

$$\sigma_{i_j+1}(v^{\ell+1}) \in \{\kappa_a^{\ell+1}, \kappa_a^{\ell+1}\},$$