

$$\sigma_\xi^{-1}(V(\mathcal{K}_a^\ell) \setminus \{\kappa_a^{\ell+1}\}) \subset V(\mathcal{P}_a^\ell) \cup V(\mathcal{P}_b^\ell),$$