

$$\sigma_{(d+1)^r} := \sigma_{(d+1, \textcolor{blue}{d+1}, \dots, d+1)},$$