$$
\begin{aligned}
\text{sum} = \quad & v_1'(1+\eta_N) + v_2'(1+\eta_{N-1}) + \ldots + v_n'(1+\eta_1) \\
& = v_1(1+\eta_{N+1}) + v_2(1+\eta_N) + \ldots + v_N(1+\eta_2) \\
& = (a_1+\alpha_1)(1+\eta_{N+1}) + (a_2+\alpha_2)(1+\eta_N) + \ldots + (a_N+\alpha_N)(1+\eta_2)
\end{aligned}
$$