$$S(k, n) = \sum_{i=0}^{n-k} (-1)^i \cdot \binom{n-k}{i}^2 \cdot (n-1)!^{2i} \cdot n!^{2n-2k-2i}.$$