3. Relations warmup

(a) only symmetric, not reflexive, antisymmetric or transitive

(b) only symmetric, not reflexive, antisymmetric or transitive

(c) reflexive and transitive, not symmetric or antisymmetric

4. Set up to relate

(a) it is not necessarily transitive of $R \cup S$

let $a, b, c, d, e \in A$

Suppose $(a,b), (b,c) \in R$, since R is transitive, by definition, (a,c) also in R

Suppose $(d,b), (b,e) \in S$ since S is transitive by definition (d,e) also in S

So $(a,b), (b,c), (a,c), (d,b), (b,e), (d,e) \in R \cup S$, in this case, we have $(d,b) \in R \cup S$ $(b,c) \in R \cup S$, but we cannot define (d,c) to be in $R \cup S$ $\therefore$ $R \cup S$ is not necessarily transitive

(b) it is necessary transitive of $R \cap S$

If $(a,b), (b,c) \in R \cap S$, by def of intersection

$(a,b) \in R$ $\quad$ $(a,b) \in S$

$(b,c) \in R$ $\quad$ $(b,c) \in S$

by their property of being transitive

$(a,c) \in R$ $\quad$ $(a,c) \in S$ $\quad$ $\therefore (a,b)(b,c) \in R \cap S$

$\therefore (a,c) \in R \cap S$ $\quad$ $\longrightarrow$ $(a,c) \in R \cap S$

(direct proof rules)

$\therefore$ $R \cap S$ is necessarily transitive if R and S are transitive