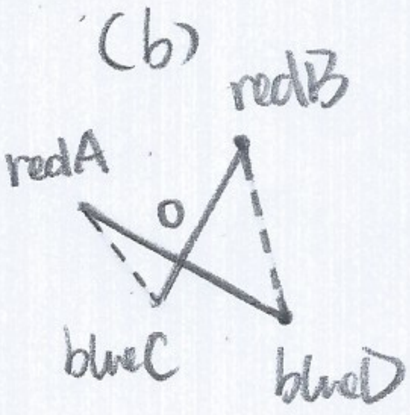
1. non-crossing matchings

(a) since there are limited numbers (n) of lines for every different collection M. they all have definite total length. no M's total length is infinite large or infinite small which is a undefined value. among those definite total length values, there must exist a value that is the smallest, thus $\exists M\, IsMinimal(M)$

(b)



Suppose there is a collection M that has two line segments cross each other (satisfies HasCrossing(M)), like picture on the left shown. AD and BC are in collection M. There is also another possibility that A connect with C and B connect with D. For geometry, recall that "in a triangular, the sum of length of two lines is bigger than length of the third line". As shown in picture, line AD and BC intersect at point O, along with 2 possible segment AC and BD, forms two triangular $\triangle AOC$ and $\triangle BOD$

In $\triangle AOC$ $|AC| < |AO| + |CO|$; in $\triangle BOD$ $|BD| < |BO| + |DO|$, $\therefore |AC| + |BD| < |AO| + |BO| + |CO| + |DO|$

$\because |AO| + |DO| = |AD|$ $|CO| + |BO| = |BC|$ $\therefore |BD| + |AC| < |AD| + |BC|$. Suppose a collection N that is very similar to M, the ONLY difference between them is that AC and BD is in N and AD and BC is in M, so as proved, the total length of N is smaller of total length of M. $\therefore$ M is not the collection having minimal total length $\therefore \forall M (HasCrossing(M) \rightarrow \neg IsMinimal(M))$

(c)

1.	$\exists M\, IsMinimal(M)$	given
2.	$\forall M (HasCrossing(M) \rightarrow \neg IsMinimal(M))$	given
3.	$IsMinimal(A)$	elim $\exists$: 1
4.	$HasCrossing(A) \rightarrow \neg IsMinimal(A)$	elim $\forall$: 2
5.	$IsMinimal(A) \rightarrow \neg HasCrossing(A)$	Contrapositive: 4
6.	$\neg HasCrossing(A)$	MP: 3, 5
7.	$\exists M \neg HasCrossing(M)$	Intro $\exists$: 6