3. Solve it

(a) step1 gcd(122, 17)

$= \gcd(17, 122 \bmod 17) = \gcd(17, 3)$ $\quad 122 = 7\times17+3$

$= \gcd(3, 17 \bmod 3) = \gcd(3, 2)$ $\quad 17 = 5\times3+2$

$= \gcd(2, 3 \bmod 2) = \gcd(2, 1)$ $\quad 3 = 1\times2+1$

$= \gcd(1, 2 \bmod 1) = \gcd(1, 0) = 1$

step2 rearrange equations

$1 = 3 - 1\times2$

$2 = 17 - 5\times3$

$3 = 122 - 7\times17$

step3 backward substitute

$1 = 3 - 1\times(17 - 5\times3)$

$= 6\times3 - 1\times17$

$= 6\times(122 - 7\times17) - 1\times17$

$= 6\times122 - 43\times17$

$\therefore 6 \bmod 17 = 6$ is the multiplicative inverse of 122 modulo 17

(b) step1 gcd(67, 43)

$= \gcd(43, 67 \bmod 43) = \gcd(43, 24)$ $\quad 67 = 1\times43+24$

$= \gcd(24, 43 \bmod 24) = \gcd(24, 19)$ $\quad 43 = 1\times24+19$

$= \gcd(19, 24 \bmod 19) = \gcd(19, 5)$ $\quad 24 = 1\times19+5$

$= \gcd(5, 19 \bmod 5) = \gcd(5, 4)$ $\quad 19 = 3\times5+4$

$= \gcd(4, 5 \bmod 4) = \gcd(4, 1)$ $\quad 5 = 1\times4+1$

$= \gcd(1, 4 \bmod 1) = \gcd(1, 0) = 1$

step2 rearrange

$1 = 5 - 1\times4$

$4 = 19 - 3\times5$

$5 = 24 - 1\times19$

$19 = 43 - 1\times24$

$24 = 67 - 1\times43$

step3 substitute

$1 = 5 * 1\times(19 - 3\times5)$

$= -1\times19 + 4\times(24 - 1\times19)$

$= 4\times24 - 5\times(43 - 1\times24)$

$= -5\times43 + 9\times(67 - 1\times43)$

$\equiv 9\times67 - 14\times43$

$\therefore 9 \bmod 43 = 9$ is the multiplicative inverse of 67 modulo 43

step4 that is $67\cdot9 \equiv 1 \pmod{43}$

by multiplicative property

we have $67\cdot9\cdot3 \equiv 3 \pmod{43}$

so any $x \equiv 9\cdot3 \pmod{43}$

$\therefore x = 27 + 43k$ for $k \in \mathbb{Z}$

$\because 0 \le x < 43$ (by def of congruency and divisibility)

$\therefore k = 0$ $\quad x = 27$

$x = 27$