Name: ____________________ Student No.: ____________________ Group A

For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties.

1. Which of the following ODE's has distinct solutions $y_1, y_2 \colon I \to \mathbb{R}$ satisfying $y_1(t_0) = y_2(t_0)$ for some $t_0 \in I$?

   ☐ $y' = y$ ☐ $y' = |y|$ ☐ $y' = y^2$ ☐ $y' = \sqrt{y}$ ☐ $y' = 1/y$

2. $y\,\mathrm{d}x + (x\ln x - xy^2)\,\mathrm{d}y = 0$ has the integrating factor

   ☐ 1 ☐ x ☐ $1/x$ ☐ y ☐ $1/y$

3. For the solution $y(t)$ of the IVP $y' = 2y - t \wedge y(0) = 1$ the value $y\left(-\frac{1}{2}\right)$ is equal to

   ☐ $\frac{4}{3}\mathrm{e}^{-1}$ ☐ $\frac{3}{2}\mathrm{e}^{-1}$ ☐ $\frac{2}{3}\mathrm{e}^{-1}$ ☐ $\frac{3}{4}\mathrm{e}^{-1}$ ☐ e^{-1}

4. For the solution $y(t)$ of the IVP $y' = \frac{\cos t}{y} \wedge y(0) = 1$ the value $y\left(\frac{\pi}{2}\right)$ is equal to

   ☐ $\sqrt{2}$ ☐ $\frac{1}{2}\sqrt{2}$ ☐ $\sqrt{3}$ ☐ $\frac{1}{2}\sqrt{3}$ ☐ 2

5. For which of the following ODE's does the set of solutions $\phi \colon \mathbb{R} \to \mathbb{R}$ <u>not</u> form a subspace of $\mathbb{R}^{\mathbb{R}}$?

   ☐ $y' = ty$ ☐ $y' = t + y$ ☐ $y'' = y'$ ☐ $y'' = y' + ty$ ☐ $y'' = 0$

6. The vector space (over $\mathbb{R}$) consisting of the matrices in $\mathbb{R}^{3\times 3}$ with all row and column sums equal to zero has dimension

   ☐ 2 ☐ 3 ☐ 4 ☐ 6 ☐ 7

7. The function $g(x) = \sum_{n=1}^{\infty} \dfrac{\sin(nx)}{n^a}$, $a \in \mathbb{Z}$, is differentiable on $\mathbb{R}$ if and only if

   ☐ $a \geq 0$ ☐ $a \geq 1$ ☐ $a \geq 2$ ☐ $a \geq 3$ ☐ $a \geq 4$

8. For which choice of $f_n(x)$ does $\sum_{n=1}^{\infty} f_n(x)$ converge uniformly on $\mathbb{R}$?

   ☐ $f_n(x) = x/n$ ☐ $f_n(x) = \mathrm{e}^{-x}/n^2$ ☐ $f_n(x) = x/n^4$
   ☐ $f_n(x) = 1/(n^2 + x^2)$ ☐ $f_n(x) = 1/(n + x^4)$

9. If $y(x)$ solves $y' = \frac{x+y}{x-y}$ then $z = y/x$ solves

   ☐ $z' = \frac{1+z^2}{(1-z)x}$ ☐ $z' = \frac{1+z}{1-z}$ ☐ $z' = z/x$ ☐ $z' = -z/x$ ☐ $z' = \frac{1+z^2}{1-z}$

10. Banach's Fixed Point Theorem applies to contractions on any subset of $\mathbb{R}^n$ that is

    ☐ open ☐ closed ☐ bounded ☐ unbounded ☐ empty

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11. A particular solution of $y'' - 4y' + 3y = 0$ is

☐ $y(t) \equiv 1$ ☐ $y(t) \equiv 3$ ☐ $y(t) = \mathrm{e}^{3t}$ ☐ $y(t) = \cos(3t)$ ☐ $y(t) = \sin(3t)$

12. The solution of $y' = y^2 + 2y$ with $y(0) = -1$ satisfies

☐ $\lim_{t\to+\infty} y(t) = -\infty$ ☐ $\lim_{t\to+\infty} y(t) = -2$ ☐ $y(1) = -1$ ☐ $\lim_{t\to+\infty} y(t) = 0$

☐ $\lim_{t\to+\infty} y(t) = +\infty$

13. The matrix norm of $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (subordinate to the Euclidean length on $\mathbb{R}^2$) is equal to

☐ 0 ☐ 1 ☐ $\sqrt{2}$ ☐ 2 ☐ 4

14. The sequence $\phi_0, \phi_1, \phi_2, \dots$ of Picard-Lindelöf iterates for the IVP $y' = y^2 \wedge y(0) = 1$ has $\phi_2(t)$ equal to

☐ $1 + t + t^2 + t^3$ ☐ $1 + t + t^2 + \frac{1}{3}t^3$ ☐ $1 + t + \frac{1}{2}t^2 + \frac{1}{3}t^3$

☐ $1 + t + \frac{1}{2}t^2 + t^3$ ☐ $1 + t$

15. The rank of $\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$ over the binary field $\mathbb{F}_2$ is equal to

☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 7

Time allowed: 45 min CLOSED BOOK ***Good luck!***

Math 286 Differential Equations (Prof. Honold) **Midterm Exam** 2018/03/28

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16. This midterm exam was

☐ too easy ☐ too difficult ☐ too long ☐ too short ■ just appropriate

Time allowed: 45 min CLOSED BOOK ***Good luck!***