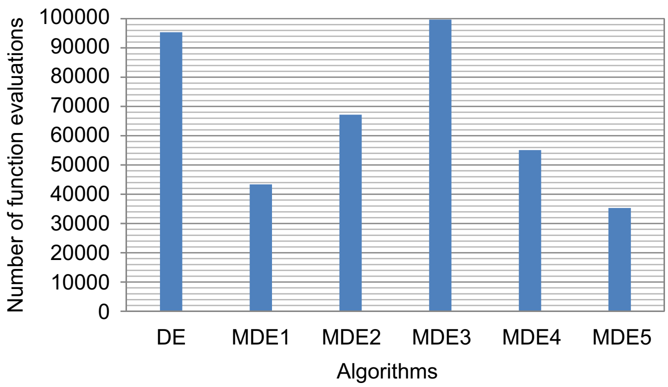

Fig. 7. Comparison of DE and modified DE algorithms in term of number of function evaluations: IEEE 4-bus model.

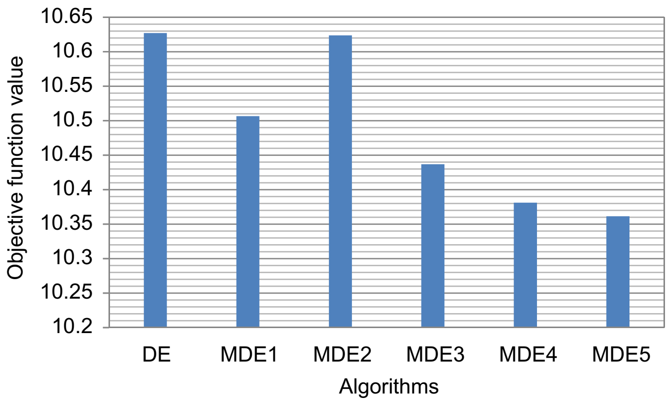

Fig. 8. Comparison of DE and modified DE algorithms in term of objective function values: IEEE 6-bus model.

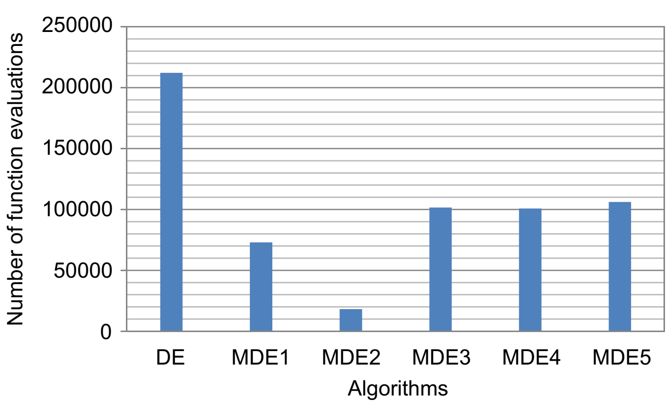

Fig. 9. Comparison of DE and modified DE algorithms in term of number of function evaluations: IEEE 6-bus model.

6. Experimental settings and results discussion

In order to make a fair comparison of all versions of DE algorithms, we fixed the same seed for random number generation so that the initial population is same for all the algorithms. We performed a number of experiments to obtain the optimum parameter settings. Considering the brevity of space, in this section we give the parameters for which we got the best results. The population size is taken as 50. The crossover constant *CR* is set as 0. 5 and the scaling factor *F* is set as 0.5 for basic DE, whereas for MDE schemes the scaling factor is a random variable, *L*, following Laplace distribution. For each algorithm, the stopping criteria is to terminate the search process when one of the following conditions is satisfied: (i) the maximum number of generations is reached (assumed 10000 generations) and (ii) $|f_{\max}-f_{\min}|<10^{-4}$ where *f* is the value of objective function. Constraints are handled according to the approach based on repair methods suggested in (Pant et al., 2009b). A total of 30 runs for each experimental setting were conducted and the best solution throughout the run was recorded as the global optimum. Results obtained by basic DE and MDE versions are also compared with previously quoted results viz. RST2 by Deep et al. (2006; Birla et al. (2006), Genetic algorithm (GA)(Dipti, 2007), self organizing migrating algorithms (SOMA) (Dipti, 2007), self organizing migrating genetic algorithm (SOMGA) (Dipti, 2007), GA with Laplace crossover and polynomial mutation (LX-POL) (Thakur, 2007) and GA with Laplace crossover and power mutation (LX-PM) (Thakur, 2007). The corresponding numerical results are recorded in Tables 8–11. Performances of DE and modified DE algorithms on IEEE 3-bus, 4-bus and 6-bus models are also illustrated with the help of bar graphs in Figs. 4–9.

The best solution obtained by DE and MDE algorithms for IEEE 3-bus model in terms of optimal decision variable values, objective function value and number of function evaluations are given in Table 8. From this Table, we can see that in terms of objective function value, all the algorithms gave more or less similar values with MDE4 and MDE5 giving slightly better result than the other algorithms. However, if we compare the NFE, then the performance of MDE5 is significantly better than all the other algorithms. MDE5 took only 38250 NFE, which is almost 50% less than the NFE's taken by other algorithms. The worst performance in terms of NFE was shown by MDE3.

The experimental results of IEEE 4-bus and 6-bus models are given in Tables 9 and 10, respectively. For the IEEE 4-bus model also, all the algorithms performed in a similar manner in terms of objective function value with MDE4 giving a marginally better value than the other algorithms. But, once again MDE5 clearly outperformed the other algorithms in terms of NFE. The worst performance was shown by MDE3, which took 99700 NFE to converge to a solution. In case of IEEE 6-bus model, the results in terms of objective function value are again parallel to each other with MDE4 and MDE5 giving a better performance than other algorithms. However, in terms of NFE, MDE2 took lowest NFE in comparison to other algorithms, while the worst performance was shown by the basic DE algorithm.

The difference in the performances of the algorithms is because of their structure. For example, the performance of MDE3 is not as good as other algorithms in terms of NFE. This is because, in MDE3, two mutant vectors $v'_{i,g+1}$ and $v''_{i,g+1}$ are generated in each iteration and the function value is evaluated for both, whereas in all the other algorithms only one mutant vector is generated for which the function is evaluated. Thus the work done by MDE3 algorithm is more than the other versions resulting in higher NFEs. MDE2, on the other hand is a greedy type of search in which the base vector is always the best vector. This concept leads the algorithm to search the vicinity of the best solution. Though such a procedure may be helpful in some cases, it may also lead to the clustering of solutions in a particular region resulting in stagnation of the performance of algorithm. Due to this reason, although MDE2 took lesser NFE for IEEE 6-bus model, its performance was not as good for the other two models. In MDE4 algorithm, two mutation operators are being used stochastically. This provides the algorithm with a choice of two mutation operators out of which one follows the Laplacean scaling factor (*L*) while the other follows a user defined scaling factor (*F*). This helps enhancing the exploration capability of the search procedure, thereby resulting in faster convergence. In MDE5, the absolute difference between the best vector and a randomly chosen vector is taken. The base vector however is a randomly chosen vector, which prevents MDE5 from becoming a purely greedy search. Thus it gives good objective function value without any extra computational efforts.

The results obtained by basic DE algorithm and the MDE versions are also compared with some of the previously quoted results available in the literature in Table 11. From these numerical results, it can be seen that MDE4 and MDE5 algorithms performed better than other algorithms for IEEE 3-bus and 6-bus models, respectively. However, in case of 4-bus model, LX-POL, which is a modified version of real coded GA, gave better performance than other algorithms. For IEEE 6-bus model, GA and SOMA gave the worst performance. Here, we have compared the algorithms only in terms of objective function value because NFEs were not evaluated in the available results.

7. Conclusion

Coordination of directional over-current relay (DOCR) is a frequently arising problem in the field of electrical engineering, which can be formulated as an optimization problem. The mathematical model of the problem is highly complex and non-linear in nature, subject to various constraints, and requires sophisticated optimization techniques for its solution. In this paper, an attempt is made to solve the IEEE 3-bus, 4-bus and 6-bus models with the help of basic DE and its modified versions. Five improved versions viz. MDE1, MDE2, MDE3, MDE4 and MDE5 are suggested and used to solve the above mentioned DOCR problem. Empirical analysis of numerical results obtained by MDE schemes and other algorithms (basic DE, RST2, GA, SOMA, SOMGA, LX-POL and LX-PM) show the competence of the proposed algorithms. Moreover, MDE schemes require only one control parameter i.e. the crossover rate (Cr), whereas most of the other techniques have more than one control parameters, which are to be fine tuned for the successful performance of an algorithm. Among the MDE schemes, the authors would recommend the use of MDE4 and MDE5 schemes for solving the complex type of problems mentioned in the present study. In future, work can be done on the development of an adaptive crossover rate, so as to make MDE a totally parameter free algorithm.

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