3. Countability

We use technique "dovetailing"

$R = \{a+bi : a, b \text{ are rational}\}$

Since we know rational numbers Q is countable. that means we can order all rational numbers in a list (by def of Countability)

Such as $Q = \{0, 1/1, -1/1, 2/1, -2/1, 1/2, -1/2, 3/1, -3/1, 2/2, -2/2, \ldots\ldots\}$

So we can set up a table below

$R = \{a+bi : a, b \text{ are rational}\}$

b \ a	0	1/1	−1/1	2/1	−2/1	1/2	−1/2	...... (in Q's order)
0	0	1	-1	2	-2	$\frac{1}{2}$	$-\frac{1}{2}$	
1/1	i	$1+i$	$-1+i$	$2+i$	$i-2$	$\frac{1}{2}+i$	$i-\frac{1}{2}$	
−1/1	$-i$	$1-i$	$-1-i$	$2-i$	$-2-i$	$\frac{1}{2}-i$	$-\frac{1}{2}-i$	
2/1	$2i$	$1+2i$	$-1+2i$	$2+2i$	$-2+2i$	$\frac{1}{2}+2i$	$-\frac{1}{2}+2i$	
−2/1	$-2i$	$1-2i$	$-1-2i$	$2-2i$	$-2-2i$	$\frac{1}{2}-2i$	$-\frac{1}{2}-2i$	
1/2	$i/2$	$1+i/2$	$-1+\frac{i}{2}$	$2+i/2$	$-2+i/2$	$\frac{1}{2}+i/2$	$-\frac{1}{2}+\frac{i}{2}$	
−1/2	$-i/2$	$1-\frac{i}{2}$	$-1-i/2$	$2-i/2$	$-2-i/2$	$\frac{1}{2}-i/2$	$-\frac{1}{2}-i/2$	
...... (in Q's order)								

a, b can be any rational number in this table

therefore by "dovetailing", we can order R in a list

$\{0, i, 1, -i, 1+i, -1, 2i, 1-i, -1+i, 2, -2i, 1+2i, \ldots\ldots\}$

So by def of countability, $R: \{a+bi, a, b \text{ are rational}\}$ is countable.