$$\sum_{j=1}^{\infty} \left(\frac{1}{\epsilon_0^{-1} + \delta\lambda_j^{\alpha/2}} \right)^2 \langle e_j, (a + \delta A^{\alpha/2})(\epsilon_0^{-1} + \delta A^{\alpha/2})^{-2}(ae_k) \rangle_{\mathcal{H}}^2 \le C\|ae_k\|_{\mathcal{H}}^2 \sum_{j=1}^{\infty} \left(\frac{1}{_0^{-1} + \delta\lambda_j^{\alpha/2}} \right)^2 < \infty,$$