Structural Equations

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1 Beam vs. Plane

Beam should be good, while the reference has a simulation on plat plane, so I follow this work.

2 Strong Form to Matrix Form

strong form of dynamic/vibration can be descripted as:

$$\Big\{ \sigma_{ji,j} + b_i = \rho \ddot{u}_i on \Omega u_i = 0 on \Gamma with fixed displacement \sigma_i j \cdot n_j = p_i on \Gamma with moving interface u_i(t=0) = u_0 \dot{u}_i(t =$$

Note,in fluid structural interaction applications, obviously there are two kind of boundaries. the fixed displacement B.C. and the fraction(fluid pressure) B.C.

Galerkin weak form obtained as:

$$\delta\Pi = \int_\Omega (\delta u_i \rho \ddot{u}_i + \delta\varepsilon_{ij}\sigma_{ij} - \delta u_i b_i) dv - \int_\Gamma \delta u_i t_i ds = 0$$

in which, t_i is boundary traction.

consider linear elasticity relationship $\sigma = C\varepsilon$, strain-displacement relationship $\varepsilon = B\hat{u}$, and shape function N to obtain displacement $u(\vec{x}, t) = N u_i$.

$$\delta\Pi = \sum \delta\Pi^e = \sum \delta u^{eT} (\int_{\Omega^e} N^T \rho N dv \ddot{u}^e + \int_{\Omega^e} B^T C B dv u^e - \int_{\Omega^e} N^T b dv - \int_{\Gamma^e} N^T t ds) = 0$$

from above, we obtain equation for an element$M^e \ddot{u}^e + K^e u^e = f^e$

where

$$M^e = \int N^t \rho N dv$$

$$K^e = \int B^T C B dv$$

$$f^e = \int N^T b dv + N^T t ds$$

3 plate element

in 2D simulation, we choice thin Kirchoff 4-node plate element.

4 traction boundary