$line: \langle H^* \boldsymbol{f}, g \rangle_{L^2} = \langle \boldsymbol{f}, H g \rangle_{\ell^2} = \sum_{k=1}^{\infty} \lambda_k g_k \langle \boldsymbol{f}, S e_k \rangle_{\ell^2} = \sum_{k=1}^{\infty} \lambda_k g_k \sum_{i=1}^{N_d} f_i$ 

 $norm_s tr$ :