Lecture 13
Instant Centers

ME 370 - Mechanical Design 1

Module 4: Instant Centers

- Review of velocity
- Instant Centers (a.k.a., instantaneous center of velocity)
  - Equation to compute # ICs
- Rules for Finding ICs
  - Linear Graphs
  - Kennedy's Theorem
  - Notation
- Practice with instant centers
- Graphical Velocity Analysis with ICs
  - Mechanical Advantage
  - Velocity Ratios
- Applications of ICs
  - Ics in Suspension Design
- Reading: Norton, Chapter 6.3, 6.4

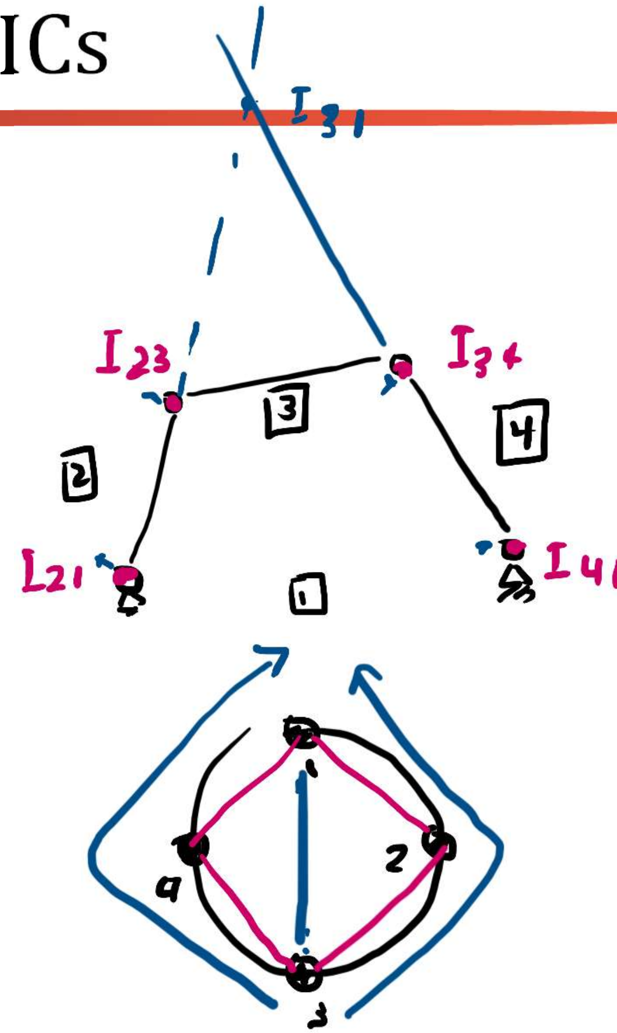
Learning Objectives

By the end of this module, you should be able to:

- Explain what an Instant Center is and recall rules to find them
- Locate Instant Centers in systems of links with pin joints and other kinds of joints
- Use Instant Centers to analyze velocity ratios and mechanical advantage

Summary of the rules to finding ICs

- **Rule 1:** Every pin joint is an IC
- **Rule 2:** 3 bodies have 3 ICs which lie in a line (Kennedy's theorem)
- **Rule 3:** The IC for a sliding joint is at infinity and is perpendicular to the sliding direction.
- **Rule 4:** For two bodies in rolling contact, the contact point is their IC
- **Rule 5:** For two bodies in rolling contact <u>with slip</u> the IC lies on the common normal, following Kennedy's Theorem



Recall: What does I_{13} mean? What is velocity @I_{1x}?

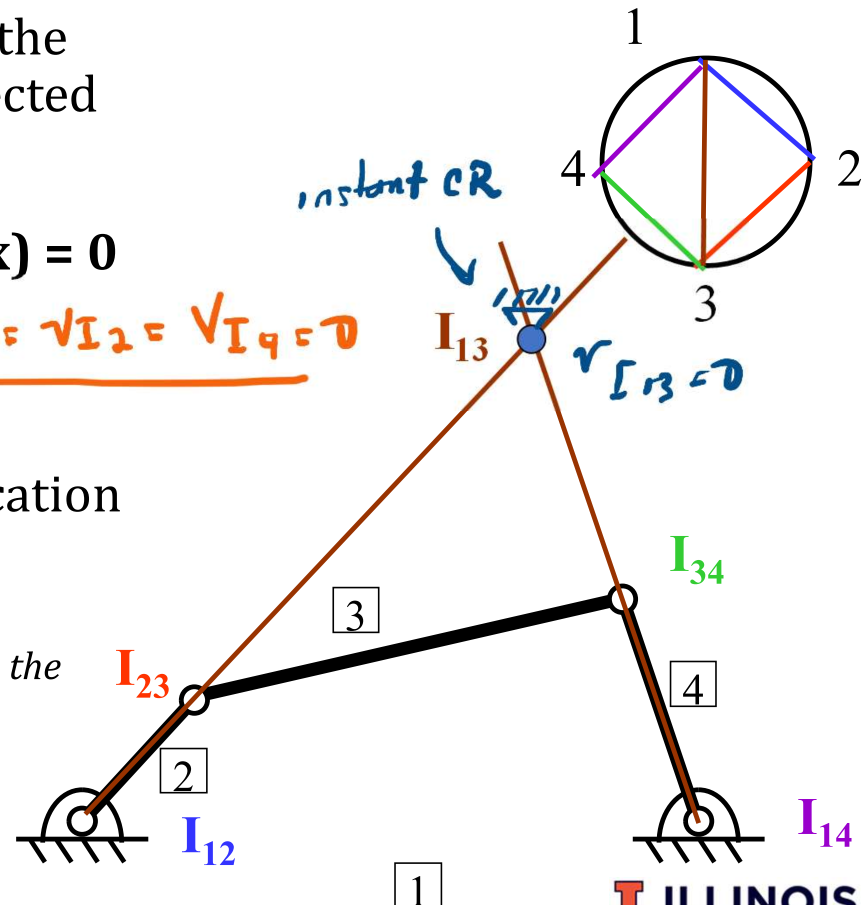
- I_{13} is a point in space where link 1 and link 3 have the same velocity (instantaneously) if they were connected

- Since link 1 (the ground) is fixed, $\mathbf{V_1 = 0}$, so $\mathbf{V(@I_1x) = 0}$ for any link x.

$\underline{V_{I_{13}} = V_{I_2} = V_{I_4}} = 0$

- Thus, at that instant, $\mathbf{I_1x}$ acts as a fixed point about which link x rotates relative to link 1, though its location changes over time.

- *Instant Center ≡ point common to both bodies in which the point has the same velocity in each body.*



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Velocity Ratios: The Inputs

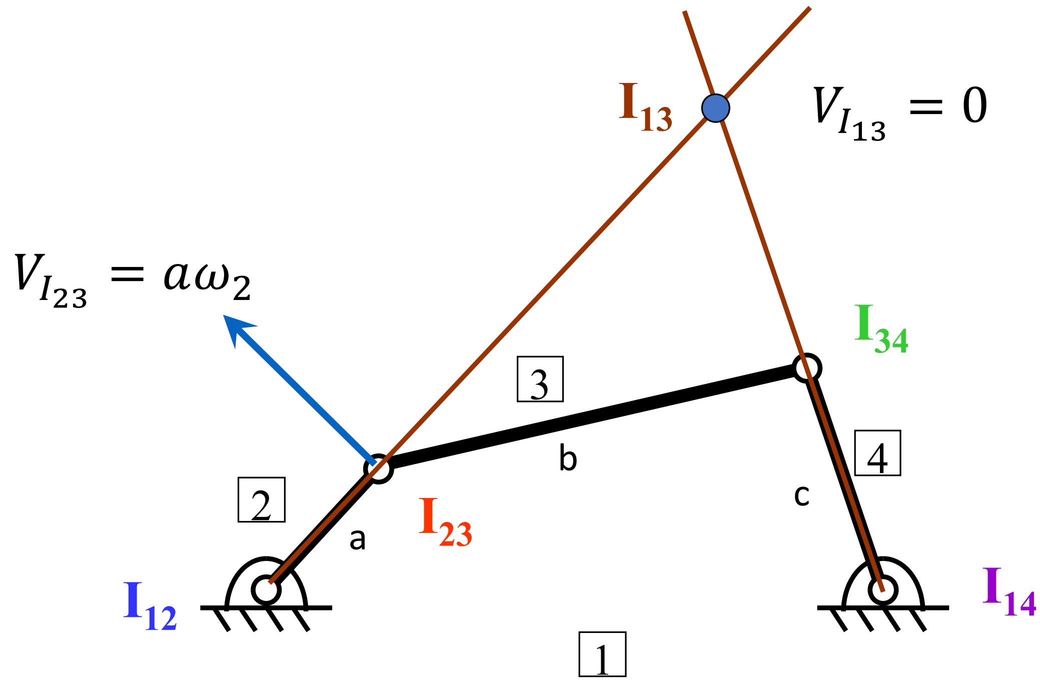
<u>Start with this knowledge.</u>

Say, we know ω_2.

As a result, we know $V_{I_{23}}$

In this system, we can find I_{13}

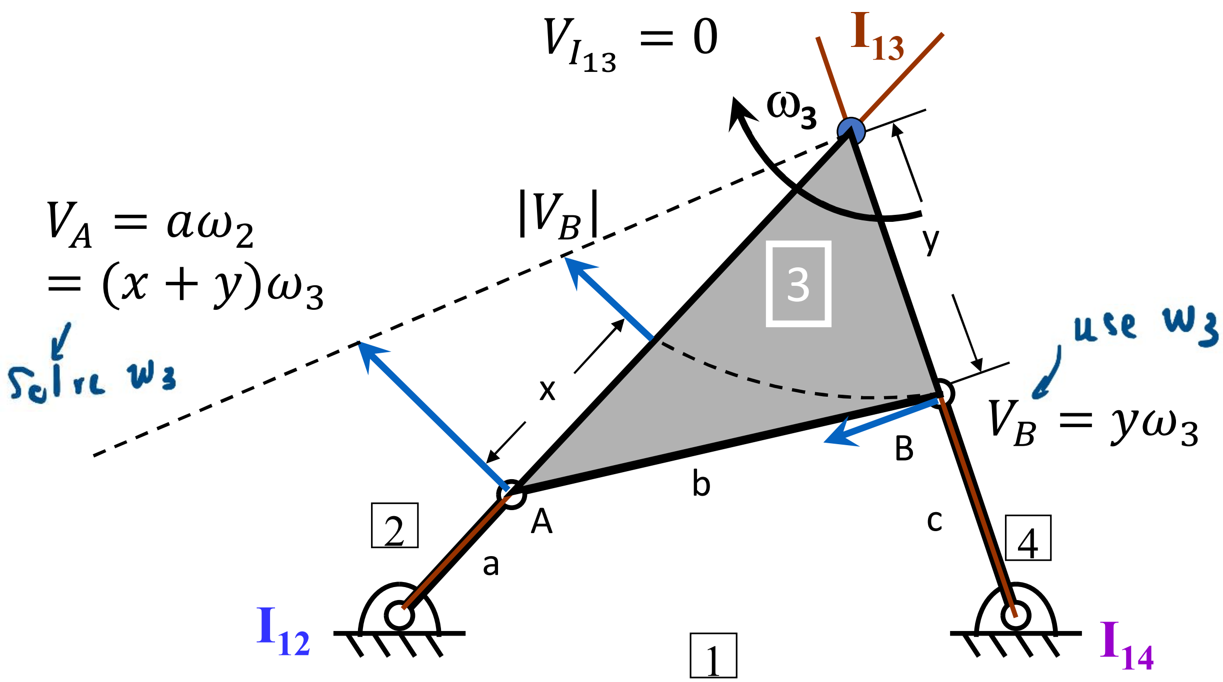
And we also know $V_{I_{13}} = 0$



Velocity Ratios: Big Idea

- **IC's allow us to compute velocities graphically**

- **Procedure:**
- Identify IC between links of interest and ground I_{13}
- IC between link and ground has V=0 $V_{I_{13}}=0$
- Link has same ω everywhere. If we know the velocity at one point, we can compute velocities everywhere
- $v = r \times \omega$



Every point in link 3 has the same ω_3

Mechanical advantage

- Machines can change the ratio of forces and velocities
- **Mechanical advantage:**

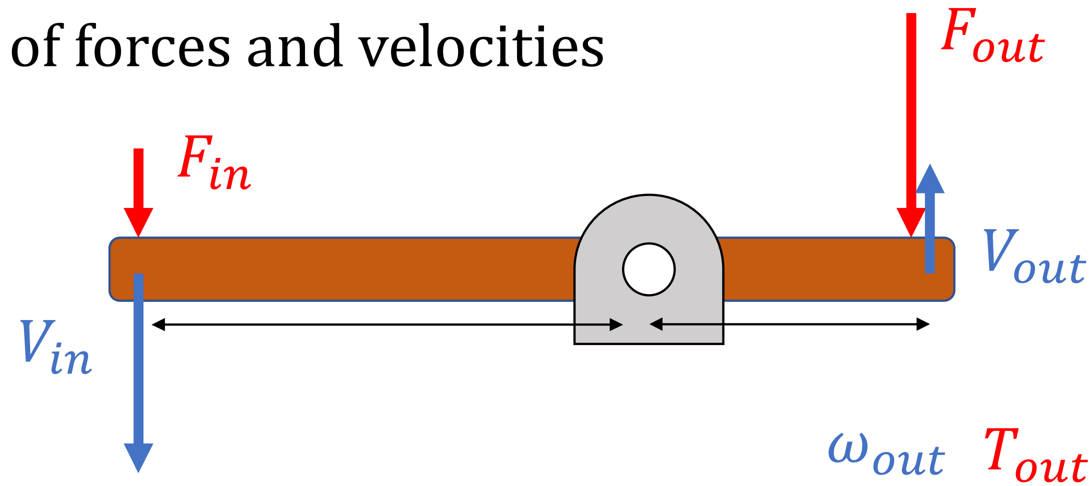
$$\text{MA} = \frac{F_{out}}{F_{in}}$$

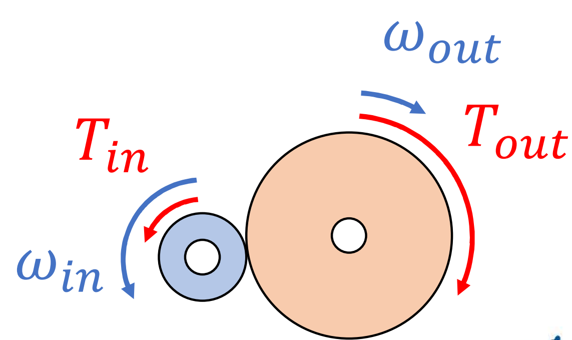
  - Remember $P = FV = T\omega$
  - $P_{in} = P_{out}$ if energy is not added or lost elsewhere

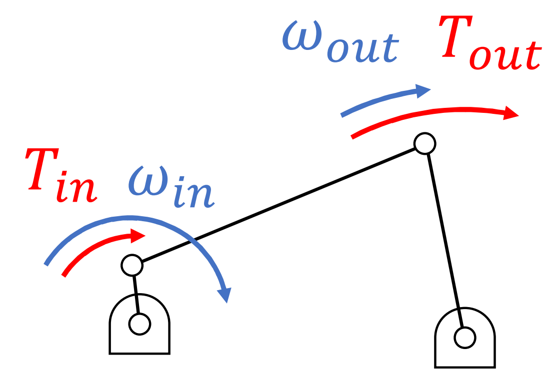
$$\Rightarrow F_{in}V_{in} = F_{out}V_{out}$$

$$\Rightarrow \frac{V_{in}}{V_{out}} = \frac{F_{out}}{F_{in}} = MA$$

  - V_{out} and F_{out} are inversely related: if one gets bigger the other gets smaller







$P_{in} = T_{in}\omega_{in} = P_{out} = \omega_{out} T_{out}$

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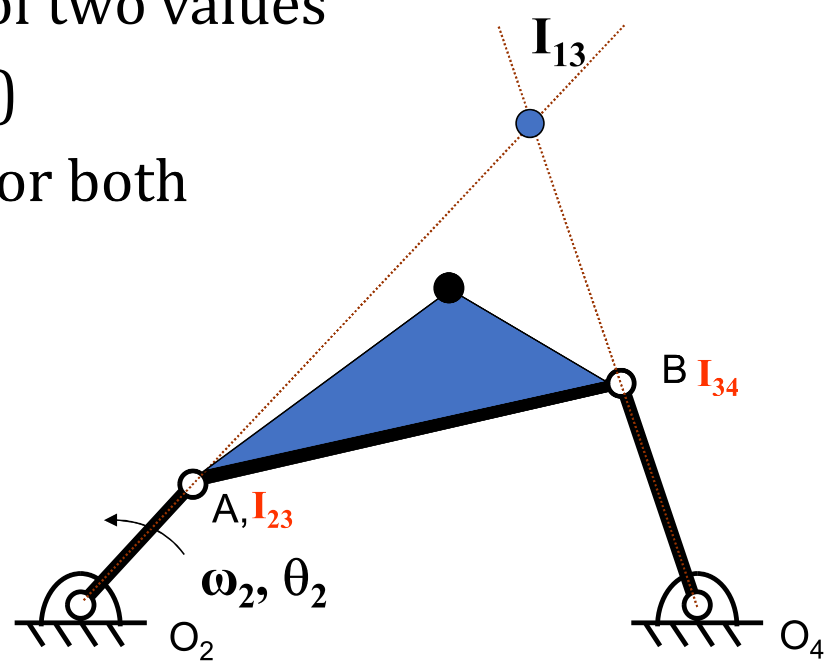
How can we use IC's?
Graphical velocity analysis

Can we use Instant Centers to easily find mechanical advantage?

Yes - We need to find the ratio of two values

- 2 Forces (How do we do this?)
- 2 Velocities (We can use IC's for both linear and angular velocity)



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How can we use IC's?
Graphical velocity analysis to solve for ω_3, ω_4, $\mathbf{V}_B$, $\mathbf{V}_C$

- **From IC's, links 2 & 3 have same velocity at A. Geometry relative to I_{13} gives:**

$v_A = \omega_2 |O_2 A| = \omega_3 |I_{13} A|$ so

$\omega_3 = \omega_2 |O_2 A| / |I_{13} A|$

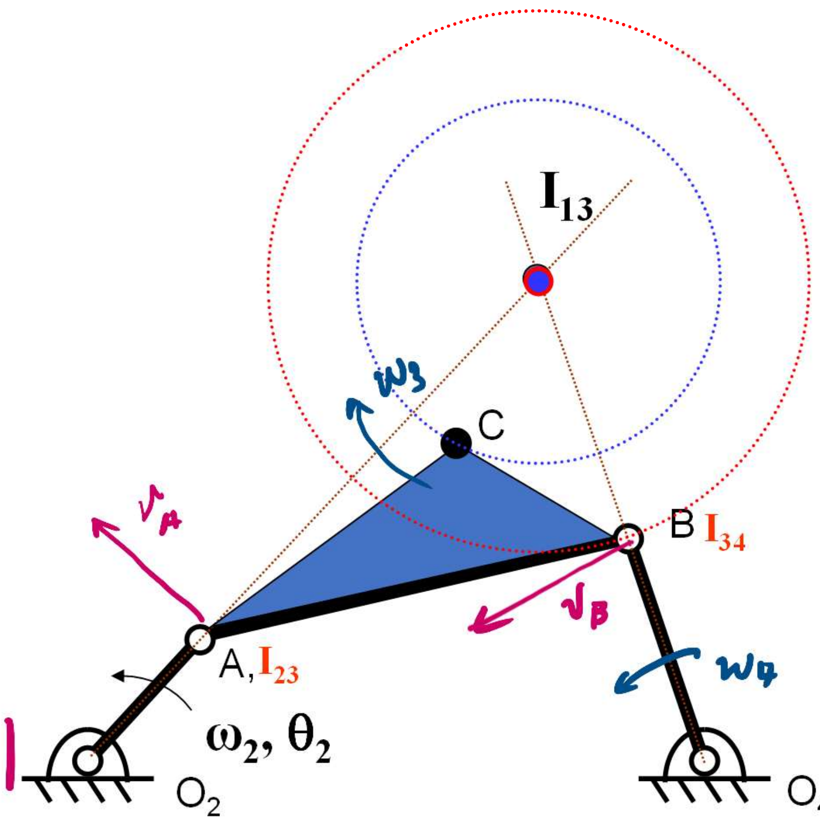
- **Using geometry:**

$v_B = \omega_3 |I_{13} B|$

- **From definition of IC @I_{34}:**

$v_B = \omega_4 |O_4 I_{34}| = \omega_3 |I_{13} B|$

$\omega_4 = \omega_3 |I_{13} B| / |O_4 I_{34}|$



ω_2 input

ω_4 output →

$MA = \frac{\omega_2}{\omega_4}$

How can we use IC's?
Graphical velocity analysis to solve for ω_3, ω_4, $\mathbf{V}_B$, $\mathbf{V}_C$

- **From IC's, links 2 & 3 have same velocity at A. Geometry relative to $\mathbf{I}_{13}$ gives:**

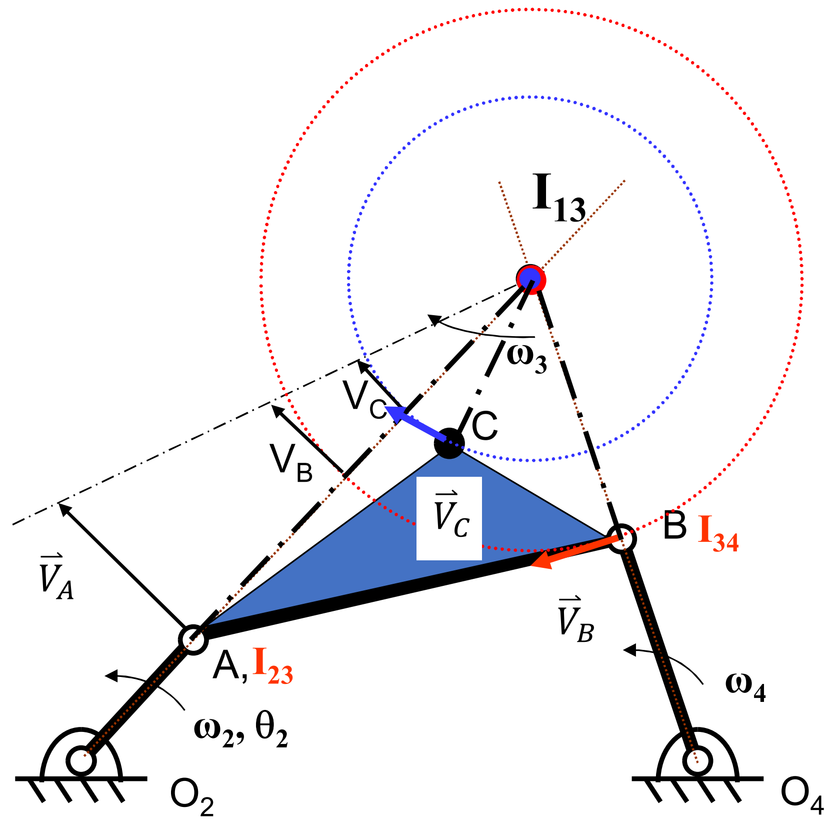
$$|\vec{V}_A| = |I_{13}A|\omega_3 \rightarrow \boxed{\omega_3 = |\vec{V}_A|/|I_{13}A|}$$

- **Using geometry:**

$$\boxed{|\vec{V}_B| = |I_{13}B|\omega_3}$$
$$|\vec{V}_C| = |I_{13}C|\omega_3$$

- **From definition of IC @$\mathbf{I}_{34}$:**

$$\boxed{\omega_4 = |\vec{V}_B|/|O_4B|}$$



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In-Class Example

- Use I_{24} (the instantaneous center of rotation of links 2 & 4) to find the velocity ratio between ω_2 and ω_4

$v_{I_{24}}$ @ Link 2 = $v_{I_{24}}$ @ Link 4 so

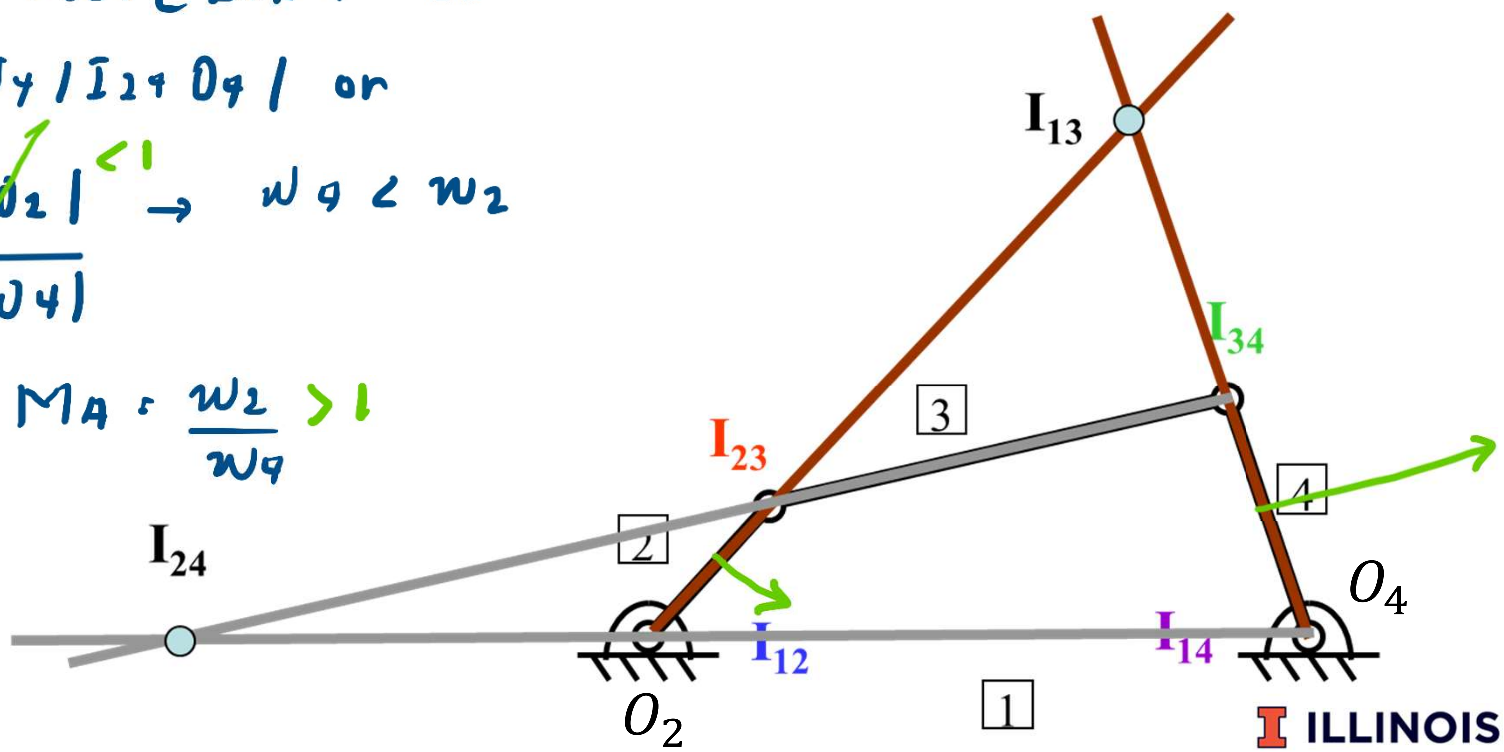
$w_2 |I_{24} O_2| = w_4 |I_{24} O_4|$ or

$w_4 = w_2 \dfrac{|I_{24} O_2|}{|I_{24} O_4|}$ <1 $\rightarrow$ $w_4 < w_2$

$w_2 \rightarrow$ input

$w_4 \rightarrow$ output

$MA = \dfrac{w_2}{w_4} > 1$



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In-Class Example

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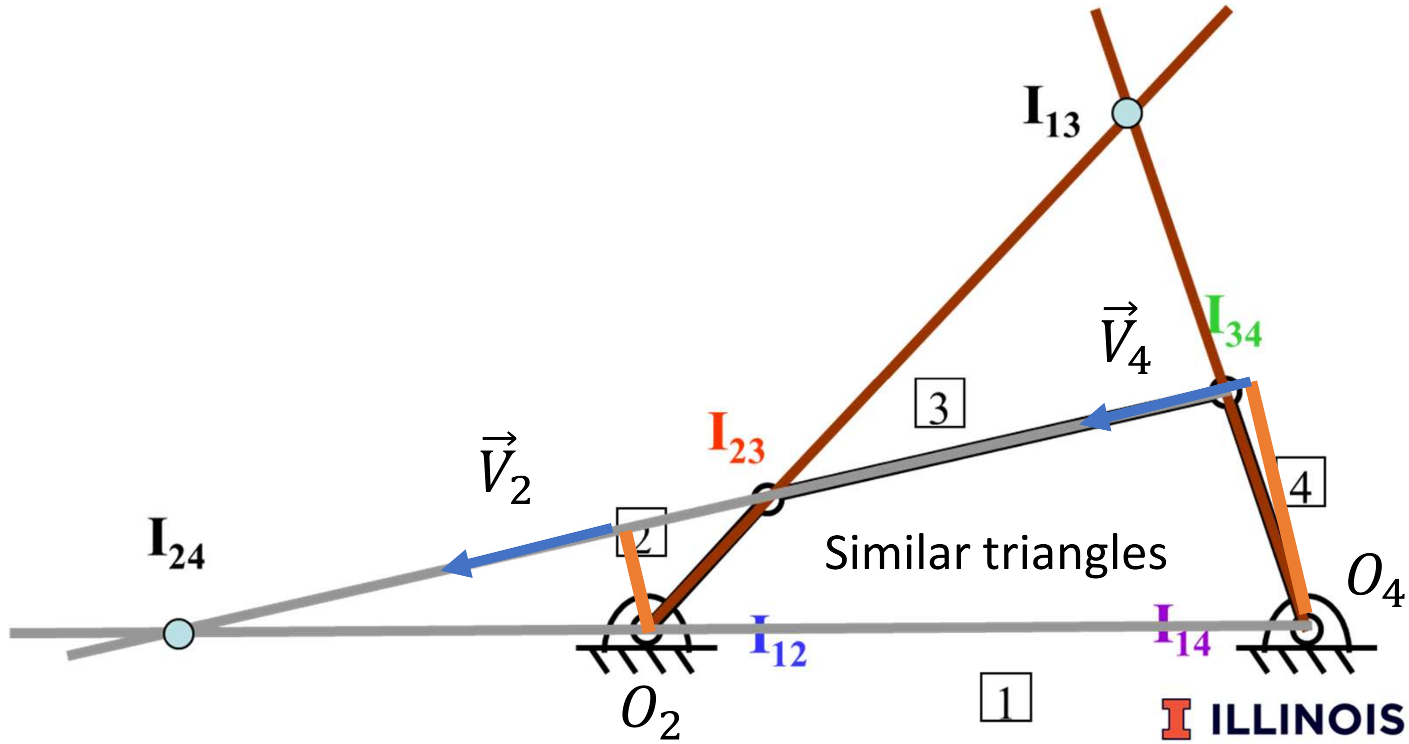
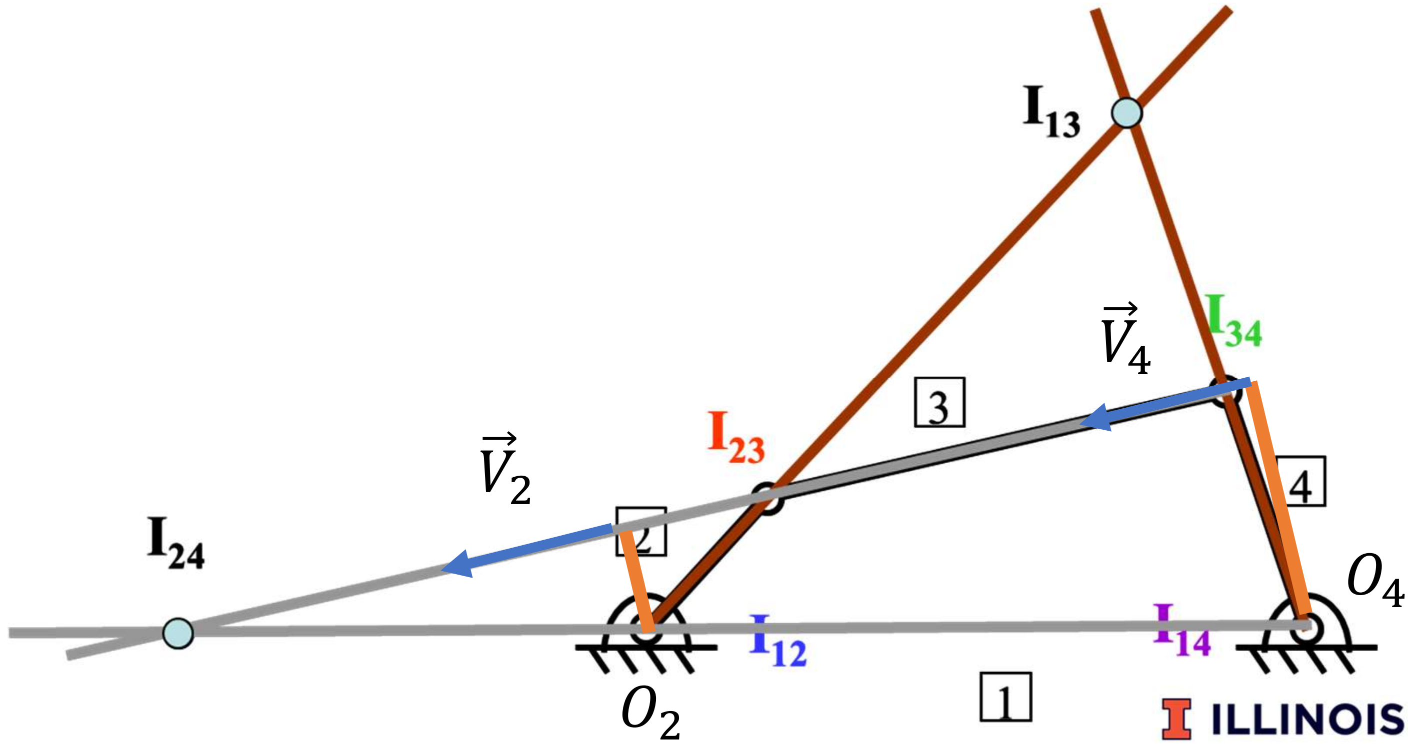


What happens if Link 2 and Link 3 become co-linear ?



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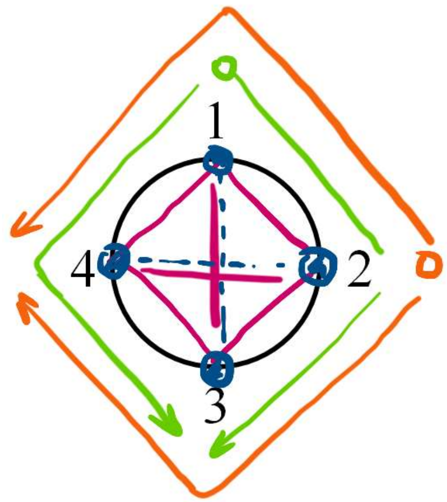
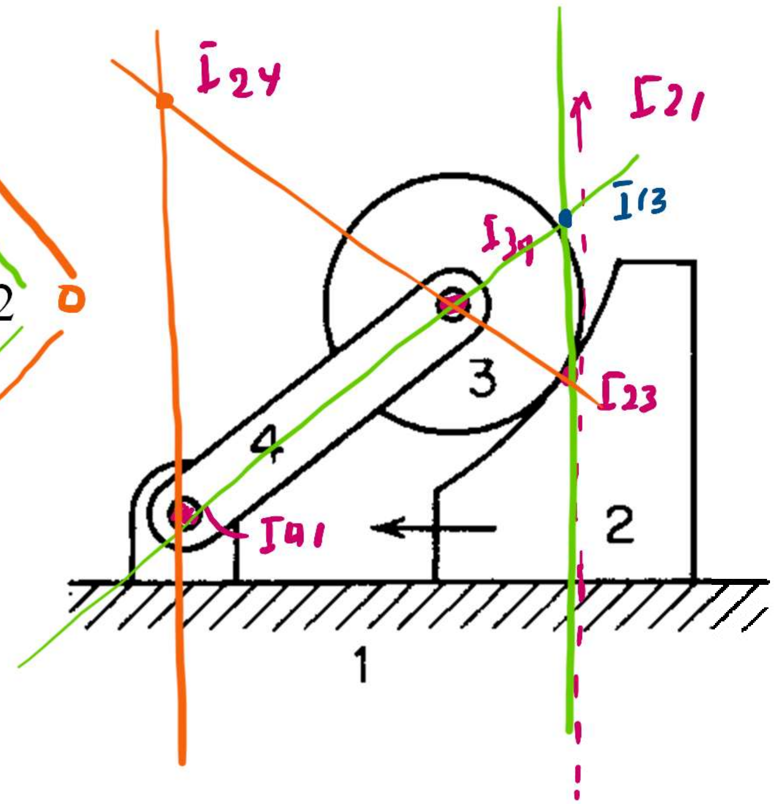
In-Class Example

- For the drawing below, assume rolling contact (without sliding) between links 2 and 3.
  - How many ICs should there be? $6 = 4(3)/2$
  - On the drawing, label all ICs between pairs of links, using the proper notation (e.g., I_{12}, I_{23}, etc.).





Instant centers in car design

https://www.youtube.com/watch?v=2Uneti_gkIw

Car & bike suspension design

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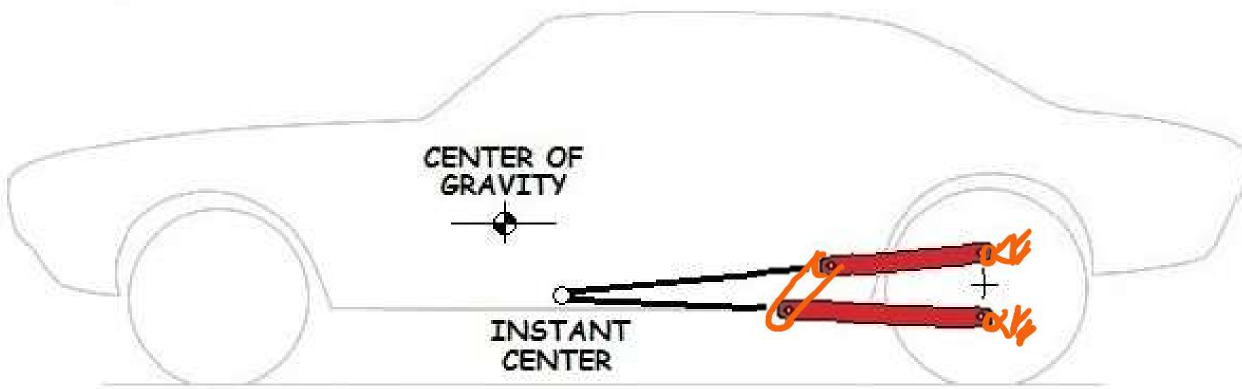
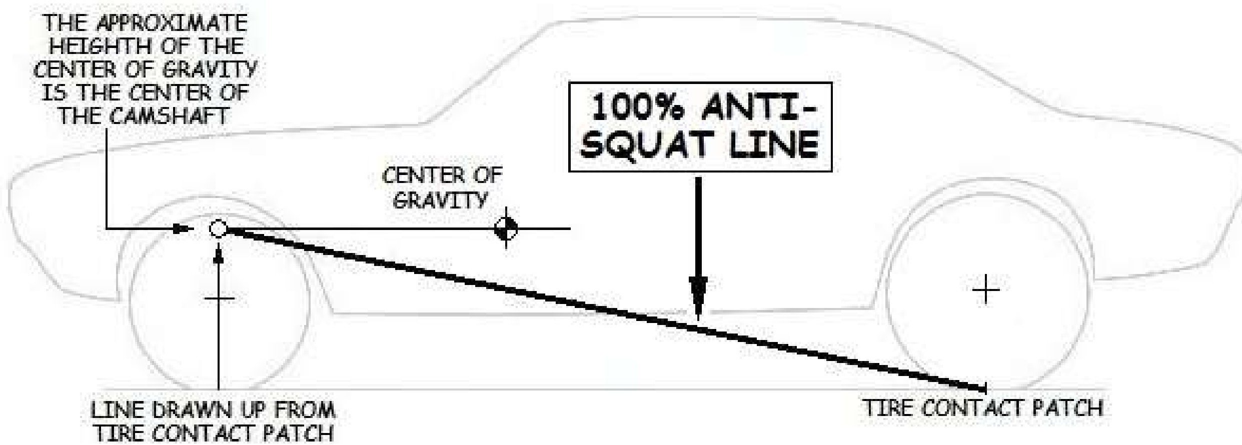
Instant centers in Suspension Design



Antisquat graphics from:
http://www.how-to-build-hotrods.com/tuning-4-link.html

For more: https://www.youtube.com/watch?v=D0bAVTiFyy4

Anti-squat line



We want all power from acceleration to translate into forward motion
Any power causing the center of gravity to move up or down (compressing suspension) reduces efficiency

Antisquat graphics from:
http://www.how-to-build-hotrods.com/tuning-4-link.html

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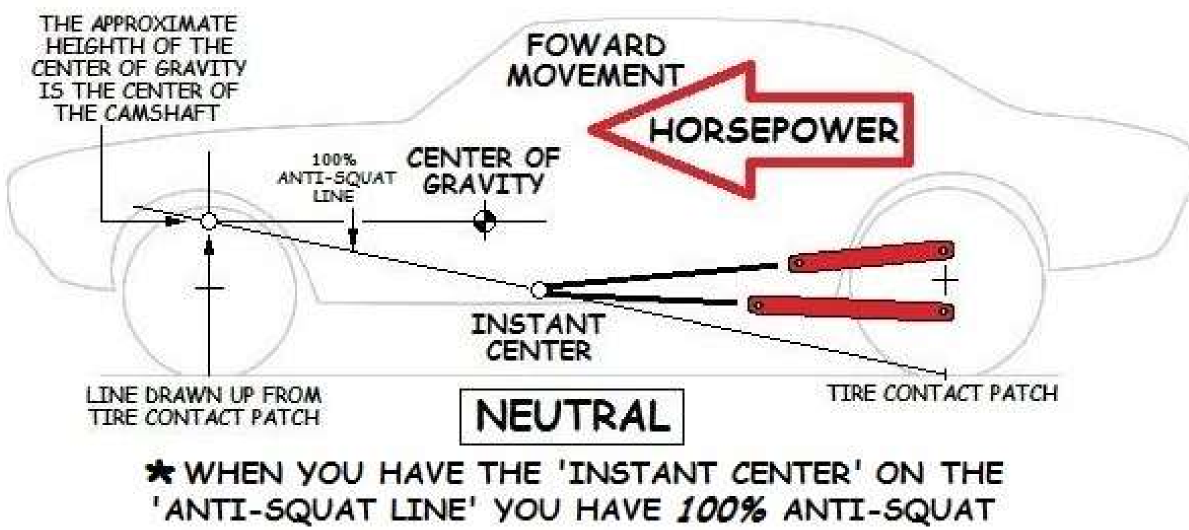
Maximum power



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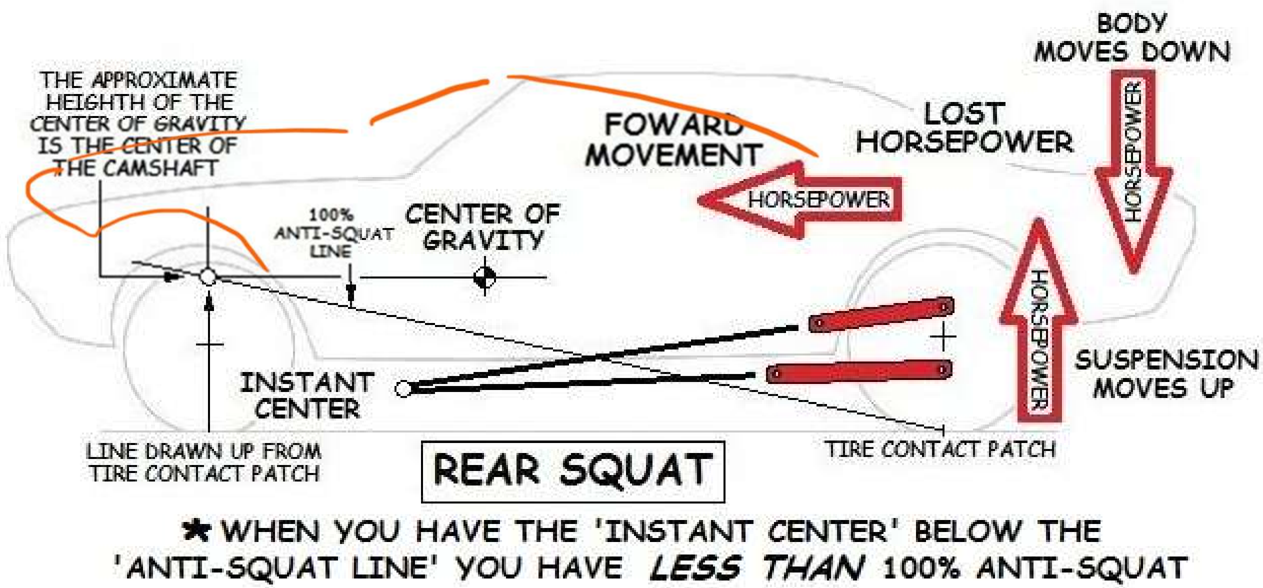
Body moves down



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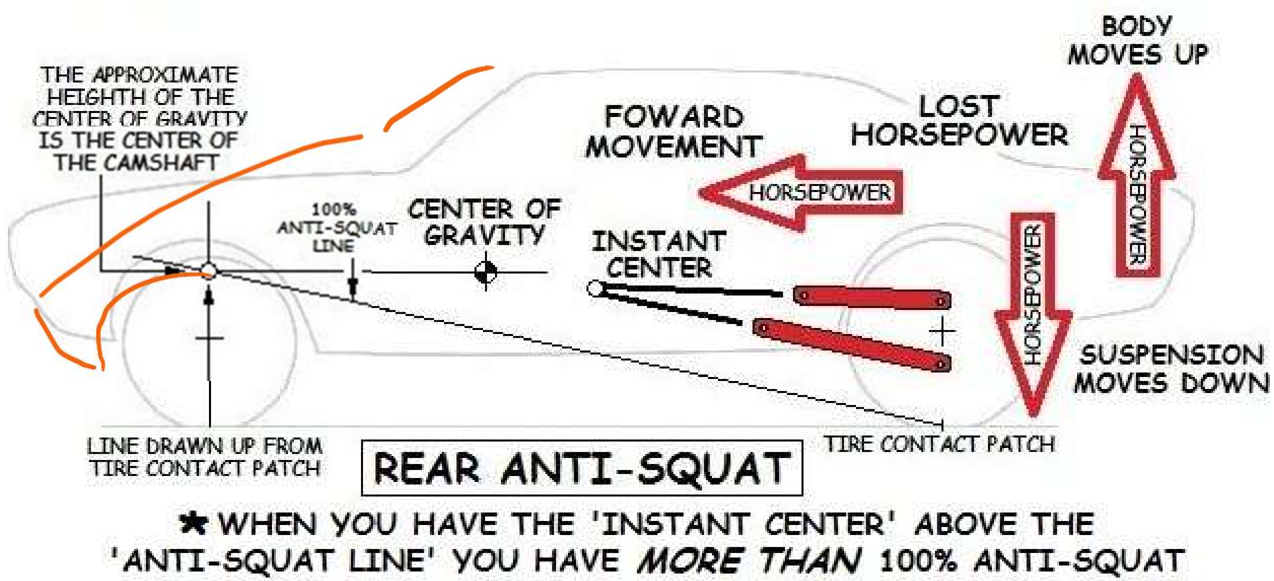
Body moves up



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