

$$\tilde{\mathcal{S}}_{jk}(A) := \sum_{\ell \in I_k} a_{i_j, \ell}, \quad j, k = 1, \dots, m.$$