$line: \tilde{\sigma}^{-1}(\kappa_a^{\ell+1}) \notin V(P_a^{\ell}) \cup V($

 $\text{norm}_s tr$: