PVA Analysis Requirements for Project Design Report

Worked Example

1) PVA

The purpose of this document is to walk you through the PVA analysis for your walker link leg mechanism. We will do a worked example of the analysis on the Jansen leg mechanism, **but you will have to apply the same logic to your team's leg mechanism**. The example will be marked in red, while the general guidance will be marked in black.

How to use this document: You should use this document for guidance, but this is not a homework assignment, where you should respond to each question. The results should instead be incorporated into your design report. The calculations in Steps 1 and 3 should become a separate appendix* with the PVA analysis calculations of your mechanism. The analyses from Steps 2 and 3 should become plots in your presentation report, as described in the project description. Use annotation to identify the contact phase and lift phases in all your plots.

* In the appendix, the write up for the PVA analysis of leg linkage should include diagrams indicating vector loops and the analytical calculations of motion. Submit as a separate PDF. It may be better to use a Word document instead of PowerPoint slides. All images and handwritten notes MUST be correct, neat, clear, and easy for graders to see to receive full credit.

Meet Strandbeest: Strandbeest is a 6-leg walker mechanism shown below.

Animation shown here: https://en.wikipedia.org/wiki/Jansen%27s_linkage

- **Gait & leg coordination:** Strandbeest is designed so the 6 legs are broken up into 3 pairs (Figure 1). All legs are driven off a single drive shaft, and the robot has a gait where 3 legs touch the ground at any given moment. Each pair of legs (front & back) face in opposite directions leading them to be 180 degrees out of phase. Each leg is a Jansen linkage, which can be operated the same in forward or reverse, which we will be able to see after performing PVA analysis. The crank shafts of each pair are set 180 degrees off from one another, so each pair of legs is 180 degrees out of phase.

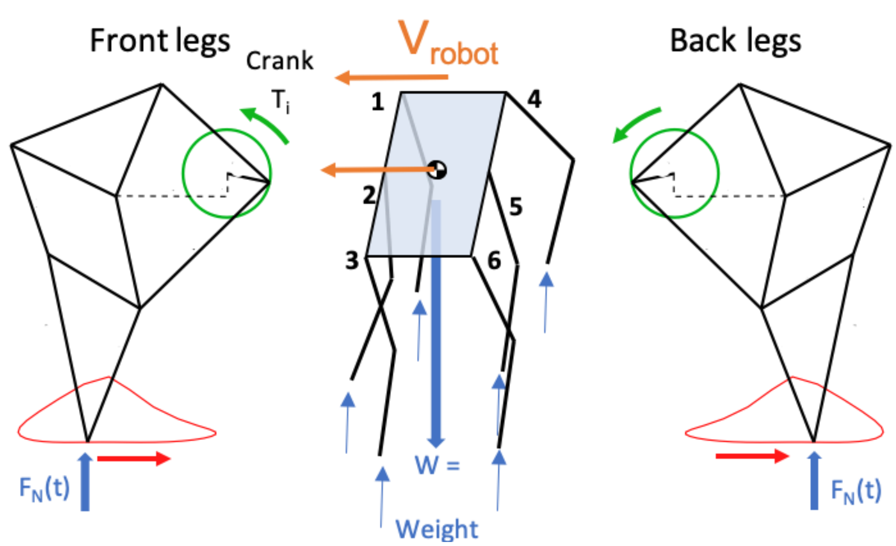

Figure 1. 6-legged Strandbeest and orientations of front and back legs and path motions for feet.

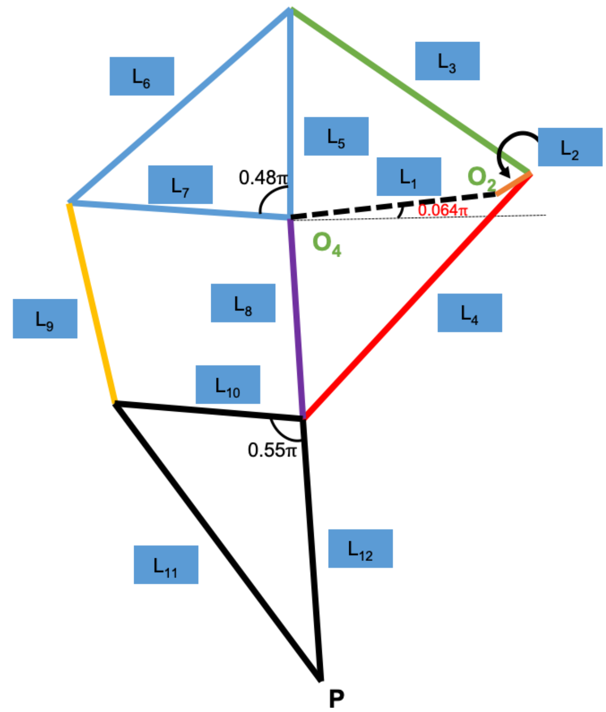

Figure 2. Theo Jansen leg mechanism defining links and geometric constraints. Note that the crank, Link 2, is the small orange link attached to O_2. Note lengths 5,6,7 marked in blue, and lengths 10,11,12 marked in black form ternary links.

Step 1: Analytical PVA of a single point P in the mechanism

To solve the PVA of the foot (point P) of the mechanism, you must identify all the vector loops, and write down the equations of motion from each loop. These equations form the basis of the computational analysis. If you use anything other than a simple 4-bar mechanism, some of the vector positions within a loop will depend on other loops. If your mechanism has more than one vector loop, it is valuable to review the PVA of 6-bar linkages in Section 4.9 of the textbook. For the purposes of the PVA exercise, assume that the leg is being driven by a **constant velocity** motor connected to the crank (Link 2) and hence the angular velocity and acceleration of Link 2 with respect to O_2 are known. Take care to be consistent in your definitions of vector directions and angles and **follow the conventions taught in lecture**.

a) Make a large diagram of your mechanism at a convenient position with all links and geometric constraints shown. Identify and label all the links with variables $L_{\#}$, with the ground being Link 1 and Link 2 being the crank connected to the motor. **Remember:** A closed triangle made of 3 links is not a mechanism; it is called a structure and behaves as a single rigid ternary link.

b) Label all relevant position vectors. **Remember**: We will use compact notation such that $\vec{R}_2 = L_2 e^{i\theta_2}$.

c) Label all relevant angular positions. **Remember**: Angular position is defined positive when going counter-clockwise from horizontal, and placed at the tail of a position vector. Identify any geometric constraints: e.g., the offset of your grounds with respect to horizontal, fixed angles in ternary links, velocities that will always be zero in a slider, or geometric constraints that relate the motion of one loop to the others. It is not necessary to write down every angle, for example the relative angles of different sides of a ternary link will always be fixed with respect to one another, and do not need to be considered independent.

d) On your diagram, identify the vector loops needed to solve the system. To choose the vector loops:

- ✓ Recall that a vector loop is defined going around counterclockwise.
- ✓ Make sure every link is included in a vector loop at least once.
- ✓ It is possible that one or more of the vector loops will not be connected to ground. This is ok as long as the vector loop is sharing a link or two with another vector loop.

✓ Structures/ternary links should not be loops by themselves (they always move the same way with respect to each other, so the loop would be redundant). Instead, use the inner edge of the ternary links in your loops, and define the angle between the links as a geometric constraint in your calculations.

Figure 3 is the diagram of the Jansen mechanism with all relevant position vectors, independent and fixed angles, and vector loops shown. The geometric constraints are the fixed internal angle of the two ternary links and the ground offset angle, all shown in both diagrams. Notice that because of the fixed angle, it is possible to write the orientation of Link 7 in terms of the orientation of Link 5. The Jansen mechanism requires three vector loops to fully define its motion.

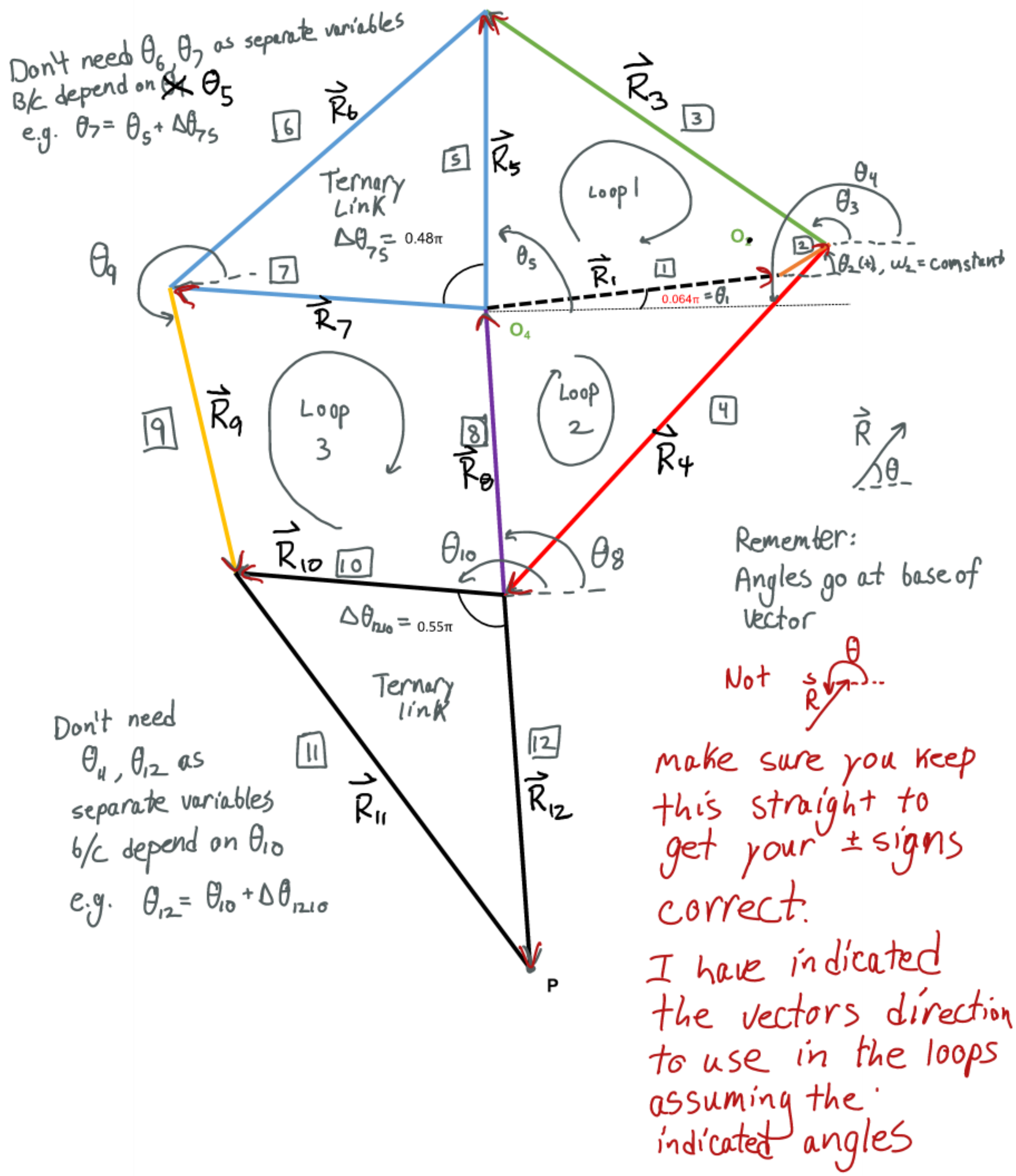

Figure 3. Example of labelled leg mechanism defining links, relevant angles, vector loops, and positive orientations for position vector and angular position.

e) Write down the vector loop equations in the complex form. Identify all known and unknown angles associated with each loop. Assume Link 1 is ground, and Link 2 is being driven as a crank, so θ_2, ω_2, α_2 are defined. Take care to be consistent in your definitions of vector directions and angles and follow the conventions taught in lecture.

Known: All link lengths, angle θ_1 is fixed, while $\Delta\theta_{75}$ $\Delta\theta_{1210}$ are geometric constraints relating the angles between different links, θ_2, ω_2, α_2 are defined by input motion of crank.
Independent Unknowns: Angles θ_3 θ_4 θ_5 θ_8 θ_9 θ_{10}
6 unknowns -> Need 6 equations total to solve for all unknowns.

There are two equivalent ways of thinking of these loops: Either as a system of equations that you solve at the same time, or as using the output unknowns of one equation as the input knowns of the next equation.

Loop 1 (starting at ground O_4, and going clockwise)

$$0 = \vec{R}_5 - \vec{R}_3 - \vec{R}_2 - \vec{R}_1 = L_5 e^{i\theta_5} - L_3 e^{i\theta_3} - L_2 e^{i\theta_2} - L_1 e^{i\theta_1}$$

Known: θ_1, θ_2; Unknown: θ_3, θ_5

Loop 2 (starting at ground O_4, and going clockwise)

$$0 = \vec{R}_1 + \vec{R}_2 + \vec{R}_4 + \vec{R}_8 = L_1 e^{i\theta_1} + L_2 e^{i\theta_2} + L_4 e^{i\theta_4} + L_8 e^{i\theta_8}$$

Known: θ_1, θ_2; Unknown: θ_4, θ_8

Loop 3 (starting at ground O_4, and going clockwise)

$$0 = -\vec{R}_8 + \vec{R}_{10} - \vec{R}_9 - \vec{R}_7 = -L_8 e^{i\theta_8} + L_{10} e^{i\theta_{10}} - L_9 e^{i\theta_9} - L_7 e^{i(\theta_5 + \Delta\theta_{75})}$$

Known: From loops 1 & 2 θ_7 $(in\ terms\ of\ \theta_5), \theta_8$; Unknown: θ_9, θ_{10}

Notice in the Jansen mechanism that two loops are independent and only depend on the output of the crank, while the third loop is dependent on the positions of the other two loops.

f) Write down the equations of motion for **position** analysis by taking the real and complex portions of each vector loop equation.

Loop 1 => Real: $L_5 \cos(\theta_5) - L_3 \cos(\theta_3) - L_2 \cos(\theta_2) - L_1 \cos(\theta_1) = 0$
Imaginary: $L_5 \sin(\theta_5) - L_3 \sin(\theta_3) - L_2 \sin(\theta_2) - L_1 \sin(\theta_1) = 0$

Loop 2 => Real: $L_1 \cos(\theta_1) + L_2 \cos(\theta_2) + L_4 \cos(\theta_4) + L_8 \cos(\theta_8) = 0$
Imaginary: $L_1 \sin(\theta_1) + L_2 \sin(\theta_2) + L_4 \sin(\theta_4) + L_8 \sin(\theta_8) = 0$

Loop 3 => Real: $-L_8 \cos(\theta_8) + L_{10} \cos(\theta_{10}) - L_9 \cos(\theta_9) - L_7 \cos(\theta_5 + \Delta\theta_{75}) = 0$
Imaginary: $-L_8 \sin(\theta_8) + L_{10} \sin(\theta_{10}) - L_9 \sin(\theta_9) - L_7 \sin(\theta_5 + \Delta\theta_{75}) = 0$

6 equations, to solve the 6 unknowns.

g) Following the same procedure, write down the equations of motion for **velocity** analysis. In each case, clearly identify the known and unknown variables. Assume that the position equations have already been solved.

Known: (1) all previous knowns; (2) All the angles are now known from the position analysis; (3) ω_2 is defined by input motion of crank; (4) From the fixed values and geometric constraints, $\omega_1 = 0, \omega_7 = \omega_5$, $\omega_{12} = \omega_{10}$.
Unknowns: $\omega_3, \omega_5, \omega_4, \omega_8, \omega_9, \omega_{10}$

Loop 1 =>
Real: $-L_5\omega_5 \sin(\theta_5) + L_3\omega_3 \sin(\theta_3) + L_2\omega_2 \sin(\theta_2) = 0$

Imaginary: $L_5\omega_5\cos(\theta_5) - L_3\omega_3\cos(\theta_3) - L_2\omega_2\cos(\theta_2) = 0$
Known: ω_2; Unknown: ω_3, ω_5

Loop 2 =>
Real: $-L_2\omega_2\sin(\theta_2) - L_4\omega_4\sin(\theta_4) - L_8\omega_8\sin(\theta_8) = 0$
Imaginary: $L_2\omega_2\cos(\theta_2) + L_4\omega_4\cos(\theta_4) + L_8\omega_8\cos(\theta_8) = 0$
Known: ω_2; Unknown: ω_4, ω_8

Loop 3 =>
Real: $L_8\omega_8\sin(\theta_8) - L_{10}\omega_{10}\sin(\theta_{10}) + L_9\omega_9\sin(\theta_9) + L_7\omega_5\sin(\theta_5 + \Delta\theta_{75}) = 0$
Imaginary: $-L_8\omega_8\cos(\theta_8) + L_{10}\omega_{10}\cos(\theta_{10}) - L_9\omega_9\cos(\theta_9) - L_7\omega_5\cos(\theta_5 + \Delta\theta_{75}) = 0$
Known: $\omega_4, \omega_7\ (= \omega_5)$; Unknown: ω_9, ω_{10}

h) Following the same procedure, write down the equations of motion for **acceleration** analysis. Assume that the position and velocity equations have already been solved.

Known: (1) all previous knowns; (2) All the angles are now known from the position analysis; (3) All the angular velocities are known from the velocity analysis; (4) $\alpha_2 = 0$ is defined by constant velocity motion of crank; (5) From the fixed values and geometric constraints, $\alpha_1 = 0, \alpha_7 = \alpha_5,\ \alpha_{12} = \alpha_{10}$
Unknowns: $\alpha_3, \alpha_5, \alpha_4, \alpha_8, \alpha_9, \alpha_{10}$

Loop 1 =>
Real: $-L_5\alpha_5\sin(\theta_5) - L_5\omega_5^2\cos(\theta_5) + L_3\alpha_3\sin(\theta_3) + L_3\omega_3^2\cos(\theta_3) + L_2\omega_2^2\cos(\theta_2) = 0$
Imaginary: $L_5\alpha_5\cos(\theta_5) - L_5\omega_5^2\sin(\theta_5) - L_3\alpha_3\cos(\theta_3) + L_3\omega_3^2\sin(\theta_3) + L_2\omega_2^2\sin(\theta_2) = 0$

Loop 2 =>
Real: $-L_2\omega_2^2\cos(\theta_2) - L_4\alpha_4\sin(\theta_4) - L_4\omega_4^2\cos(\theta_4) - L_8\alpha_8\sin(\theta_8) - L_8\omega_8^2\cos(\theta_8) = 0$
Imaginary: $-L_2\omega_2^2\sin(\theta_2) + L_4\alpha_4\cos(\theta_4) - L_4\omega_4^2\sin(\theta_4) + L_8\alpha_8\cos(\theta_8) - L_8\omega_8^2\sin(\theta_8) = 0$

Loop 3 =>
Real: $L_8\alpha_8\sin(\theta_8) + L_8\omega_8^2\cos(\theta_8) - L_{10}\alpha_{10}\sin(\theta_{10}) - L_{10}\omega_{10}^2\cos(\theta_{10}) + L_9\alpha_9\sin(\theta_9) + L_9\omega_9^2\cos(\theta_9) + L_7\alpha_5\sin(\theta_5 + \Delta\theta_{75}) + L_7\omega_7^2\cos(\theta_5 + \Delta\theta_{75}) = 0$
Imaginary: $-L_8\alpha_8\cos(\theta_8) + L_8\omega_8^2\sin(\theta_8) + L_{10}\alpha_{10}\cos(\theta_{10}) - L_{10}\omega_{10}^2\sin(\theta_{10}) - L_9\alpha_9\cos(\theta_9) + L_9\omega_9^2\sin(\theta_9) - L_7\alpha_5\cos(\theta_5 + \Delta\theta_{75}) + L_7\omega_7^2\sin(\theta_5 + \Delta\theta_{75}) = 0$

i) Write down the **position of point P** in terms of the "solved" vectors, in exponential form. Assume that the origin of the coordinate system is located at O_4.

$$\vec{R}_P = -\vec{R}_8 + \vec{R}_{12}$$
$$\vec{R}_P = -L_8 e^{i\theta_8} + L_{12} e^{i(\theta_{10} + \Delta\theta_{1210})}$$

j) Differentiate to get the **velocity** and **acceleration** of **point P** in terms of the solved vectors, again in exponential form.

$$\vec{V}_P = -\vec{V}_8 + \vec{V}_{12}$$
$$\vec{V}_P = -i\omega_8 L_8 e^{i\theta_8} + i\omega_{10} L_{12} e^{i(\theta_{10} + \Delta\theta_{1210})}$$

$$\vec{A}_P = -\vec{A}_8 + \vec{A}_{12}$$

$$\vec{A}_P = -i\alpha_8 L_8 e^{i\theta_8} + \omega_8^2 L_8 e^{i\theta_8} + i\alpha_{10} L_{12} e^{i(\theta_{10} + \Delta\theta_{1210})} - \omega_{10}^2 L_{12} e^{i(\theta_{10} + \Delta\theta_{1210})}$$

Do you want to manually solve this system of equations? **I DON'T!** In the next step, we will let the computer do the solving for us.

Step 2: Numerical PVA analysis

Getting a closed form, analytical solution to most mechanisms is difficult. Next, we will use the Python codes used in the course lab to get numerical solutions for the PVA of your leg mechanism.

- Modify the PVA code provided in lab to simulate your leg mechanism – this only requires changing the inputs of Section 1, and the plots in Section 9.
- Conduct the simulation assuming a constant rotational speed of the crank. (Use and justify the choice of crank velocity.) Pick the crank direction to correspond with the leg moving forward. Use a small enough time interval that all features of the motion are easily distinguishable. $\Delta t=0.01$ is a good starting point.
- For each plot, be sure to include axes labels and apply proper plot formatting. Combine the plots where possible for direct comparison.
- Figure 4 shows the location of each joint and numbered links. In the code, joint locations are typically provided in millimeters, but your final product should give the position, velocity, and acceleration in the standard SI units of m, m/s, and m/s^2. Check your unit conversions.

We will show the analysis and the plots that we expect you to provide using the Jansen leg mechanism. See the provided files for the MATLAB code used to produce these plots. (Python code are also now available.) Refer to the diagram below to see the initial locations for each of the joints in the mechanism (called nodes in the code).

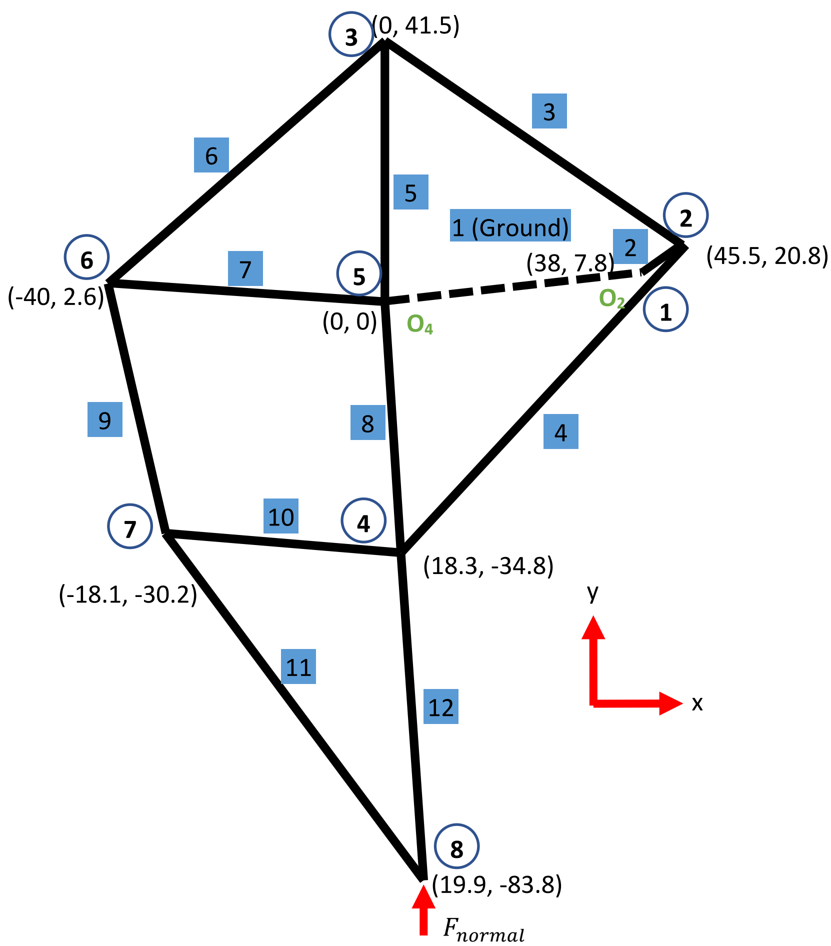

Figure 4. x-y coordinates of nodes

Once you have completed the code, make sure to provide the following in your reports.

a. Include a picture of the simulation after it has finished tracing at least one complete path for the foot. See an example plot for the Jansen mechanism below (Figure 5)

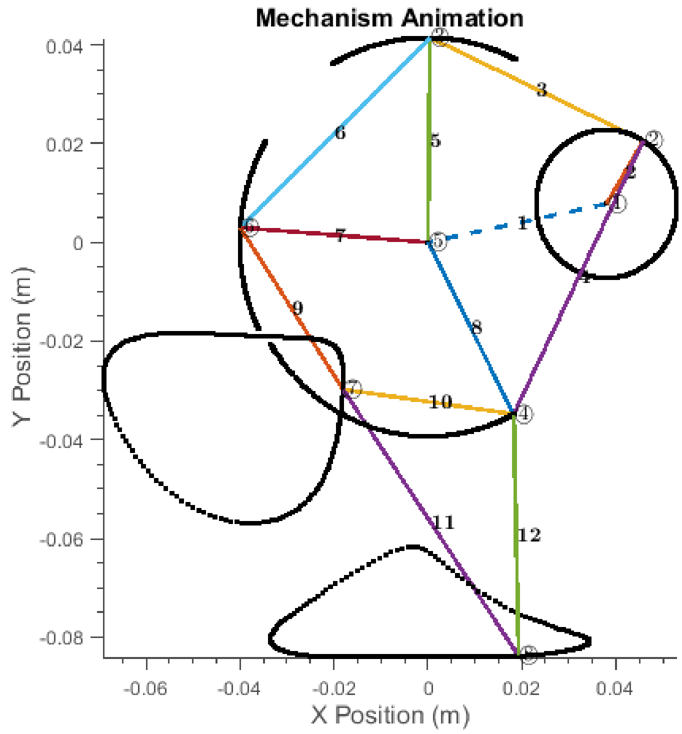

Figure 5. Plot of Jansen's linkage at one instant

b. **Plot the following time series graphs for two complete crank cycles for the foot, point P.** For each plot, be sure to include axes labels and apply proper plot formatting. Combine the plots where possible for direct comparison.

i. X-Position & Y-Position vs. Time (Figure 6)

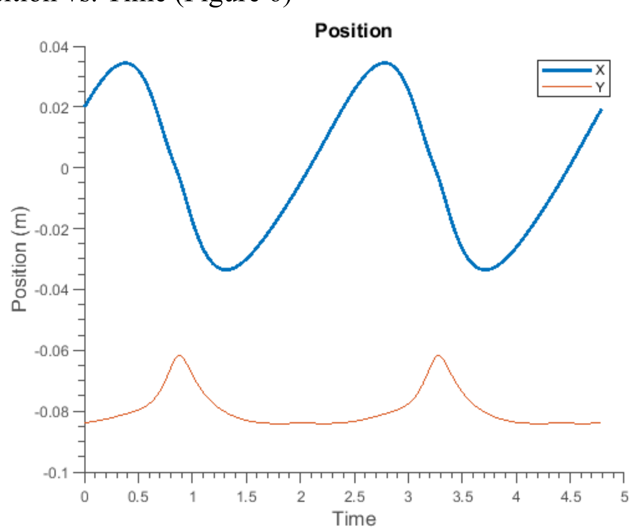

Figure 6. Results of position analysis: x and y positions as functions of time for point P.

ii. X-Velocity & Y-Velocity vs. Time

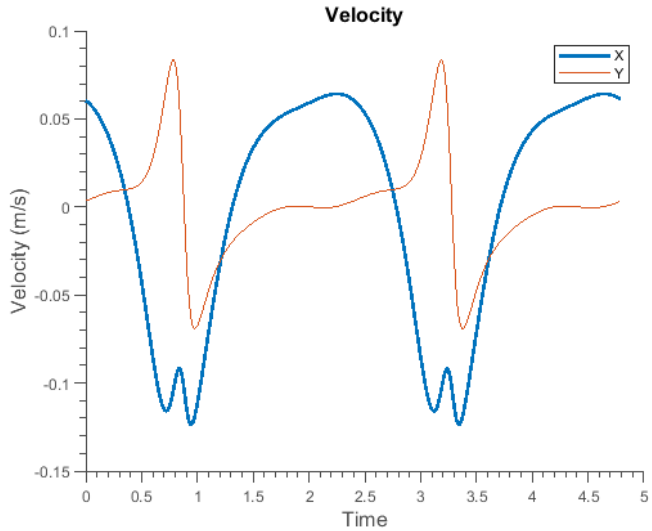

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Figure 7. Results of velocity analysis: x and y velocities as functions of time for point P

iii. X-Acceleration & Y-Acceleration vs. Time

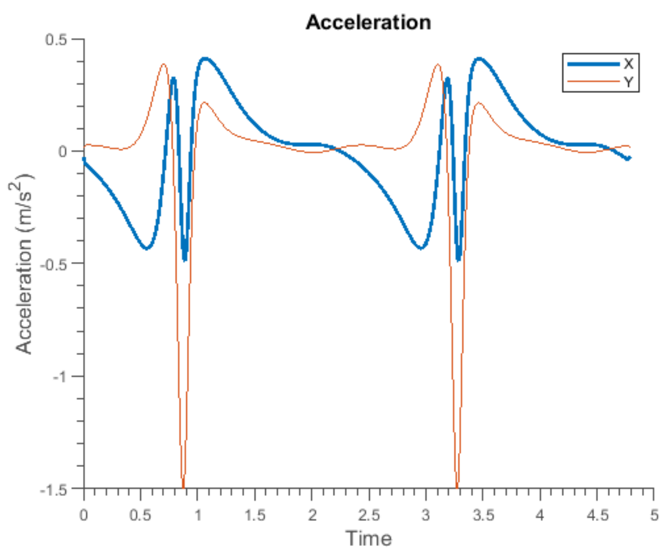

Figure 8. Results of acceleration analysis: x and y accelerations as functions of time for point P

Step 3: Interpreting and using PVA

We can use the PVA analysis to gain insight into how the robot will move, and how to modify the design to achieve the needed walking velocity.

a. Using both the animation and the PVA time series graphs that you have generated, identify when the foot is touching the ground (contact phase) and when it is in midair (lift phase). Assume the lowest points that the foot reaches to be the ground. Mark on the time axis of the graphs where the transitions between contact and lift phases occur, called the contact point and liftoff point.

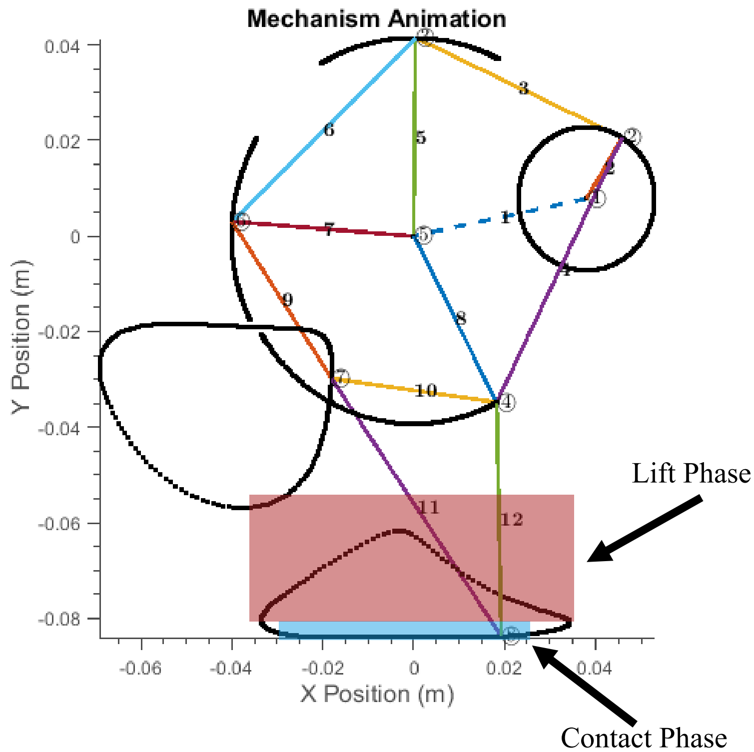

Figure 9. Identification of lift/contact phases in mechanism plot

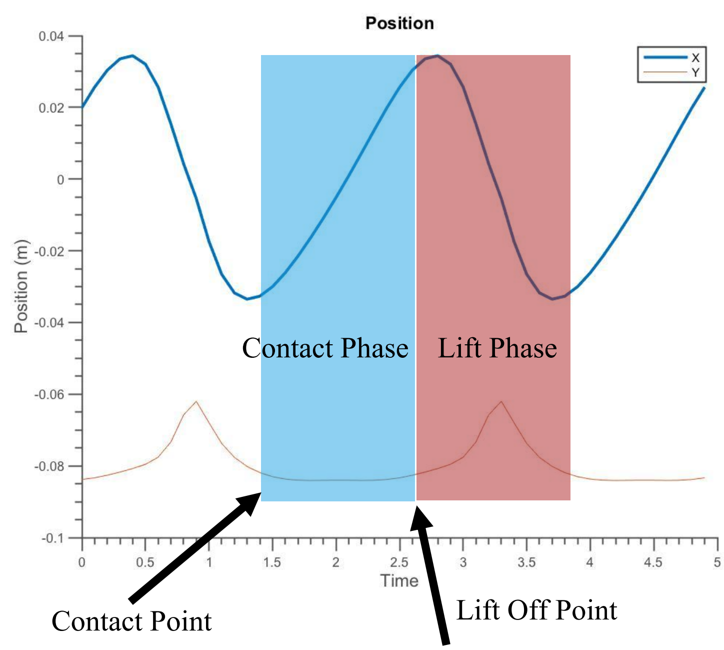

Figure 10. Identification of lift/contact phases from position diagram

b. Use your PVA analysis of leg motion to define your duty factor β, described in the walking robots background document. Identify the fraction (or percentage) of the cycle is the foot in contact with the ground. Sketch out the gait diagram that describes your robot's gait sequence, and shows how your duty cycle relates to the number of legs.

As seen in the position versus time plot above, the contact and lift phases take an equal amount of time. This means that the foot is designed to be in contact with the ground for 50% of the cycle of the crank, so

$\beta = 0.5$. The gait sequence is shown below, with the leg numbers corresponding with the first figure in this document. Note a reverse gait sequence would also be correct.

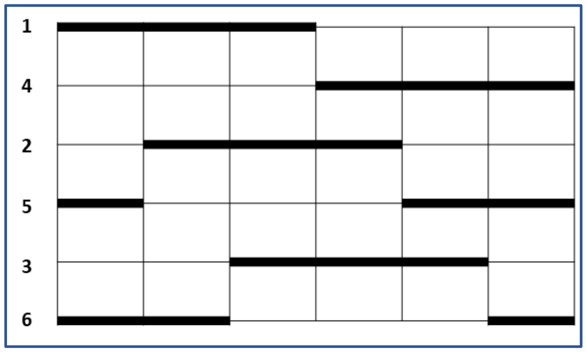

Figure 11. Gait sequence of Theo Jansen Mechanism

c. Use the data from PVA analysis to obtain one step size (stride length L_s) of your mechanism. (i.e., distance from foot contact to lift-off).

The position information for the Jansen mechanism is reproduced again below. We are looking for the change in x position between the contact point and the liftoff point. Extracting from the plot, we get:

$$L_s = x_{lift} - x_{contact} = x(t = 2.65s) - x(t = 1.5s) \approx 6.2\ cm$$

d. Relate the velocity requirements for the robot in the project description to the duty factor and stride length to estimate the required crank angular velocity ω_{crank}. Calculate the minimum required crank angular velocity of the leg. The equation from the robot gait document is:

$$V_{Robot} = \frac{L_s}{\beta T} = \frac{(L_s)(\omega_{crank})}{\beta(2\pi)}$$

$$\omega_{crank} = V_{Robot\ min} * \frac{\beta(2\pi)}{L_s}$$

$$\omega_{crank\ min} = 2\frac{m}{\min} * 0.5 * \frac{2\pi}{0.062m} = 101.29\frac{rad}{min} \approx 1.7\frac{rad}{sec}$$

e. Finally, you need to design a gear train to coordinate the motion of your legs, which means you need to know the motor angular velocity. The motor used in this project is the same motor that you used in lab. The plot of motor velocity versus torque is reproduced below (Figure 12). As you learned in the lab, the motor velocity depends on the torque on the motor. To fully solve this problem, we need to do force analysis on the leg mechanism, which will be part of future assignments. For now, assume that the torque will cause the angular velocity of the motor to move at ¼ the no-load velocity. The actual speed might be better or worse depending on details of leg design, weight, etc., but this is a good starting point which can be refined on later. Find the minimum gear ratio that will allow your design to exceed the minimum required angular velocity of the leg.

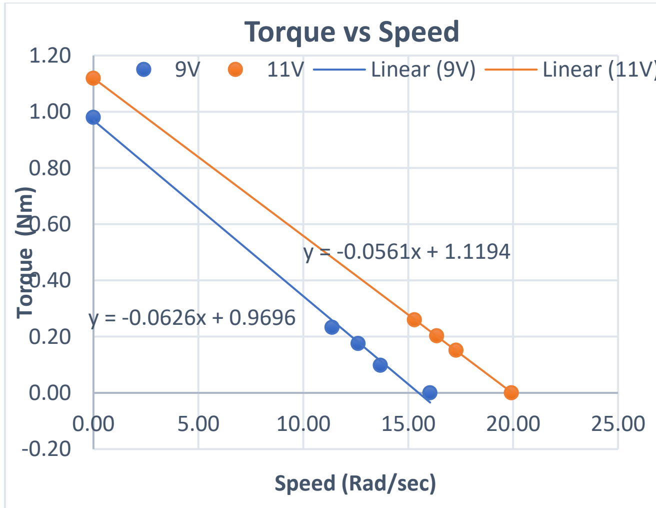

Figure 12. Motor Torque vs. Speed Graph

$$\omega_{motor\ load} \approx \frac{1}{4} * \omega_{no\ load} \approx \frac{1}{4} * 20 \approx 5\frac{rad}{sec}\ (For\ 11V\ data)$$

In this case, the assumed $\omega_{motor\ load}$ far exceeds the required ω_{crank} calculated in the previous step, which means that the required gear ratio should slow down the output RPM rather than increasing it to be able to provide sufficient torque (note that decreasing RPM implies increasing output torque).

f. From your CAD, define a gear train that exceeds the minimum required gear ratio and makes sense given the size/pitch of gears, number of teeth, and rest of your design. Calculate the final value of the gear ratio, and the predicted velocity of the robot given the assumptions above.

From step (e) above, we know that the motor speed is already high enough to far exceed the minimum robot speed. But we do not want the speed to be too high as the motor might not be able to provide the required torque (exact torque analysis will be done in subsequent DFA analysis assignments). Let's say we want to limit the robot speed to be 2 times the required minimum speed, which means the desired motor RPM ($\omega_{crank\ desired}$) should be 2* $\omega_{crank\ min}$. Gear Ratio can calculated as follows:

$$Gear\ Ratio = \frac{\omega_{motor\ load}}{\omega_{\ crank\ desired}} = \frac{5}{2 * 1.7} \approx 1.5$$

Let's assume that you were provided with two gears having 24 and 30 teeth, respectively. So, the two possible gear ratio values would be either $\frac{24}{30} = 0.8$ or $\frac{30}{24} = 1.25$. Going ahead with a final gear ratio of 1.25 (since it's closer to 1.5), the final velocity of robot can be predicted as,

$$V_{Robot\ Predicted} = \frac{L_s}{\beta T} = \frac{(L_s)(\omega_{crank})}{\beta(2\pi)} = \frac{(L_s)\left(\frac{\omega_{motor\ load}}{Final\ Gear\ Ratio}\right)}{\beta(2\pi)}$$

$$V_{\ Robot\ Predicted} = 0.062 * \frac{\frac{5}{1.25}}{.5 * 2\pi} \approx 0.079\frac{m}{sec} \approx 4.7\frac{m}{min}$$

These calculations are important, because they allow you to predict whether your design will meet the performance criteria before you start building, rather than finding out after you have made a

prototype that your design will not work. However, there were lots of assumptions that we used to make the problem tractable without force analysis. Once you have learned dynamic force analysis and know other factors like the weight of your robot, you will be able to fully predict the robot behavior.