5. Symmetry and Power

1. Let $P(n)$ be "R^n is symmetric", we'll show that $P(n)$ is true for all integers $n \geq 1$

2. Base Case $(n=1)$: R^1 is symmetric. this is true because it's been already given.

3. Inductive Hyphothesis: Suppose that $P(m)$ is true for some arbitrary integer $m \geq 1$

4. Inductive step: Goal show $P(m+1)$ is true,

which is "R^{m+1} is symmetric"

by IH, we have "R^m is symmetric" which means for every

So. if (x,y) is in R^m, then (y,x) is in R^m (by def of symmetric)

and by base case, if (x,y) is in R then (y,x) is in R

prove

So $R^{m+1} = R^m \circ R = \{(a,c) \mid \exists b$ that $(a,b) \in R$ and $(b,c) \in R^m\}$ (power of relation)

let a,b,c be some specific value

if $(a,b) \in R$ then $(b,a) \in R$

if $(b,c) \in R^m$ then $(c,b) \in R^m$

So $\exists b$ that $(c,b) \in R^m$ and $(b,a) \in R$

$R \circ R^m = \{(c,a) \mid \exists b$ that $(c,b) \in R^m$ and $(b,a) \in R\}$ by definition of power relation

$\therefore (a,c) \in R^m \circ R \rightarrow (c,a) \in R \circ R^m$ (direct proof rules)

by def of power relation

$$R^{k+1} = R^k \circ R = \underbrace{((\cdots(((R \circ R) \circ R) \circ R) \cdots \circ R)}_{k\ Rs} \circ R$$

$$= R \circ \underbrace{(R \circ (R \circ R \circ) \cdots R \circ))}_{K\ Rs} \quad \text{(associativity)}$$

$$= R \circ R^K \quad \text{(commutativity is proved)}$$

$\therefore R^k \circ R = R \circ R^k$

$\therefore (a,c) \in R^m \circ R \rightarrow (c,a) \in R^m \circ R$ (commutativity)

$R^m \circ R = R^{m+1}$

$\therefore (a,c) \in R^{m+1} \rightarrow (c,a) \in R^{m+1}$

$\therefore R^{m+1}$ is symmetric

5. Conclusion: $\therefore$ if R is a symmetric relation on Set A

R^n is also symmetric for all integer $n \geq 1$