$$\phi_{p\ell-1}(pn + r) \equiv 0 \pmod 2$$