$$B_{k\ell}^{\sigma}[a] := \begin{cases} Z_{k\ell}[a], & \text{if } \sigma = 0, S_{k\ell}[a], \\ \text{if } \sigma = 1. \end{cases}$$