$\sigma_k^{-1}(\kappa_a^{\ell+1}) \notin V(\mathcal{P}_a^{\ell}) \cup V(\mathcal{P}_b^{\ell}) \text{ for } k > \xi,$