$$s_{\lambda} \cdot s_{(\alpha)} = \sum_{\nu/\lambda \in \text{Horz(alpha)}} s_{\nu} s_{\lambda} \cdot s_{(1^r)} = \sum_{\nu/\lambda \in \text{Vert(r)}} s_{\nu}$$