

$$|\sigma_\zeta^{-1}(V(\mathcal{K}^1) \setminus (V(\mathcal{P}_a^1) \cup V(\mathcal{P}_b^1)))| \geq 2.$$