$V(\mathcal{K}_b^{\ell}) \subset \sigma_k(V(\mathcal{P}_a^{\ell})) \text{ for } k > \xi,$