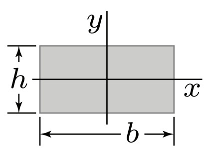
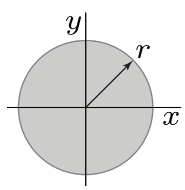
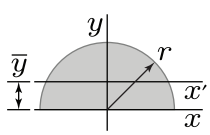
# TAM 251 EQUATION SHEET

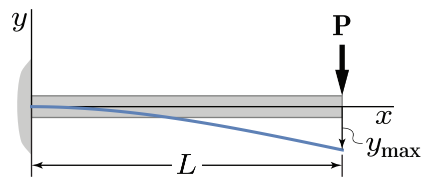
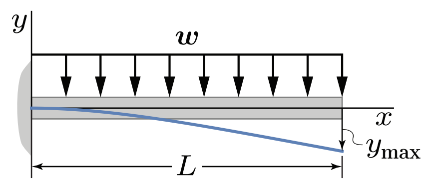
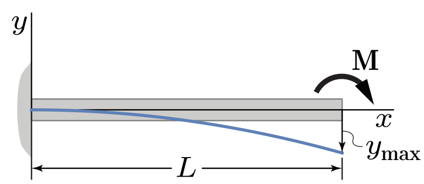
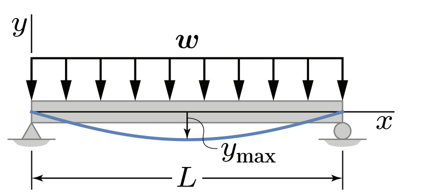
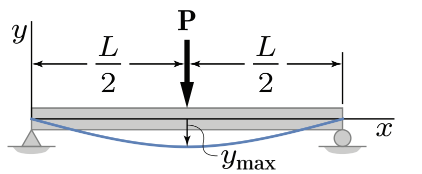
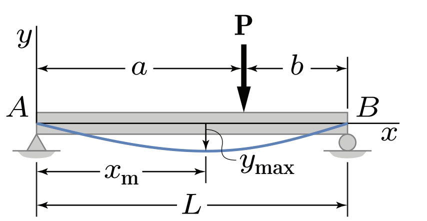
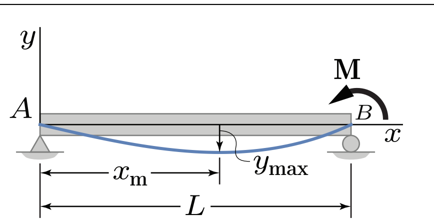
| Main Equations | |
|---|---|
| Stress | $\sigma_{avg} = \dfrac{F}{A}$ $\qquad \tau_{avg} = \dfrac{V}{A}$ |
| Strain | $\epsilon_{eng} = \dfrac{\delta}{L_0}$ $\qquad \epsilon_{true} = \ln \dfrac{L_f}{L_0}$ $\qquad \gamma = \dfrac{\delta_x}{L_y} + \dfrac{\delta_y}{L_x}$ |
| Constitutive Relations | $\sigma = E\epsilon$ $\qquad \tau = G\gamma$ |
| Material Properties (Isotropic) | $\nu = -\dfrac{\epsilon_{lat}}{\epsilon_{long}}$ $\qquad G = \dfrac{E}{2(1+\nu)}$ |
| Thermal Expansion | $\epsilon_{th} = \alpha \Delta T$ $\qquad \delta_{th} = \alpha L_0 \Delta T$ |
| Axial Loading | $\delta = \dfrac{F L_0}{E A}$ $\qquad \sigma = \dfrac{F}{A}$ |
| Torsion | $\phi = \dfrac{T L_0}{G J}$ $\qquad \tau = \dfrac{T \rho}{J}$ $\qquad \gamma = \dfrac{\phi \rho}{L_0}$ |
| Stiffness and Flexibility | $k_{\text{axial}} = \dfrac{E A}{L_0} = \dfrac{1}{f_{\text{axial}}}$ $\qquad k_{\text{torsion}} = \dfrac{G J}{L_0} = \dfrac{1}{f_{\text{torsion}}}$ |
| Bending | $\sigma = \dfrac{M c}{I}$ |
| Thin-Walled Pressure Vessels | $\sigma_h = \dfrac{p r}{t}$ $\qquad \sigma_a = \dfrac{p r}{2t}$ |
| Transverse Shear | $\tau = \dfrac{V Q}{I t}$ $\qquad q = \dfrac{V Q}{I}$ |

| Miscellaneous | | |
|---|---|---|
| Distributed Loads, Shear, & Bending Moments | $\dfrac{dV}{dx} = -w$ | $\dfrac{dM}{dx} = V$ |
| Inclined Plane: Normal Stress | $\sigma_n = \sigma_x \cos^2\theta + 2\tau_{xy}\sin\theta\cos\theta + \sigma_y \sin^2\theta$ | |
| Inclined Plane: Shear Stress | $\tau_{n,s} = (\sigma_y - \sigma_x)\sin\theta\cos\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)$ | |
| Tresca Criterion | $\lvert\sigma_1\rvert = \sigma_{\text{yield}},\ \ \lvert\sigma_2\rvert = \sigma_{\text{yield}}$ | when $\sigma_1$, $\sigma_2$ have the same sign |
| | $\lvert\sigma_1 - \sigma_2\rvert = \sigma_{\text{yield}}$ | when $\sigma_1$, $\sigma_2$ have the opposite sign |
| Von-Mises Criterion | $\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{yield}}^2$ | |

| Stress Transformations | | | |
|---|---|---|---|
| Plane Stress | $\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$ | | |
| | $\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)$ | | |
| | $\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta)$ | | |
| Mohr's Circle | $\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$ | $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$ | |
| Principal Stresses | $\sigma_1 = \sigma_{avg} + R$ | $\sigma_2 = \sigma_{avg} - R$ | $\tau_{max} = R$ |
| Plane Orientations | $\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$ | $\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$ | |

| Moments and Geometric Centroids | | | | | |
|---|---|---|---|---|---|
| | $Q = \overline{y}A$ | $I_x = \int_A y^2\, dA$ | $J_o = \int_A \rho^2\, dA$ | $\overline{y} = \frac{1}{A}\int_A y\, dA$ | |
| Rectangle |   | $I_x = \frac{1}{12}bh^3$ | | | |
| Circle |   | $I_x = \frac{\pi}{4}r^4$ | $J_z = \frac{\pi}{2}r^4$ | | |
| Semicircle |   | $I_x = \frac{\pi}{8}r^4$ | $I_{x'} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4$ | $\overline{y} = \frac{4r}{3\pi}$ | |
| Parallel Axis Theorem | $I_c = I_{c'} + Ad_{cc'}^2$ | | | | |

| Buckling | | | |
|---|---|---|---|
| Critical Load | $P_{cr} = \frac{\pi^2 EI}{(L_e)^2}$ | pinned-pinned | $L_e = L$ |
| | | pinned-fixed | $L_e = 0.7L$ |
| | | fixed-fixed | $L_e = 0.5L$ |
| | | fixed-free | $L_e = 2L$ |

| Beam Deflection | | | |
|---|---|---|---|
| Diagram | Max. Deflection | Slope at End | Elastic Curve |
| | $y_{max}$ | $\theta$ | $y(x)$ |
|   | $-\dfrac{PL^3}{3EI}$ | $-\dfrac{PL^2}{2EI}$ | $y(x) = \dfrac{P}{6EI}\left(x^3 - 3Lx^2\right)$ |
|   | $-\dfrac{wL^4}{8EI}$ | $-\dfrac{wL^3}{6EI}$ | $y(x) = -\dfrac{w}{24EI}\left(x^4 - 4Lx^3 + 6L^2x^2\right)$ |
|   | $-\dfrac{ML^2}{2EI}$ | $-\dfrac{ML}{EI}$ | $y(x) = -\dfrac{M}{2EI}x^2$ |
|   | $-\dfrac{5wL^4}{384EI}$ | $\pm\dfrac{wL^3}{24EI}$ | $y(x) = -\dfrac{w}{24EI}\left(x^4 - 2Lx^3 + L^3x\right)$ |
|   | $-\dfrac{PL^3}{48EI}$ | $\pm\dfrac{PL^2}{16EI}$ | For $0 \le x \le \frac{L}{2}$<br>$y(x) = \dfrac{P}{48EI}\left(4x^3 - 3L^2x\right)$ |
|   | For $a > b$<br>$-\dfrac{Pb\left(L^2 - b^2\right)^{3/2}}{9\sqrt{3}EIL}$<br>at $x_m = \sqrt{\dfrac{L^2 - b^2}{3}}$ | $\theta_A = -\dfrac{Pb\left(L^2 - b^2\right)}{6EIL}$<br>$\theta_B = +\dfrac{Pa\left(L^2 - a^2\right)}{6EIL}$ | For $x < a$:<br>$y(x) = \dfrac{Pb}{6EIL}\left[x^3 - x\left(L^2 - b^2\right)\right]$<br>For $x = a$:<br>$y = -\dfrac{Pa^2b^2}{3EIL}$ |
|   | $-\dfrac{ML^2}{9\sqrt{3}EI}$<br>at $x_m = \dfrac{L}{\sqrt{3}}$ | $\theta_A = -\dfrac{ML}{6EI}$<br>$\theta_B = +\dfrac{ML}{3EI}$ | $y(x) = \dfrac{M}{6EIL}\left(x^3 - L^2x\right)$ |