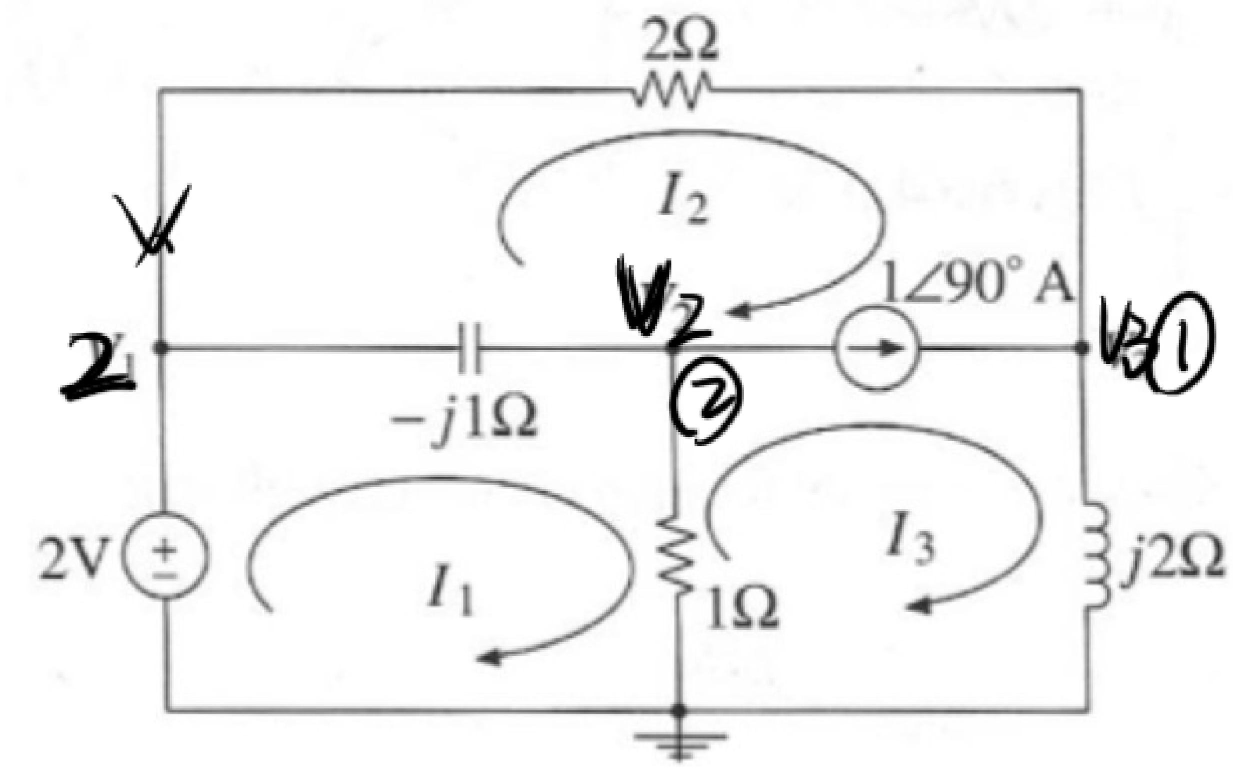
Homework 6

Due: Wednesday April 2, 2025, 11:59 pm

1. In the following circuit determine the node-voltage phasors V_1, V_2, and V_3 and express them in polar form. (20 points)



$$\begin{cases} \frac{V_2-2}{-j} + j + \frac{V_2}{1} = 0 \\ \frac{V_3-0}{2j} - j + \frac{V_3-2}{2} = 0 \end{cases}$$

$$\begin{cases} V_2 = -\frac{1}{2}(1+j) = \frac{\sqrt{2}}{2} e^{j\frac{\pi}{4}} = \frac{\sqrt{2}}{2}\angle 45^\circ \\ V_3 = 2j = 2e^{j\frac{\pi}{2}} = 2\angle 90^\circ \end{cases}$$

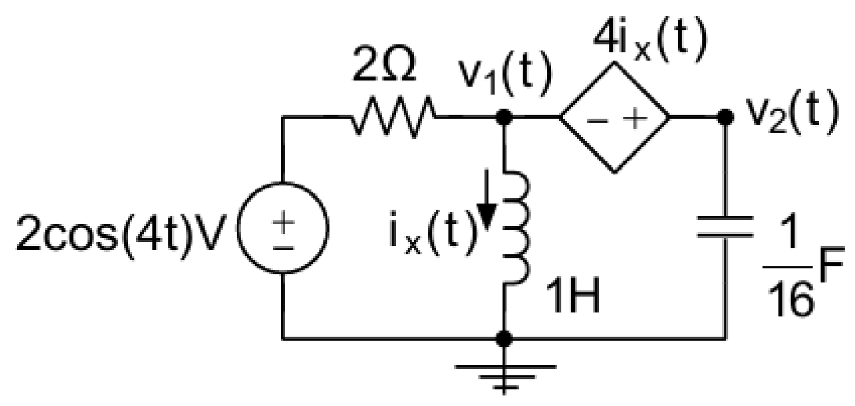
2. In the circuit shown for Problem 1, determine the loop-current phasors I_1, I_2, and I_3 and express them in polar form. (15 points)

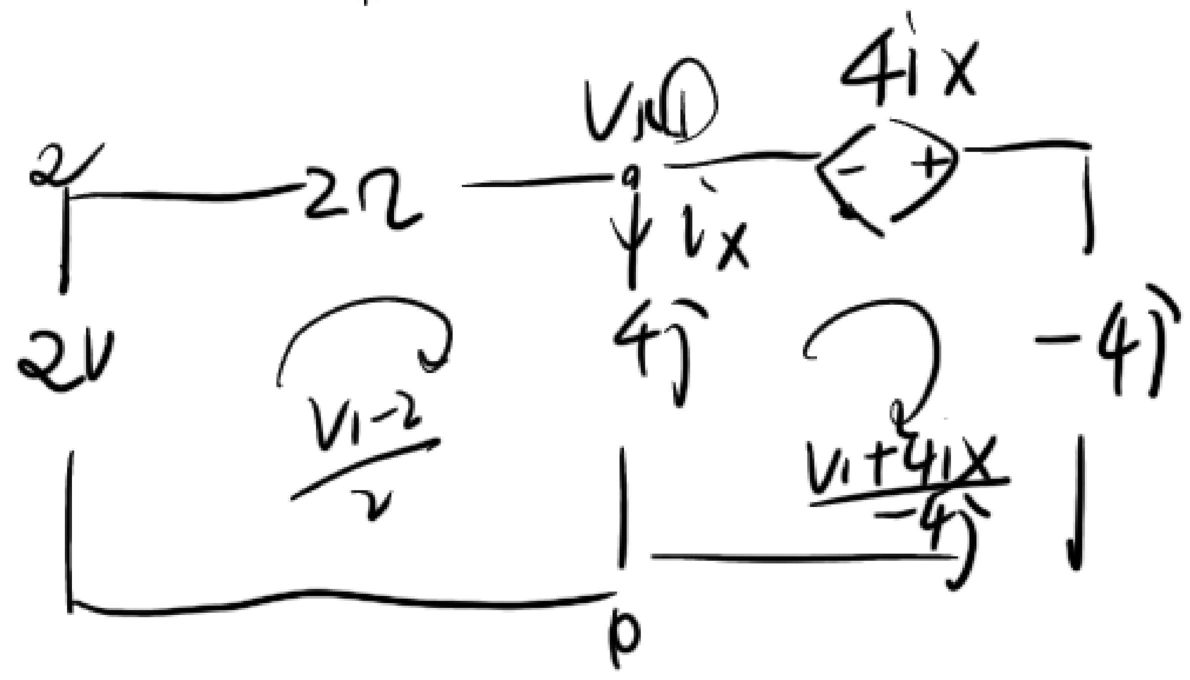
$$I_3 = \frac{V_3-0}{2j} = 1A = \angle 90^\circ$$

$$I_2 = -\left(\frac{V_3-2}{2}\right) = -(j-1) = \sqrt{2}\, e^{j\frac{5}{4}\pi} = \sqrt{2}\angle -45^\circ$$

$$I_1 = \frac{V_2-2}{-j} - I_2 = \frac{3+j}{2} = \frac{\sqrt{10}}{2} e^{j\arctan\frac{1}{3}} = \frac{\sqrt{10}}{2}\angle 18.4^\circ$$

3. Use the phasor method to determine $v_1(t)$ in the following circuit: (15 points)

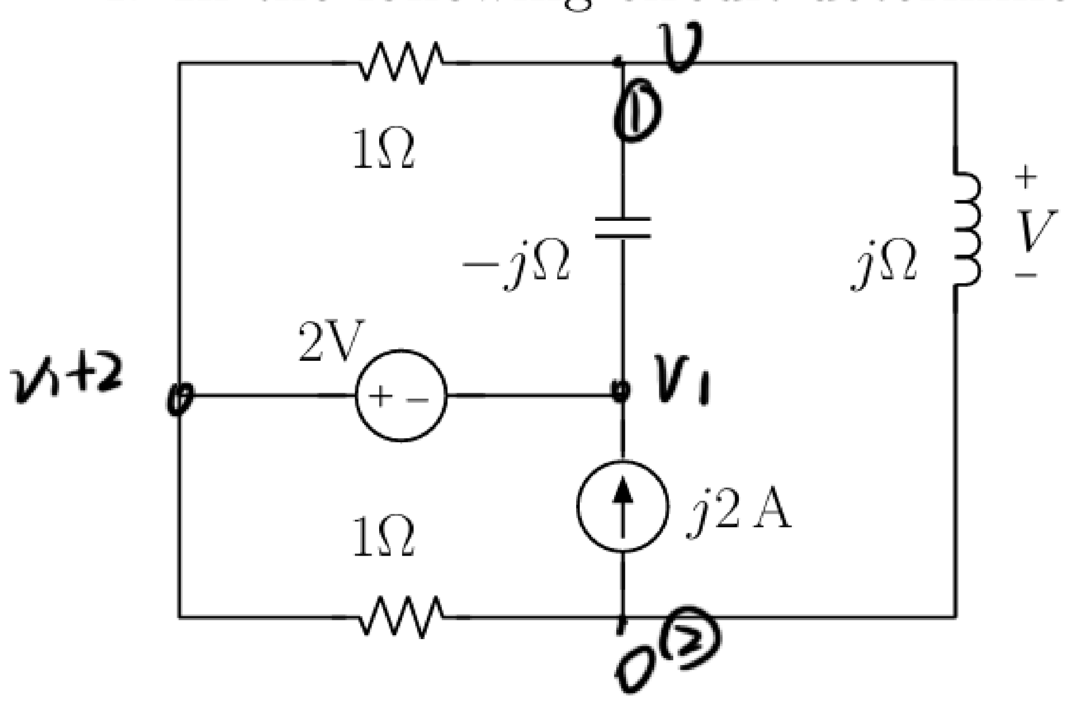




$$\frac{1}{j\omega C} = -4j$$

$$\begin{cases} \frac{V_1-2}{2} = 4j + \frac{V_1+4i_x}{-4j} \\ \frac{V_1-0}{4j} = i_x \end{cases} \Rightarrow V_1 = \frac{4}{3}V$$

4. In the following circuit determine the phasor V and express it in polar form: (15 points)

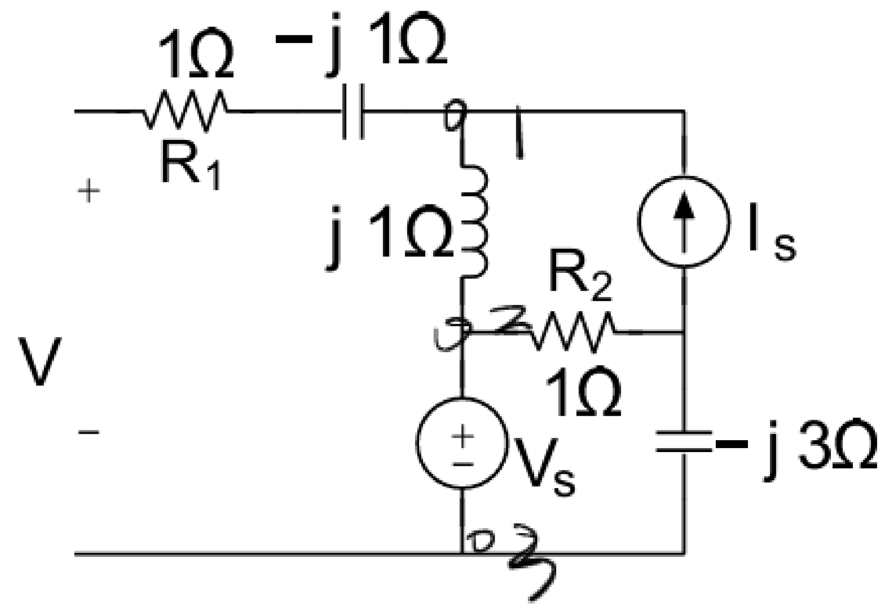


① $\frac{V-V_1}{-j} + \frac{V-0}{j} + \frac{V-V_1-2}{1} = 0 \Rightarrow V = \frac{-4-2j}{5}$

② $\frac{0-V_1-2}{1} + 2j + \frac{0-V}{+j} = 0$ $\quad = \frac{\sqrt{20}}{5} e^{j\arctan(\frac{1}{2}-\pi)}$

$= \frac{\sqrt{20}}{5} \angle -153.4°$

5. Use the following network to answer (a) through (d): (20 points)



(d) $V_T = 4-2j$ (V) $Z_T = 1(\Omega)$ $\quad I_N = 4-2j$ A

$P = \frac{|V_T|^2}{8R_T} = \frac{20}{8} = \frac{5}{2}$ W

(a) Determine the phasor V when $I_s = 0$.

(b) Determine the phasor V when $V_s = 0$.

(c) Determine V when $V_s = 4\,\text{V}$ and $I_s = -2\,\text{A}$, and calculate the average power absorbed in the resistors.

(d) What is the Thevenin equivalent and the available average power of the network when $V_s = 4\,\text{V}$ and $I_s = -2\,\text{A}$?

(a) $V = V_S$ ∵两头断路. $V_1 = V_2$

b) $V = V_1 - V_2 = I_S \cdot j$

c) $V_T = V_S + I_S j = (4-2j)$

$R_T = 1\Omega$.

$P_{R_1} = 0$ ∵ No V_1

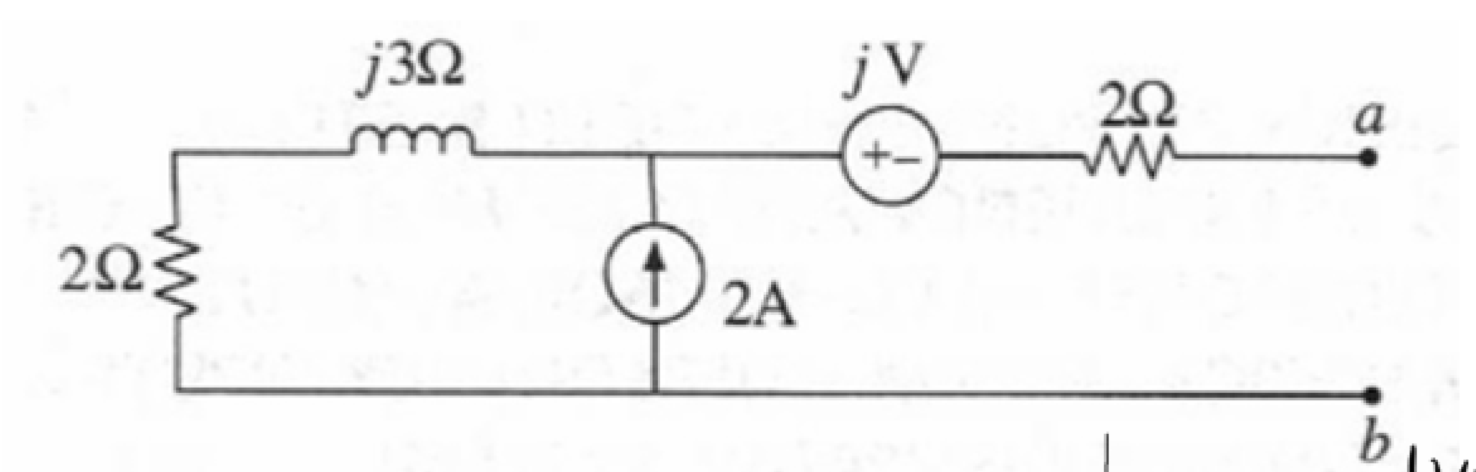
$-2 = \frac{V_1-4}{1} + \frac{V_1-0}{-3j}$

$V_{R_2} = \frac{7-9j}{5}$

$\therefore P_{R_2} = \frac{|V_{R_2}|^2}{2R}$

$= \frac{\left(\frac{\sqrt{130}}{1\times 5}\right)^2}{2} = \frac{130}{50} = \frac{13}{5}$ W

6. Determine the impedance Z_L of a load that is matched to the following network at terminals a and b, and determine the net power absorbed by the matched load: (15 points)



$V_T = +2(2+3j) - j = 4+5j$

$j - (2-i)(3j+2) - 2i = 0$

$\Rightarrow i_N = \frac{5j+4}{3j+4}$

$\Rightarrow Z_T = 3j+4$

$Z_L = 4-3j$

$P_a = \frac{|V_T|^2}{8R_T}$

$= \frac{16+25}{8\times 4}$

$= \frac{41}{32}$ W