$d(\sigma,\tau) \le d(\sigma,\tau_*) + d(\tau_*,\tau) \le 4n^3(1+f(n)) + 8n^3(1+2f(n))(n+1) = 8n^4(1+o(1)).$