

## 2. Constructing four grammars

(a)  $S \rightarrow 0S1 \mid 1S0 \mid 0S0 \mid 1S1 \mid 1$

(b)  $S \rightarrow 1S1 \mid C$   $C$  generates all binary strings that have same number of 0s and 1s with all 0s before 1s, it enables part "0<sup>n</sup>"  
 $C \rightarrow 0C1 \mid \epsilon$  be always after part "1<sup>m</sup>" which can be added in rules of variable  $S$  only

(c)  $S \rightarrow 0S1S0 \mid C$

$C \rightarrow D \mid D1 \mid 1D \mid 0110$

$D \rightarrow E \mid E1 \mid 1E \mid 010$

$E \rightarrow E0 \mid 00$

$E$  generates binary strings contains at least 2, at most infinite zeros,  $D$  generates binary strings with at least 2 0s and at most one 1 if "1" is at the beginning or end, and a special case "010".

$C$  generates binary strings with at least 2 0s and at most 2 1s if "1" is at beginning or the end, and a special case "0110"

This grammar works because it ensures at least 2 0s and at most 2 1s in special and general cases, then add whatever 0s you like to the beginning and end of string.

(d)  $S \rightarrow 1S1 \mid S11 \mid 11S \mid 10C \mid C$

$C \rightarrow 0C1 \mid C11 \mid 00C \mid \epsilon$

$C$  generates binary strings with  $n$  0s and  $p$  1s where  $n \equiv p \pmod{2}$ , this grammar

works because in rules of  $S$  and  $C$ ,  $p \pmod{2}$  is always same with  $m \pmod{2}$  by making addition to  $p$  and  $m$  to be same. I can add 2 to  $m$  without addition to  $p$ , add 2 to  $p$  without addition to  $m$ , or add 1 to  $m$  and add 1 to  $p$  at same time.