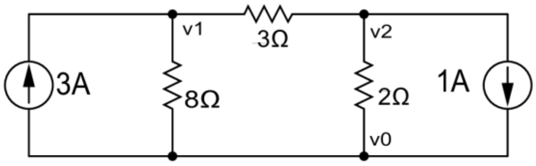
Homework 2

Due: Wednesday March 5, 2025, 11:59 pm

1. (a) Write nodal equations for the circuit below, assuming that $v_0 =$ ground. Solve for v_1 and v_2.

   (b) Write nodal equations for the same circuit, but this time, assume that $v_1 =$ ground. Solve for v_0 and v_2.

   (c) How are your answers to parts (a) and (b) related?



Solution:

(a) Taking v_0 as reference (ground), apply KCL to node v_1:

$$\begin{aligned} \frac{v_1}{8} + \frac{v_1 - v_2}{3} &= 3 \\ 11v_1 - 8v_2 &= 72. \end{aligned}$$

Similarly, apply KCL to node v_2:

$$\begin{aligned} \frac{v_2 - v_1}{3} + \frac{v_2}{2} + 1 &= 0 \\ 5v_2 - 2v_1 &= -6. \end{aligned}$$

Solve for v_1 and v_2, we get

$$\begin{aligned} v_1 &= 8V \\ v_2 &= 2V. \end{aligned}$$

(b) Taking v_1 as reference (ground) instead, apply KCL to node v_0:

$$\begin{aligned} \frac{v_0}{8} + \frac{v_0 - v_2}{2} \quad +3 &= 1 \\ 5v_0 - 4v_2 + 16 &= 0. \end{aligned}$$

Similarly, apply KCL to node v_2:

$$\begin{aligned} \frac{v_2}{3} + \frac{v_2 - v_0}{2} + 1 &= 0 \\ 5v_2 - 3v_0 &= -6. \end{aligned}$$

Solve for v_0 and v_2, we get

$$\begin{aligned} v_0 &= -8V \\ v_2 &= -6V \end{aligned}$$

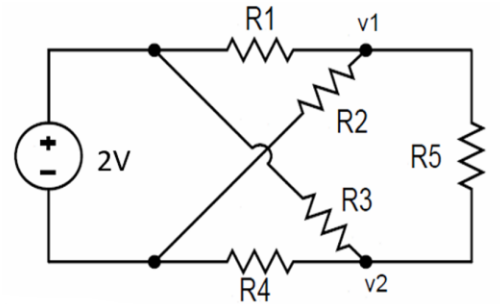
(c) Observation: The relative relationship between v_0, v_1, and v_2 are remained in question (a) and (b), regardless of reference point.
Numerically:

$$\begin{aligned} v_1 - v_0 &= 8 - 0 = 0 + 8 = 8V \\ v_2 - v_1 &= 2 - 8 = -6 - 0 = -6V. \end{aligned}$$

2. The circuit shown below is called a bridge or ladder circuit; variations of this circuit are used in many signal processing applications. If R_1 is a variable resistor, and if all of the other resistors are constant, it is possible to vary the "output voltage" $v_1 - v_2$ across a wide range of values by adjusting R_1.

   (a) Use nodal analysis to nd v_1 and v_2 in the following circuit, given that $R_1 = 2k\Omega$, $R_2 = 8k\Omega$, $R_3 = 4k\Omega$, $R_4 = 4k\Omega$, and $R_5 = 4k\Omega$. Hint: you do not need to know the current through the source.

   (b) Assume that R_2 through R_5 are as given in part (a), but R_1 is variable. What value of R_1 results in $v_1 = v_2$?



Solution:

(a) Pick the "-" end of voltage source as reference (ground). Apply KCL to node v_1:

$$\frac{v_1 - 2}{R_1} + \frac{v_1}{R_2} + \frac{v_1 - v_2}{R_5} = 0$$

Similarly, apply KCL to node v_2:

$$\frac{v_2 - 2}{R_3} + \frac{v_2}{R_4} + \frac{v_2 - v_1}{R_5} = 0$$

Given that $R_1 = 2k\Omega$, $R_2 = 8k\Omega$, $R_3 = 4k\Omega$, $R_4 = 4k\Omega$, and $R_5 = 4k\Omega$, solve for v_1 and v_2, we get

$$\begin{aligned} v_1 &= \frac{28}{19}V \\ v_2 &= \frac{22}{19}V. \end{aligned}$$

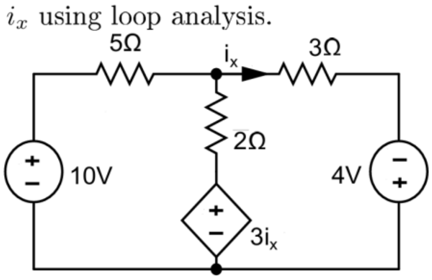
(b) Using the equations in part (a), substitute the value of R_2 through R_5 only and let $v_1 = v_2$, we get:

$$\begin{aligned} \frac{v_1 - 2}{R_1} + \frac{v_1}{8} &= 0 \\ \frac{v_1 - 2}{4} + \frac{v_1}{4} &= 0 \end{aligned}$$

Solve for v_1 and R_1:

$$\begin{aligned} R_1 &= 8k\Omega \\ v_1 &= 1V \end{aligned}$$

3. Find i_x using loop analysis.



Solution:

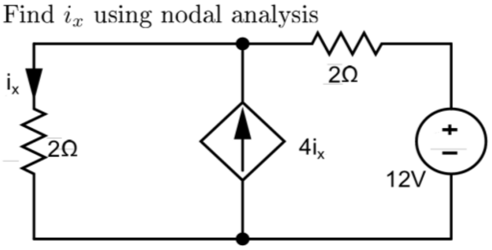
Apply KVL (loop analysis) to the 2 loops. We can generate 2 equations:

$$\begin{aligned} 3i_x + (-4) + (-3i_x) + 2(i_x - i_y) &= 0 \\ 5i_y + 2(i_y - i_x) + 3i_x + (-10) &= 0 \end{aligned}$$

Solve for i_x and i_y, we get

$$\begin{aligned} i_x &= 3A \\ i_y &= 1A. \end{aligned}$$

4. Find i_x using nodal analysis



Solution:

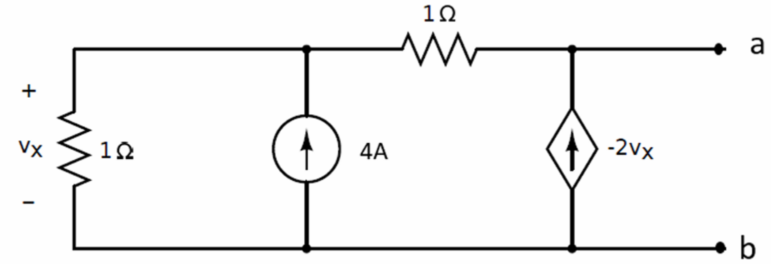
Apply KCL (node analysis) to the 2 loops. We can generate 2 equations:

$$\begin{aligned} \frac{v_x}{2} + \frac{v_x - 12}{2} &= 4i_x \\ i_x &= \frac{v_x}{2} \end{aligned}$$

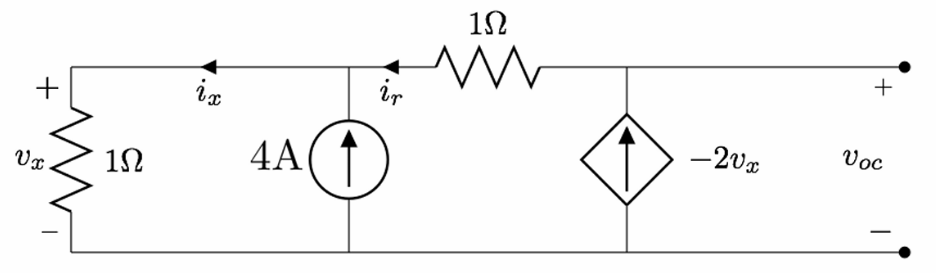
Solve for i_x and v_x, we get

$$\begin{aligned} i_x &= -3A \\ v_x &= -6V. \end{aligned}$$

5. Determine the Thevenin equivalent of the following network between nodes a and b:



Solution:



To find the open-circuit voltage, we construct a KCL equation at the node above the 4A source, yielding

$$i_x = i_r + 4.$$

Clearly the dependent source current must run through the top resistor, so

$$i_r = -2v_x.$$

The current through the left resistor can be computed by Ohm's law, giving

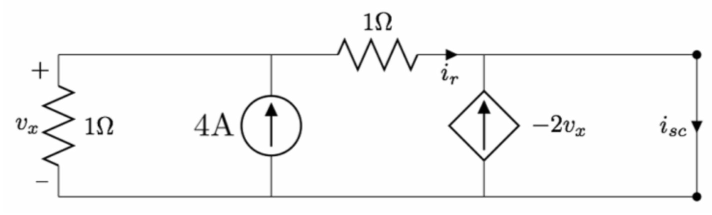
$$i_x = v_x/1 = v_x.$$

Thus we have

$$\begin{aligned} v_x &= 4 - 2v_x \\ 3v_x &= 4 \\ v_x &= \frac{4}{3}\text{V}. \end{aligned}$$

The open-circuit voltage is clearly seen to be the sum of the voltages across the resistors. Thus, we have

$$\begin{aligned} v_{oc} &= v_x + 1i_r \\ &= 4/3 - 2v_x \\ &= 4/3 - 8/3 \\ &= -\frac{4}{3}\text{V}. \end{aligned}$$



To find the short-circuit current, we construct a KCL equation at node a, yielding

$$i_{sc} = i_r - 2v_x.$$

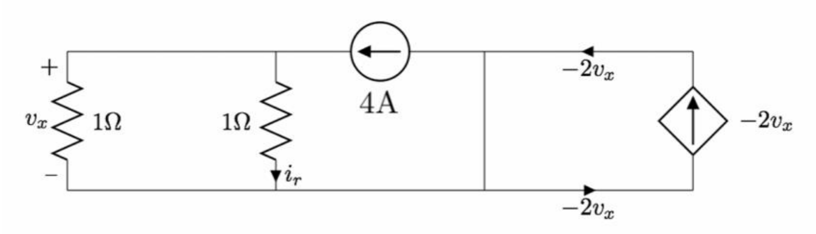
We may then note that the voltage across both the resistors is the same since they are in parallel. Thus

$$i_r = v_x/1 = v_x$$

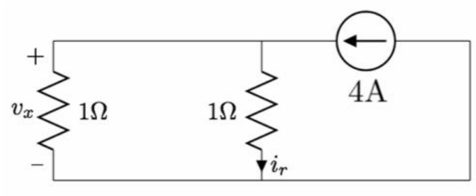
and

$$i_{sc} = i_r - 2i_r = -i_r.$$

Note that the dependent source does not contribute in this equivalent circuit, since it takes as much current from the node as it supplies as shown in the diagram below.



Thus it may be ignored for computational purposes. This yields the following equivalent diagram.



The remaining current from the 4A source is equally split between the 1Ω resistors. Therefore

$$\begin{aligned} i_r &= 2\text{A} = -i_{sc} \\ i_{sc} &= -2\text{A}. \end{aligned}$$

The Thevenin resistance is then given by $R_{Th} = \frac{v_{oc}}{i_{sc}} = \frac{-4/3}{-2} = \frac{2}{3}$, and the Thevenin equivalent is given below.