$S(n-1,n) = n!^2 - (n-1)!^2 = (n-1)!^2(n^2-1).$