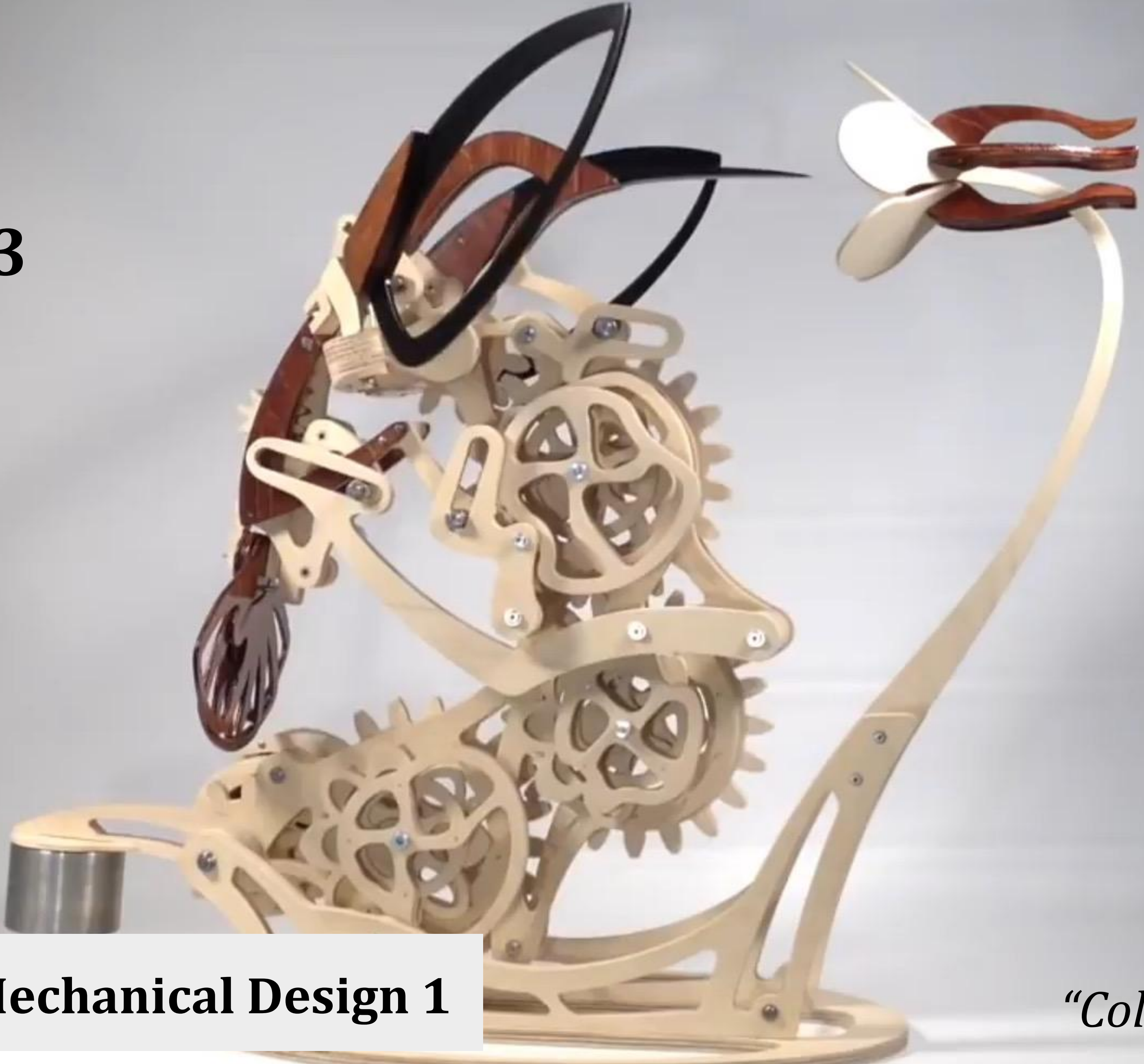
Module 9:
Lecture 27
Balancing - 3



ME 370 - Mechanical Design 1

"Colibri" by Derek Hugger

** www.youtube.com/watch?v=1scj5sotD-E*

Lecture 27: Balancing 3

Activities & Upcoming Deadlines

- **General:**
  - **Innovation Studio** anonymous feedback form (see QR code), responses may result in more space to work on projects for future classes.
  - Will close Friday 12/12 at 7 pm. Make spare parts for P2D4 before then.
  - Studio’s official hours are 11:00 AM – 10:00 PM M-F. Do not go early or stay late. Possible grade penalties.
- **Week 14:**
  - HW 14 (VW): due Tuesday 12/9
  - HW 15 (Balancing): will be posted by Tuesday. Due Tuesday 12/16
- **Project 2:**
  - **P2D3 (2-minute Video):**
    - All students rank 5 videos in your lab section by Tuesday Dec 9. See Canvas Assignment. Mandatory
  - **P2D4 (Performance Event):**
    - Grading rubric posted
    - Sign up for 10-minute demonstration timeslot on Google Sheet. Arrive 15 minutes early to have your device verified for Mechanical Design requirements.
  - **P2D5 (Final Report):** Guidelines and rubric are posted.
- **Next lecture:** Last class: View top 12 videos – **attendance mandatory**

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Module 9 Topics

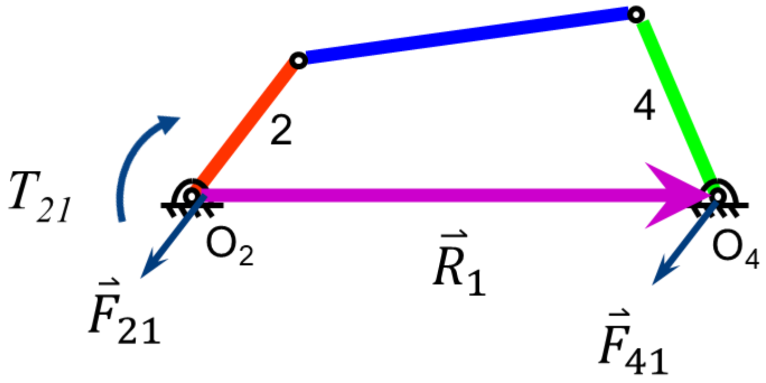
- Balancing of Rotating Machinery (Text 11.8, 11.11, 12)
  - Shaking force and torque (11.8)
  - Shaking moment (12.6)
  - Static balancing (12.1)
  - Dynamic balancing (12.2)
  - Balancing more complex planar mechanisms (12.3+)
    - Single piston engine
    - 4-bar linkage
  - Flywheels (11.11)
    - Torque variation
    - Motor selection
    - Sizing a flywheel

Cover today

Balancing more complex planar mechanisms

- Would like to have:
  - Min shaking forces $\vec{F}_s = \vec{F}_{21} + \vec{F}_{41}$
  - Min shaking torques $T_s = T_{21} = -T_{12}$
  - Min shaking moment $\vec{M}_s = \vec{T}_{21} + (\vec{R}_1 \times \vec{F}_{41})$
  - Uniform input torque

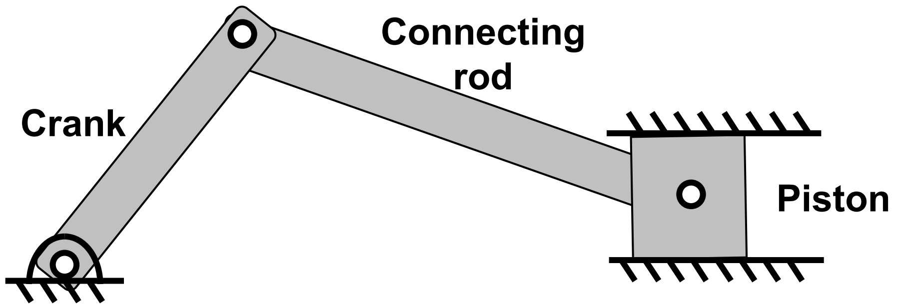
Generally competing: Make one better at the expense of the others



- Forced to do one of the following:
  - Static balance & accept the results on others
  - Make some tradeoff between all factors
- Ideally move CG of each link to axes of rotation
  - Then each link is statically balanced (so CG of link is stationary)
- Problem: coupler's center of rotation is constantly moving
- Solution: move CG of entire mechanism to be stationary
  - Then entire mechanism is "statically balanced" $\sum\vec{F} = 0 \rightarrow \sum m_i\vec{R}_i = 0$
  - Probably not "dynamically balanced" $\sum\vec{M} = 0 \rightarrow \sum m_i\vec{R}_i l_i = 0$

Example: Single Piston Engine (Norton 13.4)

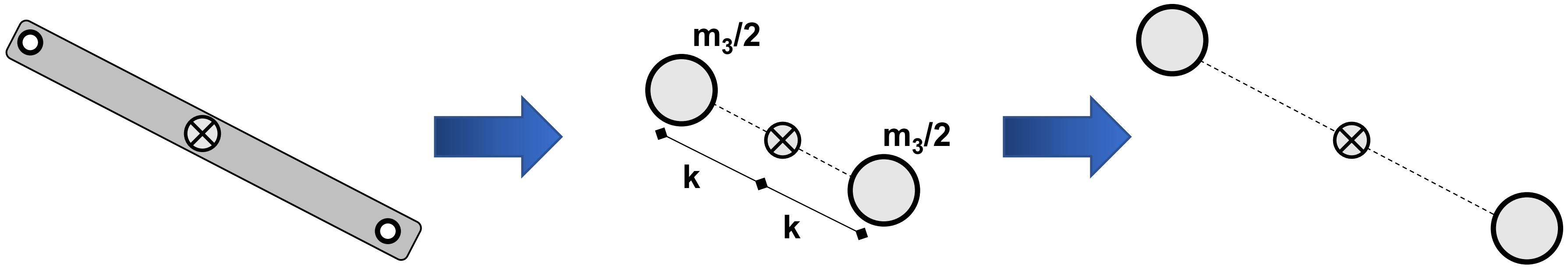
Goal: move CG's to points of rotation



- Focus on connecting rod

As an approximation, can be idealized as …

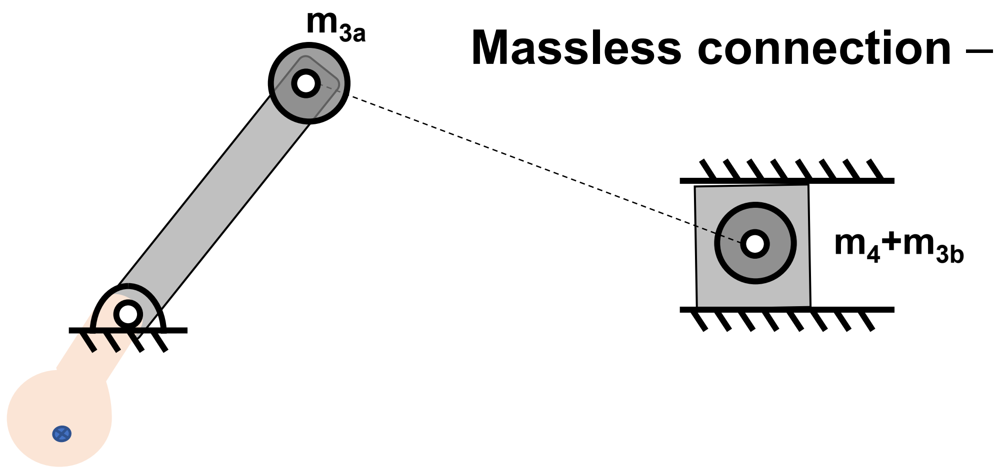
m_{3a} – same motion as link 2

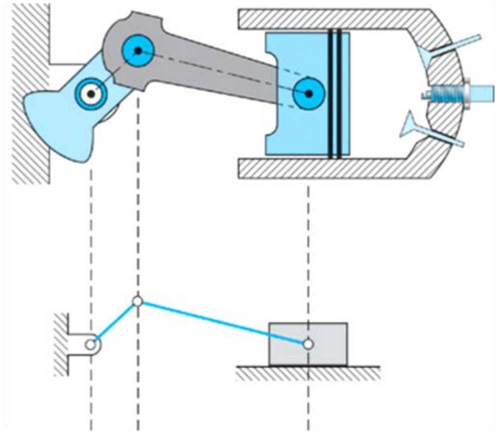


same motion as link 4 - **m_{3b}**

$\mathbf{m_3, I_{CG3}}$ | $\mathbf{m_3, I_{CG3}}$ | $\mathbf{m_3, I_{CG} > I_{CG3}}$

Example – Single Piston Engine

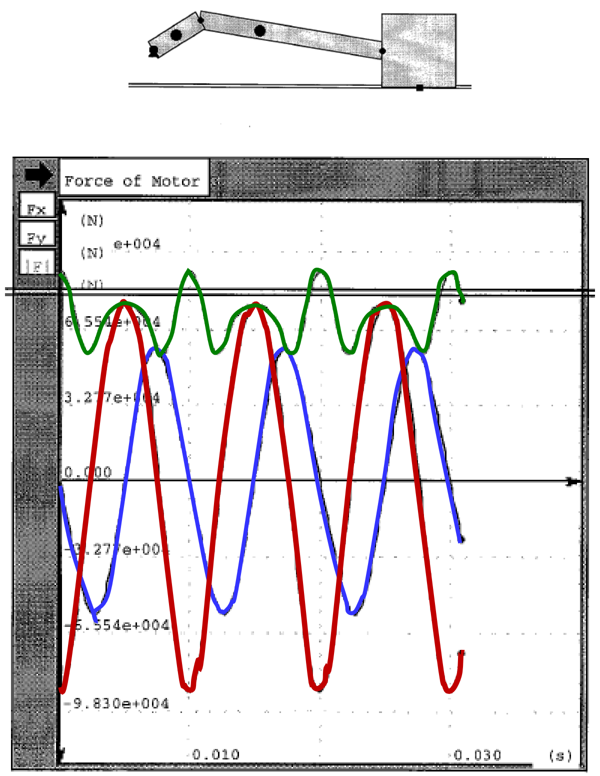




- Add balancing weight to crankshaft
  - If $(crank + m_{3a})$ is statically balanced, all shaking force is from $(m_4 + m_{3b})$
  - More common to overbalance the crank
    - Reduces $\left|\vec{F}_s\right|dt$

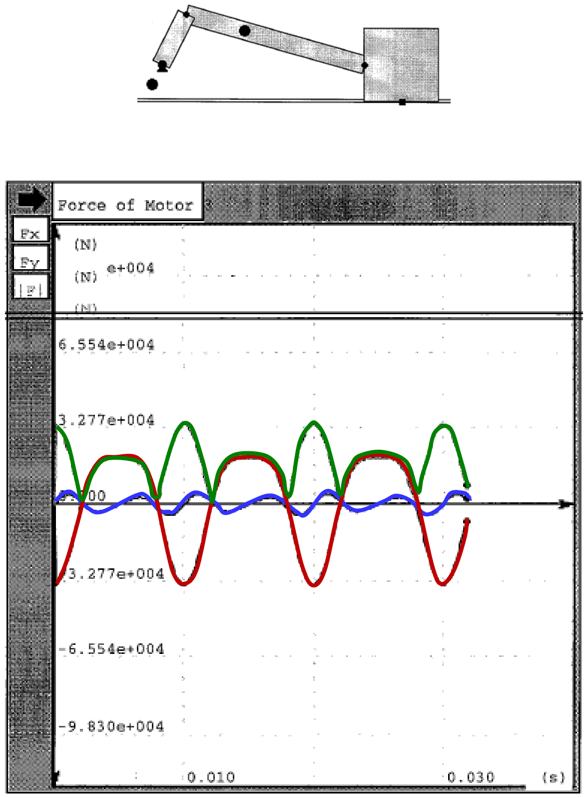
Balancing Single Piston Engine

Unbalanced



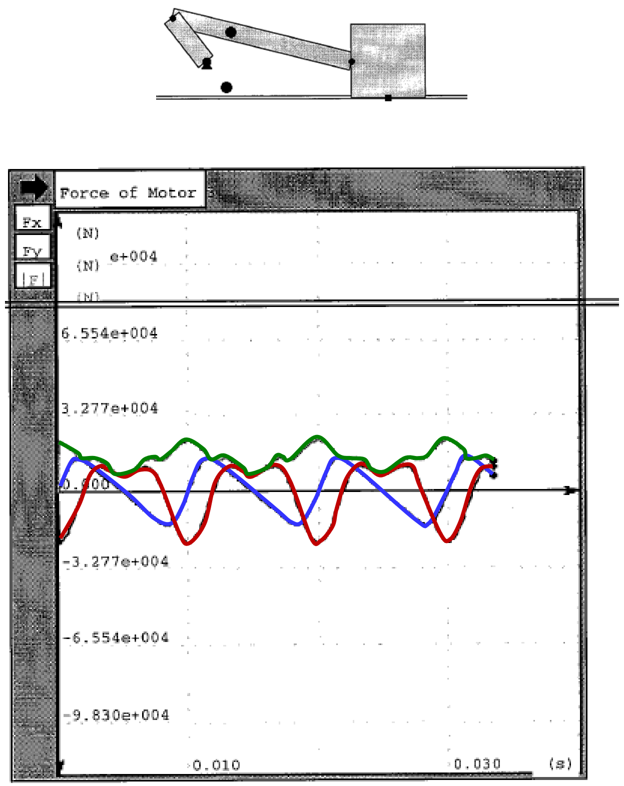
F_x F_y |F|

Balanced



F_x F_y |F|

Overbalanced



F_x F_y |F|

Balancing 4-bar Linkage

- Can individually balance rotating links by earlier methods
- Coupler presents a problem!

  Can you contemplate a situation in which the mass center of the coupler is stationary

  … probably not!
- Any mechanism will have a global mass center that we can force to be stationary to achieve static balance
  - Can get $\sum \vec{F} = 0$
  - But not $\sum \vec{M} = 0$

Balancing 4-bar Linkage

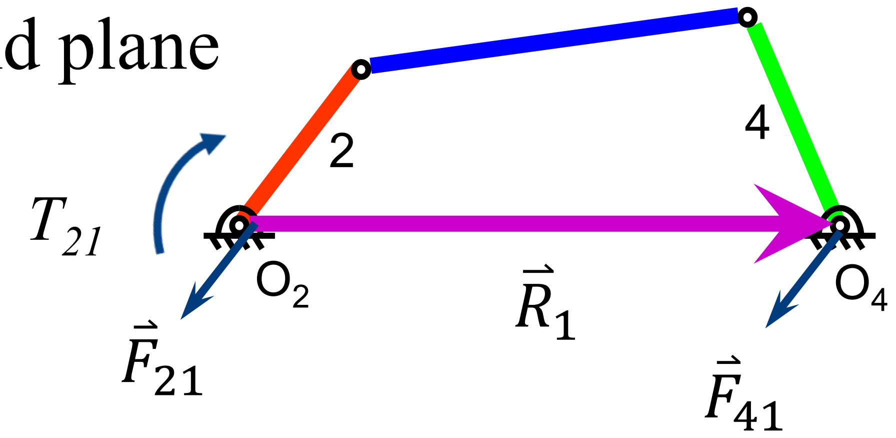
- Shaking force
  - Sum of all forces acting on the ground plane
  - In a four- bar mechanism:

    $\sum \vec{F}_s = \vec{F}_{21} + \vec{F}_{41}$

    can eliminate!
- Shaking moment
  - Sum of all moments acting on the ground plane
  - In a four- bar mechanism:

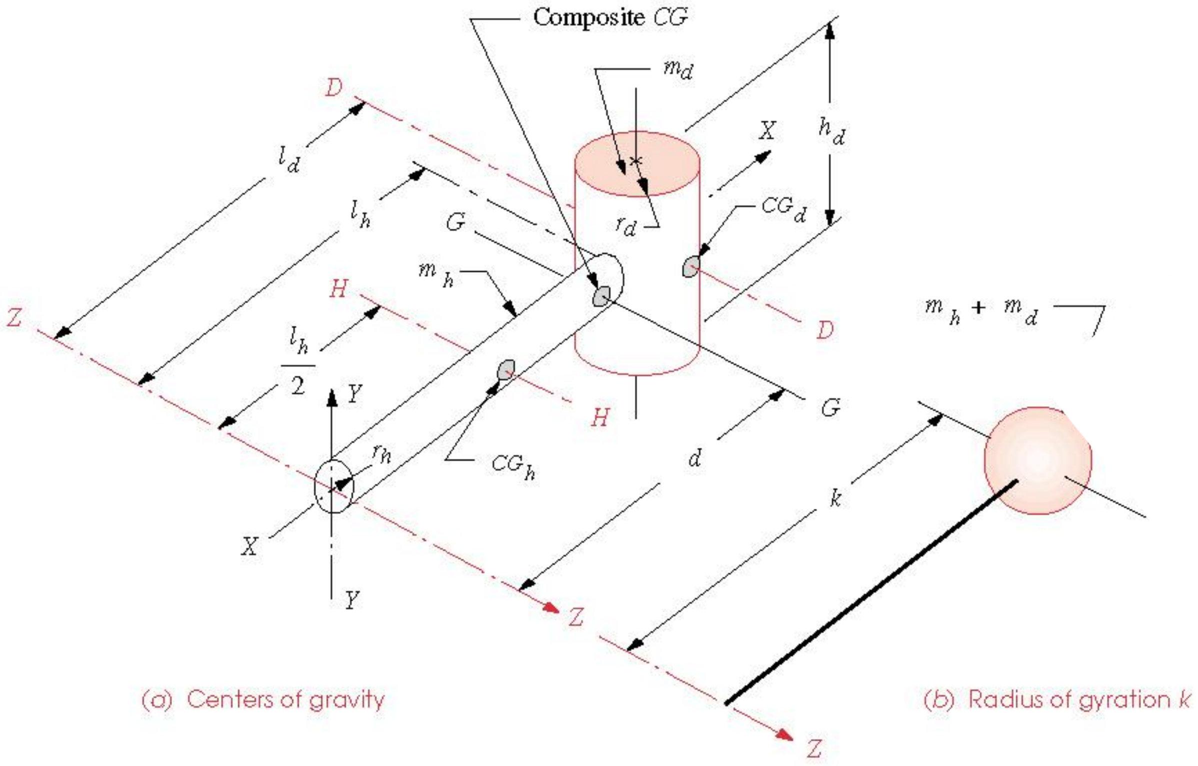
    $\vec{M}_s = \vec{T}_{21} + (\vec{R}_1 \times \vec{F}_{41})$

    cannot eliminate!



Review: Mass Moment & Global CG

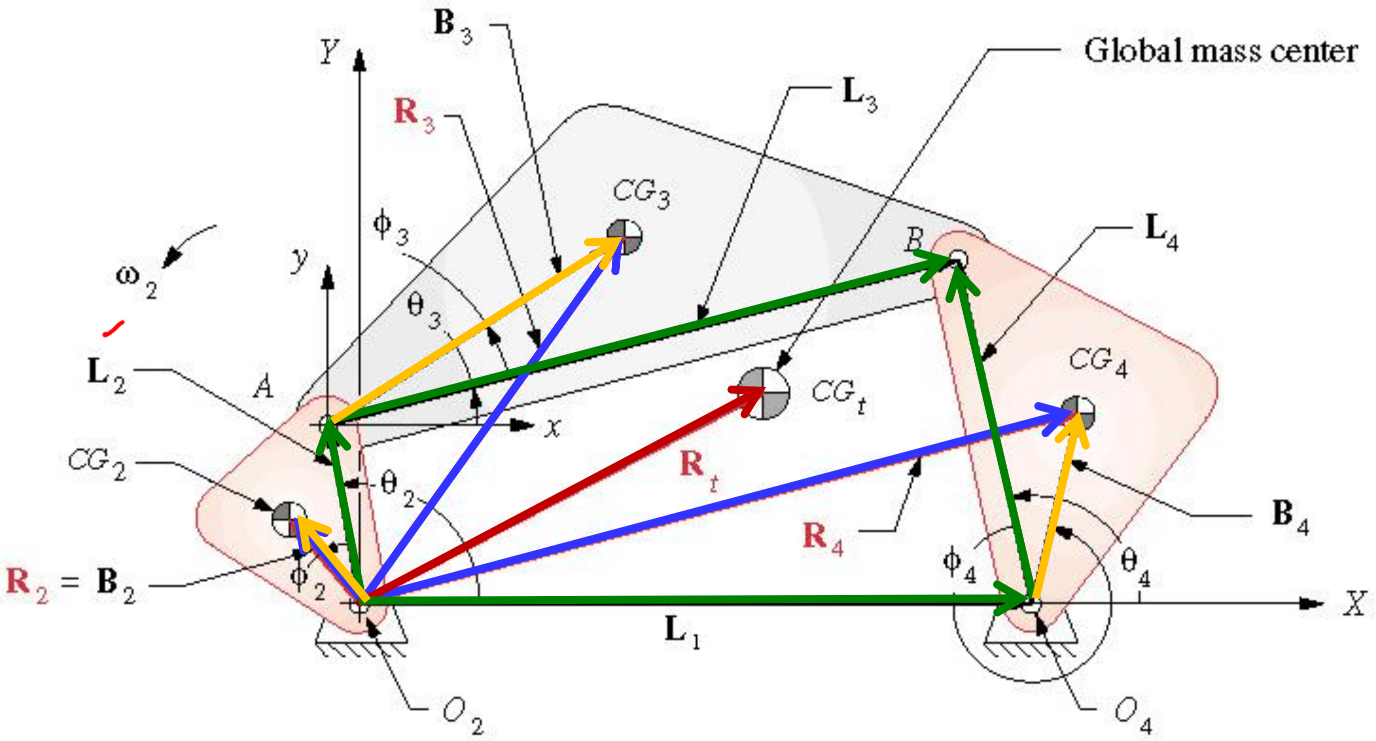
- Mass of an object causes a moment about a chosen axis
- Mass Moment = mass x distance from axis



$$\sum M_{zz} = m_h \frac{l_h}{2} + m_d l_d = (m_h + m_d) d$$

$$\text{Global CG} = d = \frac{m_h \frac{l_h}{2} + m_d l_d}{(m_h + m_d)}$$

Balancing 4-bar Linkage (Norton §12.3)



Blue vectors represent position vectors of CG of each link from O_2

Red vector represents position vector of Global CG

Yellow vectors represent local position vectors of a link's CG within the link. It has length b_i and angle ϕ_i

Green represent the vector loop based on the link lengths l_i

Find location of total CG. Sum the mass moments* about O_2 of each link rotating about O_2

$$\sum M_{O_2} = \sum m_i \vec{R}_i = m_2\, \vec{R}_2 + m_3 \vec{R}_3 + m_4 \vec{R}_4 = m_t \vec{R}_t$$

$$\vec{R}_2 = b_2 e^{j(\theta_2+\phi_2)}$$
$$\vec{R}_3 = l_2 e^{j(\theta_2)} + b_3 e^{j(\theta_3+\phi_3)}$$
$$\vec{R}_4 = l_1 e^{j(\theta_1)} + b_4 e^{j(\theta_4+\phi_4)}$$

*Not sum of moments:
$\vec{M}_{O2} = \vec{R}_i \times \vec{F}_i$

Vector Loop equation

$$\vec{L}_2 + \vec{L}_3 - \vec{L}_4 - \vec{L}_1 = 0$$

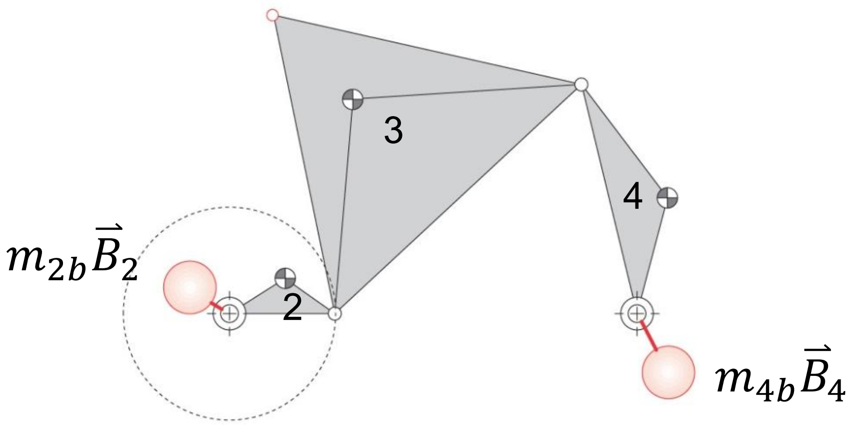
$\vec{L}_1 = l_1 e^{j(\theta_1)}$ $\quad \vec{L}_3 = l_3 e^{j\boxed{(\theta_3)}}$

$\vec{L}_2 = l_2 e^{j\boxed{(\theta_2)}}$ $\quad \vec{L}_4 = l_4 e^{j\boxed{(\theta_4)}}$

F

Balancing 4-bar Linkage: Summary (For derivation, see Norton §12.3)

- Will get 4 equations, so typical approach involves:
  - Assuming coupler remains unchanged
  - Solving for $m_2\vec{B}_2$ and $m_4\vec{B}_4$ mass and location of CG for links 2 and 4
    - Eliminate θ_3
    - Force $m_t\vec{R}_t$ to be constant (stationary), we get the following 4 equations that need to be solved to determine new links 2 and 4, which include balancing weights to each link:



Balance masses m_{2b} and m_{4b} (red) can accommodate for equations if links 2 and 4 already defined.

$$(m_2b_2)_x = m_3\left(b_3\frac{l_2}{l_3}\cos\phi_3 - l_2\right)$$

$$(m_2b_2)_y = m_3\left(b_3\frac{l_2}{l_3}\sin\phi_3\right)$$

$$(m_4b_4)_x = -m_3b_3\frac{l_4}{l_3}\cos\phi_3$$

$$(m_4b_4)_y = -m_3b_3\frac{l_4}{l_3}\sin\phi_3$$

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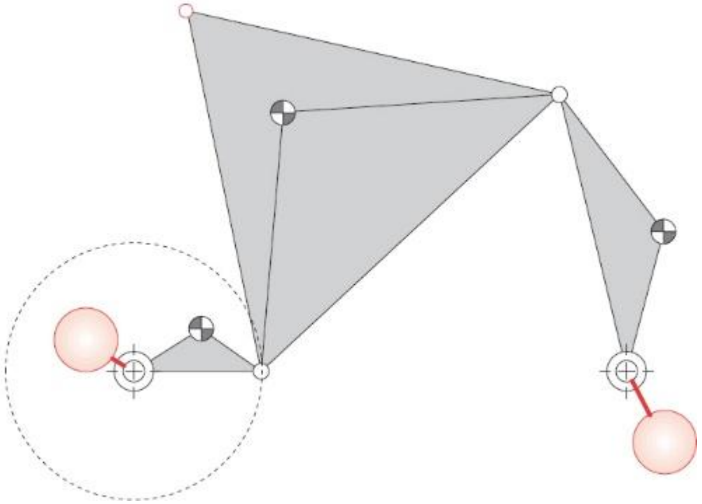
Balancing 4-bar Linkage (See for full derivation Norton §12.3)

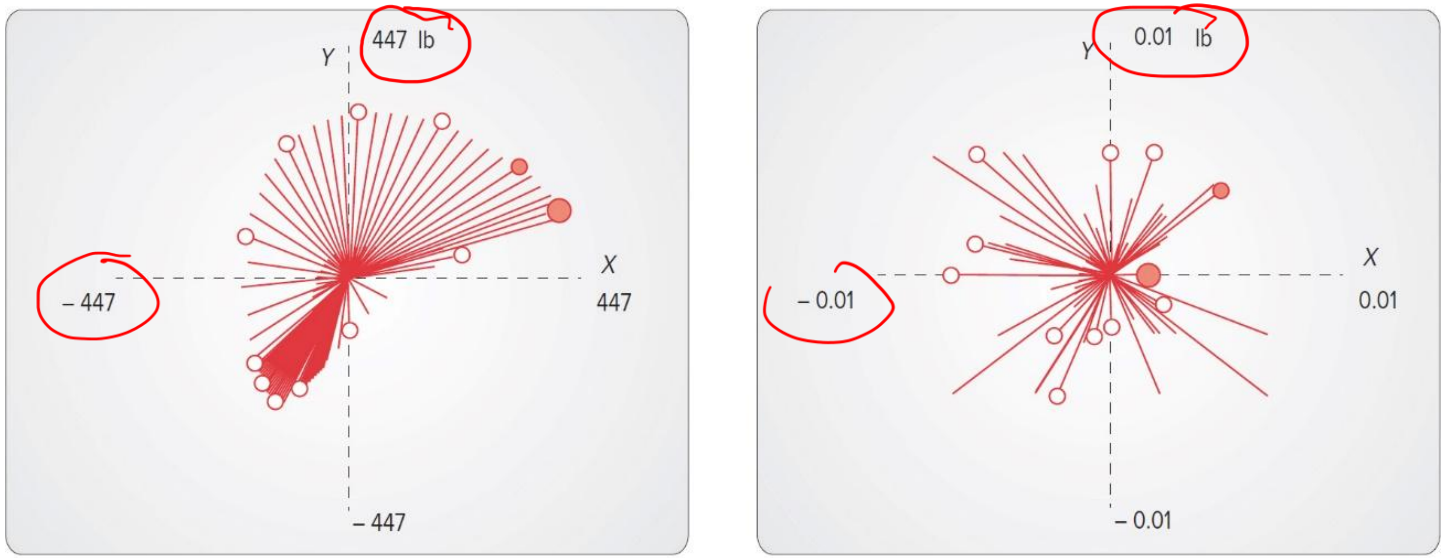
L1 = 19 in
L2 = 5
L3 = 15
L4 = 10

Cplrpt = 13
@ 63°

ω_2 = 50 rad/sec
0 to 360
by 5 deg



Note the magnitudes in each plot!



(a) Shaking force with linkage unbalanced F_s

(b) Shaking force with linkage balanced

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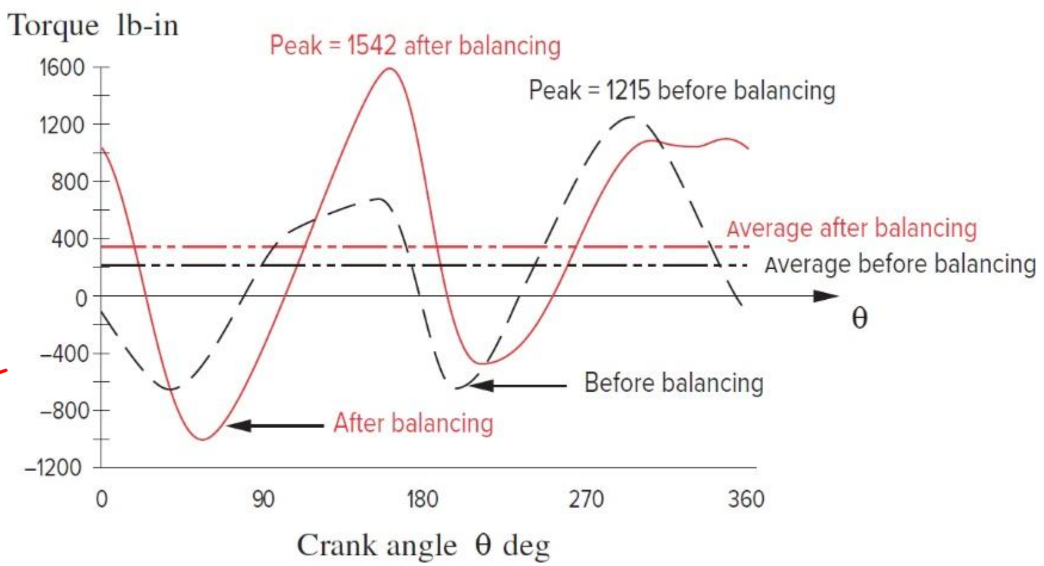
Effect of Balancing on Input Torque

$$T = I\alpha$$

- What happens to I after adding balancing weights or modifying links to achieve global balancing?
  - Generally increases
- What happens to Input Torque (T_{12})?
  - Will vary over time
- Will this affect dynamic forces for the crank?
  - No. No angular acceleration on input link

$\omega = const$



- Will this affect dynamic forces for the rocker?
  - Yes, the balanced moment of inertia is typically greater than the unbalanced; need larger drive torque to compensate
- How do we fix this?
  - Add a flywheel to even out input torque
- Note: We have unbalanced individual links to gain a global balance!

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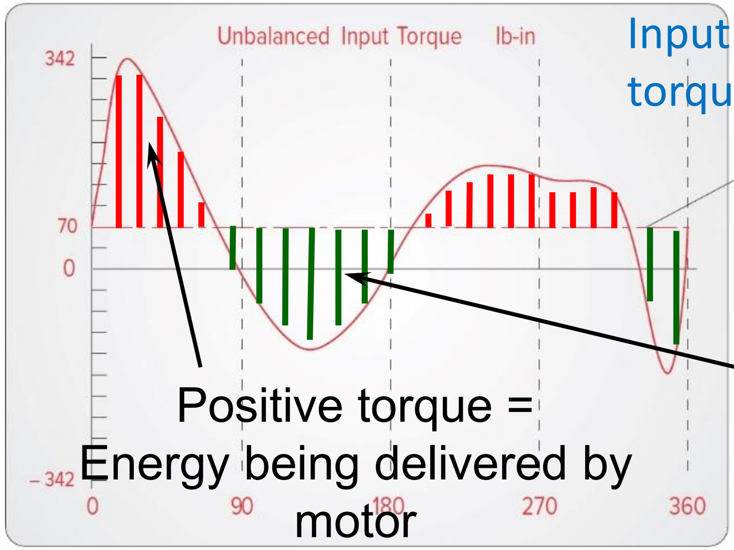
Torque Variation

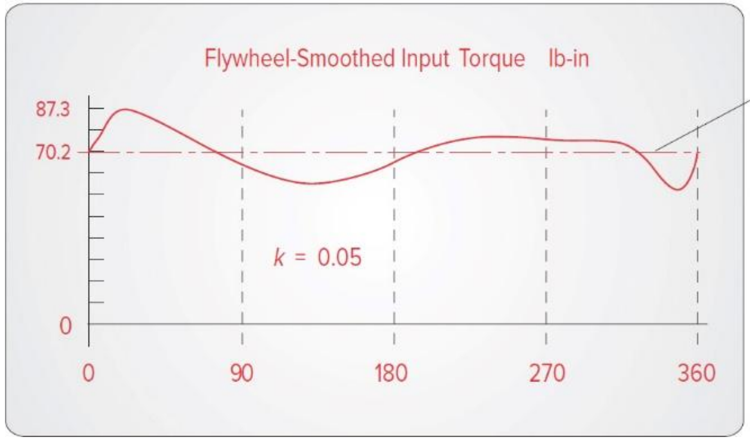
Flywheel used to smooth out torque oscillations by storing kinetic energy



Input (crank) torque

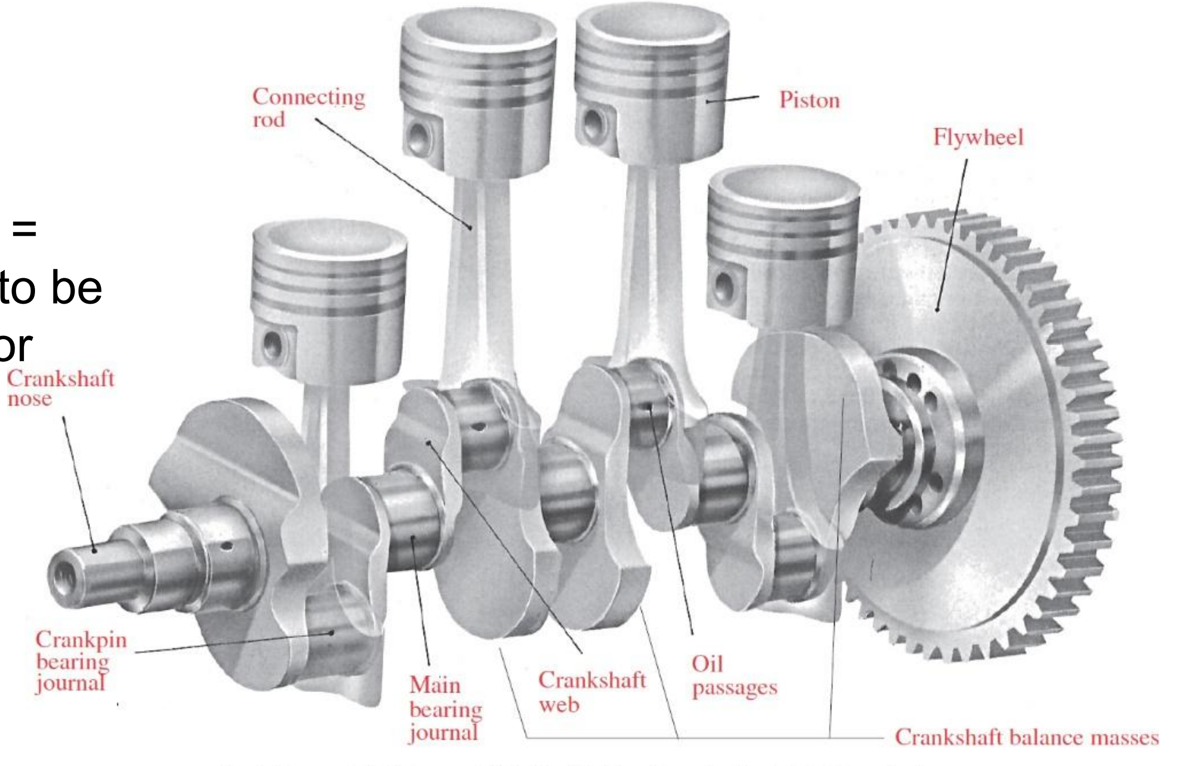
Positive torque = Energy being delivered by motor

Negative torque = Energy attempting to be returned to motor



Note the magnitudes in each plot!

Crankshaft from inline 4-cylinder internal combustion engine highlights much of the design knowledge learned in ME 370



Note how each piston slider-crank mechanism is balanced by crankshaft balance masses

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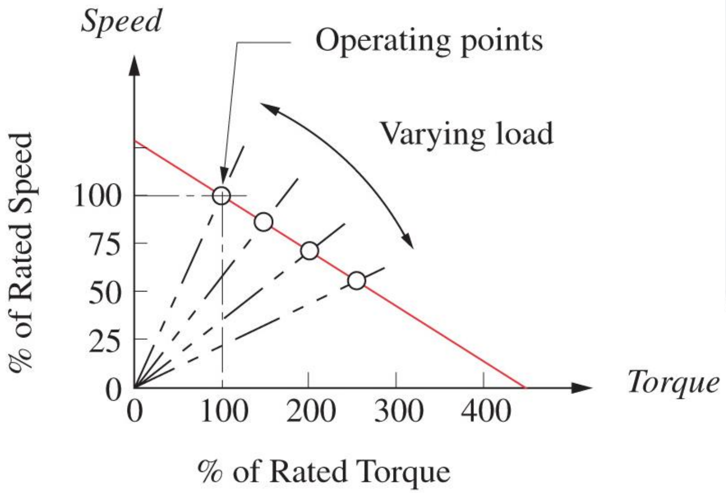
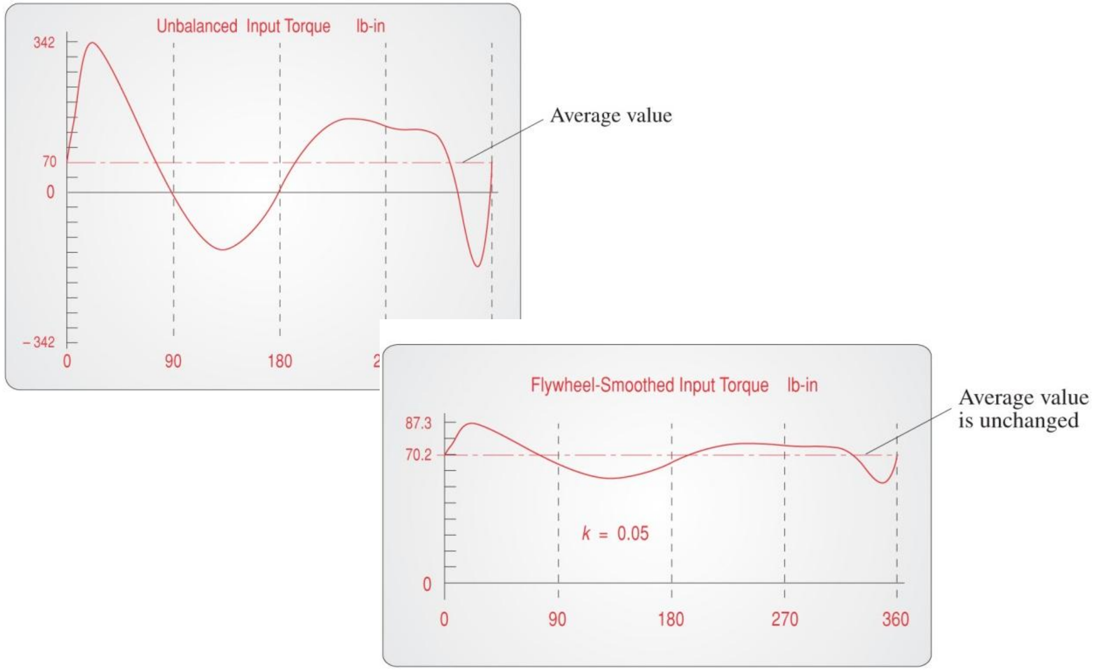
Motors and Flywheels (Norton, Chapter 11.11)

- Motors operate with a given speed-torque relationship (high speed → low torque)



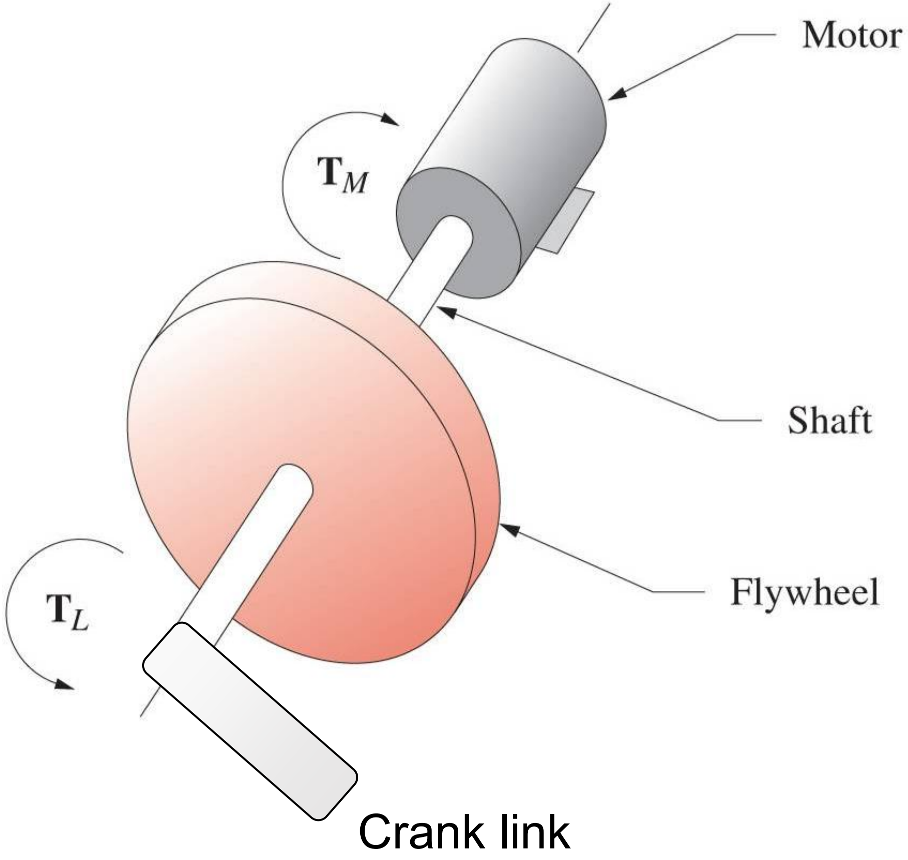


- Flywheels are used to smooth out these oscillations by storing kinetic energy

Motor Selection

- We want to select our motors around some reasonable value of average torque:
  T_M = Drive (motor) torque
  T_L = Input (load) torque
  I = Moment of inertia of ***all*** rotating mass

$I = I_{sys}$
$= I_{link} + I_{motors}$
$+ I_{flywheel}$



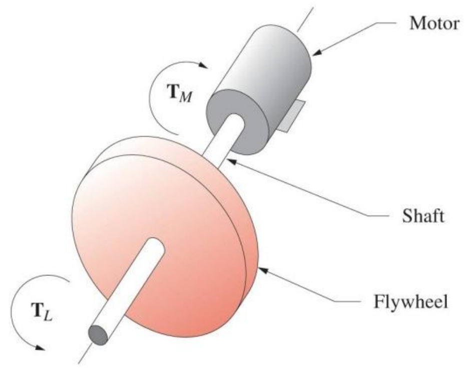
$$\sum T = T_L - T_M = I\alpha$$

we want: $T_M = T_{avg}$

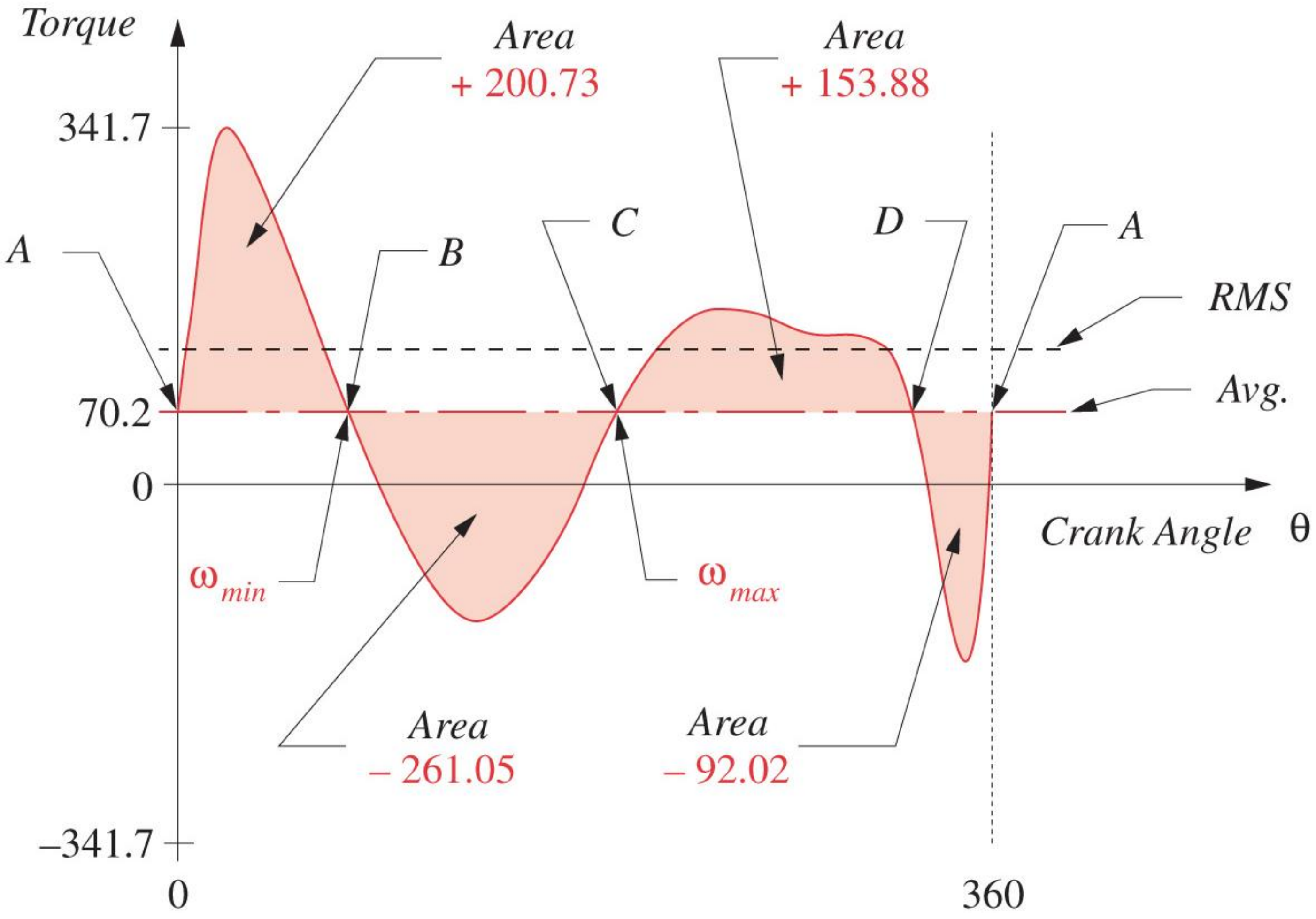
$$\sum T = T_L - T_{avg} = I\alpha$$

Sizing a Flywheel (see for full derivation: Norton, Chapter 11.11)

$$\begin{aligned} T_L - T_{avg} &= I\alpha \\ &= I\frac{d\omega}{dt} \\ &= I\frac{d\omega}{dt}\frac{d\theta}{d\theta} \\ &= I\frac{d\theta}{dt}\frac{d\omega}{d\theta} \\ &= I\omega\frac{d\omega}{d\theta} \\ \left(T_L - T_{avg}\right)d\theta &= I\omega d\omega \end{aligned}$$



Max and Min angular velocities are related to negative and positive energy exchanges between the motor and load



Sizing a Flywheel

- Integrate to find the change in energy between the min and max drive shaft angular velocities

$$\int_{\theta(\omega_{min})}^{\theta(\omega_{max})} (T_L - T_{avg}) d\theta = \int_{\omega_{min}}^{\omega_{max}} (I\omega) d\omega$$

- The only way we can extract energy from the system is to slow it down (recall $E = \frac{1}{2} I\omega^2$)
- Exactly constant shaft angular velocity is difficult to achieve: the best we can do is to minimize the speed variation $(\omega_{\max} - \omega_{\min})$ by increasing I with a flywheel

Sizing a Flywheel

Note: E = area under torque curve from $\theta @ \omega_{max}$ to $\theta @ \omega_{min}$

- Factor the expression for energy change:

$$E = \frac{1}{2} I\left(\omega_{max}^2 - \omega_{min}^2\right) = \frac{1}{2} I(\omega_{max} + \omega_{min})(\omega_{max} - \omega_{min})$$

- If the torque-time relationship is harmonic (or nearly so), then the average angular velocity is

$$\omega_{avg} = \frac{1}{2} \cancel{I}(\omega_{max} + \omega_{min})$$

- So the energy change is

$$E = I\omega_{avg}(\omega_{max} - \omega_{min})$$

Sizing a Flywheel

- Define the **coefficient of fluctuation** as

$$k = \frac{\omega_{max} - \omega_{min}}{\omega_{avg}}$$

  - This is a measure of how far away the extreme values are from the average
  - Design parameter: usually chosen around *k* = 1 to 5 (for 1-5%)
- The change in energy is then $E = kI\omega_{avg}^2$
- So the necessary moment of inertia ***of the new system*** is

$$I = \frac{E}{k\omega_{avg}^2}$$

Sizing a Flywheel

- The total moment of inertia is composed of the shaft, the motor rotor, the crank (only), and the flywheel – do not forget to account for the other three contributions

$$I = I_{orig\ sys} + I_{flywheel} = (I_s + I_m + I_c) + I_{flywheel}$$

- Maximizing $I_{flywheel}$ while minimizing material (keeping the mass low) has the majority of the mass at large radii
  - Large and thin better than small and thick
  - Use dense metals in the rim (iron and steel)