$$\sigma_{i+1}^{-1}(\sigma_i(v_a^{\ell})) \notin V(\mathcal{P}_a^{\ell}),$$