Numerical Analysis HW3

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Problem 1

> Use the most accurate three-point formula to determine each missing entry in the following tables.

a.

x	$f(x)$	$f'(x)$
1.1	9.025013	
1.2	11.02318	
1.3	13.46374	
1.4	16.44465	

本题含有两个边界处的导数的求解，所以对$x_1=1.1$和$x_4=1.4$应该使用边界公式，即课本formula 4.4：$f'(x_0)=\frac{1}{2h}[-3f(x_0)+4f(x_0+h)-f(x_0+2h)]+\frac{h^2}{3}f^{(3)}(\xi_0)$

其余两个值应该使用formula 4.5：$f'(x_0)=\frac{1}{2h}[f(x_0+h)-f(x_0-h)]+\frac{h^2}{3}f^{(3)}(\xi_0)$

此处计算时省略去含 h 的二次项，计算得：

导数	计算值
$f'(x_1)$	17.769705
$f'(x_2)$	22.193635
$f'(x_3)$	27.10735
$f'(x_4)$	32.15085

b.

x	$f(x)$	$f'(x)$
8.1	16.94410	
8.3	17.56492	
8.5	18.19056	
8.7	18.82091	

和a题同理，仅需将 h 的值修改为0.2，即可计算出：

导数	计算值
$f'(x_1)$	3.09205
$f'(x_2)$	3.11615
$f'(x_3)$	3.139975
$f'(x_4)$	3.163525

Problem 2

> Suppose that $N(h)$ is an approximation to $M$for every $h>0$ and that
>
> $M=N(h)+K_1h^2+K_2h^4+K_3h^6+\cdots,$
>
> for some constants $K_1,K_2,K_3,\ldots$ Use the values $N(h)$, $N(\frac{h}{3})$, and $N(\frac{h}{9})$ to produce an $O(h^6)$ approximation to M.

因为$M=N(h)+K_1h^2+K_2h^4+K_3h^6+\cdots$

令$h=\frac{h}{3}$，代入原式，计算化简得：

$M=N_2(h)-K_2\frac{h^4}{9}-K_3\frac{10}{81}h^6$

其中：$N_2(h)=N_1(\frac{h}{3})+\frac{1}{8}[N_1(\frac{h}{3})-N_1(h)]$

重复操作，得到：

$M=N_3(h)+\frac{K_3}{729}h^6$

其中：$N_3(h) = N_2(\frac{h}{3}) + \frac{1}{80}[N_2(\frac{h}{3}) - N_2(h)])$

这样就得到了 $O(h^6)$ 情况下对 M 的估计值。

事实上，重复两次$h = \frac{h}{3}$后，$N_2(h)$和$N_3(h)$即使用了$N(h)$，$N(\frac{h}{3})$和$N(\frac{h}{9})$

Problem 3

> Approximate the following integrals using the Trapezoidal rule and Simpson's rule, respectively.

a. $\int_{-0.25}^{0.25} (cosx)^2 dx$

梯形法：$\int_{-0.25}^{0.25} (cosx)^2 dx \approx 0.25(2cos^2(0.25)) = 0.46939564047$

辛普森法：$\int_{-0.25}^{0.25} (cosx)^2 dx \approx \frac{1}{6}(4 \times 1 + 2cos^2(0.25)) = 0.97959709365$

b. $\int_{-0.5}^{0} xln(x+1)dx$

梯形法：$\int_{-0.5}^{0} xln(x+1)dx \approx 0.25(0 - 0.5ln(0.5)) = 0.08664339757$

辛普森法：$\int_{-0.5}^{0} xln(x+1)dx \approx \frac{1}{6}(0 - 0.5ln(0.5) - 4 \times 0.25ln(0.75)) = 0.105709$

c. $\int_{0.75}^{1.3} ((sinx)^2 - 2xsinx + 1)dx$

梯形法：$\int_{0.75}^{1.3} ((sinx)^2 - 2xsinx + 1)dx \approx 0.13799948743$

辛普森法：$\int_{0.75}^{1.3} ((sinx)^2 - 2xsinx + 1)dx \approx -0.02330703191$

d. $\int_{e}^{e+1} \frac{1}{xlnx} dx$

梯形法：$\int_{e}^{e+1} \frac{1}{xlnx} dx \approx 0.5(0.20478890379 + 0.36787944117) = 0.28633417248$

辛普森法：$\int_{e}^{e+1} \frac{1}{xlnx} dx \approx \frac{1}{3}(0.57266834496 + 1.06335436971) = 0.54534090489$

Problem 4

> Use Romberg integration to compute $R_{3,3}$ for the following integrals.

由:

$R_{3,3}=R_{3,2}+\frac{1}{15}(R_{3,2}-R_{2,2}),R_{3,2}=R_{3,1}+\frac{1}{3}(R_{3,1}-R_{2,1}),R_{2,2}=R_{2,1}+\frac{1}{3}(R_{2,1}-R_{1,1})$
知：需计算 $R_{k,1}\ (k=1,2,3)$,

并有：$R_{3,3}=\frac{1}{45}(64R_{3,1}-20R_{2,1}+R_{1,1})$

a. $\int_{-1}^{1}(cosx)^2dx$

$R_{1,1}=0.58385316345$

$R_{2,1}=1.29192658173$

$R_{3,1}=1.88815259173$

$\Rightarrow R_{3,3}=2.12415749777$

b. $\int_{-0.75}^{0.75}xln(x+1)dx$

$R_{1,1}=0.47660504647$

$R_{2,1}=0.23830252323$

$R_{3,1}=0.25352005857$

$\Rightarrow R_{3,3}=0.26524085179$

c. $\int_{1}^{4}((sinx)^2-2xsinx+1)dx$

$R_{1,1}=2.07432430412$

$R_{2,1}=-0.89554449179$

$R_{3,1}=-0.71555514005$

$\Rightarrow R_{3,3}=-0.57356255165$

d. $\int_{e}^{2e}\frac{1}{xlnx}dx$

$R_{1,1}=0.64765402729$

$R_{2,1}=-0.56099642568$

$R_{3,1}=0.53560896904$

$\Rightarrow R_{3,3}=1.02577903465$

Problem 5

> Use Euler’s method to approximate the solutions for each of the following initial-value problems.

a. $y' = y/t - (y/t)^2, 1 \le t \le 2, y(1) = 1, with\ h = 0.1$

直接求解该常微分方程得：$y(x) = \dfrac{x}{c + lnx}$，代入初值条件得：$c = 1$

使用欧拉方法计算得每个点的对应值为：

i	t_i	w_i	$y(t_i)$
2	1.2	1.0082645	1.0149523
4	1.4	1.0385147	1.0475339
6	1.6	1.0784611	1.0884327
8	1.8	1.1232621	1.1336536
10	2.0	1.1706516	1.1812322

b. $y' = 1 + y/t + (y/t)^2, 1 \le t \le 3, y(1) = 0, with\ h = 0.2$

直接求解该常微分方程得：$y(x) = xtan(c + log(x))$，代入初值得：$c = 0$

使用欧拉方法计算得每个点的对应值为：

i	t_i	w_i	$y(t_i)$
2	1.4	0.4388889	0.4896817
4	1.8	1.0520380	1.1994386
6	2.2	1.8842608	2.2135018
8	2.6	3.0028372	3.6784753
10	3.0	4.5142774	5.8741000

Problem 6

> 写一个程序，来计算sin(x)函数的（6,6）级帕德逼近，正确答案应该是：
>
> $$[6/6]_{\sin(x)} = \frac{(12671/4363920) * x^5 - (2363/18183) * x^3 + x}{1 + (445/12122) * x^2 + (601/872784) * x^4 + (121/16662240) * x^6}$$

```
function PadeApprox=Pade_Approximation(G,r)
error(nargchk(2,2,nargin));
error(nargoutchk(1,1,nargout));
```

```
if ~isa(G,'tf') || ~isscalar(G)
    error('Input needs to be a SISO transfer function');
end
if ~isreal(r) || (fix(r)~=r) || (r<1) || (r>n)
    error('Invalid value of reduced model order')
end
[num,den]=tfdata(G,'v');
D_fact=num(1)/den(1);
num=num-D_fact*den;
num1=num(end:-1:1)/den(1);
den1=den(end:-1:1)/den(1);
n=length(den1)-1;
c(1)=num1(1)/den1(1);
for i=2:min(n,2*r)
    c(i)=(num1(i)-sum(den1(2:(i)).*c(end:-1:1)))/den1(1);
end
if (2*r)>n
    for i=n+1:(2*r)
        c(i)=-sum(den1(2:n+1).*c(end:-1:(end-n+1)))/den1(1);
    end
end
%Finding Coefficients
C1=c(repmat((r+1:-1:2),r,1)+repmat((0:(r-1))',1,r));
b=-inv(C1)*c(1:r)';
a(r)=0;
for i=1:r
    a(i)=c(i:-1:1)*b(1:i);
end
b=[1 b(end:-1:1)'];
a=a(end:-1:1);
[a,b]=tfdata(tf(a,b)+D_fact,'v');
b=b.*(abs(b)>1e-6);
a=a.*(abs(a)>1e-6);
PadeApprox=tf(a,b);
```

得到的结果为：$\dfrac{43x(12671x^4-567120x^2+4363920)}{1331x^6+126210x^4+6728400x^2+183284640}$

上下通分即得到原式

要验证为12阶泰勒展开式，只需两边取等号，化简知等式成立。

Problem 7

> Consider the following data:

i	x_i	y_i
1	0	6
2	2	8
3	4	14
4	5	20

1.Compute the linear least squares polynomial approximation for this data.

由最小二乘计算公式可得，线性条件下的拟合结果为：$y = 2.711864407x + 4.542372881$

2.Compute the error E of the above approximation.

y_i	$\hat{y}$
6	4.542372881
8	9.966101695
14	15.38983051
20	18.10169492

由$\int_a^b [f(x) - P_n(x)]^2 dx$可计算得：

$E = 11.52542372$