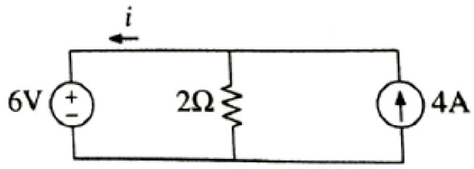
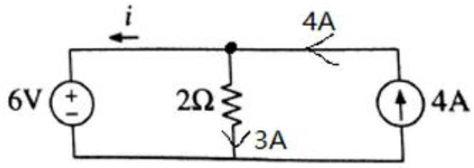
Homework 1

Due: Wednesday February 26, 2025, 11:59 pm

1. Determine i in the following circuit.



Solution:



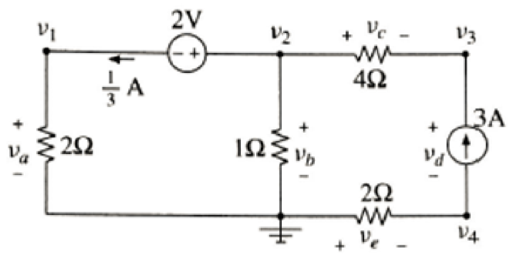
The voltage drop across the resister is 6V. By Ohm's law, the current across the resister is

$$I = \frac{6V}{2\Omega} = 3A$$

Applying KCL, we have

$$i = 4A - 3A = 1A.$$

2. **(a)** In the following circuit, determine all of the unknown element and node voltages.
   (b) What is the voltage drop in the circuit from the reference to node 4?
   (c) What is the voltage rise from node 2 to node 3?
   (d) What is the voltage drop from node 1 to the reference?



Solution:

(a)

$$v_1 = \frac{1}{3}A \cdot 2\Omega = \frac{2}{3}V.$$

$$v_2 = \frac{2}{3}V + 2V = \frac{8}{3}V.$$

$$i_b = \frac{v_2}{1\Omega} = \frac{8}{3}A.$$

$$v_3 = v_2 - 4\Omega \cdot (-3)A = \frac{44}{3}V.$$

$$v_4 = 0 - 2\Omega \cdot 3A = -6V.$$

$$v_a = v_1 = \frac{2}{3}V.$$

$$v_b = v_2 = \frac{8}{3}V.$$

$$v_c = v_2 - v_3 = -12V.$$

$$v_d = v_3 - v_4 = \frac{62}{3}V.$$

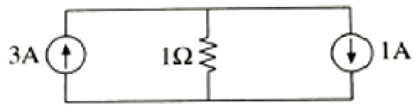
$$v_e = 0 - v_4 = 6V.$$

(b) Voltage drop from reference to node 4: $V_{drop} = v_e = 6V$.

(c) Voltage rise from node 2 to node 3: $V_{rise} = v_3 - v_2 = 12V$.

(d) Voltage drop from node 1 to reference: $V_{drop} = v_1 = \frac{2}{3}V$.

3. In the following circuit, one of the independent current sources is injecting energy into the circuit, while the other one is absorbing energy. Identify the source that is injecting the energy absorbed in the circuit and confirm that the sum of all absorbed powers equals to zero.



Solution:

By KCL, the current across the resister (from the top to the bottom) is: $I = 3A - 1A = 2A$.

Voltage drop across the resister: $V = 2A \cdot 1\Omega = 2V$.

Power of the left source: $P_{left} = 2V \cdot (-3)A = -6W$.

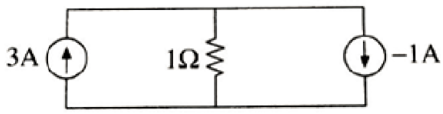
Power of the right source: $P_{right} = 2V \cdot 1A = 2W$.

Power absorbed by resister: $P_r = 2A \cdot 2V = 4W$.

Hence, the left current source is injecting energy.

The sum of all absorbed powers is $P_{left} + P_{right} + P_r = 0$.

4. Calculate the absorbed power for each element in the following circuit and determine which elements inject energy into the circuit.



Solution:

By KCL, the current across the resister (from the top to the bottom) is: $I = 3A - (-1A) = 4A$.

Voltage drop across the resister: $V = 4A \cdot 1\Omega = 4V$.

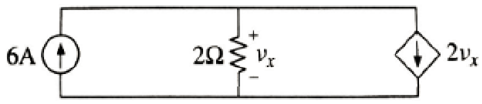
Power of the left source: $P_{left} = 4V \cdot (-3)A = -12W$.

Power of the right source: $P_{right} = 4V \cdot (-1A) = -4W$.

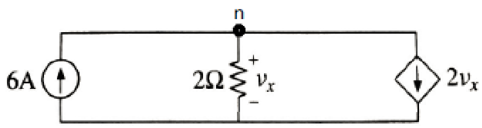
Power absorbed by resister: $P_r = 4A \cdot 4V = 16W$.

Hence, both current sources are injecting energy.

5. In the circuit given, determine v_x and calculate the absorbed power for each circuit element. Which element is injecting the energy absorbed in the circuit?



Solution: Applying KCL to node n.



$$6A = 2v_x + \frac{v_x}{2\Omega}$$

Solving the above equation, we get

$$v_x = \frac{12}{5}V$$

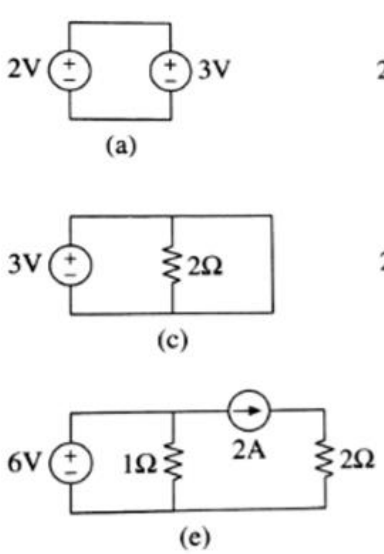
Power of the independent source: $P_{left} = v_x \cdot (-6)A = -\frac{72}{5}W$.

Power of the dependent source: $P_{right} = v_x \cdot 2 \cdot v_x = \frac{288}{25}W$.

Power of the resister $P_r = \frac{v_x}{2\Omega} \cdot v_x = \frac{72}{25}W$.

Hence, the independent source is injecting energy.

6. Some of the following circuits violate KVL/KCL and/or basic definitions of two-terminal elements given in Section 1.3. Identify these ill-specified circuits and explain the problem in each case.



Solution:

(a) Violates KVL.

(b) Correct.

(c) Violates KVL.

(d) Violates KCL.

(e) Correct.

7. (a) Let $A = 3 - j3$. Express A in exponential form.
   (b) Let $B = -1 - j1$. Express B in exponential form.
   (c) Determine the magnitudes of $A + B$ and $A - B$.
   (d) Express AB and A/B in rectangular form.

Solution:

(a) $|A| = \sqrt{3^2 + 3^2} = 3\sqrt{2}$. $\angle A = \tan^{-1}(\frac{-3}{3}) = -\frac{\pi}{4}$. $A = 3\sqrt{2}e^{-j\frac{\pi}{4}}$

(b) $|B| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$. $\angle B = \pi + \tan^{-1}(1) = \frac{5\pi}{4}$. $B = \sqrt{2}e^{j\frac{5\pi}{4}}$

(c) $|A + B| = |3 - j3 + (-1 - j1)| = |2 - j4| = \sqrt{2^2 + 4^2} = 2\sqrt{5}$
$|A - B| = |3 - j3 - (-1 - j1)| = |4 - j2| = \sqrt{4^2 + 2^2} = 2\sqrt{5}$

(d) $AB = (3 - j3)(-1 - j1) = -6$
$A/B = \frac{3-j3}{-1-j1} = \frac{(-1+j1)(3-j3)}{(-1+j1)(-1-j1)} = \frac{j6}{2} = j3$

8. (a) Determine the rectangular forms of $7e^{j\frac{\pi}{4}},\ 7e^{-j\frac{\pi}{4}},\ 5e^{j\frac{3\pi}{4}},\ 5e^{-j\frac{3\pi}{4}}$.

   (b) Simplify $P = 2e^{j\frac{5\pi}{4}} - 2e^{-j\frac{5\pi}{4}},\ Q = 8e^{-j\frac{\pi}{4}} - 8e^{j\frac{\pi}{4}}$, and $R = \frac{e^{j\frac{3\pi}{4}}}{e^{-j\frac{\pi}{4}}}$

Solution:

(a) $7e^{j\frac{\pi}{4}} = 7\cos\frac{\pi}{4} + j7\sin\frac{\pi}{4} = \frac{7\sqrt{2}}{2} + j\frac{7\sqrt{2}}{2}$
$7e^{-j\frac{\pi}{4}} = 7\cos(-\frac{\pi}{4}) + j7\sin(-\frac{\pi}{4}) = \frac{7\sqrt{2}}{2} - j\frac{7\sqrt{2}}{2}$
$5e^{j\frac{3\pi}{4}} = 5\cos\frac{3\pi}{4} + j5\sin\frac{3\pi}{4} = -\frac{5\sqrt{2}}{2} + j\frac{5\sqrt{2}}{2}$
$5e^{-j\frac{3\pi}{4}} = 5\cos(-\frac{3\pi}{4}) + j5\sin(-\frac{3\pi}{4}) = -\frac{5\sqrt{2}}{2} - j\frac{5\sqrt{2}}{2}$

(b) $P = 2(e^{j\frac{5\pi}{4}} - e^{-j\frac{5\pi}{4}}) = 2(\cos\frac{5\pi}{4} + j\sin\frac{5\pi}{4} - (\cos(-\frac{5\pi}{4}) + j\sin(-\frac{5\pi}{4}))) = 4\cdot j\sin\frac{5\pi}{4} = -j2\sqrt{2}$
$Q = 8e^{-j\frac{\pi}{4}} - 8e^{j\frac{\pi}{4}} = 8\cdot -j\sqrt{2} = -j8\sqrt{2}$
$R = e^{j(\frac{3\pi}{4}-(-\frac{\pi}{4}))} = e^{j\pi} = -1$