

# Ch1-3 Viscosity and Surface Tension

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## Objectives:

- Viscosity (Apply and Analyze)
- Surface tension (Identify and Explain)

Reading: 1.6, 1.9

For solid body,  
 $\tau = \gamma G, \tau \sim \gamma$

Recall: "Big Idea"

$$\sum \underset{\substack{\uparrow \\ \text{pressure} \\ \text{gravity} \\ \text{viscous effect} \\ \text{surface tension}}}{F} = m \underset{\substack{\uparrow \\ \text{Acceleration} \\ \text{or} \\ \text{Inertia}}}{a}$$

Poll: Dimensions of  $\mu$ ?

- A)  $F L^2 T^{-1}$
- ★ B)  $F L^{-2} T$
- C)  $F^{-1} L^2 T$
- D)  $F^{-1} L^2 T^{-1}$

$$\mu = \frac{\tau}{\dot{\gamma}} \doteq \frac{F/L^2}{\frac{1}{T}} \doteq \frac{F}{L^2} \cdot T \doteq F L^{-2} T$$

Unit (SI):  $\frac{N}{m^2} \cdot s \doteq Pa \cdot s$

For a fluid, shear stress due to viscosity:

$$\tau = \mu \frac{du}{dy} = \mu \dot{\gamma}$$

where  $\dot{\gamma} = \frac{du}{dy}$  is shear strain rate or shear rate.  
 time rate of the change in shear strain!

Poll: Dimensions of  $\dot{\gamma}$ ?

- A)  $L/T$
- B)  $L^2/T$
- C)  $L/T^2$
- ★ D)  $1/T$

$$\dot{\gamma} = \frac{du}{dy} \doteq \frac{L/T}{L} \doteq \frac{1}{T}$$

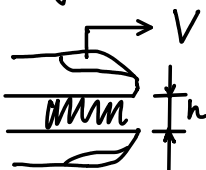
The unit is "inverse second"

$$\dot{\gamma} = \frac{\text{change in strain}}{\text{change in time}} \doteq \frac{1}{T}$$

Not Hz.

Example: honey shear force.

Given Honey, b/t thumb and forefinger. Newtonian fluid.

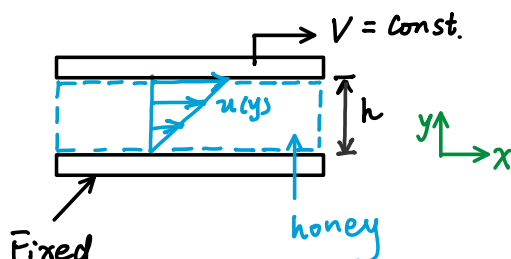


honey area: A

Find Estimate shear force

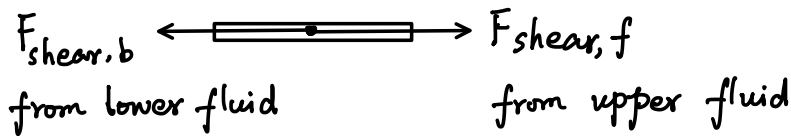
Solution

Model



FBD for a slice of honey.

x direction: area:  $A$



"Big Idea"  $\sum \vec{F} = m \vec{a}$   
 $\uparrow$  viscous effect  $\uparrow$  0

Shear force:

$$\vec{F} = \tau A \quad \dot{\gamma} = \frac{du}{dy} = \frac{v-0}{h-0} = \frac{v}{h}$$

$$= (\mu \dot{\gamma}) A$$

$$= \left( \mu \frac{v}{h} \right) A$$

$$\text{So: } \boxed{\vec{F} = \mu \frac{v}{h} A}$$

\* Sanity check.  $\mu \frac{v}{h} A$

\* Dimension check  $\doteq (FL^{-2}T) \left( \frac{L}{T} \right) L^2$

\* Unit check.  $\doteq FL^{-2+1-1+2} T^{-1+1} \doteq FL^0 T^0 \doteq F$

W/ numbers:

$$\mu \approx 10 \text{ Pa}\cdot\text{s}, \quad A \approx 1 \text{ cm}^2, \quad v \approx 2 \frac{\text{cm}}{\text{s}}, \quad h \approx 0.1 \text{ mm}$$

$$\vec{F} = \left( 10 \frac{\text{N}}{\text{m}^2} \cdot \text{s} \right) \frac{\left( 2 \cdot 10^{-2} \frac{\text{m}}{\text{s}} \right)}{0.1 \cdot 10^{-3} \text{ m}} 1 \cdot 10^{-4} \text{ m}^2$$

$$= 0.2 \text{ N}$$

$$1 \text{ N} \approx 0.225 \text{ lb}$$

(lbf)

$$0.2 \text{ N} \approx 0.045 \text{ lb}$$

↗ (lbf)

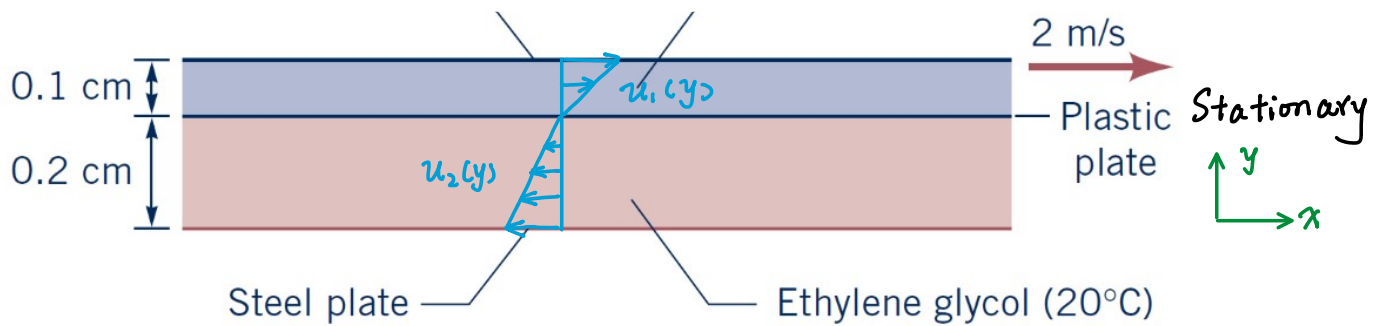
Roughly the weight of two cherries.

Example: HW Problem 1.74

Three large plates are separated by thin layers of ethylene glycol and water, as shown in the figure below. The top plate moves to the right at 2 m/s. At what speed and in what direction must the bottom plate be moved to hold the center plate stationary?

Copper plate

Water (20°C)



### Solution

$x$  direction.

FBD for middle plate:

$F_{\text{shear, eg}}$  from EG flow

$F_{\text{shear, w}}$  from water flow

"Big Idea"  $\sum F = ma$

viscous effect  $\uparrow$

$0$

$$\text{Thus, } F_{\text{shear, w}} - F_{\text{shear, eg}} = 0$$

$$F_{\text{shear, w}} = F_{\text{shear, eg}}$$

$$F_{\text{shear, w}} = \tau_w A = \left( \mu_w \frac{du_1}{dy} \right) A$$

$$F_{\text{shear, eg}} = \tau_{eg} A = \left( \mu_{eg} \frac{du_2}{dy} \right) A$$

Stop here. Students need to move ahead by themselves.

### Surface Tension $\sigma$

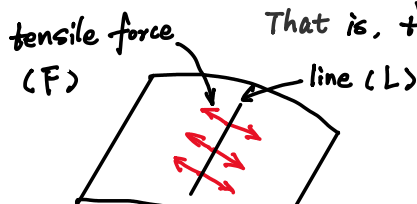
What is surface tension? Show two youtube videos.



Key idea:

- \* "Surface": interface b/t two immiscible fluids, like air-and-water,
- \* "Tension": tensile force acting in the plane of surface along any line in the surface.

That is, the surface is like a stretched membrane.



"stretched membrane"



\* Surface tension is the intensity of the molecular attraction per unit length along any line in the surface.

Poll: Dimensions of surface tension?

A)  $F \cdot L$

B)  $F \cdot L^2$

★ C)  $F/L$

D)  $F/L^2$

So in other words, surface tension is the tensile force per unit length along any line in the surface (interface).