

$$\phi_{p^\ell-1}(pn+r)\equiv 0\pmod{2}$$