$$\exp\left(-\sum_{k=1}^{N_d}\log\sigma_k\right) = \prod_{k=1}^{N_d}\sigma_k^{-1} \in L^1_{\mu_0}(\mathcal{H}_u \times \mathbb{R}^+; \mathbb{R})$$