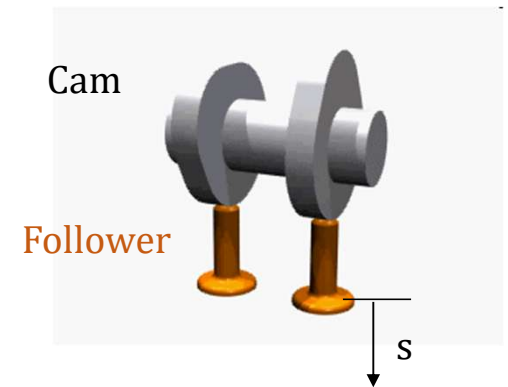


Cam Motion

Purpose:

- Transform rotational motion to translational (typically) motion
- Produce desired repetitive output motion, or displacement (s)
 - Function generator
 - Intermittent motion
- More expensive than gears
- Wears more easily, needs lubricant



https://en.wikipedia.org/wiki/Cam#/media/File:Nockenwelle_ani.gif

See these URLs for examples of motions and how to build own from paper

<http://www.robives.com/mechanisms/cams> this URL has old Adobe Flash images, but has good explanation of cam design considerations

<https://www.robives.com/mechanism/cam/> this newer URL shows multiple types of cam designs

Lecture 18

Simple Motion Control



ME 370 - Mechanical Design 1

Module 6 topics: Motors, Cams and Motion Control

- **Motors**

- DC motor principle
- DC motor model
- Linear Motor Model
 - Constraints
 - Behavior in time
 - Gearboxes
- Motor Parameters
- Power and Efficiency

- Cam and Follower

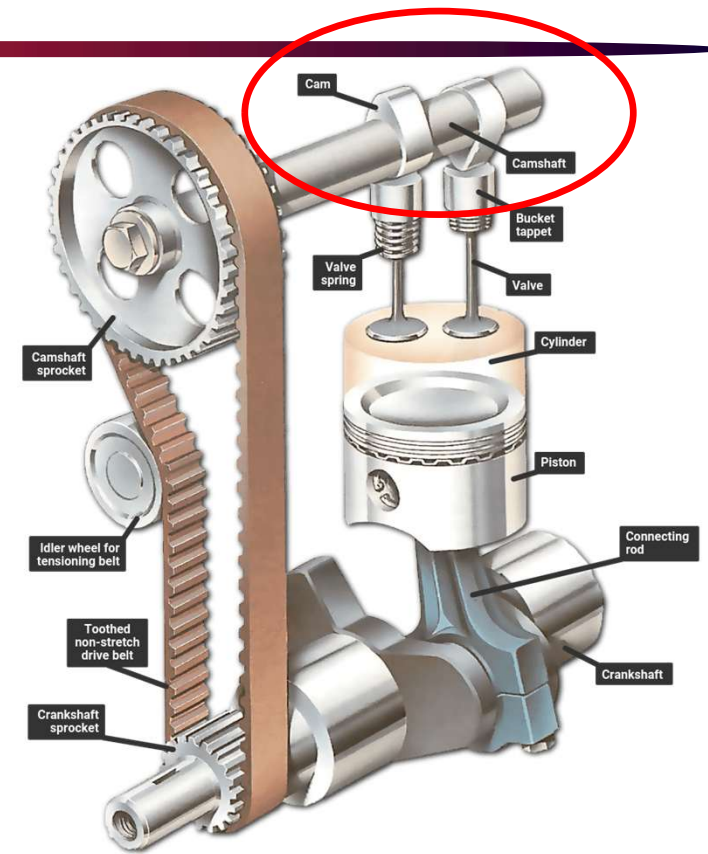
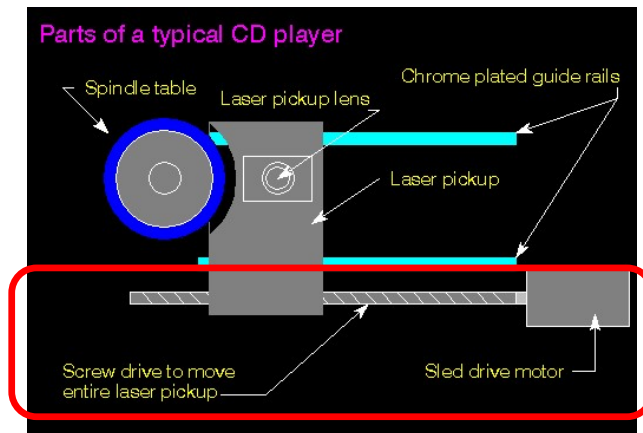
- Types of Motion
- Types of Follower
- Practical Considerations

- **Motion control**

- Simple Motion Control Dwell-Rise-Dwell motions:
 - Fundamental Law of Cam (Motion) Design
 - Simple Harmonic Motion
 - Sinusoidal Acceleration (i.e., Cycloidal Displacement)
- Advanced Motion Control
 - Additional Dwell-Rise-Dwell motions:
 - Trapezoidal acceleration
 - Modified Trapezoidal acceleration
 - Modified Sine acceleration
 - 3-4-5 Polynomial Rise Displacement
 - 4-5-6-7 Polynomial Rise Displacement
 - Rise-Fall-Dwell motions:
 - Cycloidal Motion
 - Double Harmonic
 - 3-4-5-6 Polynomial

Recall: Cam Usage

- Conventional cam design:
 - sewing machine
 - 4-stroke internal combustion engine
- New designs:
 - Replace cams with actuators
 - Pneumatic, electro-mechanical
 - Example: Laser positioning in CD drive



<https://www.howacarworks.com/basics/the-engine-how-the-valves-open-and-close>

Types of Motion Constraints

1) Critical extreme position (CEP):

- Define end positions of follower, but not path

2) Critical path motion (CPM)

- Define path (and possibly velocity and acceleration)

Dwell:

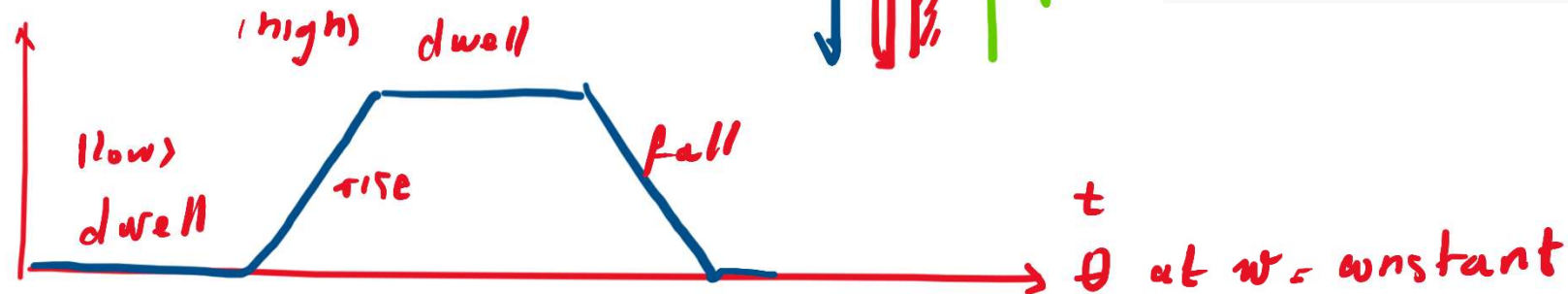
- No output motion for specified input motion



Critical Extreme Position (CEP): Use piecewise functions to produce **motion program**

Examples Motion Programs:

- Rise-Fall
- Rise-Fall-Dwell
- Rise-Dwell-Fall-Dwell



See these URLs for examples of motions and how to build own from paper

<http://www.robives.com/mechanisms/cams> this URL has old Adobe Flash images, but has good explanation of cam design considerations

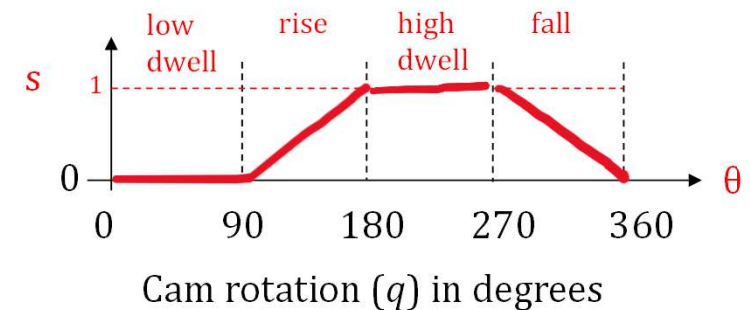
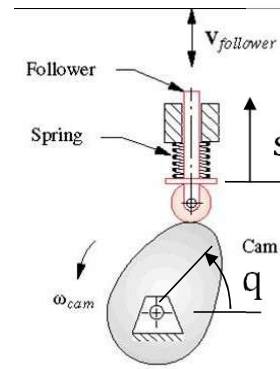
<https://www.robives.com/mechanism/cam/> this newer URL shows multiple types of cam designs

Example 1: Dwell-Rise-Dwell-Fall using Linear Functions

$$S = m\theta + b \quad \theta = 90^\circ \rightarrow S = 0$$

$$\theta = 180^\circ \rightarrow S = 1$$

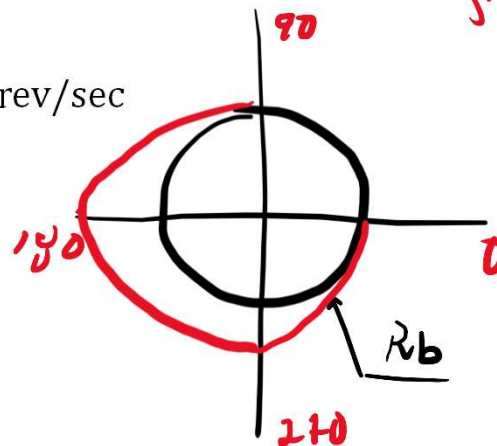
- **Dwell**
 - at zero displacement for 90 degrees (low dwell)
- **Rise**
 - 1 inch in 90 degrees
- **Dwell**
 - at 1 inch for 90 degrees (high dwell)
- **Fall**
 - 1 inch in 90 degrees
- Cam's ω
 - 2π radians/sec = 1 rev/sec



Exercise: Write down a piecewise equation of motion.

$$S(\theta) = \begin{cases} 1: 0, & 0 \leq \theta < 90^\circ \\ 2: \frac{\theta - 90}{90}, & 90 \leq \theta < 180^\circ \\ 3: 1, & 180 \leq \theta < 270^\circ \\ 4: \frac{360 - \theta}{90}, & 270 \leq \theta < 360^\circ \end{cases}$$

$$r(\theta) = S(\theta) + R_b$$

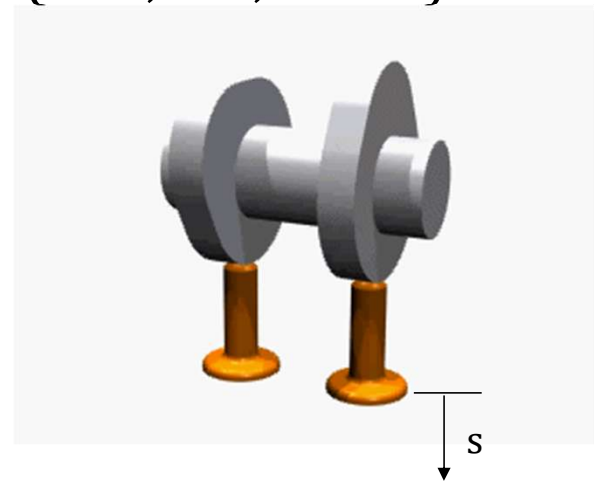


How to Design the Motion in Cams

- The following should be considered when selecting the mathematical expressions that define each piece of motion (rise, fall, dwell) for reliable operation:

- Displacement (s)
- Velocity (v)
- Acceleration (a) → Relates to force
- Jerk (j) → Relates to force change

Excitations of harmonics



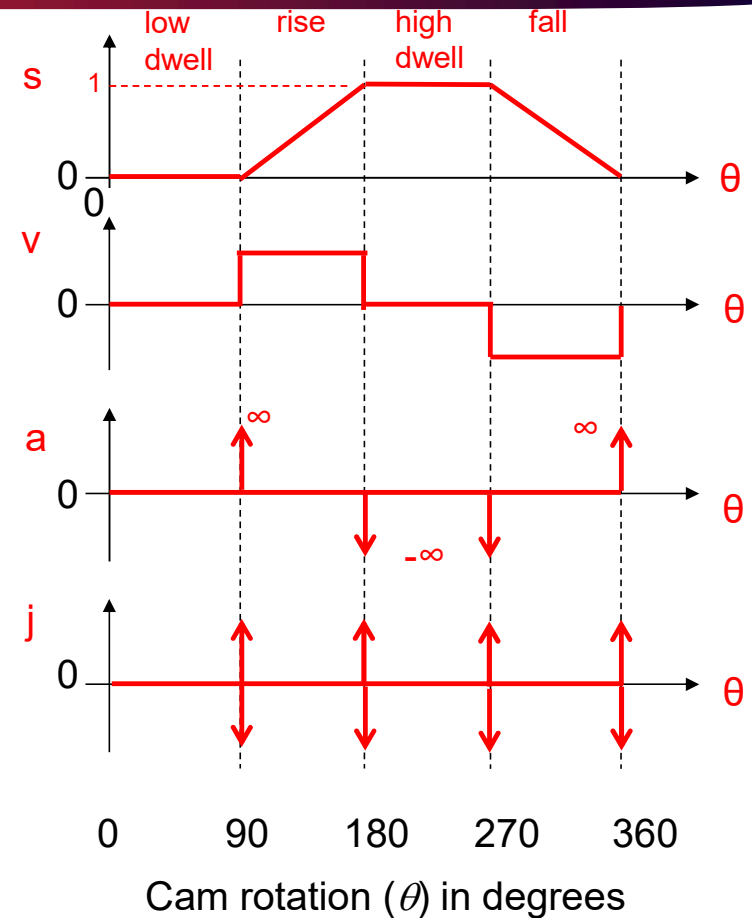
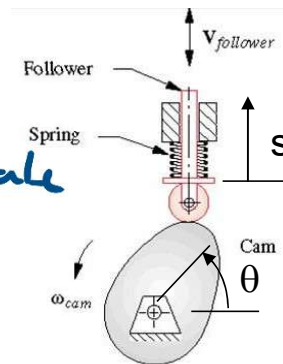
- Both the behavior of these functions **within** the piece of motion and **in between** different pieces of motion are important.

Example: Dwell-Rise-Dwell-Fall using Linear Functions

- **Dwell**
 - at zero displacement for 90 degrees (low dwell)
- **Rise**
 - 1 inch in 90 degrees
- **Dwell**
 - at 1 inch for 90 degrees (high dwell)
- **Fall**
 - 1 inch in 90 degrees
- Cam's ω
 - 2π radians/sec = 1 rev/sec

1a)

all frequencies participate
excites harmonics!



A note on angular position, θ , and time, t

We can convert between angular position θ and time t easily because ω is constant.

Given: $s = s(\theta)$

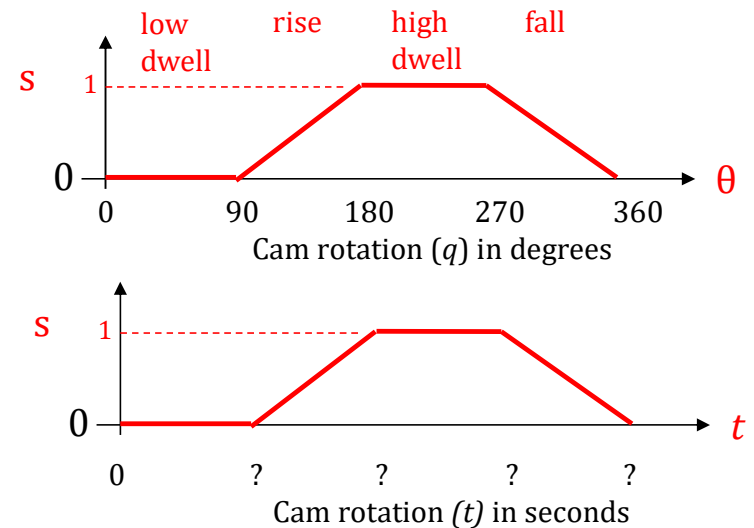
- Convert $s(\theta)$ to $s(t)$ by replacing θ by t and β by t' .
 - β is the angular interval of rise (or fall).
 - t' is the time interval of rise (or fall).

Ex: $\omega = 1 \text{ rpm} = 6 \text{ deg/sec}$.

- If $\beta = 120 \text{ deg}$, what is $t' = ?$

$$t' = \frac{\beta}{\omega} = 90 / 6 = 15 \text{ s}$$

$$\beta = 120^\circ$$



Motion Program and Practical Considerations

- **Motion Program:**

defined by combining several separate functions (piecewise function).

- **Any discontinuity in the velocity results in infinite acceleration.**

- $F = ma \rightarrow$ Infinite force \rightarrow Infinite stress

- **Any discontinuity in acceleration results in infinite jerk.**

- $dF/dt = mj$, therefore, dF/dt goes to infinity.
- Such spikes in jerk can excite harmonics in the spring-mass system of the follower \rightarrow vibration in the mechanism.

- **Points where a dwell is connected to a rise or a fall:**

- A **dwell** always has zero velocity and acceleration.
- Piecewise functions for rise and fall **must match** this zero velocity and acceleration at the points where they connect to the dwell (for avoiding discontinuity).



Fundamental Law of Cam (Motion) Design

The cam function (= follower motion) must be continuous through first and second derivatives of displacement across entire interval (360°).

- ∴ requires 3rd order continuity
 - Displacement (C^0 continuity)
 - Velocity (C^1 continuity)
 - Acceleration (C^2 continuity)
- In other words, position, velocity and acceleration should be continuous across the entire interval (360°).
- **Corollary:** The jerk function must be finite across the entire interval (360°).

Example 2: Simple Harmonic Motion

$$s = \frac{h}{2} \left[1 - \cos \pi \frac{\theta}{\beta} \right]$$

$$v = \frac{\pi h}{\beta^2} \sin \pi \frac{\theta}{\beta}$$

$$a = \frac{\pi^2 h}{\beta^3} \cos \pi \frac{\theta}{\beta}$$

$$j = \frac{\pi^3 h}{\beta^4} \sin \pi \frac{\theta}{\beta}$$

β : total angle of rise interval

θ : cam shaft angle

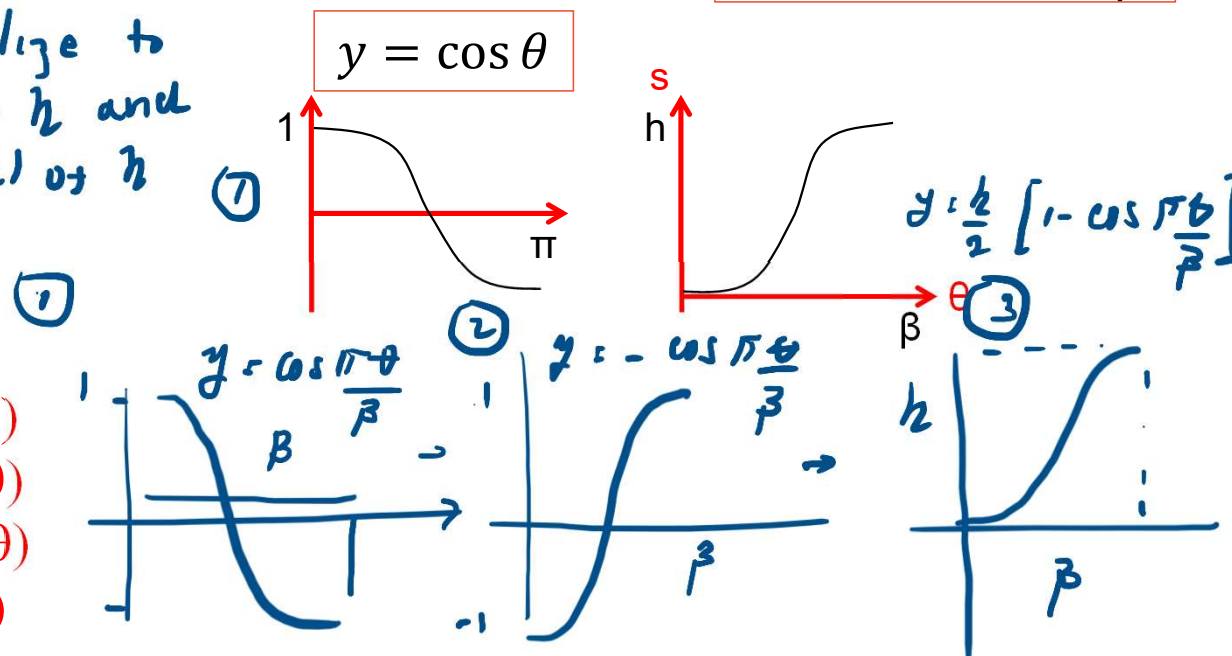
h : total rise displacement

$$s = \frac{h}{2} \left[1 - \cos \pi \frac{\theta}{\beta} \right]$$

normalize to
rise of h and
interval of β

$$y = \cos \theta$$

$$\begin{aligned} s &= s(\theta) \\ v &= v(\theta) \\ a &= a(\theta) \\ j &= j(\theta) \end{aligned}$$



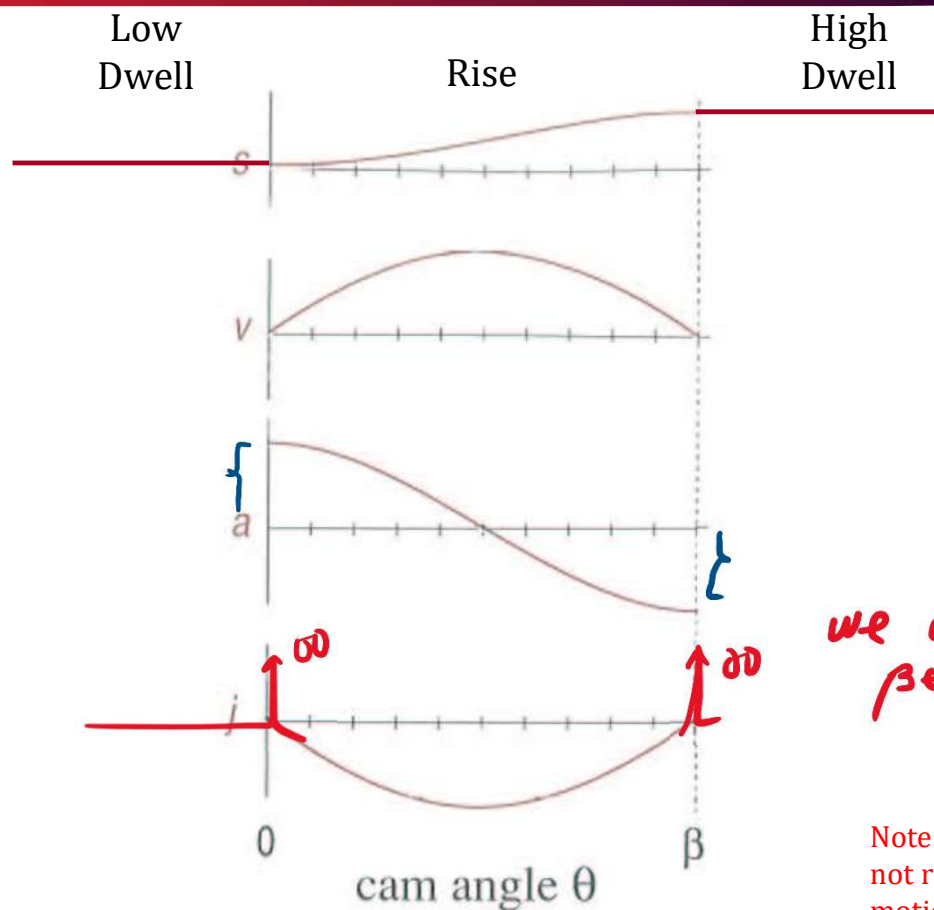
Example 2: Simple Harmonic Motion: Dwell-Rise-Dwell

$$s = \frac{h}{2} \left[1 - \cos \pi \frac{\theta}{\beta} \right]$$

$$v = \frac{\pi h}{\beta^2} \sin \pi \frac{\theta}{\beta}$$

$$a = \frac{\pi^2 h}{\beta^3} \cos \pi \frac{\theta}{\beta}$$

$$j = \frac{\pi^3 h}{\beta^4} \sin \pi \frac{\theta}{\beta}$$



we can do better.

Note: this design does not represent full 360° motion

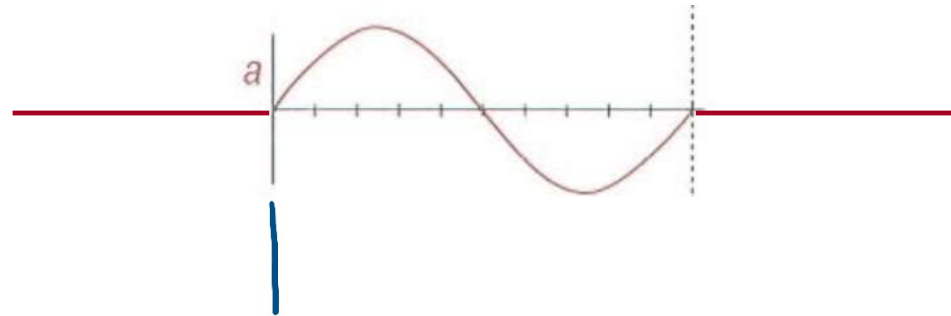
How can we get rid of spikes in jerk?

Design for acceleration function instead of displacement function

- Must have zero acceleration at ends (where it meets dwell)

Possible Solution:

- **Sinusoidal Acceleration (i.e., Cycloidal Displacement)**
 - Full-period sine function within rise (fall) interval



Cycloidal Displacement

$$\text{Let, } a = C \sin\left(2\pi \frac{\theta}{\beta}\right)$$

where C will be determined by boundary conditions.

- Equations for displacement, velocity, acceleration, and jerk will depend on boundary conditions of specific design.

Cycloidal Displacement - example

For

$$a = C \sin\left(2\pi \frac{\theta}{\beta}\right)$$

with boundary conditions of:

$$v = 0 \text{ at } \theta = 0 \text{ and } \theta = \beta$$

$$s = 0 \text{ at } \theta = 0,$$

$$s = h \text{ at } \theta = \beta$$

$$\text{Get } s = h \left[\frac{\theta}{\beta} - \frac{1}{2\pi} \sin\left(2\pi \frac{\theta}{\beta}\right) \right]$$

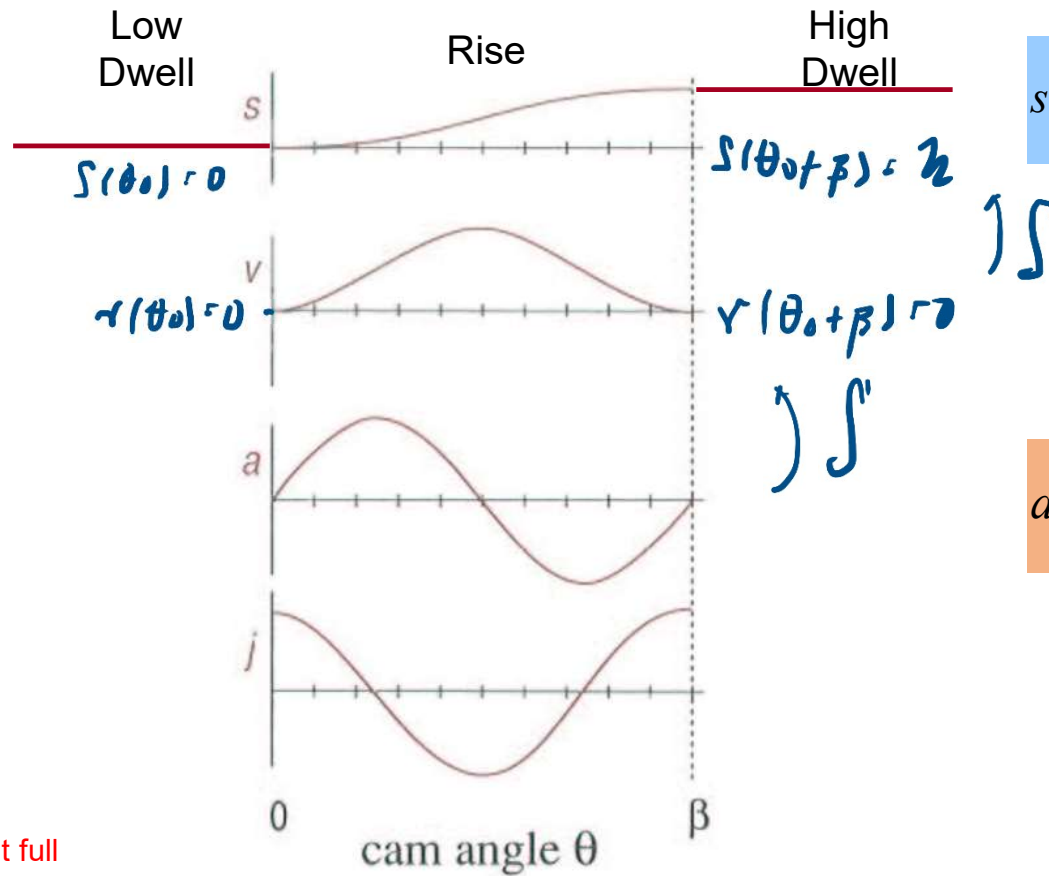
use $v = \frac{ds}{d\theta}$

$$v = \frac{h}{\beta} \left[1 - \cos\left(2\pi \frac{\theta}{\beta}\right) \right]$$

$$a = 2\pi \frac{h}{\beta^2} \sin\left(2\pi \frac{\theta}{\beta}\right)$$

$$j = 4\pi^2 \frac{h}{\beta^3} \cos\left(2\pi \frac{\theta}{\beta}\right)$$

Cycloidal displacement (Sinusoidal Acceleration)



$$s = h \left[\frac{\theta}{\beta} - \frac{1}{2\pi} \sin \left(2\pi \frac{\theta}{\beta} \right) \right]$$

$$a = 2\pi \frac{h}{\beta^2} \sin \left(2\pi \frac{\theta}{\beta} \right) \quad (\text{rate}).$$

Note: this design
does not represent full
360° motion