

$$\tilde{\beta}_0(\mathfrak{n}) = \begin{cases} \sigma_3 \sigma_3^2 \sigma_3 \sigma_2^{n-4} \sigma_1 \sigma_2^3 \sigma_1 & n \text{ is odd;} \\ \sigma_3 \sigma_2^3 \sigma_3 \sigma_2^{n-3} \sigma_1 \sigma_2^3 \sigma_1 & n \text{ is even.} \end{cases}$$