2. $f(A)=\begin{cases}\frac{1}{2} & A\in(-1,1)\\ 0 & else\end{cases}$

$EX(t)=\frac{1}{2}\int_{-\infty}^{\infty}A\sin(t+\frac{\pi}{4})\frac{1}{2}dA+\frac{1}{2}\int_{-\infty}^{\infty}A\sin(t-\frac{\pi}{4})\cdot\frac{1}{2}dA$

$=\frac{1}{4}\sin(t+\frac{\pi}{4})\int_{-\infty}^{\infty}A\,dA+\frac{\sin(t-\frac{\pi}{4})}{4}\int_{-\infty}^{\infty}A\,dA=0.$

$R_X(t,t+\tau)=EX(t)X(t+\tau)=EA^2\sin(t+\theta)\sin(t+\tau+\theta)=EA^2E\sin(t+\theta)\sin(t+\tau+\theta)$

$E(A^2)=D(A)+E^2(A)=\frac{4}{12}+0=\frac{1}{3}$

$E\sin(t+\theta)\sin(t+\tau+\theta)=\frac{1}{2}E\sin(t+\frac{\pi}{4})\sin(t+\tau+\frac{\pi}{4})+\frac{1}{2}E\sin(t-\frac{\pi}{4})\sin(t+\tau-\frac{\pi}{4})$

$=\frac{1}{2}\left(\frac{\cos(\tau)-\cos(2t+\frac{\pi}{2}+\tau)}{2}+\frac{\cos(\tau)-\cos(2t+\tau-\frac{\pi}{2})}{2}\right)$

$=\frac{1}{4}(2\cos(\tau)+\sin(2t+\tau)-\sin(2t+\tau))=\frac{1}{4}\cdot2\cos(\tau)=\frac{1}{2}\cos(\tau)$

$\therefore R_X(t,t+\tau)=EA^2E\sin(t+\theta)\sin(t+\tau+\theta)=\frac{1}{3}\cdot\frac{1}{2}\cos(\tau)=\frac{1}{6}\cos(\tau)$

只与τ有关，而与t无关

$\therefore\{X(t),-\infty<t<\infty\}$是平稳过程.

4 (1) $\mu_{X(t)}=EX(t)=E(A\sin t-B\cos t)=\sin t\,EA-\cos t\,EB=\mu\sin t-\mu\cos t$

$=\sqrt{2}\mu(\frac{\sqrt{2}}{2}\sin t-\frac{\sqrt{2}}{2}\cos t)=\sqrt{2}\mu\sin(t-\frac{\pi}{4})$

$R_X(t,t+\tau)=EX(t)X(t+\tau)=E(A\sin t-B\cos t)(A\sin(t+\tau)-B\cos(t+\tau))$

$=E(A^2\sin t\sin(t+\tau)-AB\sin t\cos(t+\tau)-AB\sin(t+\tau)\cos t+B^2\cos t\cos(t+\tau))$

$EA^2=EB^2=\sigma^2\quad EAB=E(A)E(B)=\mu^2$

接上式 $=\sigma^2\sin t\sin(t+\tau)-\mu^2\sin t\cos(t+\tau)-\mu^2\sin(t+\tau)\cos t+\sigma^2\cos t\cos(t+\tau)$

$=\sigma^2\cos(t+\tau-t)-\mu^2\sin(t+t+\tau)$

$=\sigma^2\cos\tau-\mu^2\sin(2t+\tau)$

(2) $\because\{X(t);-\infty<t<\infty\}$是宽平稳过程.

$\therefore\mu_X(t)$为常数. $R_X(t,t+\tau)$只和τ有关.

$\therefore\mu=0.$

(3) $E(A)=1\times\frac{1}{2}+1\times(-\frac{1}{2})=0.\quad E(A^2)=1$

$X(0)=-B\qquad X(\frac{\pi}{4})=\frac{\sqrt{2}}{2}(A-B)$

$X(0)$:

	-1	1
P	$\frac{1}{2}$	$\frac{1}{2}$

$X(\frac{\pi}{4})$

	-2	0	2
P	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$\because$ $X(0)$ 和 $X(\frac{\pi}{4})$ 不服从同一分布

$\therefore$ $\{X(t)\}$ 不是平平稳过程.

6. $\mu_{Y(t)} = EY(t) = E(\int_0^t X(u)\,du) = E(X\sin t) = E(X)E(\sin t) = \sin t$

$\int_0^t X(u)\,du = \int_0^t X\cos u\,du = X\sin t$

$R_{XY}(s,t) = EX(s)Y(t) = EX\cos s\int_0^t X(u)\,du = EX^2\cos s\sin t$

$= EX^2 \cdot \cos s \cdot \sin t = 4\cos s\sin t$

$EX^2 = D(X) + (EX)^2 = 3+1 = 4$

9. (1) $\mu_X(t) = EX(t) = E(\sqrt{2}X\cos t + Y\sin t) = \sqrt{2}\cos t\,EX + \sin t\,EY$

$EX = \int_{-1}^1 x(1-|x|)\,dx = \int_0^1 x(1-x)\,dx + \int_{-1}^0 x(1+x)\,dx$

$= \int_0^1 x\,dx - \int_0^1 x^2\,dx + \int_{-1}^0 x\,dx + \int_{-1}^0 x^2\,dx$

$= \frac{1}{2} - \frac{1}{3} - \frac{1}{2} + \frac{1}{3} = 0$

$f_Y(x) = \begin{cases} \frac{1}{2} & -1<x<1 \\ 0 & \text{else} \end{cases}$ $\quad E(Y) = 0.$

$\therefore$ $\mu_X(t) = \sqrt{2}\cos t\,EX + \sin t\,EY = 0$

$R_X(t,t+\tau) = EX(t)X(t+\tau) = E(\sqrt{2}X\cos t + Y\sin t)(\sqrt{2}X\cos(t+\tau) + Y\sin(t+\tau))$

$= E(2X^2\cos t\cos(t+\tau) + \sqrt{2}XY\cos t\sin(t+\tau) + \sqrt{2}XY\sin t\cos(t+\tau) + Y^2\sin t\sin(t+\tau))$

$= 2\cos t\cos(t+\tau)EX^2 + \sqrt{2}E(XY)\cos t\sin(t+\tau) + \sqrt{2}E(XY)\sin t\cos(t+\tau) + E(Y^2)\sin t\sin(t+\tau)$

$EXY = EXEY = 0$

$EX^2 = \int_{-1}^1 x^2(1-|x|)\,dx = 2\int_0^1 x^2(1-x)\,dx = 2(\int_0^1 x^2\,dx -$

$\int_0^1 x^3\,dx) = 2(\frac{1}{3}x^3\Big|_0^1 - \frac{1}{4}x^4\Big|_0^1) = 2(\frac{1}{3}-\frac{1}{4}) = 2\times\frac{1}{12} = \frac{1}{6}$

$EY^2 = DY + (EY)^2 = \frac{1}{3}$

$DY = \frac{4}{12} = \frac{1}{3} \quad EY = \frac{-1+1}{2} = 0$

$R_X(t,t+\tau) = \frac{1}{3}\cos t\cos(t+\tau) + \frac{1}{3}\sin t\sin(t+\tau)$

$= \frac{1}{3}\cos(t+\tau-t) = \frac{1}{3}\cos\tau$

$\because \mu_X(t) = 0$ 为常数 $R_X(t,t+\tau) = \frac{1}{3}\cos\tau$ 只和τ有关

$\therefore \{X(t)\}$ 是平稳过程.

(2) $\langle X(t)\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}X(t)\,dt = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}(\sqrt{2}X\cos t + Y\sin t)\,dt$

$= \lim_{T\to\infty}\frac{1}{2T}(2\sqrt{2}X\sin T + 2Y\sin T) = \lim_{T\to\infty}\frac{(\sqrt{2}X+Y)\sin T}{T} = 0$

$\because \langle X(t)\rangle = EX(t) = 0$

$\therefore$ 均值具有各态历经性.

(3) $\langle X(t)X(t+\tau)\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}X(t)X(t+\tau)\,dt = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}(\sqrt{2}X\cos t + Y\sin t)(\sqrt{2}X\cos(t+\tau) + Y\sin(t+\tau))\,dt$

$= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}[2X^2\cos t\cos(t+\tau) + Y^2\sin t\sin(t+\tau) + \sqrt{2}XY\cos t\sin(t+\tau) + \sqrt{2}XY\sin t\cos(t+\tau)]\,dt$

$= \lim_{T\to\infty}\frac{1}{2T}\left(2X^2\int_{-T}^{T}\frac{\cos(\tau)+\cos(2t+\tau)}{2}dt + Y^2\int_{-T}^{T}\frac{\cos(\tau)-\cos(2t+\tau)}{2}dt + \sqrt{2}XY\int_{-T}^{T}\sin(2t+\tau)\,dt\right)$

$= \lim_{T\to\infty}\frac{1}{2T}\left(X^2\int_{-T}^{T}\cos(\tau)+\cos(2t+\tau)\,dt + \frac{1}{2}Y^2\int_{-T}^{T}\cos(\tau)-\cos(2t+\tau)\,dt - \frac{\sqrt{2}}{2}XY(\cos(2T+\tau)-\cos(-2T+\tau))\right)$

$= \lim_{T\to\infty}\frac{1}{2T}\left(X^2\cdot 2T\cos(\tau) + \frac{X^2}{2}\sin(2T+\tau) - \frac{X^2}{2}\sin(-2T+\tau) + Y^2T\cos(\tau) - \frac{1}{4}Y^2\sin(2T+\tau) + \frac{1}{4}Y^2\sin(-2T+\tau) - \frac{\sqrt{2}}{2}XY(\cos(2T+\tau)-\cos(-2T+\tau))\right)$

$= \lim_{T\to\infty} X^2\cos(\tau) + \frac{X^2}{4T}\sin(\tau) - \frac{X^2}{4T}\sin(\tau) + \frac{Y^2}{2}\cos(\tau) - \frac{Y^2}{8T}\sin(\tau) + \frac{Y^2}{8T}\sin(\tau) - \frac{\sqrt{2}}{4T}XY\cos\tau + \frac{\sqrt{2}}{4T}\cos\tau$

$= \lim_{T\to\infty} X^2\cos(\tau) + \frac{Y^2}{2}\cos(\tau) = (X^2 + \frac{Y^2}{2})\cos(\tau) \neq R_X(t,t+\tau)$

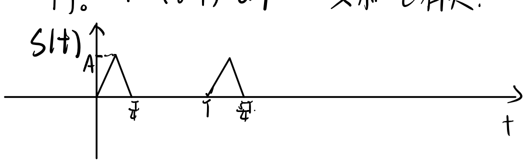
$\therefore$ 不具有各态历经性.

10、(1) $\mu_X(t) = EX(t) = Es(t+\theta) = \int_0^T s(t+\theta)\frac{1}{T}\,d\theta = \frac{1}{T}\int_t^{t+T}s(\varphi)\,d\varphi$

$\because s(t)$ 有周期性 $\therefore \frac{1}{T}\int_t^{t+T}s(\varphi)\,d\varphi = \frac{1}{T}\int_0^T s(\varphi)\,d\varphi = \frac{1}{T}\cdot\frac{T}{4}\cdot A\cdot\frac{1}{2} = \frac{A}{8}$

$R_X(t,t+\tau) = E[s(t+\theta)s(t+\tau+\theta)] = \int_0^T s(t+\theta)s(t+\tau+\theta)\cdot\frac{1}{T}\,d\theta = \frac{1}{T}\int_t^{t+T}s(\varphi)s(\varphi+\tau)\,d\varphi$

$= \frac{1}{T}\int_0^T s(\varphi)s(\tau+\varphi)\,d\varphi$ 只和τ有关.



$s(t) = \begin{cases} \frac{8A}{T}t & 0\le t\le\frac{T}{8} \\ -\frac{8A}{T}(t-\frac{T}{4}) & \frac{T}{8}<t\le\frac{T}{4} \\ 0 & \frac{T}{4}<t\le T \end{cases}$

$$R_X\left(\frac{T}{8}\right) = \frac{1}{T}\int_0^T s(\varphi)\, s\left(\frac{T}{8}+\varphi\right) d\varphi = \frac{1}{T}\int_0^{\frac{T}{8}} \frac{8A}{T}x\left[-\frac{8A}{T}\left(x+\frac{T}{8}\right)-\frac{T}{4}\right] dx$$

$$= -\frac{64A^2}{T^3}\int_0^{\frac{T}{8}} x\left(x-\frac{T}{8}\right) dx = -\frac{64A^2}{T^3}\left(-\frac{1}{8^3}\cdot\frac{1}{6}T^3\right) = 64A^2\cdot\frac{1}{8^3}\cdot\frac{1}{6} = \frac{A^2}{48}$$

(2) $\langle X(t)\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} X(t)\, dt = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} s(t+\theta)\, dt = \lim_{T\to\infty}\frac{1}{2T}\cdot\frac{A}{8}\cdot T\cdot 2 = \frac{A}{8}$