Lectures 12-13

Instant Centers

ME 370 - Mechanical Design 1

Lecture 10: Instant Centers

Last Time: 3-point synthesis (Specified moving points, Alternate Moving Points) **9/30/24**

Today (Reading, Norton Ch 6.3 + 6.4):

A. Instant Centers
B. Rules to find Instant Centers

Activities & Upcoming Deadlines

- **Week 6:**
  - **HW 5 (IC's) released Wednesday**
  - **Lab 6 (Project 1 Final Presentation)**
    - ***Grading rubric on Canvas***
    - ***Upload 3-slide presentation, 1-page instruction manual, and Expense Report to Gradescope immediately before lab***
    - ***Peer Evaluation#2: CATME – opens the day after Lab 6. Due in one week after lab*** **- mandatory**

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Module 4: Instant Centers

- Review of velocity
- Instant Centers (a.k.a., instantaneous center of velocity)
  - Equation to compute # ICs
- Rules for Finding ICs
  - Linear Graphs
  - Kennedy's Theorem
  - Notation
- Practice with instant centers
- Graphical Velocity Analysis with ICs
  - Mechanical Advantage
  - Velocity Ratios
- Applications of ICs
  - Ics in Suspension Design
- Reading: Norton, Chapter 6.3, 6.4

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Recall Basic Questions:

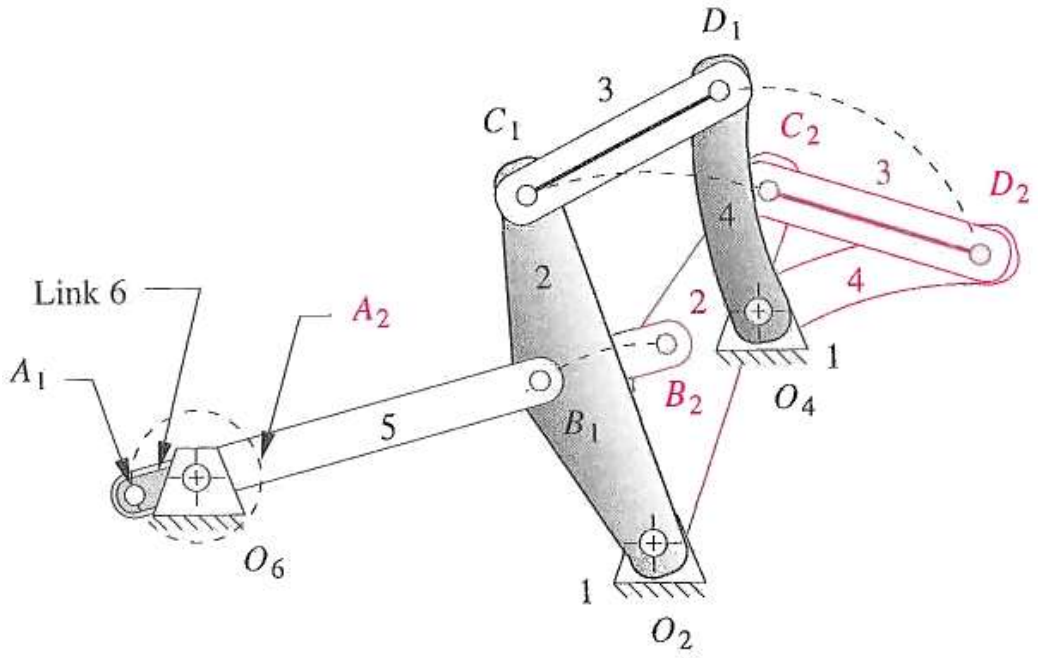
- Can we determine the motion of a mechanism by just looking at it?
- With an infinite variety of possible mechanisms can we determine which ones are useful?
- How will adding, removing, or changing links, or joints, change the functionality of a mechanism?

Recall Basic Questions:

- Can we determine the motion of a mechanism by just looking at it?
  - Identify DOF, Grashoff Condition, Path Synthesis
  - Instant Centers (IC's)
- With an infinite variety of possible mechanisms can we determine which ones are useful?
  - Toggle positions, transmission angle, Position Synthesis
- How will adding, removing, or changing links, or joints, change the functionality of a mechanism?
  - Transformation rules, Path Synthesis
  - Instant Centers (IC's)

Synthesizing Link Positions

- We can create a dyad driver to move the coupler (link 3) from one **position** to another
- Our approach is to use GLS, which gives us a “cookbook” procedure to create a mechanism that has the proper motion.
- What if we care about both **position** and **velocity**?



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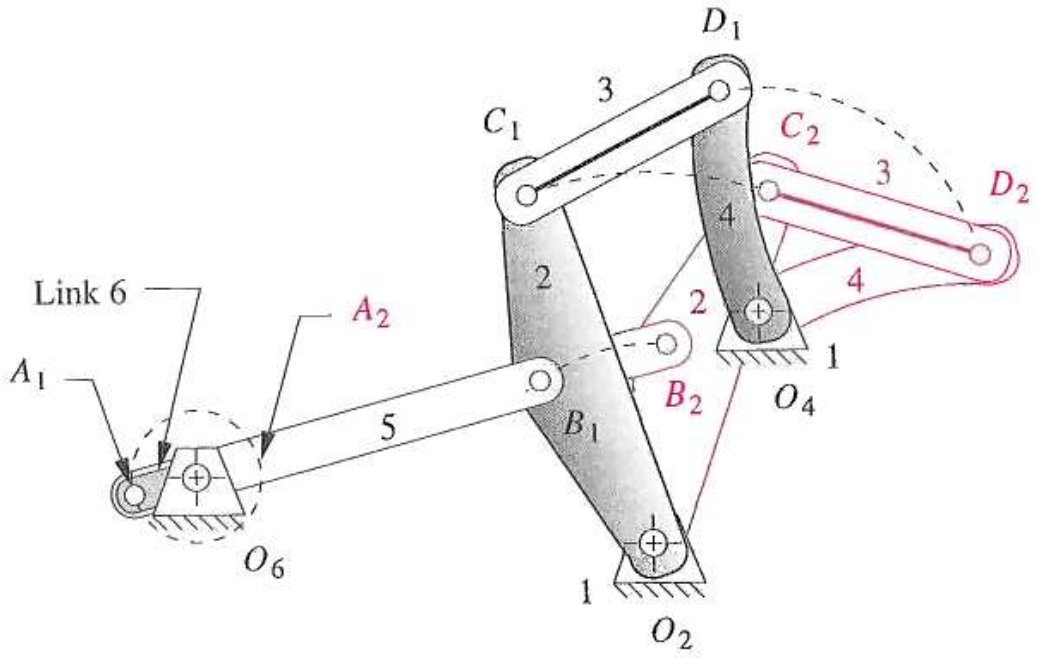
Synthesizing Link Velocities

- We can create a dyad driver to move the coupler (link 3) from one **position** to another
- Our approach is to use GLS, which gives us a “cookbook” procedure to create a mechanism that has the proper motion.
- What if we care about both **position** and **velocity**?
- Is there an easy way to synthesize the velocities here?
- Solution: Instant Centers



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Learning Objectives

By the end of this module, you should be able to:

- Explain what an Instant Center is and recall rules to find them
- Locate Instant Centers in systems of links with pin joints and other kinds of joints
- Use Instant Centers to analyze velocity ratios and mechanical advantage

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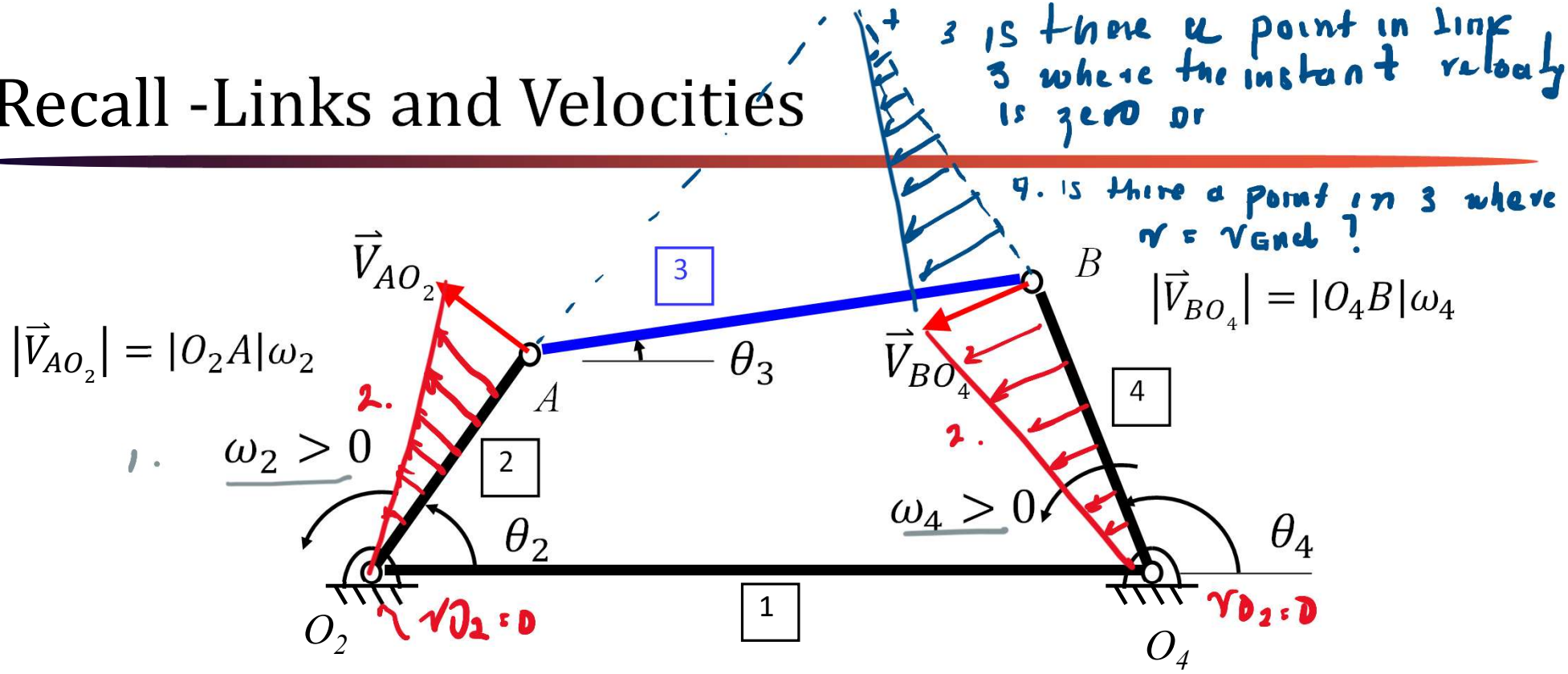
Recall -Links and Velocities



A link i has an angular (rotational) velocity ω_i which is the same everywhere on the link

The translational (linear) velocity $\vec{V}_{PO_j}$ relative to ground point O_j depends on the position of the point P on the link, e.g., pts A and B have different translational velocities, $\vec{V}_{AO_2}$ vs. $\vec{V}_{BO_4}$

The magnitude of the velocity is related to the distance of P from the ground point, e.g., $\vec{V} = \vec{r} \times \vec{\omega} \Rightarrow |\vec{V}_{AO_2}| = |O_2A|\omega_2$

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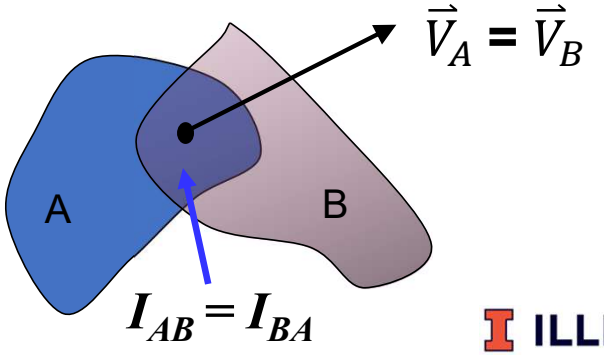
What are Instant Centers?

- An instant center (IC) is a point that is common to two bodies in plane motion
  - Naming Convention: $I_{AB} = I_{BA}$
- At the IC, both bodies have the same ***instantaneous*** translational velocity (at that instant in time).
- An IC does not have to be located ***ON*** either of the bodies.
- Each pair of bodies has an IC so for n bodies the number of ICs is:

$$\#IC = \frac{n(n-1)}{2}, \text{ where } n = \text{\# bodies}$$

For 4–bar: $\#IC = \frac{4(4-1)}{2} = 6$



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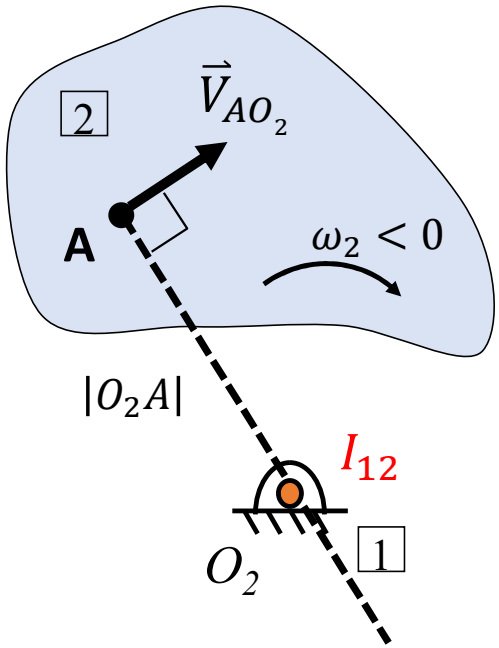
Instant Center of One Link and Ground



- The instant center between one body and the ground is its instant center of rotation.

- a body does not need to be pinned to ground to have an IC with it

- $IC_{i,j} = ICR_j$

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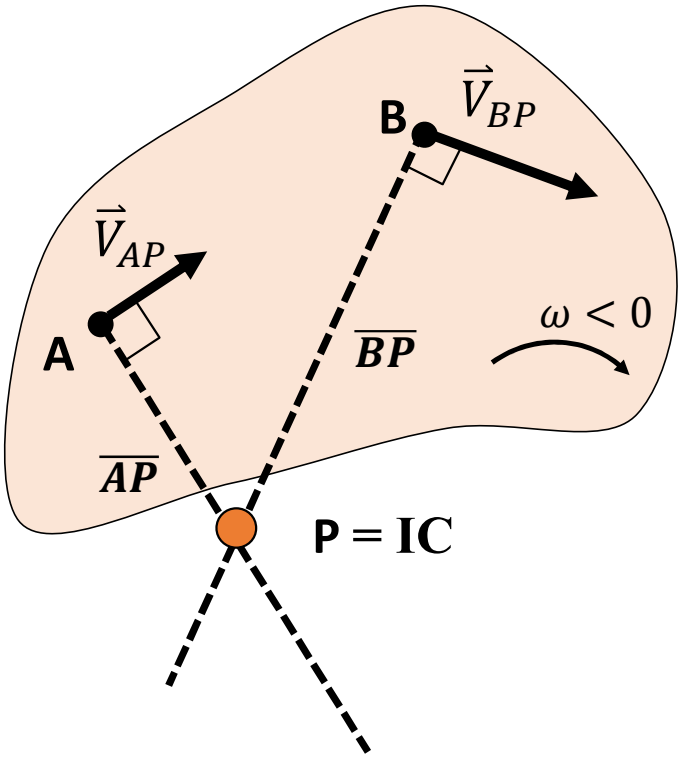
Instant Center of a Single Link



- To find the instant center for a single body when the velocities of 2 points are known, we take advantage of the fact that the linear velocities of all points in a rotating body are perpendicular to their radii of rotation.

- What happens if both points have the same velocity magnitude and direction?

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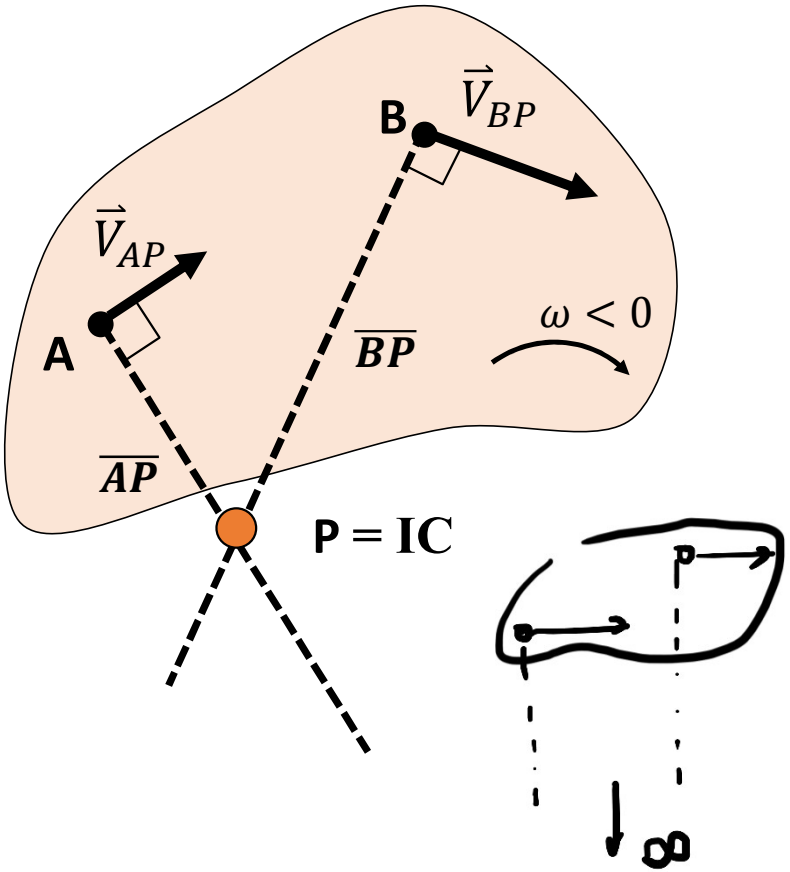
Instant Center of a Single Link



- To find the instant center for a single body when the velocities of 2 points are known, we take advantage of the fact that the linear velocities of all points in a rotating body are perpendicular to their radii of rotation.

- What happens if both points have the same velocity magnitude and direction?
- The link is a slider, and the IC is at infinity

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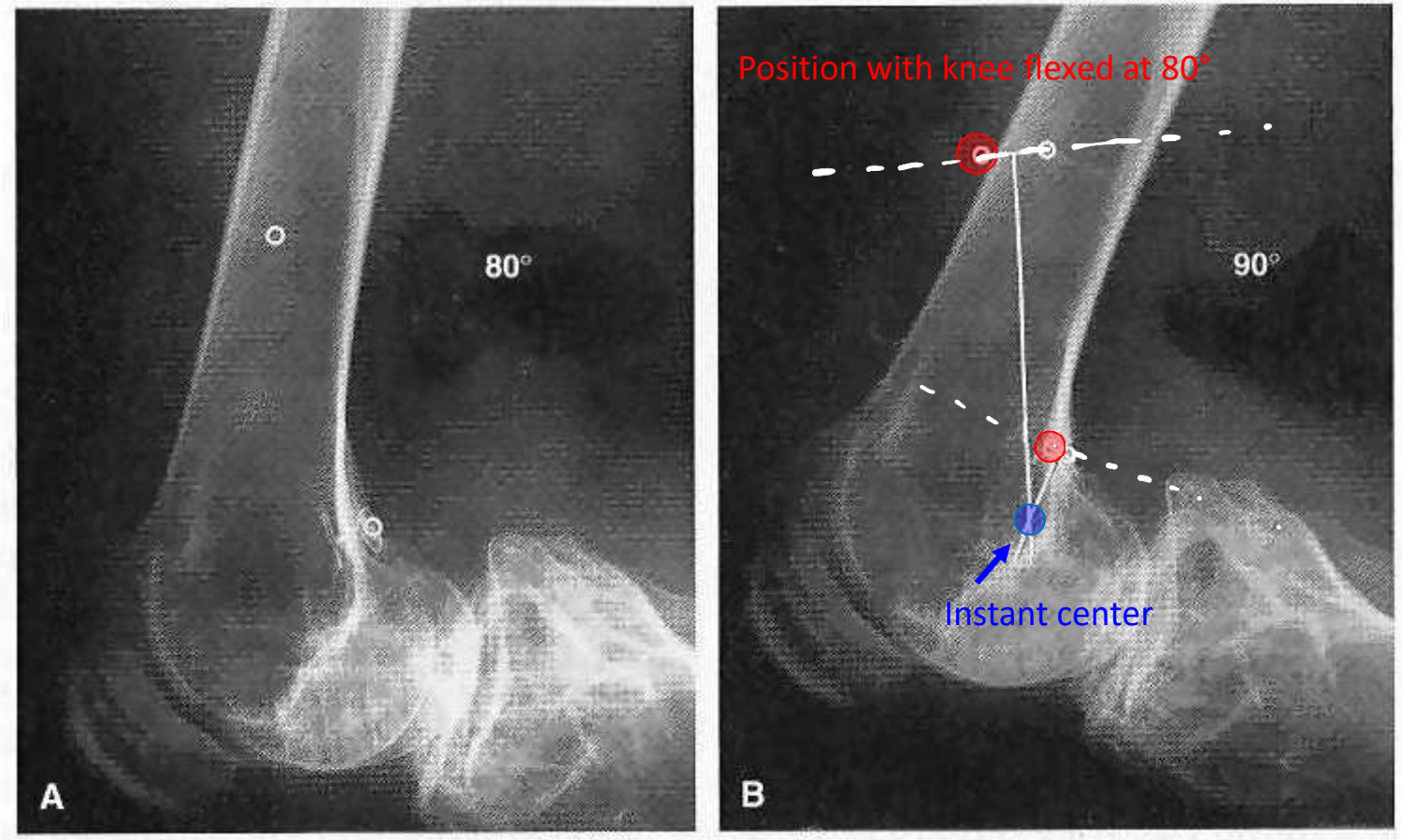
Example: The Knee Instant Center of Two Bodies



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Instant centers of the knee

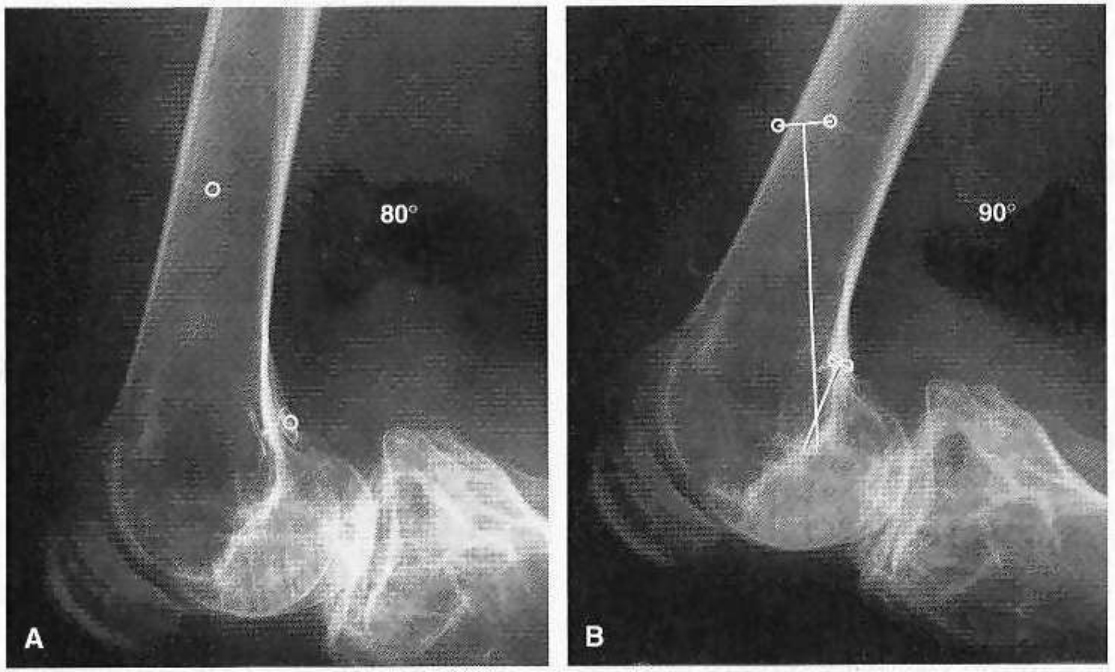

FIG. 7-4

Locating the instant center. **A,** Two easily identifiable points on the femur are designated on a roentgenogram of a knee flexed 80°. **B,** This roentgenogram is compared with a roentgenogram of the knee flexed 90°, on which the same two points have been indicated. The images of the tibiae are superimposed, and lines are drawn connecting each set of points. The perpendicular bisectors of these two lines are then drawn. The point at which these perpendicular bisectors intersect is the instant center of the tibiofemoral joint for the motion between 80 and 90° of flexion. *Courtesy of Ian Goldie, M.D. University of Gothenburg, Gothenburg, Sweden.*

Nordin & Frankel 2001

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Instant centers of the knee

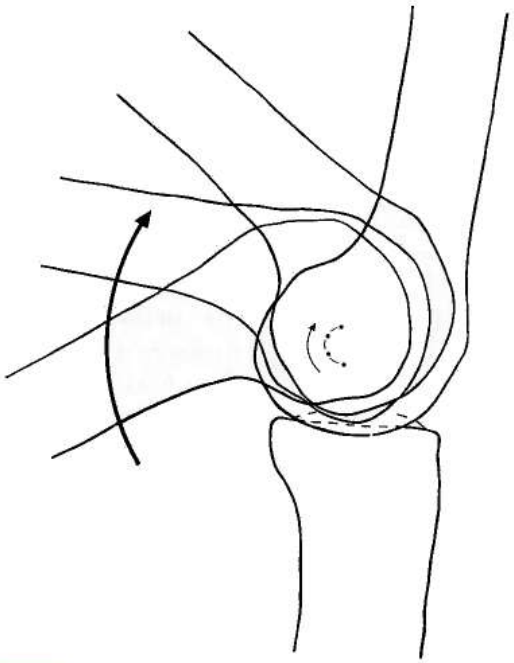

FIG. 7-5

Semicircular instant center pathway for the tibiofemoral joint in a 19-year-old man with a normal knee.

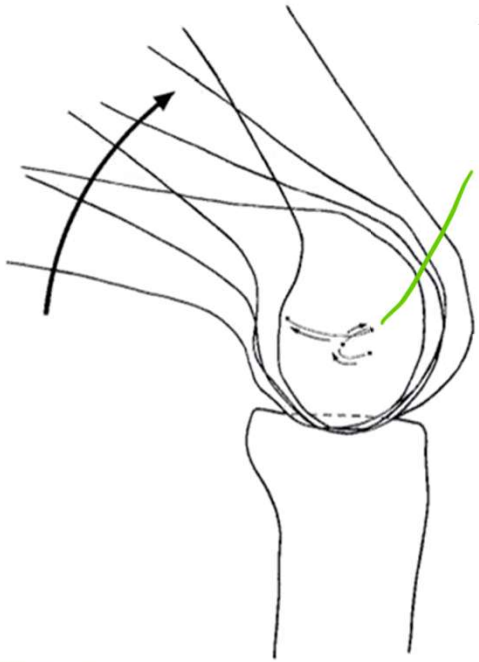

FIG. 7-7

Abnormal instant center pathway for a 35-year-old man with a bucket-handle derangement. The instant center jumps at full extension of the knee. *Adapted from Frankel, V.H., Burstein, A.H., & Brooks, D.B. (1971). Biomechanics of internal derangement of the knee. Pathomechanics as determined by analysis of the instant centers of motion.* J Bone Joint Surg, 53A, *945.*

Nordin & Frankel 2001

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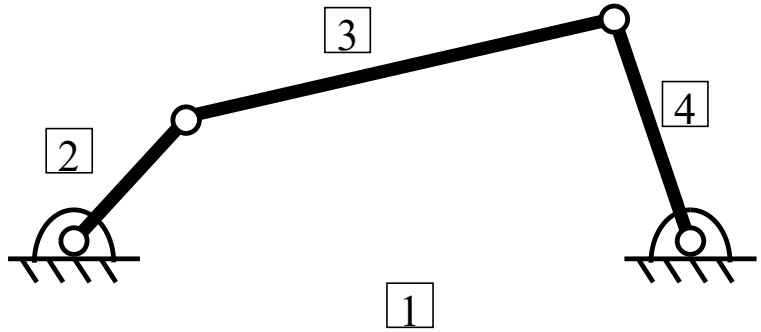
What are the rules to finding ICs?

We can use geometric rules to find ICs given a kinematic diagram.
Can you spot any ICs immediately by looking at this 4-bar linkage?



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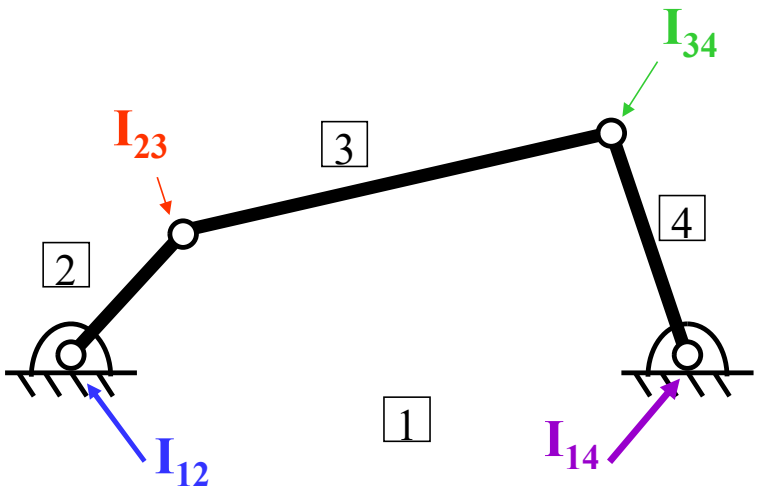
What are the rules to finding ICs? **Rule 1**

- **Rule 1:** Every pin joint is an IC

How many more ICs do we need to find?



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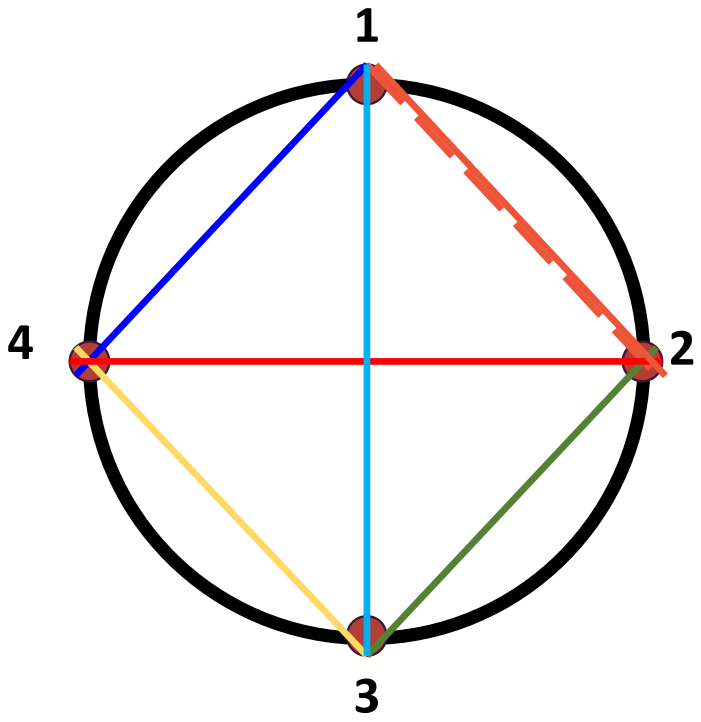
How do we keep track of them?

- Linear Graph (from graph theory) of joint connections
  - Use for bookkeeping
  - Draw a circle
  - # points on perimeter = # links
  - Draw lines between points identifying the corresponding IC. ("connect the dots")
    - Use solid line *after* have found IC
    - Use dashed if *still trying* to find IC (Rules 2-5)
      - Make solid *after* finding IC
  - # of Lines = # of ICs
    - $\#IC = \frac{n(n-1)}{2}$

  (e.g., for a four-bar mechanism, you should have 6 lines)



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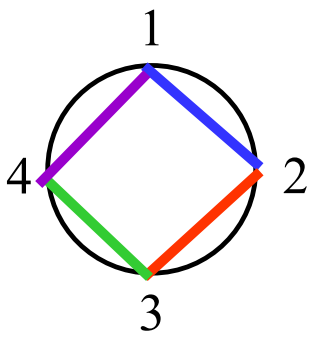

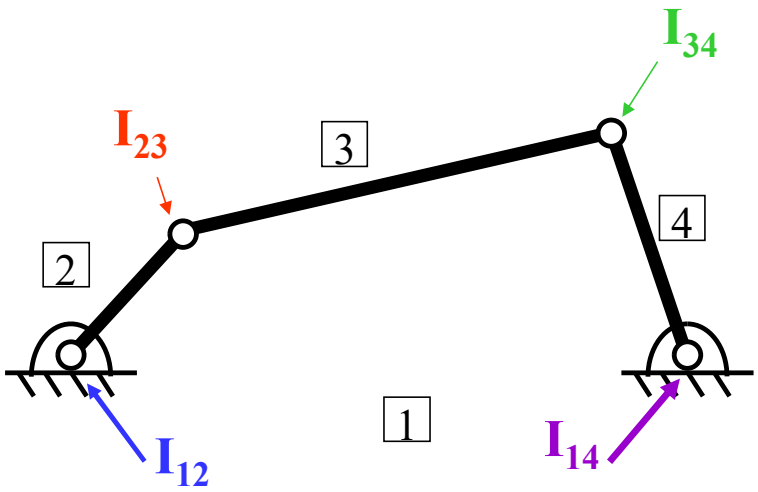

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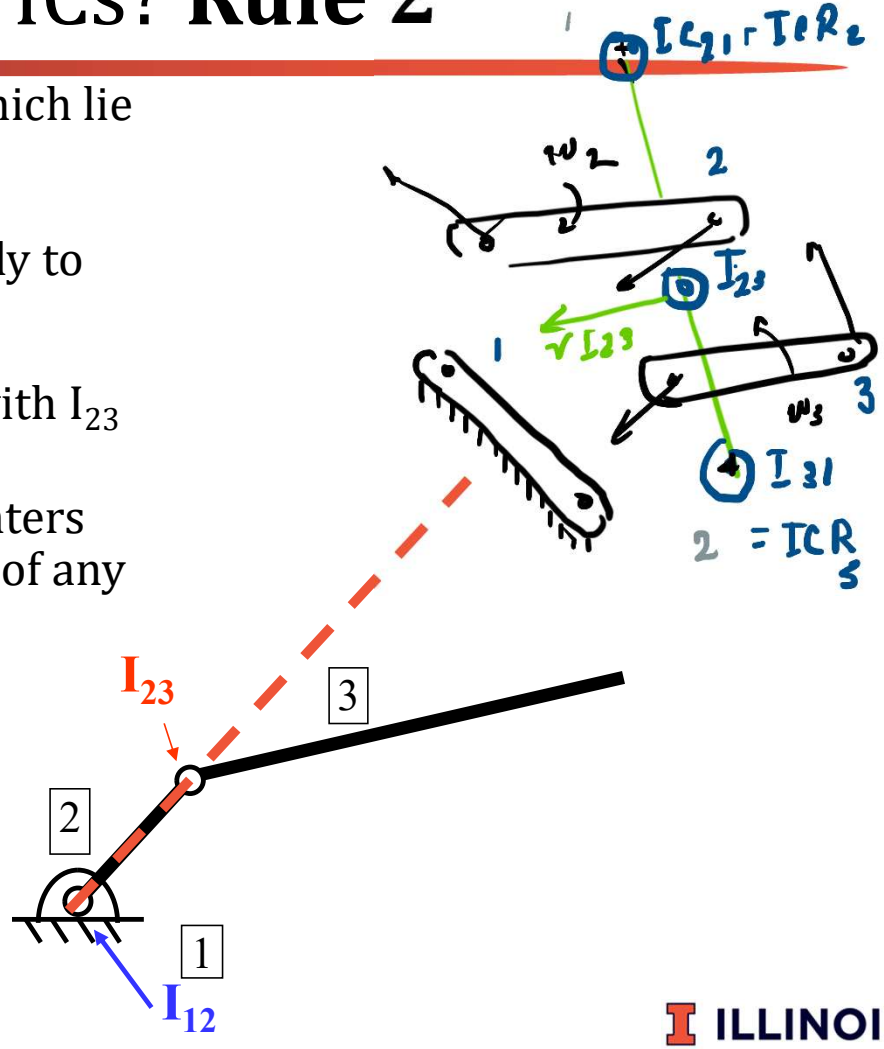
What are the rules to finding ICs? **Rule 2**

- **Rule 2:** 3 bodies in a plane will have exactly 3 ICs which lie in a line **(Kennedy's theorem)**
  - These bodies do not have to be connected directly to each other
  - The 3 ICs will lie on the same straight line
    - e.g., I_{13} lies on the same line connecting I_{12} with I_{23}
- Use line graph to determine triple sets of instant centers
  - i.e., intersection of lines identifies exact location of any third IC
- Contraction of indices gives instant center

  Ex. I_{12}, I_{23} gives I_{13}

  I_{14}, I_{43} gives I_{13}



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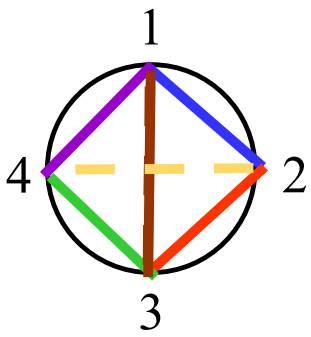
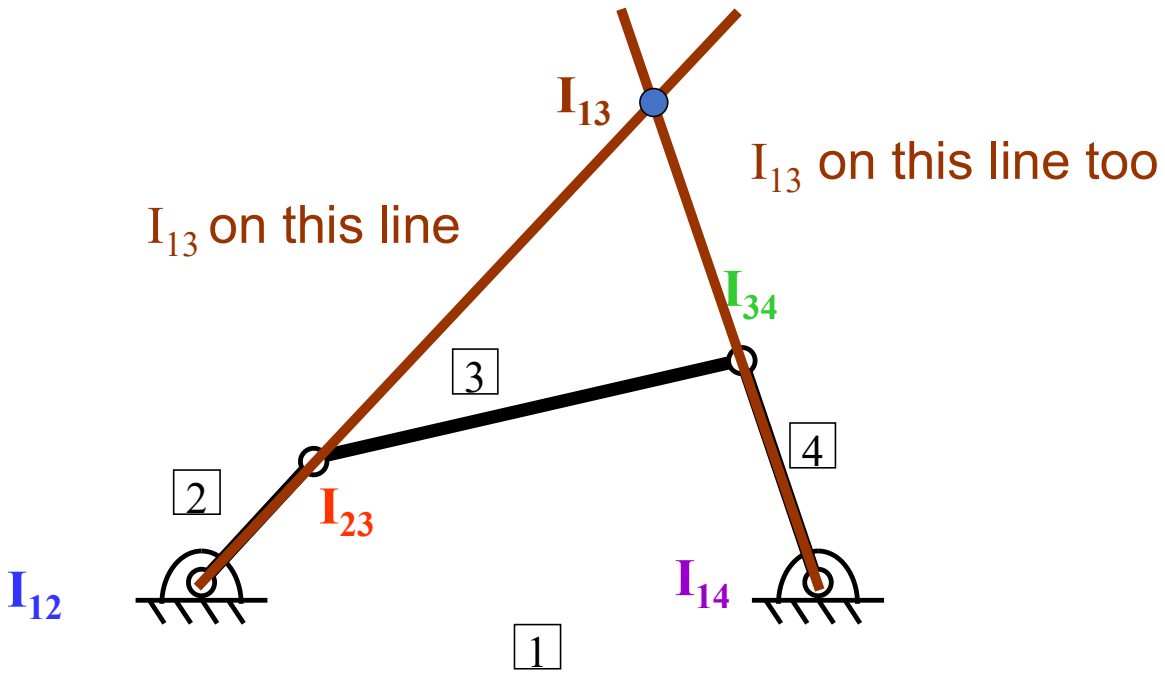


I_{13} lies on same line of I_{12}, I_{23}

I_{13} lies on same line of I_{14}, I_{43}



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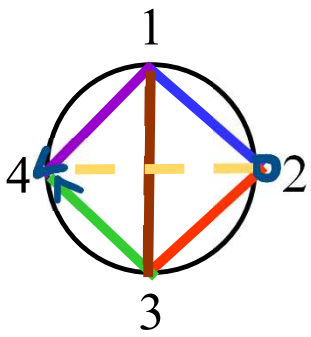
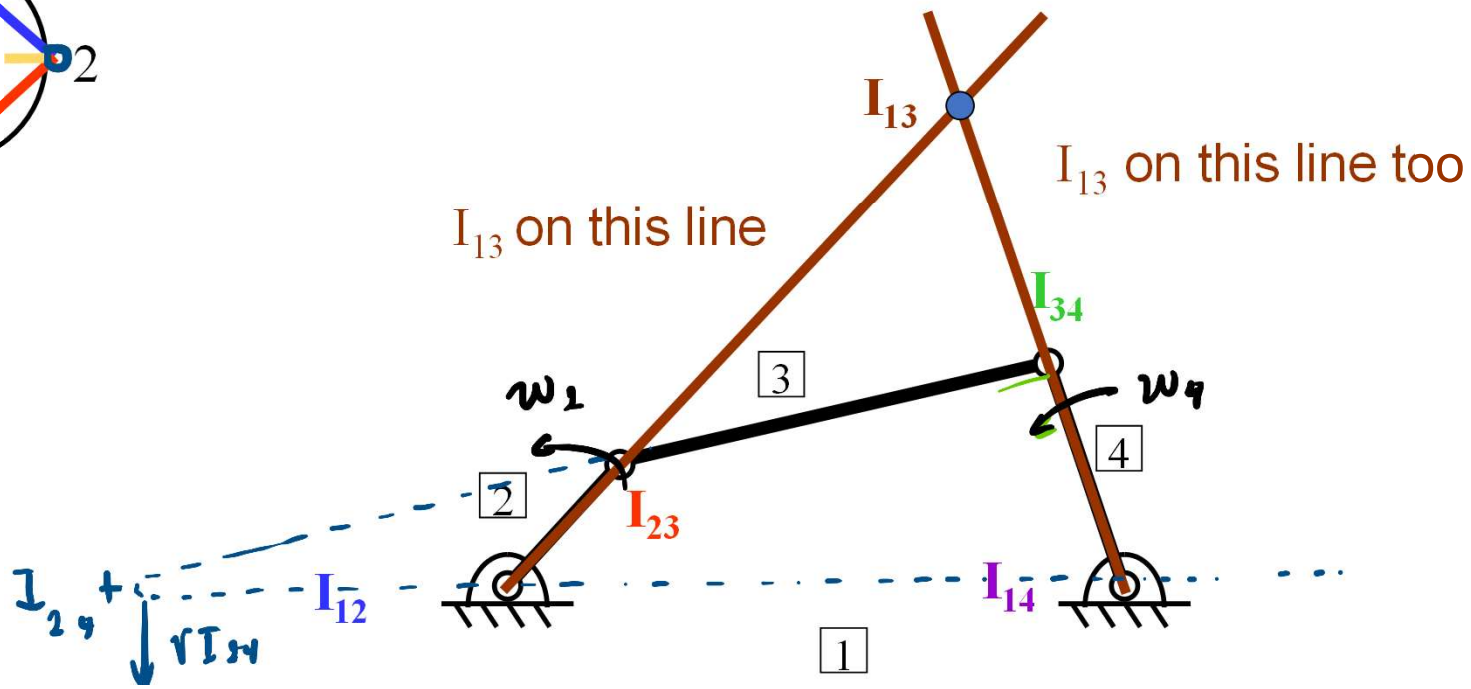
Exercise:

- Where is I_{24} ?

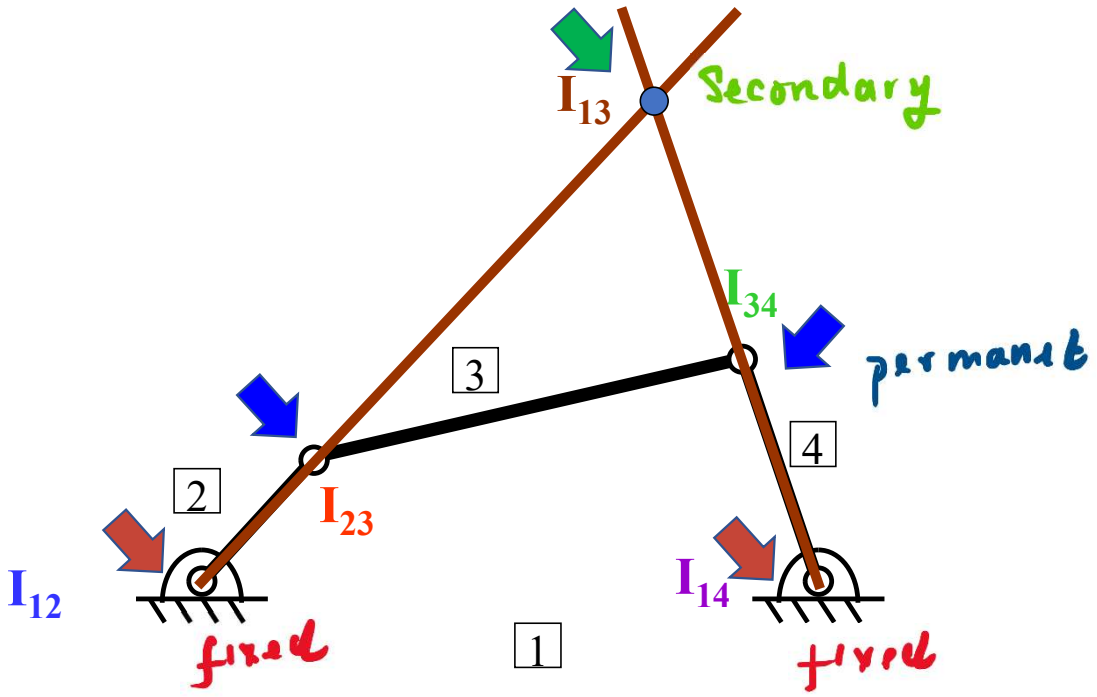
which body spins faster 2 or 4.





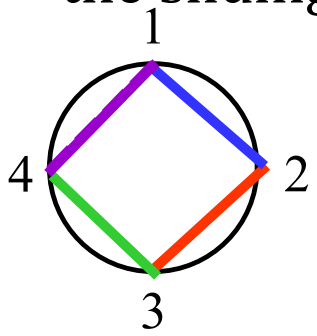
Types of ICs

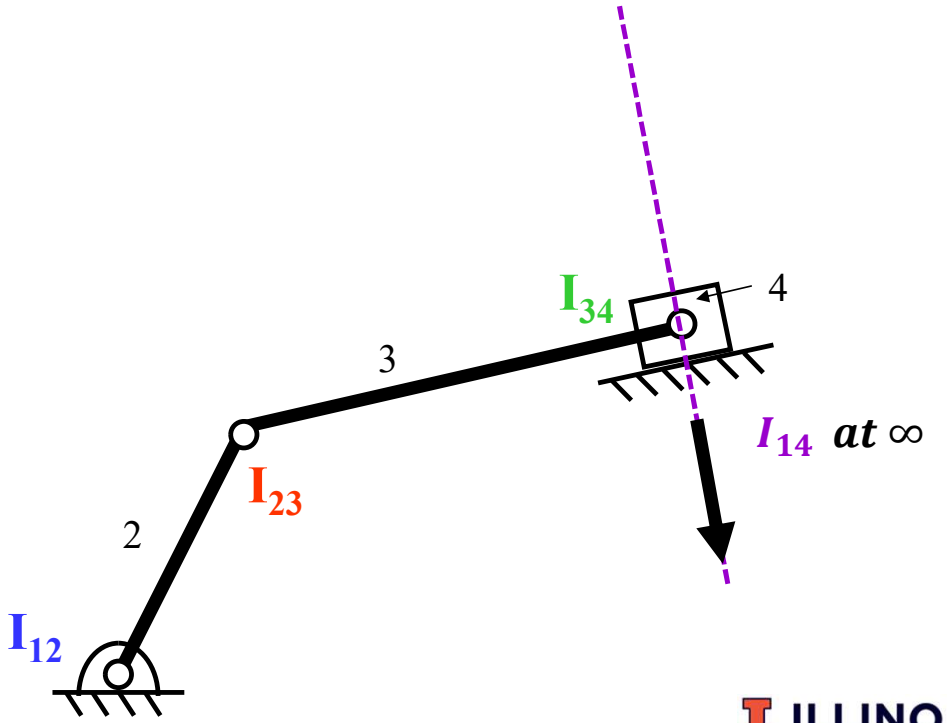
- Primary
  - Fixed – Do not move (e.g., IC at GND nodes)
  - Permanent- Can move with the linkage, but their location is fixed relative to the linkage (e.g., IC at Pin Joint)
- Secondary
  - Can move and their location changes as the linkage moves (e.g., Kennedy Theorem ICs)



What are the rules to finding ICs?

- **Rule 3:** The IC for a sliding joint is at infinity and is perpendicular to the sliding direction.





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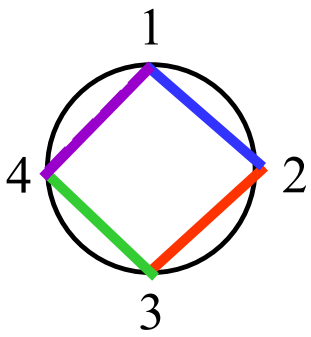
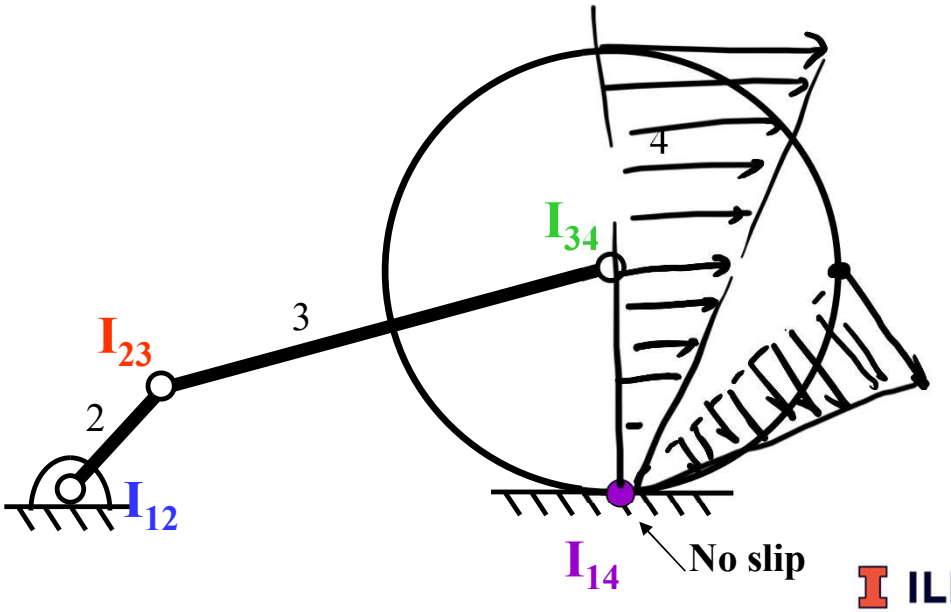
What are the rules to finding ICs?

- **Rule 4:** For two bodies in rolling contact, the contact point is their IC





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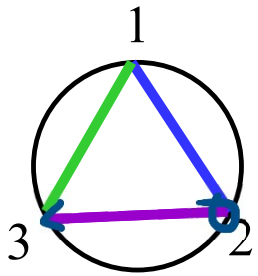
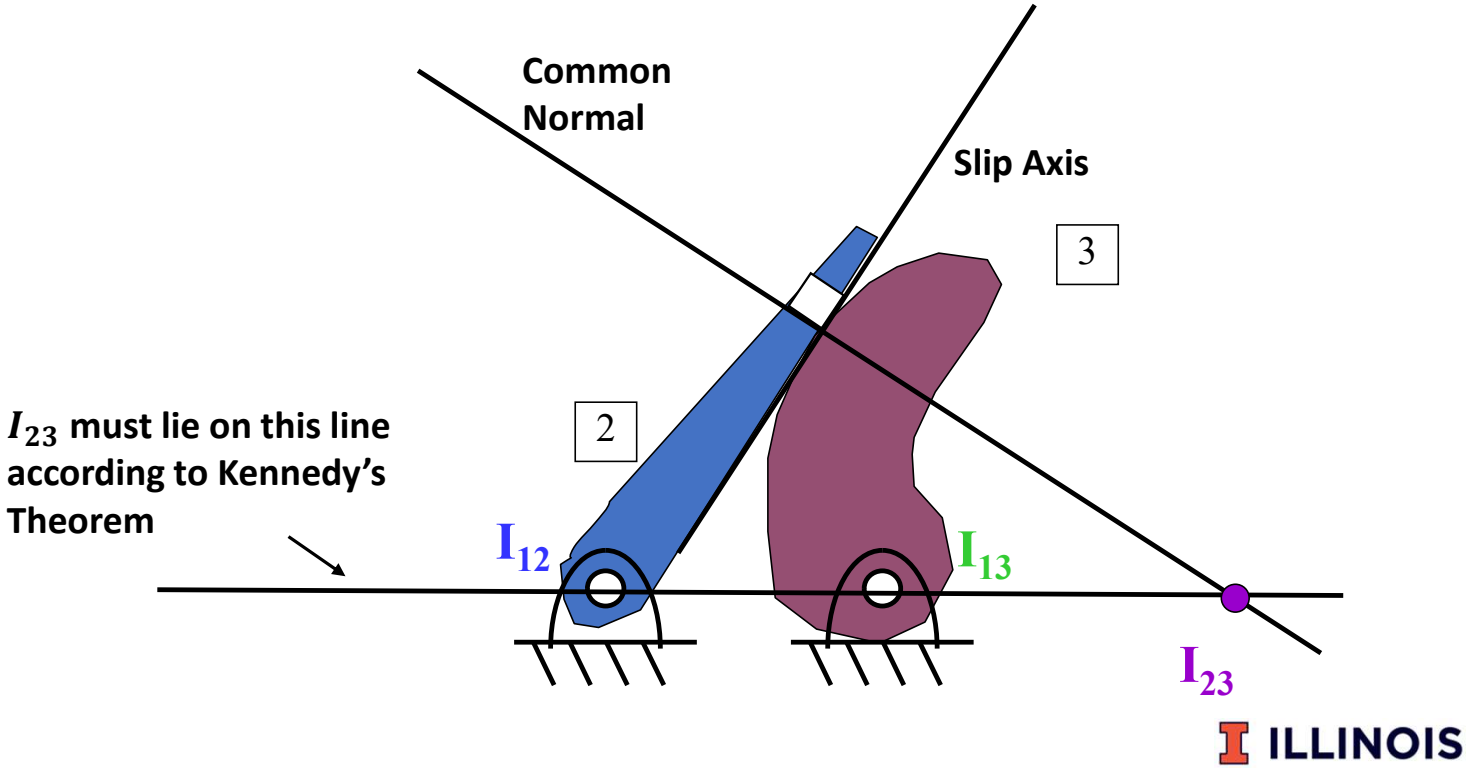
What are the rules to finding ICs?

- **Rule 5:** For two bodies in rolling contact <u>with slip</u> the IC lies on the common normal, following Kennedy's Theorem





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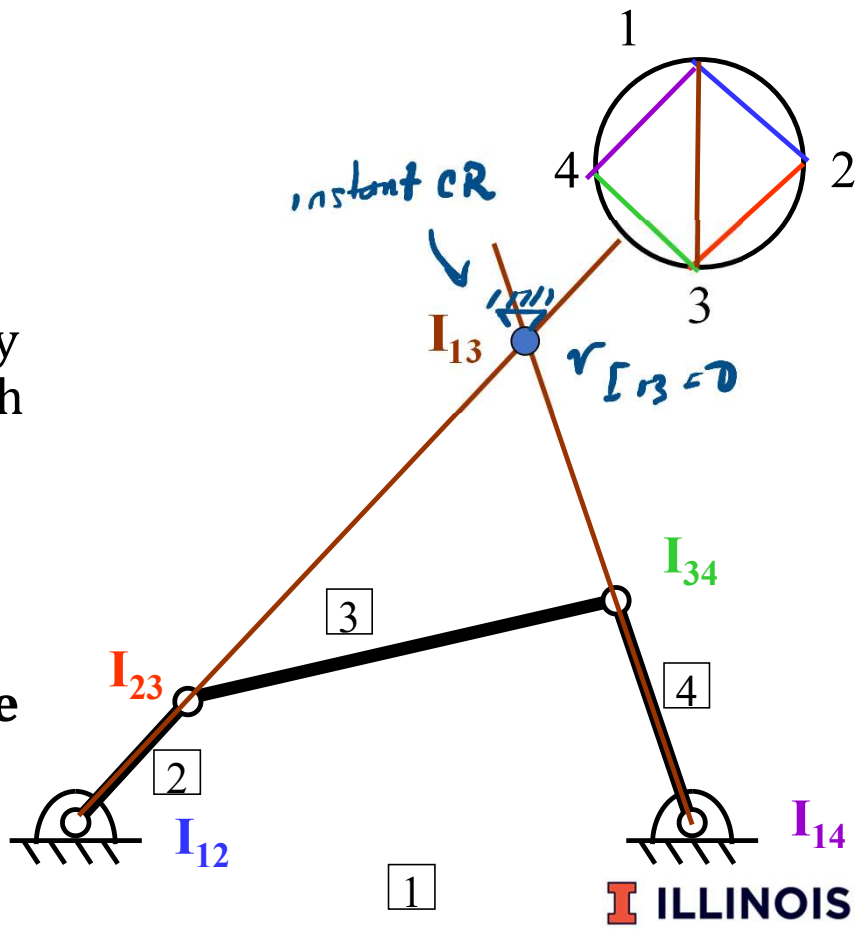
Summary of the rules to finding ICs

- **Rule 1:** Every pin joint is an IC
- **Rule 2:** 3 bodies have 3 ICs which lie in a line (Kennedy's theorem)
- **Rule 3:** The IC for a sliding joint is at infinity and is perpendicular to the sliding direction.
- **Rule 4:** For two bodies in rolling contact, the contact point is their IC
- **Rule 5:** For two bodies in rolling contact with slip the IC lies on the common normal, following Kennedy's Theorem

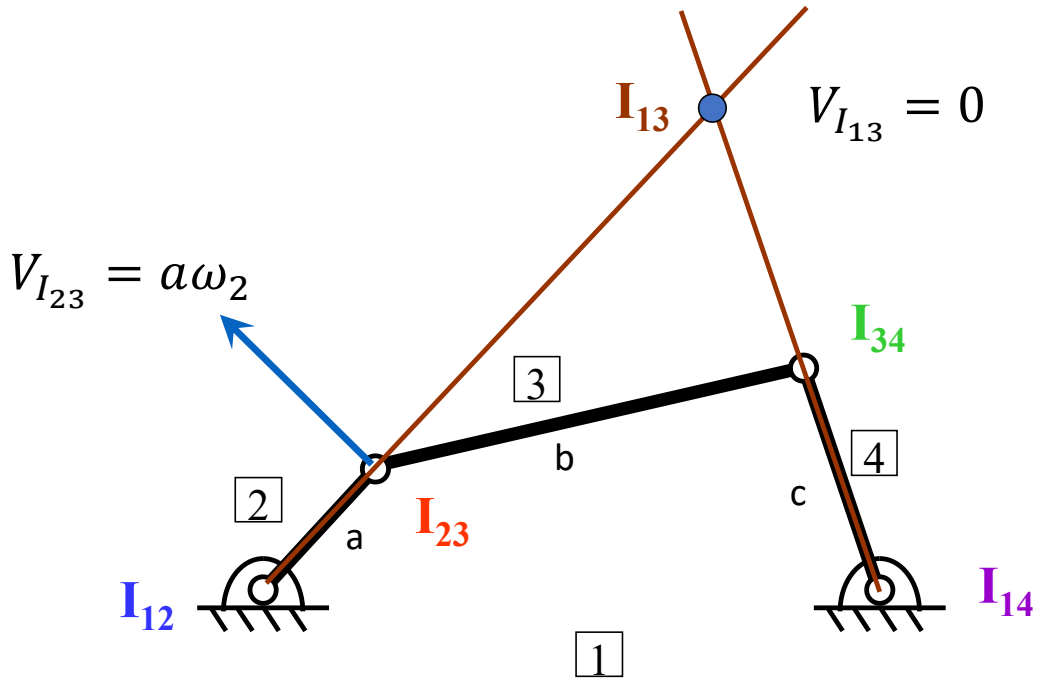
What does I_{13} mean? What is velocity @I_{1x}?

- I_{13} is a point in space where link 1 and link 3 have the same velocity (instantaneously) if they were connected
- Note that always, $V(@I_{1x}) = 0$
  (where x is any other link number)
  - Since Link 1 (ground) is fixed, i.e., $V_1 = 0$, then by definition*, the velocity of any IC associated with link 1 is zero at **THIS INSTANT IN TIME**.
    - Location of I_{1x} will change at next instant of time.
- **So since $V(@I_{1x}) = 0$, then I_{1x} can be considered to be a "fixed point" about which link *x* is in pure rotation with respect to link 1**

** Instant Center ≡ point common to both bodies in which the point has the same velocity in each body.*



Velocity Ratios: The Inputs



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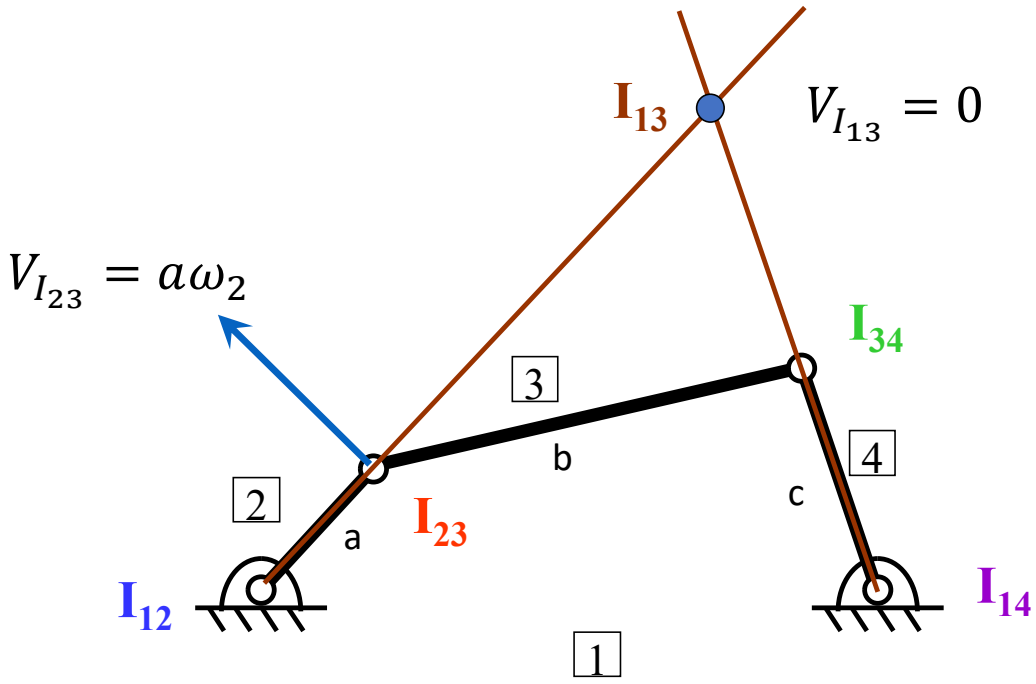
Velocity Ratios: The Inputs

Start with this knowledge.
Say, we know ω_2.
As a result, we know $V_{I_{23}}$

In this system, we can find I_{13}
And we also know $V_{I_{23}} = 0$



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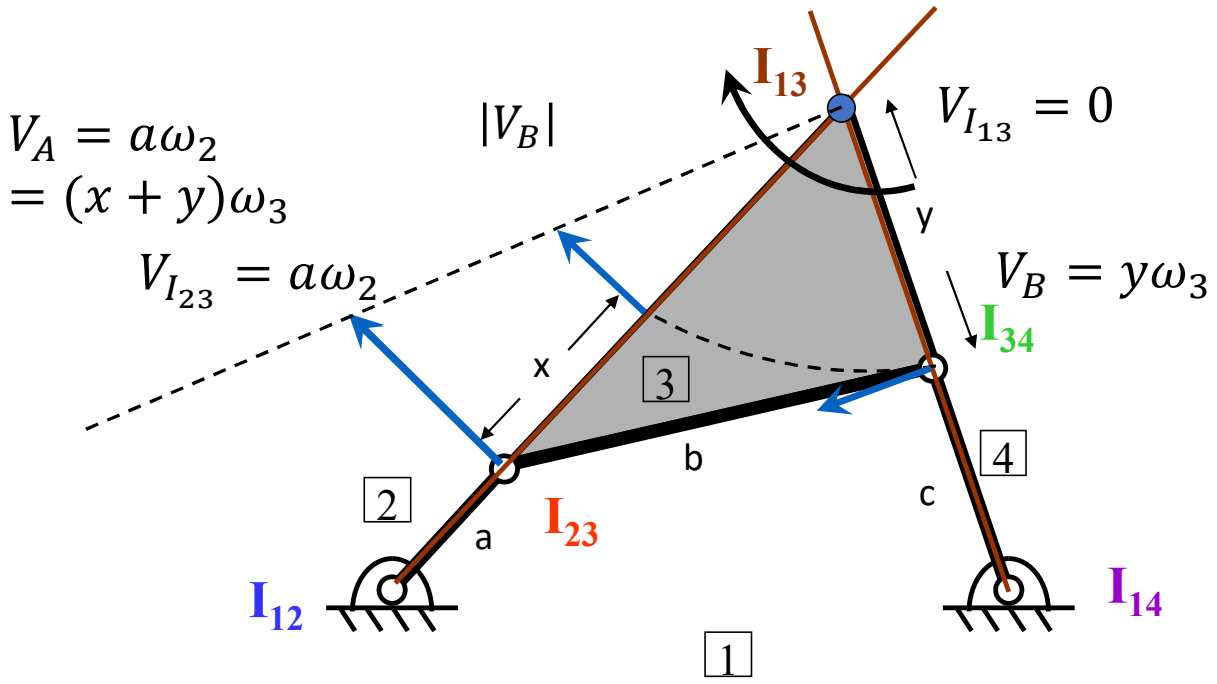
Velocity Ratios: A Virtual Link



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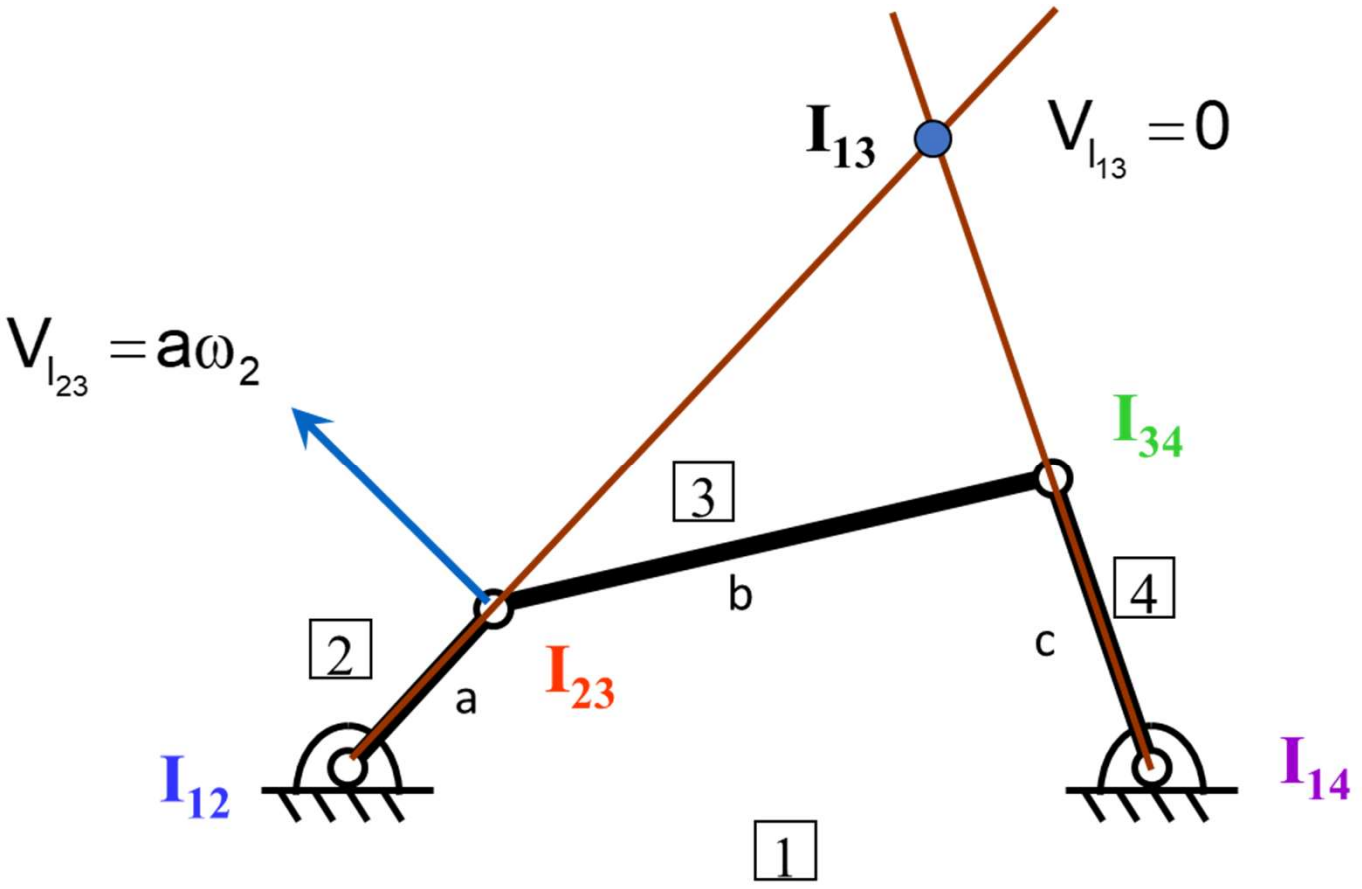

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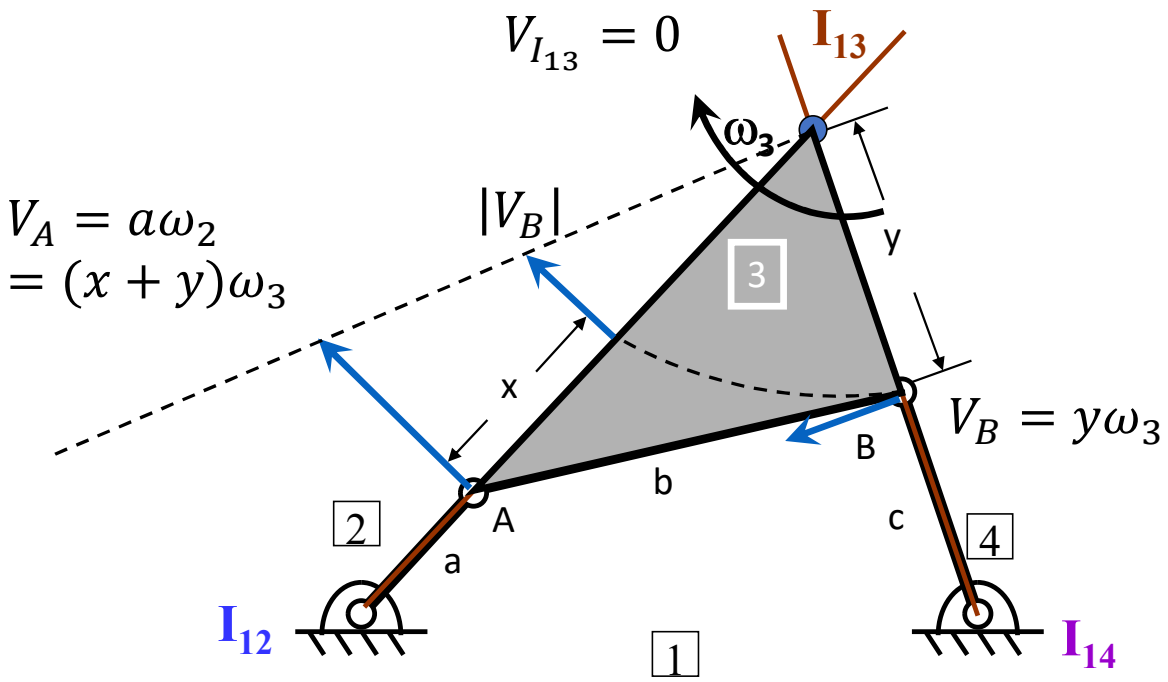

Every point in link 3 has the same ω_3

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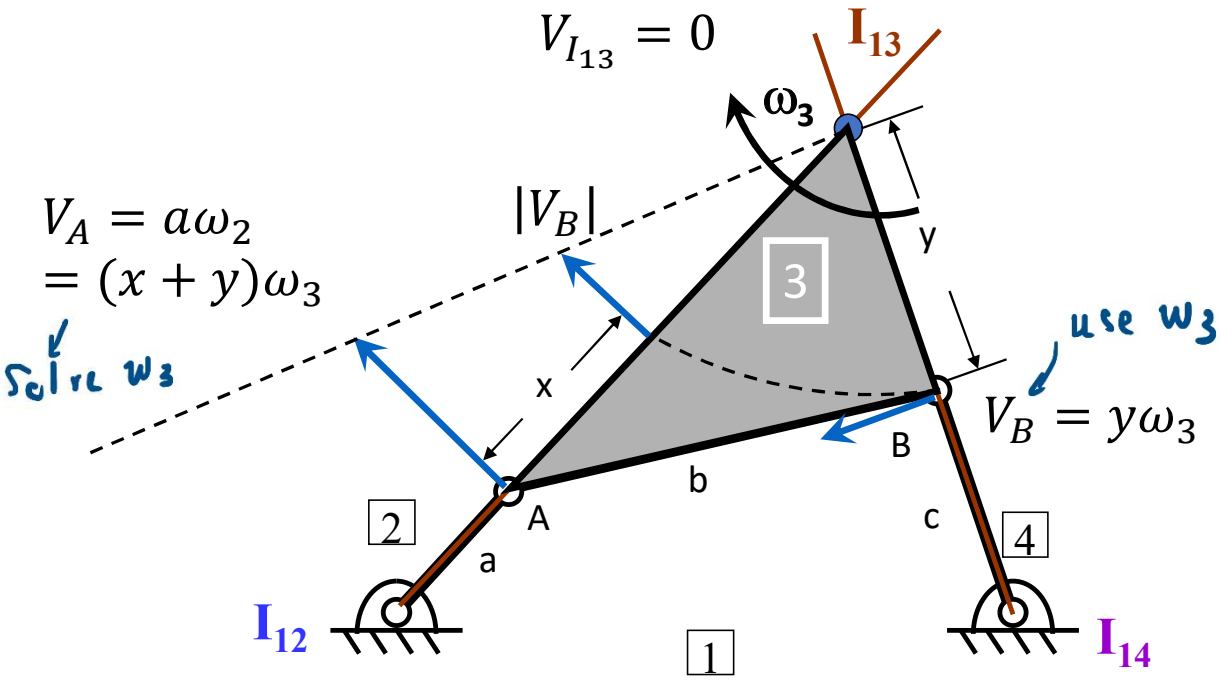
Velocity Ratios: Big Idea

- **IC's allow us to compute velocities graphically**

- **Procedure:**
- Identify IC between links of interest and ground
- IC between link and ground has V=0
- Link has same ω everywhere. If we know the velocity at one point, we can compute velocities everywhere
- $v = r \times \omega$



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