$$(\mathbb{S}^3 \setminus \{(0,-i)\}, \alpha_{\mathbb{S}^3}) \to (\mathbb{R}^3, dz' + x'dy' - y'dx') \cong \mathbb{C} \times \mathbb{R}; (z_1, z_2) \mapsto \left(\frac{iz_1}{i+z_2}, \frac{-\Re(z_2)}{|i+z_2|^2}\right); (\cos\theta, \sin\theta) \mapsto \left(\frac{iz_1}{1+z_2}, \frac{-\Re(z_2)}{|i+z_2|^2}\right)$$