$$V(\mathcal{K}_b^\ell) \subset \sigma_k(V(\mathcal{P}_a^\ell)) \text{ for } k > \xi,$$