$\sigma_{i+1}^{-1}(\sigma_i(v_b^{\ell})) \in N_{X_L}(v_b^{\ell}) \cap V(\mathcal{P}_b^{\ell}),$