

$$\mu_j \in V(\mathcal{K}_a^{\ell+1}) \implies \sigma_{i_j+1}(v^{\ell+1}) = \kappa_b^{\ell+1} \text{ and } \mu_j \in V(\mathcal{K}_b^{\ell+1}) \implies \sigma_{i_j+1}(v^{\ell+1}) = \kappa_a^{\ell+1},$$