

$$S_1:=\sum_{n=1}^N\frac{1}{n^s}+\mathfrak{O}^*(\varepsilon/6),\qquad S_2:=\sum_{k=0}^K\frac{C_k(\textcolor{blue}{q})}{a^k}+RS_K+\mathfrak{O}^*(A_{\mathfrak{I}}\varepsilon/3).$$