$$\sigma_{i+1}^{-1}(V(\mathcal{K}^\ell)) \subset V(\mathcal{P}_a^\ell) \cup V(\mathcal{P}_b^\ell) \text{ and} \sigma_{i+1}(\{v_a^\ell, v_b^\ell\}) \not\preceq V(\mathcal{K}^\ell),$$