

$$\phi_{p^\ell-1}(pn+r) \equiv 0 \pmod{2}$$