12. (1) $P(N(1)=1, N(4)>1) = P(N(1)=1)P(N(3)>0) = P(N(1)=1)[1-P(N(3)=0)]$

$= 10e^{-10}(1-e^{-30})$

(2) $P(3<W_3\leq 4 \mid W_1=1, W_2=2) = P(1<W_1\leq 2) = P(N(1)=0)P(N(2)\geq 1 \mid N(1)=0)$

$= P(N(1)=0)P(N(1)\geq 1) = P(N(1)=0)[1-P(N(1)=0)]$

$= e^{-10}(1-e^{-10})$

(3) P(个人理赔钱数超过5500) $= \frac{1}{2}$

P(个人理赔钱数低于5500) $= \frac{1}{2}$

将$N(t)$分解为$N_1(t)$和$N_2(t)$，分别是$(0,t]$理赔钱数低于5500的客户与高于5500的客户

$\lambda = \lambda_1+\lambda_2 = p_1\lambda + p_2\lambda \quad p_1=p_2=\frac{1}{2}$

$\lambda_1=\lambda_2=\frac{1}{2}\lambda=5$

$\therefore P = P(N_2(t)\geq 1) = 1-P(N_2(t)=0) = 1-e^{-5t}$

14. (1) $N(t)=N_1(t)+N_2(t) \qquad \lambda=\lambda_1+\lambda_2=3$

$P(N(1)=2) = \frac{9e^{-3}}{2}$

(2) $\lambda_3=\lambda_4=\frac{\lambda}{2}=1.5$ 分别为小于1kg和大于1kg的强度

$P = P(N_3(1)=2)P(N_4(1)=2) = \left(\frac{1.5^2e^{-1.5}}{2}\right)^2 = \frac{81}{64}e^{-3}$

(3) $\lambda_5=\frac{\lambda}{3}=1 \quad \lambda_6=\lambda-\lambda_5=2$ 分别为钓到1kg以上鲫鱼与其他情况的强度

$P = P(N_5(1)=1)P(N_5(2)=2 \mid N_5(1)=1)P(N_6(1)=0)P(N_6(2)=2 \mid N_6(1)=1) = (e^{-1}e^{-2})^2 = e^{-6}$

(4) $P = \left(\frac{1}{2}\times\frac{2}{1+2}\right)^2 = \frac{1}{9}$

15. $N(t)-N(s) \sim \pi\left(\int_s^t \lambda(h)dh\right)$

(1) $P(N(2)=3) = \frac{2^3e^{-2}}{3!} = \frac{4}{3}e^{-2}$

(2) $P(N(1)=2, N(2)=4) = P(N(1)=2)P(N(2)=4 \mid N(1)=2) = \frac{(\frac{1}{2})^2e^{-\frac{1}{2}}}{2!}\cdot\frac{(\frac{3}{2})^2e^{-\frac{3}{2}}}{2!} = \frac{9}{64}e^{-2}$

(3) $P(N(1)=2 \mid N(2)=4) = \frac{P(N(1)=2, N(2)=4)}{P(N(2)=4)} = \frac{\frac{9}{64}e^{-2}}{\frac{2^4e^{-2}}{4!}} = \frac{27}{128}$

17. (1) $P\{B(3.6)\leq 1 \mid B(1.6)=0.8, B(2.39)=-0.1\} = P\{B(3.6)-B(2.39)\leq 1.1\}$

$B(3.6)-B(2.39) \sim N(0,1.21) = N(0,1.1^2)$

$P\{B(3.6)-B(2.39)\leq 1.1\} = \Phi(1)$

(2) $Cov(B(8)-B(4), B(6)) = Cov(B(8),B(6)) - Cov(B(4),B(6))$

$= 6-4 = 2$

(3) $2B(1) \sim N(0,4)$

$D(2B(1)+B(2)) = D(2B(1)) + D(B(2)) + 2Cov(2B(1), B(2))$

$= 4+2+2\min\{4,2\}$

$= 10$

21. (1) $P(B(\frac{1}{10})\geq 1.5 \mid B(\frac{1}{6})=2, B(\frac{1}{4})=2.4) = P(\frac{1}{10}\tilde{B}(10)\geq 1.5 \mid \frac{1}{6}\tilde{B}(6)=2, \frac{1}{4}\tilde{B}(4)=2.4)$

$= P(\tilde{B}(10)\geq 15 \mid \tilde{B}(6)=12) = P(\tilde{B}(10)-\tilde{B}(6)\geq 3)$

$= 1-\Phi\left(\frac{3}{\sqrt{10-6}}\right) = 1-\Phi(1.5)$

(2) $\tilde{B}(4)=9.6 \quad \tilde{B}(6)=12 \qquad \tilde{B}(10)=\tilde{B}(10)-\tilde{B}(6)+12 \sim N(12,4)$

$B(\frac{1}{10}) \sim \frac{1}{10}\tilde{B}(10) \sim N(\frac{6}{5}, \frac{1}{25})$

23. (1) $B(t) \sim N(0,t)$

$P(|B(t)|\leq x) = P(-x\leq B(t)\leq x) = 2\Phi(\frac{x}{\sqrt{t}})-1$

(2) $P(\max_{0\leq s\leq t} B(s)-B(t)\leq x) = P(\max_{0\leq s\leq t}(B(s)-B(t))\leq x) = P(\max_{0\leq s\leq t} B(t-s)\leq x)$

$= P(\max_{0\leq u\leq t} B(u)\leq x) = 1-P(\max_{0\leq u\leq t} B(u)>x) = 1-2P(B(t)>x)$

$= 1-2(1-\Phi(\frac{x}{\sqrt{t}})) = 2\Phi(\frac{x}{\sqrt{t}})-1$