1. Keep track of the leftovers

1. Let $Q(n)$ be " $\sum_{i=2}^{n} \frac{1}{(i-1)i} = \frac{i-1}{i}$ ", we'll show $Q(n)$ is true for all integers $n \geq 2$ by Induction

2. Base case ($n=2$): $\sum_{i=2}^{2} \frac{1}{(i-1)i} = \frac{1}{(2-1)2} = \frac{1}{2} = \frac{2-1}{2}$ therefore $Q(2)$ is true

3. Inductive Hypothesis: suppose that $Q(m)$ is true for some arbitrary integer $m \geq 2$

4. Inductive step: Goal: Show $Q(m+1)$ is true.

Which is $\sum_{i=2}^{m+1} \frac{1}{(i-1)i} = \frac{m}{m+1}$

$\sum_{i=2}^{m} \frac{1}{(i-1)i} = \frac{m-1}{m}$ by IH. we add $\frac{1}{m(m+1)}$ to both sides

$$\sum_{i=2}^{m} \frac{1}{(i-1)i} + \frac{1}{m(m+1)} = \frac{m-1}{m} + \frac{1}{m(m+1)}$$

$$\sum_{i=2}^{m+1} \frac{1}{(i-1)i} = \frac{(m-1)(m+1)+1}{m(m+1)} = \frac{m^2-1+1}{m(m+1)}$$

$$= \frac{m^2}{m(m+1)} = \frac{m}{m+1}$$ which is exactly $Q(m+1)$

5. So $Q(n)$ ::= " $\sum_{i=2}^{n} \frac{1}{(i-1)i} = \frac{i-1}{i}$ " is true for all integers $n \geq 2$

let $P(n)$ be " $\sum_{i=2}^{n} \frac{1}{(i-1)i} < 1$ " Since $\frac{i-1}{i} < 1$

we can get $Q(n) \rightarrow P(n)$. we proved that $Q(n)$ is true for every integer $n \geq 2$, So by modus ponens, $P(n)$ is true for every integers $n \geq 2$

$\therefore \sum_{i=2}^{n} \frac{1}{(i-1)i} < 1$ for all integers $n \geq 2$

Assume $Q(n)$ is true

$\therefore \sum_{i=2}^{n} \frac{1}{(i-1)i} = \frac{i-1}{i}$ since $\frac{i-1}{i} < 1$

$\therefore \sum_{i=2}^{n} \frac{1}{(i-1)i} < 1$. $\therefore P(n)$ is true

so by direct proof rule