

$$\sum_{j=1}^\infty \left(\frac{1}{\epsilon_0^{-1}+\delta \lambda_j^{\alpha/2}}\right)^2 \left\langle e_j, (a+\delta A^{\alpha/2})(\epsilon_0^{-1}+\delta A^{\alpha/2})^{-2}(ae_k)\right\rangle_{\mathcal{H}}^2 \quad 36mu \leq C\|ae_k\|_{\mathcal{H}}^2 \sum_{j=1}^\infty \left(\frac{1}{\epsilon_0^{-1}+\delta \lambda_j^{\alpha/2}}\right)^2 < \infty,$$