$$V(\mathcal{K}_a^\ell) \cup \{\eta_1\} = \sigma_{i_j}(V(\mathcal{P}_a^\ell)),$$