Ch1-2 Dimensions and Viscosity

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Objectives:

Reading: 1.2 – 1.6

- Dimensions and Units (Apply and Analyze)
- Viscosity (Identify and Explain)
- Density, Specific Weight, Specific Gravity (Text 1.4, study by yourself). (Apply and Analyze)

Recall:

Fluid: a ①substance that ②deforms ③continuously when acted on by ④a shear stress of ⑤any magnitude.

Mechanics: force and motion.

Big Idea: $\sum F = ma$

Dimensions and Units:

Poll: A) Units   B) Dimensions

① Are meters, kilograms, and seconds units or dimensions? A

② Are length, mass, and time units or dimensions? B

Common dimensions in fluid mechanics		Units (SI)
Mass,	M	kg
Length,	L	m
Time,	T	s
Force,	F	N

FLT System

MLT System

"Big Idea"

$F = ma$

$(m = F/a)$

$$\boxed{F \doteq M\frac{L}{T^2}, \quad M \doteq F\frac{T^2}{L}}$$

■ Table 1.1 Dimensions Associated with Common Physical Quantities

	FLT System	MLT System
Acceleration	LT^{-2}	LT^{-2}
Angle	$F^0L^0T^0$	$M^0L^0T^0$
Angular acceleration	T^{-2}	T^{-2}
Angular velocity	T^{-1}	T^{-1}
Area	L^2	L^2
Density	$FL^{-4}T^2$	ML^{-3}
Energy	FL	ML^2T^{-2}
Force	F	MLT^{-2}
Frequency	T^{-1}	T^{-1}
Heat	FL	ML^2T^{-2}
Length	L	L
Mass	$FL^{-1}T^2$	M
Modulus of elasticity	FL^{-2}	$ML^{-1}T^{-2}$
Moment of a force	FL	ML^2T^{-2}
Moment of inertia (area)	L^4	L^4
Moment of inertia (mass)	FLT^2	ML^2
Momentum	FT	MLT^{-1}
Power	FLT^{-1}	ML^2T^{-3}
Pressure	FL^{-2}	$ML^{-1}T^{-2}$
Specific heat	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	FL^{-3}	$ML^{-2}T^{-2}$
Strain	$F^0L^0T^0$	$M^0L^0T^0$
Stress	FL^{-2}	$ML^{-1}T^{-2}$
Surface tension	FL^{-1}	MT^{-2}
Temperature	Θ	Θ
Time	T	T
Torque	FL	ML^2T^{-2}
Velocity	LT^{-1}	LT^{-1}
Viscosity (dynamic)	$FL^{-2}T$	$ML^{-1}T^{-1}$
Viscosity (kinematic)	L^2T^{-1}	L^2T^{-1}
Volume	L^3	L^3
Work	FL	ML^2T^{-2}

Example:

$\boxed{\text{Given}}$ Density $\rho = \dfrac{m}{\forall}$ ← mass, ← volume

$\boxed{\text{Find}}$ i) dimensions of ρ in MLT system

ii) dimensions of ρ in FLT system

$\boxed{\text{Solution}}$

i) $\rho \doteq \dfrac{M}{L^3}$

ii) $\rho \doteq \dfrac{M}{L^3} \doteq ML^{-3}$ and $M \doteq F\dfrac{T^2}{L} \doteq FL^{-1}T^2$

Thus, $\rho \doteq (FL^{-1}T^2)(L^{-3}) \doteq FL^{-4}T^2$

General homogeneous equation and Restricted homogeneous equation

- "Homogeneous" here means dimensionally homogeneous.
- An equation is dimensionally homogeneous if the dimensions of its left side are the same as the dimensions of its right side.

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* General homogeneous equation: valid for any unit systems

* Restricted homogeneous equation: restricted to particular unit system.

Unit systems
Read Text 1.2.1

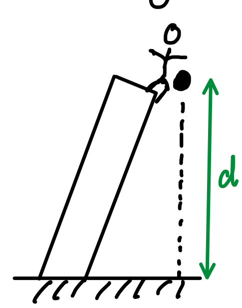
$\begin{cases} \text{SI : International System} \\ \text{BG : British Gravitational System (slug)} \\ \text{EE : English Engineering System (lbm)} \end{cases}$ Mass: 1 slug = 32.174 lbm

* How to tell the general or restricted homogeneous equation?

Example:

$\boxed{\text{Given}}$ Freely Falling Body

① $d = \dfrac{gt^2}{2}$ ② $d = 16.1\,t^2$



Gallileo's Leaning Tower of Pisa Experiment d: distance, t: time, g: gravitational acceleration

$\boxed{\text{Find}}$ i) The dimensions of the constant in the two equations.

ii) Is it a general or restricted homogeneous equation?

$\boxed{\text{Solution}}$

① $d = \frac{gt^2}{2}$

$$2 = gt^2d^{-1} \doteq \frac{L}{T^2}T^2L^{-1} \doteq L^0T^0 \quad \text{dimensionless}$$

Since the constant of 2 is dimensionless, the equation $d = \frac{gt^2}{2}$ is valid for any unit system. That is, the equation $d = \frac{gt^2}{2}$ is a general homogeneous equation.

② $d = 16.1t^2$

$$16.1 = dt^{-2} \doteq LT^{-2} \quad \text{dimensional}$$

Since the constant of 16.1 is dimensional, the equation $d = 16.1t^2$ is restricted to particular unit system. That is, the equation $d = 16.1t^2$ is a restricted homogeneous equation.

What unit system is the equation $d = 16.1t^2$ restricted to?

For BG or EE unit system, we have $g = 32.2\ ft/s^2$

$$d = \frac{gt^2}{2} = \frac{32.2\ ft/s^2}{2}t^2 = (16.1\ ft/s^2)t^2$$

The equation $d = 16.1t^2$ is restricted to d in ft and t in s.

$\boxed{\text{Comments}}$

* The key is the dimensions of the constant in the equation.

Dimensionless constant ⟶ a general homogeneous equation.

Dimensional constant ⟶ a restricted homogeneous equation.

Viscosity:

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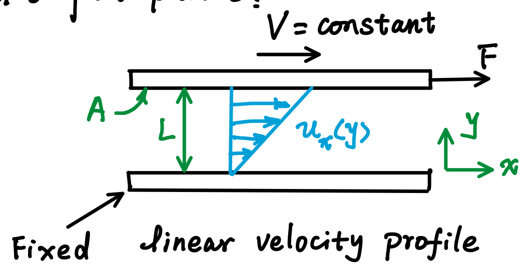
Key observations of fluid in motion:

- Viscosity leads to an internal friction that resists relative sliding motion of fluid, e.g. thumb-forefinger test.
- Fluids stick to boundaries: $u|_{boundary} = 0$. No-slip condition

Couette flow b/t flat plates:



Observations:

$F \sim V$

$$\frac{F}{A} \sim \frac{V-0}{L-0}$$

$$\tau_{yx} \sim \frac{du}{dy}$$

This is used to define Newtonian fluids.

* Fluids for which the shear stress τ_{yx} is linearly related to (i.e. proportional to) the velocity gradient $\frac{du}{dy}$ are designated as Newtonian fluids after Isaac Newton (1642 - 1727).
* All the other behaviors $\longrightarrow$ Non-Newtonian fluids.

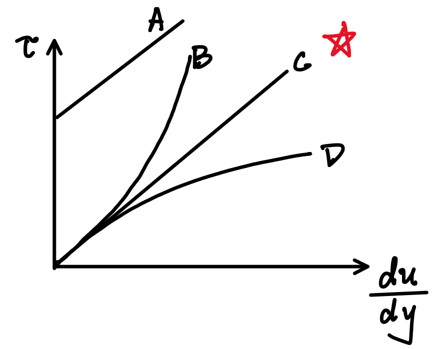
For the abovementioned linear relationship, if we would like to get an equation, we need a proportional coefficient. μ.

$$\tau_{yx} = \mu \cdot \frac{du}{dy}$$ Newton's viscosity law

Shear stress $\left(\frac{\text{Force}}{\text{Area}}\right)$

Viscosity (Proportionality coefficient)

Velocity gradient (Local slope of velocity profile, Rate of shear strain, or shear rate) (See supplements)

Poll: 

Which one is a Newtonian fluid?

* The slope in the $\tau - \frac{du}{dy}$ diagram is the viscosity.
* For Newtonian fluids, the viscosity is independent to the velocity gradient $\frac{du}{dy}$.

Typical Newtonian fluids: water, alcohol, gasoline, air, etc.

Typical Non-Newtonian fluids: paint, water-corn starch mixture, slime, etc.

B: shear thickening fluid.

D: shear thinning fluid.