

$$|\sigma_{\xi+1}^{-1}(V(\mathcal{K}_a^\ell)) \setminus (V(\mathcal{P}_a^\ell) \cup V(\mathcal{P}_b^\ell))| \geq 2,$$