$\sigma^{-1}(V(\mathcal{K}^{\ell}) \setminus \{\kappa_a^{\ell+1}, \kappa_b^{\ell+1}\}) \subset V(X^{\ell}),$