

$$S(k,n)=\sum_{i=0}^{n-k}(-1)^i\cdot\binom{n-k}{i}^2\cdot(n-1)!^{2i}\cdot i!\cdot n!^{2n-2k-2i}.$$