

3. Formal Proofs

- (a)
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| 1. | $(P \wedge \neg q) \vee (\neg P \wedge q)$ | given |
| 2. | $r \rightarrow \neg s$ | given |
| 3. | $(S \wedge P) \rightarrow r$ | given |
| 4. | $S \rightarrow \neg r$ | Contrapositive: 2 |
| 5. | $\neg r \rightarrow \neg(S \wedge P)$ | Contrapositive: 3 |
| 6. | $\neg r \rightarrow (\neg S \vee \neg P)$ | de Morgan's Law: 5 |
| 7.1 | S | Assumption |
| 7.2 | $\neg r$ | MP: 4, 7.1 |
| 7.3 | $\neg S \vee \neg P$ | MP: 6, 7.2 |
| 7.4 | $\neg(\neg S)$ | double negation: 7.1 |
| 7.5 | $\neg P$ | Elim V: 7.3, 7.4 |
| 7.6 | $\neg P \vee q$ | Intro V: 7.5 |
| 7.7 | $\neg(P \wedge \neg q)$ | de Morgan Law: 7.6 |
| 7.8 | $\neg P \wedge q$ | Elim V: 1, 7.7 |
| 7.9 | q | Elim A: 7.8 |
| 7. | $S \rightarrow q$ | Double Proof Rule |

- (b)
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|-------|--|---------------------|
| 1.1 | $(\neg P \rightarrow q) \wedge (r \rightarrow \neg q)$ | Assumption |
| 1.2 | $P \rightarrow q$ | Elim A: 1.1 |
| 1.3 | $r \rightarrow \neg q$ | Elim A: 1.1 |
| 1.4 | $q \rightarrow \neg r$ | Contrapositive: 1.3 |
| 1.5.1 | P | Assumption |
| 1.5.2 | q | MP: 1.2, 1.5.1 |
| 1.5.3 | $\neg r$ | MP: 1.4, 1.5.2 |
| 1.5 | $P \rightarrow \neg r$ | Double proof rule |
| 1.6 | $r \rightarrow \neg P$ | Contrapositive: 1.5 |

1. $((P \rightarrow q) \wedge (r \rightarrow \neg q)) \rightarrow (r \rightarrow \neg P)$ Double proof Rule