

$$\begin{aligned}
B_{k,\ell}^1 &\leq C\|(a+\delta A^{\alpha/2})^{-1}e_k\|_{\mathcal{H}}\|(a+\delta A^{\alpha/2})^{-1}e_\ell\|_{\mathcal{H}} \\
&\leq C\|(\underline{a}+\delta A^{\alpha/2})^{-1}e_k\|_{\mathcal{H}}\|(\underline{a}+\delta A^{\alpha/2})^{-1}e_\ell\|_{\mathcal{H}} \\
&= C\frac{1}{\underline{a}+\delta\lambda_k^{\alpha/2}}\frac{1}{\underline{a}+\delta\lambda_\ell^{\alpha/2}}.
\end{aligned}$$