Module 7

Lecture 22:

# Dynamic Force

Analysis (DFA) - 3

"Colibri" by Derek Hugger

www.youtube.com/watch?v=1scj5sotD-E

Fourbar slider-crank

(2) Draw free-body diagram of each segment.

On each link, label local coordinate systems, external joint forces & torques, kinematics, internal forces & torques, and position vectors. Note equal and opposite forces on links at joints. Label forces such that force vector $\vec{F}_{ij}$ represents the force of link i **on** link j and is applied at the common joint on link j.

T_{12} (external torque, motor)

T_{21}
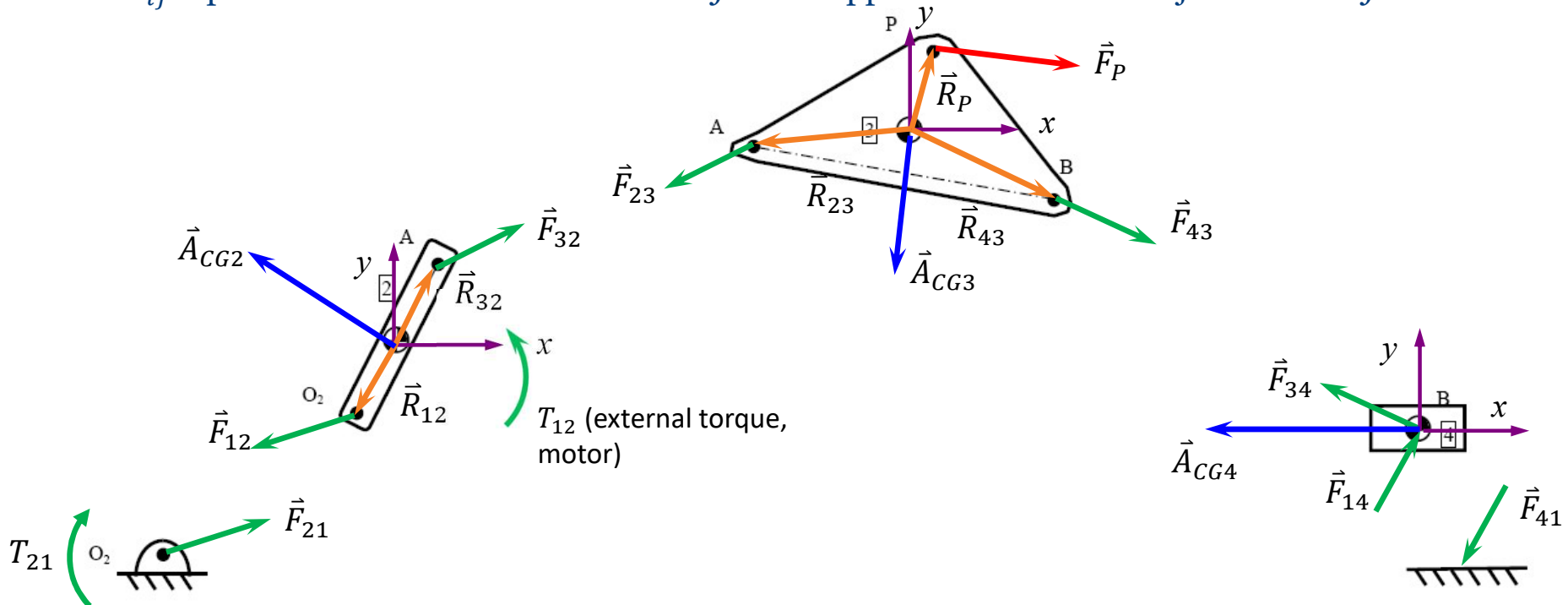
Fourbar slider-crank

(3) Symbolically write out equations of motion for each moving link

For Link 3:

$$\sum \vec{F}_i = m_i \vec{A}_{CGi} \qquad \sum T_{iz} = I_{CGi}\alpha_i$$

$$\sum \vec{F} = \vec{F}_P + \vec{F}_{23} + \vec{F}_{43} = m_3 \vec{A}_{CG3}$$

$$\sum T_z = (\vec{R}_P \times \vec{F}_P) + (\vec{R}_{23} \times \vec{F}_{23}) + (\vec{R}_{43} \times \vec{F}_{43}) = I_{CG3}\alpha_3$$

Recall: $\vec{F}_{23} = -\vec{F}_{32}$

$$F_{23x} + F_{43x} + F_{px} = m_3 A_{CG3x}$$

(new variables are introduced with positive sign)

$$-F_{32x} + F_{43x} + F_{Px} = m_3 A_{CG3x}$$

$$-F_{32y} + F_{43y} + F_{Py} = m_3 A_{CG3y}$$

Add 2 more unknowns, and 3 more equations

$$(-R_{23x}F_{32y} + R_{23y}F_{32x}) + (R_{43x}F_{43y} - R_{43y}F_{43x}) + (R_{Px}F_{Py} - R_{Py}F_{Px}) = I_{CG3}\alpha_3$$

Torque of F32   torque of F43   Torque of Ext form

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Fourbar slider-crank

For Link 4:

$$\sum \vec{F}_i = m_i \vec{A}_{CGi} \qquad \sum T_{i\,z} = I_{CGi}\alpha_i$$

→ *does not rotate so*

$$\alpha_4 = 0$$

$$A_{CG4y} = 0$$

$$R_{ij} = 0$$

No torque balance necessary!

all force pass through CG

$$0 = 0$$

$\vec{F}_{34}$

$\vec{A}_{CG3}$

$\vec{F}_{14}$
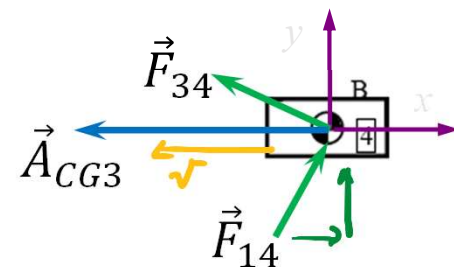
$F_{14x} = \mu F_{14y}$

Friction on slider:

not an unknown

← $F_{14x} = \pm\mu F_{14y} = -\text{sign}(V_{CG4x})\,\mu F_{14y}$

$$F_{14x} - F_{43x} = m_4 A_{CG4x} \rightarrow \pm\mu F_{14y} - F_{43x} = m_4 A_{CG4x}$$

→ $\Sigma f_{xc} = m A_{G4ax}$

$$F_{14y} - F_{43y} = 0$$

Add 1 unknown, and 2 more equations
→ 8 unknowns, 8 equations
(note: originally 9 unknowns, but can find F_{14x} through friction equation)

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Gearset

Find: internal forces between links, and driving torque T_{12}

1. *Draw complete system. Label points, dimensions, external forces & torques, kinematics.*
   - Take the Gear as body 2 and the Pinion as body 3.
   - Consider the torque at the pinion, T_{13}, to be known.

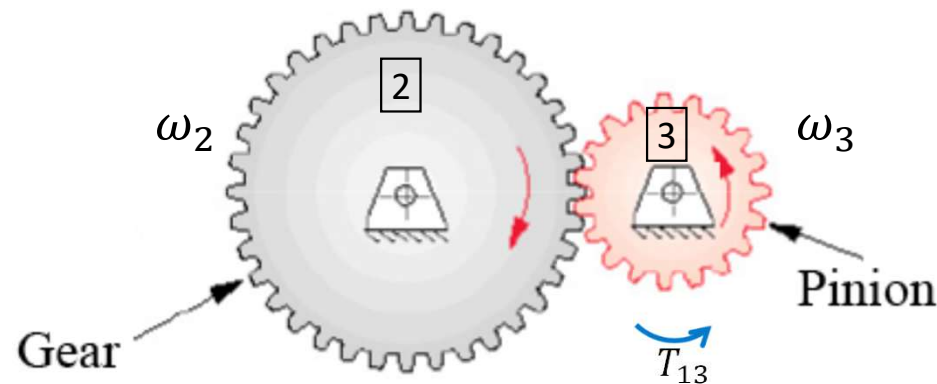Knowns:

T_{13}

For $i = 2, 3$:

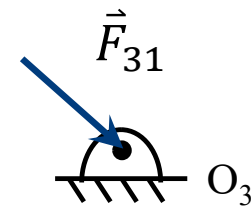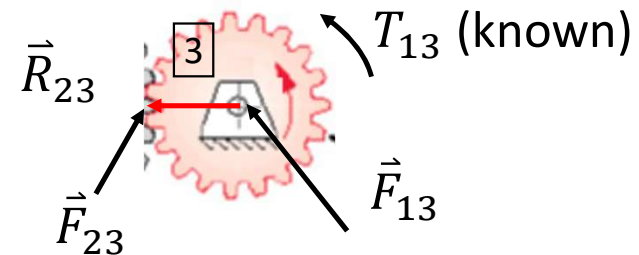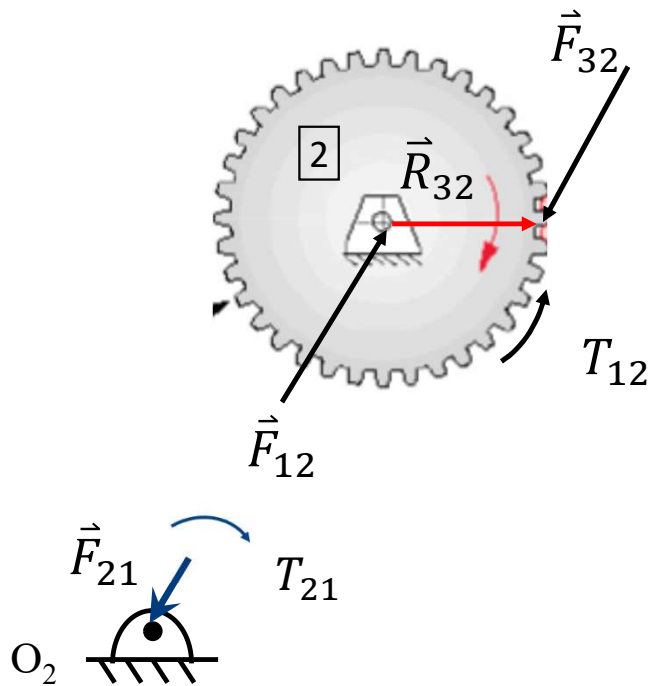$\theta_i, \omega_i, \alpha_i$

$\vec{A}_{CGi} = 0$

Unknowns:

T_{12}

$\vec{F}_{12}, \vec{F}_{13}, \vec{F}_{23}$

ω_2 2   3 ω_3

Pinion

Gear

T_{13}

Gearset

$\vec{F}_{32}$

$\boxed{2}$ $\vec{R}_{32}$

T_{12}

$\vec{F}_{12}$

$\vec{F}_{21}$ T_{21}

O_2

$\vec{R}_{23}$ $\boxed{3}$ T_{13} (known)

$\vec{F}_{13}$

$\vec{F}_{23}$

$\vec{F}_{31}$

O_3

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Gearset

3. Write the set of equations for each of the gears. You should have 3 equations for each gear. List the unknowns.

Gear

$$F_{12x} + F_{32x} = m_2 A_{CG2x}$$

$$F_{12y} + F_{32y} = m_2 A_{CG2y}$$

$$T_{12} + (R_{32x}F_{32y} - R_{32y}F_{32x}) = I_{CG2}\alpha_2$$

$$F_{13x} - F_{32x} = m_3 A_{CG3x}$$

Pinion

$$F_{13y} - F_{32y} = m_3 A_{CG3y}$$

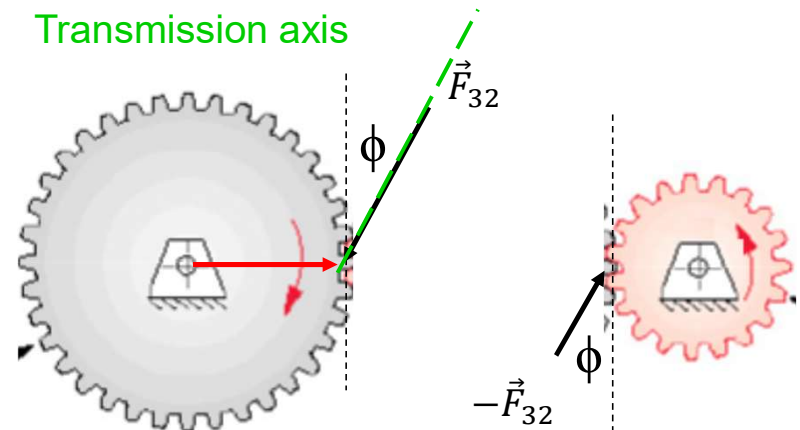$$T_{13} - (R_{23x}F_{32y} - R_{23y}F_{32x}) = I_{CG3}\alpha_3$$
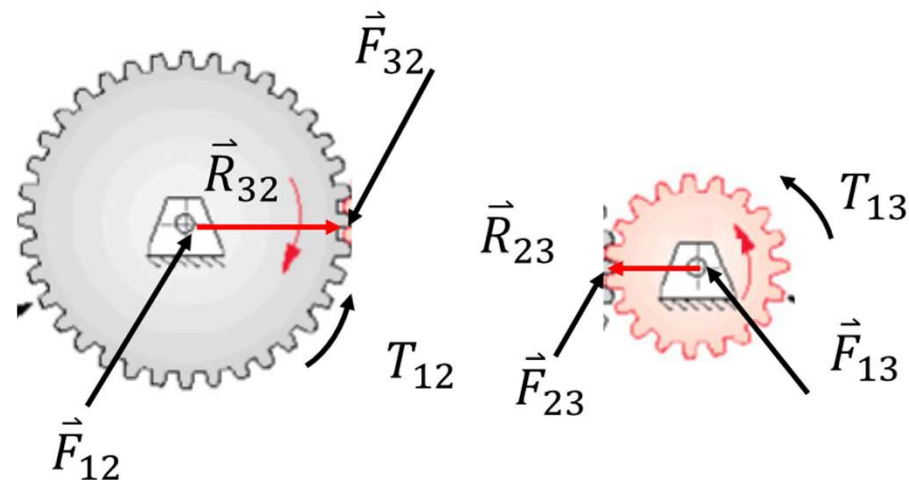
Gearset – identifying constraints

Too many unknowns, need an additional constraint

Fixed pressure angle adds constraint for gear set:

$$F_{32x} =$$

Transmission axis

$\vec{F}_{32}$

ϕ

$-\vec{F}_{32}$

ϕ

where ϕ is the pressure angle from gear tooth geometry
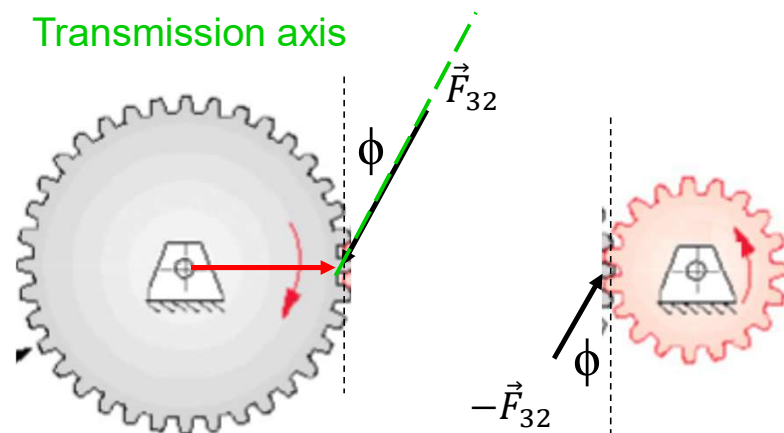
Gearset – identifying constraints

Too many unknowns, need an additional constraint

Fixed pressure angle adds constraint for gear set:

Transmission axis

$$F_{32x} = \tan(\phi)\left|F_{32y}\right|$$
$$= \tan(\phi)\,\text{sign}\,(F_{32y})F_{32y}$$
$$= cF_{32y}$$
$$F_{32x} = cF_{32y}$$

$$\Rightarrow c = \tan(\phi)\,\text{sign}(F_{32y})$$

where ϕ is the pressure angle from gear tooth geometry

Gearset

(1) $F_{12x} + F_{32x} = mA_{CG2x}$

(2) $F_{12y} + F_{32y} = mA_{CG2y}$

(3) $T_{12} + (R_{32x}F_{32y} - R_{32y}F_{32x}) = I_{CG2}\alpha_2$

(4) $F_{13x} - F_{32x} = mA_{CG3x}$

(5) $F_{13y} - F_{32y} = mA_{CG3y}$

(6) $T_{13} - (R_{23x}F_{32y} - R_{23y}F_{32x}) = I_{CG3}\alpha_3$

(7) $F_{32x} = cF_{32y}$

$c = \tan(\phi)\,\text{sign}(F_{32y})$

Matrix solves for 6 unknowns. Use equation 7 to solve for F_{32x}

$$\begin{bmatrix} 1 & 0 & c & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & X & 0 & 0 & 1 \\ 0 & 0 & -c & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & Y & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} F_{12x} \\ F_{12y} \\ F_{32y} \\ F_{13x} \\ F_{13y} \\ T_{12} \end{Bmatrix} = \begin{Bmatrix} m_2 A_{CG2x} \\ m_2 A_{CG2y} \\ I_{CG2}\alpha_2 \\ m_3 A_{CG3x} \\ m_3 A_{CG3y} \\ I_{CG3}\alpha_3 - T_{13} \end{Bmatrix}$$

$X = R_{32x} - cR_{32y}$ 　　 $Y = cR_{23y} - R_{23x}$

(5) Insert known/given values for variables in [A] & {C}.

(6) Solve for unknown forces and torques in {B} (typically internal joint forces and torques) using {B} = [A]⁻¹ {C}.

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