

$$\mathcal{S}_l = \{(z_l, e) \in \mathcal{X}^{N+1} : e = 0\}$$