Module 5

Lecture 15

Gears – Part 2

ME 370 - Mechanical Design 1

“Colibri” by Derek Hugger

** www.youtube.com/watch?v=1scj5sotD-E*

Lecture 15: Gears 2

Topics: 10/15/25 Gears – Part 2 (Norton Chap 9)

Activities & Upcoming Deadlines

- **HW:**
    - **HW 7 (IC #1):** to be posted soon, due Tuesday 10/21

- **Lab 8: PVA analysis** – Meet in 1001 MEL
    - This is an **Individual Student** Lab.
    - Pre-lab: READ all lab materials prior to lab time, submit pre-lab assignment
    - Post-lab due date: delayed 1 week to Week 10 (Week of 10/29)

- **Project 2:**
    - P2D1 (Conceptual Design Review) – Meet in 1001 MEL
        - Grading Rubric & Presentation template (and design resources) have been posted on Canvas
        - Propose two possible designs for the legged dispensing robot (include positioning of the motor and battery holder). Also include theme images if possible
        - Extra credit: High quality functional prototype of each leg mechanism (+0.5 pt each)

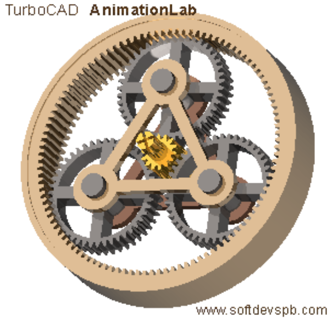
Next lecture: Module 6: Motors (Chapter 2.19), Cams and Motion Control (Chapter 8). Lectures 16-19

Recall: Angular Velocity Ratio (m_V) & Torque Ratio (m_T)

$$\boxed{m_V = \frac{\omega_{out}}{\omega_{in}}}$$

$$\boxed{m_V = \frac{\omega_{out}}{\omega_{in}} = \pm\frac{r_{in}}{r_{out}} = \pm\frac{d_{in}}{d_{out}} = \pm\frac{N_{in}}{N_{out}}}$$

Negative sign if external gearset
Positive if internal gearset
Gear ratio is usually the same as the angular velocity ratio



Internal gearset

External gearset

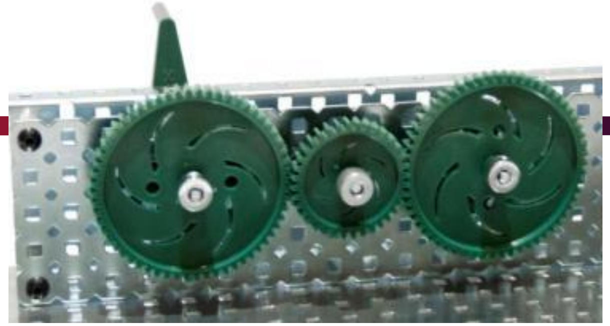
$$\boxed{m_T \equiv \frac{T_{out}}{T_{in}}}$$

$$\boxed{\therefore m_T = \frac{T_{out}}{T_{in}} = \frac{\omega_{in}}{\omega_{out}} = \frac{1}{m_V}}$$

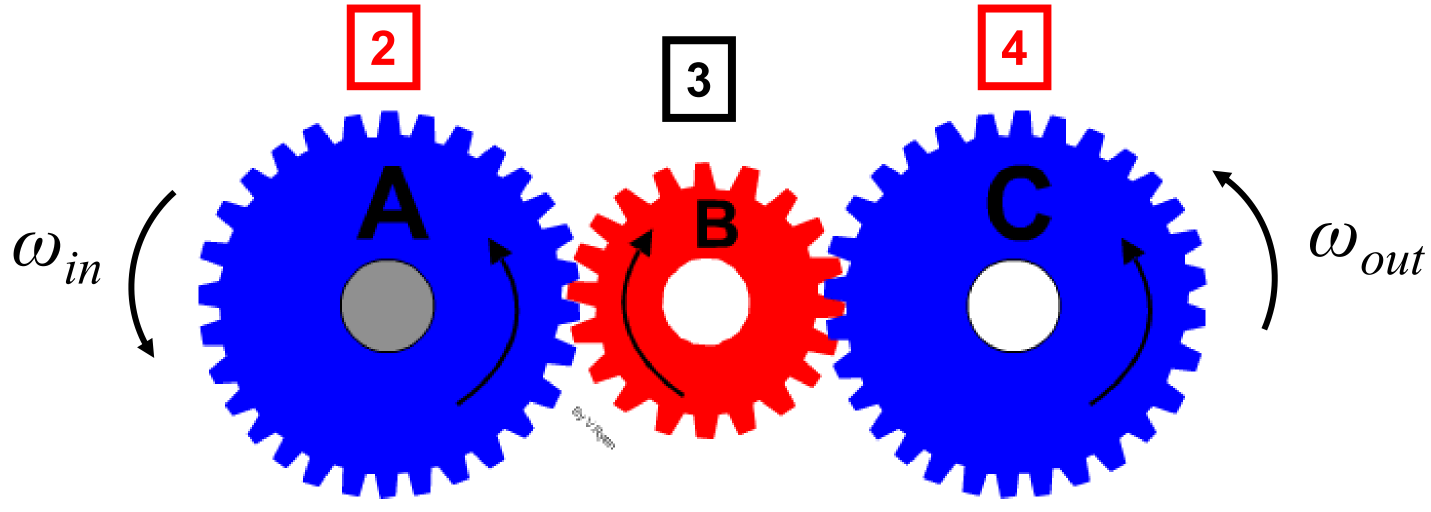
If assume 100% efficiency

Recall: Simple gear train

All gears on different shafts



https://quizlet.com/255781174/simple-gear-train-with-idler-diagram/



https://technologystudent.com/gears1/gears2.htm

$$m_V = \left(-\frac{N_2}{N_3}\right)\left(-\frac{N_3}{N_4}\right)$$

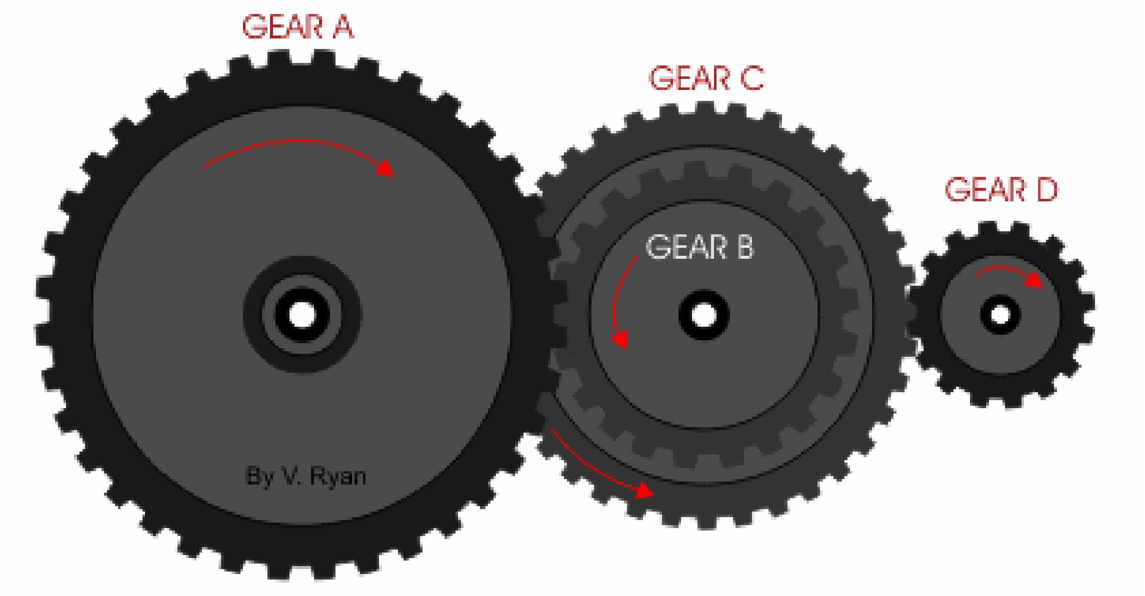
$$\boxed{m_v = \frac{N_2}{N_4} = \frac{N_{in}}{N_{out}}}$$

Note: individual gearsets are negative since external gearsets

- Gear 3 is called an idler gear.
  - Has no contribution to gear ratio.
  - Only changes direction of input to output.
  - Odd # gears: input & output same direction
  - Even # gears: input & output opposite direction
- Typically do not use more than one idler gear. If want to cover longer distances, then use chain or belt.

ILLINOIS

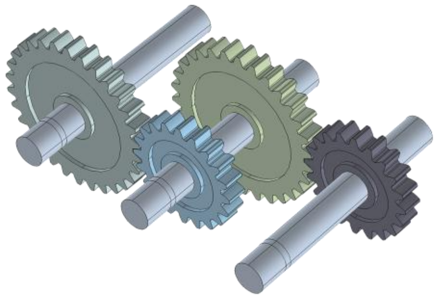
Recall: Compound gear trains



http://www.technologystudent.com/gears1/gears8.htm

A & C are drivers (in)
B & D are driven (out)

Some gears share same shaft



https://images.app.goo.gl/PwfqXyj7cR33pdzF8

$$m_V = \left(-\frac{N_A}{N_B}\right)\left(-\frac{N_C}{N_D}\right) = \frac{N_A N_C}{N_B N_D}$$

$$\boxed{m_V = \pm\frac{\prod(N_{\text{driver gears}})}{\prod(N_{\text{driven gears}})}}$$

Exercise: Salad spinner gears

$$m_v = \frac{N_2}{N_4} = \frac{N_{in}}{N_{out}}$$

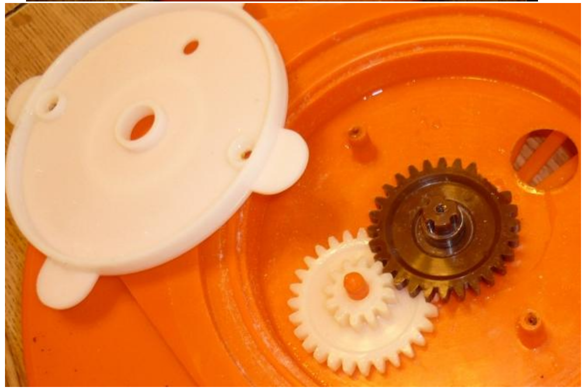
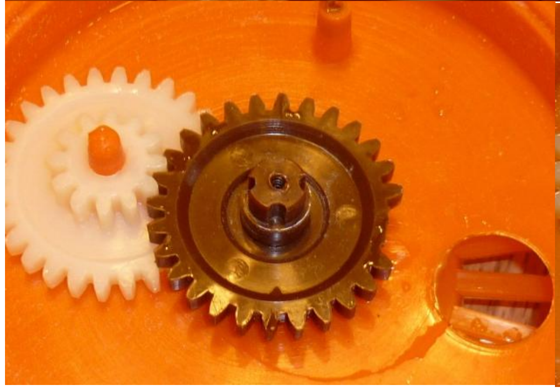
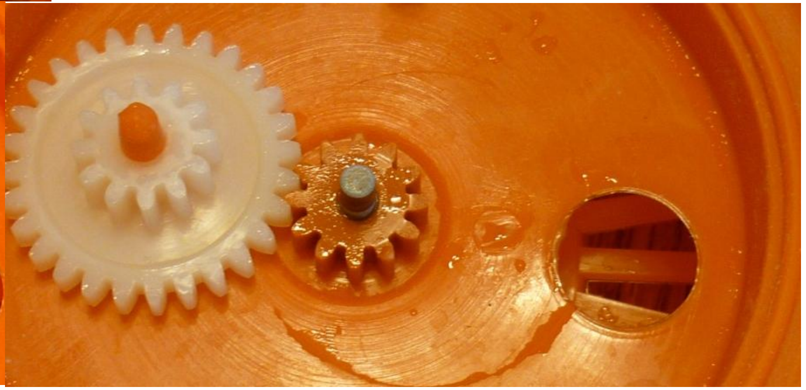

$$m_V = \pm \frac{\prod(\text{\# teeth on driver gears})}{\prod(\text{\# teeth on driven gears})}$$

Join Code: **370**



What is the gear ratio?

Brown 28 (driver)
White 12 (driven)
White 28 (driver)
Orange 12 (driven)

A: 28/12
B: (28+28)/(12+12)
C: (28*28)/(12*12)
D: (12*12)/(28*28)

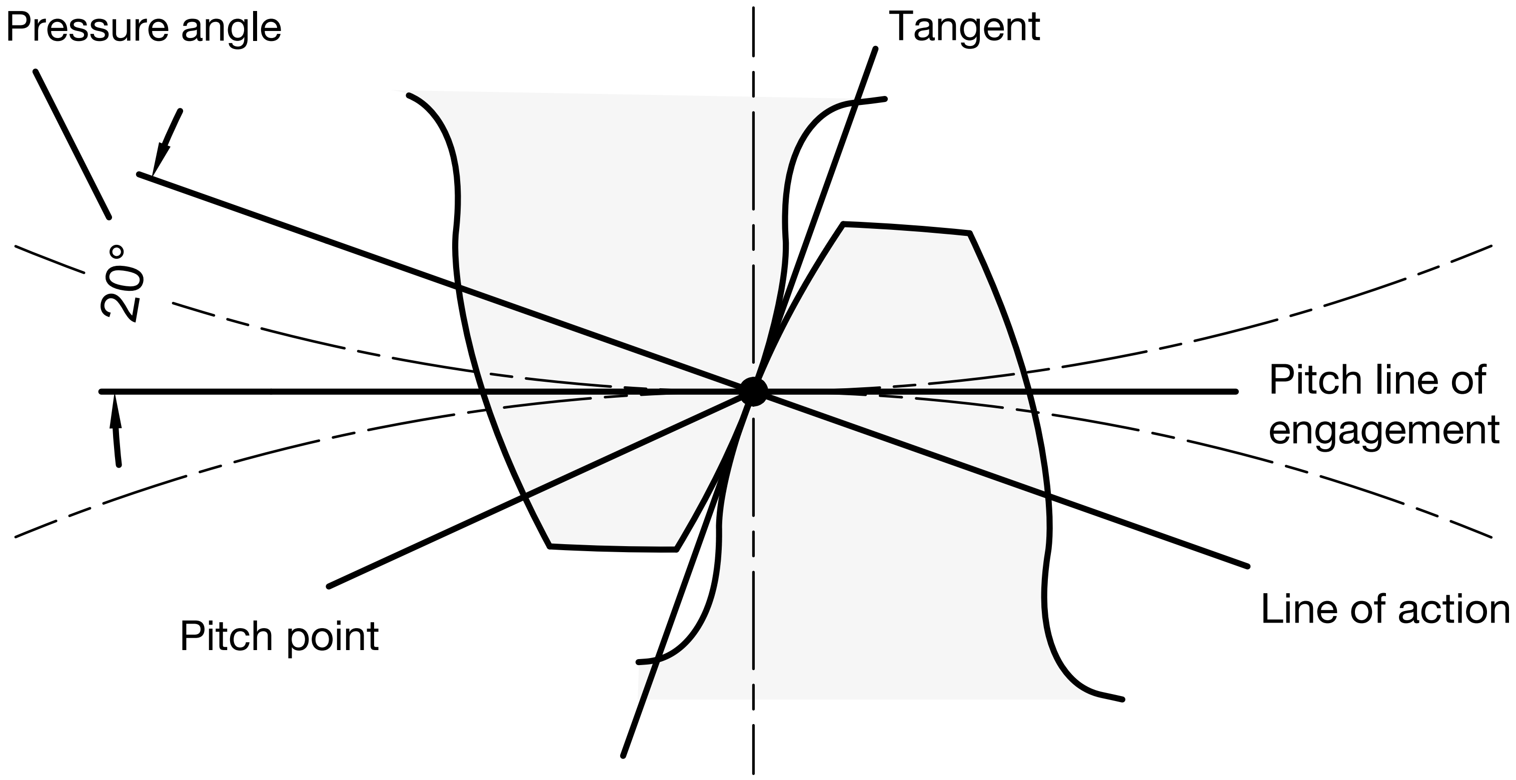
ILLINOIS

Gear "pitch" p_d and p_c

- For proper meshing, all gears in a gear train must have the same "pitch"
- Three ways of defining pitch *(D is gear pitch diameter):*

Diametral Pitch	Circular Pitch	Module (metric)
$p_d = \frac{N}{D}$	$p_c = \frac{\pi D}{N}$	$m = \frac{D}{N} = \pi\, p_c$
Number of *teeth per inch* along its pitch diameter.	Approximate distance between teeth measured along the pitch circle.	Number of π millimeters between teeth. i.e. : A mod 1 gear has a tooth spacing of π mm and A mod 2 gear has a tooth spacing of 2π mm

Gear pressure angle



- Pressure angle is the angle between the line of action and the pitch line of engagement
- ϕ = 14.5° (legacy) 20° (modern)
- Larger angle: additional pressure on bearing, but stronger gears

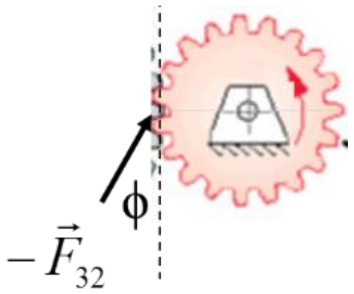
Selection of commercially available gears

- For a proper mesh, following **must** match between gears:
  1. **Diametral pitch** ("pitch"): # of teeth per inch of pitch diameter

$$P = \frac{N_2}{d_2} = \frac{N_3}{d_3}$$

  2. **Pressure angle**

     $\phi = 20^{\circ}$ most common



Another useful web site:

http://www.rushgears.com/Tech_Tools/PartSearch8/partSearch.php?gearType=SPUR

Example of catalog of spur gears

http://www.grainger.com/Grainger/wwg/search.shtml?searchQuery=spur+gear&op=search&Ntt=spur+gear&N=0&sst=subset

Gear,Spur,10 Pitch
Item # 1L974
Spur Gear, 10 Pitch, Pitch Dia 1.400 In, Face Width 1.000 In, Number of Teeth 14, Bore Dia 0.625 In, Outside Dia 1.600 In, Overall Length 1.630 In

Gear Ratio

- **Gearset:** combination of 2 gears
- Gear ratio is usually the same as the angular velocity ratio:

$$\therefore m_V = \pm\frac{\omega_{out}}{\omega_{in}} = \pm\frac{r_{in}}{r_{out}} = \pm\frac{d_{in}}{d_{out}} = \pm\frac{N_{in}}{N_{out}}$$

- Gear ratio is sometimes reported as a relationship between the output gear and the input gear:

Gear ratio = output : input

- To create large gear ratios, gears are often connected together in **gear trains (i.e., simple and compound gear trains)**

Gear ratio for worm gears

- Depends on:
  - Worm gear: Number of teeth
  - Worm: number of thread or “starts” (e.g., single, double, triple start).

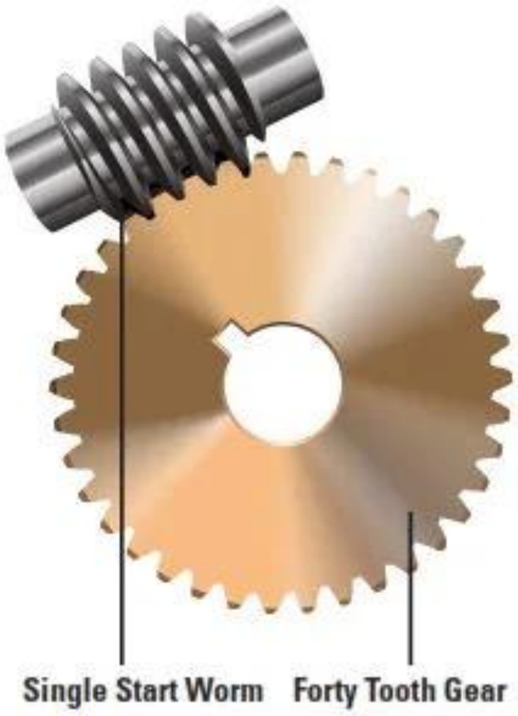


Figure 1 – Worm gear assembly

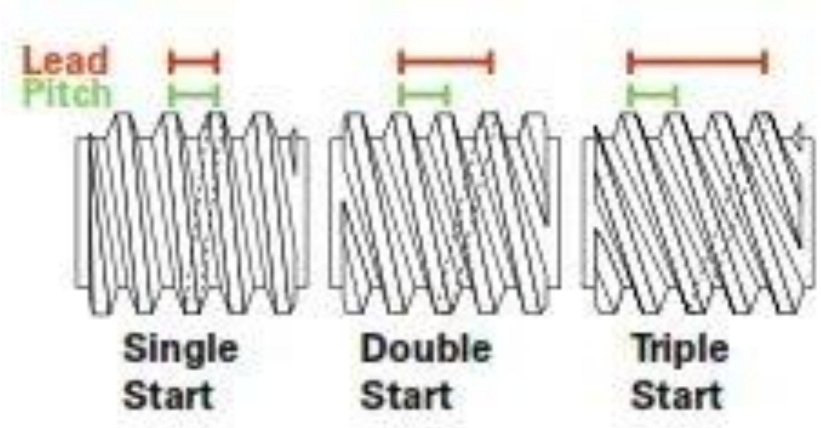


Figure 2 – Multiple starts on worm gears

Example:
40 tooth worm gear, respective gear ratios would be 40:1, 20:1, 13.333:1

https://www.bodine-electric.com/blog/introduction-to-worm-gearmotors/

How to design a good gear

https://cloudfront-us-east-1.images.arcpublishing.com/coxohio/BEPPRM2NN5DJJOXNTTWTNDGSA4.jpg

Fundamental Law of Gearing

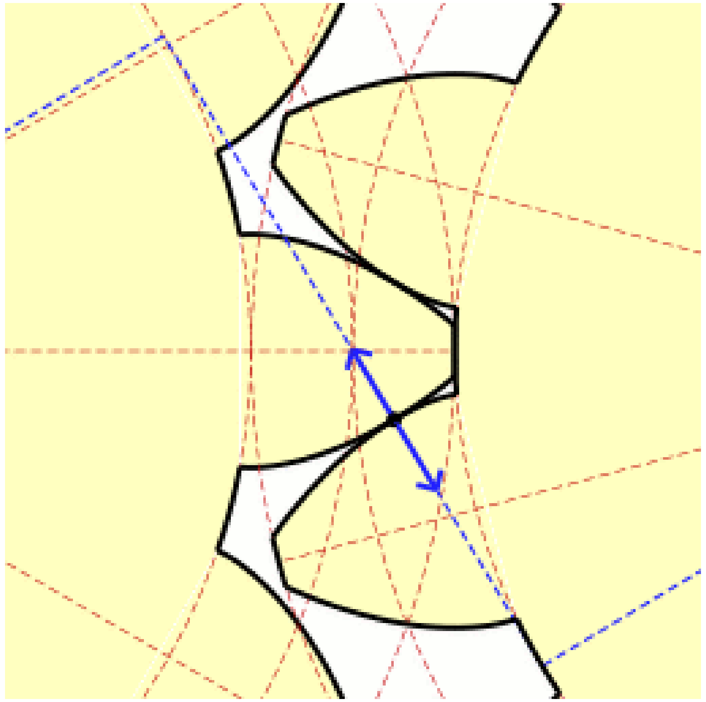
- ***Angular velocity ratio (m_V) between gears remains constant throughout the mesh.***

$$m_v = \frac{\omega_{out}}{\omega_{in}}$$

- **Geometric definition:**

  To maintain a constant velocity ratio between the gears, the common normal at the point of contact between meshing teeth must always intersect line of centers at fixed point, which is called the pitch point.

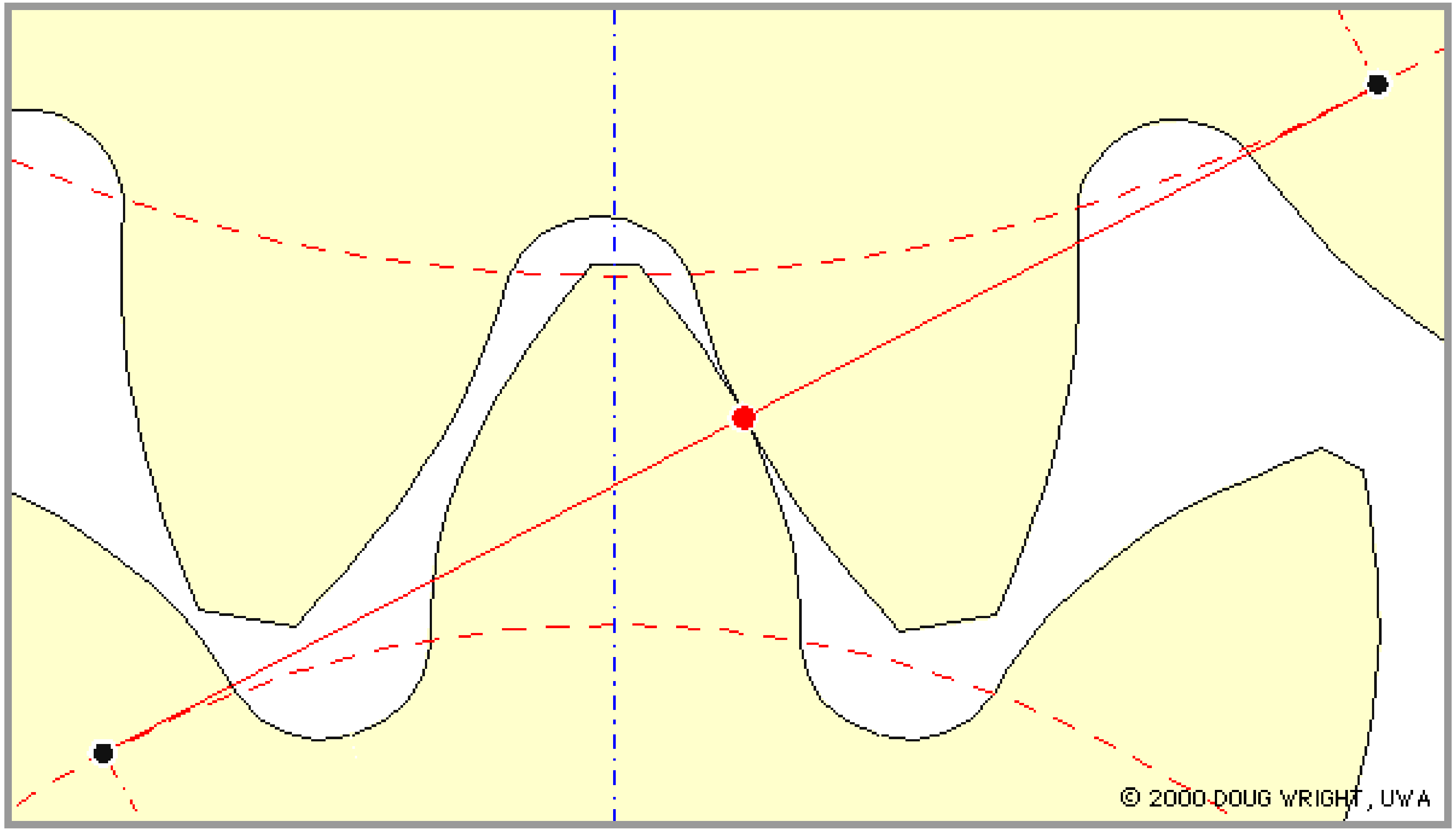
- The basis of gear and gear train design.
- Ensures smooth velocity transmission between gears.



http://upload.wikimedia.org/wikipedia/commons/c/c2/Involute_wheel.gif

ILLINOIS

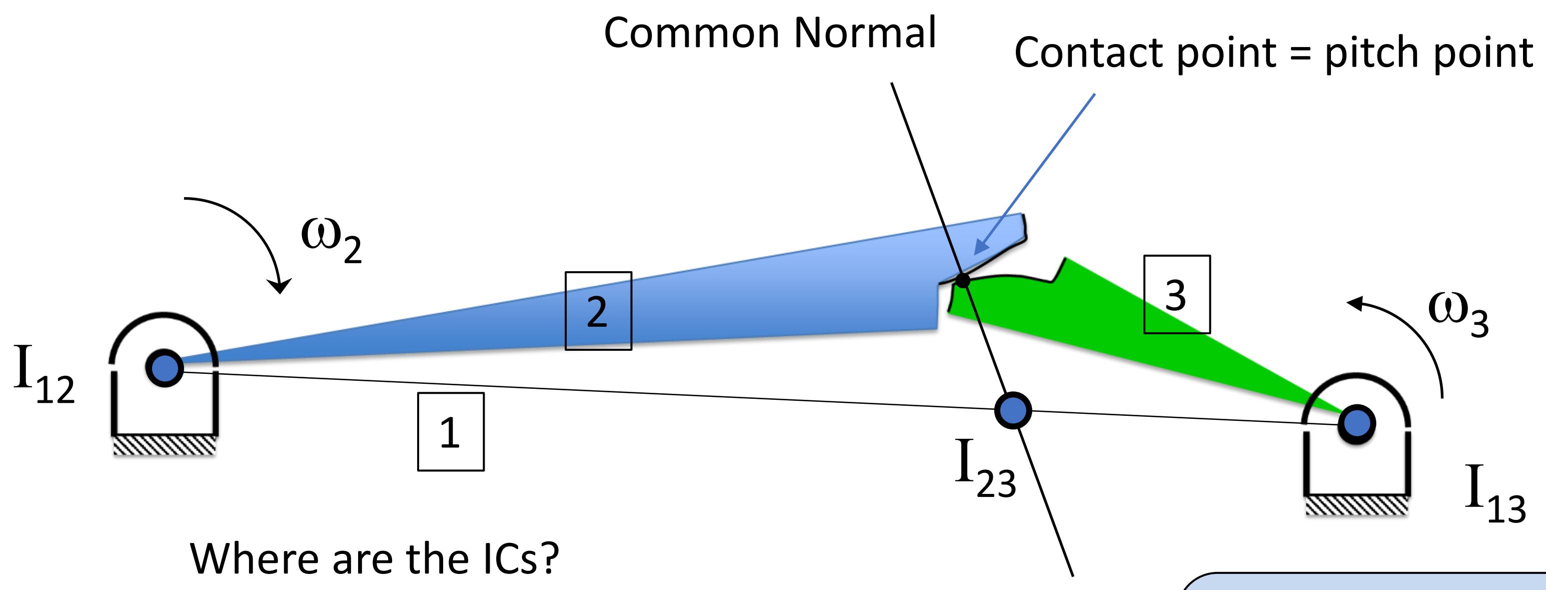
Fundamental Law of Gearing



Angular velocity ratio (m_V) between gears remains constant throughout the mesh.

http://www-mdp.eng.cam.ac.uk/web/library/enginfo/textbooks_dvd_only/DAN/gears/meshing/gearAnimation.gif

Law of Gearing: Recalling ICs



Where are the ICs?

Calculate $\omega_{3,}$ given ω_2

$$V_p = V_{in} = V_{out}$$

$$\overline{I_{12}I_{23}}\omega_2 = \overline{I_{13}I_{23}}\omega_3$$

$$\therefore \omega_3 = \frac{\overline{I_{12}I_{23}}}{\overline{I_{13}I_{23}}}\omega_2$$

Rule #5:
For 2 bodies in rolling contact,`
the IC lies on the common normal
following Kennedy's theorem

Under what conditions will ω_3 remain constant if ω_2 is constant?

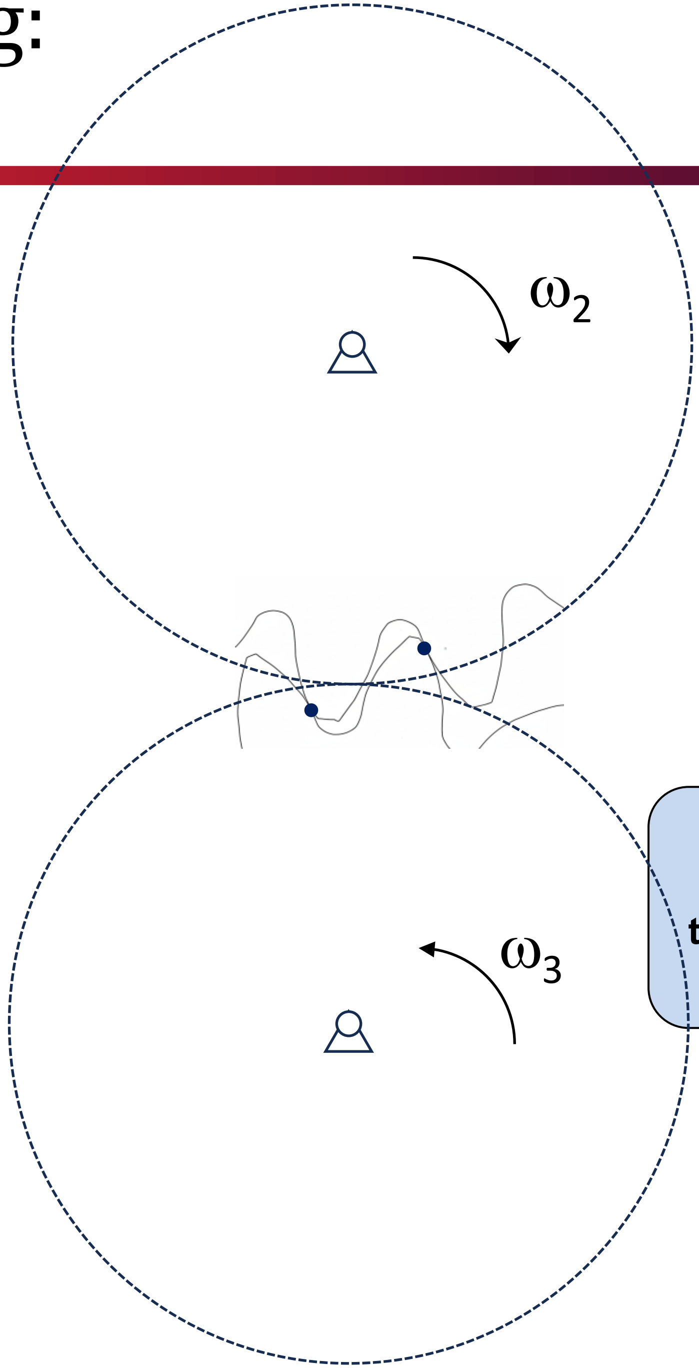
I_{23} does not move. For this to happen, the common normal to the contacting teeth must not move as the gears rotate
→ Fundamental Law of Gearing

In class exercise: Law of Gearing: Recalling ICs

Instructions:
- Identify the three bodies in the gear mechanism.
- Label the instant centers of velocity between the two gears and the ground.
- Draw the line of centers (**I_{12}–I_{13}**).
- Draw the common normal at the point (or points) of contact.
- Is the common normal constant?

- Find the instant center of velocity between the two gears (**I_{23}**) — *Hint: use Rules 2 and 5 of instant centers.*
- Does the location of **I_{23}** change over time?

- Using the location of the instant center and the velocity of the driver gear, determine the velocity of the driven gear.
- Does the velocity of the driven gear change over time?



Rule #5:
For 2 bodies in rolling contact the IC lies on the common normal following Kennedy's theorem

ILLINOIS

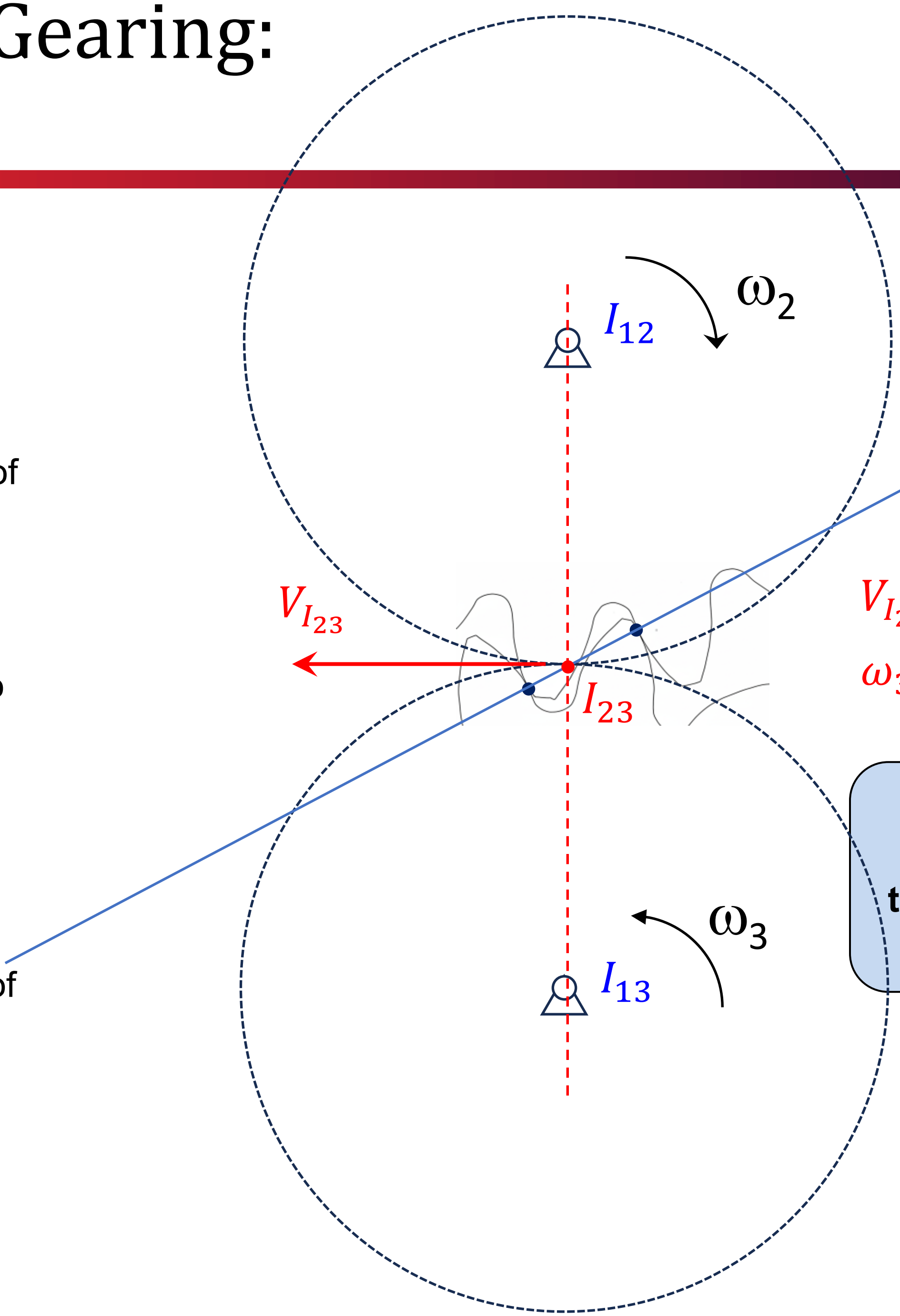
In class exercise: Law of Gearing: Recalling ICs

Instructions:

- Identify the three bodies in the gear mechanism.
- Label the instant centers of velocity between the two gears and the ground.
- Draw the line of centers ($\mathbf{I_{12}}$–$\mathbf{I_{13}}$).
- Draw the common normal at the point (or points) of contact.
- Is the common normal constant?

- Find the instant center of velocity between the two gears ($\mathbf{I_{23}}$) — *Hint: use Rules 2 and 5 of instant centers.*
- Does the location of $\mathbf{I_{23}}$ change over time?

- Using the location of the instant center and the velocity of the driver gear, determine the velocity of the driven gear.
- Does the velocity of the driven gear change over time?



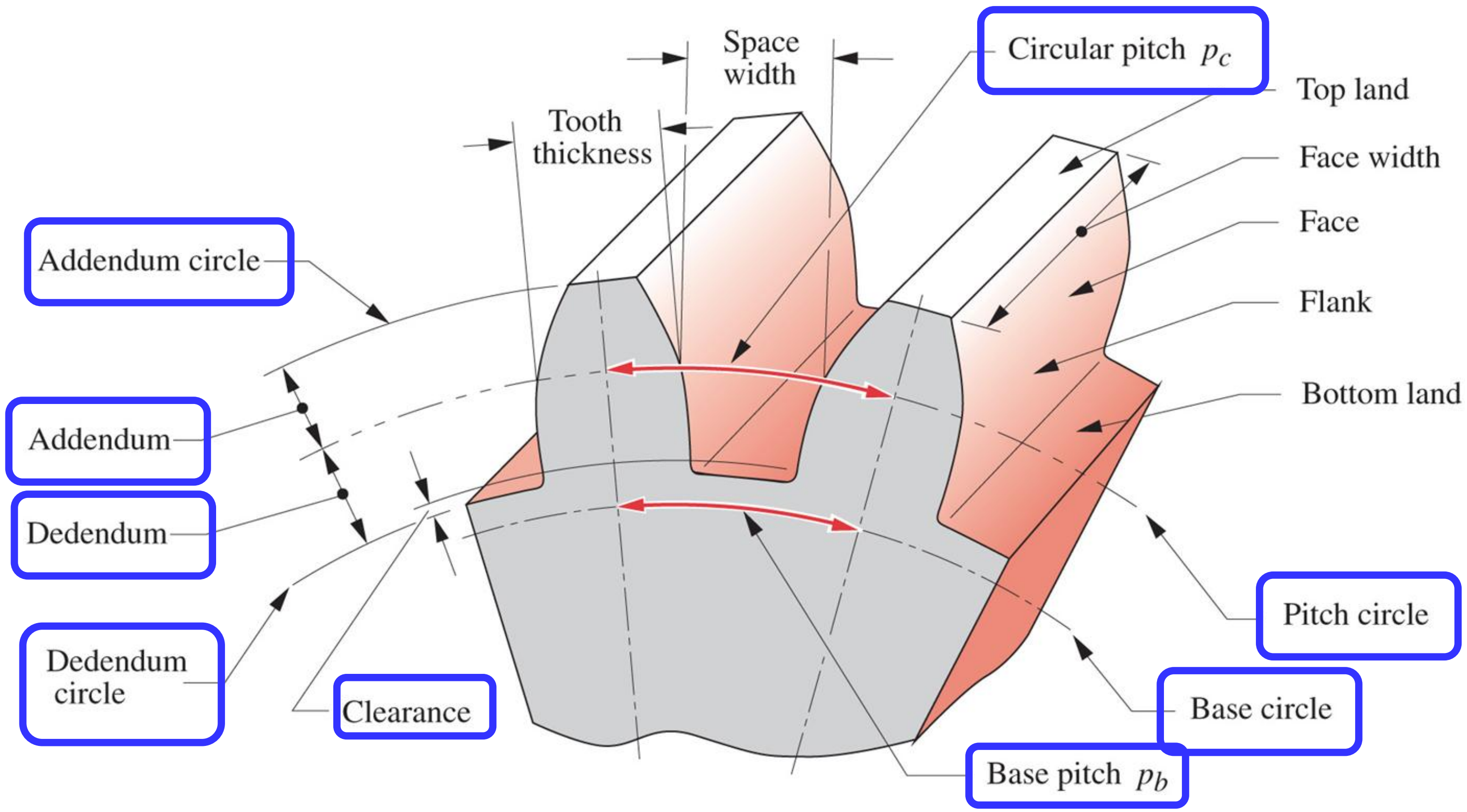
$$V_{I_{23}} = \omega_2|I_{12}I_{23}| = \omega_3|I_{13}I_{23}|$$

$$\omega_3 = \frac{\omega_2|I_{12}I_{23}|}{|I_{13}I_{23}|}$$

Rule #5:
For 2 bodies in rolling contact the IC lies on the common normal following Kennedy's theorem

Involute Tooth Shape

- Most common shape, adds additional features to enable meshing
- Robust to centerline distance inaccuracy



$$p_c = \frac{\pi d}{N}$$

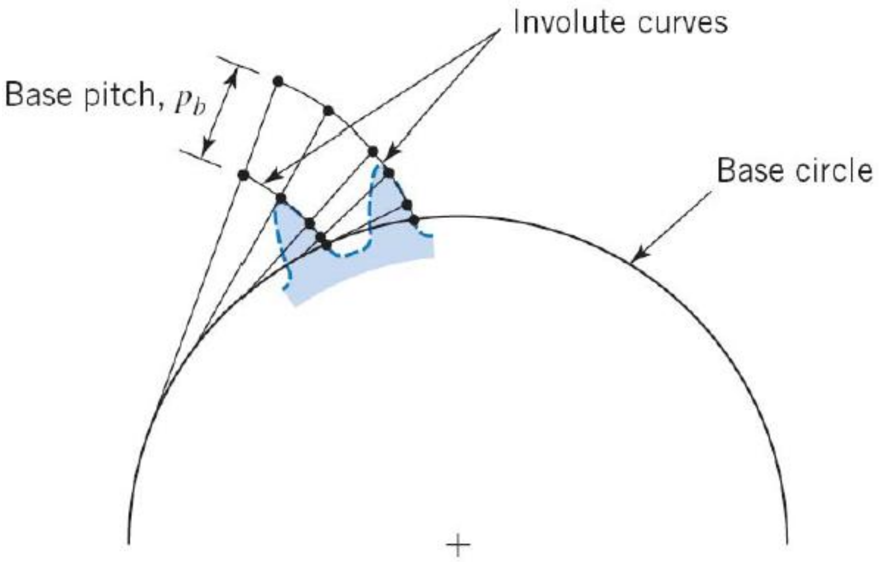
Where d=pitch diameter

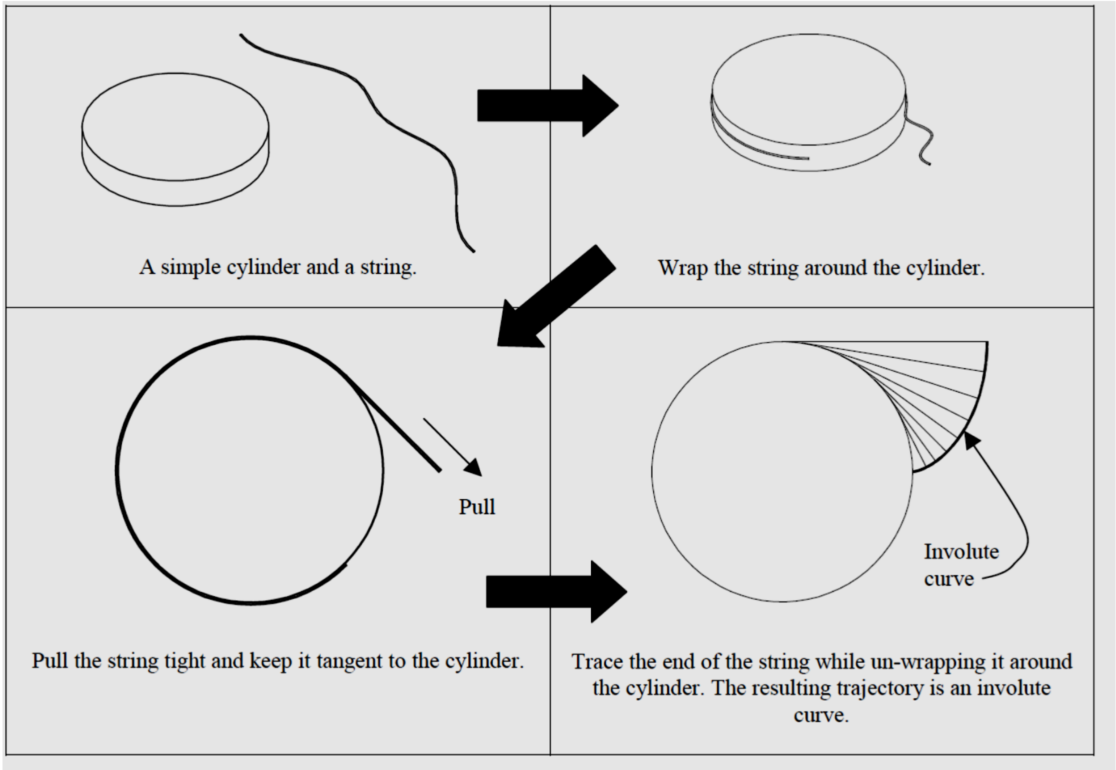
N=# teeth

Involute Tooth Shape

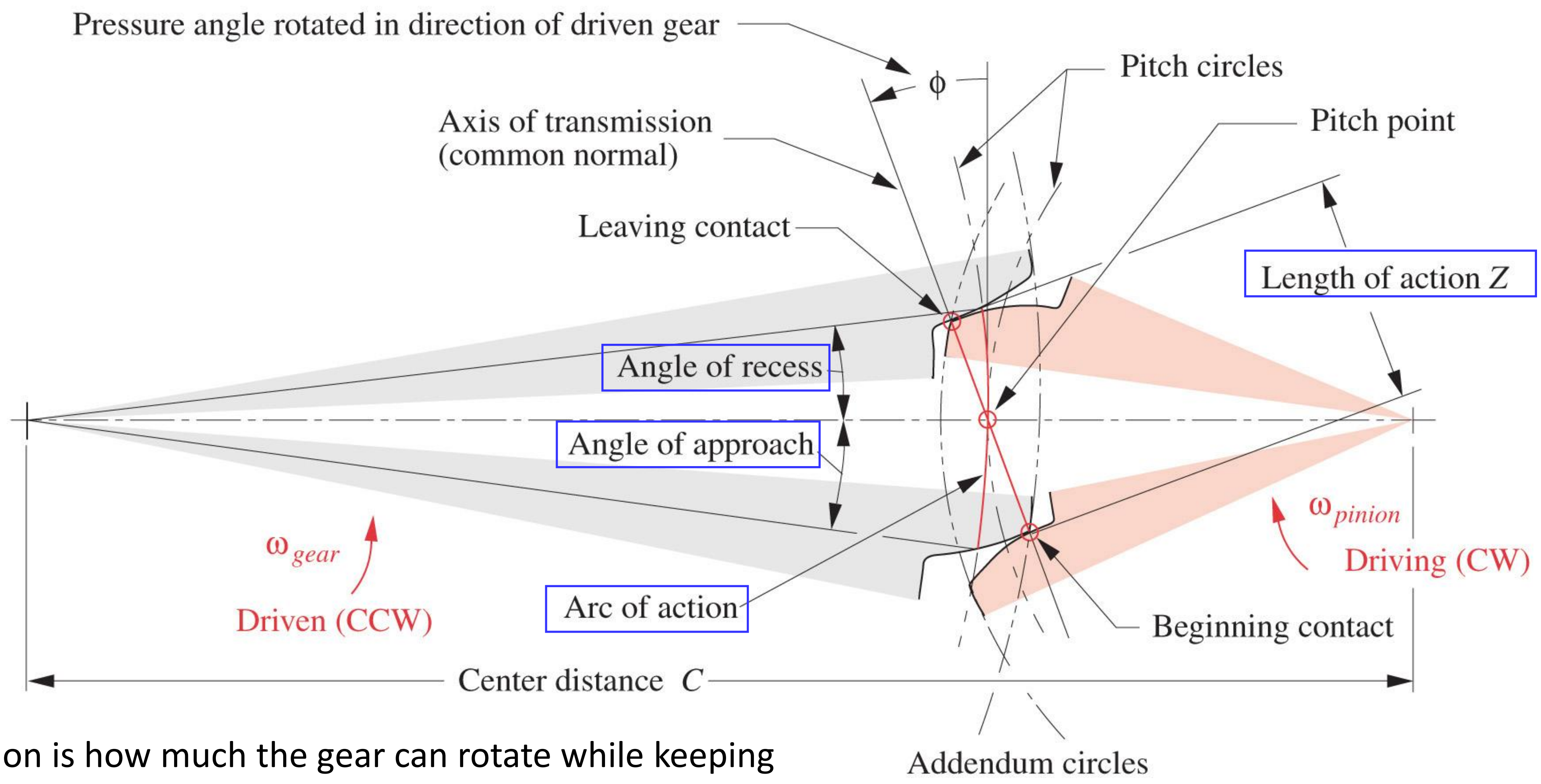
Most common geometry for gears

- Satisfies Fundamental Law of Gearing.
- Velocity ratio does not change in case of a center-to-center distance inaccuracy.
- Curve generated by any point on a taut thread as it unwinds from a circle



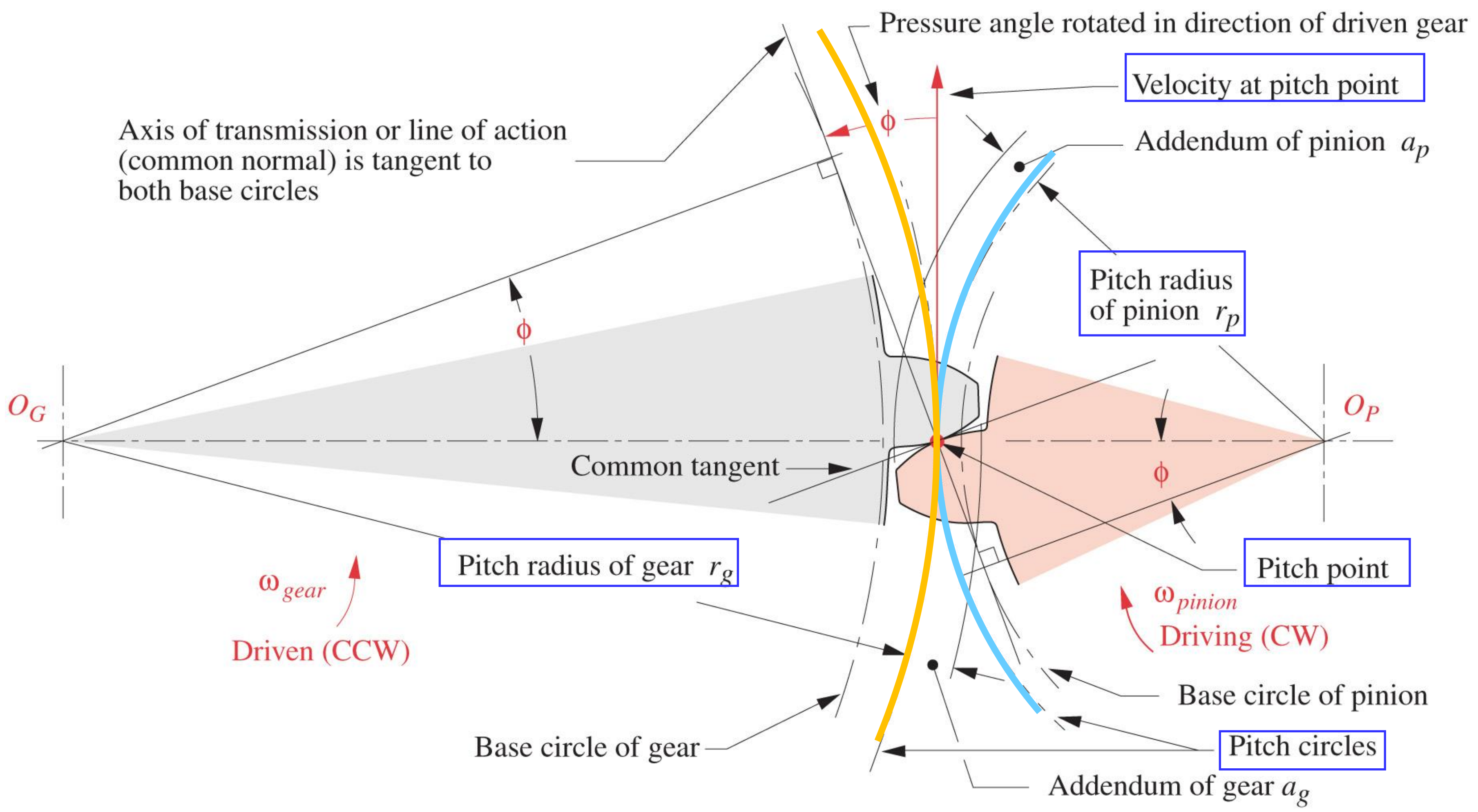


Key features of involute toothed gears – arc of action



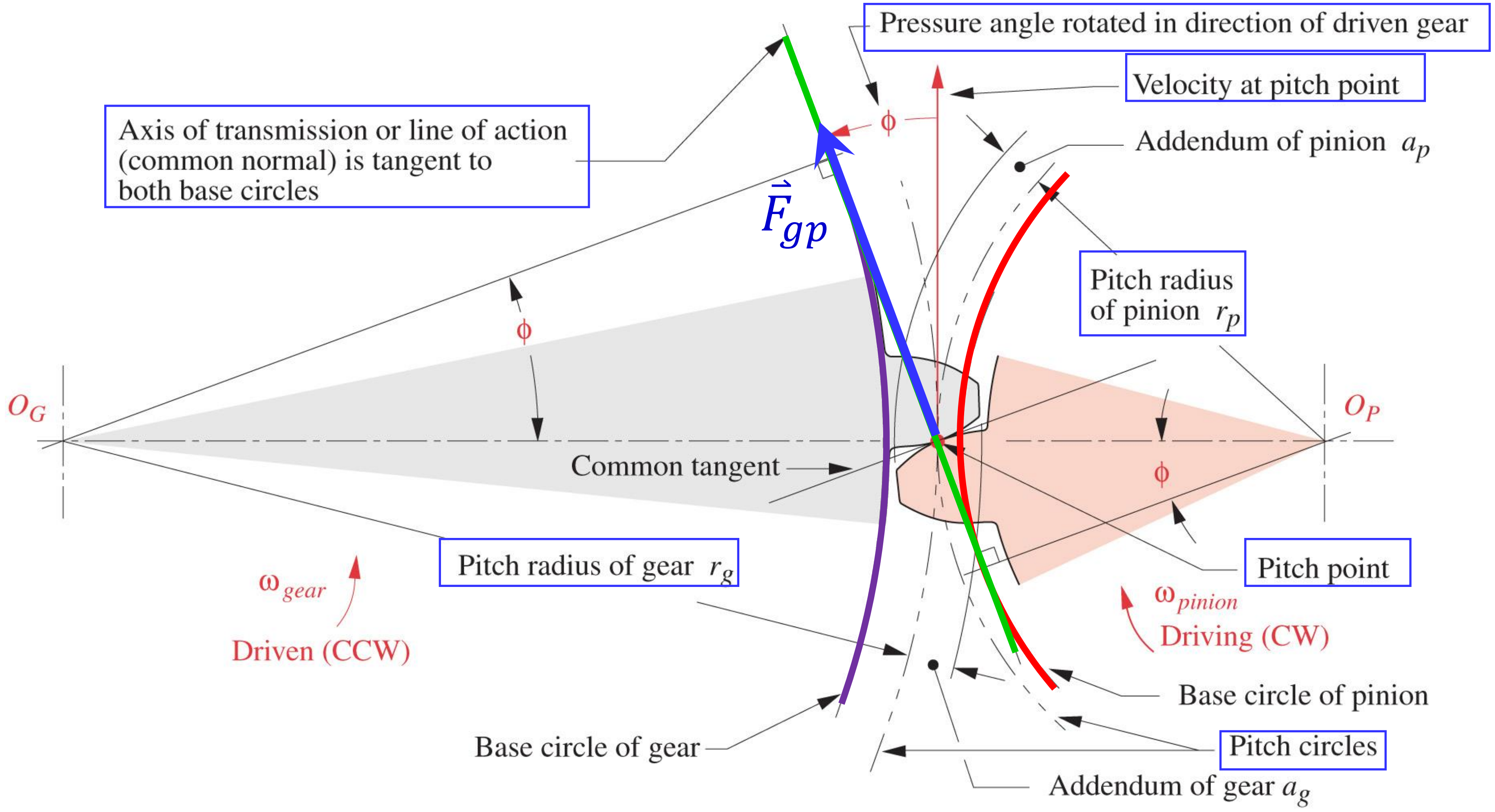
Arc of action is how much the gear can rotate while keeping two specific teeth in contact

Key features of involute toothed gears - velocity

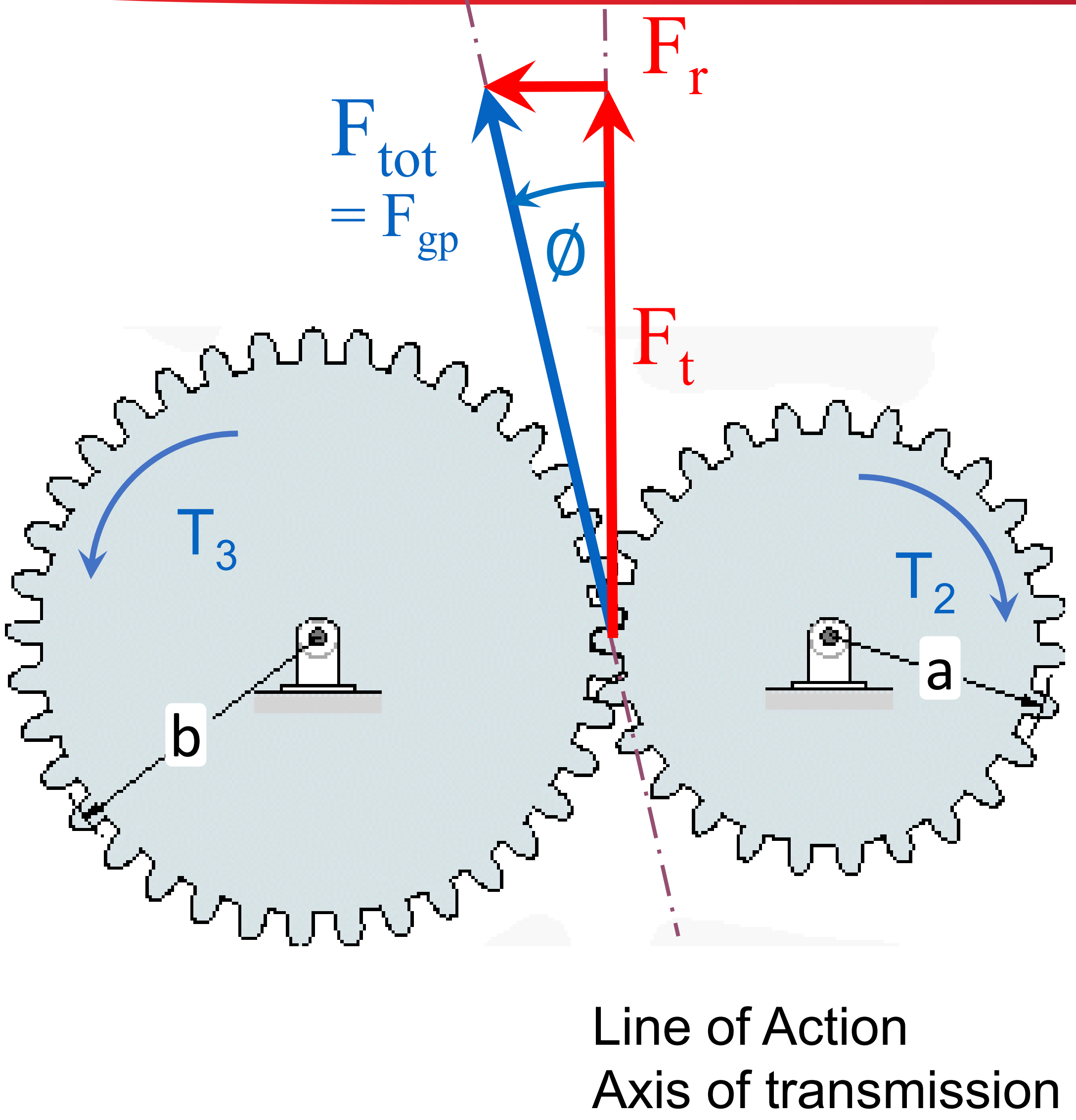


Key features of involute toothed gears - forces

$\phi = 20^{\circ}$ most common



Transverse versus radial forces in gears



$\emptyset$ Pressure angle of the gear, e.g., 20°

F_t Tangential (Transmitted) force

F_r Radial force

Power is transmitted by the tangential force

$$T_2 = F_t a$$
$$T_3 = F_t b$$

No power is transmitted by the radial force

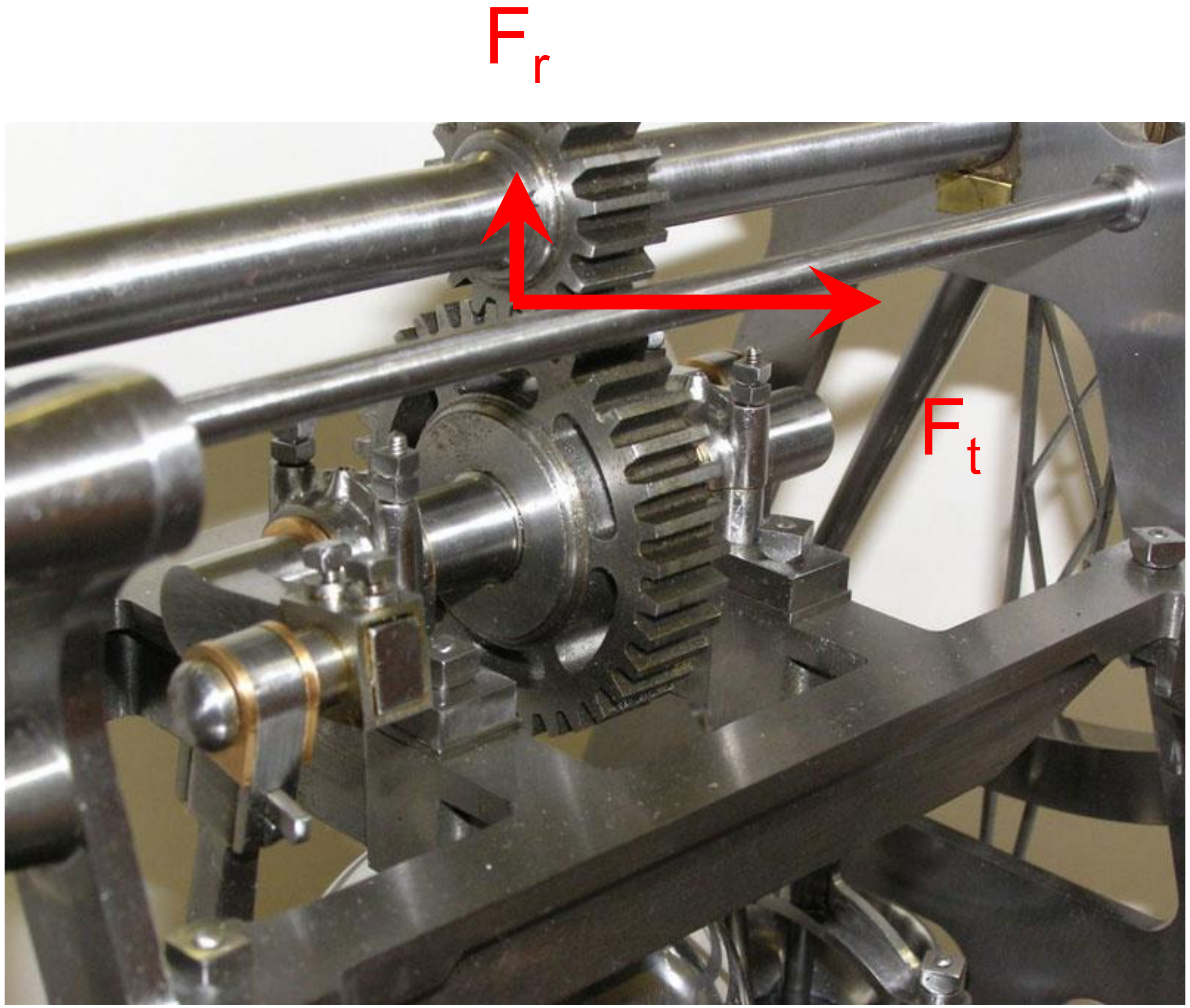
$$F_r = F_{tot} \sin \emptyset = F_t \tan \emptyset$$

Radial forces push gears apart!

$F_r \sim 1/3\, F_t$

ILLINOIS

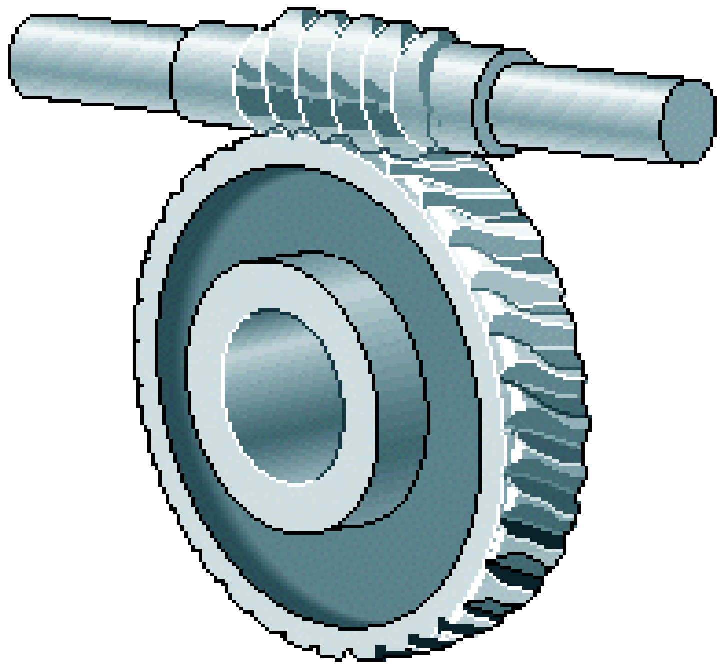
Poor design in gear systems:

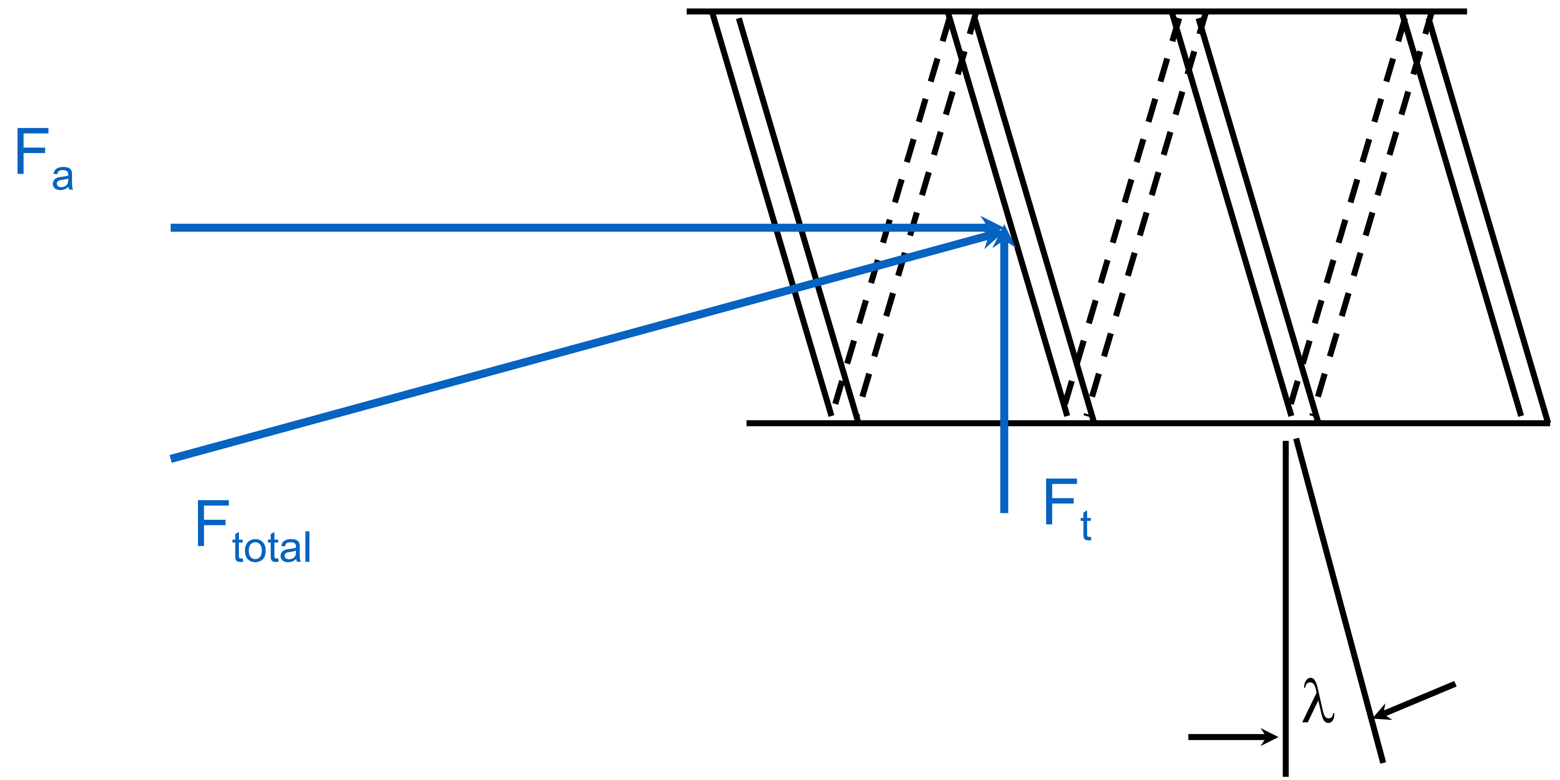


Deflection of a beam under perpendicular force

$$\Delta x \propto \frac{L^3}{EI} F$$

Worm Gear Forces





$\lambda \sim 10^o$

$F_t = F_{total} \cos \phi_n \sin \lambda$

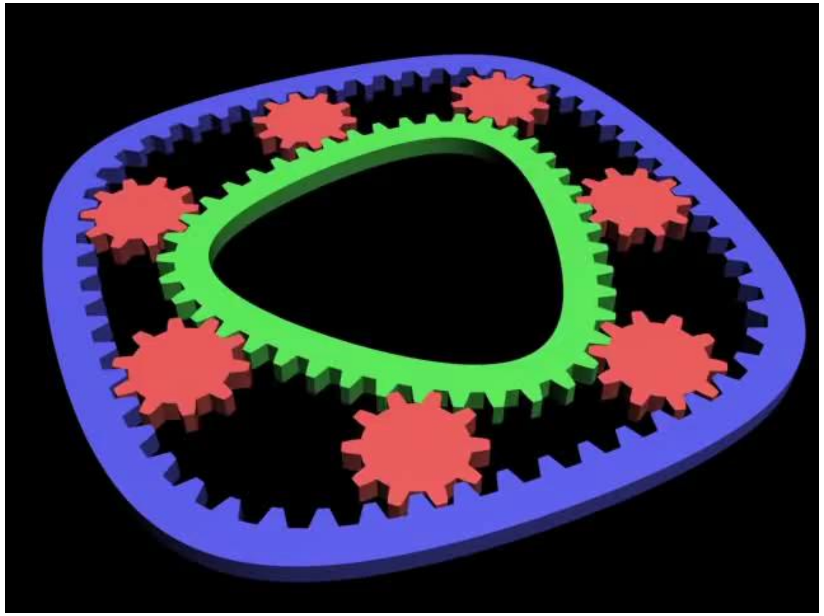
$F_a \sim 5\, F_t$

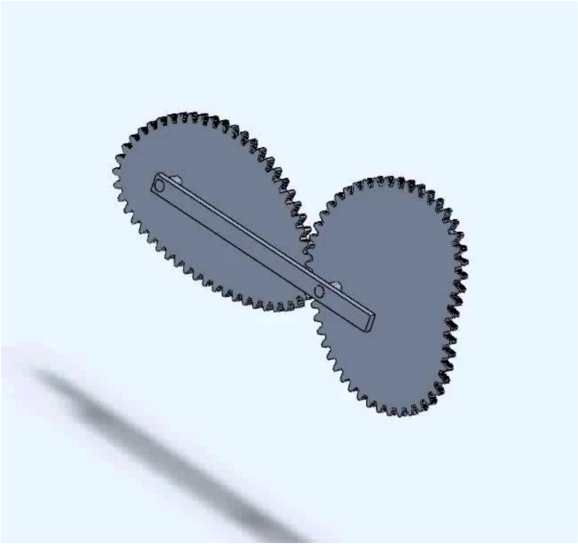
Axial force on worm is large so cannot ignore friction forces !

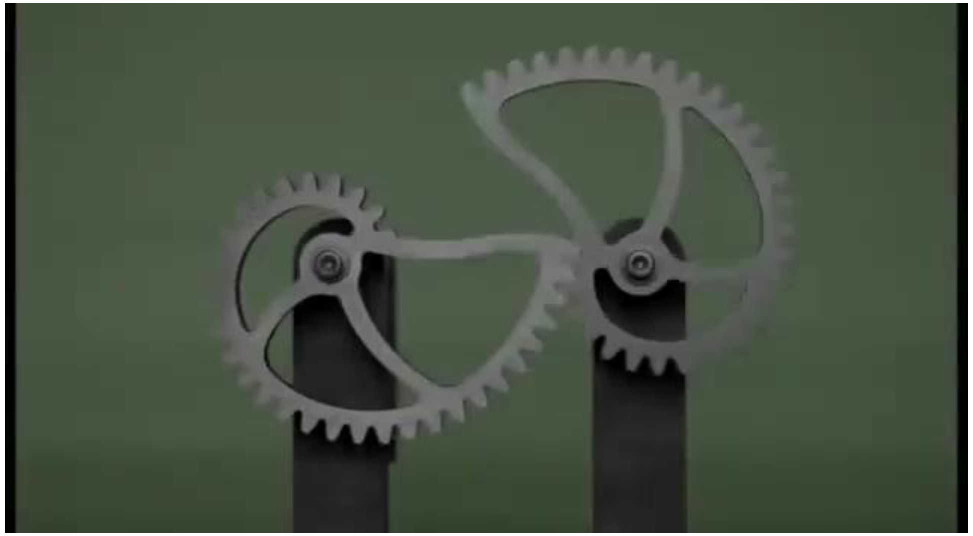
So how do I design a gear?

- Drawing involute curves and designing gears in a CAD program is doable, but difficult. Not recommended
  - Creo: https://www.youtube.com/watch?v=QjkiC1OJIdU
- The good news is there are many CAD programs or online resources with gear generators where you can just define parameters, and the gears are automatically generated. You can export the SVG files to other CAD programs as needed
  - Fusion 360: https://www.youtube.com/watch?v=MtK6yK0NRM0
  - Excellent online program: https://geargenerator.com
- Finally, another option is to download an existing gear CAD file (e.g. from McMaster), and modify it. This works well for more complex designs, e.g. bevel gears.

Non circular gears…



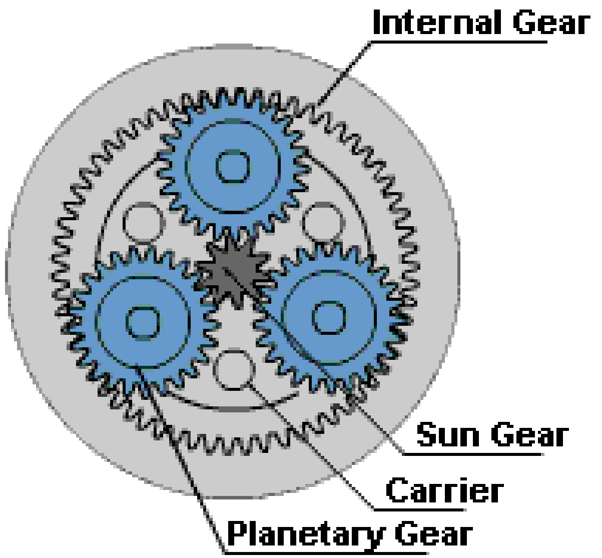




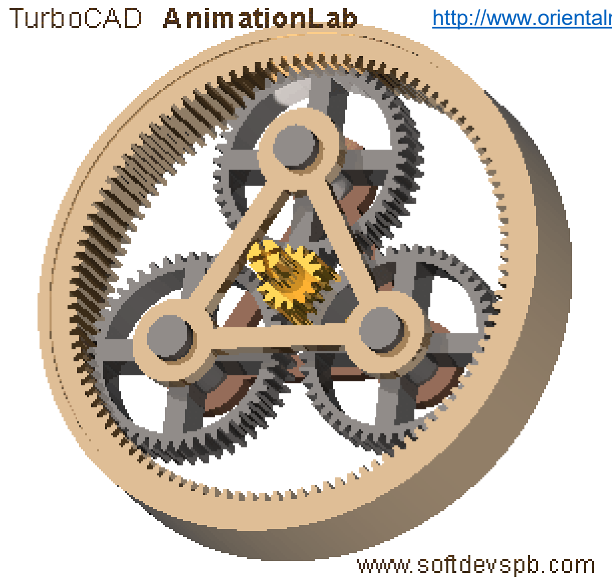
Planetary gear trains

Consist of a *sun gear* in the middle, a *ring gear* outside, and *planetary gears* in-between that are held by a *carrier or arm*.

1. 2-DOF system (simple gear train DOF is 1).
2. Two inputs and one output
3. Compact design to increase gear ratio.
4. Gear train can be rugged due to multiple planets.



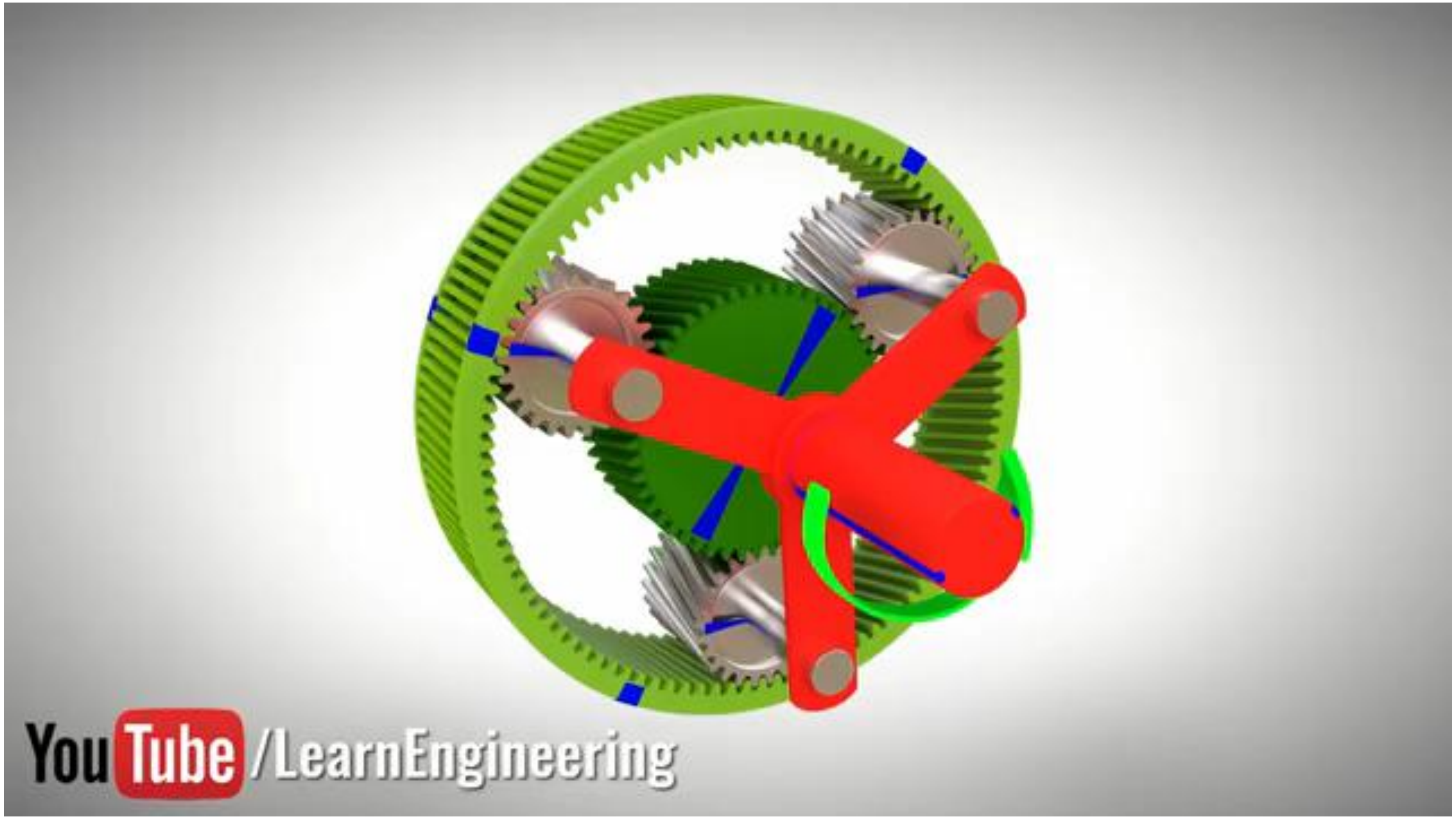
http://www.orientalmotor.com/newsletter/PN-Geared.htm



http://www.benchtophybrid.com/PG_Intro.html

Planetary gear motion

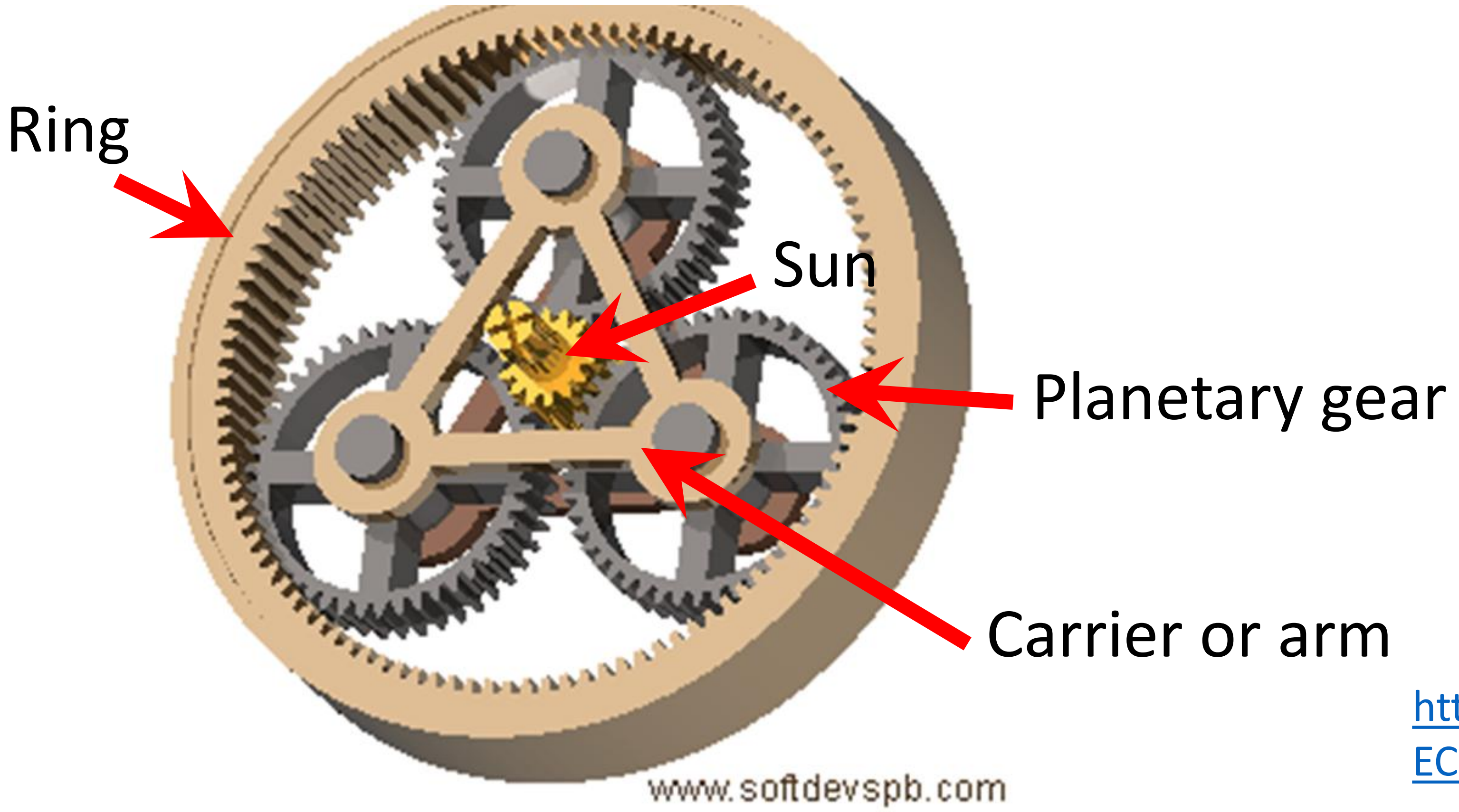
https://www.youtube.com/watch?v=ARd-Om2VyiE&t=4s



Additional resource on the use of planetary gears in automatic transmissions
https://www.youtube.com/watch?v=u_y1S8C0Hmc

Planetary Gear Trains

Four different types of motion based on which gears (sun, ring, planetary via carrier or arm) are held stationary.



https://youtu.be/ECljAo1q1RQ

Planetary Gear Trains

- The gear ratio depends on which gear is held fixed

	Input	**Output**	**Stationary**	**Calculation** $\omega_{in}/\omega_{out} = 1/m_v$
A	Sun (**S**)	Planet Carrier (**C**)	Ring (**r**)	$1 + \frac{N_r}{N_s}$
B	Planet Carrier (**C**)	Ring (**r**)	Sun (**S**)	$1/\left(1 + \frac{N_s}{N_r}\right)$
C	Ring (**r**)	Planet Carrier (**C**)	Sun (**S**)	$1 + \frac{N_s}{N_r}$
D	Sun (**S**)	Ring (**r**)	Planet Carrier (**C**)	$-\frac{N_r}{N_s}$

https://en.wikipedia.org/wiki/Epicyclic_gearing

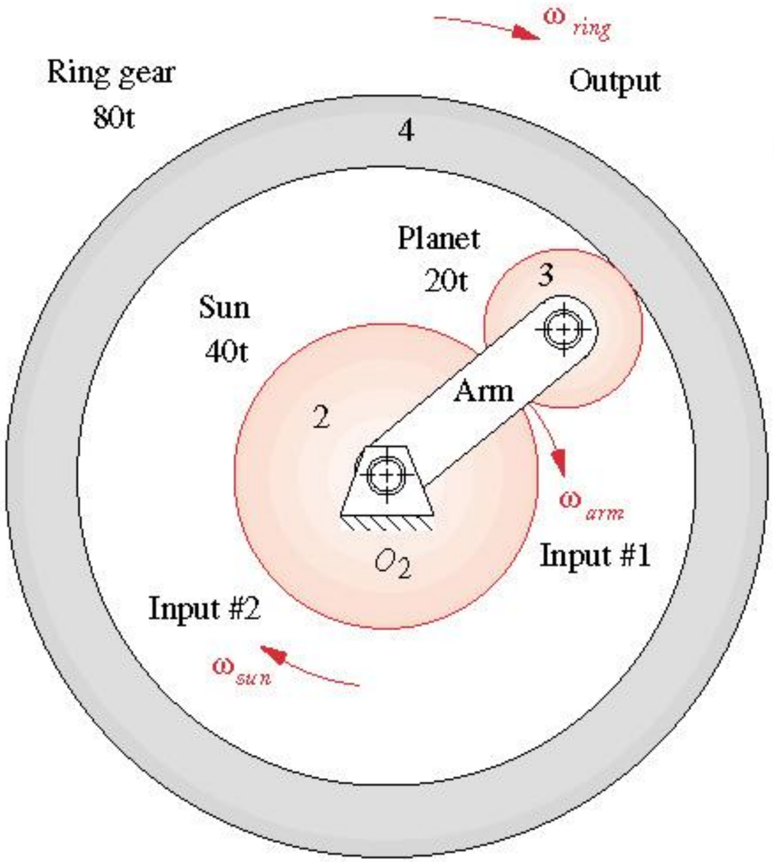
Fundamental planetary gear equation

$$\omega_r = -\frac{N_s}{N_r}\omega_s + \left(1 + \frac{N_s}{N_r}\right)\omega_c$$

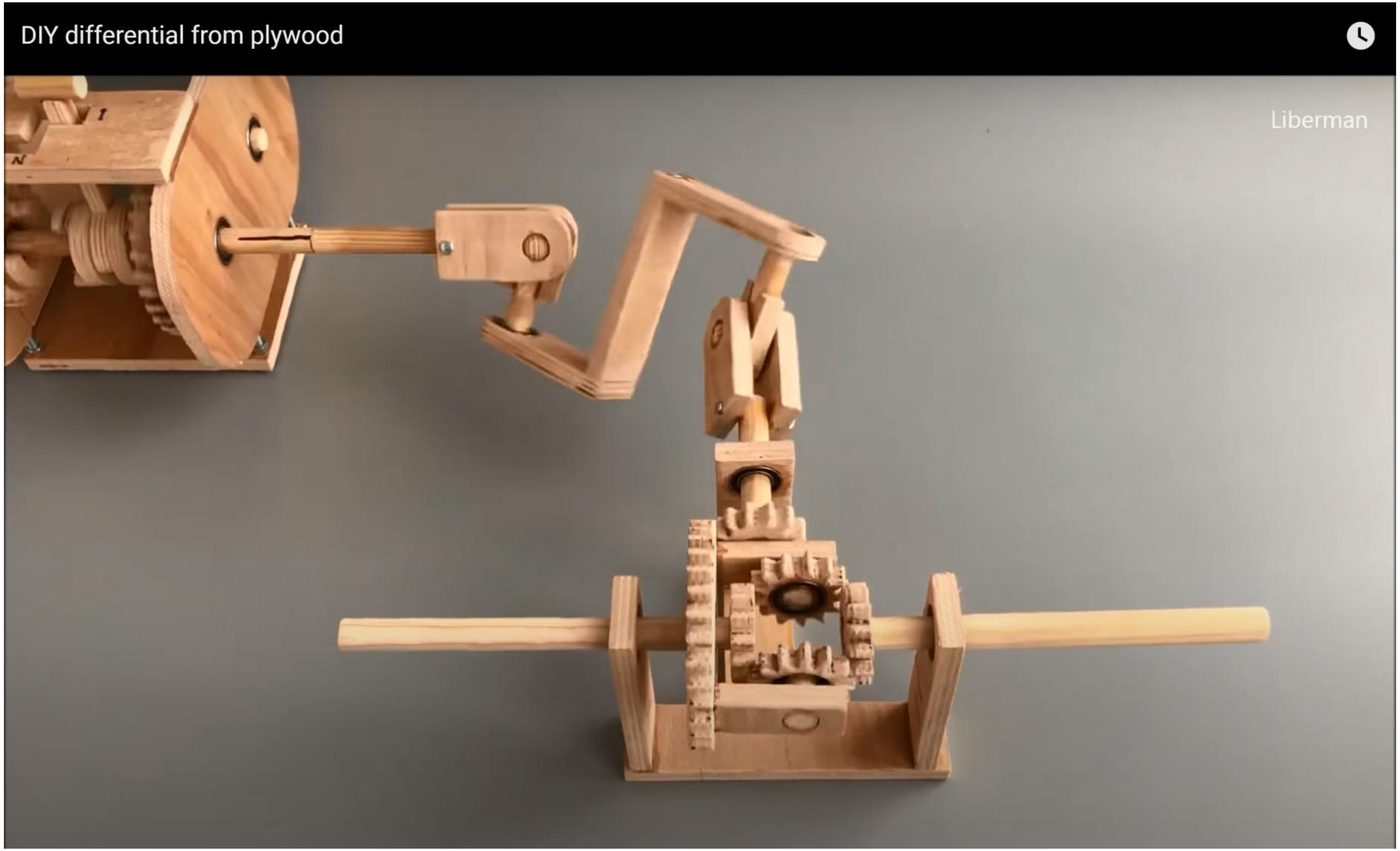
$$\boxed{N_s\,\omega_s + N_r\,\omega_r = (N_s + N_r)\,\omega_c}$$

or, $\quad -\frac{N_r}{N_s} = \frac{\omega_s - \omega_c}{\omega_r - \omega_c}$

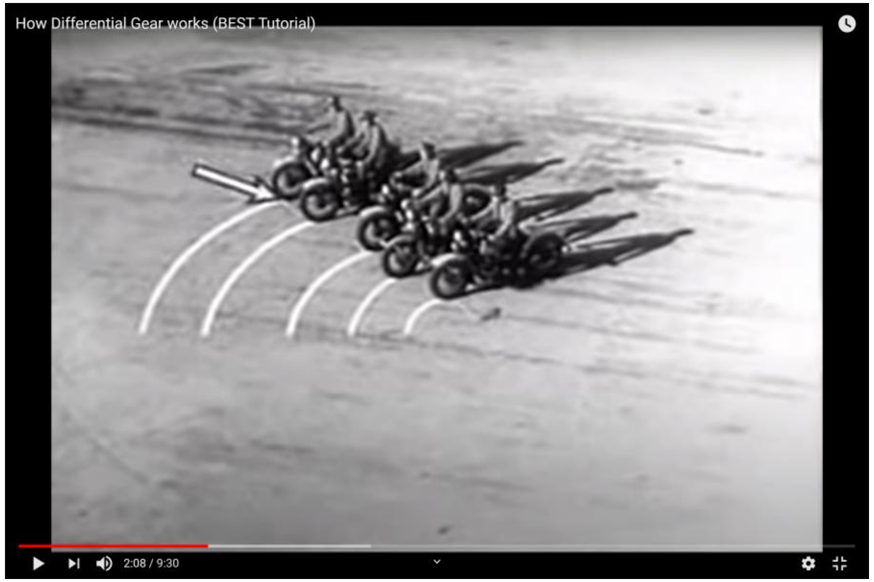
Planet size (i.e. N_p) does not matter!



Differential Gear Trains



https://youtu.be/pSGpddX3MPw?t=210



How Differential Gear Trains Work by Chevrolet

https://youtu.be/K4JhruinbWc?t=532

9.5 min, but very useful for understanding the differential gear train design

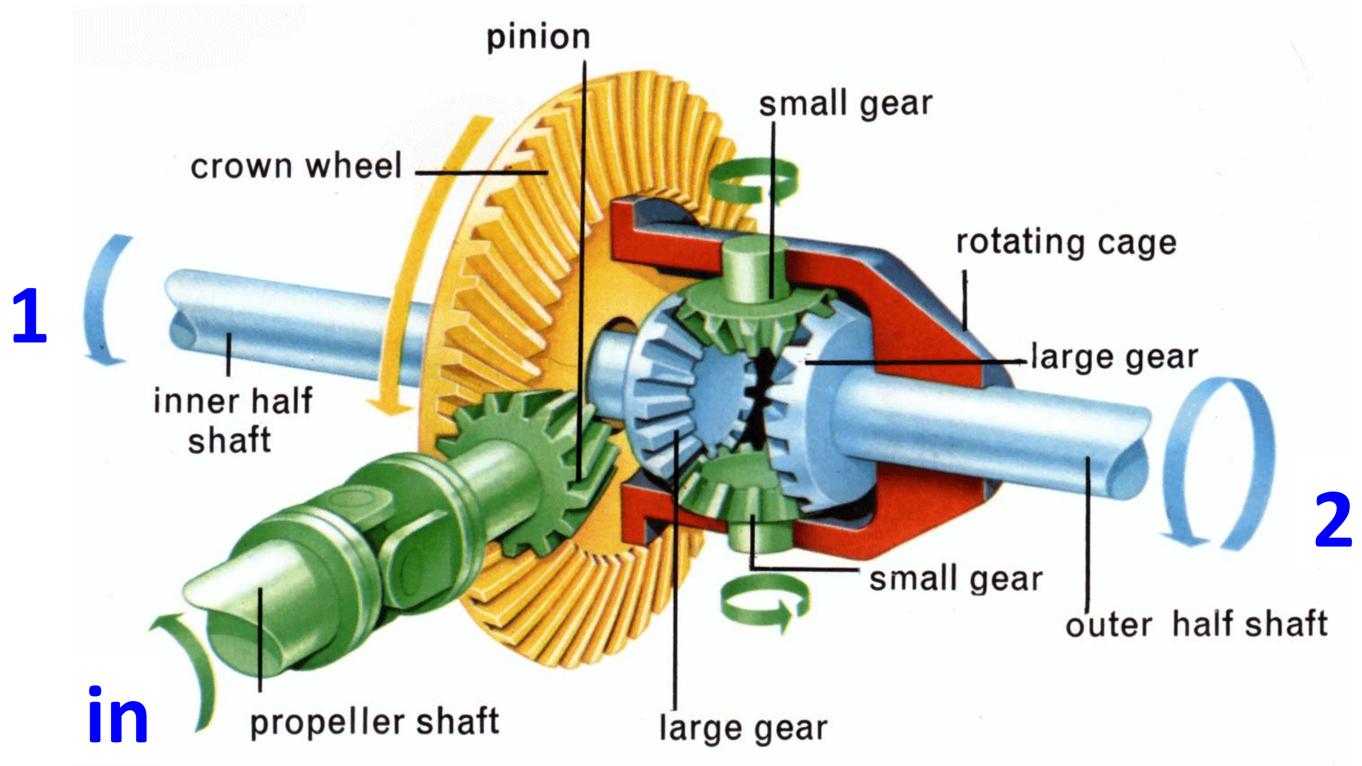
ILLINOIS

Differential Gear Trains

Green input pinion = 14 teeth

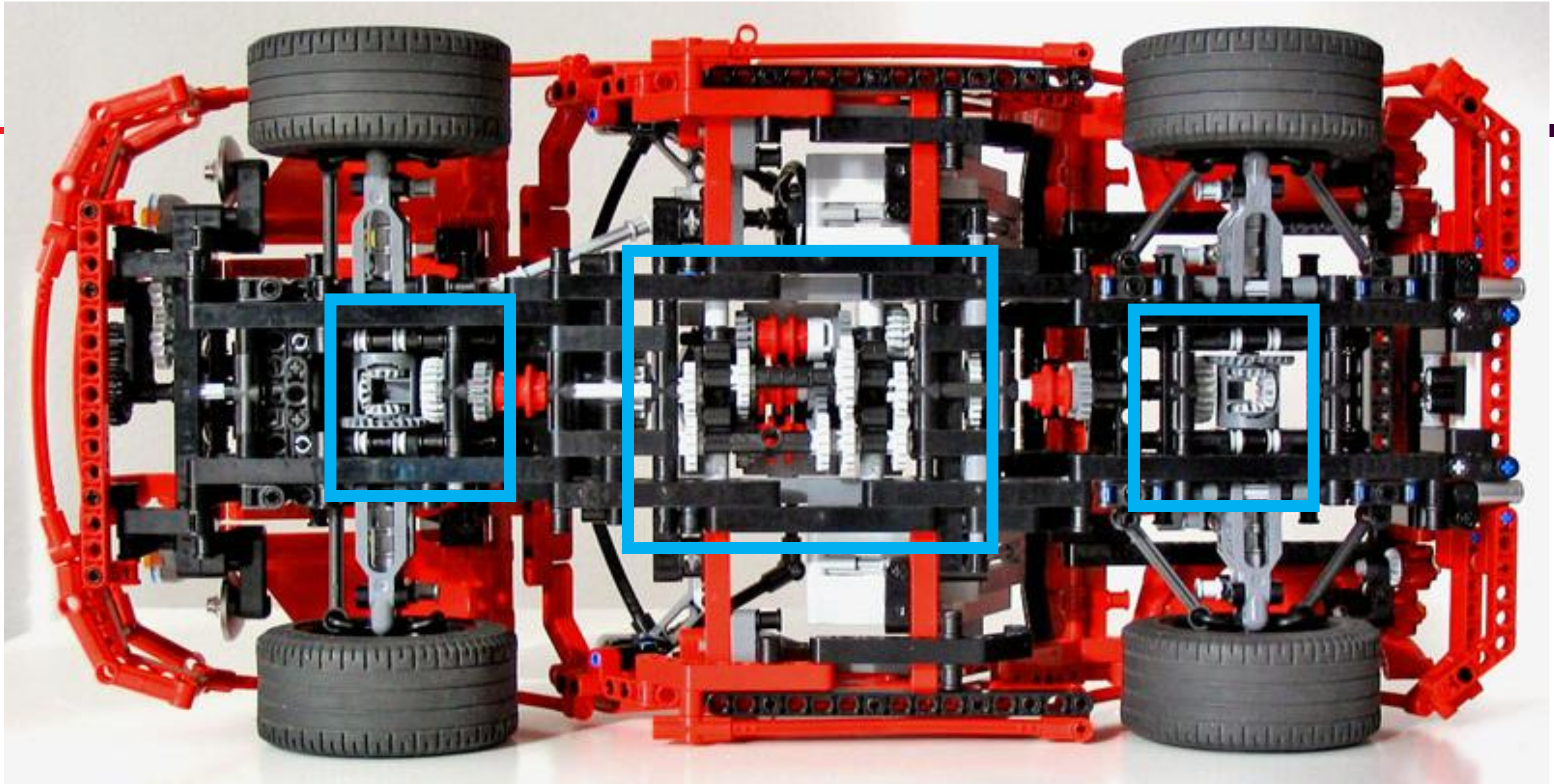
Yellow crown gear = 56 teeth

gear ratio 1:4



$$\omega_{in} = \frac{N_{input}}{N_{crown}} \cdot \frac{\omega_1 + \omega_2}{2}$$

$$T_{in} = \frac{N_{crown}}{N_{input}} \cdot \left(\frac{2\omega_1}{\omega_1 + \omega_2} T_1 + \frac{2\omega_2}{\omega_1 + \omega_2} T_2\right)$$



We can see several important gear elements here.

- Transmission with various gear trains (center)
- Two differential gear trains (front and rear)
- Additionally, linkage mechanisms in the suspension connected to the wheels

Useful Links

- Playlist of different gear systems: https://www.youtube.com/playlist?list=PLA1B93B13A9F9A7C5
- Check out video 9 of hypocycloidal gear train for a very interesting gear train design
- How do trains stay on tracks around a curve?
  - https://www.youtube.com/watch?v=y7h4OtFDnYE