$$\sigma_i^{-1}(\{\kappa_a^\ell, \kappa_b^\ell\}) \subset \sigma_i^{-1}(V(\mathcal{K}^{\ell-1})) \setminus (V(\mathcal{P}_a^{\ell-1}) \cup V(\mathcal{P}_b^{\ell-1})),$$