

#### 4. Irregularity

(a) Let set  $A = \{0^n 1^m : m \leq n\}$ . So we want to prove  $A$  is not regular. Suppose for contradiction that some DFA,  $M$ , recognizes  $A$ .

Let  $S = \{0^n : n \geq 1\}$ . Since  $S$  is infinite and  $M$  has finitely many states, so there must exist strings  $0^{a_1} \in S$  and  $0^{b_1} \in S$  for  $a \neq b$  that end in the same state of  $M$ .

Consider appending  $1^c$  to both strings, where  $c > a$ ,  $c > b$ .

Note that  $0^{a_1} 1^c 0^a$  where  $c > a$ , must belong to  $A$ , so that  $0^{a_1} 1^c 0^a \in A$ , but  $0^{b_1} 1^c 0^a \notin A$  since  $a \neq b$ . But they both

end up in the same state of  $M$ , call it  $q$ . Since  $0^{a_1} 1^c 0^a \in A$  state  $q$  must be an accept state but then  $M$  would incorrectly accept  $0^{b_1} 1^c 0^a \notin A$ , which is an error.

So  $M$  does not recognize  $A$ .

Since  $M$  was arbitrary, no NFA recognizes  $A$  and  $A$  is not regular.