$\sigma_{i+1}^{-1}(\sigma_i(v_a^{\ell})) \in N_{X_L}(v_a^{\ell}) \cap V(\mathcal{P}_a^{\ell}),$