

$$s_\lambda \cdot s_{(\alpha)} = \sum_{\nu/\lambda \in \text{Horz}(\alpha)} s_\nu s_\lambda \cdot s_{(1^r)} = \sum_{\nu/\lambda \in \text{Vert}(\text{r})} s_\nu$$