$\phi_{p\ell-1}(pn+r) \equiv 0 \pmod{2}$