Numerical Analysis HW4

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Problem 1:

> Construct the Lagrange interpolating polynomials for the following functions, and find a bound for the error on the interval $[x_0, x_n]$.

a.

x_0	x_1	x_2
0	0.3	0.6
y_0	y_1	y_2
$e^0 cos 0$	$e^{0.3} cos 0.3$	$e^{0.6} cos 0.6$
1	1.82189401012	3.31847864546

$$L_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\,y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\,y_1 + \frac{(x-x_2)(x-x_0)}{(x_2-x_1)(x_2-x_0)}\,y_2$$
$$\implies P_2(x) = -11.22388889x^2 + 3.8105x + 1$$
$$error\ bound:\ 0.11371294$$

b.

x_0	x_1	x_2
2	2.4	2.6
y_0	y_1	y_2
$sin(ln2)$	$sin(ln2.4)$	$sin(ln2.6)$
0.01209740541	0.0152792174	0.01667604225

同理于a题，由拉格朗日二次三点插值得：

$$P_2(x) = -0.1306344167x^2 + 0.8969979335x - 0.63249693$$
$$error\ bound: \ 9.45762 \times 10^{-4}$$

Problem 2:

> Let $P_3(x)$ be the interpolating polynomial for the data $(0,0)$, $(0.5,y)$, $(1,3)$, and $(2,2)$. The coefficient of x_3 in $P_3(x)$ is 6. Find y.

由插值公式可得

$$P_3(x) = 0 + \frac{(x-0)(x-1)(x-2)}{(0.5-0)(0.5-1)(0.5-2)}y + \frac{(x-0)(x-1)(x-0.5)}{(1-0)(1-0.5)(1-2)} \times 3 + \frac{(x-0)(x-2)(x-0.5)}{(2-0)(2-1)(2-0.5)} \times 2$$

显然，x^3的系数为$\frac{8y-16}{3}$

$\therefore \ y = \frac{34}{8} = 4.25$

Problem 3:

a.

> Neville's method is used to approximate $f(0.5)$, giving the following table. Determine $P_2 = f(0.7)$.

$x_0 = 0$	$P_0 = 0$		
$x_1 = 0.4$	$P_1 = 2.8$	$P_{0,1} = 3.5$	
$x_2 = 0.7$	P_2	$P_{1,2}$	$P_{0,1,2} = \frac{27}{7}$

由插值公式得：$P_2 = f(0.5) = 4$。

b.

> Suppose $x_j = j$, for $j = 0, 1, 2, 3$ and it is known that

$$P_{0,1}(x) = 2x+1, P_{0,2}(x) = x+1,\ and\ P_{1,2,3}(2.5) = 3.$$

Find $P_{0,1,2,3}(2.5)$.

由插值公式得：$P_{0,1,2,3}(2.5) = 2.875$

Problem 4

For a function f , the forward-divided differences are given by

$x_0 = 0.0$	$f[x_0]$		
		$f[x_0, x_1]$	
$x_1 = 0.4$	$f[x_1]$		$f[x_0, x_1, x_2] = \frac{50}{7}$
		$f[x_1, x_2] = 10$	
$x_2 = 0.7$	$f[x_2] = 6$		

Determine the missing entries in the table.

由向前差分公式计算得：

$f[x_0] = f(x_0) = 1$

$f[x_1] = f(x_1) = 3$

$f[x_0，x_1] = 5$

Problem 5

Determine the natural cubic spline S that interpolates the data $f(0) = 0, f(1) = 1,$ and $f(2) = 2$.

由三次样条插值公式，$S(x) = x\ on\ [0, 2]$.

Problem 6

Proof that a strictly diagonally dominant matrix is invertible.

使用反证法，假设A不可逆，则有$det(A) = 0$

$AX = 0$ 有非零解，设为$X = (x_1, x_2, \dots, x_n)^T$

且令$|x_k| = max|x_i|$

根据假设，有$\Sigma_{j=1}^{n} a_{kj}x_j = 0$

从而$|a_kk||x_k| = \Sigma_{j!=1} a_{kj}x_j$

而根据A为严格对角优势矩阵

$|a_k k||x_k| \geq |x_k|\Sigma_{j \neq 1}|a_{kj}| > \Sigma_{j \neq 1}|a_{kj}||x_j| \geq \Sigma_{j \neq k} a_{kj} x_j$

两式矛盾，所以A可逆。

$Q.E.D.$