

$$S(n-1, n) = n!^2 - (n-1)!^2 = (n-1)!^2(n^2-1).$$