2. Runtime... Better Go catch it!

1. Let $P(n)$ be "$T(n) \le 20cn$" we'll prove that $P(n)$ is true for all integers $n \ge 0$ by strong induction

2. Base Case: $P(0)$: $T(0)=0$ (given)

$\therefore T(0) \le 20 \cdot c \cdot 0$ so $P(0)$ is true

3. Inductive Hyphosis: assume that for arbitrary integer $k \ge 0$ $P(j)$ is true for every integer j from 0 to k

4. Inductive step goal: show $P(k+1)$ is true which is $T(k+1) \le 20c(k+1)$.

① case 1 $k+1 \le 20$ $T(k+1) \le c$ (given) $\because k \ge 0$ $k+1 > 0$

$\therefore T(k+1) \le c \cdot 20(k+1)$ (algorithm) $P(k+1)$ is true here

② case 2 $k+1 > 20$ $T(k+1) = T(\lfloor \frac{3(k+1)}{4} \rfloor) + T(\lfloor \frac{k+1}{5} \rfloor) + c(k+1)$

$4 < \frac{k+1}{5} < \frac{3(k+1)}{4} < k+1$ by IH $\begin{cases} T(\lfloor \frac{3(k+1)}{4} \rfloor) \le 20c \lfloor \frac{3(k+1)}{4} \rfloor \\ T(\lfloor \frac{k+1}{5} \rfloor) \le 20c \lfloor \frac{k+1}{5} \rfloor \end{cases}$

and $0 \le \lfloor x \rfloor \le x$ (given)

$\therefore T(\lfloor \frac{3(k+1)}{4} \rfloor) + T(\lfloor \frac{k+1}{5} \rfloor) \le 20c \lfloor \frac{3(k+1)}{4} \rfloor + 20c \lfloor \frac{k+1}{5} \rfloor$

$0 \le \lfloor \frac{3(k+1)}{4} \rfloor \le \frac{3(k+1)}{4}$ $\quad 0 \le \lfloor \frac{k+1}{5} \rfloor \le \frac{k+1}{5}$

$\therefore T(\lfloor \frac{3(k+1)}{4} \rfloor) + T(\lfloor \frac{k+1}{5} \rfloor) \le 20c \cdot \frac{3(k+1)}{4} + 20c \cdot \frac{k+1}{5}$

$\le 20c \cdot \frac{19}{20}(k+1)$

$\le 19c(k+1)$

$\therefore T(k+1) \le 19c(k+1) + c(k+1)$

$\le 20c(k+1)$ $\quad \therefore P(k+1)$ is true here

5. Thus $P(n)$ is true for all integers $n \ge 0$ by strong induction

$\therefore T(n) \le 20cn$ for all integers $n \ge 0$