$$S(\beta) = \Delta_{N+1} S_0(\beta_0) \Delta_{N+1} \doteq \beta(\sigma_N ... \sigma_2 \sigma_1^3 \sigma_2 ... \sigma_N)$$