$$r \le \frac{\|\mathbf{a}^{(i)}\|}{\sigma_{\min}(A^{(i)})}.$$