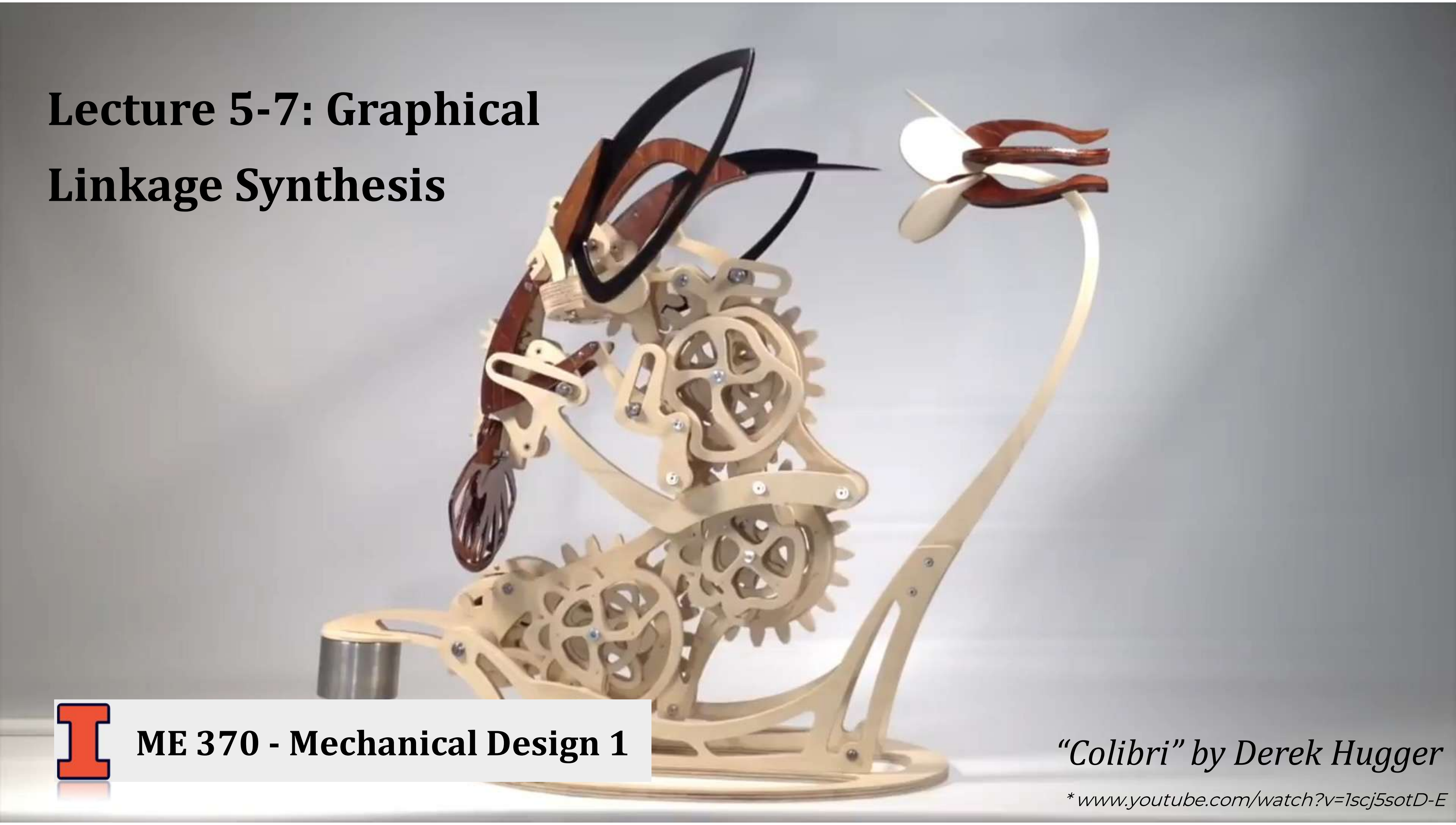
Lecture 5-7: Graphical Linkage Synthesis



ME 370 - Mechanical Design 1

"Colibri" by Derek Hugger

** www.youtube.com/watch?v=1scj5sotD-E*

Theo Jansen's Strandbeests

Beautiful example of Path Synthesis
https://www.youtube.com/watch?v=zYGVYLzN06g

Topic 2: Graphical Linkage Synthesis

- Synthesis techniques
  - Generation types and strategies
- Motion synthesis: Two-position synthesis
  - Rocker output
  - Coupler output
  - Rotopoles
- Dyad drivers
- Quick return mechanisms
- Motion synthesis: Three-position synthesis
  - Specified moving points
  - Alternate moving points
- Path synthesis
  - Coupler curves

Kinematic or Mechanism Synthesis

- How do we design a mechanism to achieve desired functionality?

- **Kinematic (or Mechanism) synthesis**, determines the size and configuration of mechanisms that shape the flow of power through a mechanical system, or machine, to achieve a desired performance.

Kinematic synthesis - procedure

- Define desired motion
  - e.g. dispense candy, walking gait
- Choose mechanism type
  - e.g., crank-rocker, slider-crank
- Specify geometry
  - e.g., link lengths, type & # of joints
- Avoid undesirable behaviors
  - e.g., toggle positions, change points

Choosing and refining mechanisms

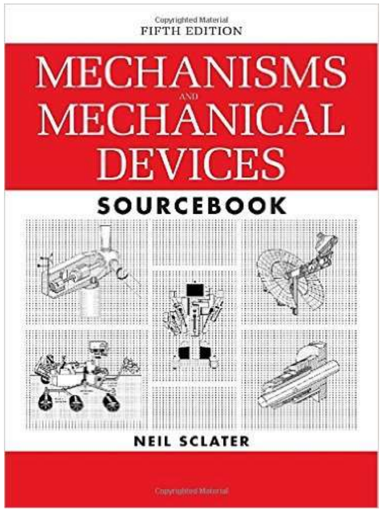
- How to design a mechanism?

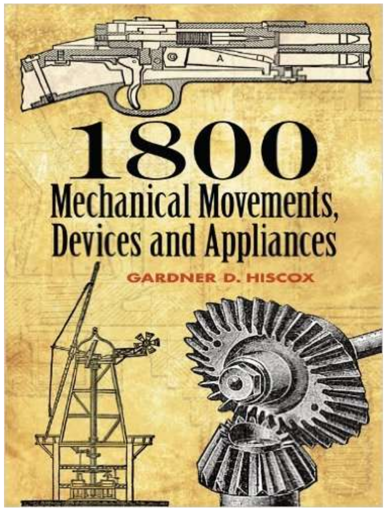
Research existing solutions

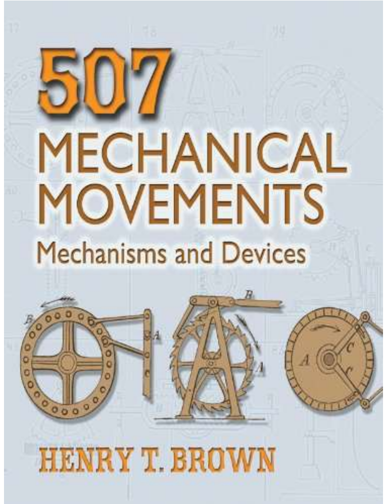
A day of deep historical searching is worth 6 months in the lab

Compilations of solutions to different kinds of motion

1. Online and books are often divided by mechanism type (crank-rocker, slider-crank, 4- or 6-bar mechanism, using gears, using chains, etc,). What do you want?







2. Or, there are collections of ways to achieve common motions, like straight-line mechanisms…

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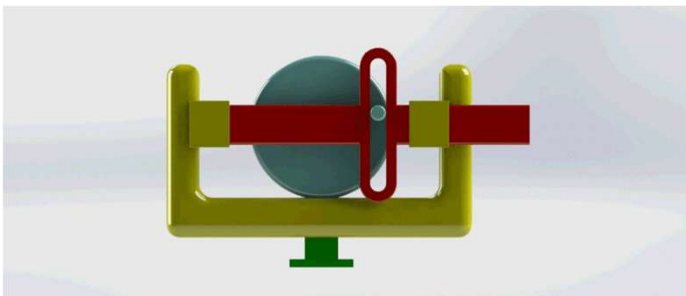
Example: Straight line Motion

Chebyshev linkage

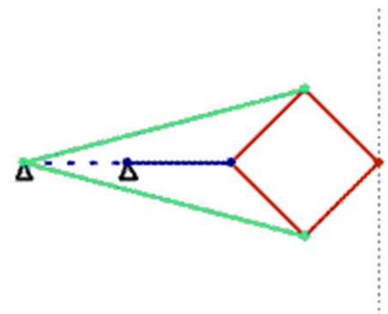
https://www.youtube.com/watch?v=o3oczQU8QIY

Other Straight-Line Mechanisms

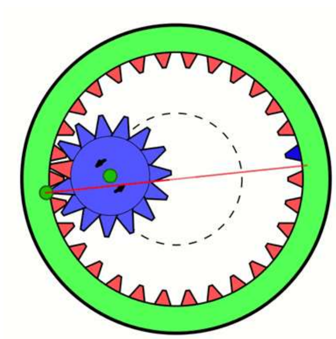
Scotch Yoke



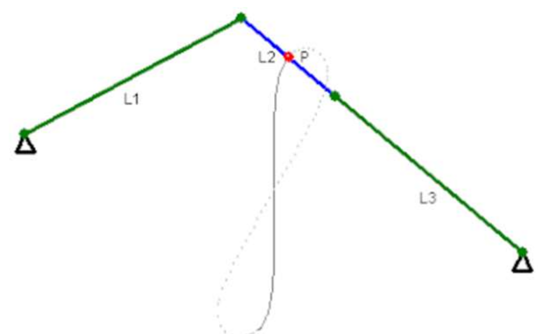
Peaucellier linkage



Hypocycloidal Gears



Watts linkage



Choosing and refining mechanisms

- How to design a mechanism?

Research existing solutions

Analyze / Understand

Adapt / Improve

- Size, power, efficiency, constraints
- Linkages ok?
- Sliders ok? Friction?
- Does it need to be exactly a straight line, or is approximate ok?
- Is it ok to have part of the motion be not a straight line?

Adapting and improving linkage design

Computational Design of Mechanical Characters

S. Coros[1] B.Thomaszewski[1]
G. Noris[1] S. Sueda[2] M. Forberg[2]
R. Sumner[1] W. Matusik[3] B. Bickel[1]

[1]Disney Research Zurich [2]Disney Research Boston [3]MIT CSAIL

Disney Research

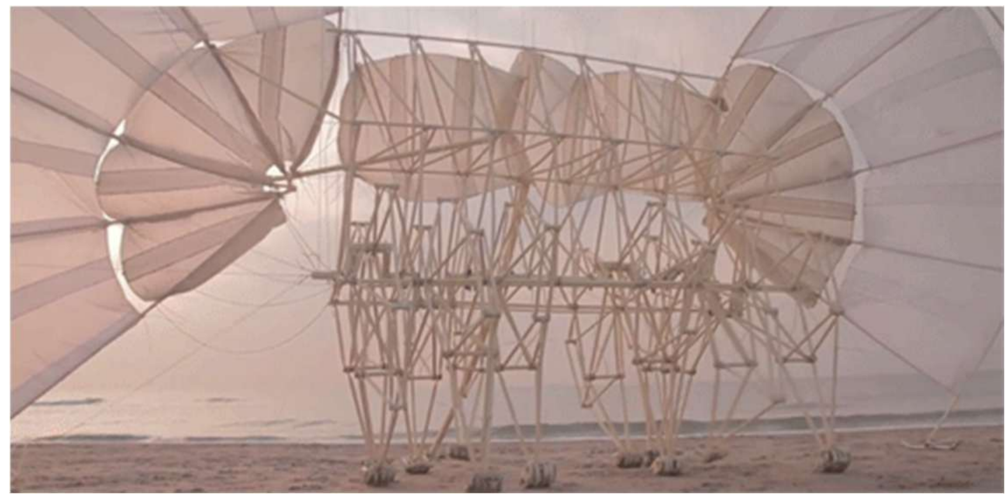
Generation types

- **Function generation**
  - Correlates input to output
  - $f(\theta_{crank})$
- **Path generation**
  - Control of a point to follow prescribed path
  - Not concerned with link orientation
  - e.g., coupler curves, straight-line mechanisms
  - Only prescribes the position of a single point on a link
- **Motion generation**
  - Control of a line (or link) to follow prescribed set of positions
  - Prescribes position **and** orientation of the link

Theo Jansen’s Strandbeests

Animaris Umerus

Animaris Vulgaris



Beautiful example of <u>Path Synthesis</u>

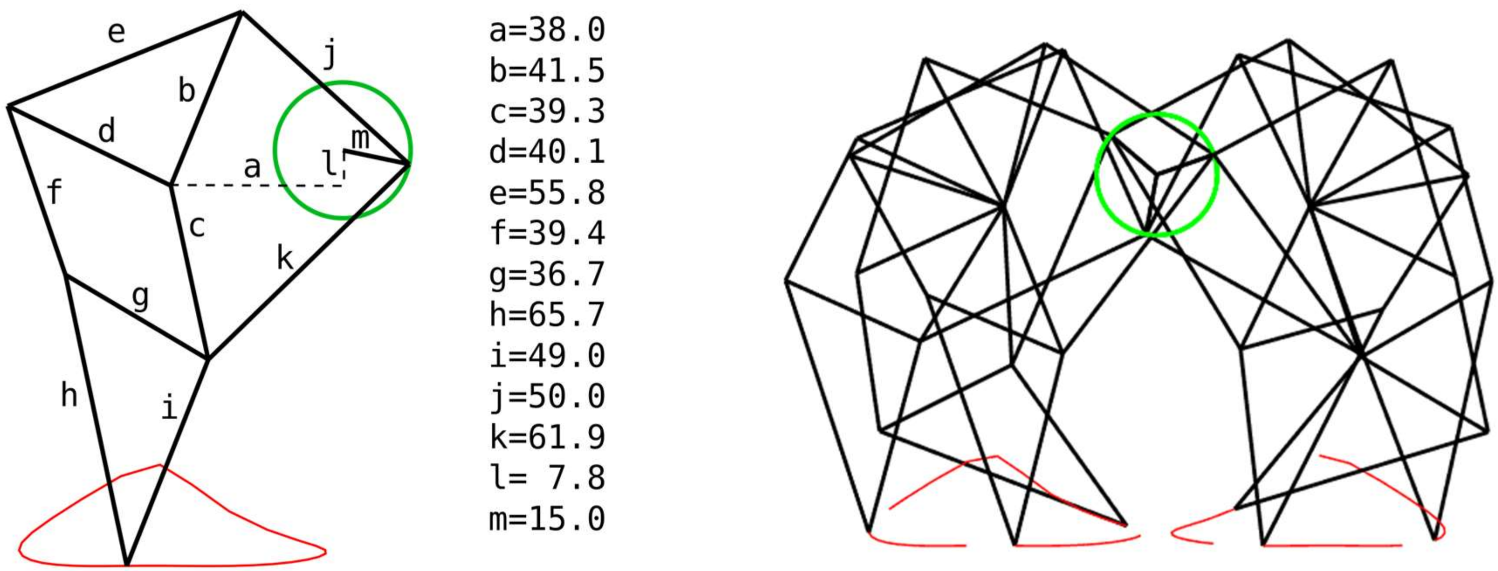
https://www.youtube.com/watch?v=zYGVYLzN06g

https://www.facebook.com/CenterforBioDiv/videos/10155746398505460/

Photo and gif credits: https://www.wired.com/2015/09/heres-chance-play-wind-powered-strandbeest/

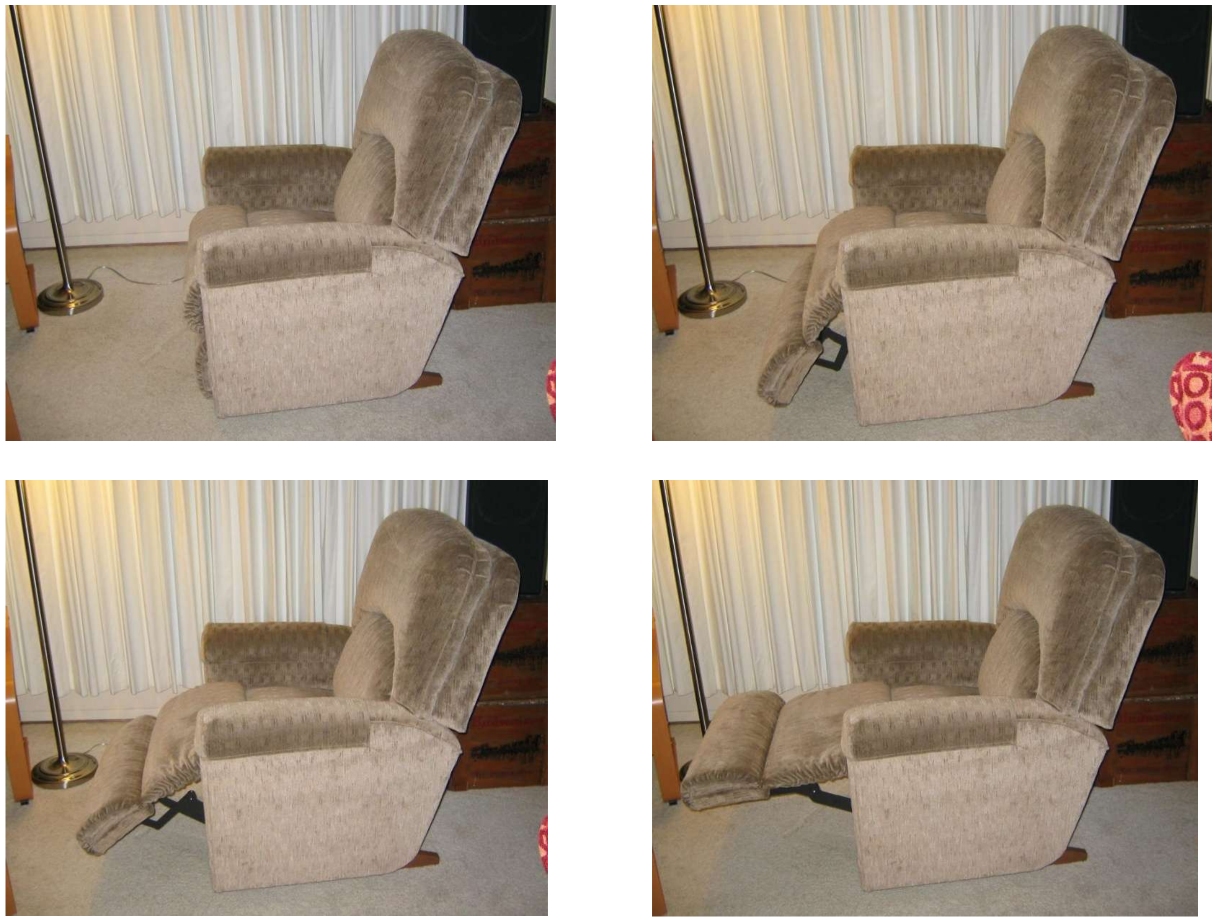
Example: Path Generation

- Jansen Leg mechanism



Example: the recliner

Is the movement of the foot rest on the recliner an example of path generation or motion generation?



Linkage Synthesis – *creating* a mechanisms for an *output*

1. ***Graphical Motion Generation*** – have a *link* follow prescribed *positions.*
   - Create 2-position and 3-position
   - Add dyad drivers and utilizing quick return mechanisms
   - How to fix designs with toggle point problems
2. ***Path Generation*** – have a *point* follow a prescribed *path.*
   - Existing solutions, books, and look up tables
   - Using computer software to fine-tune or optimize a 4-bar output path

Graphical Linkage Synthesis Tools

- We will design mechanisms to achieve desired motion using graphical tools and the principles of geometry
- We need the following tools. Pull them out and have them ready

Ruler

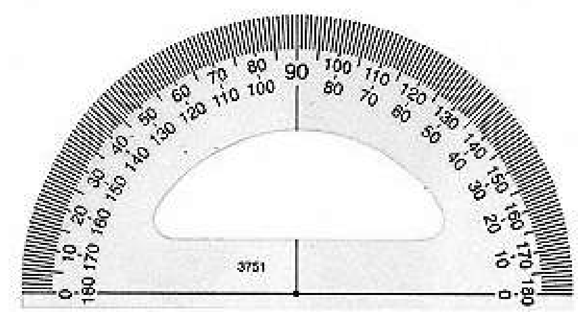
- Draw straight lines
- Measure size

Compass

- Draw constant R curves
- Bisect lines between points

Protractor

- Measure angles



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Graphical Linkage Synthesis

Goals:

Design a mechanism to achieve desired motion

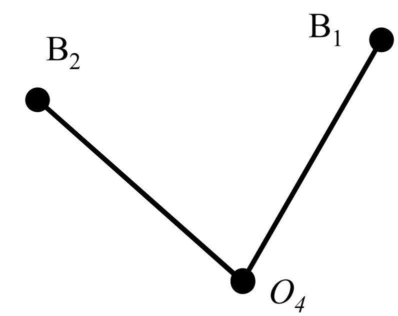
- Two position synthesis
  - Rocker output
  - Coupler output
  - Rotopole
- Three position synthesis

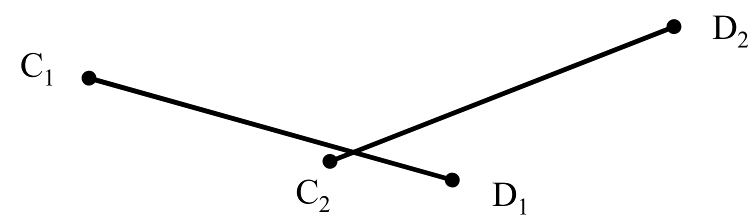
Be able to control the limits and positions of motion

- Dyad driver
- Alternative moving points

Be able to vary the timing of motion

- Quick return mechanisms





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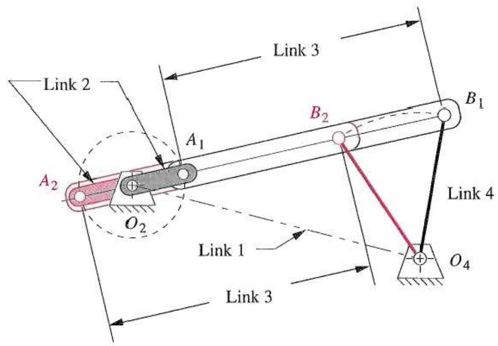
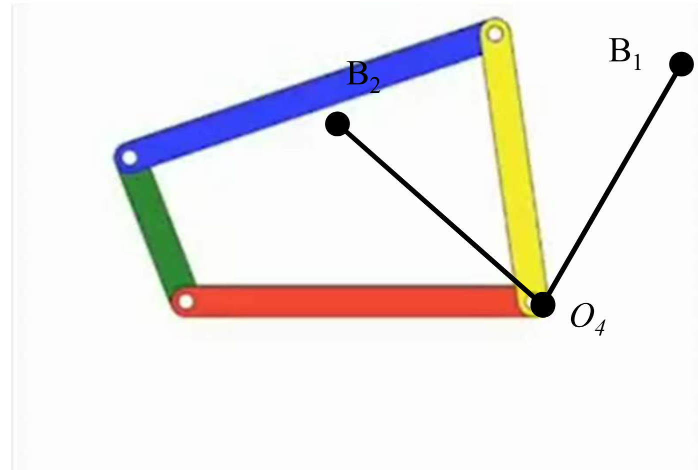
Two-position synthesis

Rocker output

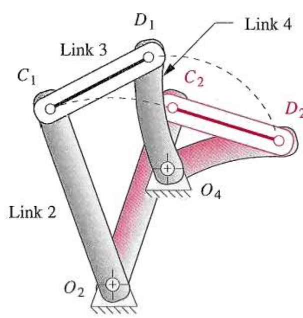
- Most suitable for when you want a Grashof crank-rocker
- Function generator
    - Output is the two angular positions of the rocker

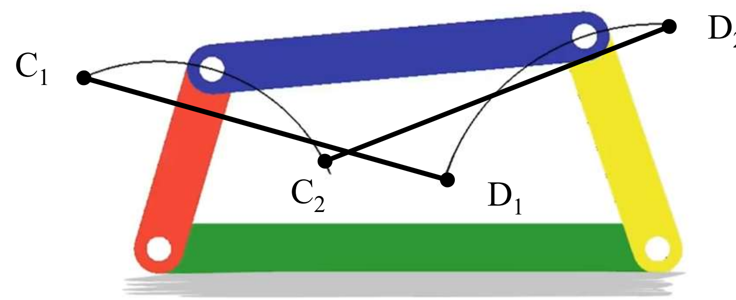




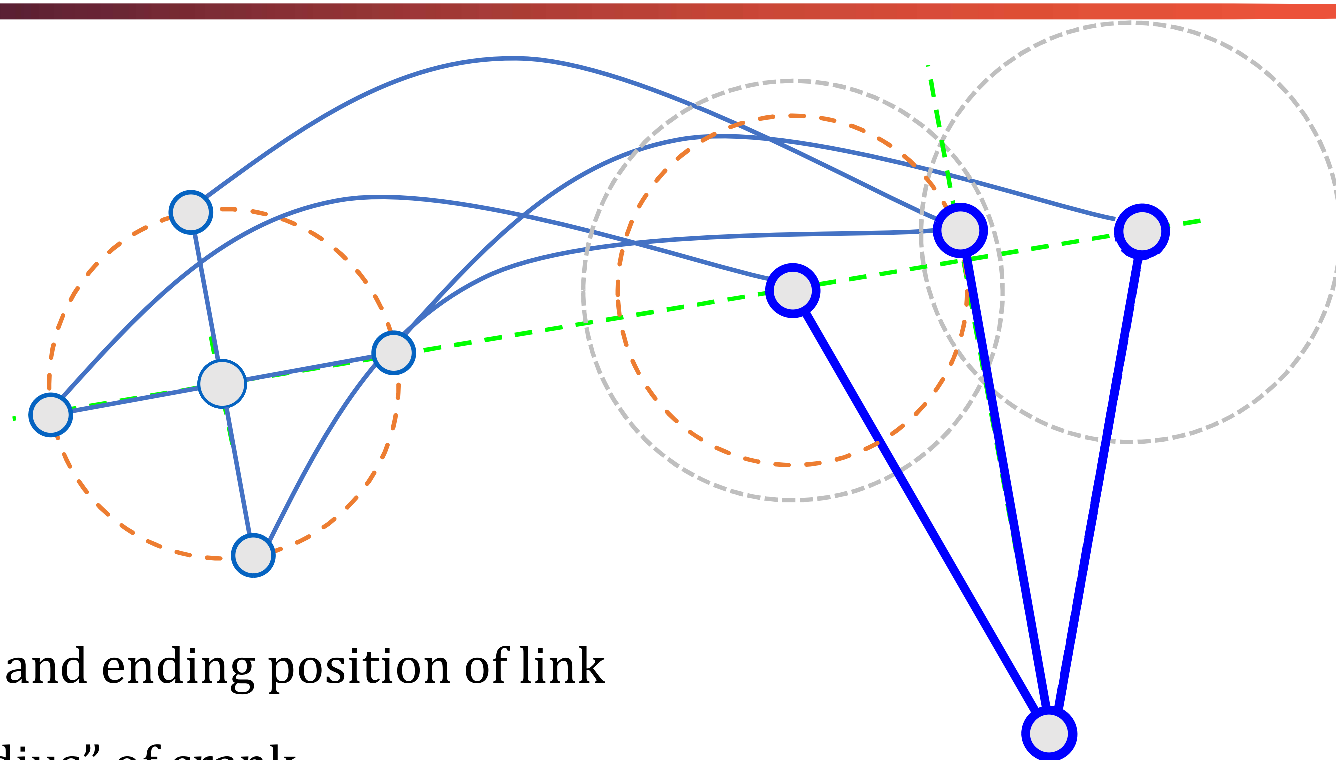
Coupler output

- Motion generator
    - Two positions of a line in the plane are the output
    - Often triple rocker





Graphically designing a Crank rocker

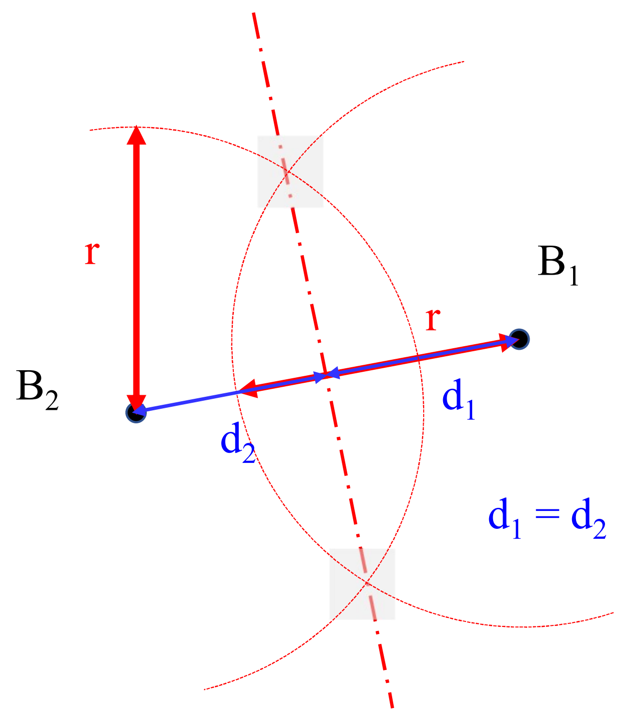


1. Bisect starting and ending position of link
2. Determine “radius” of crank
3. Select location of crank

How to bisect a line with a compass

To evenly divide (bisect) the line

- Choose distance between compass points to be > ½ of line length
- Put point end of compass at one end of the line to be divided (pt. B_1)
- Lightly draw a semi-circle that intersects the line
- Put point end of compass at the other end of the line to be divided (pt. B_2)
- Lightly draw another semi-circle that intersects the line
- Lightly draw a construction line that connects the intersection of the two semi-circles. The intersection of this construction line with the given line is the bisecting point.



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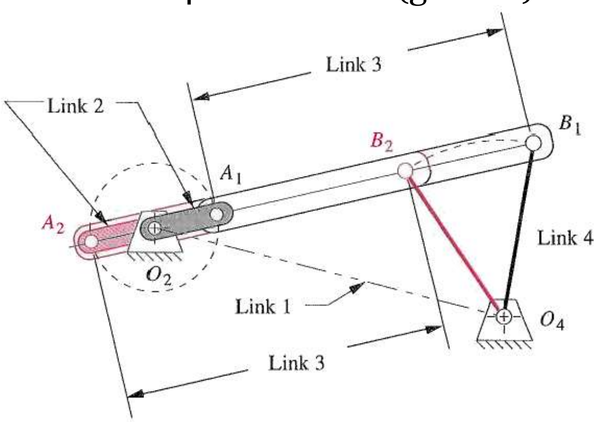
Rocker output

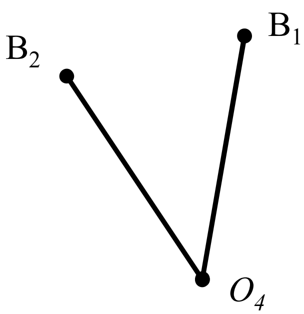
Given: Two rocker positions B_1O_4 and B_2O_4 (link 4), design a 4-bar linkage that will obtain both positions.

Synthesis Steps:

1. Extend B_1B_2.
2. Pick O_2 along line
3. Bisect B_1B_2.
4. Distance is radius around O_2
5. Label A_1 & A_2
6. Check for Grashof condition (**S + L < P + Q**)
7. If non-Grashof, redo steps 2-5.

Note: O2O4 forms link 1 (ground)





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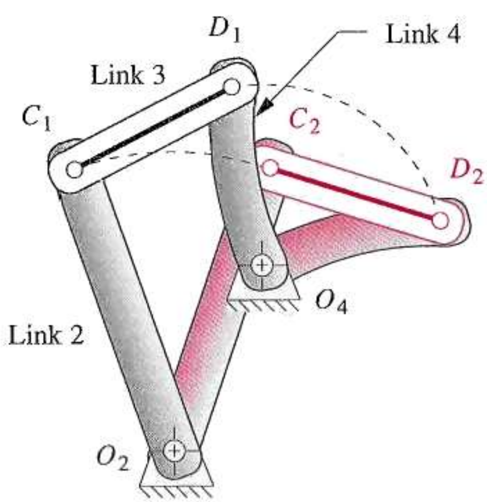
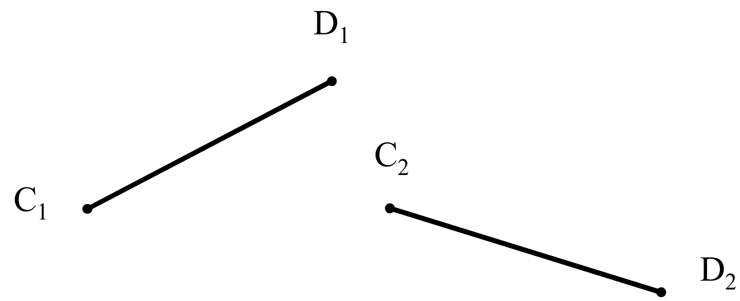
Coupler output

Given: Two coupler positions C_1D_1 and C_2D_2 (link 3), design a 4-bar linkage that will obtain both positions.

Synthesis Steps:

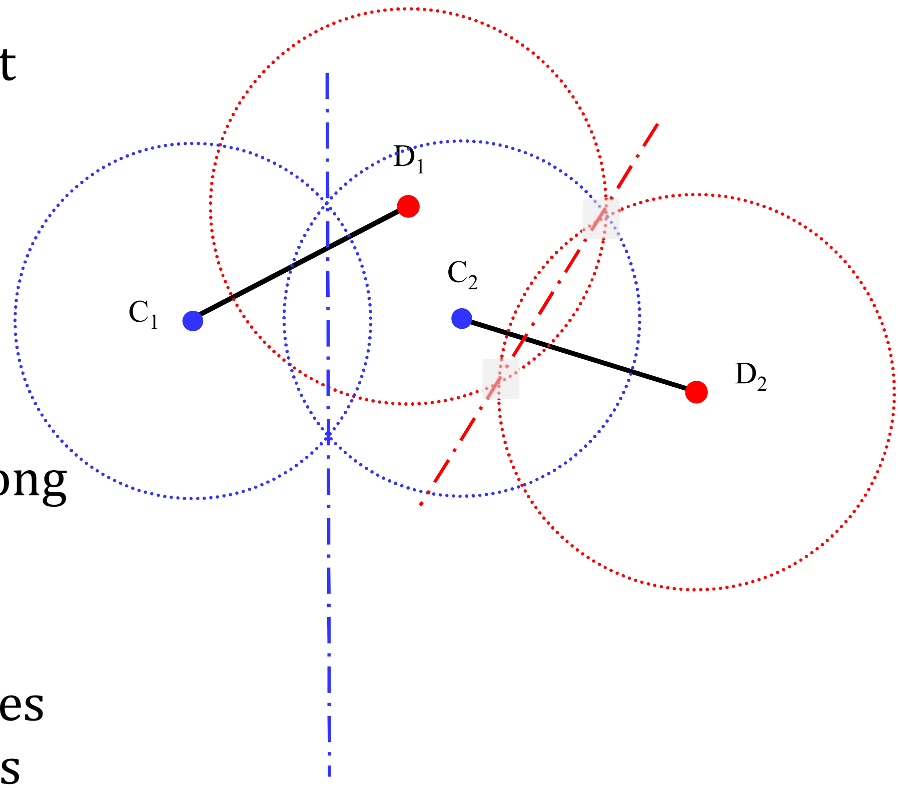
1. Bisect C_1C_2. Pick O_2 along line → forms link 2 (O_2C)
2. Bisect D_1D_2. Pick O_4 along line → forms link 4 (O_4D)
3. Check for Grashof condition





Lots of design freedom with 2-position synthesis

- The pin-joint of the first link can be located ANYWHERE along the bisecting line.
- The pin-joint of the second link can be located ANYWHERE along the bisecting line.
- There are an unlimited number of 4-bar linkages that can accomplish this movement



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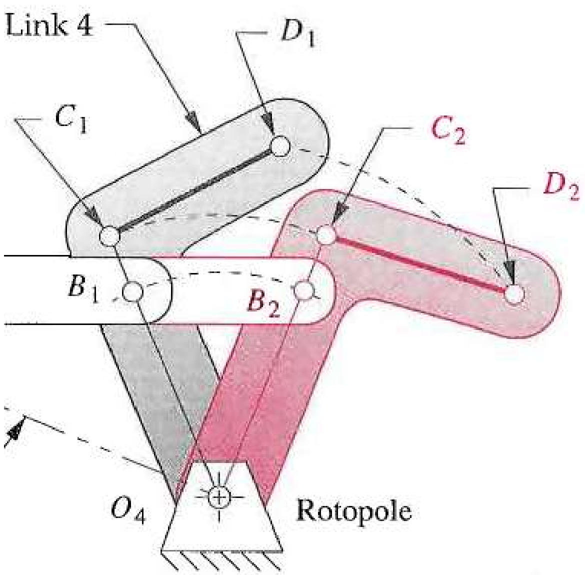
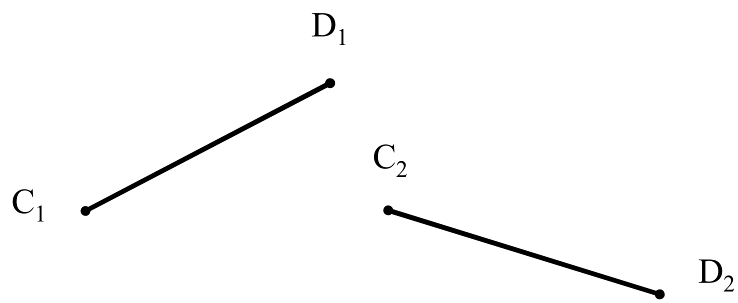
Two-position synthesis: Rotopole

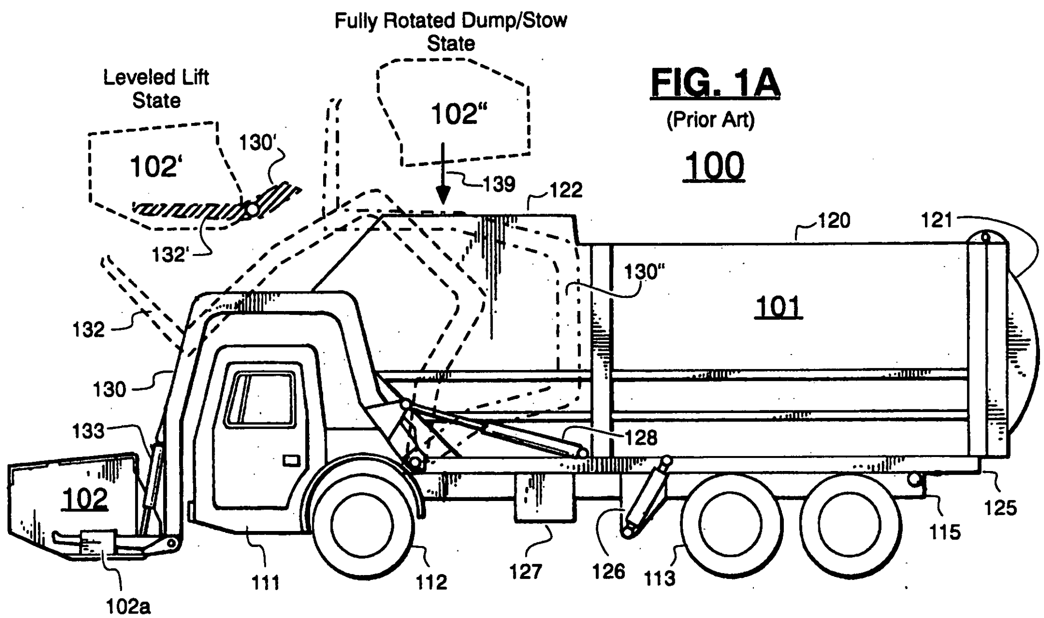
Advantage: simpler & stronger mechanism
Disadvantage: more specific

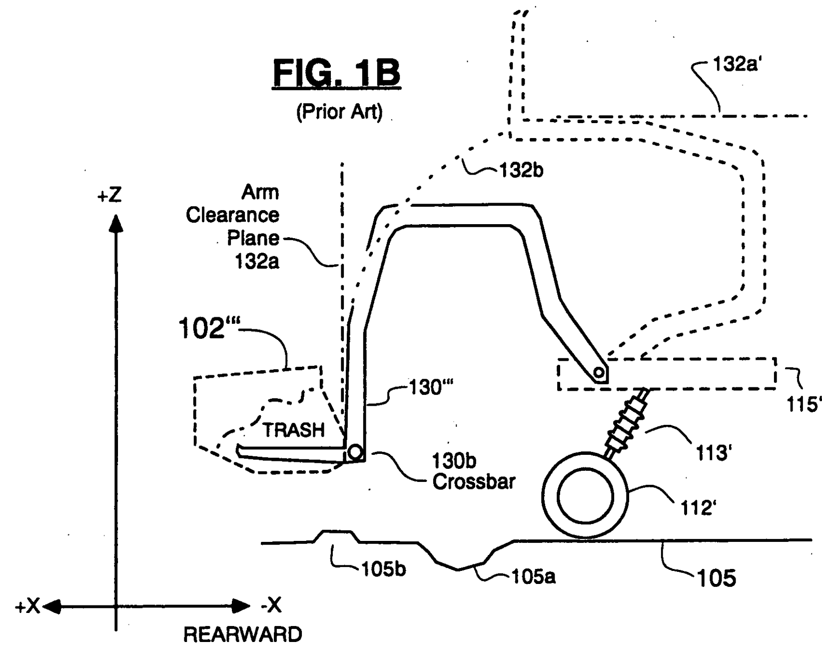
Rotopole: common ground rotation point – converts two-position **__coupler output__** motion to two-position **__rocker output__**.
Then treat as crank-rocker.





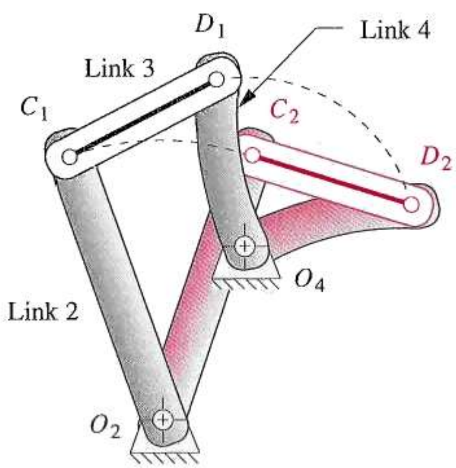
Rotopole example: Garbage truck

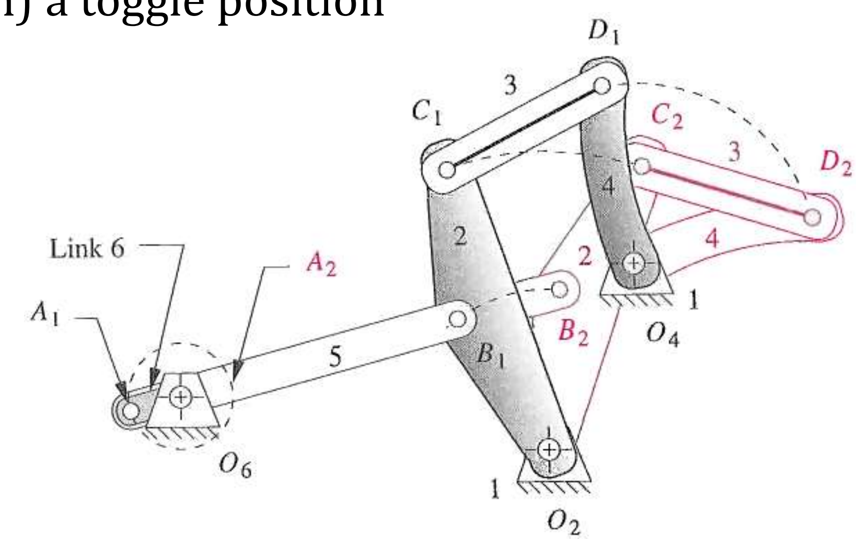




Dyad drivers

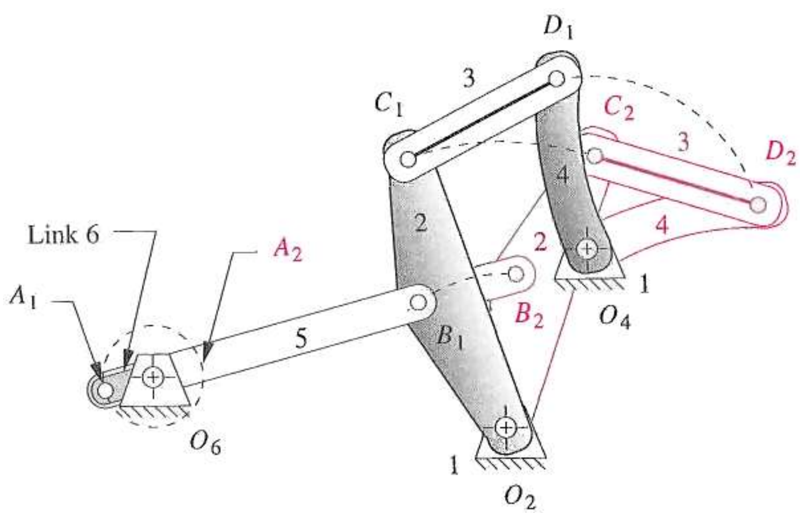
- 2-bar chain added to drive an existing mechanism
- Can be attached to any rocker
- Creates a crank-rocker
- Useful for:
  - Adding reciprocating motion to rocker mechanisms
  - Driving a mechanism out of (or through) a toggle position

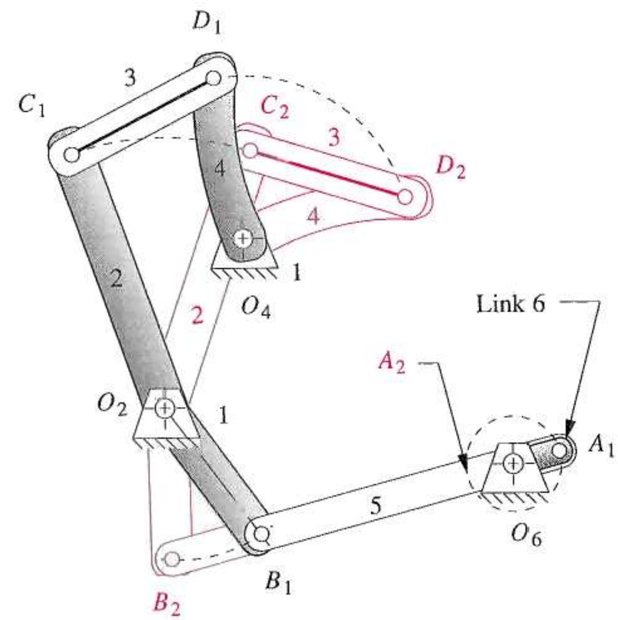




Possible dyad configurations

- **Important:** Many possible driver designs for a given linkage





Dyad synthesis

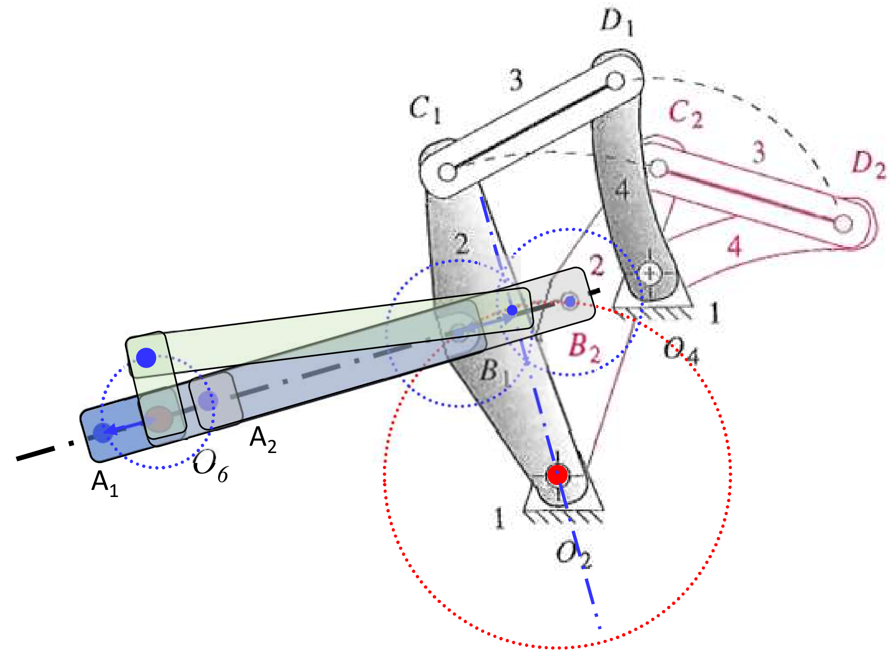
***Add:* 2-bar chain added to drive an existing mechanism.**

Synthesis Steps:

1. Pick B_1 anywhere on an existing link (link 2 here)
2. Draw arc about O_2 through B_1. Note: B_2 will be on this arc but in position 2.
3. Complete steps 1-6 for a 2-position rocker output synthesis

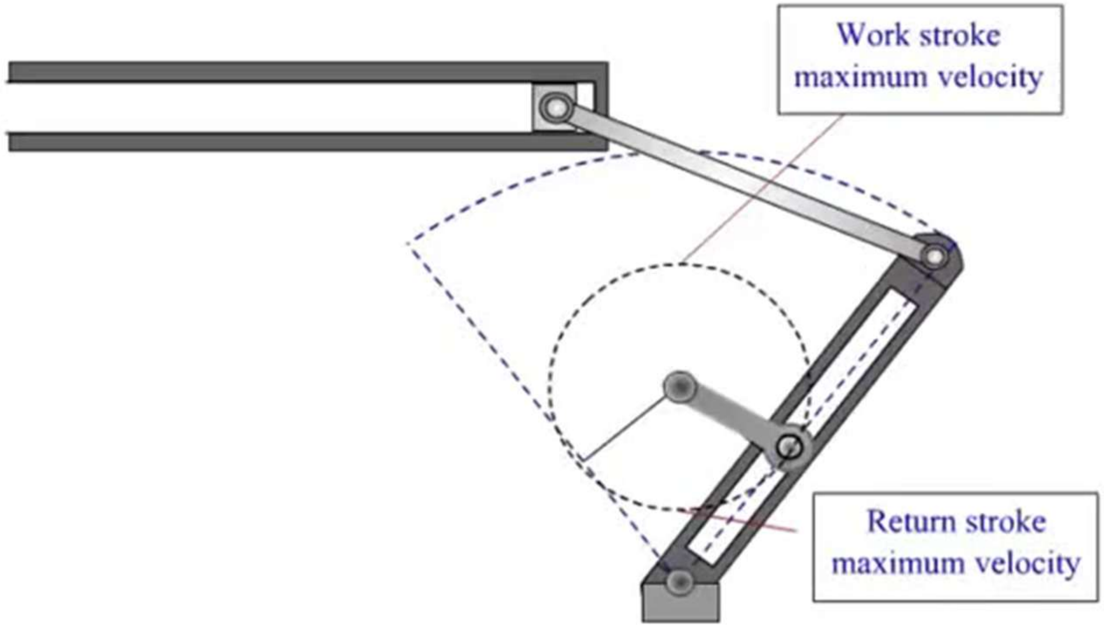
2-position rocker output synthesis steps:

1. Extend B_1B_2.
2. Pick O_6.
3. Bisect B_1B_2. Use distance as radius around O_6.
4. Label A_1 & A_2.
5. Check for Grashof condition with dyad. Measure lengths of links: O_6A_2 (dyad crank), A_2B_2 (dyad coupler), B_2O_2 (dyad rocker), O_6O_2 (dyad ground).
6. If non-Grashof, redo 3-6.



Quick return motion

- Assuming cranks turn at a constant rate, then the **arc swept area** (of the crank rotation) determines the speed ratio between directions.

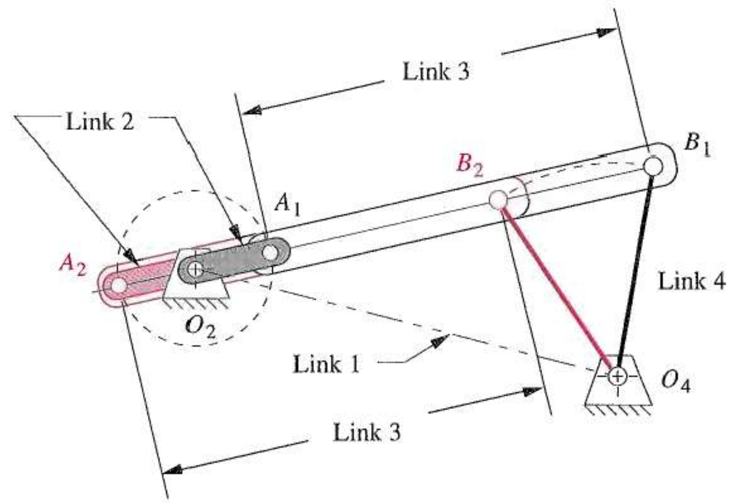


https://www.youtube.com/watch?v=nZCSvbuVU6E

Quick-return mechanisms

In this mechanism the time back and forth are the same because point O_2 is along B_1B_2

What happens O_2 if is not along B_1B_2?



Determining quick-return: time ratio T_R

1. Determine graphically the arc swept areas between the motions forward and back

α: return, β: forward

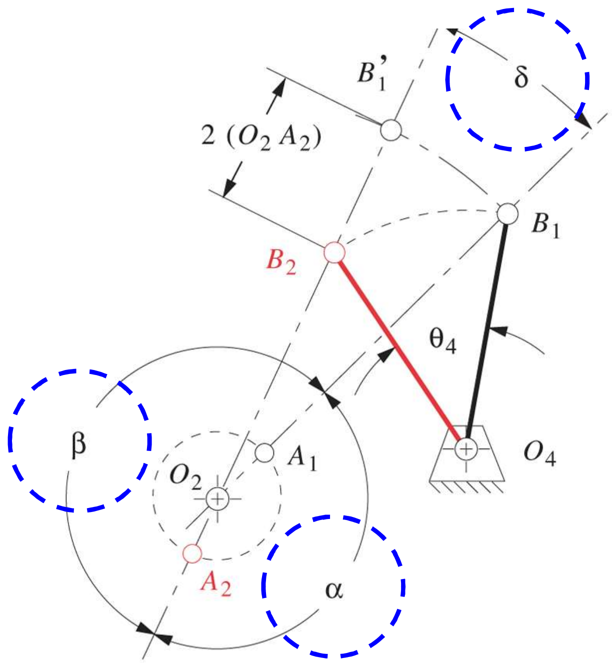
1. Check values knowing δ is the angle between the extended coupler positions:

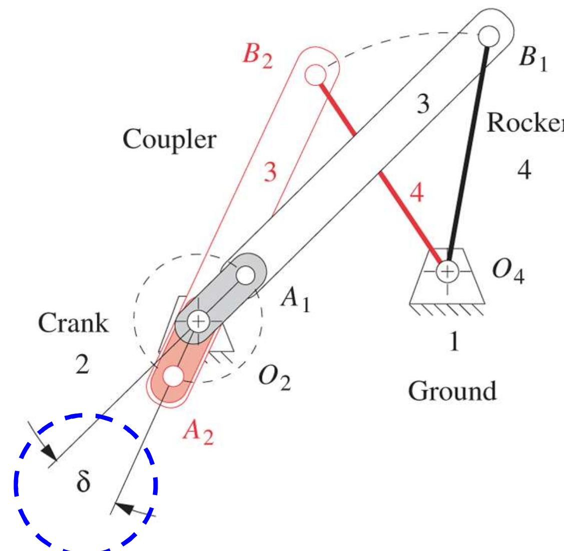
$$\alpha + \beta = 360$$

$$\delta = (\beta - \alpha)/2$$

2. Quick return ratio gives the time or speed ratio between motions.

$$T_R = \alpha / \beta$$





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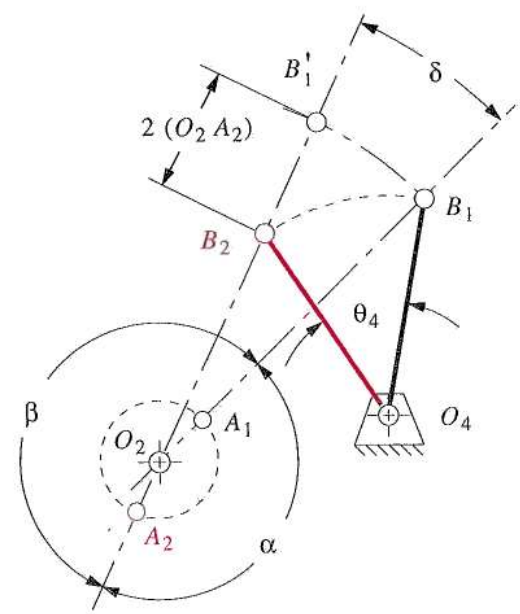
Synthesizing quick-return mechanisms

1. For a given T_r, calculate α, β and δ
2. Draw a line through point B_1 at any convenient angle
3. Draw a line through B_2 at angle δ from the first line, label intersection O_2 (O_2O_4 defines ground link)
4. Draw an arc centered at O_2 from B_1 to cut the extended line O_2B_2. Label intersection as B_1'
5. The length of the crank is 0.5 (B_2B_1')
6. Draw an arc centered at O_2 with a radius= 0.5 (B_2B_1'). Label A_1 and A_2 along O_2B_1 and O_2B_2, respectively.
7. Check for Grashof. If non-Grashof, repeat 2-6.



(a) Construction of a quick-return Grashof crank-rocker

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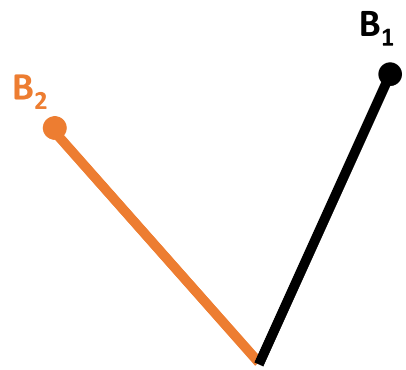
Example of constructing a quick return

1. For a given TR calculate α, β and δ
2. Draw a line through point B1 at any convenient angle
3. Draw a line through B_2 at angle δ from the first line, label intersection O_2 (O_2O_4 defines ground link)
4. Draw an arc centered at O_2 from B_1 to cut the extended line O_2B_2. label intersection B_1'
5. The length of the crank is 0.5 B_2B_1'
6. Draw an arc centered at O2 with a radius= 0.5 B_2B_1'. Label A_1 and A_2 along O_2B_1 and O_2B_2, respectively. These points define the extreme positions of the crank rocker.
7. Measure out link lengths. Check for Grashof. If non Grashof repeat 2-6



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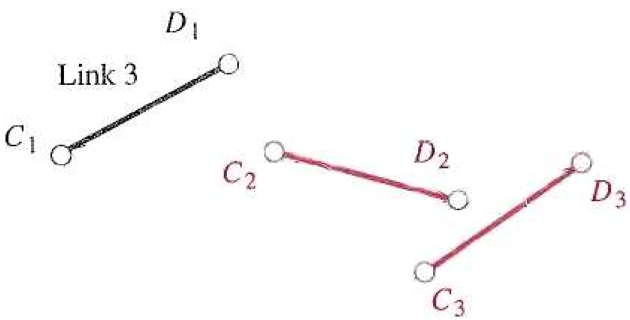
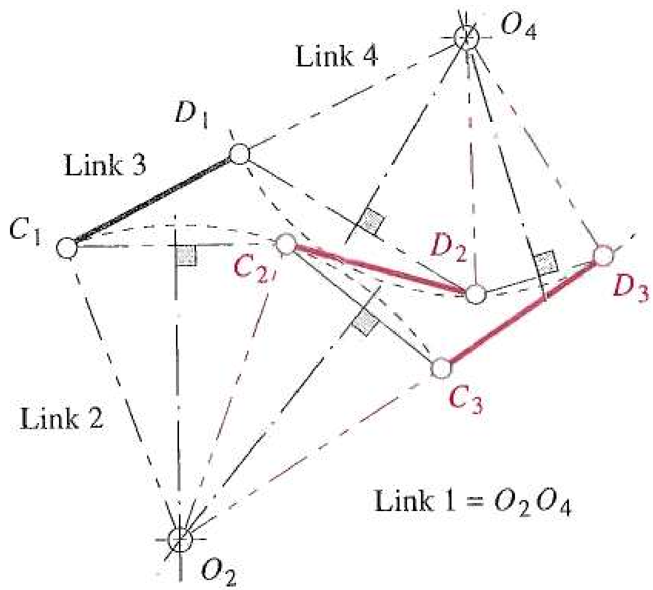
Three-position synthesis: Specified moving points

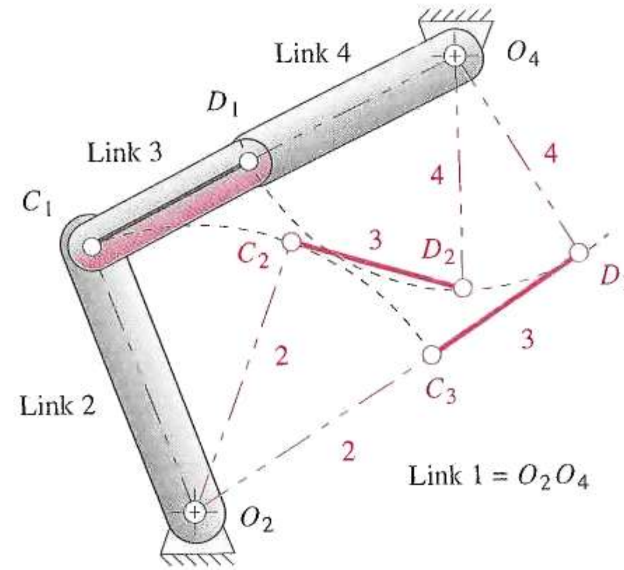
- To find ground point O_2:
  - Find intersection of bisections of C_1C_2 and C_2C_3
- Repeat for O_4 using point D
- Example 3-5 in Norton 3.4

problem



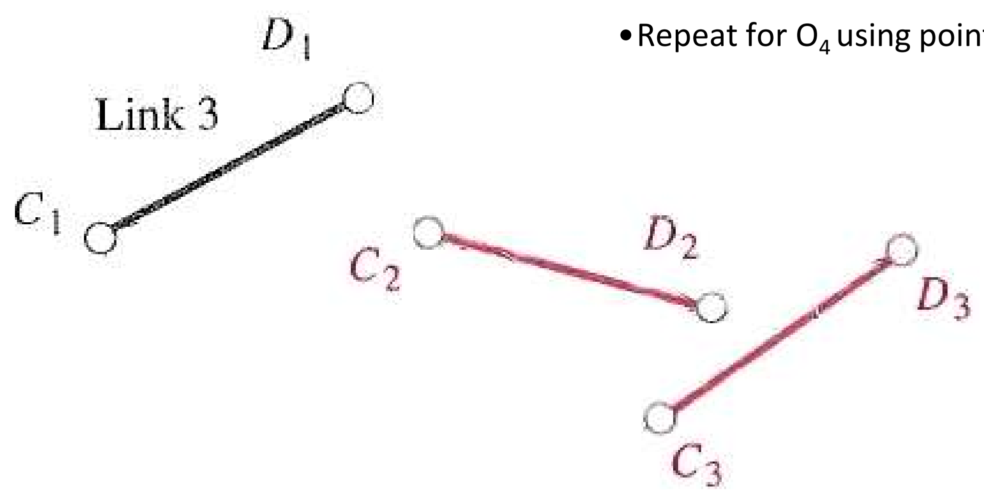
solution





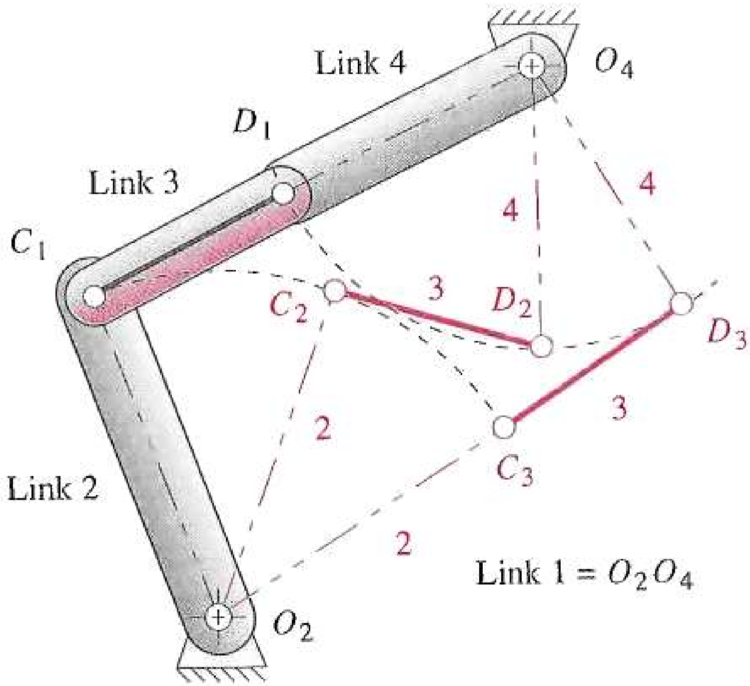
Specified moving points

- Example 3-5 in Norton section 3.4



- To find ground point O_2:
  - Find intersection of bisections of C_1C_2 and C_2C_3
- Repeat for O_4 using point D

Specified moving points – final design



Note toggle positions in positions 1 & 3

- Can we use a dyad to drive out of the toggle positions?

- What happens if the previous technique places the fixed points (O_2 and O_4) in undesirable locations?

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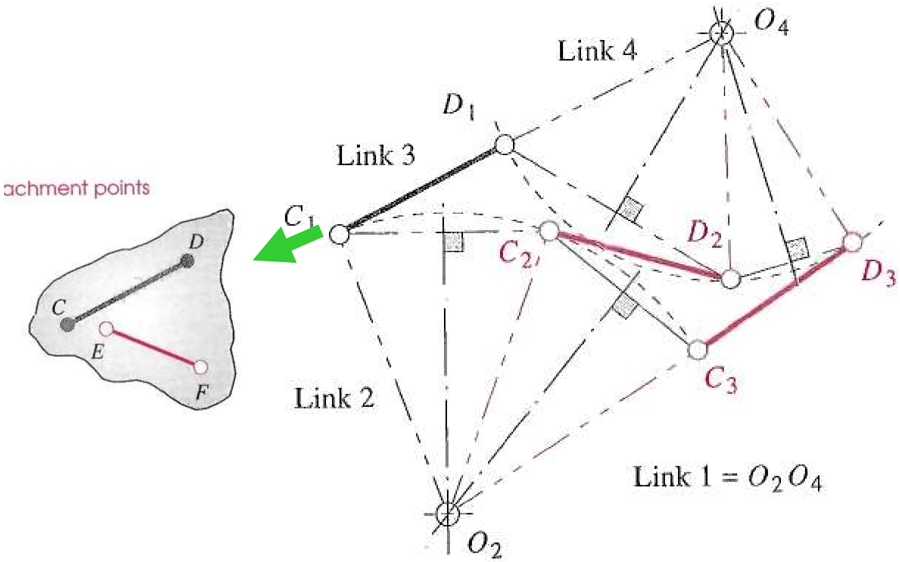
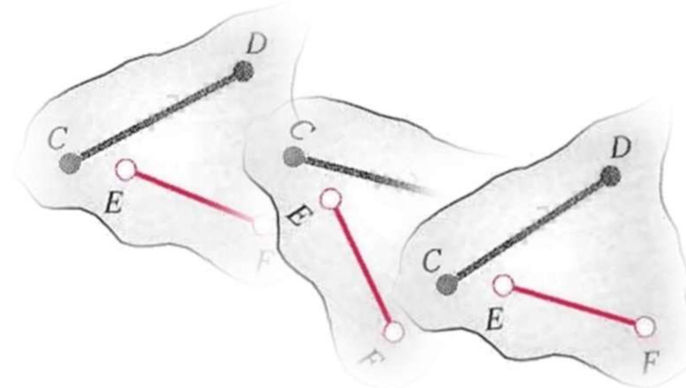
Solution: Alternative moving points

Change the *link size* & *node locations* → must change the *node path*

problem





Follow same procedure as before, but use new nodes EF to define locations of O_2 and O_4

Example 3-6 in Norton section 3.4

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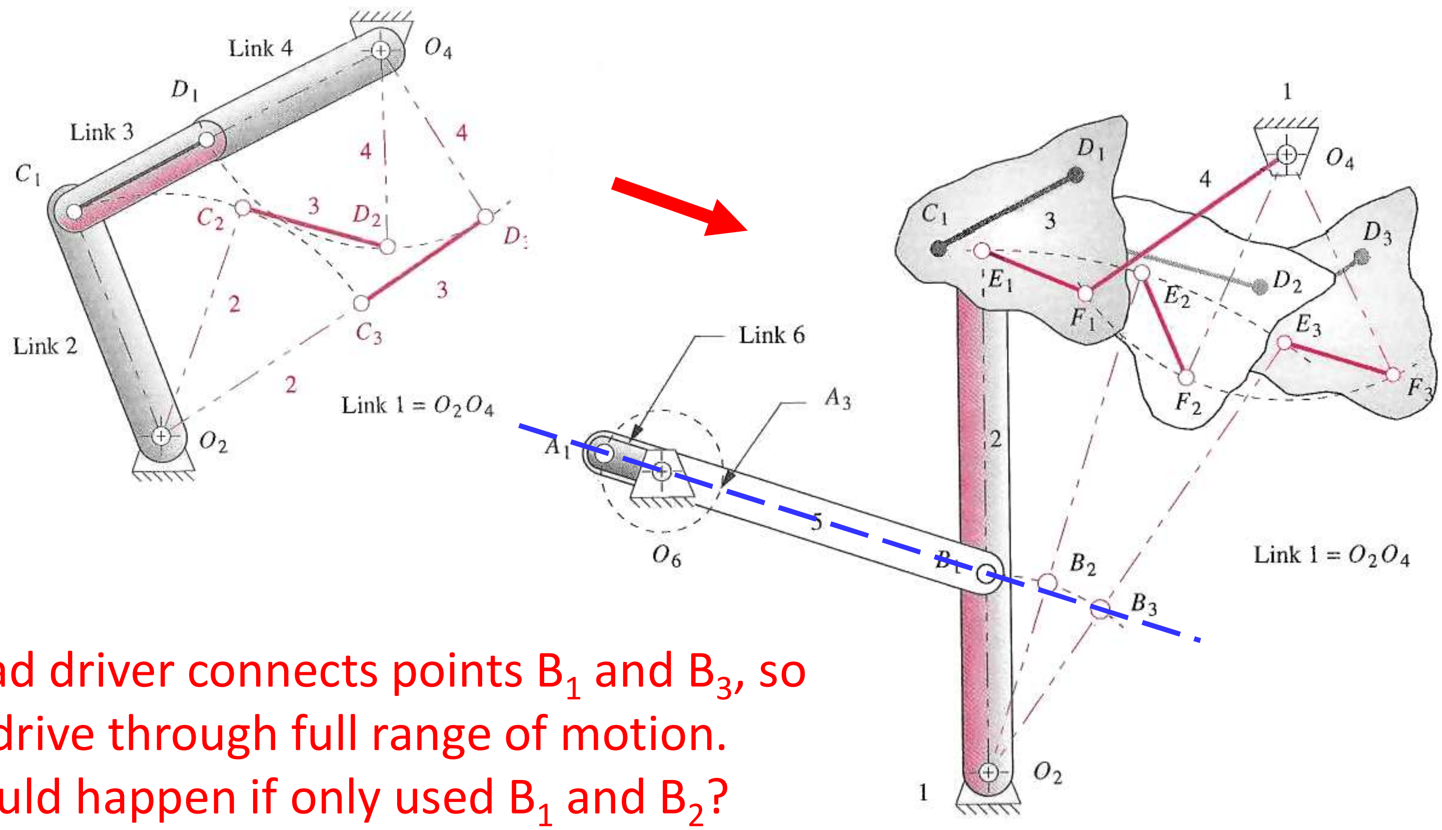
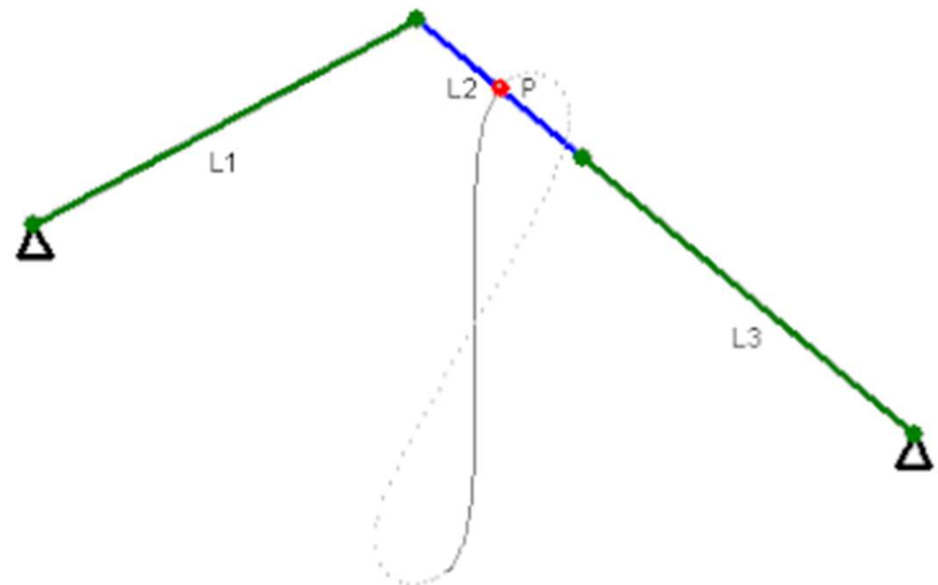
Alternate moving points – final design



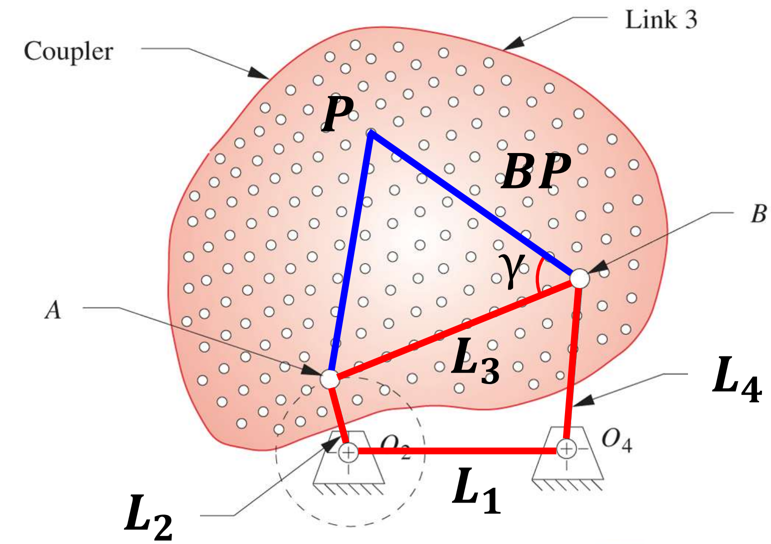
Note: dyad driver connects points B_1 and B_3, so that can drive through full range of motion. What would happen if only used B_1 and B_2?

Path Synthesis or Path Generation

Path Synthesis/Generation: Designing a mechanism to make a point follow a prescribed path

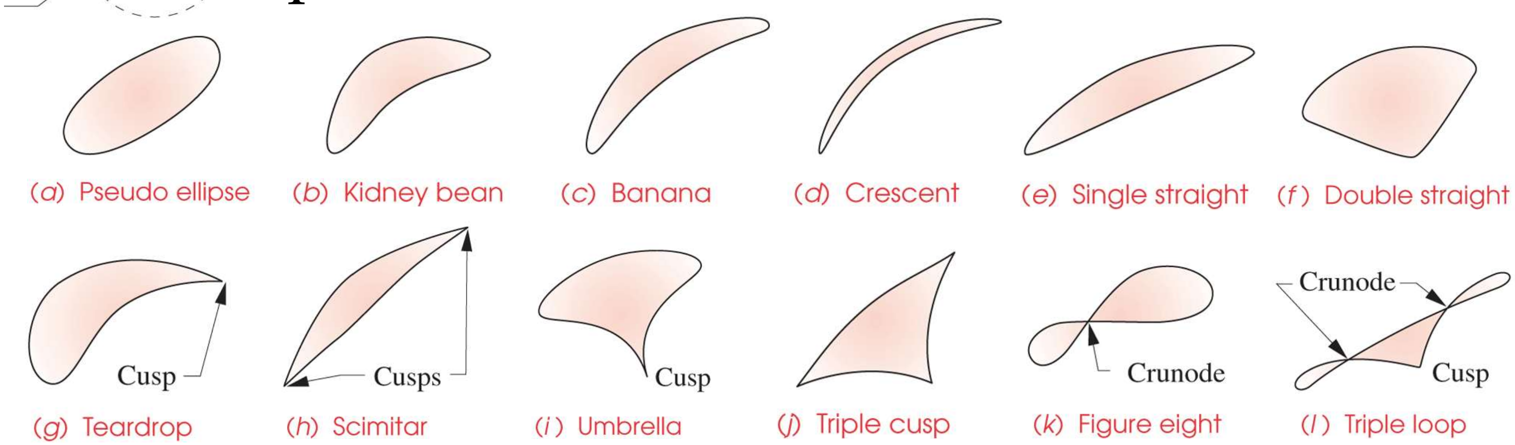


Four-bar mechanism – coupler curves…



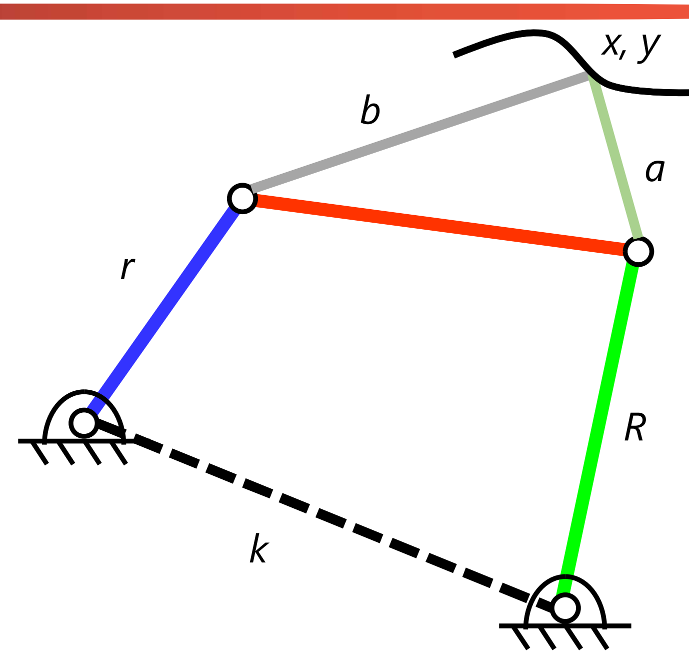
- Put a pen in any hole, *such as "P,"* in link-3.
- As link-2 rotates it creates an "output drawing."
- As the *length* of *link-2* or *link-4*, or the "*location*" in *link-3* changes, you will get "families" of different output
  - Visualizing Paths as a function of 4-bar geometry

http://dynref.engr.illinois.edu/aml.html
https://demonstrations.wolfram.com/CouplerCurvesOfAFourBarLinkage/



Analytical Path Generation of coupler points

- The equation describing a point on the coupler curve is of 6th order.
- It is complex and can be written in many forms.
- Such complex path generation is usually solved graphically, or by numerical trial-and-error.
- The Beyer form is written as:



$$
\begin{aligned}
& a^2\left[(x-k)^2+y^2\right]\left(x^2+y^2+b^2-r^2\right)^2 \\
& -2ab\left[\left(x^2+y^2-kx\right)\cos\gamma+ky\sin\gamma\right]\left(x^2+y^2++b^2-r^2\right)\left[(x-k)^2+y^2+a^2-R^2\right] \\
& +b^2\left(x^2+y^2\right)\left[(x-k)^2+y^2+a^2-R^2\right]^2 \\
& -4a^2b^2\left[\left(x^2+y^2-kx\right)\sin\gamma-ky\cos\gamma\right]^2=0
\end{aligned}
$$

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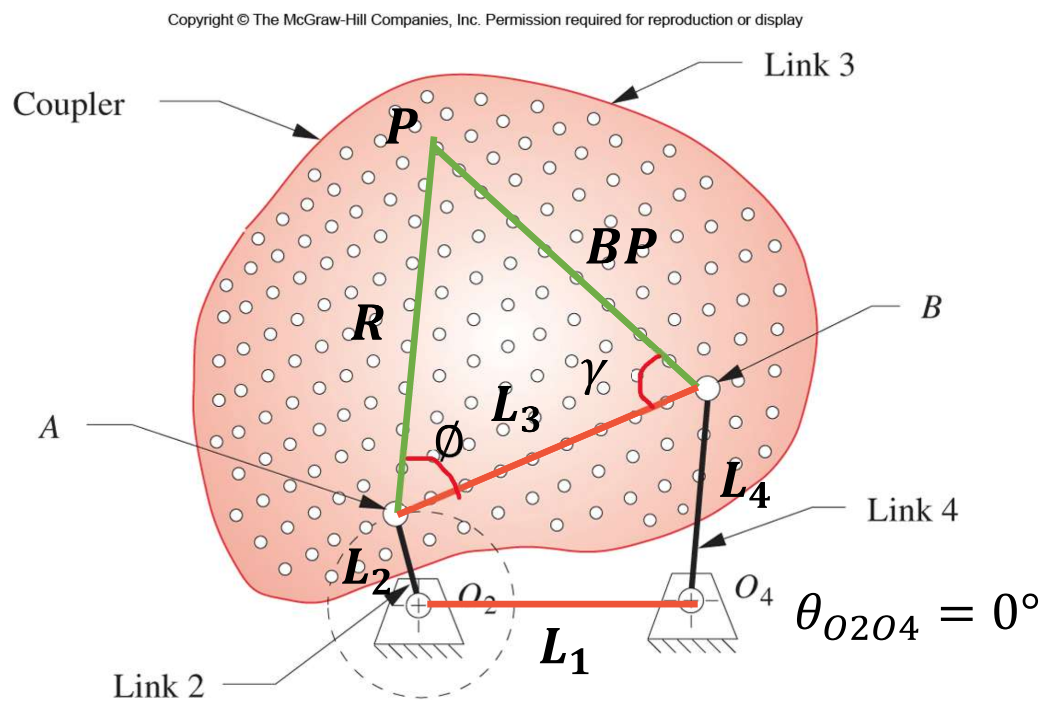
Path Generation: Coupler Points

Determining independent parameters to define the size, orientation, and shape of a coupler curve

To determine mechanism:

To determine path position P:

Independent parameters:



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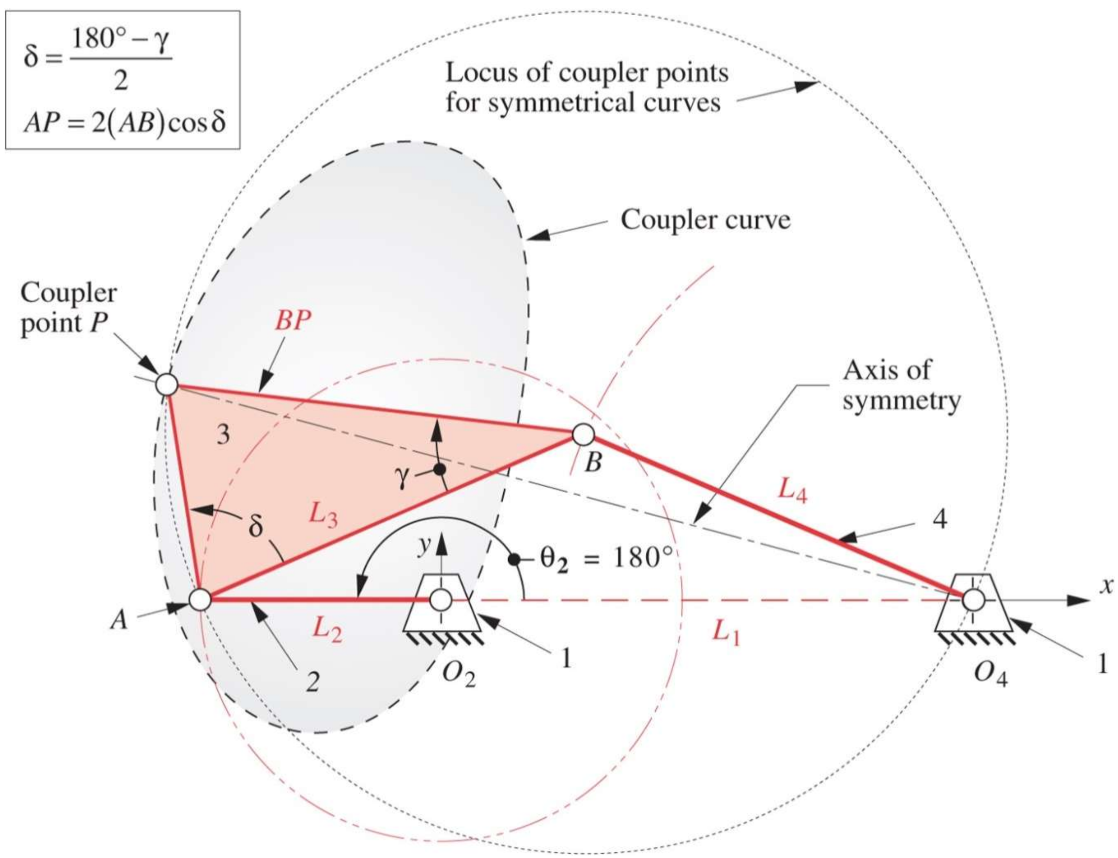
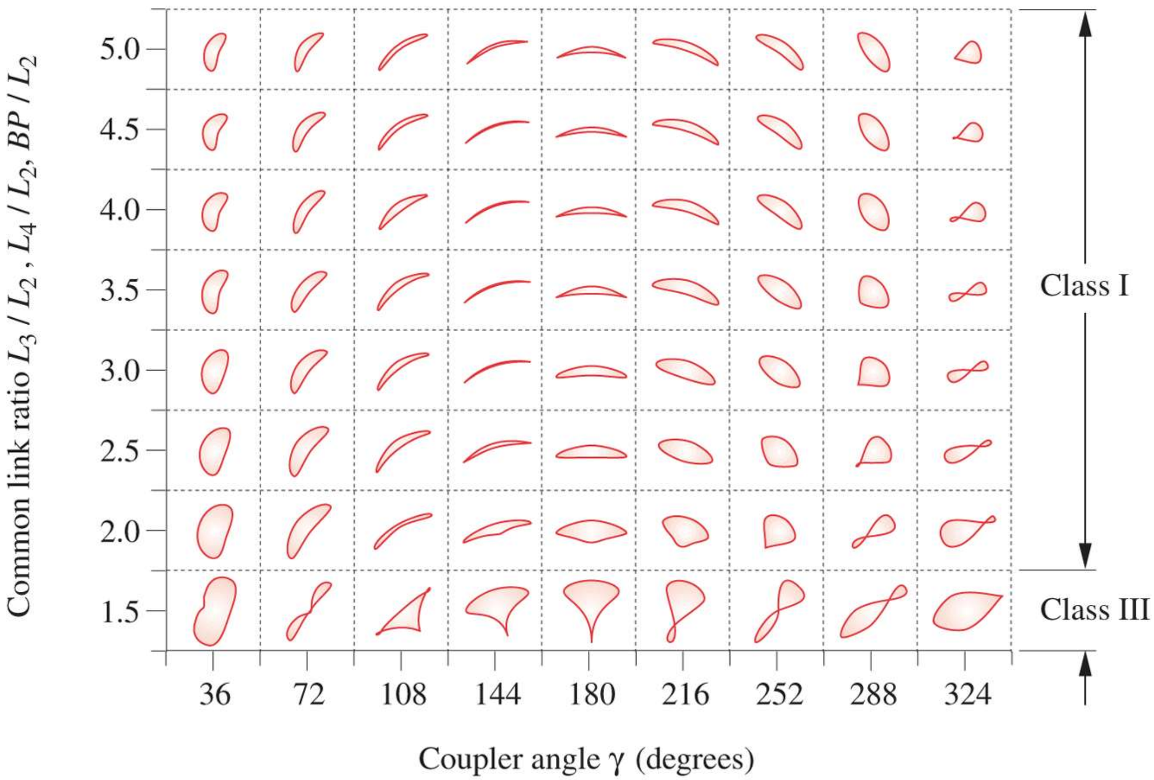
Five input parameter of coupler curve tables



1. Ground link ratio L_1 / L_2
2. Coupler link ratio L_3 / L_2
3. Output link ratio L_4 / L_2
4. Offset link ratio BP / L_2
5. Offset angle γ

Coupler curves for symmetric 4-bar linkages

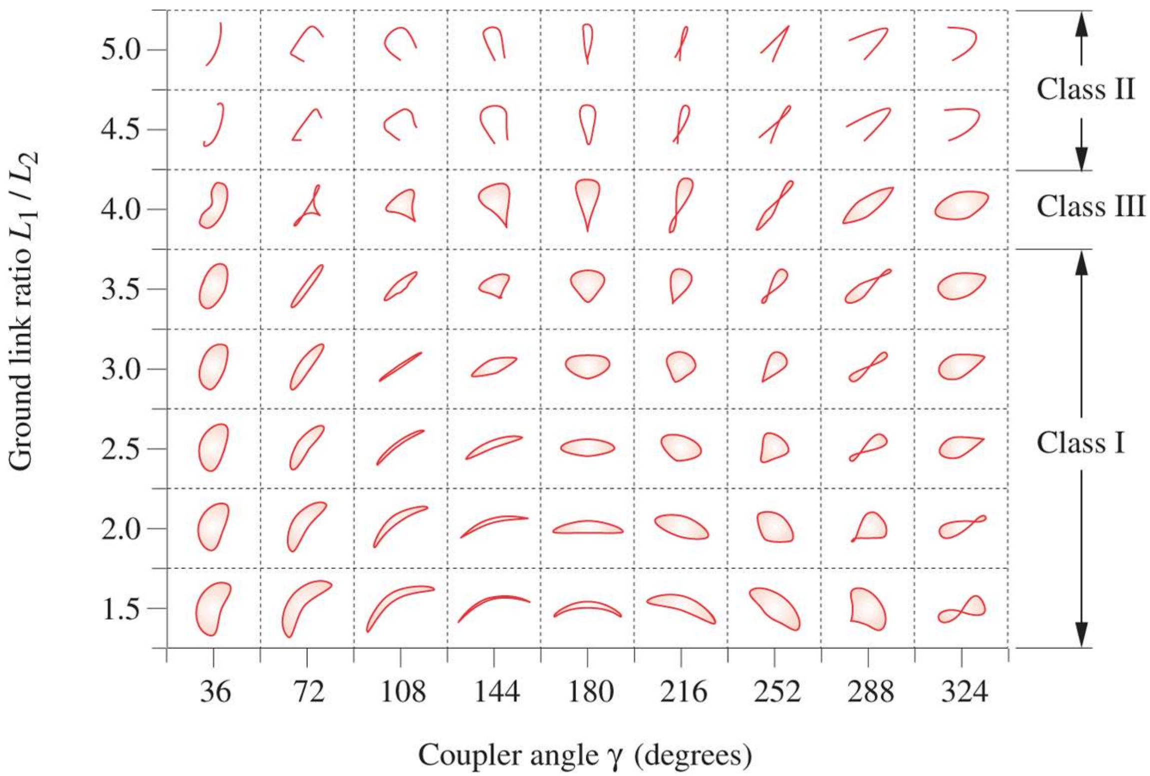


(*a*) Variation of coupler curve shape with common link ratio and coupler angle for a ground link ratio $L_1 / L_2 = 2.0$

- We can use lookup tables and atlases to find the mechanism parameters that will give us the desired curve

$L_1/L_2 = 2$ for all graphs shown

Coupler curves for symmetric 4-bar linkages



(*b*) Variation of coupler curve shape with ground link ratio and coupler angle for a common link ratio $L_3 / L_2 = L_4 / L_2 = BP / L_2 = 2.5$

$$^{L_3}/_{L_2} = {}^{L_4}/_{L_2} = {}^{BP}/_{L_2} = 2.5$$

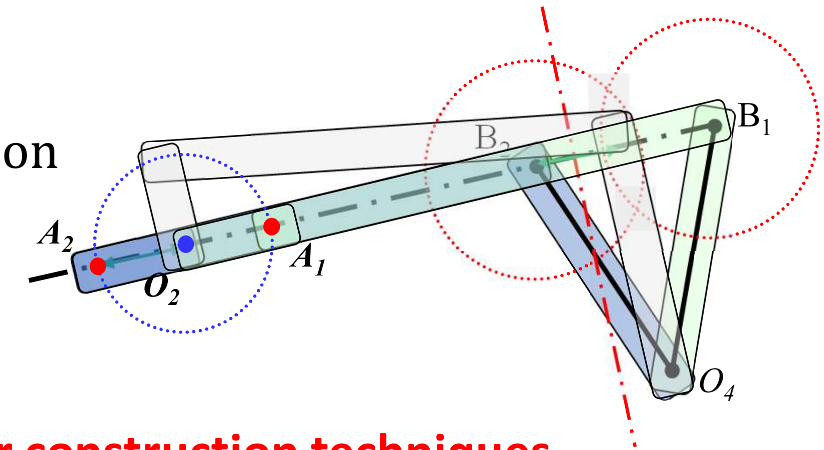
for all graphs

Using lookup tables for path synthesis

1. Define the motion you want
2. Use the look up table to find the motion that most closely resembles the motion you desire. Read off the mechanism parameters that give that motion and use them as a starting point for your design
3. Examine how the path changes by varying the different mechanism parameters along each axis.
4. Adjust the motion by varying the mechanism parameters and test using simulation or prototyping until desired motion is achieved.

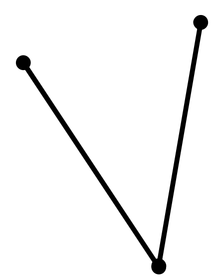
Precision is important in synthesis

- Poor precision means:
  - Designed mechanism might not give desired motion
  - Assembly may not be possible
  - Might have incorrect timing
- Two major mistakes during GLS:



To small/big of mechanism

Poor construction techniques



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