$$\mathcal{S}_l = \{ (z_l, e) \in \mathcal{X}^{N+1} \colon e = 0 \}$$