2. $X-Y+Y=X$

$(X\setminus\{y\})\cup\{y\}$

$= \{x:(x\in X)\wedge(x\notin\{y\})\}\cup\{x: x\in\{y\}\}$

$= \{x: ((x\in X)\wedge(x\notin\{y\}))\vee(x\in\{y\})\}$

$= \{x: ((x\in X)\vee(x\in\{y\}))\wedge((x\notin\{y\})\vee(x\in\{y\}))\}$

$= \{x: ((x\in X)\vee(x\in\{y\}))\wedge T\}$

$= \{x: (x\in X)\vee(x\in\{y\})\}$

(a) let n be an arbitrary element in set $(X\setminus\{y\})\cup\{y\}$, as shown in left n satisfies: $(n\in X)\vee(n\in\{y\})$, there exists a condition that $n=y$ and $y\notin X$, which means $n\notin X$

$\therefore \exists x((x\in(X\setminus\{y\}\cup\{y\}))\wedge\neg(x\in X))$

$\exists x\,\neg(\neg x\in(X\setminus\{y\}\cup\{y\})\vee(x\in X))$

$\neg\forall x(x\in(X\setminus\{y\})\cup\{y\}\to x\in X)$

$\therefore (X\setminus\{y\})\cup\{y\}\nsubseteq X$

(b) let n be an arbitrary element in set X, $\therefore x\in X$, as definition, it is also an element of $(x\in X)\vee(x\in\{y\})$, which is $(X\setminus\{y\})\cup\{y\}$

$\therefore \forall x(x\in X\to x\in((X\setminus\{y\})\cup\{y\}))$ $\therefore X\subseteq(X\setminus\{y\})\cup\{y\}$

3. Power sets

(a) Consider following example

$S:\{1,2\}$ $P(S)=\{\{1\},\{2\},\{1,2\},\emptyset\}$

$T:\{2,3\}$ $P(T)=\{\{2\},\{3\},\{2,3\},\emptyset\}$

$S\cap T=\{2\}$ $P(S\cap T)=\{\{2\},\emptyset\}$

$\therefore P(S)\cup P(T)\cup P(S\cap T)=\{\{1\}\{2\}\{3\}\{1,2\}\{2,3\},\emptyset\}$

$S\cup T=\{1,2,3\}$

$P(S\cup T)=\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\emptyset\}$

as shown, $\{1,2,3\}$ and $\{1,3\}$ are elements of $P(S\cup T)$ but not $P(S)\cup P(T)\cup P(S\cap T)$

$\therefore P(S\cup T)\neq P(S)\cup P(T)\cup P(S\cap T)$

(b) as definition

① $S\cap T=\{x:(x\in S)\wedge(x\in B)\}$

$\therefore S\cap T\subseteq S$ $S\cap T\subseteq T$

$S\cap T$ is subset of both S and T

$\therefore$ the subset of $S\cap T$ is also subsets of S and subsets of U

$\therefore P(S\cap T)\subseteq P(S)$ $P(S\cap T)\subseteq P(T)$

② let n be an arbitrary element of $P(S\cap T)$, by definition of subset it should also be element set of $P(S)$ and $P(T)$

$\therefore n\in P(S\cap T)\to \boxed{n\in P(S)\wedge n\in P(T)=n\in P(S)\cap P(T)}$

③ let m be an arbitrary element set of $(P(S)\cap P(T))$, which means m should be both subset of S and subset of T, so elements of m should be in S and T (if an element is just in S but not in T, it will not be an element of subset of T) $\therefore m\in P(S\cap T)$

$\therefore m\in(P(S)\cap P(T))\to m\in P(S\cap T)$

④ as shown above, $P(S\cap T)=P(S)\cap P(T)$