

$$\exp\left(-\sum_{k=1}^{N_d}\log\sigma_k\right)=\prod_{k=1}^{N_d}\sigma_k^{-1}\in L^1_{\mu_0}(\mathcal{H}_u\times\mathbb{R}^+;\mathbb{R})$$