Lecture 9
PVA Part 2

“Colibri” by Derek Hugger

* www.youtube.com/watch?v=1scj5sotD-E

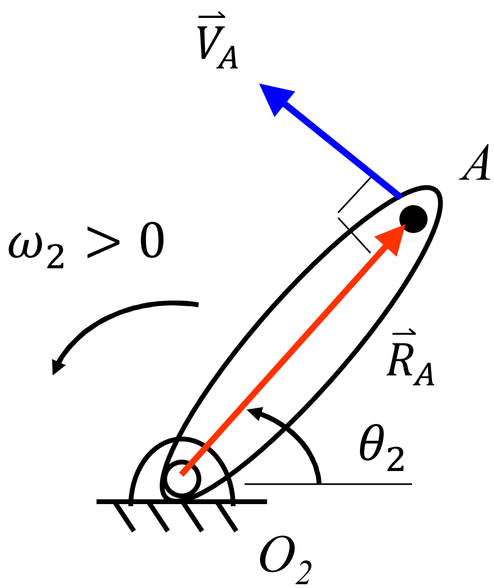
ME 370 - Mechanical Design 1

PVA Topics

- Vector notation (Complex and Compact)
- Analytical analysis method
  - **P**osition analysis
  - **V**elocity analysis
  - **A**cceleration analysis
- PVA analysis of a moving point
- Vector loop equation
- PVA analysis of a four-bar linkage
- PVA analysis of other four-bar mechanisms
  - Offset slider-crank
  - Inverted offset slider-crank
- PVA analysis of mechanisms > four Links

Acceleration analysis: use compact notation to find acceleration

$\vec{A}_A = ?$



$$\vec{A}_A = \frac{d\vec{V}_A}{dt} = \frac{d}{dt}\left(j\omega_2 a e^{j\theta_2}\right)$$

$$= j\frac{d}{dt}\left(w_2(t)\, a\, e^{j\theta_2(t)}\right)$$

$$= j\frac{dw_2}{dt} a\, e^{j\theta_2} + j\, w_2 a j e^{j\theta_2} w_2$$

$$j \cdot j = -1$$

$$= j\alpha_2 \underbrace{a e^{j\theta_2}}_{R_2} - w_2^2 a e^{j\theta_2}$$

$$= j\alpha_2 \vec{R}_2 - w_2^2 \vec{R}_2$$

tangential normal.

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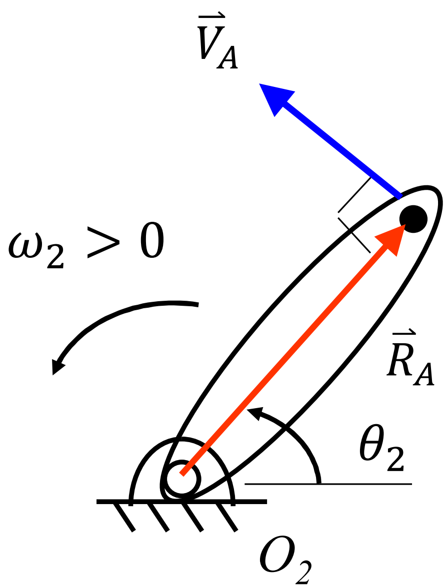
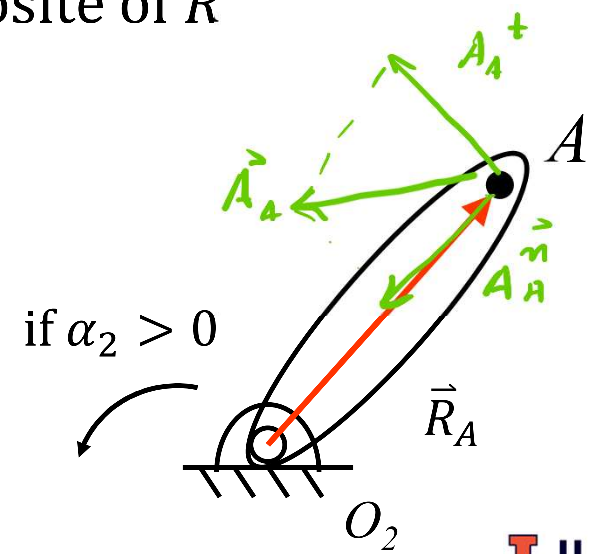
Which direction do $A^t{}_A$, $A^n{}_A$ point?

$$\vec{A}_A = j\alpha_2\vec{R}_2 - \omega_2^2\vec{R}_2$$
$$= \vec{A}_A^t + \vec{A}_A^n$$



$\vec{A}_A^t$ depends on sign of α_2
$\vec{A}_A^n$ is always opposite of $\vec{R}$



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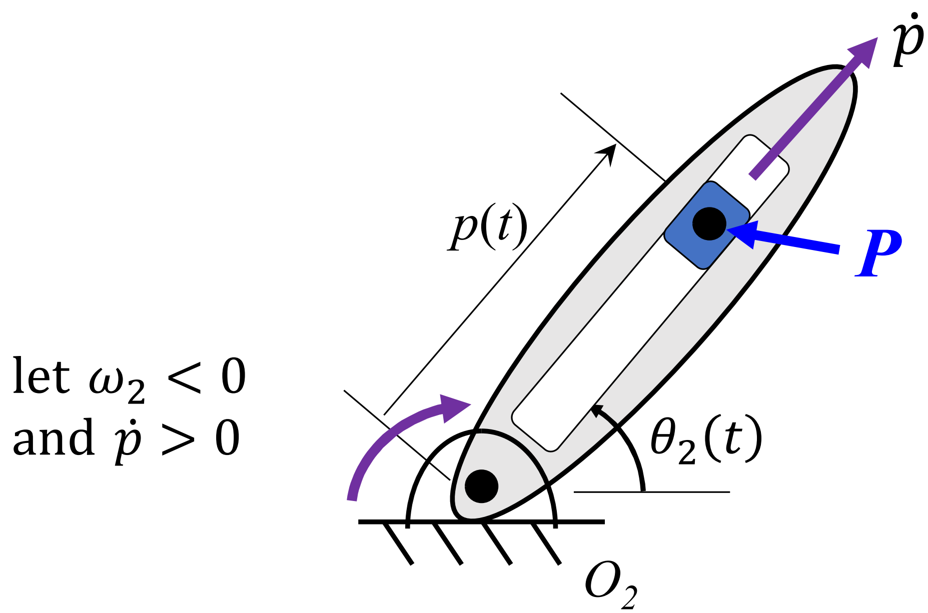
PVA analysis of a moving point

- Example: slider block in rotating link
- 2 DOF

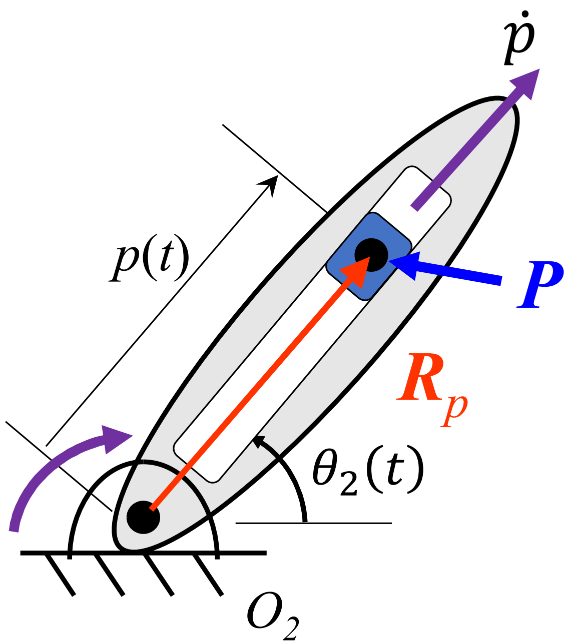


Position vector

$$\vec{R}_P = ?$$

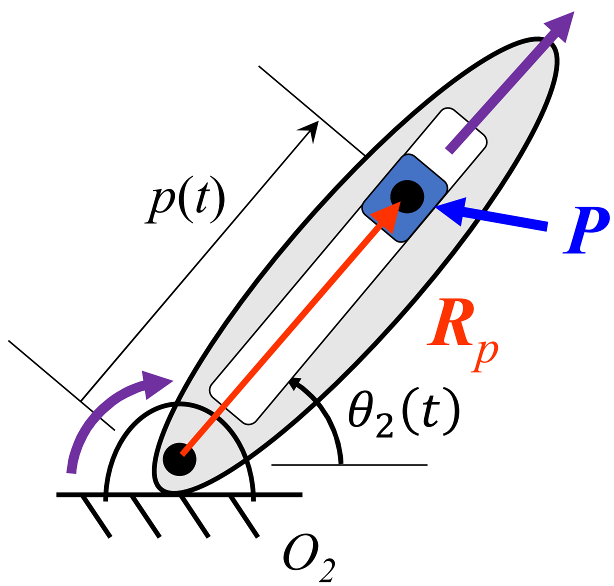
$\vec{R_p} = p(t)\,e^{j\theta_2(t)}$

let $\omega_2 < 0$
and $\dot{p} > 0$



Velocity vector

let $\omega_2 < 0$
and $\dot{p} > 0$



$$\vec{V}_P = ?$$

$$\vec{V}_P = \frac{d}{dt}\vec{R}_P$$

$$v_p = \frac{d}{dt}\left(p(t)\, e^{j\theta_2(t)}\right)$$

$$= \frac{d e^{j\theta_2(t)}}{dt}\, p(t) + \frac{dp(t)}{dt}\, e^{j\theta_2(t)}$$

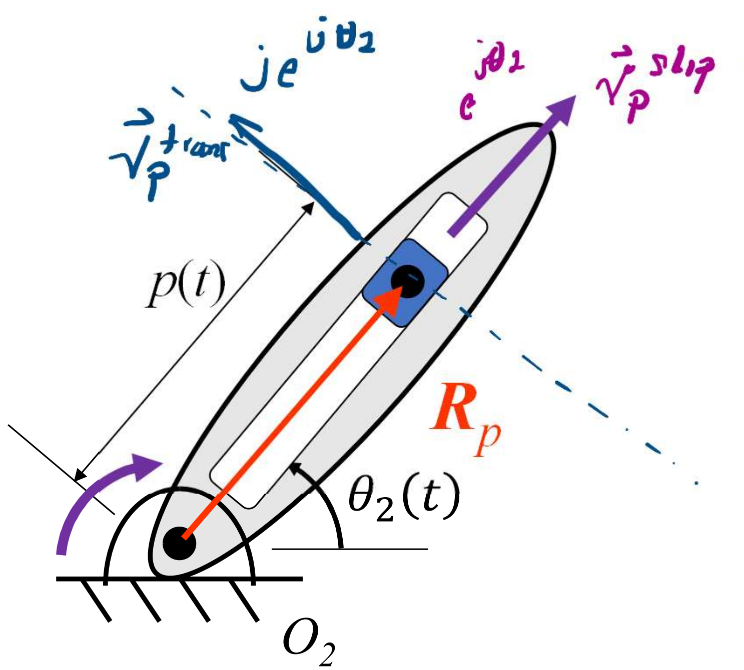
$$= j\dot{\theta}_2 e^{j\theta_2} p + \dot{p} e^{j\theta_2(t)}$$

$$= \underbrace{j\omega_2 \tilde{R_p}}_{v_{tang}} + \underbrace{\dot{p} e^{j\theta_2(t)}}_{\bar{v}_{slip}}$$

What does $\boldsymbol{V}_P$ look like?

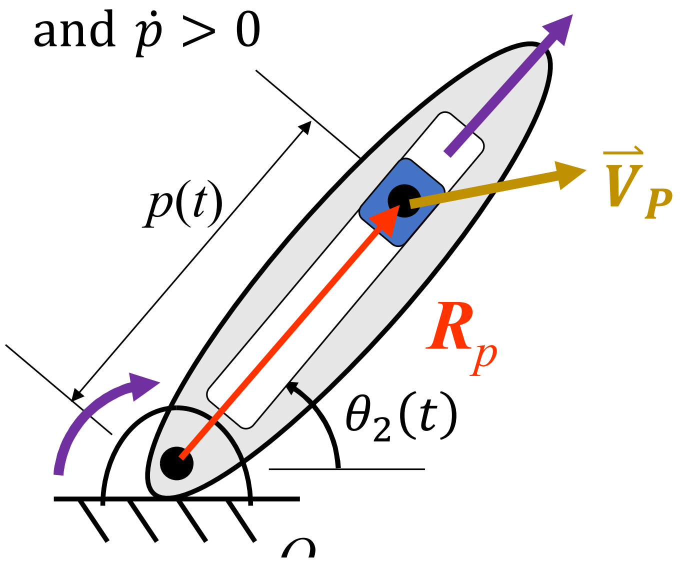
$$\vec{V}_P = j\omega_2 \vec{R}_P + \dot{p}e^{j\theta 2}$$
$$= \vec{V}_P^{trans} + \vec{V}_P^{slip}$$

let $\omega_2 < 0$
and $\dot{p} > 0$



Acceleration vector

let $\omega_2 < 0$
and $\dot{p} > 0$



$$\vec{A}_P = ?$$

$$\vec{A}_P = \frac{d}{dt}\vec{V}_P = \frac{d}{dt}\left(p(t) j\omega_2 e^{j\theta_2(t)} + \dot{p} e^{j\theta_2}\right)$$

$$= \dot{p} j \omega_2 e^{j\theta_2} + p \frac{d}{dt}\left(j\omega_2(t) e^{j\theta_2(t)}\right) + \ddot{p} e^{j\theta_2} + \dot{p} j \omega_2 e^{j\theta_2}$$

$$p \underbrace{j^2}_{-1} \omega_2^2 e^{j\theta_2 t} \qquad p j \alpha_2 e^{j\theta_2}$$

Similar

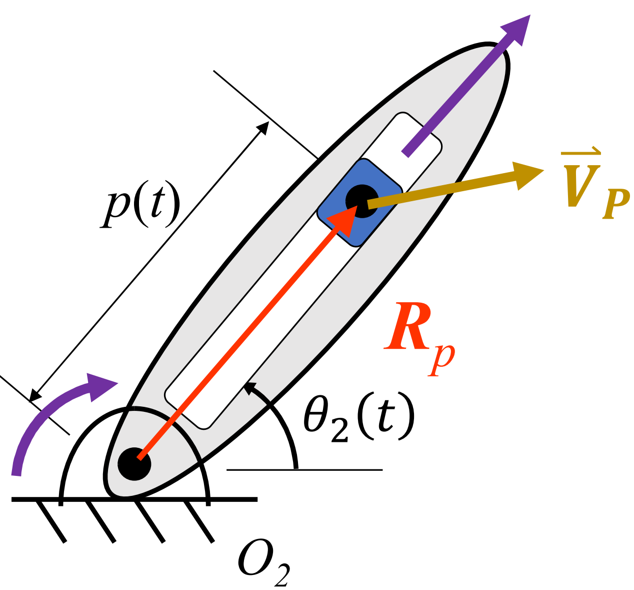
$$= \ddot{p} e^{j\theta_2} + 2\dot{p} j \omega_2 e^{j\theta_2} - \omega_2^2 \underbrace{p e^{j\theta_2}}_{R_p} + j \alpha_2 \underbrace{p e^{j\theta_2}}_{R_p}.$$

Acceleration vector

$$\vec{A}_P = ?$$

$$\vec{A}_P = \frac{d}{dt}\vec{V}_P = \frac{d}{dt}\left(j\omega_2 p(t) e^{j\theta_2(t)} + \dot{p}e^{j\theta_2}\right)$$

Product Rule (5 terms)



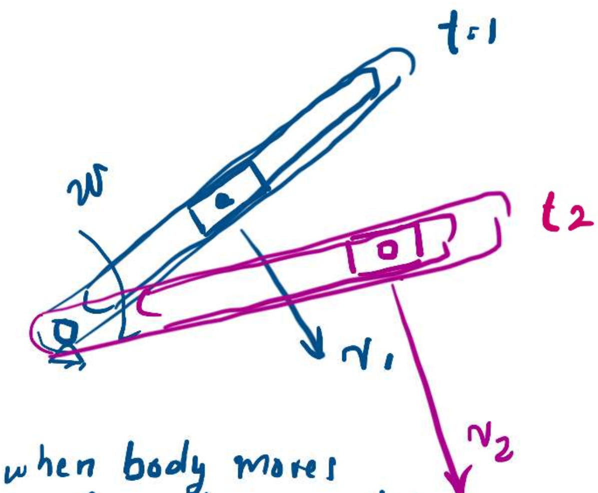
let $\omega_2 < 0$
and $\dot{p} > 0$

$$= \ddot{p}e^{j\theta 2} + j2\omega_2\dot{p}e^{j\theta 2} - \omega_2^2\vec{R}_P + j\alpha_2\vec{R}_P$$

$$= \vec{A}_A^{slip} + \vec{A}_A^{Coriolis} + \vec{A}_A^{n} + \vec{A}_A^{t}$$

What does A_P look like?

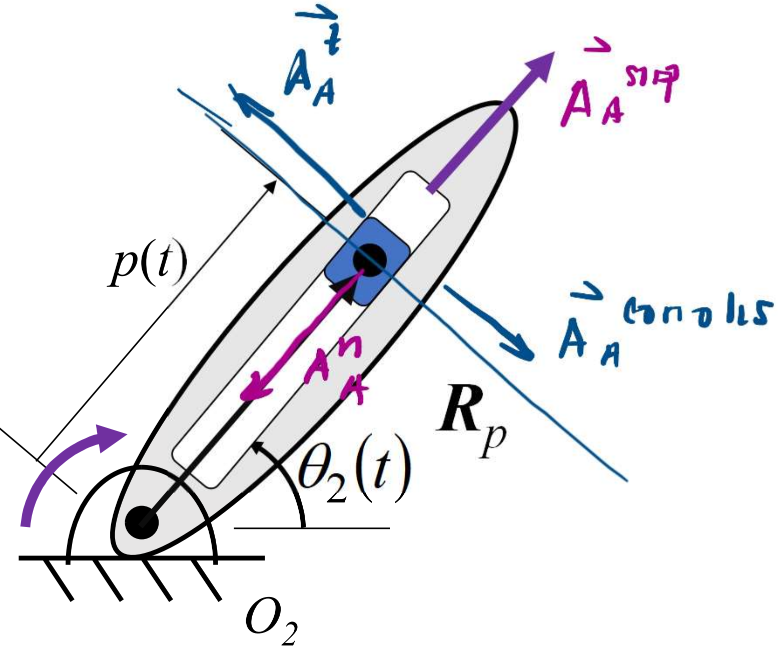
$$\vec{A}_P = \ddot{p}e^{j\theta 2} + j2\omega_2\dot{p}e^{j\theta 2} - \omega_2^2\vec{R}_P + j\alpha_2\vec{R}_P$$
$$= \vec{A}_A^{slip} + \vec{A}_A^{Coriolis} + \vec{A}_A^{n} + \vec{A}_A^{t}$$



when body moves w.r.t center of rotation velocity changes in direction and magnitud $2\omega_2\dot{p}$

let $\omega_2 < 0$
and $\dot{p} > 0$

let $\alpha_2 > 0$
and $\ddot{p} > 0$



Review of Terms

Term	Symbol	Expression	Direction w.r.t to $\vec{R}$	Cause
Transmission Velocity	$\vec{V}^t$	$j\omega\vec{R}$	Perpendicular	Rotation
Slip Velocity	$\vec{V}^{slip}$	$\dot{p}e^{j\theta}$	Parallel	Sliding
Transmission Acceleration	$\vec{A}^t$	$j\alpha\vec{R}$	Perpendicular	Rotation
Normal Acceleration	$\vec{A}^n$	$-\omega^2\vec{R}$	Parallel	Rotation
Slip Acceleration	$\vec{A}^{slip}$	$\ddot{p}e^{j\theta}$	Parallel	Sliding
Coriolis Acceleration	$\vec{A}^{coriolis}$	$j2\omega\dot{p}e^{j\theta}$	Perpendicular	translating in a rotating reference frame

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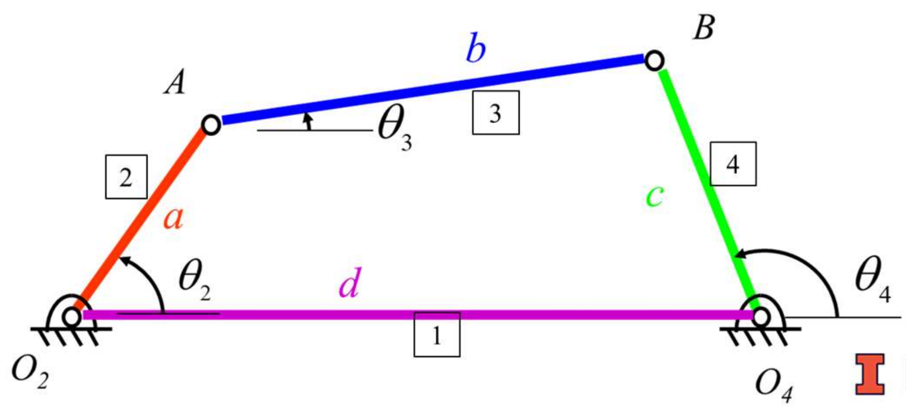
PVA analysis of a 4-bar linkage

- **Given** an existing 4-bar linkage and the angular PVA of one link (a, b, c, d, θ_2, ω_2, and α_2)
  - **Find** the angular PVA of the other links (θ_3, θ_4, ω_3, ω_4, α_3, and α_4).
- Create analytical equations that can be solved on a computer.
- Use these results
  - To determine points of zero and peak velocity or acceleration
  - To compute kinetic energy: $KE = \frac{1}{2}mv^2$
  - To compute forces and torques on links ($F = ma$, $T = I\alpha$)
- How?
  - Use vector loops



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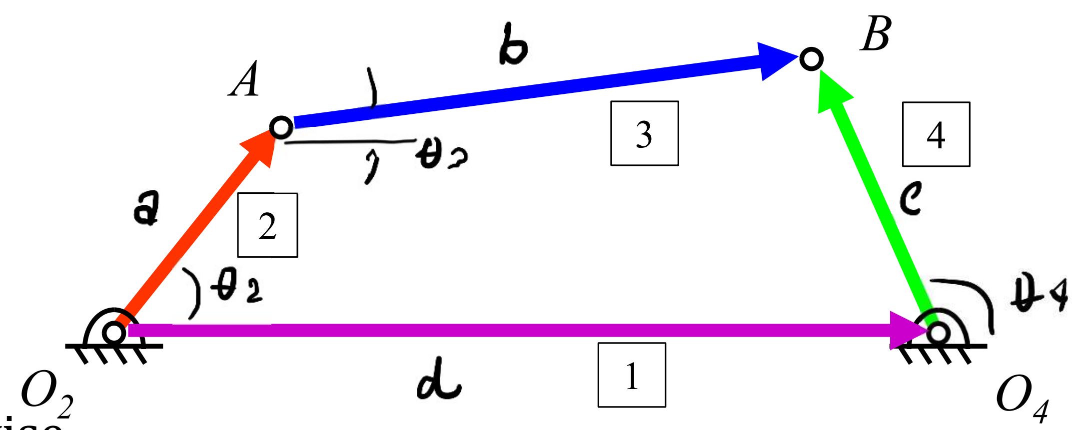
Position Vector Loop

- Vectors generally have tails at ground point
- Definitions of angles:
  - Positive magnitude: Counter-clockwise from horizontal
  - Placed at tail of vector
  - $\theta_1 = 0$, when O_2 parallel to O_4
- Vector loop equation: (clockwise)

$$\vec{R}_2 + \vec{R}_3 = \vec{R}_1 + \vec{R}_4 \quad \text{or}$$

$$\vec{R}_2 + \vec{R}_3 - \vec{R}_1 - \vec{R}_4 = 0$$

$$a e^{j\theta_2} + b e^{j\theta_3} - d e^{j\theta_1} - c e^{j\theta_4} = 0$$

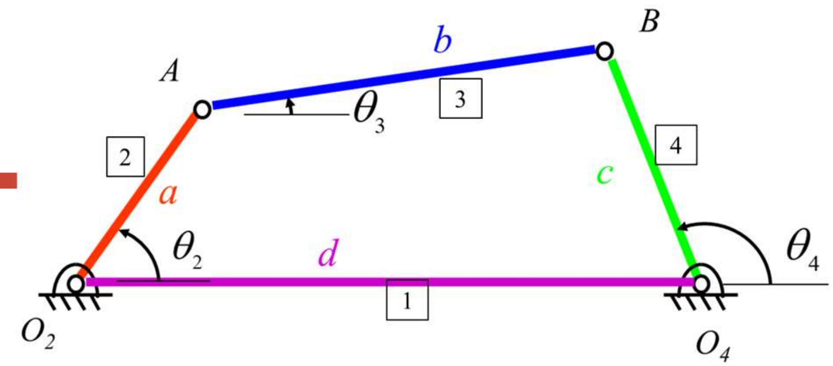


Recall:

$$\text{e.g., } \vec{R}_2 = \vec{R}_A = \vec{R}_{AO_2} = a e^{j\theta_2}$$

Label order means that this vector points from point O_2 to point A

How to solve for θ_3 and θ_4?



$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - \cancel{de^{j\theta_1}} = 0$$

Apply Euler's formula: $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$

Recall: $\theta_1 = 0$

$$a(\cos\theta_2 + j\sin\theta_2) + b(\cos\theta_3 + j\sin\theta_3) - c(\cos\theta_4 + j\sin\theta_4) - d = 0$$

Real: $a\cos\theta_2 + +b\cos\theta_3 - c\cos\theta_4 - d = 0$

Im: $a\sin\theta_2 + +b\sin\theta_3 - c\sin\theta_4 = 0$

2 Eqns
2 Unknowns

θ_2 is given, θ_3 and θ_4 are unknowns

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Solutions for θ_3 and θ_4

$$(\theta_4)_{1,2} = 2\tan^{-1}\left[\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right]$$

$$(\theta_3)_{1,2} = 2\tan^{-1}\left[\frac{-E \pm \sqrt{E^2 - 4DF}}{2D}\right]$$

where,

$$A = \cos\theta_2 - \left(\frac{d}{a}\right) - \left(\frac{d}{c}\right)\cos\theta_2 + \left(\frac{a^2 - b^2 + c^2 + d^2}{2ac}\right)$$

$$B = -2\sin\theta_2$$

$$C = \left(\frac{d}{a}\right) - \left(\frac{d}{c} + 1\right)\cos\theta_2 + \left(\frac{a^2 - b^2 + c^2 + d^2}{2ac}\right)$$

$$D = \cos\theta_2 - \left(\frac{d}{a}\right) + \left(\frac{d}{b}\right)\cos\theta_2 + \left(\frac{c^2 - d^2 - a^2 - b^2}{2ab}\right)$$

$$E = -2\sin\theta_2$$

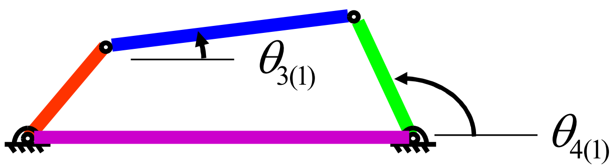
$$F = \left(\frac{d}{a}\right) + \left(\frac{d}{b} - 1\right)\cos\theta_2 + \left(\frac{c^2 - d^2 - a^2 - b^2}{2ab}\right)$$

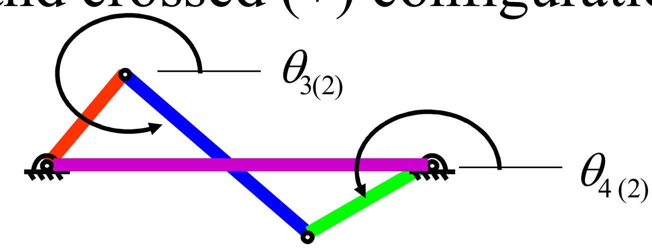
See Norton 4.5

Solutions for θ_3 and θ_4

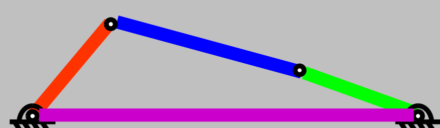
$$(\theta_4)_{1,2} = 2\tan^{-1}\left[\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right] \qquad (\theta_3)_{1,2} = 2\tan^{-1}\left[\frac{-E \pm \sqrt{E^2 - 4DF}}{2D}\right]$$

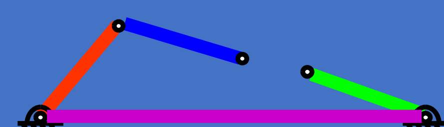
If $B^2 > 4AC$ → 1,2 are open (-) and crossed (+) configurations





If $B^2 = 4AC$ → solutions are real and equal; toggle points





If $B^2 < 4AC$ → complex conjugates; no physical solution