Ch1-4 Surface Tension

Objectives: Reading: 1.9

- Surface Tension ⟨Apply and Analyze⟩

Recall:

Surface Tension σ

* "Surface": interface.
* "Tension": tensile force along any line in the surface.
* Intensity of molecular attraction (tensile force) per unit length along any line in the surface.

Poll. Which is the SI unit of Surface tension?

A) N·m

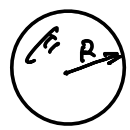
B) $N \cdot m^2$

C) N/m

D) N/m^2

Example. Pressure in a droplet.

$\boxed{\text{Given}}$ Spherical water droplet in air



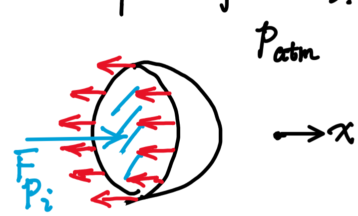
$\boxed{\text{Find}}$ Pressure inside due to Surface tension. (Neglect gravity)

$\boxed{\text{Solution}}$

Big Idea. $\Sigma F = ma$ · ~~Acce.~~

- Pressure
- ~~gravity~~
- ~~Viscous~~ effect
- Surface tension

Cut the sphere for FBD.



$$\Sigma F_x = m \cancel{a_x}^{\,0}$$

$$p_i \cdot (\pi R^2) - \sigma \cdot (2\pi R) = 0 \Longrightarrow \boxed{p_i = \frac{2\sigma}{R}}$$

Gage pressure   Force per unit length   Length

$\boxed{\text{Comments}}$

Analogy with temperature in K and °C.

p_{abs} is relative to vacuum, like T in K relative to 0 K.

p_{gage} is relative to p_{atm}, like T in °C relative to 273.15 K.

* $p_i = p_{i,abs} - p_{atm,abs}$

Gage pressure is usually used in fluid mechanics.

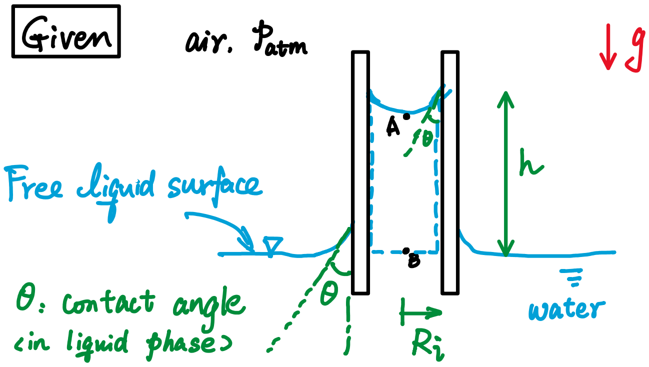
For the gage pressure, $p_{atm} = 0$, so the force effect due to p_{atm}

usually does not show up in the analysis in fluid mechanics.

* $p_i \sim \sigma$
* $p_i \sim 1/R$: A smaller droplet has larger p_i.

Example: Capillary Rise.

$\boxed{\text{Given}}$



$\theta < 90°$, "hydrophilic"

$\theta > 90°$, "hydrophobic"

Poll: Which one has a larger pressure?

A) A

B) B

$p_B = p_{atm}$, $p_A < p_{atm}$.

(See Ch1-4 Supplements)

Or

Poll: compare p_A and p_{atm}

A) $p_A = p_{atm}$

B) $p_A > p_{atm}$

C) $p_A < p_{atm}$

$\boxed{\text{Find}}$ Height h.

$\boxed{\text{Solution}}$

Big Idea

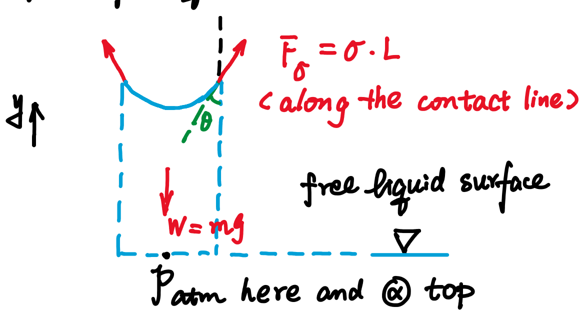
$$\Sigma F = m\cancel{a}^{\,0}$$

- pressure ?
- gravity
- ~~viscous~~ viscous effect
- Surface tension

· Acce

Gravity is balanced by surface tension.

FBD for liquid column.



$$\Sigma F_y = m a_y$$

$$F_\sigma \cos\theta - W = 0$$

$m = \forall \rho$

$$\sigma \cdot 2\pi R_i \cos\theta - mg = 0$$

$$2\pi \sigma R_i \cos\theta - \pi R_i^2 h \rho g = 0$$

$$2\sigma \cos\theta - R_i h \rho g = 0$$

$$\boxed{h = \frac{2\sigma\cos\theta}{R_i \rho g}}$$

Comments

* $h \sim \sigma$
* $h \sim \cos\theta$. so. $h \to max$ @ $\theta = 0^\circ$. i.e. perfect wetting like water on clean glass.
* $\rho g \doteq \frac{\text{weight}}{\text{Volume}}$, specific weight $\gamma = \rho g$ (Text. 1.4)
* $h \sim \frac{1}{R_i}$, a small radius gives a big height.

Numbers ?

e.g. water in a clean glass tube

$\sigma \approx 73 \cdot 10^{-3}$ N/m, $\rho \approx 1 \cdot 10^3$ kg/m^3,

$\theta \approx 0^\circ$, $g = 9.81$ N/kg. $R_i = 1$ mm

$$h = \frac{2 \cdot (73 \cdot 10^{-3}\ N/m) \cdot \cos 0^\circ}{(1 \cdot 10^{-3} m)(10^3\ kg/m^3)(9.81\ N/kg)}$$

$$= 149 \cdot 10^{-4}\ m$$

$$= 14.9\ mm$$

Show Fig. E1.8 in text. Effect of surface tension decresses dramatically with tube diameter. Only when tube diameter is quite small, surface tension does matter.

For $D \approx 30$ mm, $h = 1$ mm.

Key strategies of fluid mechanics discussed in Chapter 1:

- Isolate the object or body involving fluid appropriately to perform FBD analysis and apply the "Big Idea" of $\Sigma F = ma$ to move forward.

  An appropriate definition of the object or body involving fluid should expose the physical quantities of interest.