Lecture 2-4

Planar Mechanisms 1-3

ME 370 - Mechanical Design 1

"Colibri" by Derek Hugger

** www.youtube.com/watch?v=1scj5sotD-E*

Planar mechanism kinematics (Chapter 2)

- Planar Kinematics
- Identifying links and joints
- Degrees of Freedom
- Mechanism mobility
  - Degrees of freedom in a mechanism
- Gruebler's Equation (Gruebler-Kutzbach Equation)
- Mechanisms, structures, and preloaded structures
- Four-bar linkage mechanisms
  - Crank-rocker, double crank, double rocker, crank-slider
- Transformation Rules
- Isomers
- Gruebler's Paradoxes
- Grashof Condition
- Inversions
- Toggle Position
- Transmission angle (section 3.3)

PLANAR MECHANISM KINEMATICS

- **Kinematics:**
- Study of motion
  - position, velocity, acceleration
- We'll predominately do only 2-D (planar) designs and analyses

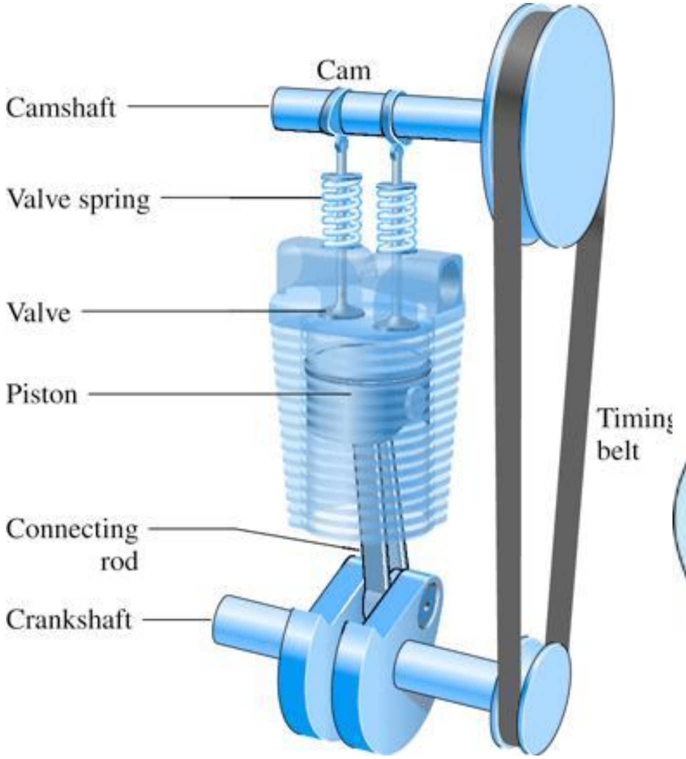
- **Mechanism**:
- Device that transforms motions to some desirable pattern and typically develops very low forces and transmit little power.

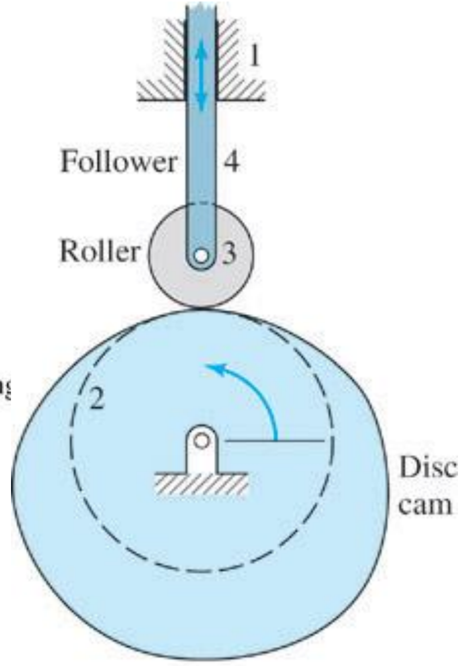
high power → machines

Types of planar (2-D) motion

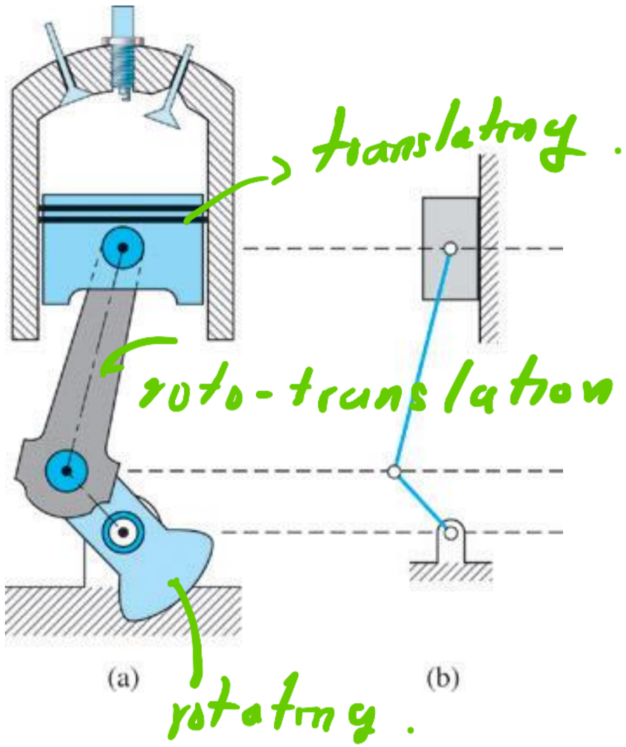
- Rotation
- Translation
- Complex motion = rotation + translation

Video on 4 stroke diesel engine
http://youtu.be/fTAUq6G9apg





(a)



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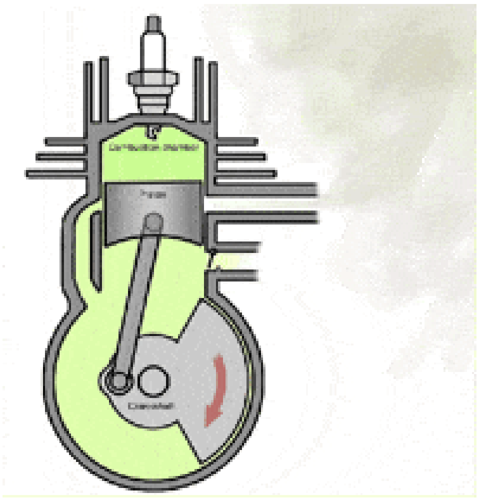
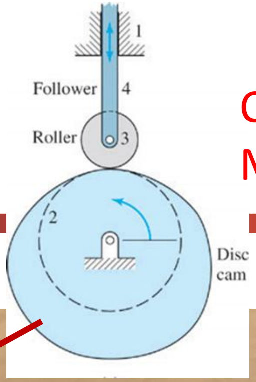


https://images.app.goo.gl/YLNrbxypMmN8mCoGA

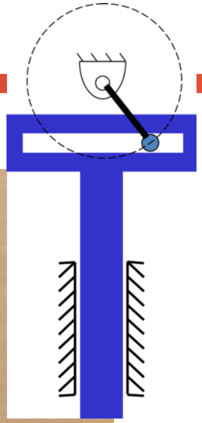
Engineered Mechanisms: battery-operated sewing machine

What types of mechanisms are contained in this sewing machine?

Engineered Mechanisms

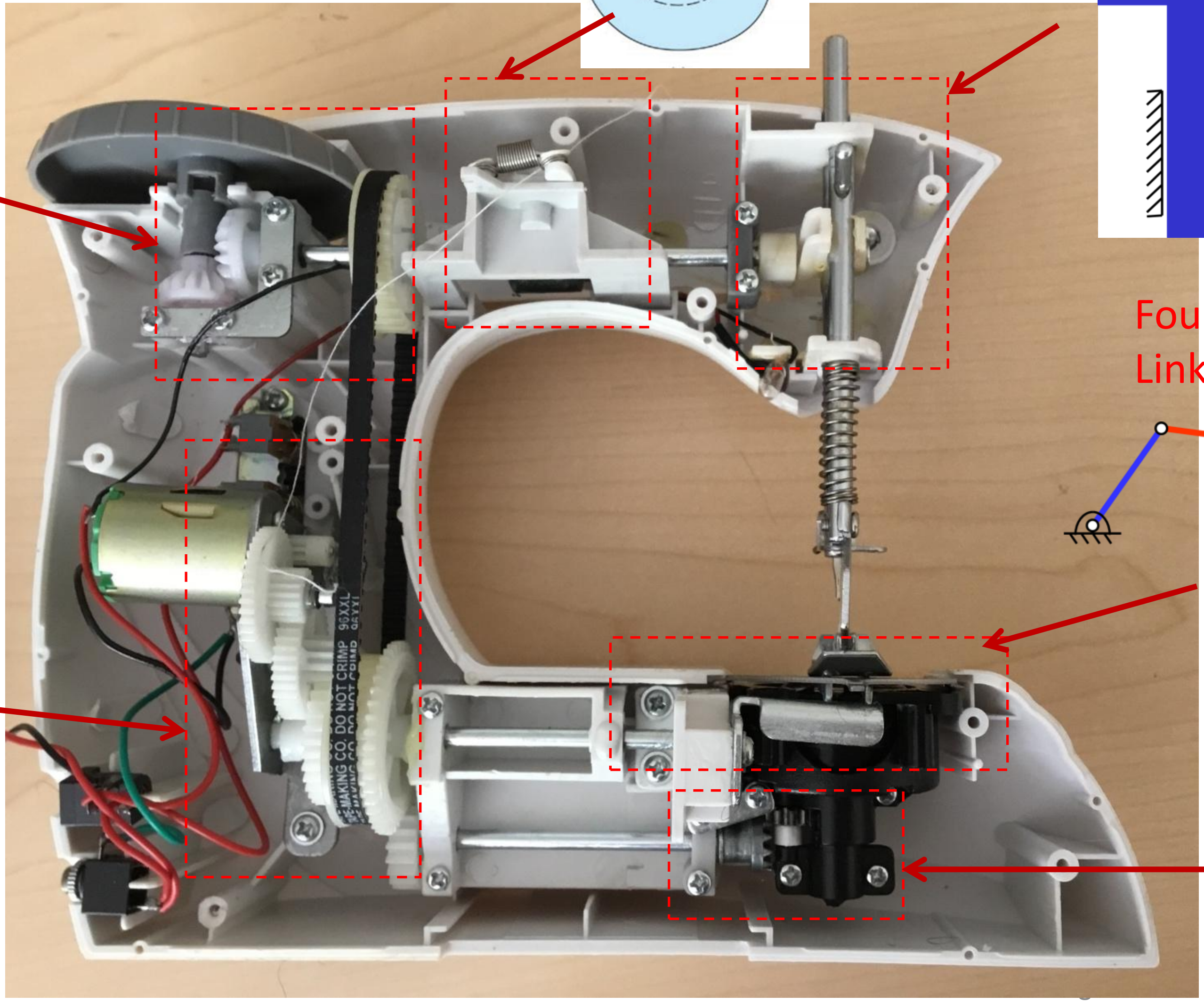


Cam-Follower Mechanism

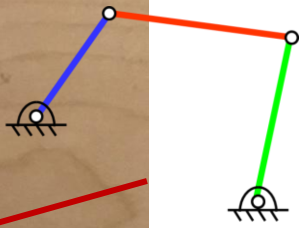


Scotch- Yoke Mechanism

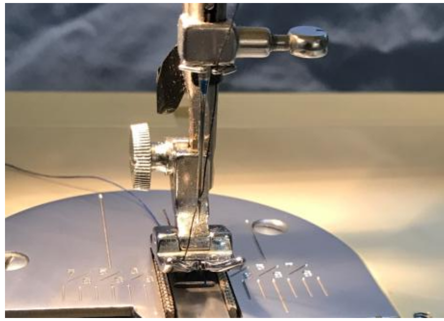
Gearing



Four-bar Linkage



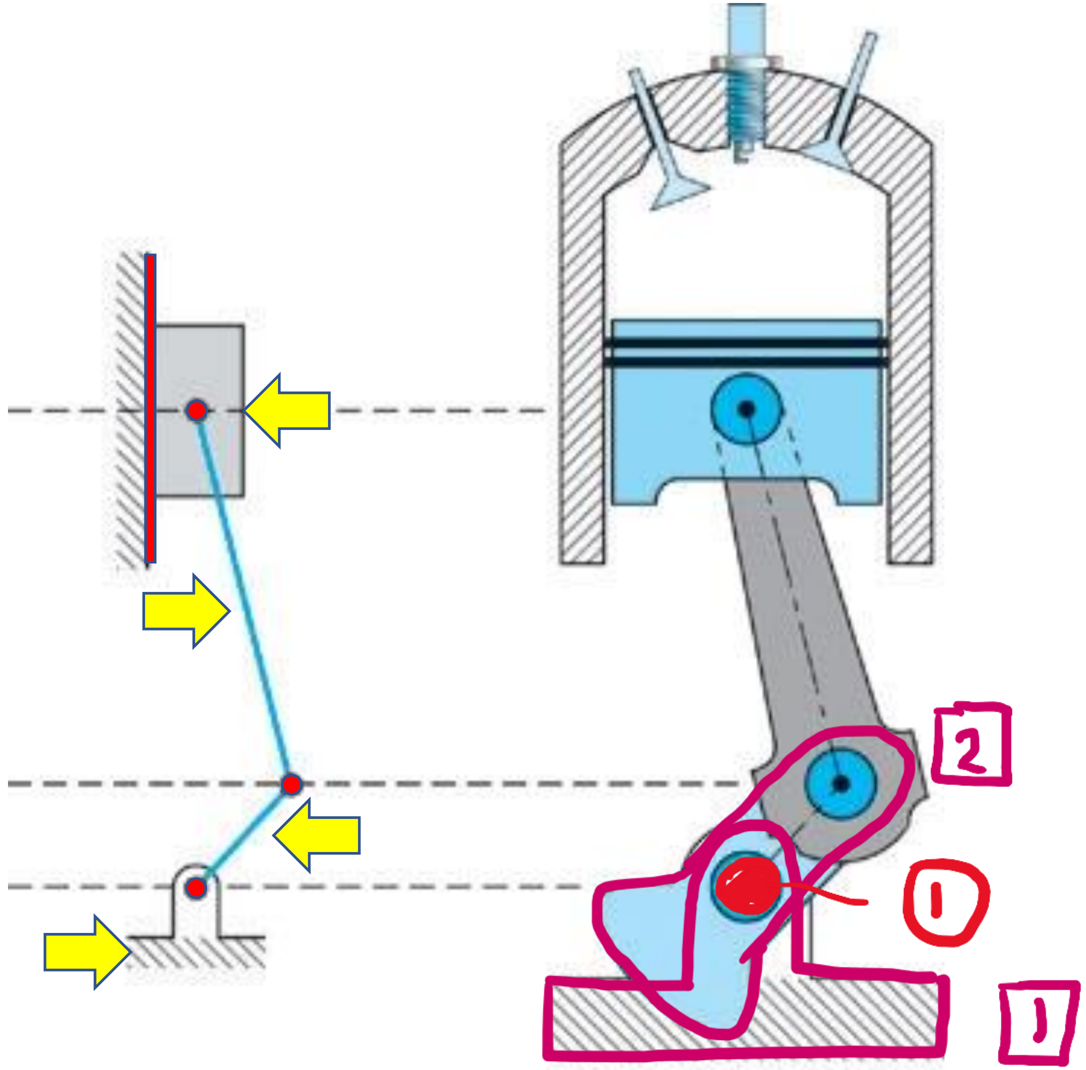
Feed-dog mechanism



Gear Train System (Gear Box)

Gearing

Linkages & Joints



- Links are rigid
- Joints are where links connect or contact other objects
- We represent complex mechanisms with simplified kinematic diagrams, which emphasize only the position and connectivity of links and joints

* Figures from Norton, Design of Machinery, 5th Ed, 2012

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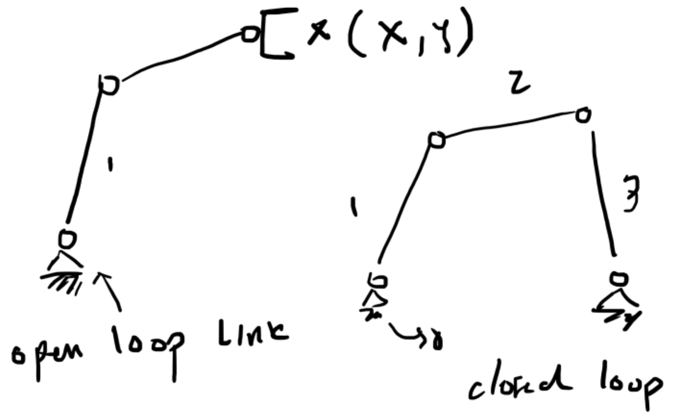
What is a Linkage?

- A physical assemblage of links and joints
- Basic building blocks of mechanisms
- Machine element #1
- *Can be opened (e.g. Arm) or closed (e.g. loop)*



Link

+

Joint

→ Linkage

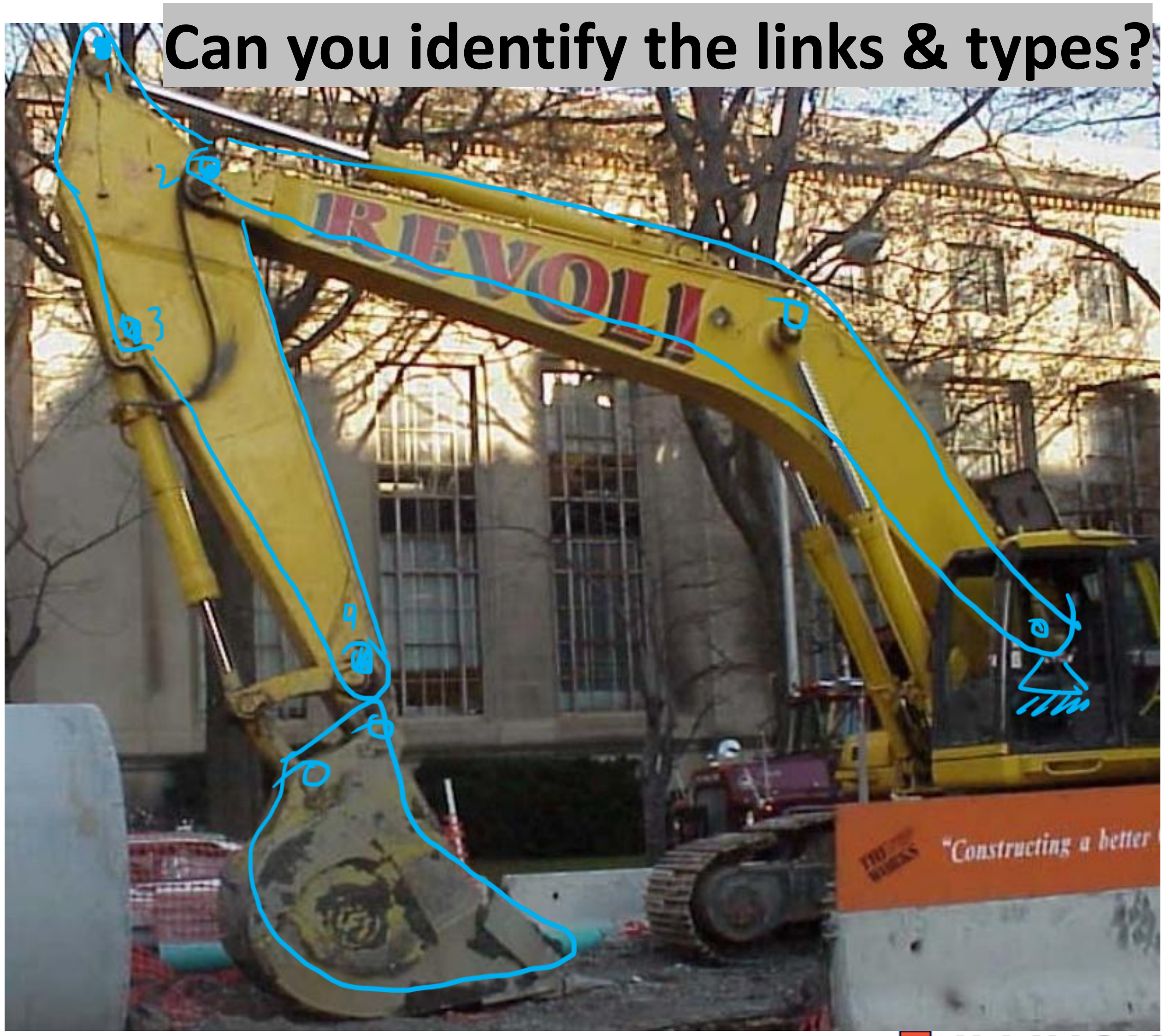
Kinds of links

"Link": Rigid body with at least 2 nodes
"Node": point of attachment to other links

- Binary link has 2 nodes:

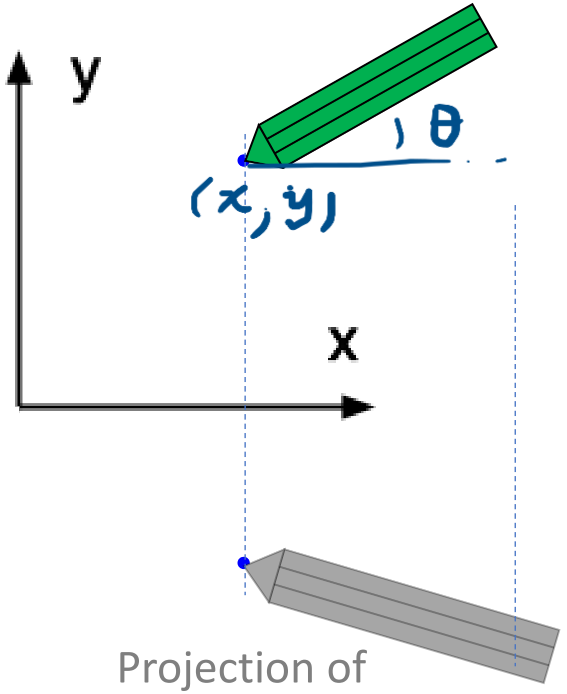
- Ternary link has 3 nodes:

- Quaternary link has 4 nodes:



Degrees of Freedom

- # of *independent coordinates* required to define the position and orientation of an object.
- Choice of coordinates is not unique, but must be independent (and minimal).

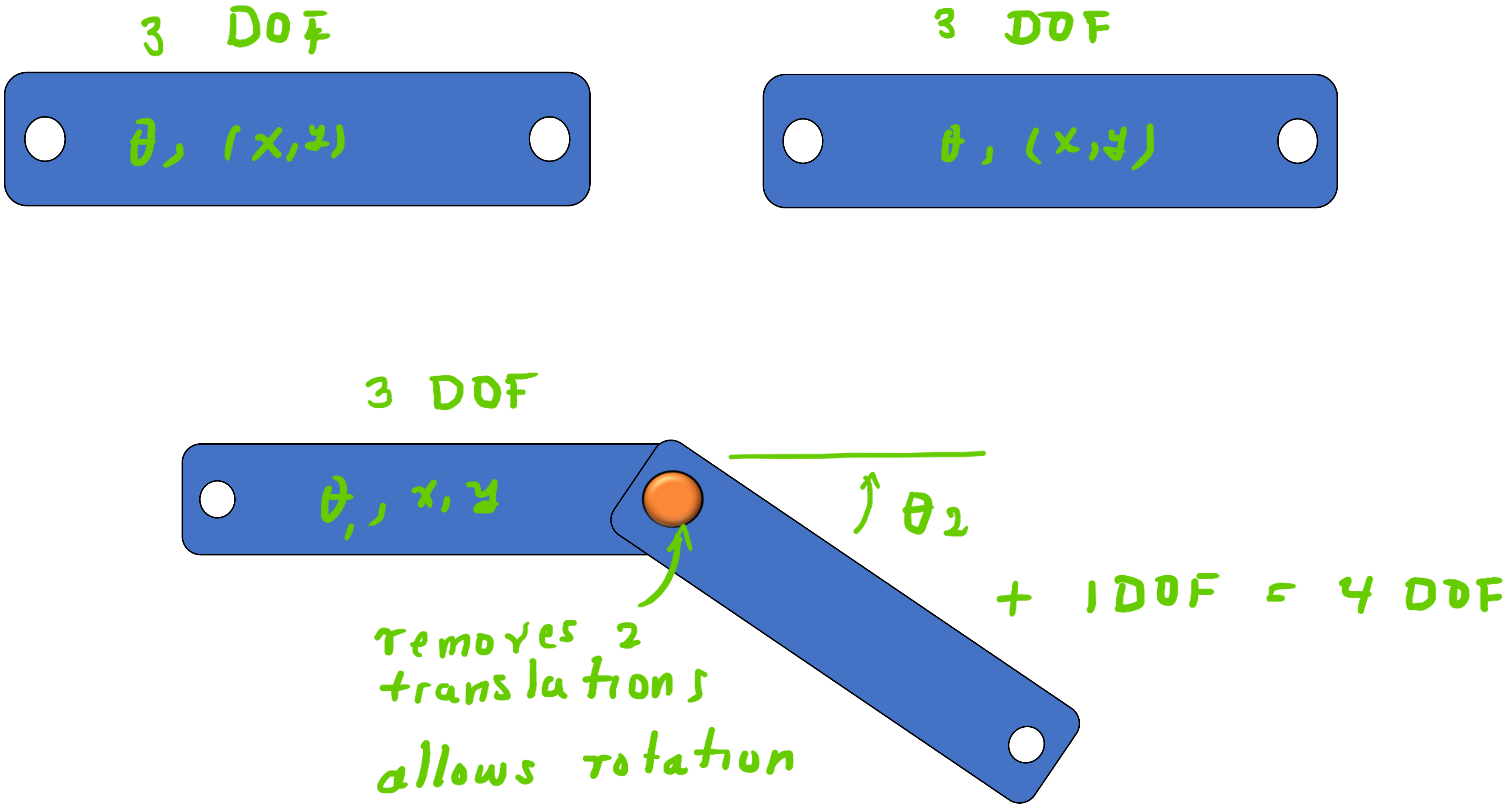


Projection of pencil in X-Z plane

Object DOF in 2D = ? 3 DOF → θ, (x, y)

Object DOF in 3D = ? 6 DOF → $\underbrace{(\theta, \varphi, \psi)}_{\text{rotations}}$, $\underbrace{(x, y, z)}_{\text{translations}}$

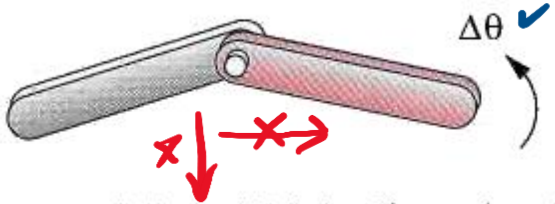
Removing degrees of freedom by connecting links with joints



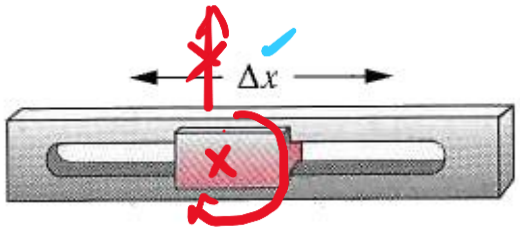
Types of Joints – classified by # degrees of freedom (DOF)

- **Full joint:** Allows 1 DOF at the joint and removes 2 DOF
- Examples:
  - *pin (rotating) joint*
  - *slider (prismatic) joint*
  - *pure rolling joint* – no slippage, e.g., gears

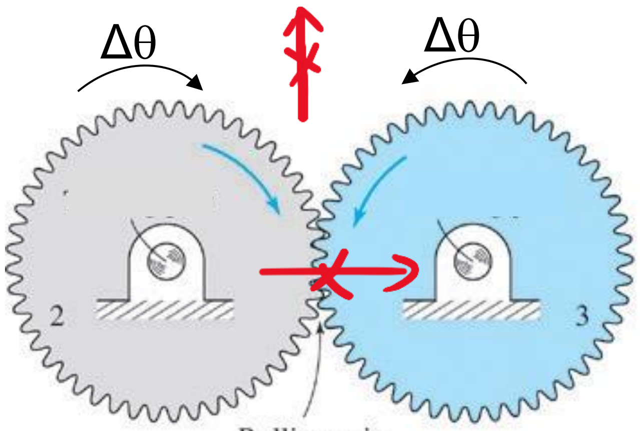
- **Half joint:** Allows 2 DOF at the joint, and removes 1 DOF, in planar mechanisms
- Examples
  - *pin in slot*
  - *rolling contact* – sliding



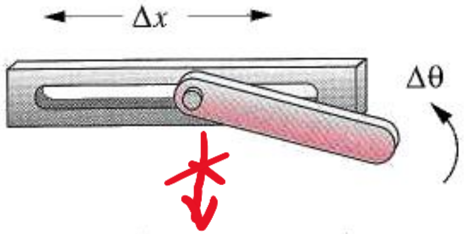
Rotating full pin (R) joint (form closed)



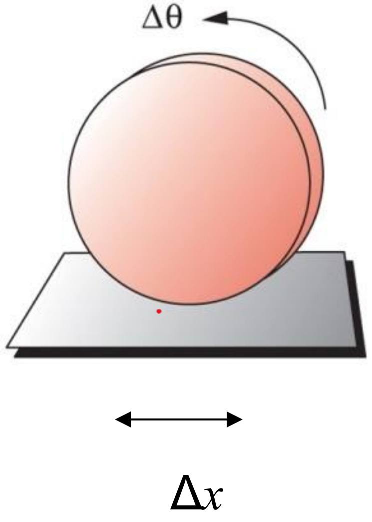
Translating full slider (P) joint (form closed)



Rolling pair
(or friction gears, no slippage)

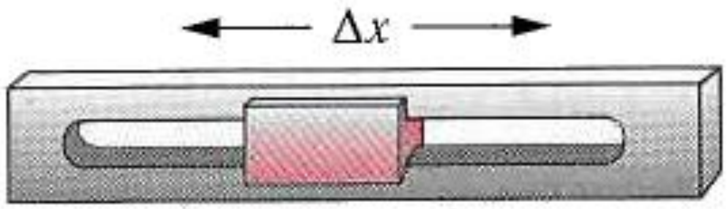


Pin in slot (form closed)

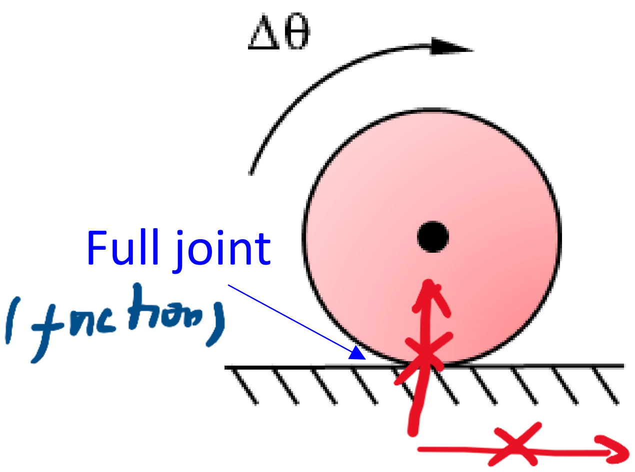


Kinds of Joints – the full joint

- Full joint: allows 1 DOF at the joint, and removes 2 DOF from the mechanism



Rolling Link

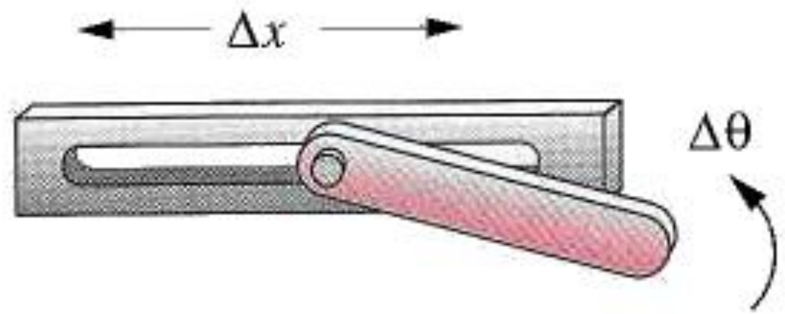


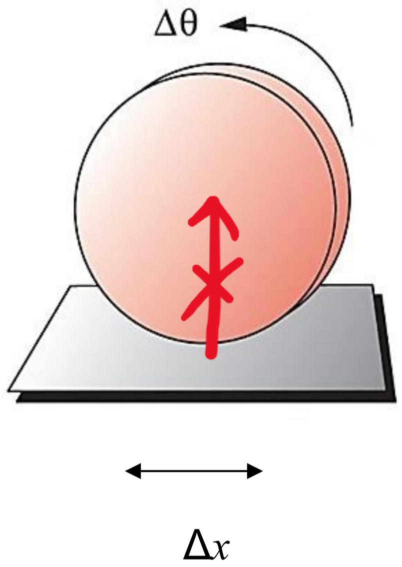
3 DOF for red link minus 2 DOF from full joint = 1 DOF to determine position of red link

* Figures from Norton, Design of Machinery, 5th Ed, 2012

Kinds of Joints – the half joint

- Half joint: allows 2 DOF at the joint, and removes only 1 DOF from the mechanism





with sliding

3 DOF for red link minus 1 DOF from half joint = 2 DOF to determine position of red link

Basic questions:

- Can we determine the motion of a mechanism by just looking at it?
- With an infinite variety of possible mechanisms can we determine which ones are useful?
- How will adding, removing, or changing links, or joints, change the functionality of a mechanism?

Mechanism Mobility or DOF, in 2D

- A Link has 3 Degrees of Freedom (DOF)
- A system of ***n*** unconnected links has 3n DOF
- How many DOF are removed when we:
  - fix
    Ground a link - 3
  - Connect links with a **Full joint** - 2
  - Connect links with a **Half joint** - 1

Gruebler's Equation

- How to compute mobility or #DOF?

$\#DOF = 3(n\text{-}1) - 2J_1 - J_2$ $= 3n - 2J_1 - J_2 - 3$

fixed / Grounded link.

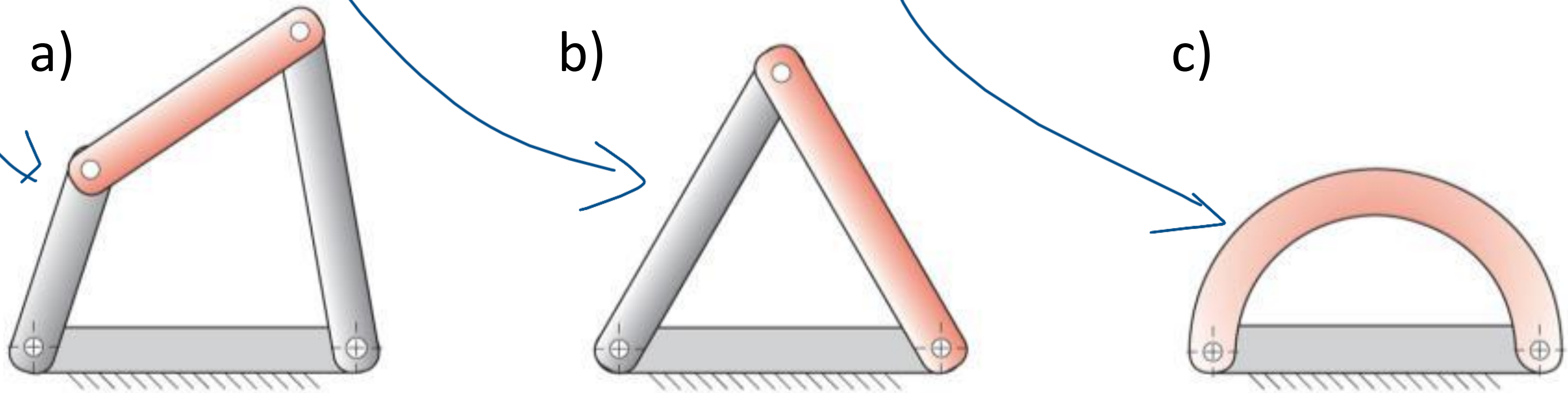
where n = # links

J_1 = # full joints

J_2 = # half joints

Calculating DOF and what it means

Mechanisms, structures, and preloaded structures: $DOF = 3(n\text{-}1) - 2J_1 - J_2$



$n = 4, J_1 = 4, J_2 = 0$

$DOF = 3(n\text{-}1) - 2J_1 - J_2$

$DOF = 1$

$n = 3, J_1 = 3, J_2 = 0$

$DOF = 3(n\text{-}1) - 2J_1 - J_2$

$DOF = 0$

$n = 2, J_1 = 2, J_2 = 0$

$DOF = 3(n\text{-}1) - 2J_1 - J_2$

$DOF = -1$

* Figures from Norton, Design of Machinery, 5th Ed, 2012

Kinematic diagram and labeling formalism

- A kinematic diagram of a mechanism is a sketch that helps to keep track of the number links, full joints, and half-joints.
  - Use hashed lines to represent ground.
  - Ground link = Link 1, no matter where connected.
- We use certain symbols to help identify these components of a mechanism.
- Gruebler's Equation:
  $DOF = 3(n-1) - 2J_1 - J_2$

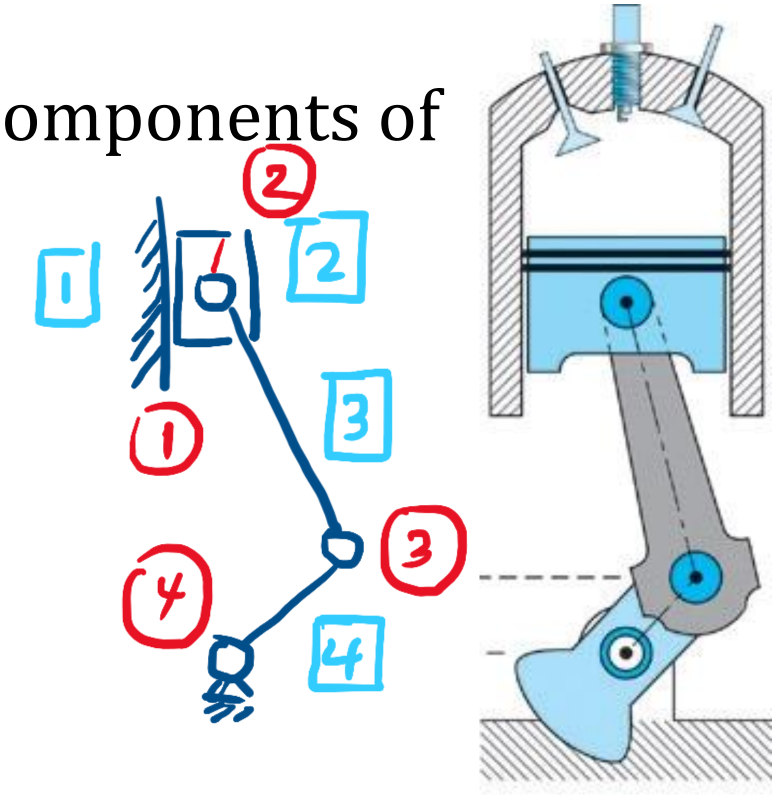
$= 3(3) - 2(4)$

$= 1$

□ = link = 4

○ = full joint (J_1) = 4

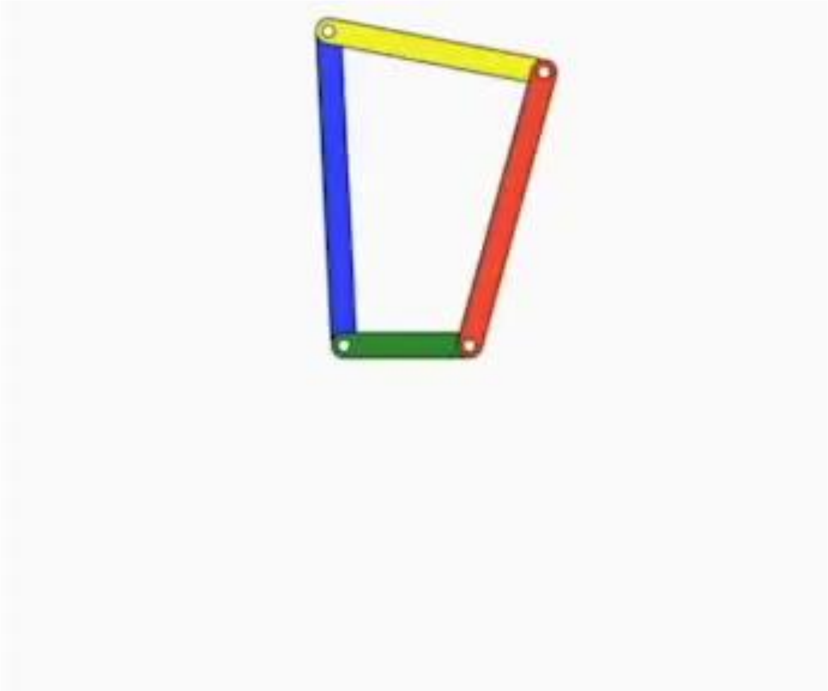
◇ = half joint (J_2) = 0



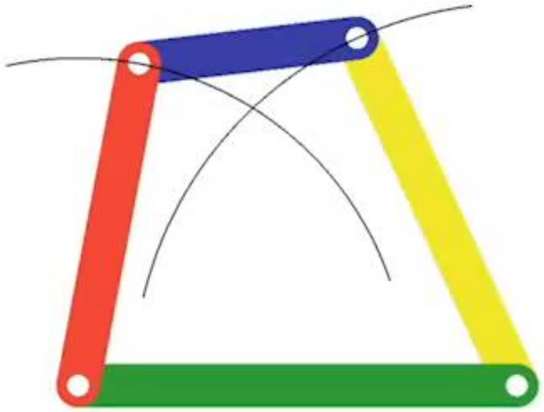
Four-bar linkage – basic building block mechanism

Crank-Rocker

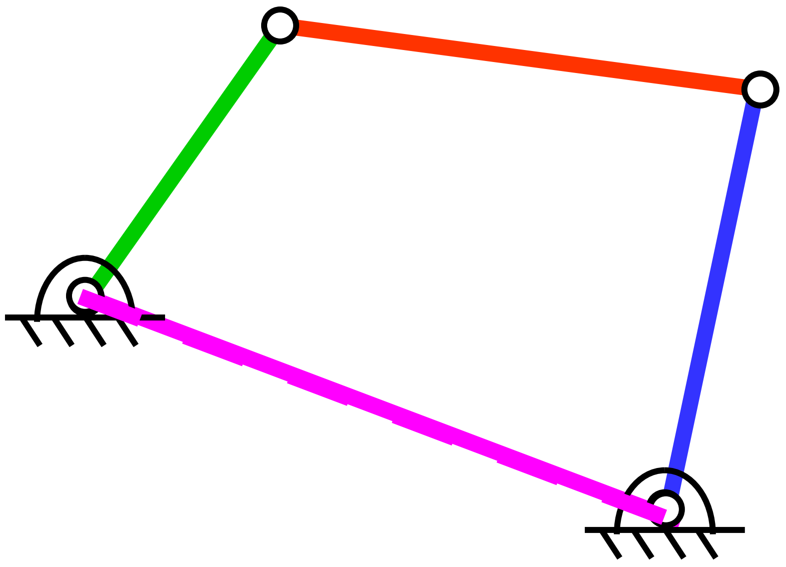
Double Crank



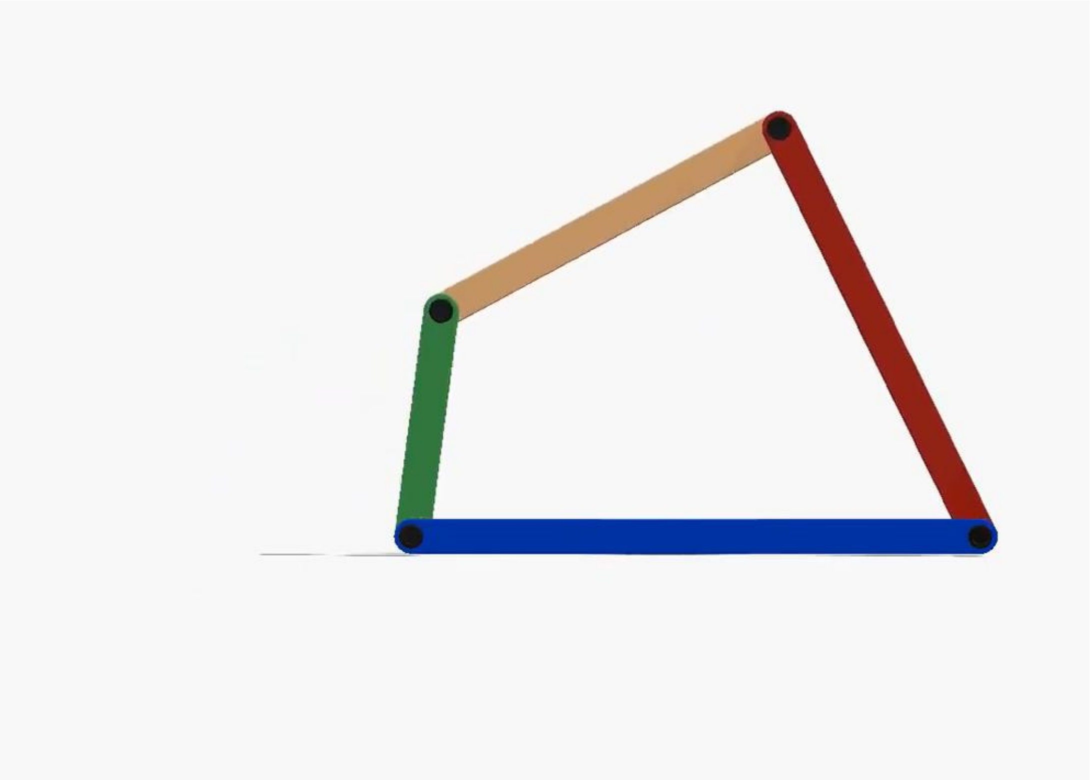
Double Rocker

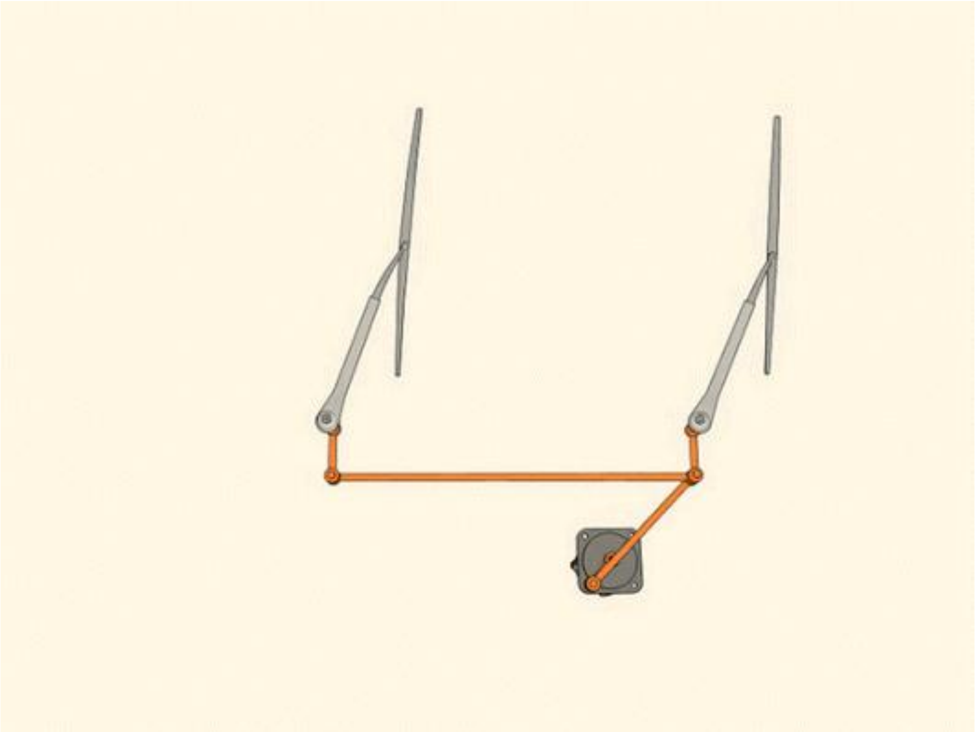


Four-bar linkage (crank-rocker)



Crank-rocker example: Windshield wiper

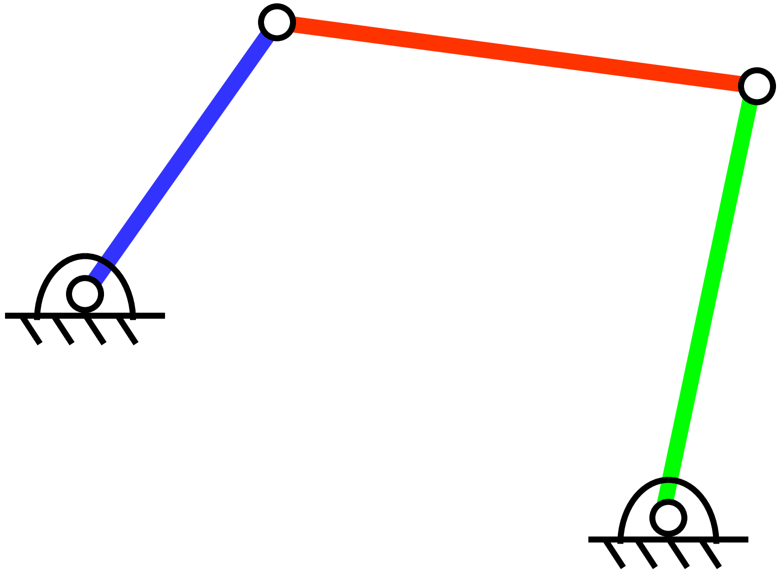




https://www.youtube.com/watch?v=UwYmWgsjtEQ

https://www.youtube.com/watch?v=D_KIOr_TpcM

Four-bar linkage: Identify links, joints and # DOF



Accounting Method for determining DOF:
Draw symbols on a mechanism and count up links and joints

□ = link (n)

○ = full joint (J_1)

◇ = half joint (J_2)

Other 4-bars - Vise Grips - Rocker-rocker

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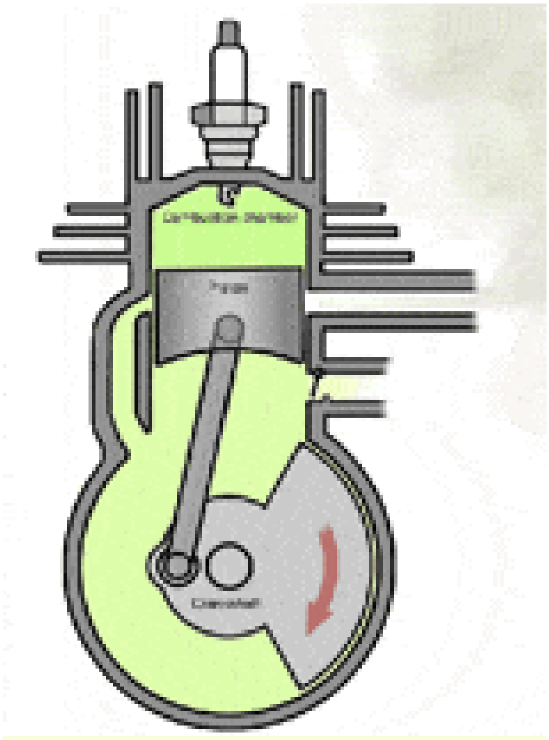
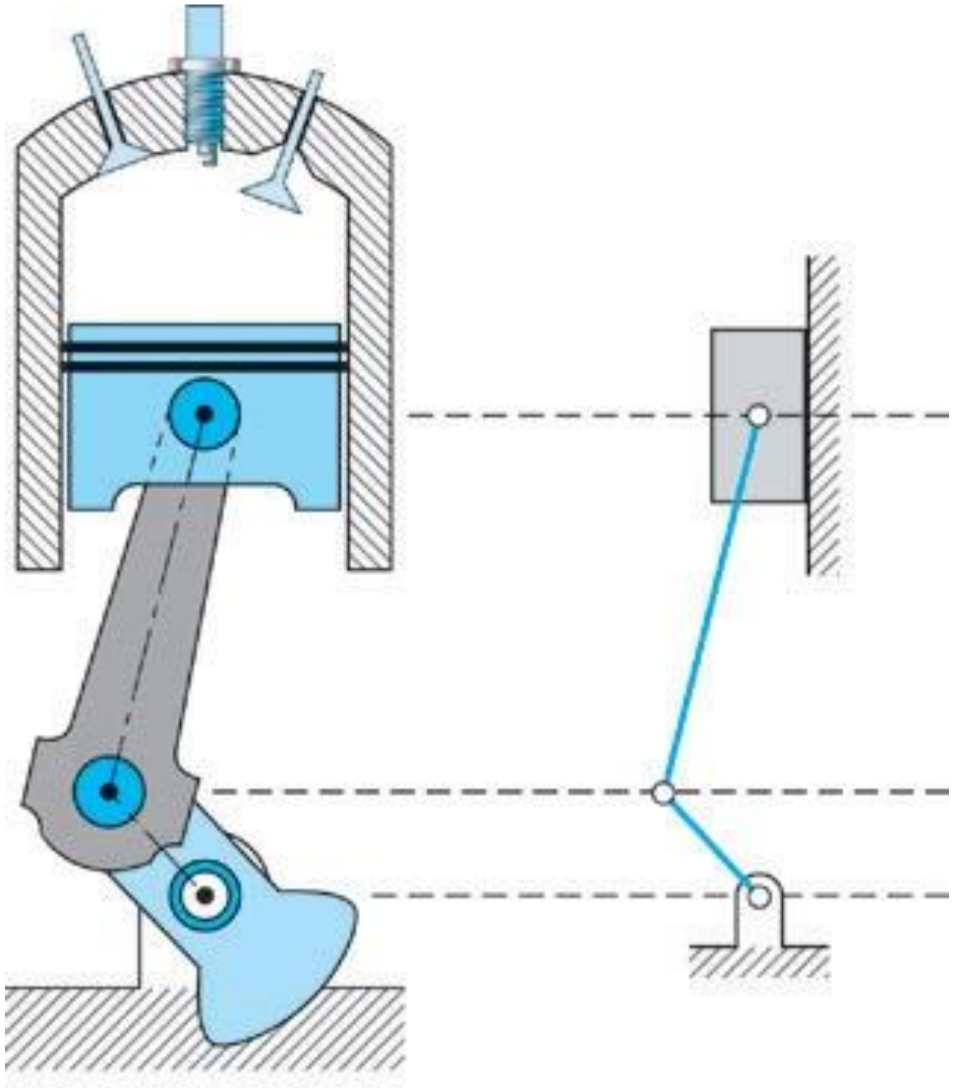
Other 4-bars. Piston: Crank-Slider

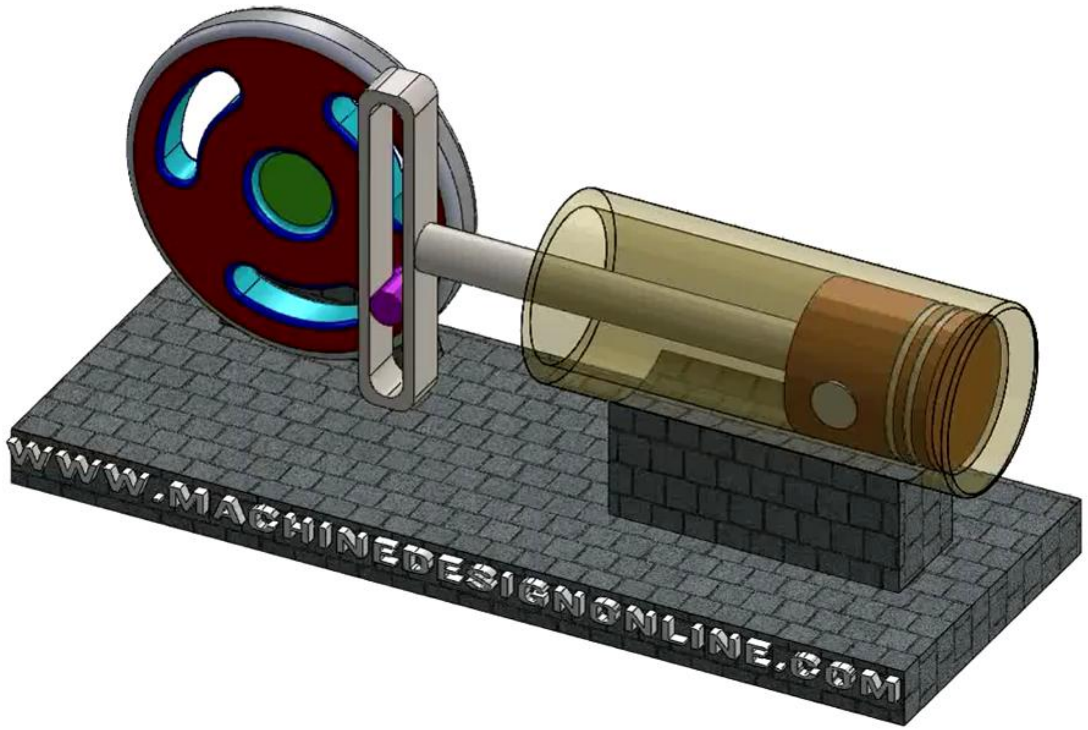




https://images.app.goo.gl/YLNrbxypMmN8mCoGA

Other 4-bars. Corkscrew: Rocker-slider

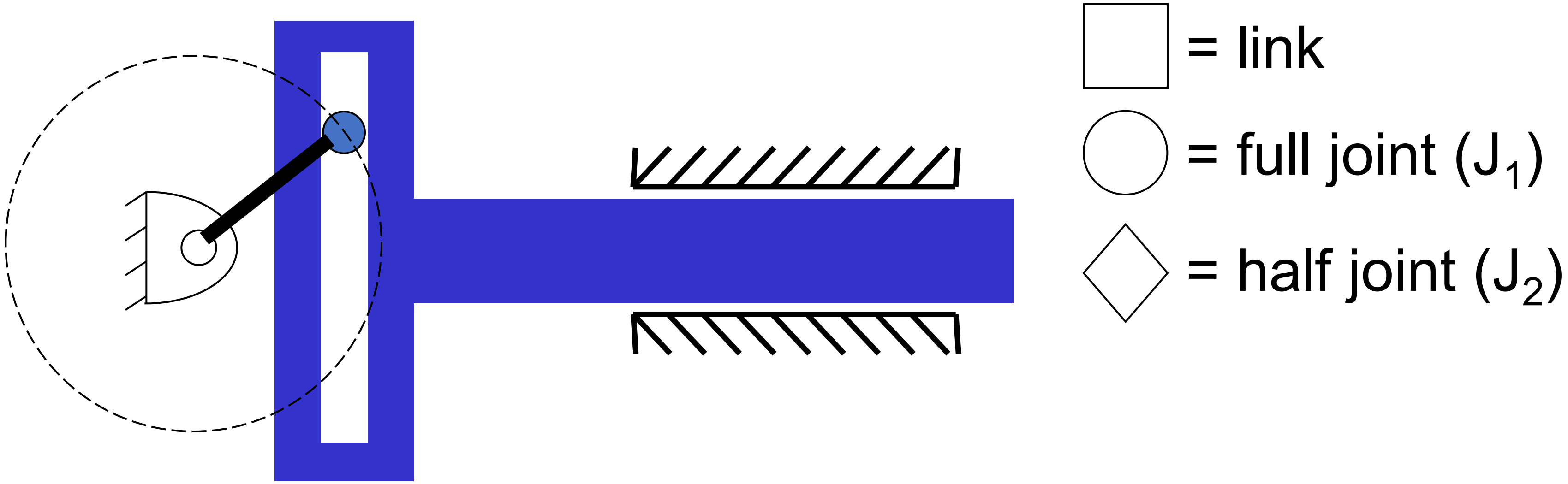
Example: Scotch yoke mechanism



Video of Scotch yoke mechanism
http://youtu.be/_K4PSV4MO70

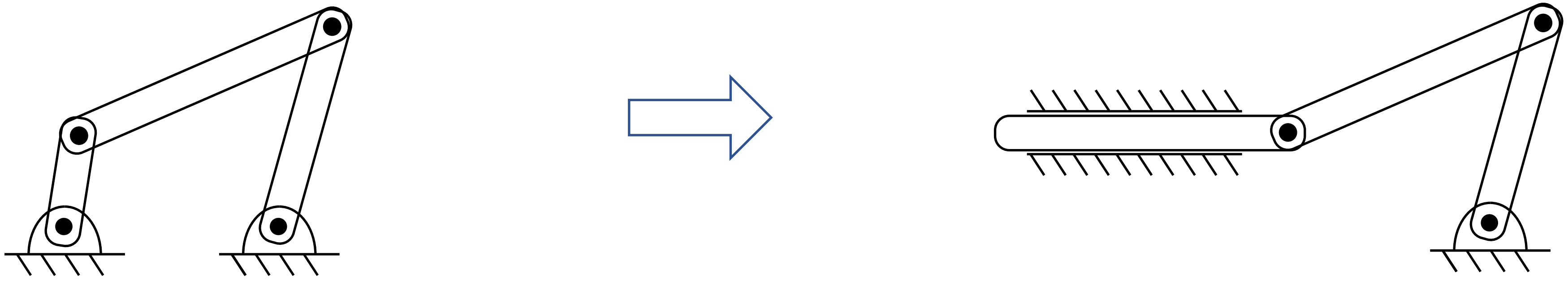
Practice: Scotch yoke

Identify links, joints and calculate # DOF by using Gruebler's equation

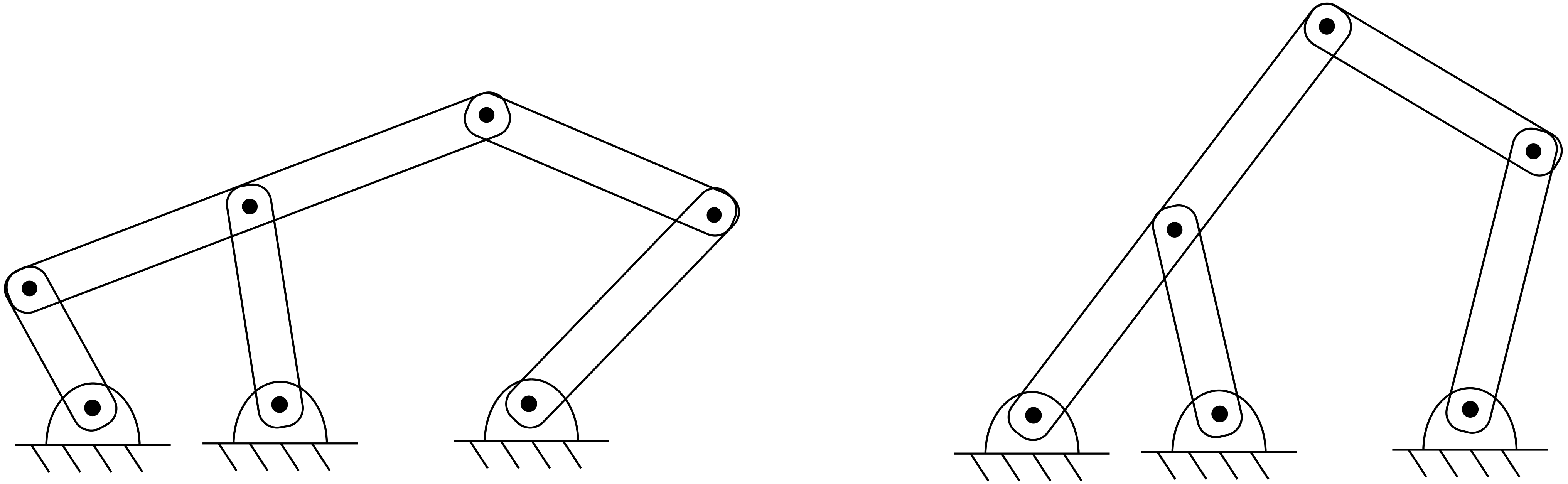


Four Transformation rules & their effect on DOF

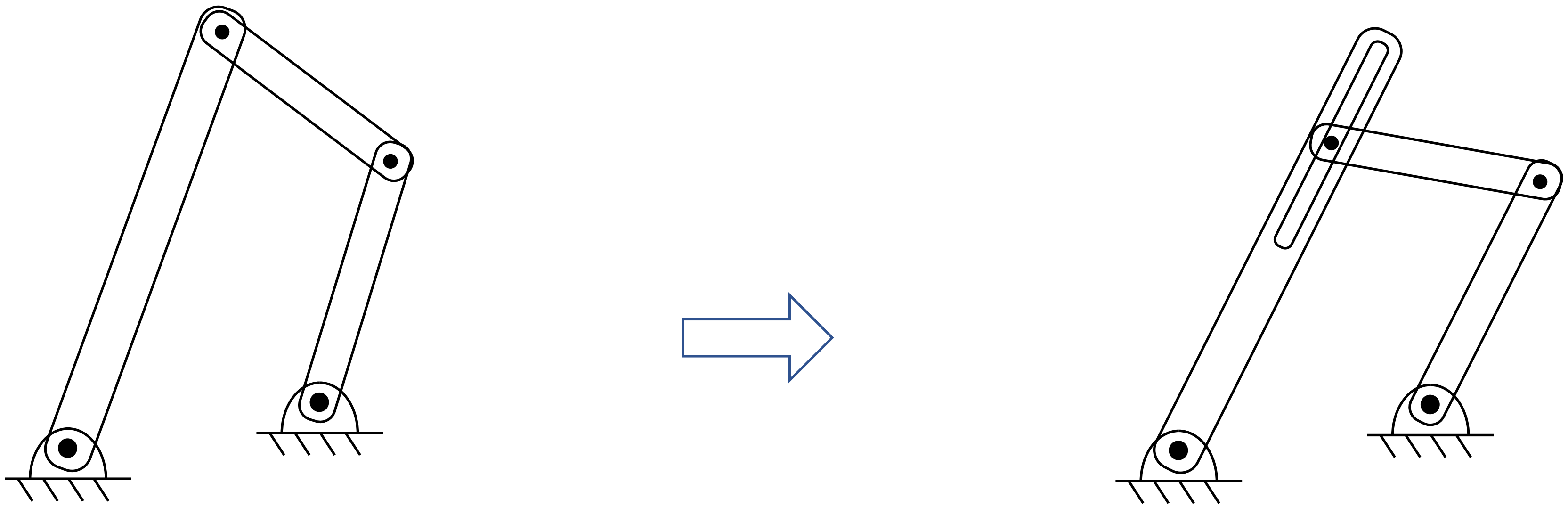
Rule 1: replace a *pin joint* with a *sliding joint* → *no change in DOF*



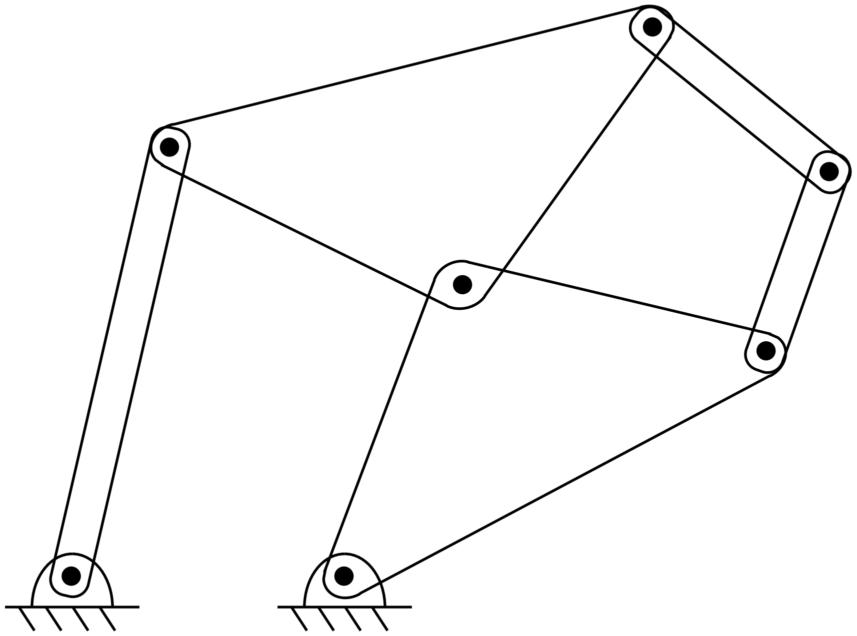
Rule 2: remove a link & full-joint→ *DOF is decreased 1*

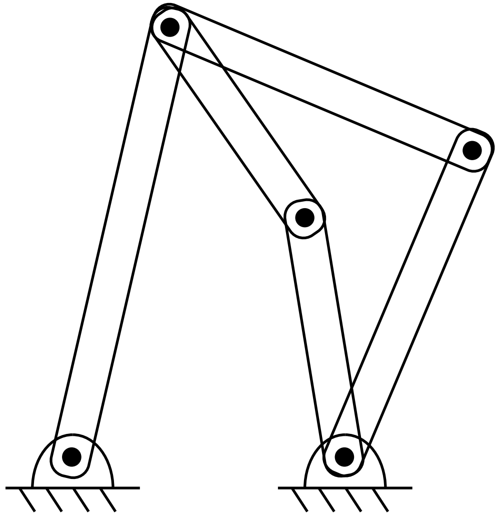


Rule 3: replace a full-joint with half-joint → *DOF increased by 1*



Rule 4: nodes can be combined to create higher order multi-joints
→ *no change in DOF*

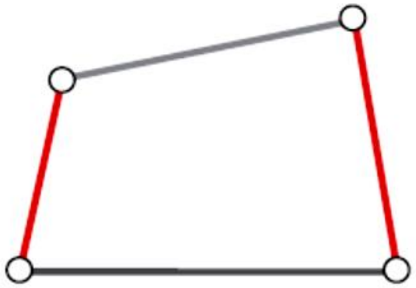




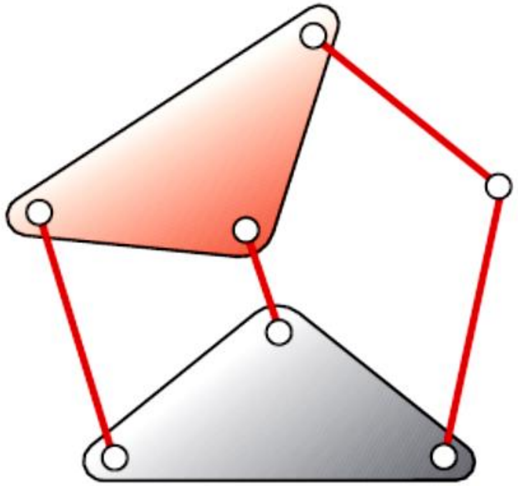
Isomers of 1-DOF mechanisms

Number of Valid Isomers

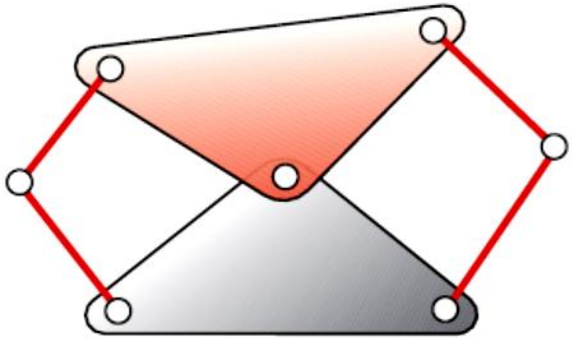
Links	Valid Isomers
4	1
6	2
8	16
10	230
12	6856



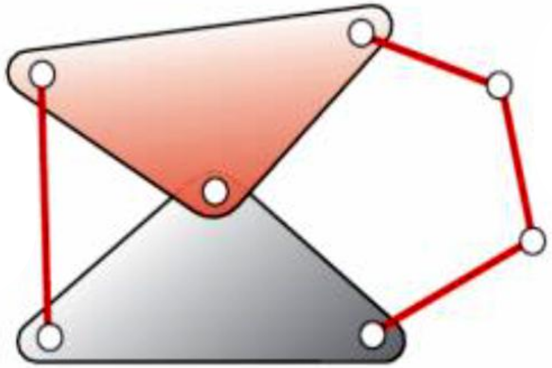
The only fourbar isomer



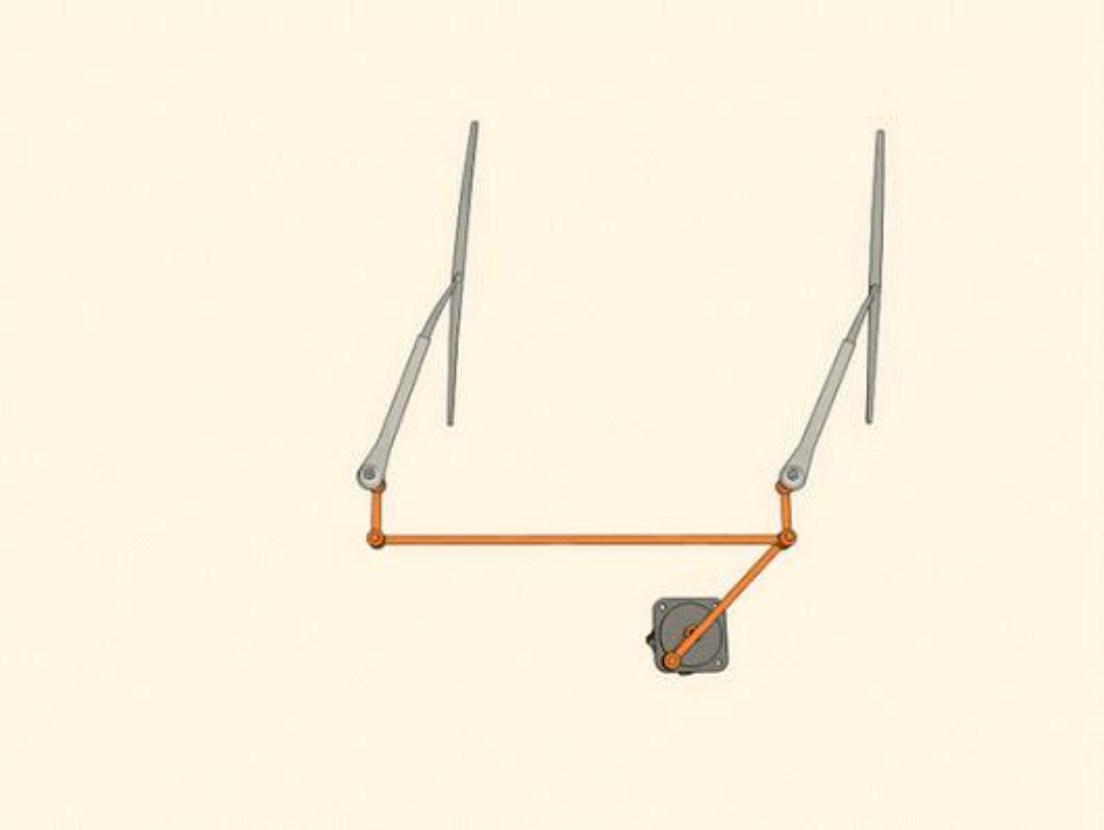
Stephenson's sixbar isomer

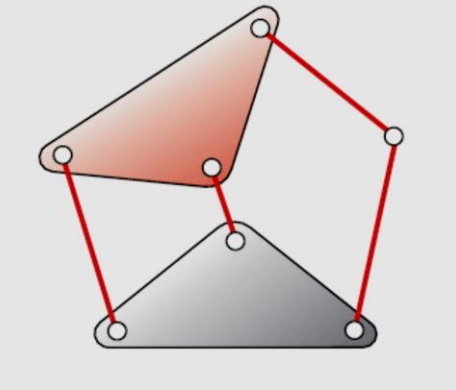


Watt's sixbar isomer

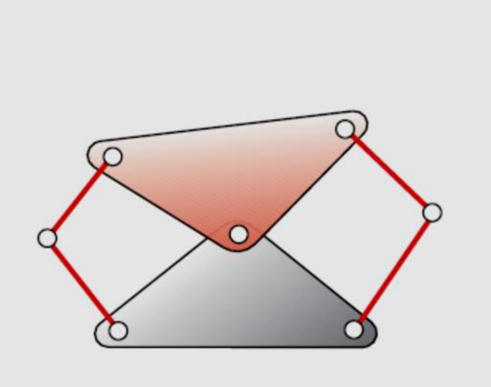


Practice: Identify Sixbar Isomer





Stephenson's sixbar isomer



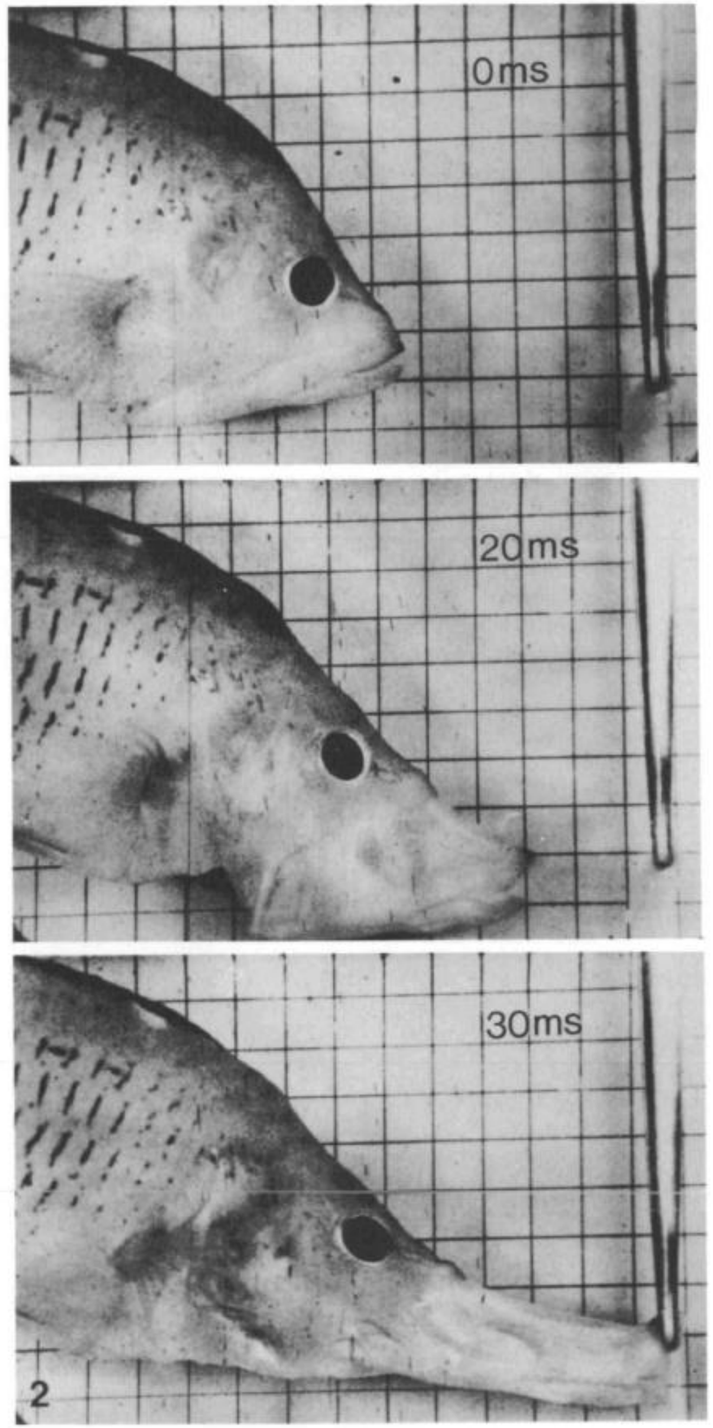
Watt's sixbar isomer

Planar mechanisms in Nature

http://www.youtube.com/watch?v=pDU4CQWXaNY

Sling Jaw Wrasse: A Unique Feeding Mechanism

- The *Epibulus insidiator* possesses the most extreme jaw protrusion ever measured in fishes
- This motion may be understood through kinematic model, combining <u>fourbar</u> and <u>sixbar</u> linkages



Westneat, J. Exp. Biol. ***159****, 165-184 (1991))*

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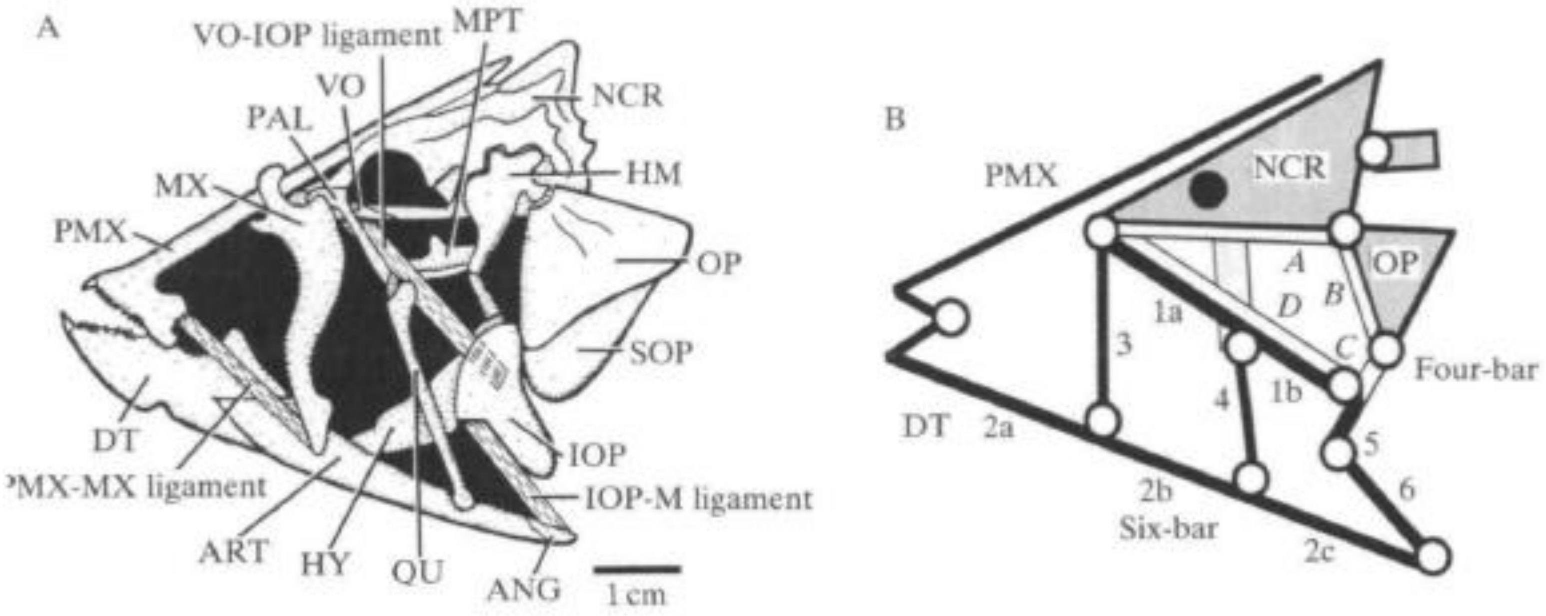
Exercise: Linkage Biomechanics

Can you find the 4 and 6 bar linkages?
The sixbar is which isomer?

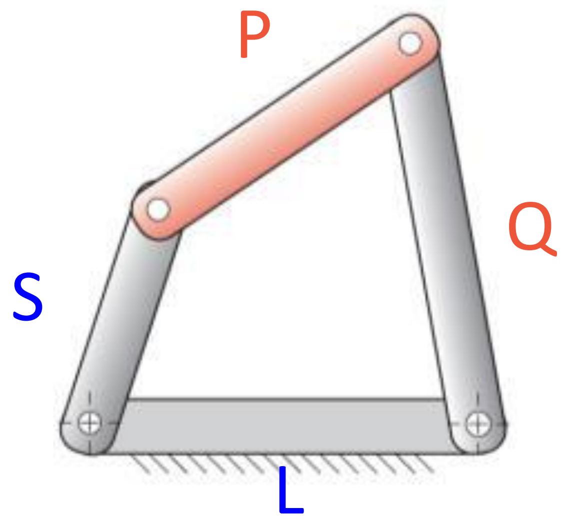


Westneat, J. Exp. Biol. ***159****, 165-184 (1991))*

Grashof condition: Mechanism length and allowed motion

Define:

- S – shortest link
- L – longest link
- P, Q – remaining links



$S + L \leq P + Q$: Grashof condition

$S + L < P + Q$: Class 1 (Grashof)

$S + L > P + Q$: Class 2 (non-Grashof)

$S + L = P + Q$: Class 3 (special-case Grashof)

Class 1: $S + L < P + Q$

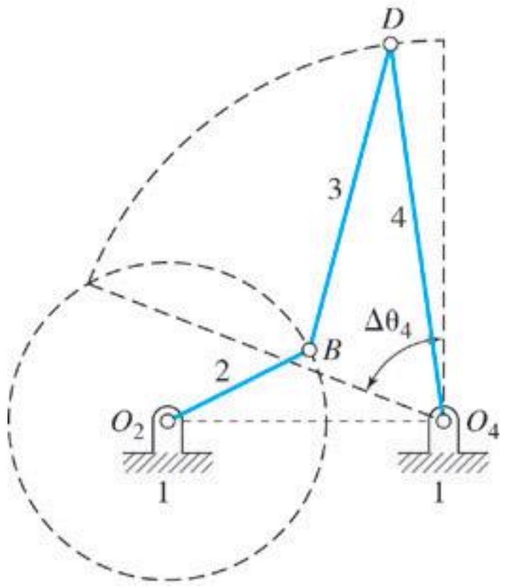
"Satisfies the Grashof condition"

- At least one link *will* be able to make a *full rotation*

Crank-Rocker

S** is always the **crank

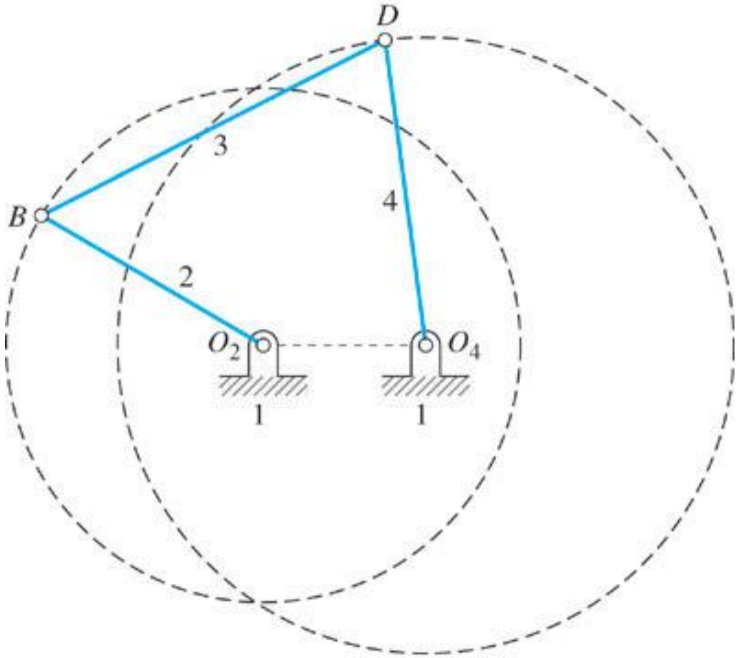
Only Link 2 rotates



Double Crank

S** is always **ground

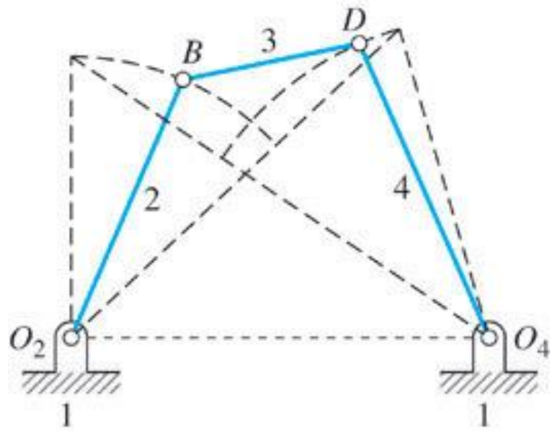
Link 2, 3, and 4 rotate



Double Rocker

S** is always the **coupler

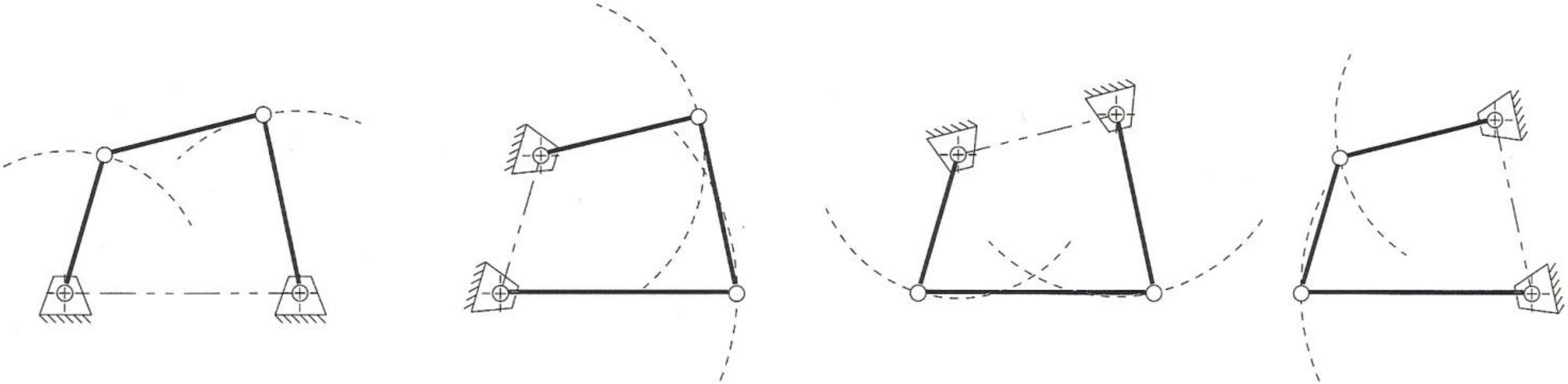
Only Link 3 rotates



Class 2: $S + L > P + Q$

"DOES NOT Satisfy the Grashof condition"

- *No link* will be able to make a full rotation.
- All four inversions are triple rockers:

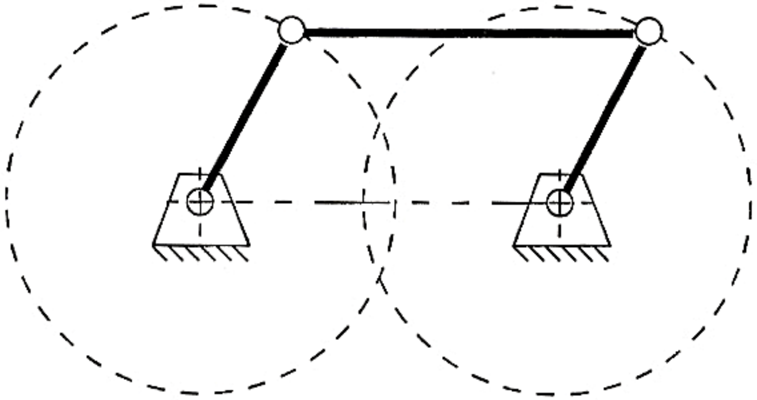


Class 3: S + L = P + Q

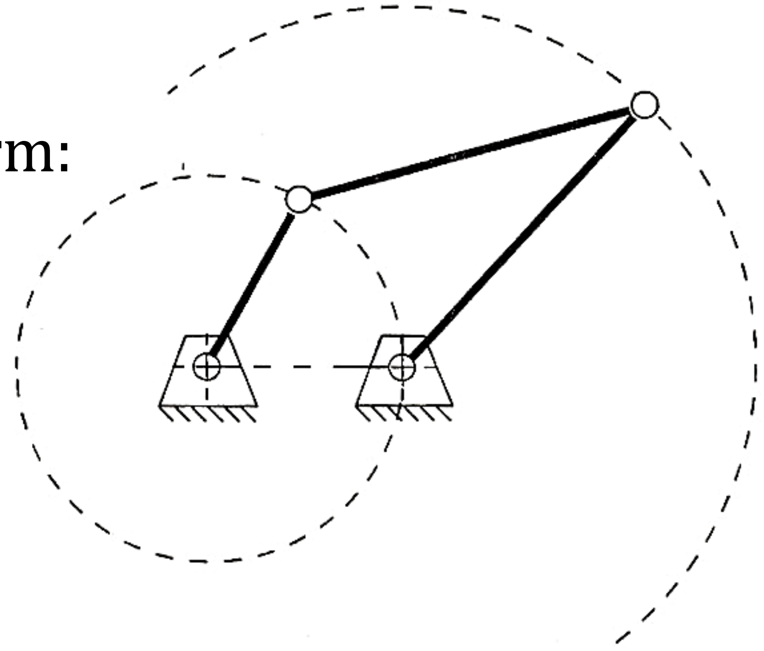
"Special Case Grashof"

- *At least one link* will make a full rotation (like Class 1). Two Forms:
  - *Parallelogram* – shortest links are opposite each other
  - *Delta* – shortest links are adjacent to each other
- *Problem*: output has a "*change point*" where links are in a line and output direction is indeterminant

Parallelogram:



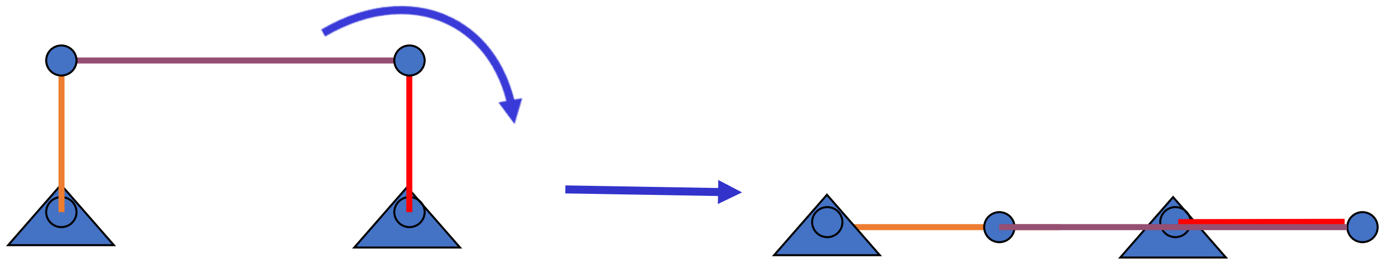
Delta Form:



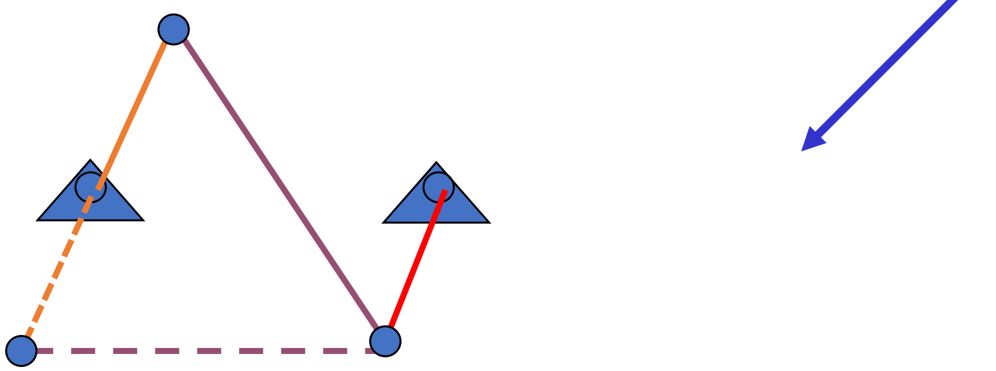
https://www.youtube.com/watch?v=h8bz4ni6mdY

Grashof Class 3: $S + L = P + Q$

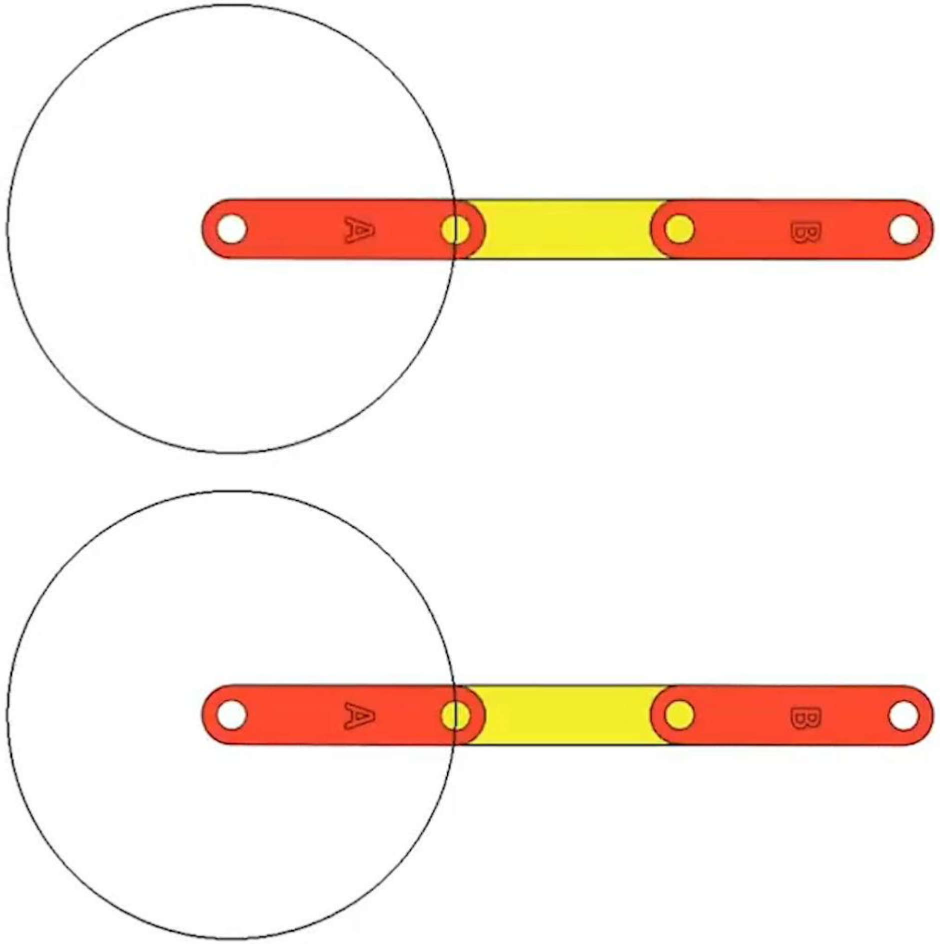
Will have **change points** twice per revolution, where all links become co-linear.



⇒ Output is indeterminate



Parallelogram or Anti-Parallelogram form

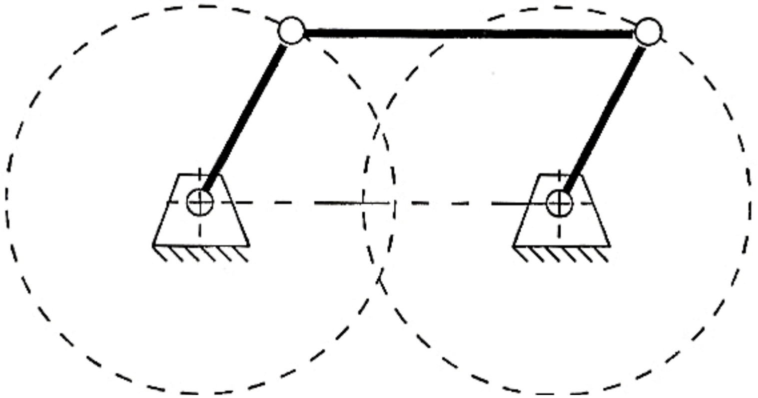


https://www.youtube.com/watch?v=h8bz4ni6mdY

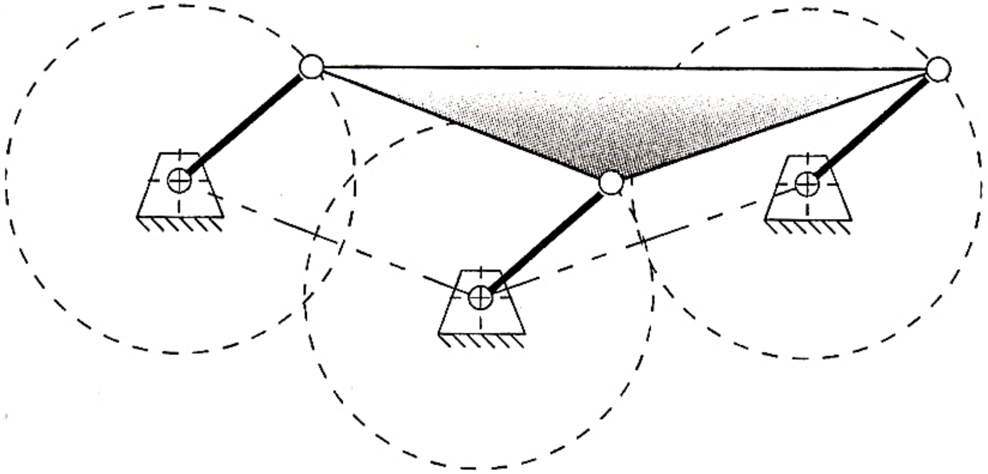
ILLINOIS

Class 3: **S + L = P + Q**

- *Problem*: output has a "*change point*" where links are in a line and output direction is indeterminant



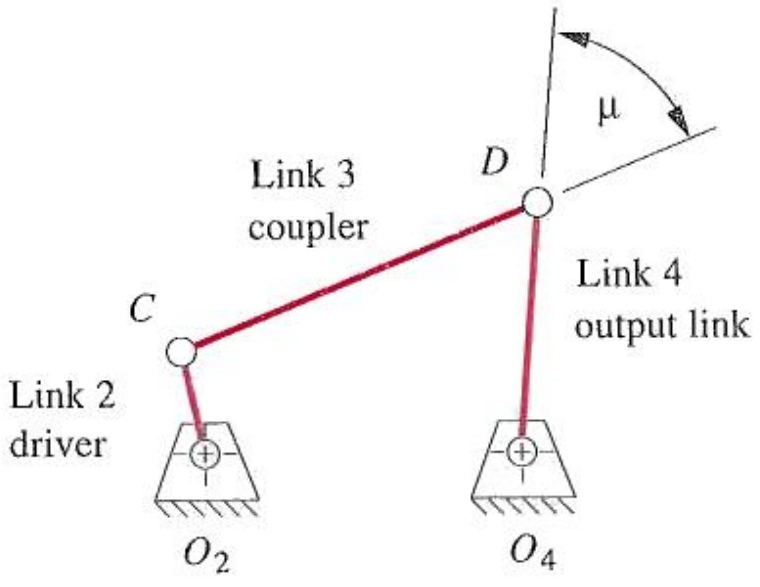
Solution to change point uncertainty is to add a link to the coupler:



Toggle Position

- Two links are co-linear
- No further motion possible in given direction
    - Toggle position holds jaws closed
- Must drive different links to open and close
- Check designs for possible toggle positions!

Transmission angle, μ (Norton, Section 3.3)



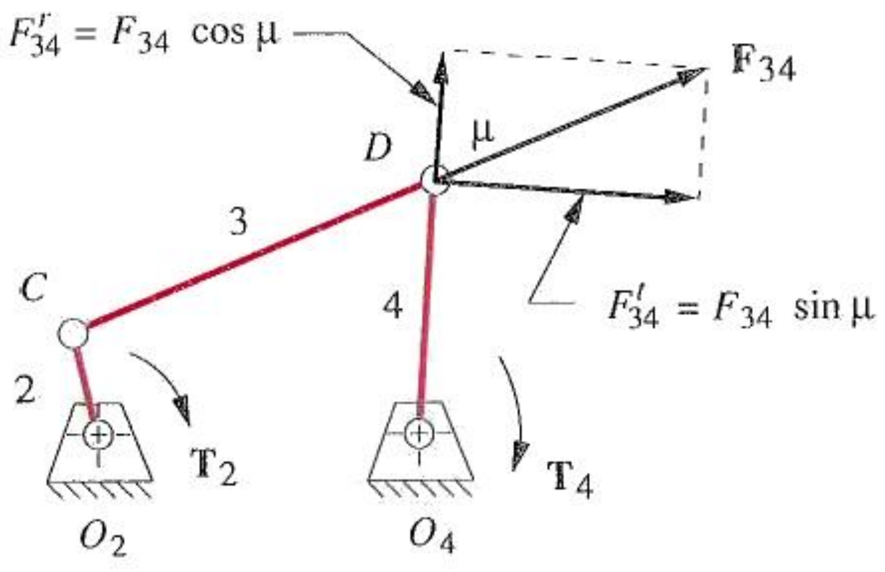
(a) Linkage transmission angle μ

- **<u>Acute</u>** angle between coupler and output links
- Measure of quality of force transmission at joint

Transmission angle, μ

- Coupler only transmits force along its axis (F_{34}).
  - F^t determines torque on output (rocker)
  - F^r determines tension/compression on rocker and joints D and O_4 ➔ friction
- Design rule:

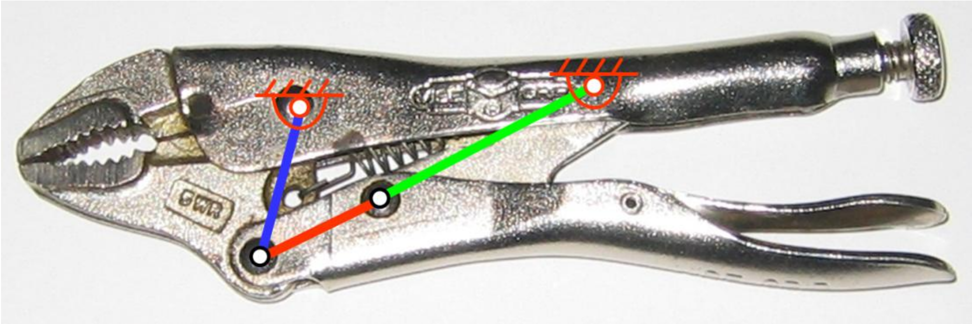
  Try to keep $90^\circ > \mu > 40^\circ$



(b) Static forces at a linkage joint

Limiting Conditions in Mechanism Motion

- Toggle positions



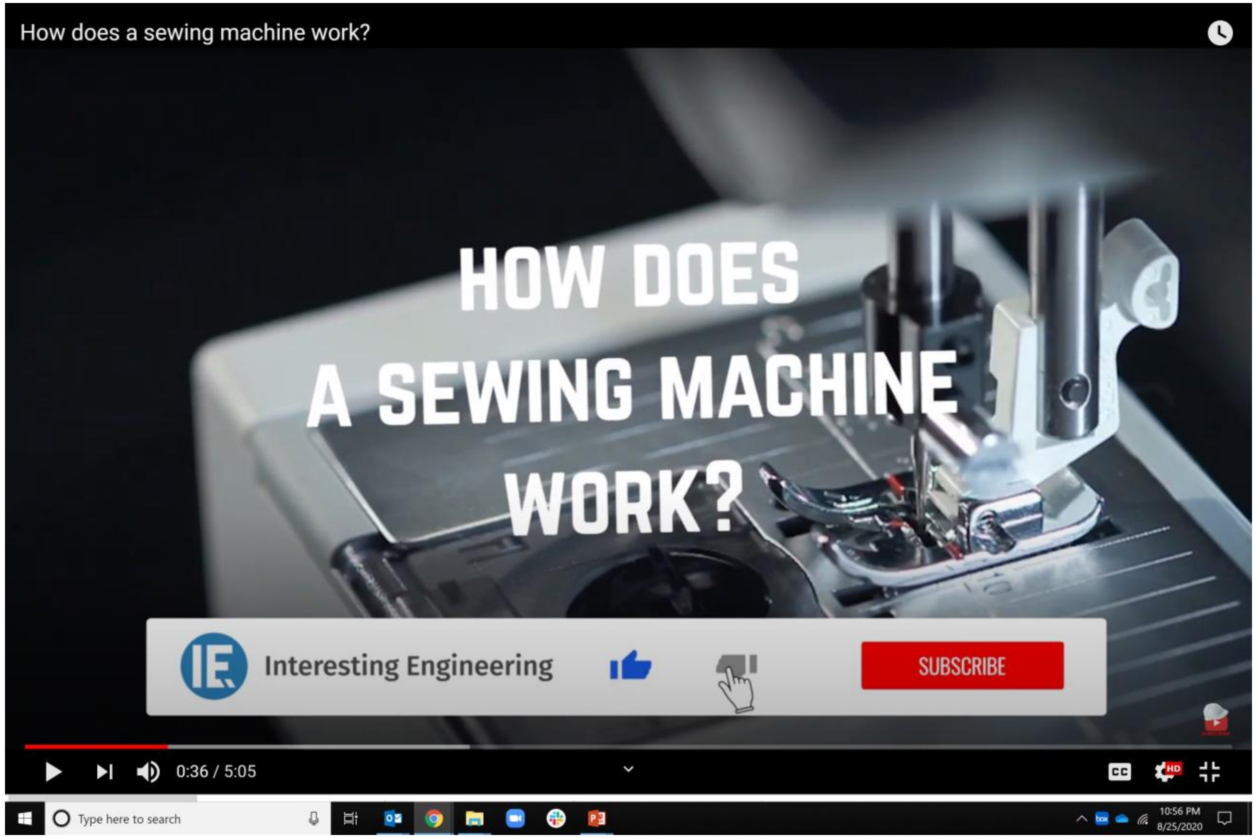
- Change points

- Transmission angles (more on forces later in semester)
  - Typically 90° > μ_{min} > 40°

Norton, Section 3.3

Extra fun videos

Sewing machines: amazing mechanisms



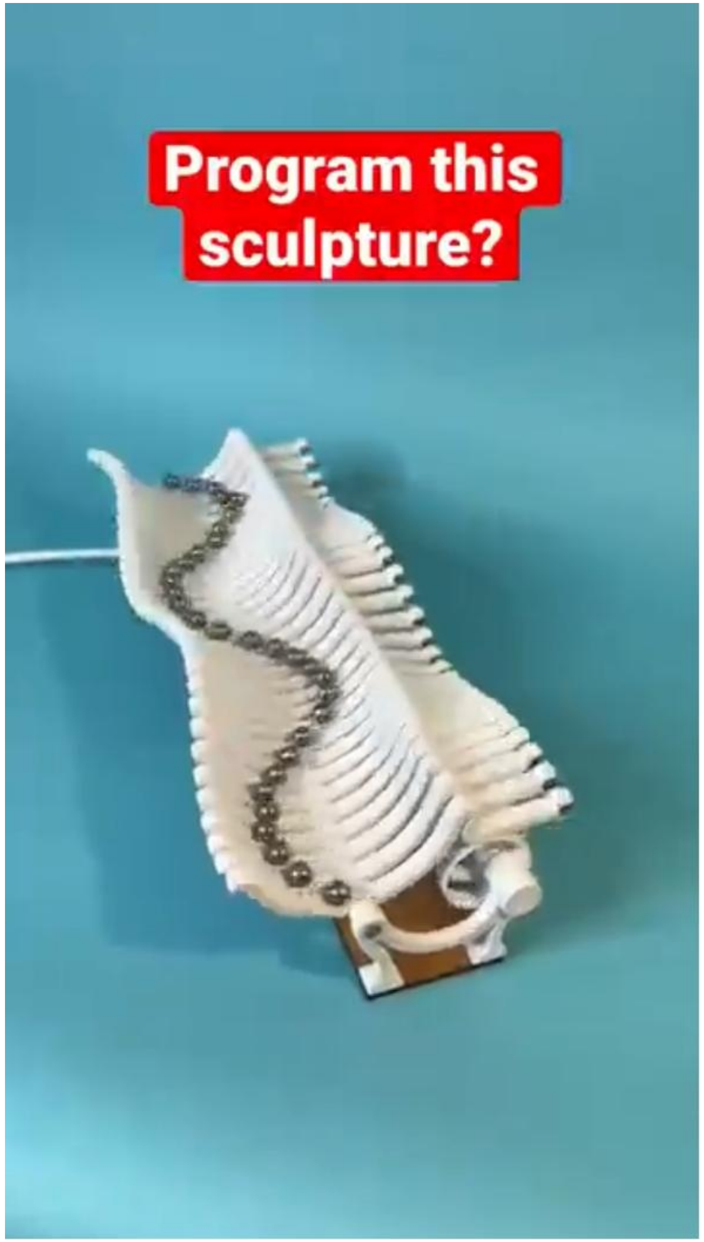
https://youtu.be/7RfGqGiEGTU

Details of 3 key mechanisms from 2:44 -3:54 min

How traditional Cuckoo clocks are made

https://www.youtube.com/watch?v=0GAmegD5MIQ

- Ideas for programable or reconfigurable mechanisms



https://www.youtube.com/shorts/JIV7mBg3Rls

Kinetic sculptures made by Bob Potts

https://youtu.be/5RGl5ZXnoIs?si=VVBUW7KKVZnYt-4h