$$B_{k,\ell}^{1} \leq C \|(a + \delta A^{\alpha/2})^{-1} e_{k}\|_{\mathcal{H}} \|(a + \delta A^{\alpha/2})^{-1} e_{\ell}\|_{\mathcal{H}} \leq C \|(\underline{a} + \delta A^{\alpha/2})^{-1} e_{k}\|_{\mathcal{H}} \|(\underline{a} + \delta A^{\alpha/2})^{-1} e_{\ell}\|_{\mathcal{H}} = C \frac{1}{\underline{a} + \delta \lambda_{k}^{\alpha/2}} \underline{a} + C \|(\underline{a} + \delta A^{\alpha/2})^{-1} e_{k}\|_{\mathcal{H}} \|(\underline{a} + \delta A^{\alpha/2})^{-1} e_{\ell}\|_{\mathcal{H}} = C \frac{1}{\underline{a} + \delta \lambda_{k}^{\alpha/2}} \underline{a} + C \|(\underline{a} + \delta A^{\alpha/2})^{-1} e_{k}\|_{\mathcal{H}} \|(\underline{a} + \delta A^{\alpha/2})^{-1} e_{\ell}\|_{\mathcal{H}} = C \frac{1}{\underline{a} + \delta \lambda_{k}^{\alpha/2}} \underline{a} + C \|(\underline{a} + \delta A^{\alpha/2})^{-1} e_{\ell}\|_{\mathcal{H}} = C \|(\underline{a}$$