Student No.:

Group A

For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties.						
1.	$y_2(t_0)$ for some	$t_0 \in I$?		as $y_1, y_2 \colon I \to \mathbb{R}$ sati $ y' = \sqrt{y}$		
2.	$y dx + (x \ln x - x)$	$(xy^2) dy = 0$ has the	integrating factor	<u></u> у	1/y	
3.	For the solution $\frac{4}{3}e^{-1}$		$= 2y - t \wedge y(0) = 1$ $\qquad \qquad $	I the value $y\left(-\frac{1}{2}\right)$ is	equal to	
4.	For the solution $\sqrt{2}$			the value $y\left(\frac{\pi}{2}\right)$ is equal to $\frac{1}{2}\sqrt{3}$	al to	
5.	of $\mathbb{R}^{\mathbb{R}}$?			tions $\phi \colon \mathbb{R} \to \mathbb{R} \text{ not } f$ $y'' = y' + ty$		
6.		ce (over \mathbb{R}) consister that dimension $\boxed{3}$		s in $\mathbb{R}^{3\times3}$ with all results in \mathbb{R}^{6}	ow and column	
7.		n=1		able on $\mathbb R$ if and only		
8.		~		$a \ge 3$ niformly on \mathbb{R} ?	a ≥ 4	
	$\int f_n(x) = x/n$ $\int f_n(x) = 1/n$		$\int_{0}^{\infty} f_n(x) = e^{-x}/n^2$ $\int_{0}^{\infty} f_n(x) = 1/(n + 1)$		$\int f_n(x) = x/n^4$	
9.		$z' = \frac{x+y}{x-y}$ then $z = y/z$ $z' = \frac{1+z}{1-z}$				
10.	Banach's Fixed open	Point Theorem ap	plies to contraction bounded	as on any subset of \mathbb{R} unbounded	n that is empty	

11. A particular solution of $y'' - 4y' + 3y = 0$ is						
$y(t) \equiv 1$ $y(t) \equiv 3$ $y(t) = e^{-t}$	$y(t) = \cos(3t) \qquad y(t) = \sin(3t)$					
12. The solution of $y' = y^2 + 2y$ with $y(0) = -1$ satisfies						
	$-2 \qquad \boxed{y(1) = -1} \qquad \boxed{\lim_{t \to +\infty} y(t) = 0}$					
$\lim_{t \to +\infty} y(t) = +\infty$						
13. The matrix norm of $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (subordinate t	to the Euclidean length on \mathbb{R}^2) is equal to					
01	$\boxed{\sqrt{2}}$ $\boxed{}$ 2					
14. The sequence $\phi_0, \phi_1, \phi_2, \dots$ of Picard-Linde $\phi_2(t)$ equal to	löf iterates for the IVP $y' = y^2 \wedge y(0) = 1$ has					
	$-t^2 + \frac{1}{3}t^3 \qquad \qquad \boxed{1 + t + \frac{1}{2}t^2 + \frac{1}{3}t^3}$					
$\boxed{1+t+\frac{1}{2}t^2+t^3} \qquad \boxed{1+t}$						
15. The rank of $\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$ over the bins	ary field \mathbb{F}_2 is equal to					
1 2	3 4 7					
Time allowed: 45 min CLOS	SED BOOK Good luck!					

Student No.: ____ Group A Name: __ For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties. 1. Which of the following ODE's has distinct solutions $y_1, y_2 : I \to \mathbb{R}$ satisfying $y_1(t_0) =$ $y_2(t_0)$ for some $t_0 \in I$? y' = y y' = |y| $y' = y^2$ $y' = \sqrt{y}$ y' = 1/y2. $y dx + (x \ln x - xy^2) dy = 0$ has the integrating factor 1/y3. For the solution y(t) of the IVP $y' = 2y - t \wedge y(0) = 1$ the value $y\left(-\frac{1}{2}\right)$ is equal to $\frac{3}{3}e^{-1}$ $\frac{3}{2}e^{-1}$ $\frac{3}{4}e^{-1}$ 4. For the solution y(t) of the IVP $y' = \frac{\cos t}{y} \wedge y(0) = 1$ the value $y\left(\frac{\pi}{2}\right)$ is equal to 2 5. For which of the following ODE's does the set of solutions $\phi \colon \mathbb{R} \to \mathbb{R}$ <u>not</u> form a subspace of $\mathbb{R}^{\mathbb{R}}$? y' = ty y' = t + y y'' = y' y'' = y' + ty y'' = 06. The vector space (over \mathbb{R}) consisting of the matrices in $\mathbb{R}^{3\times3}$ with all row and column sums equal to zero has dimension 7. The function $g(x) = \sum_{n=1}^{\infty} \frac{\sin(nx)}{n^a}$, $a \in \mathbb{Z}$, is differentiable on \mathbb{R} if and only if $a \ge 1$ 8. For which choice of $f_n(x)$ does $\sum_{n=1}^{\infty} f_n(x)$ converge uniformly on \mathbb{R} ? $\int f_n(x) = x/n$ $\int f_n(x) = e^{-x}/n^2$ $\int f_n(x) = 1/(n^2 + x^2)$ $\int f_n(x) = 1/(n + x^4)$ $\int f_n(x) = x/n^4$ $f_n(x) = x/n$ 9. If y(x) solves $y' = \frac{x+y}{x-y}$ then z = y/x solves $z' = \frac{1+z^2}{(1-z)x} \qquad \qquad z' = \frac{1+z}{1-z} \qquad \qquad z' = z/x \qquad \qquad z' = -z/x \qquad \qquad z' = \frac{1+z^2}{1-z}$ 10. Banach's Fixed Point Theorem applies to contractions on any subset of \mathbb{R}^n that is

bounded

open

closed

empty

unbounded

11. A particular solution of $y'' - 4y' + 3y = $ $y(t) \equiv 1 \qquad y(t) \equiv 3 \qquad y(t)$		$y(t) = \sin(3t)$
12. The solution of $y' = y^2 + 2y$ with $y(0)$ $\lim_{t \to +\infty} y(t) = -\infty$ $\lim_{t \to +\infty} y(t) = +\infty$		$\lim_{t\to+\infty}y(t)=0$
13. The matrix norm of $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (subording)	inate to the Euclidean length $\sqrt{2}$	on \mathbb{R}^2) is equal to 2
	Lindelöf iterates for the IVP $1 + t + t^2 + \frac{1}{3}t^3$ $1 + t$	$y' = y^2 \wedge y(0) = 1$ has
15. The rank of $\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$ over the second se	ne binary field \mathbb{F}_2 is equal to	47
16. This midterm exam was too easy too difficult	too long too short	just appropriate
Time allowed: 45 min	CLOSED BOOK	Good luck!