$\sigma_{\xi}^{-1}(\kappa_a^{\ell}) \notin V(\mathcal{P}_a^{\ell}) \setminus \{v_a^{\ell}\},$