$\sigma_{\xi}^{-1}(V(\mathcal{K}_a^{\ell}) \setminus {\kappa_a^{\ell+1}}) \subset V(\mathcal{P}_a^{\ell}) \cup V(\mathcal{P}_b^{\ell}),$