(b) Let set $A = \{0^n . n = k^2 \; k = N\}$ so we will prove B is not regular

Suppose for contradiction that some DFA, M, recognizes A

Let $S = \{0^m : m > 0\}$ since S is infinite and M has finitely many states, there must be two strings, 0^a and 0^b for some $a \neq b$ that end in the same state in M and let $a > b$

consider appending 0^{a^2-a} to both strings

note that $0^{a^2-a+a} = 0^{a^2} \in A$. Consider a^2-a+b.

Since $a > b > 0$ and $a.b \in \mathbb{Z}$

$\therefore a > 1 \quad b > 0$

$\therefore 1-a < 0 < b$

$\therefore a^2-a+1-a < a^2-a+b$

where $a^2-a+1-a = a^2-2a+1 = (a-1)^2$

and $a > b \;\therefore a^2-a+b < a^2$

$\therefore (a-1)^2 < a^2-a+b < a^2$

Since there doesn't exist and integer k such that $a-1 < k < a$

$\therefore a^2-a+b$ cannot be a perfect square of any integer

which means $0^{a^2-a+b} \notin A$

But they both end up in the same state of M. call it q. Since $0^{a^2-a+a} \in A$.

State q must be an accept state but then M would incorrectly accept $0^{a^2-a+b} \notin A$ which is an error

So M doesn't recognizes A

Since M was arbitrary, no NFA recognizes A and A is not regular.