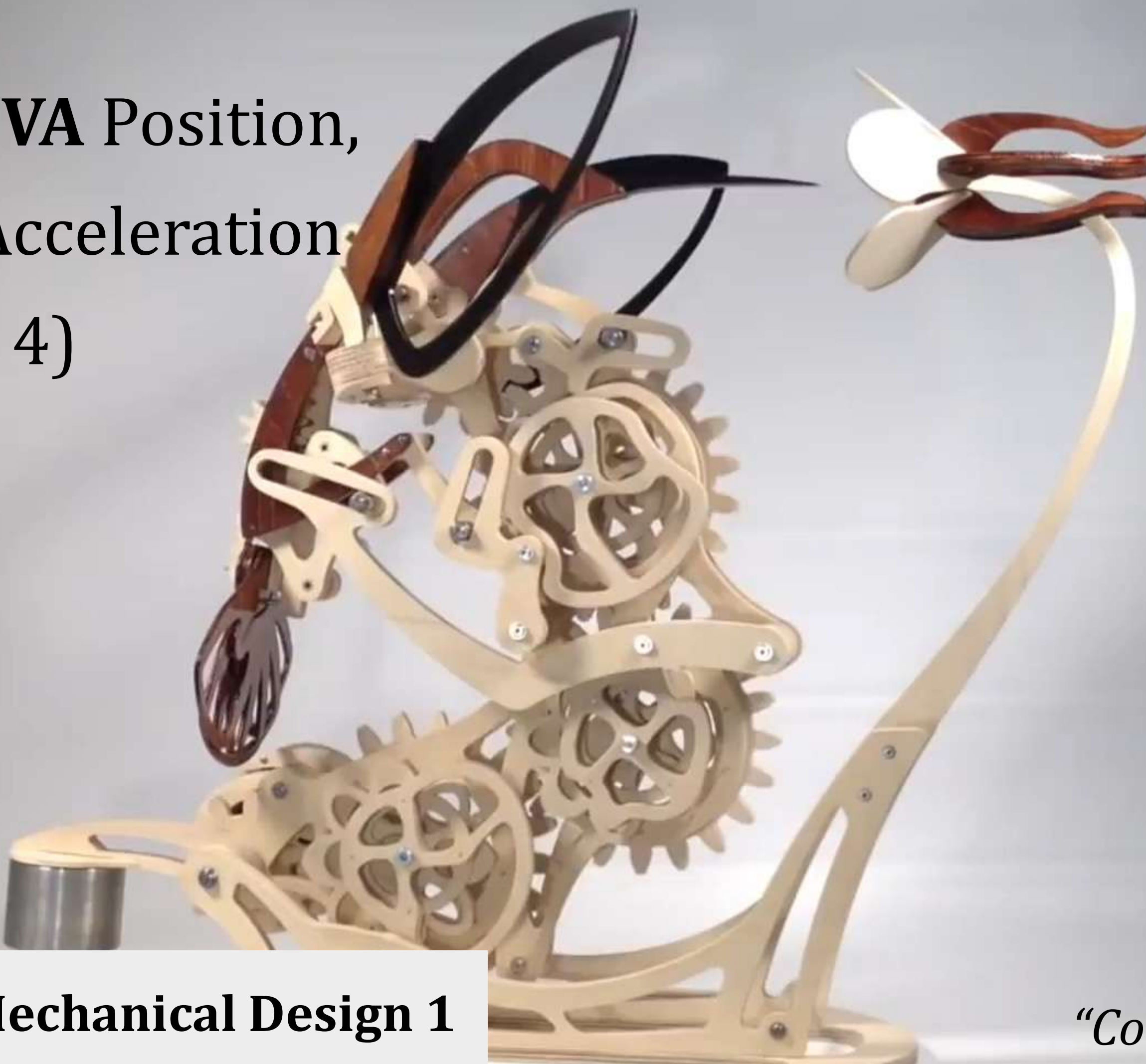
Module 3

Lecture 11: PVA Position, Velocity, and Acceleration Analysis (Part 4)



ME 370 - Mechanical Design 1

"Colibri" by Derek Hugger

** www.youtube.com/watch?v=1scj5sotD-E*

Lecture 11: Position, Velocity, Acceleration

Topics: 10/1/25 PVA Analysis – Part 4 (Norton Chap 4, 6, 7)

Announcements

- Lab attendance is mandatory for ALL weekly labs (except Labs 13 & 14). Starting with Lab 6, failure to attend lab (without prior approved excuse) will result in a grade penalty* on the individual student's overall lab grade. Additionally, note that 2% of the final grade is also attributed to "Class Participation (based on lectures and labs, project review sessions, overall effort)". *Grade penalty: first occurrence (-20% of team lab grade), higher penalty for more absences.

Activities & Upcoming Deadlines

- **Lab 6:** work on Project 1. Attendance is required
- **HW 5 (PVA #1):** Due Tuesday 10/7
- **Project 1:**
  - P1D5 materials (rubric, slide template, expense report spreadsheet, team contribution statements) posted
  - Upload slide presentation, CAD animation video, and team contribution statement. Note that if you change your materials between submission and presentation, only the submitted files will be used for grading. Be prepared to present your slides and show the final working dispenser.

Next lecture: 10/6/25 – Start Module 4 Instant Centers (Norton Chap 6.3 and 6.4)

PVA Topics

1. Vector notation (Complex and Compact)
2. Analytical analysis method
   - **P**osition analysis
   - **V**elocity analysis
   - **A**cceleration analysis

Covered in Lecture 8

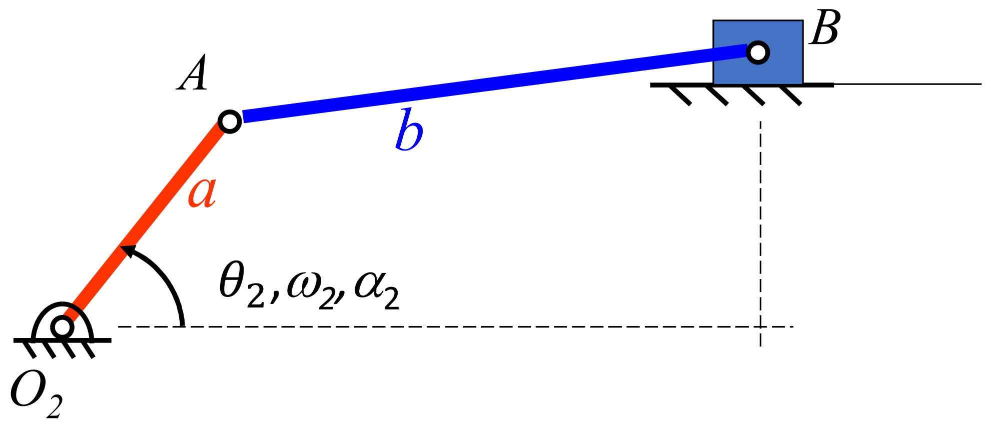
3. PVA analysis of a moving point
4. PVA analysis of a four-bar linkage
   - Vector loop equation

Covered in Lecture 9, 10

5. PVA analysis of other four-bar mechanisms
   - Offset slider-crank
   - Inverted offset slider-crank
6. PVA analysis of mechanisms > four Links

PVA for (offset) crank-slider

Offset = slider axis does not pass through crank pivot



- Given: a, b, c, θ_2, ω_2, α_2
- Solve for $d, \dot{d}, \ddot{d}$, θ_3, ω_3, α_3
- How?
  - Use Vector loop equations for Pos, Vel, or Accel vectors for mechanism
  - Write in compact vector notation ($\vec{R}_{AO_2} = ae^{j\theta_2(t)}$).
  - Apply Euler's Identity : $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$
  - Separate into Real and Imaginary parts
  - Solve simultaneously

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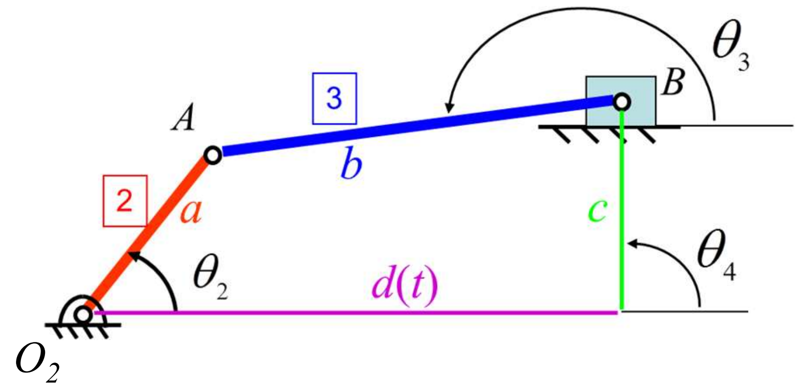
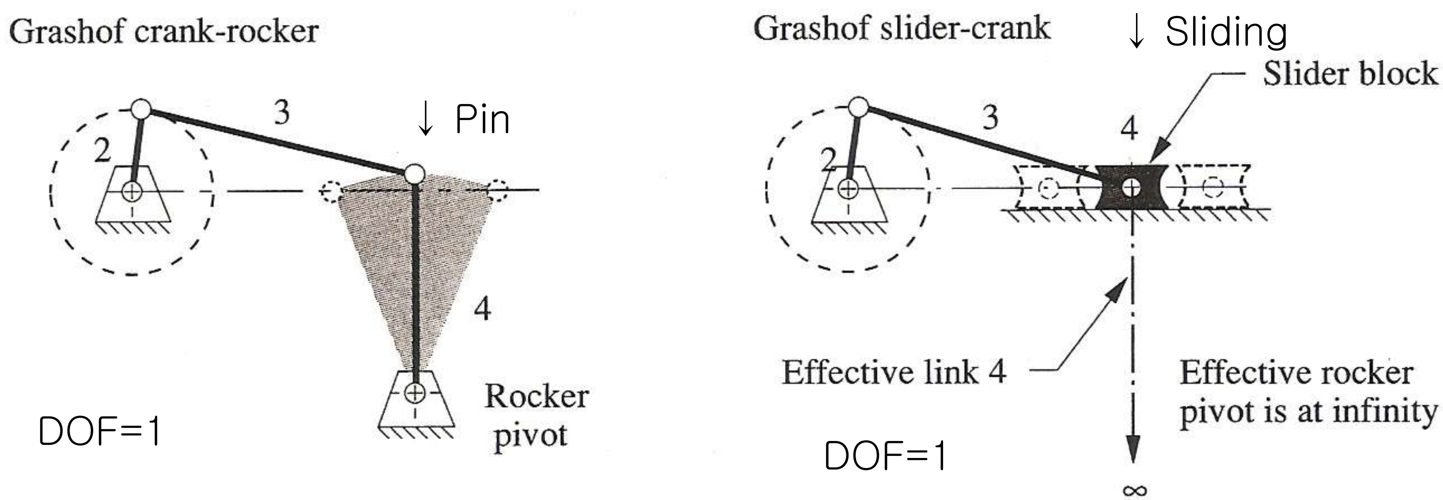
Transformation Rules (section 2.9)
Example: RULE 1

(replace pin with sliding joint, no change in # DOF)



n=4, J_1=4

n=4, J_1=4



(a) Transforming a fourbar crank-rocker to a slider-crank

PVA for (offset) crank-slider: Position Vector Loop

Step 1: How to setup the vector loop?

2 loop Eq

$\vec{R_A} = \vec{R_s} + \vec{R_{BA}}$ and

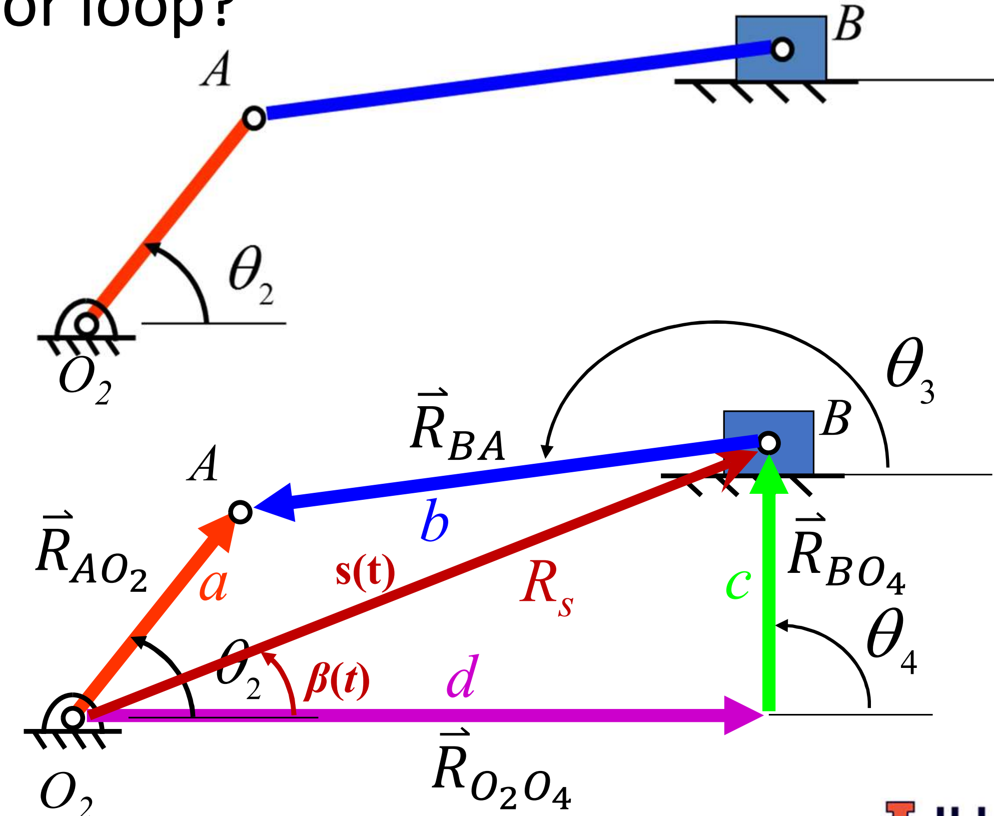
$\vec{R_s} = \vec{R_{O_4O_2}} + \vec{R_B}$

so

$\vec{R_A} = \vec{R_{O_4O_2}} + \vec{R_B} + \vec{R_{BA}}$ or

$\vec{R_A} - \vec{R_{O_4O_2}} - \vec{R_B} - \vec{R_{BA}} = 0$

$ae^{j\theta_2} - be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_1} = 0$



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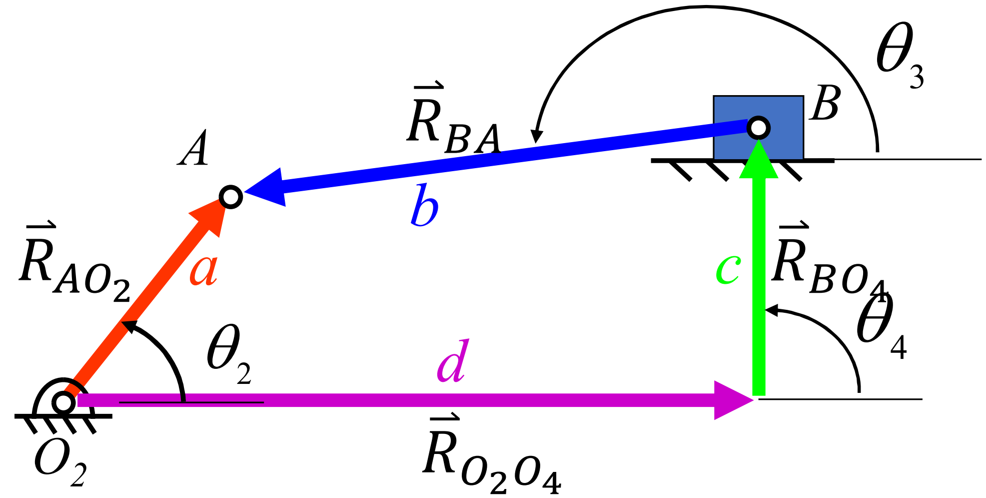
PVA for (offset) slider-crank

- Step 2: Find Real and Imaginary Components



- Position analysis (Solve for d, θ_3)

What are the values for θ_1 and θ_4 ?

$$ae^{j\theta_2} - be^{j\theta_3} - ce^{j\cancelto{90^\circ}{\theta_4}} - de^{j\cancelto{0^\circ}{\theta_1}} = 0$$

$$e^{\pm j\theta} = y \pm j\sin\theta$$

Real: $$a\cos\theta_2 - b\cos\boxed{\theta_3} - \boxed{d} = 0$$

Im: $$a\sin\theta_2 - b\sin\boxed{\theta_3} - c = 0$$

Repeat for velocity and acceleration analyses

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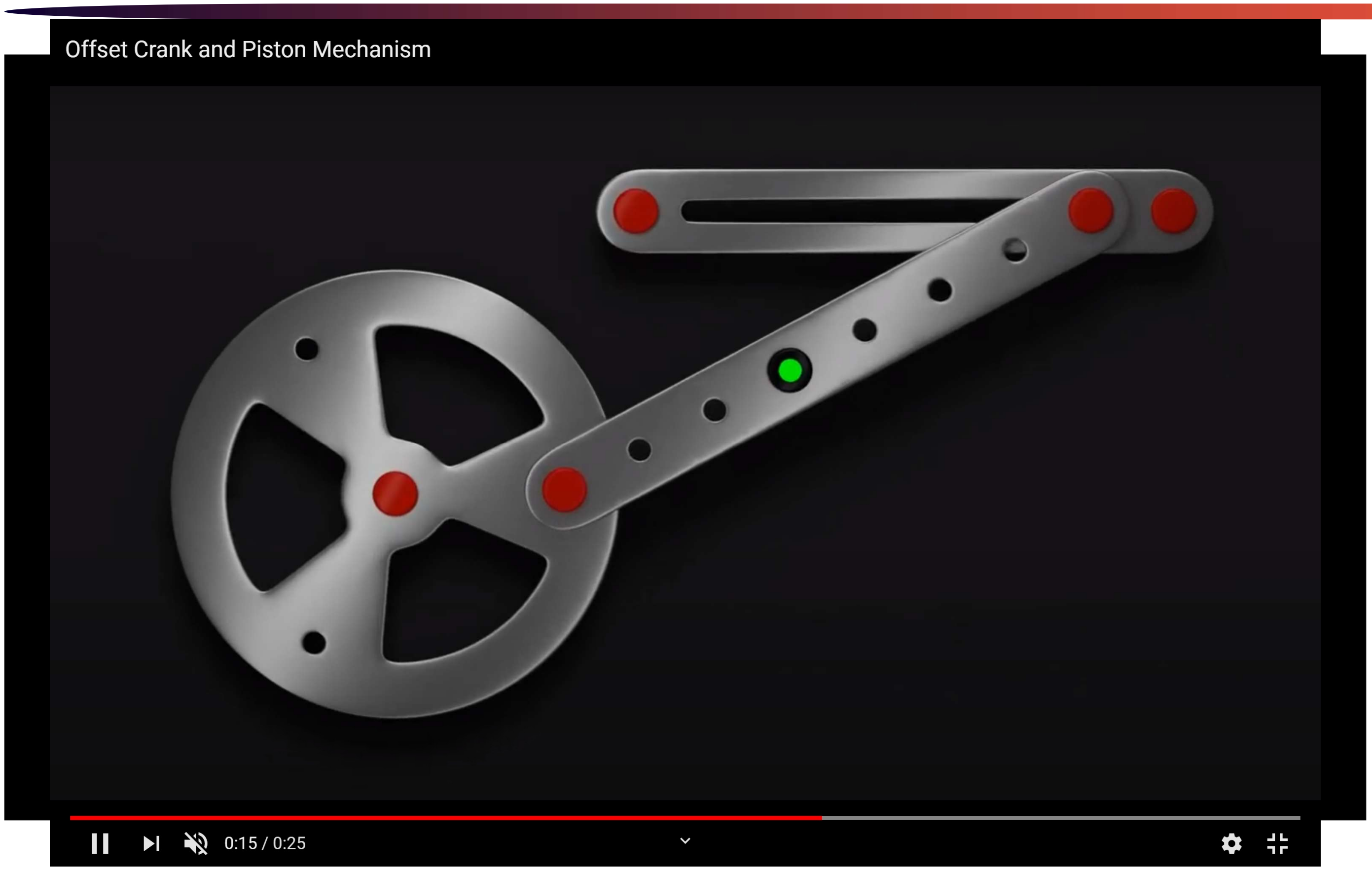
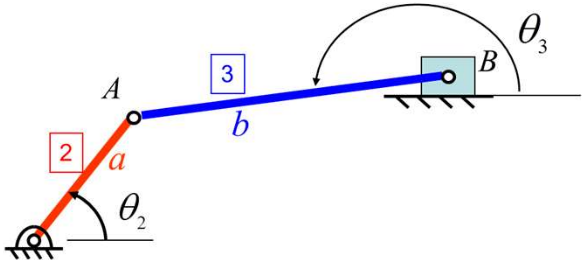
(offset) crank-slider





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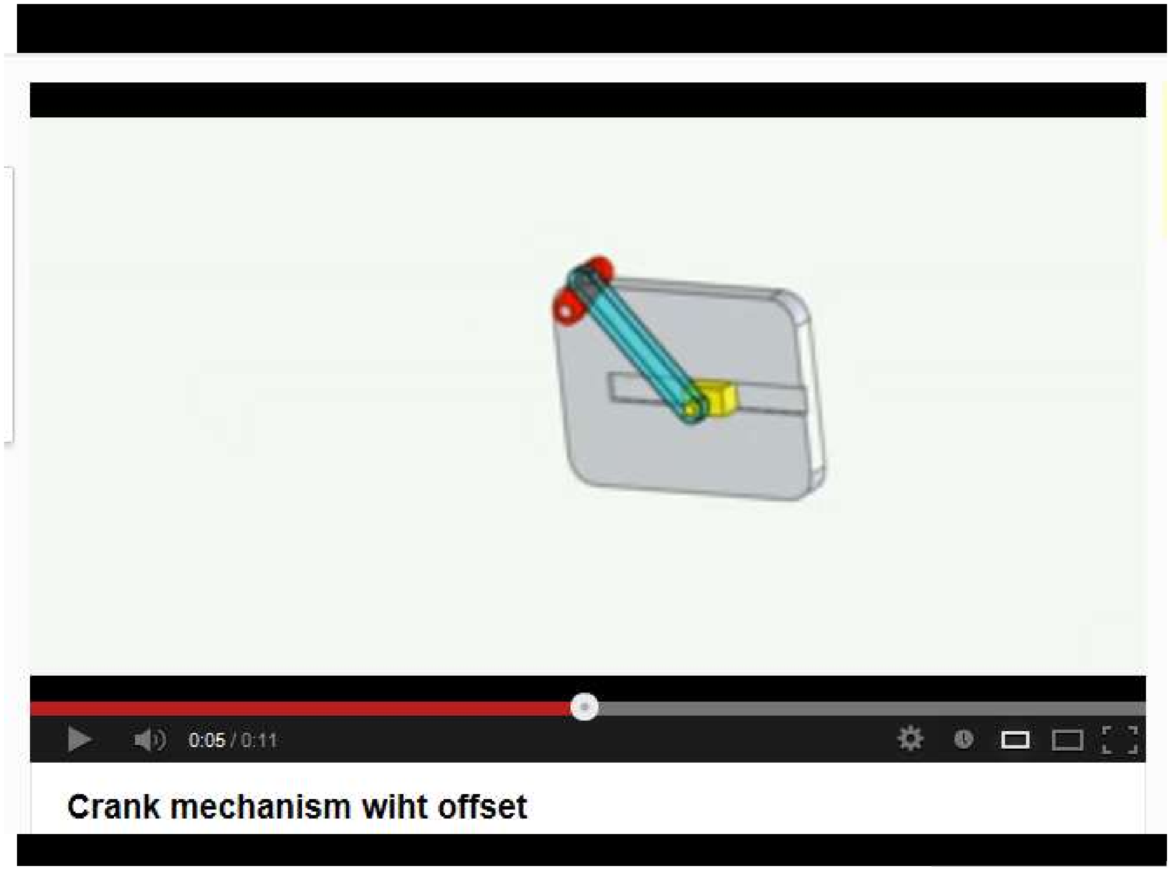

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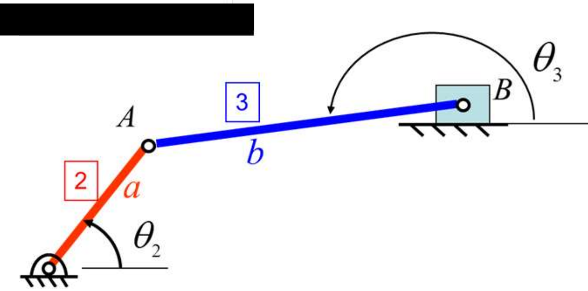
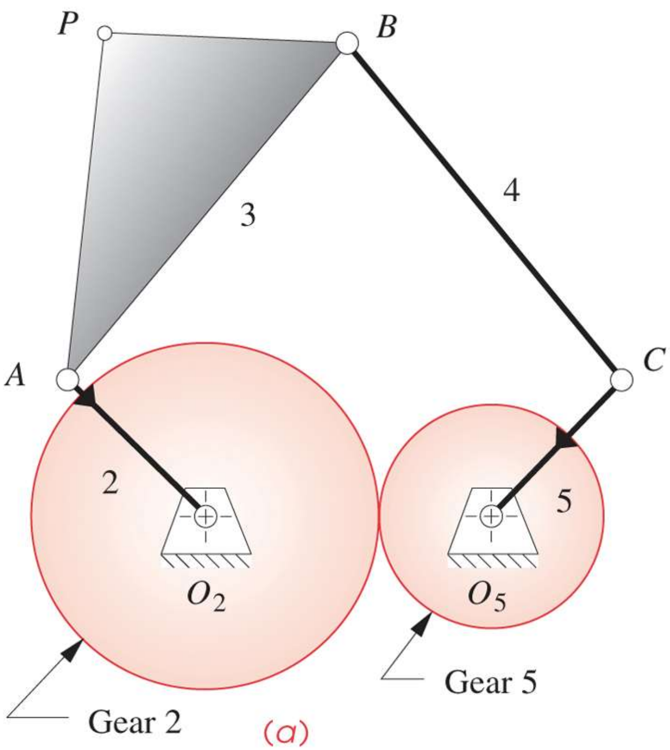


How about Mechanisms with more than 4 links?

If you are given θ_2, what are the values for θ_3, θ_4, θ_5 ?



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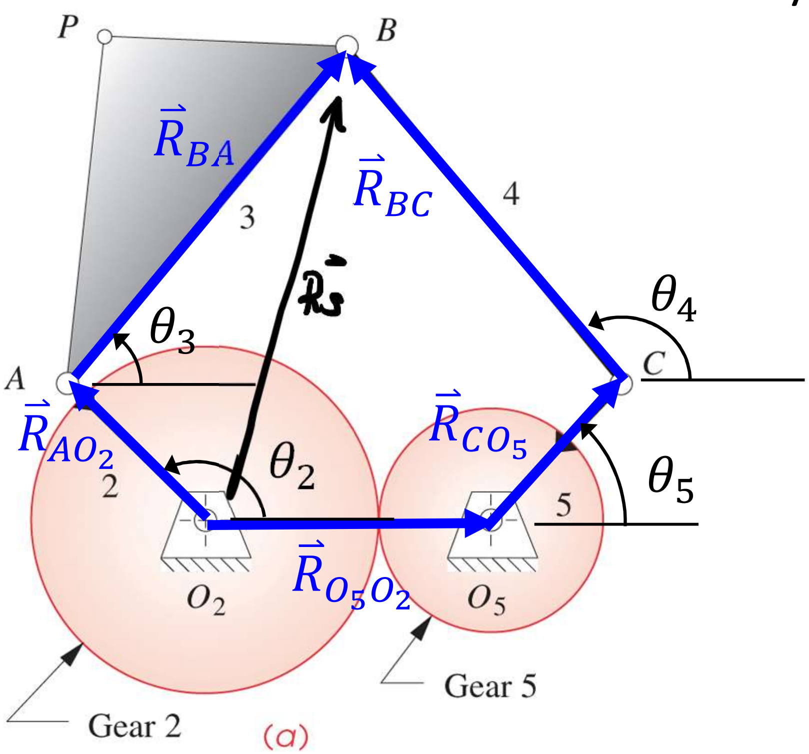
How about Mechanisms with more than 4 links?



If you are given θ_2, what are the values for θ_3, θ_4, θ_5 ?

$$\vec{R}_{AO_2} + \vec{R}_{BA} - \vec{R}_{BC} - \vec{R}_{CO_5} - \vec{R}_{O_5O_2} = 0$$

Follow usual approach:
Get **2 equations based on Real, Im** parts

Need 3rd equation:
Use phase angle between gears (ϕ), defined as

$$\boldsymbol{\phi = \theta_5 - \lambda\theta_2}$$

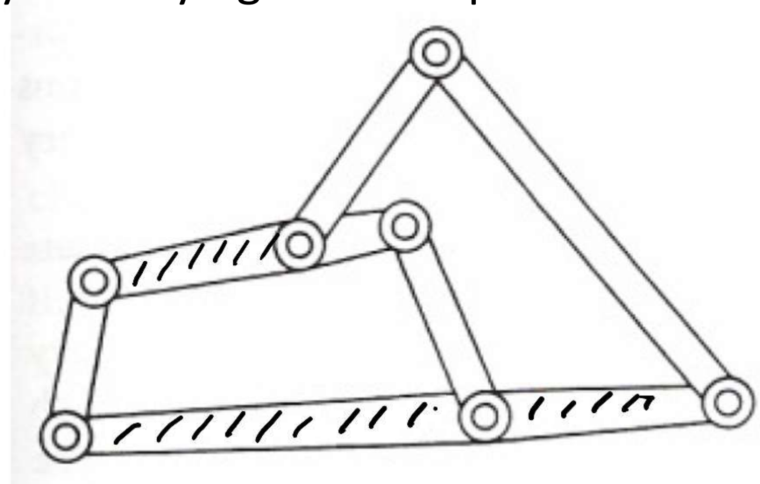
where gear ratio: $\lambda = \pm\frac{r_{in}}{r_{out}} = \pm\frac{r_2}{r_5} = \pm\frac{d_2}{d_5} = \pm\frac{N_2}{N_5} = \frac{\omega_5}{\omega_2} = \frac{\omega_{out}}{\omega_{in}}$

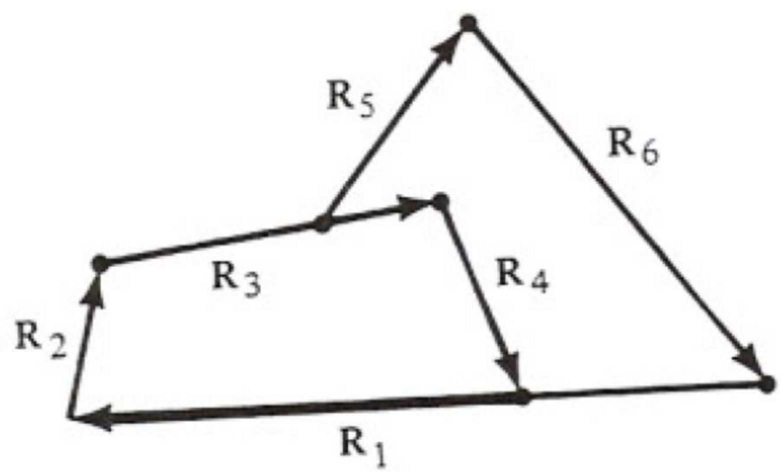
r: pitch circle radius, d: pitch circle diameter, N: # teeth, ω: angular velocity

Mechanisms with multiple vector loops

What loops would you create if you are given θ_2 and asked to solve for θ_6?

Try identifying 4-bar loops in more complex 1-DOF planar mechanisms



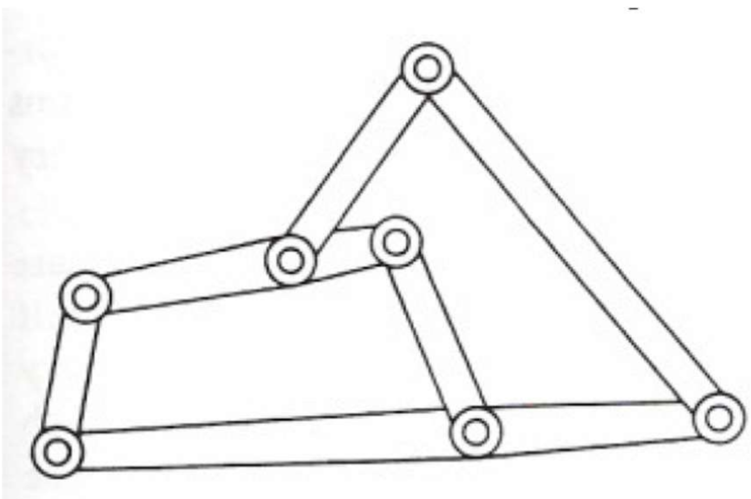


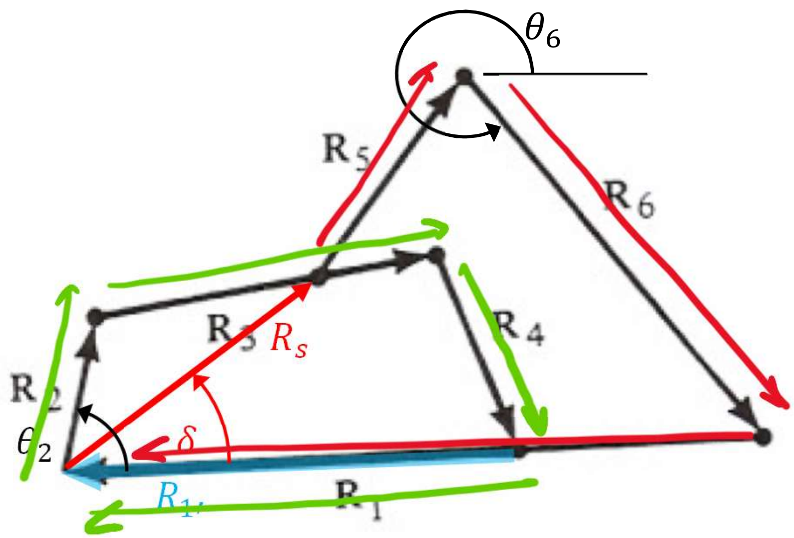
How many DOF?

n = 6, J_1 = 7 → DOF = 1

Mechanisms with multiple vector loops

What loops would you create if you are given θ_2 and asked to solve for θ_6?

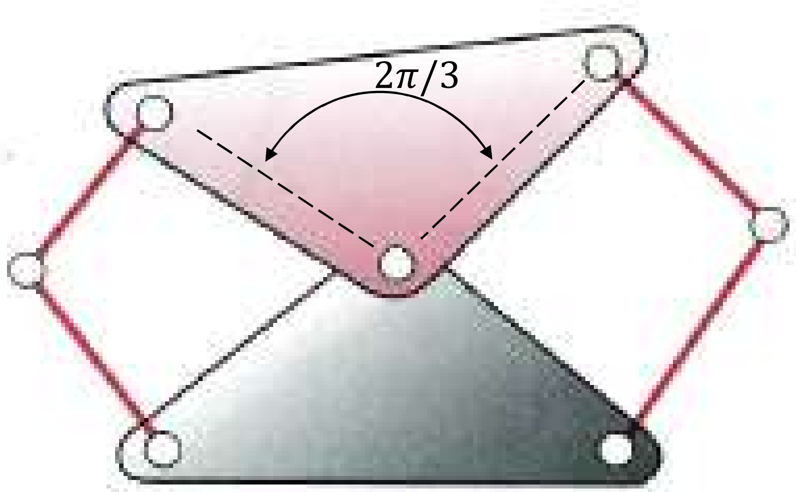




First Loop	$R_2 + R_3 + R_4 + R_{1'} = 0$	Solve for θ_3 θ_4
Second Loop	$R_s + R_5 + R_6 + R_1 = 0$	Solve for θ_6 (δ and Rs are known)

Mechanisms with multiple vector loops

Where are the loops?



How many DOF?

$n = 6$, $J_1 = 8 \rightarrow$ DOF = 1

Try identifying 4-bar loops in more complex 1-DOF planar mechanisms

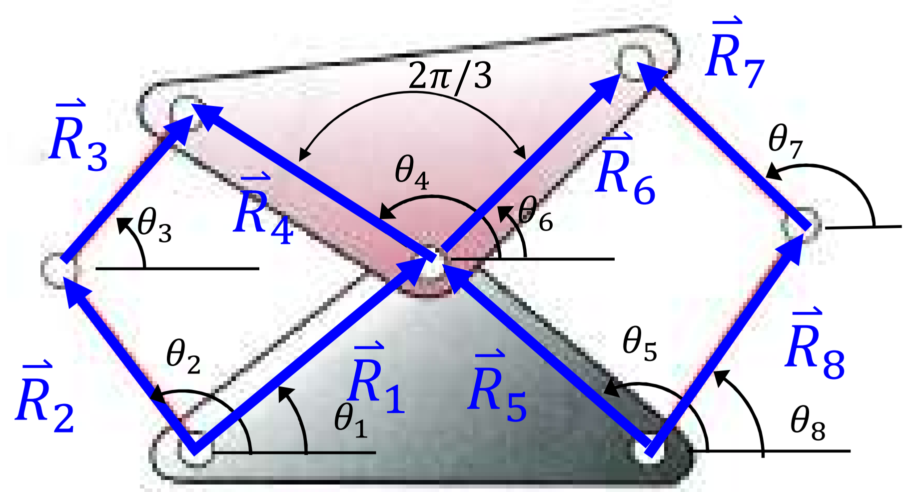
Rules for vector loops:

- All links need to be included in a loop
- Ternary or higher link should not be in a loop
- Include only inner edge of ternary links
- Use trig rules to define one loop as dependent on the other

Mechanisms with multiple vector loops

Where are the loops?

Try identifying 4-bar loops in more complex 1-DOF planar mechanisms



Rules for vector loops:

- All links need to be included in <u>a</u> loop
- Ternary or higher link should not be in a loop
- Include only inner edge of ternary links
- Use trig rules to define one loop as dependent on the other

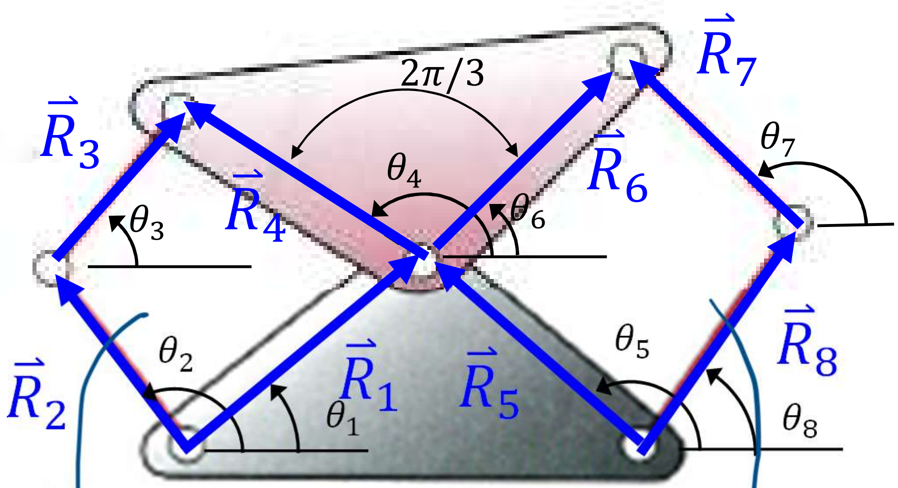
How many DOF?

n = 6, J_1 = 8 → DOF = 1

Need extra equation:

$\theta_6 = \theta_4 - \frac{2\pi}{3}$ Geometric constraint

Mechanisms with multiple vector loops



θ_2 is the input
2 Eq → 2 unknowns
θ_3, θ_4

2 Eq → 3 unknowns
$\theta_6, \theta_7, \theta_8$

Why to need the geometric constraint?

$$\boxed{\theta_6 = \theta_4 - \frac{2\pi}{3}}$$

Loop 1: $\vec{R}_2 + \vec{R}_3 - \vec{R}_4 - \vec{R}_1 = 0$
$r_2 e^{j\theta_2} + r_3 e^{j\theta_3} - r_4 e^{j\theta_4} - r_1 e^{j\theta_1} = 0$
θ_1 = constant
θ_3 and θ_4 unknown
(2 eqns from Re, Im)

Loop 2: $\vec{R}_5 + \vec{R}_6 - \vec{R}_7 - \vec{R}_8 = 0$
$r_5 e^{j\theta_5} + r_6 e^{j\theta_6} - r_7 e^{j\theta_7} - r_8 e^{j\theta_8} = 0$
θ_5 = constant
θ_6, θ_7 and θ_8 unknown
(2 eqns from Re, Im + 1 geometric constraint)

also θ_6 and θ_4 are related. so 5x5

Summary

- A 4-bar mechanism is a basic building block of more complex planar mechanisms
- We have studied how to set up the vector loop equation for the 4-bar mechanism and solve it
- We also saw that successive differentiation of this equation allows us to solve for the velocity and accelerations of the links in the mechanism
- We saw how to use the same solution strategy for variants of the 4-bar mechanism
- By identifying 4-bar loops in more complex 1-DOF planar mechanisms, we can extend this technique to a larger set of mechanisms.

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