$$\sigma_{i_j+1}(v^{\ell+1}) \in \left\{\kappa_a^{\ell+1}, \kappa_b^{\ell+1}\right\},$$