$$\mathtt{sum} = v_1'(1+\eta_N) + v_2'(1+\eta_{N-1}) + \dots + v_n'(1+\eta_1) = v_1(1+\eta_{N+1}) + v_2(1+\eta_N) + \dots + v_N(1+\eta_2) = (a_1+\alpha_1)(1+\eta_{N+1}) + (a_2+\alpha_2) + \dots + v_N(1+\eta_N) +$$