couple fluid and structural

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1 system governing equation

In each time step, SPH for fluid, we calculate velocity of each particle, and then update the other properties of that point, such as its displacement,density and pressure; and FEM for stuctural, we calculate beam deflection from the equation, and then update other properties, such as stress, strain.

so in this couple system, the self-dependent variable are particles' velocity vector $v(x, y, t)$ and FEM node position vector $w(x, t)$. the governing equation are:

$$\rho_i \dot{v_i} = \Delta\sigma_i + g_i + f_i^{s2f}$$

$$\frac{d^2}{dx^2}(EI\frac{d^2 w_I}{dx^2}) - \rho_I \ddot{w}_I = f_I^{f2s}$$

the two equations are coupled by f^{s2f} and f^{f2s}. as one FEM node correspond to multi fluid particles, while each fluid particles correspond to at most 2 FEM node, so these two forces are actually not equivalent.

Note in this simple case, all beam nodes play dual-roles as FEM node as well as interface nodes.

one way is set interface node - fluid particle Pair $< i, I >$, here i stands for fluid particle, and I stands for interface node, which obtain:

$$f^{s2f} = \sum_i repulse\_force(< i, I >)$$

$$f^{f2s} = \sum_I -repulse\_force(< i, I >)$$

2 approximation of equations

for sph particle velocity vector:

$$\frac{du}{dt} = \sum_j m_i(\frac{\sigma_i}{\rho_i^2} + \frac{\sigma_j}{\rho_j^2})\nabla w_{ij} + \sum_j \frac{m_j}{\rho_j} f_j^{s2f} w_{ij}$$

Note to update the approximation fluid equation, we need calculate ρ_i, w_{ij} at first.

for beam deflection displacement:

as before, we adopt Hermite shape function $w = \sum_{k=1}^{4} \Delta_k \theta_k$, then

$$\sum_{J=1}^{4} EI(\theta_I, \theta_J)\Delta_J - \rho\ddot{\Delta}_J l(\theta_I, \theta_J) = f_I^{f2s}$$

the other parts of sph , FEM algorithm, please check ¡sph.pdf¿ and ¡beam2.pdf¿