

$$\begin{aligned} &\sum_{j=1}^\infty \left(\frac{1}{\epsilon_0^{-1}+\delta\lambda_j^{\alpha/2}}\right)^2 \left\langle e_j, (a+\delta A^{\alpha/2})(\epsilon_0^{-1}+\delta A^{\alpha/2})^{-2}(ae_k)\right\rangle_{\mathcal{H}}^2 \\ &\leq C\|ae_k\|_{\mathcal{H}}^2 \sum_{j=1}^\infty \left(\frac{1}{\epsilon_0^{-1}+\delta\lambda_j^{\alpha/2}}\right)^2 < \infty, \end{aligned}$$