

$$F(k,n)=\binom{n}{k}^2\cdot k!\cdot (n-1)!^{2k}\cdot S(k,n),$$