Math 286 Differential Equations Plus (Prof. Honold) **Midterm Exam** 2021/04/01

Name: ____________________ Student No.: ____________________ Group A

For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties.

1. Which of the following ODE's has distinct solutions $y_1, y_2 \colon I \to \mathbb{R}$ satisfying $y_1(t_0) = y_2(t_0)$ for some $t_0 \in I$?

   ☐ $y' = \sin(t\,y^2)$ ☐ $yy' = 0$ ☐ $y' = |ty|$ ☐ $y' = y\sqrt{t}$ ☐ $y' = \sqrt{|ty|}$

2. $(x^4y^2 - y)\,\mathrm{d}x + (x^2y^4 - x)\,\mathrm{d}y = 0$ has the integrating factor

   ☐ $1/(xy)$ ☐ 1 ☐ $1/(xy)^2$ ☐ $1/(xy^2)$ ☐ $1/(x^2y)$

3. The family of curves $y = c/x^2$, $c \in \mathbb{R}$ satisfies the ODE

   ☐ $\mathrm{d}y = x^{-2}\,\mathrm{d}x$ ☐ $\mathrm{d}y = 2x^{-3}\,\mathrm{d}x$ ☐ $2xy\,\mathrm{d}x + x^2\,\mathrm{d}y = 0$ ☐ $\mathrm{d}x = \mathrm{d}y$
   ☐ $2yx^{-3}\,\mathrm{d}x - x^{-2}\,\mathrm{d}y = 0$

4. For the solution $y(t)$ of the IVP $y' = y^3 - 7y + 6$, $y(0) = 0$ the limit $\lim_{t\to+\infty} y(t)$ equals

   ☐ -3 ☐ -2 ☐ -1 ☐ 1 ☐ 2

5. For the solution $y(t)$ of the IVP $y' = (y/t) - 1$, $y(1) = \ln 2$ the value $y(2)$ is equal to

   ☐ 0 ☐ 1 ☐ 2 ☐ $\ln 2$ ☐ $2\ln 2$

6. For the solution $y(t)$ of the IVP $y' = y^2\mathrm{e}^{-t}$, $y(0) = -1$ the value $y(-1)$ is equal to

   ☐ e ☐ $1/(\mathrm{e}-2)$ ☐ $\mathrm{e}+2$ ☐ $1/(\mathrm{e}+2)$ ☐ $\mathrm{e}-2$

7. For the solution $y \colon (0, +\infty) \to \mathbb{R}$ of the IVP $t^2y'' + 2t\,y' - 2y = 1$, $y(1) = 0$, $y'(1) = 1$ the value $y(2)$ is equal to

   ☐ $\frac{13}{8}$ ☐ 1 ☐ $\frac{13}{24}$ ☐ $\frac{19}{8}$ ☐ $\frac{19}{24}$

8. The power series $\sum_{k=1}^{\infty} 2^k z^{k^2}$ has radius of convergence

   ☐ 0 ☐ $\frac{1}{2}$ ☐ 1 ☐ 2 ☐ ∞

9. The smallest integer s such that $f_s(x) = \sum_{k=1}^{\infty} \frac{\cos(k^2x)}{k^s}$ is differentiable on $\mathbb{R}$ is equal to

   ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

10. If $y(t)$ solves $y' = t^2y + t\,y^2$ then $z = 1/y(t)$ solves

    ☐ $z' = -t^2z$ ☐ $z' = -z^2 + t$ ☐ $z' = t^2/z + t/z^2$ ☐ $z' = -z^2$
    ☐ $z' = -t^2z - t$

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11. The sequence $\phi_0, \phi_1, \phi_2, \dots$ of Picard-Lindelöf iterates for the IVP $y' = y + 2t,\ y(0) = -2$ has $\phi_2(t)$ equal to

☐ $-2 + 2t + \frac{1}{3}t^3$ ☐ $-t^2 + \frac{1}{3}t^3$ ☐ $-2 - 2t + \frac{1}{3}t^3$ ☐ $t^2 + \frac{1}{3}t^3$
☐ $1 + t + \frac{3}{2}t^2 + \frac{1}{3}t^3$

12. $y'' - 4y' + 4y = 2t + e^{2t}$ has a particular solution $y_p(t)$ of the form

☐ $c_0 + c_1 t + c_2 e^{2t}$ ☐ $c_0 + c_1 t + c_2 t^2 e^{2t}$ ☐ $c_0 t + c_1 t^2 e^{2t}$ ☐ $c_0 + c_1 t$
☐ $c_0 t + c_1 e^{2t}$

13. Maximal solutions of $y' = y^2 + y$ satisfying $y(0) > 0$ are defined on an interval of the form

☐ (a, b) ☐ $[a, b]$ ☐ $(a, +\infty)$ ☐ $(-\infty, b)$ ☐ $(-\infty, +\infty)$
with $a, b \in \mathbb{R}$.

14. For $\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$, the matrix $e^{\mathbf{A}t}$ is equal to

☐ $\begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix}$ ☐ $\begin{pmatrix} e^t & e^{-t} \\ e^t & e^{-t} \end{pmatrix}$ ☐ $\begin{pmatrix} e^t & e^t \\ e^{-t} & e^{-t} \end{pmatrix}$
☐ $\begin{pmatrix} 1+t & t \\ -t & 1-t \end{pmatrix}$ ☐ $\begin{pmatrix} 1+t & -t \\ t & 1-t \end{pmatrix}$

15. The matrix norm of $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ (subordinate to the Euclidean length on $\mathbb{R}^2$) is equal to

☐ 0 ☐ 1 ☐ $\sqrt{2}$ ☐ 2 ☐ 4

Time allowed: 60 min CLOSED BOOK ***Good luck!***