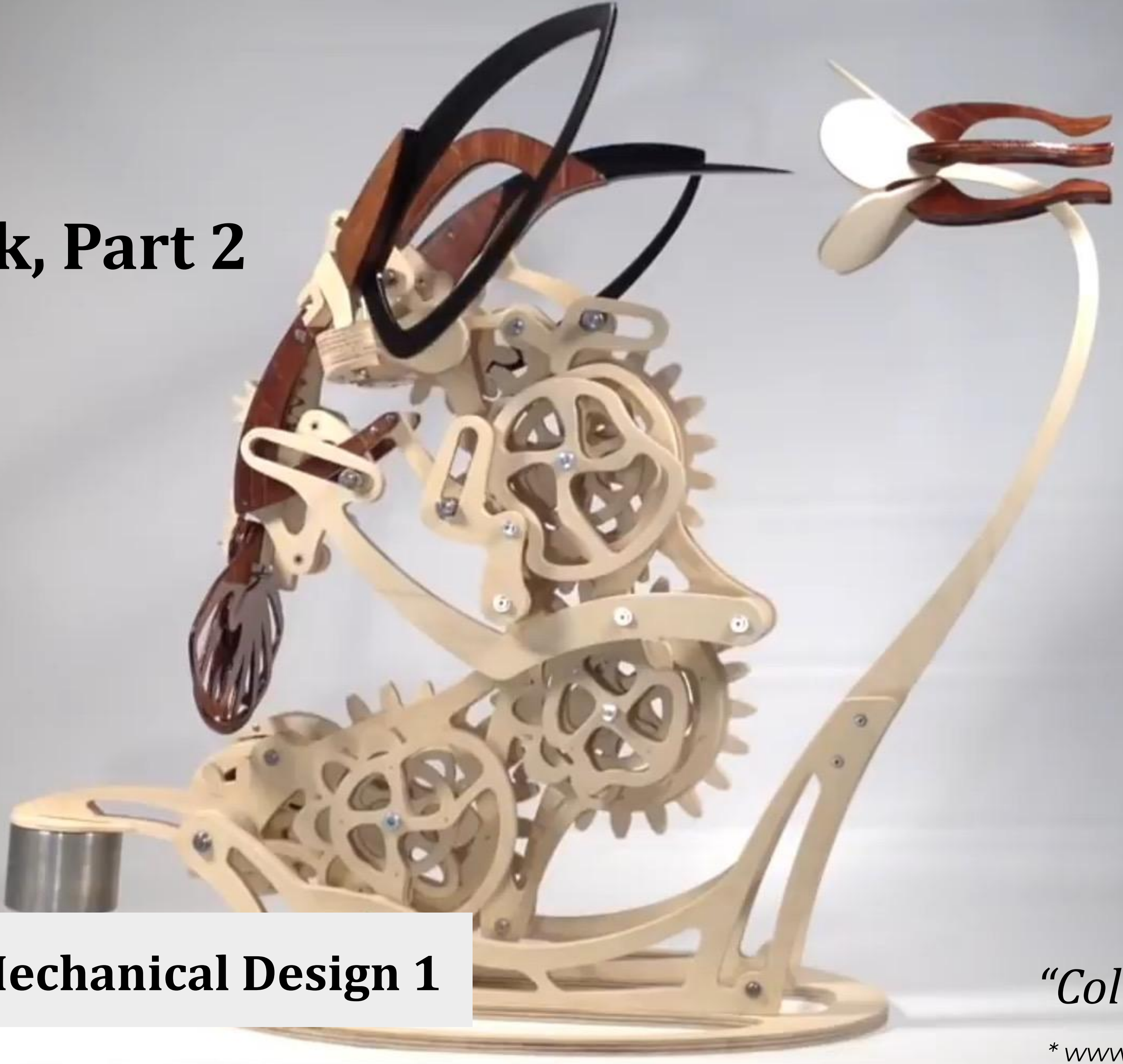
Lecture 24

Module 8:

Virtual Work, Part 2



ME 370 - Mechanical Design 1

“Colibri” by Derek Hugger

* www.youtube.com/watch?v=1scj5sotD-E

Lecture 24: Virtual Work - 2

Today Module 8, Part 2 Virtual Work (Reading, Norton Chapter 10.14-10.15 and 11.10)

Activities & Upcoming Deadlines

- **Week 13:**
  - **HW 11 (Motor, Cam, Motion 2):** due Tuesday 11/18
  - HW 12 (DFA) and HW 13 (Course Reflections–short assignment): to be posted Tuesday, **BOTH** due Tuesday 12/1
  - **Lab 11 (Dynamic Force Analysis with Python)** – Post-lab due the night before your lab section during the week of Nov 19 (delayed by 2 weeks due to P2D2 during Lab 12).
  - **Lab 13:** Work on Project 2 (Optional attendance) – given the status of most projects in P2D2, teams are STRONGLY suggested to use lab time to continue working on your project
- **Project 2:**
  - **CATME peer evaluation for P2D2:** due this week
  - **P2D3 (2-minute Video):** Due Sunday 12/7 by 11:59 pm. Consider collecting photos/videos of your team working on the robot and evolution in design process.

Virtual Work Topics

- Energy Method
  - Principle of Virtual Work
  - Total power for system of *n* moving links
  - Vector equation for estimating external applied forces and torques
- Examples of solving for input torque
  - Four-bar linkage
  - Simple gear set

Recall: Virtual work power equation

- Use to solve for only externally (applied) forces and/or torques
- No need to solve the series of equations from DFA

Due to external forces and torques on links

Due to inertial properties of links

$$\sum_{i=2}^{n}\left(\vec{\mathbf{F}}_i \cdot \vec{\mathbf{v}}_i\right) + \sum_{i=2}^{n}\left(\vec{\mathbf{T}}_i \cdot \vec{\boldsymbol{\omega}}_i\right) = \sum_{i=2}^{n}\left(\vec{m}_i \vec{\mathbf{A}}_{CGi} \cdot \vec{\mathbf{v}}_{CGi}\right) + \sum_{i=2}^{n}\left(I_{CGi}\vec{\boldsymbol{a}}_{\boldsymbol{i}} \cdot \vec{\boldsymbol{\omega}}_{\boldsymbol{i}}\right)$$

Velocity at point of application of external force, *not* CG!

Velocity, accelerations, and moment of inertia at link's CG!

Review: Methods for determining velocity using instant centers

Graphical velocity analysis to solve for ω_3, ω_4, $\mathbf{V}_B$, $\mathbf{V}_C$

From PVA we know:

$$\vec{V}_A = j\omega_2 \vec{R}_2, \text{ and } \vec{R}_2 = |O_2 A| e^{j\theta_2}$$

$$\Longrightarrow |\vec{V}_A| = |O_2 A| \omega_2$$

From IC's, links 2 & 3 have same velocity at A. Geometry relative to I_{13} gives:

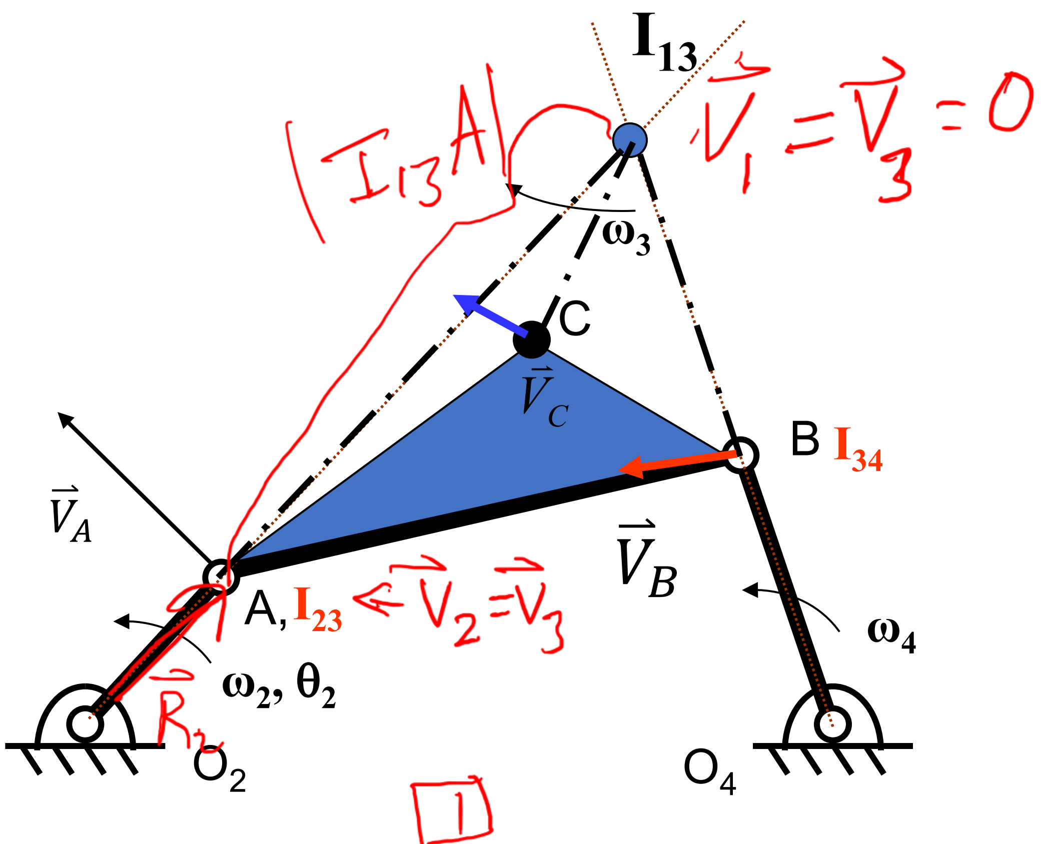
$$|\vec{V}_A| = |I_{13}A|\omega_3 \Longrightarrow \boxed{\omega_3 = |\vec{V}_A| / |I_{13}A|}$$

Using geometry:

$$\boxed{\begin{aligned} |\vec{V}_B| &= |I_{13}B|\omega_3 \\ |\vec{V}_C| &= |I_{13}C|\omega_3 \end{aligned}}$$

From definition of IC @I_{34}:

$$\boxed{\omega_4 = |\vec{V}_B| / |O_4 B|}$$



Review: Velocity ratios

Use I_{24} (the instantaneous center of rotation of links 2 & 4) to find the velocity ratio between ω_2 and ω_4

$$\vec{V}_2 = \vec{V}_4$$

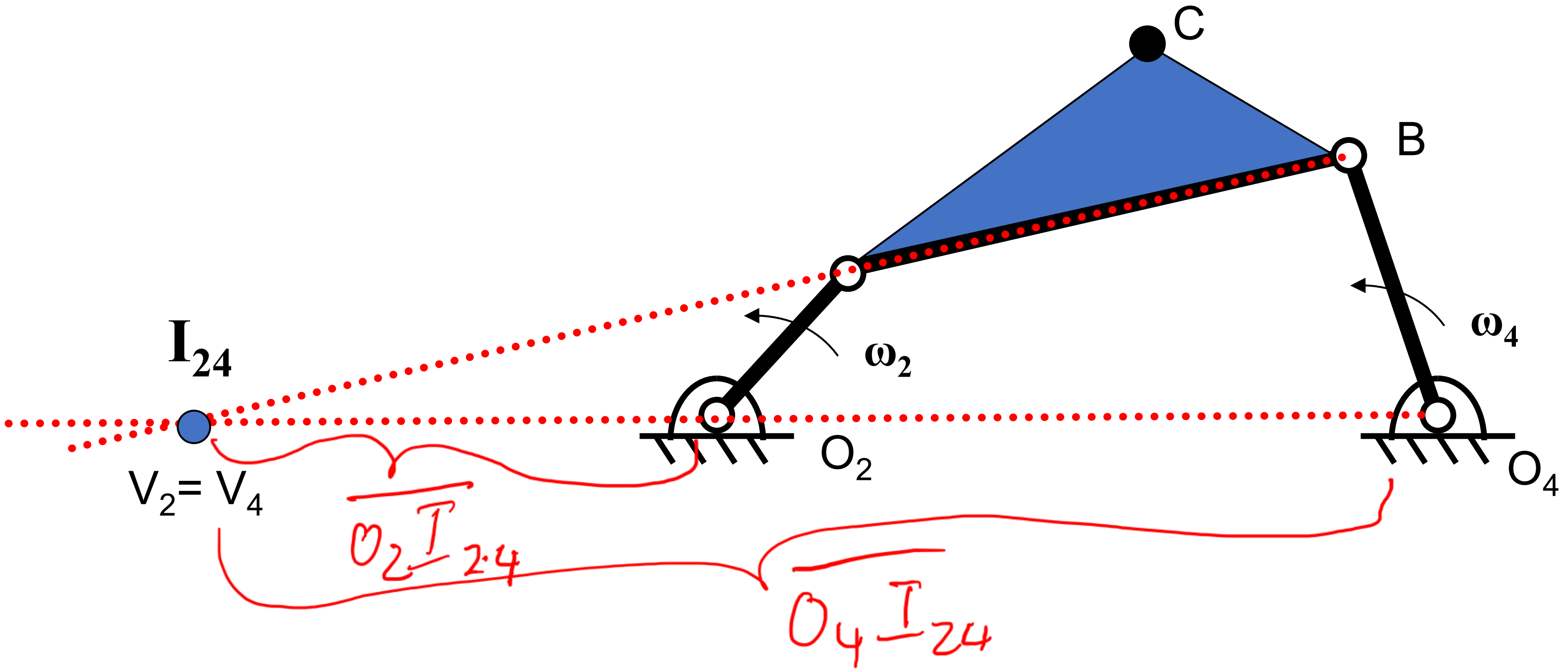
$$\omega_2|O_2I_{24}| = \omega_4|O_4I_{24}|$$

$$\frac{\omega_4}{\omega_2} = \frac{|O_2I_{24}|}{|O_4I_{24}|}$$

If link 2 is input:

$$\frac{\omega_4}{\omega_2} = \frac{\omega_{output}}{\omega_{input}}$$

"velocity ratio"



Review: Torque ratios & Mechanical Advantage

- **Power in a rotating mechanical system:** $P = T\omega$

  Neglecting losses:

$$P_{in} = P_{out} \rightarrow T_{in}\omega_{in} = T_{out}\omega_{out}$$

$$\frac{T_{out}}{T_{in}} = \frac{\omega_{in}}{\omega_{out}}$$

"torque ratio"

Thus torque ratio is the inverse of the velocity ratio

- **Mechanical advantage (*m_A*) for translating system:** $m_A = \dfrac{F_{out}}{F_{in}}$

- **Therefore mechanical advantage for a rotating system is:**

$$\left.\begin{aligned} m_A &= \frac{F_{out}}{F_{in}} \\ F_{out} &= \frac{T_{out}}{r_{out}} \end{aligned}\right\} m_A = \left(\frac{T_{out}}{T_{in}}\right)\left(\frac{r_{in}}{r_{out}}\right) = \left(\frac{\omega_{in}}{\omega_{out}}\right)\left(\frac{r_{in}}{r_{out}}\right)$$

Exercise 2 : Gearset

Recall DFA procedure:

Find: internal forces between links, and driving torque T_{12}

1. *Draw complete system. Label points, dimensions, external forces & torques, kinematics.*

- Take the Gear as body 2 and the Pinion as body 3.
- Consider the torque at the pinion, T_{13}, to be known.

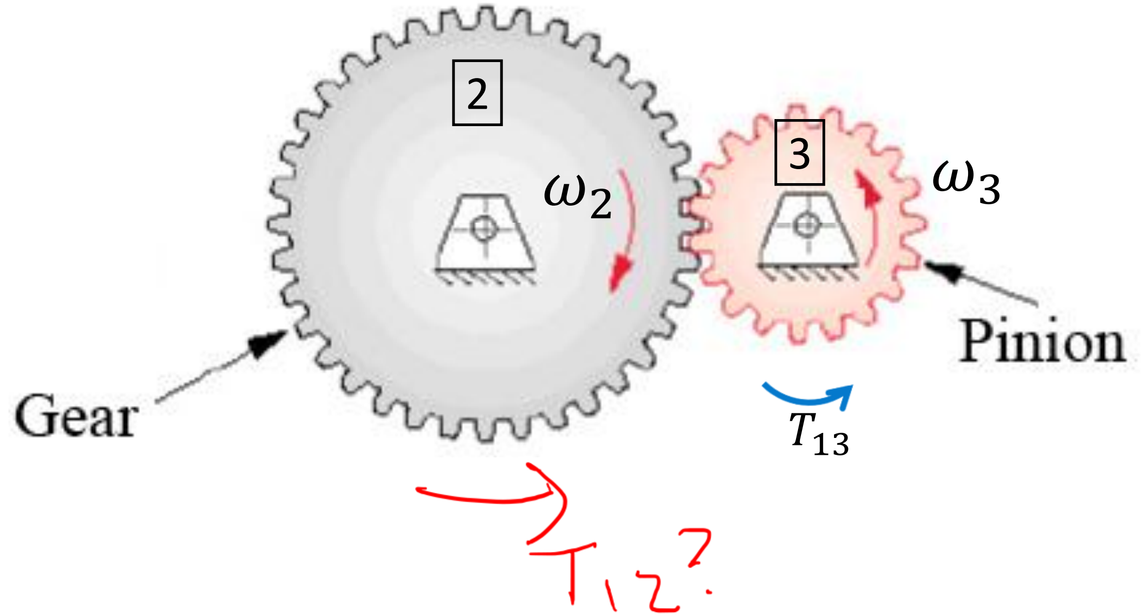
Knowns:

T_{13}

For $i = 2, 3$:

$\theta_i, \omega_i, \alpha_i$

$\vec{A}_{CGi} = 0$



Unknowns:

T_{12}

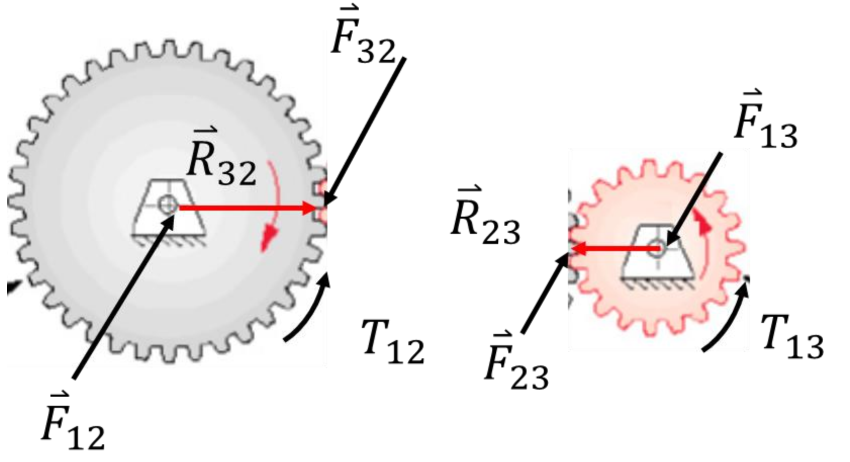
$\vec{F}_{12}, \vec{F}_{13}, \vec{F}_{23}$

7 unknowns ,
Need 7 equations

(4) Convert to matrix format **[A]** **{B}** = **{C}**

(5) Insert known/given values for variables in **[A]** & **{C}**.

(6) Solve for unknown forces and torques in **{B}** using **{B}** = **[A]**$^{-1}$ **{C}**.



$(1) F_{12x} + F_{32x} = mA_{CG2x}$

$(2) F_{12y} + F_{32y} = mA_{CG2y}$

$$\boxed{(3) T_{12} + (R_{32x}F_{32y} - R_{32y}F_{32x}) = I_{CG2}\alpha_2}$$

$(4) F_{13x} - F_{32x} = mA_{CG3x}$

$(5) F_{13y} - F_{32y} = mA_{CG3y}$

$$\boxed{(6) T_{13} - (R_{23x}F_{32y} - R_{23y}F_{32x}) = I_{CG3}\alpha_3}$$

$(7) F_{32x} = cF_{32y}$

$c = \tan(\phi)\,\mathrm{sign}(F_{32y})$

$$\begin{bmatrix} 1 & 0 & c & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & X & 0 & 0 & 1 \\ 0 & 0 & -c & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & Y & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} F_{12x} \\ F_{12y} \\ F_{32y} \\ F_{13x} \\ F_{13y} \\ T_{12} \end{Bmatrix} = \begin{Bmatrix} m_2A_{CG2x} \\ m_2A_{CG2y} \\ I_{CG2}\alpha_2 \\ m_3A_{CG3x} \\ m_3A_{CG3y} \\ I_{CG3}\alpha_3 - T_3 \end{Bmatrix}$$

$X = R_{32x} - cR_{32y}$ $\qquad$ $Y = cR_{23y} - R_{23x}$

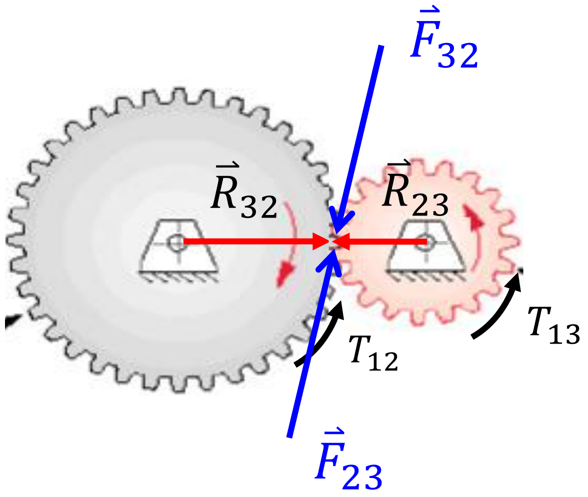
Let’s consider only solving for external torque T_{12}

In this (special) case, we can solve for the output torque with two equations

•Start with rewritten versions of equations (3) and (6)

$$-\cancelto{0}{R_{32y}}F_{32x} + R_{32x}F_{32y} + T_{12} = I_{CG2}\alpha_2 \qquad (3')$$

$$\cancelto{0}{R_{23y}}F_{32x} - R_{23x}F_{32y} = I_{CG3}\alpha_3 - T_{13} \qquad (6')$$



•Multiply (6’) by the ratio of gear radii $\left(\frac{R_{32x}}{R_{23x}}\right)$:

$$-R_{32x}F_{32y} = \left(\frac{R_{32x}}{R_{23x}}\right) I_{CG3}\alpha_3 - \left(\frac{R_{32x}}{R_{23x}}\right) T_{13} \qquad (6”)$$

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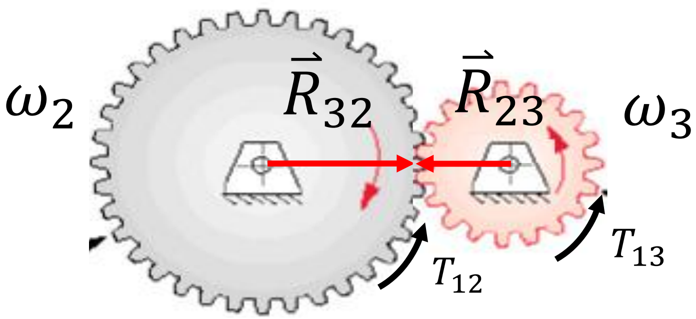
Gearset – Torque Solution



Combine (3') and (6") to create one solution equation

$$T_{12} = I_{CG2}\alpha_2 + \left(\frac{R_{32x}}{R_{23x}}\right) I_{CG3}\alpha_3 - \left(\frac{R_{32x}}{R_{23x}}\right) T_{13}$$

Use the geometry of the gears to get the following expression to solve for T_{12} based on the number of teeth per gear (N_i):

Recall, from definition of diametral pitch: $\frac{N_2}{N_3} = \pm \frac{R_{32x}}{R_{23x}}$

$$\boxed{T_{12} = I_{CG_2}\alpha_2 - \left(\frac{N_2}{N_3}\right) I_{CG_3}\alpha_3 + \left(\frac{N_2}{N_3}\right) T_{13}} \quad 1$$

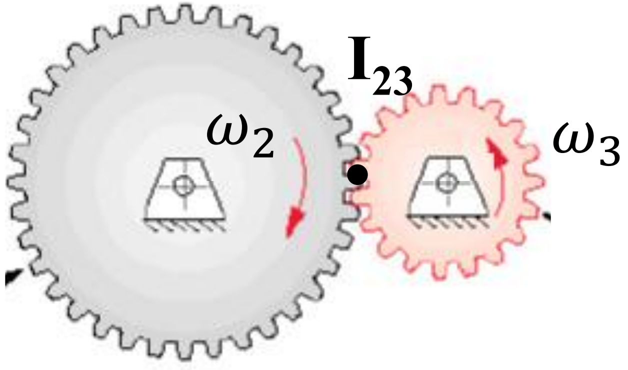
Which sign?

− external

even # gears

Recall: Gearset – Velocity Relationship

Relative motion of two gears is two rolling circles without slip.
The instant center is at the contact point between gears.



$$@\ I_{23}:\ V_3 = V_2 \rightarrow R_{23x}\omega_3 = R_{32x}\omega_2$$

$$\frac{\omega_3}{\omega_2} = \frac{R_{32x}}{R_{23x}}$$

This relationship btw (out/in) is the "velocity ratio"

From definition of diametral pitch $\dfrac{N_2}{N_3} = -\dfrac{R_{32x}}{R_{23x}}$

Even # external gears

$$\frac{\omega_3}{\omega_2} = -\frac{N_2}{N_3} \qquad \text{(A)}$$

Therefore, velocity ratio and number of teeth ratio are also related.

Negative because gears rotate in opposite directions

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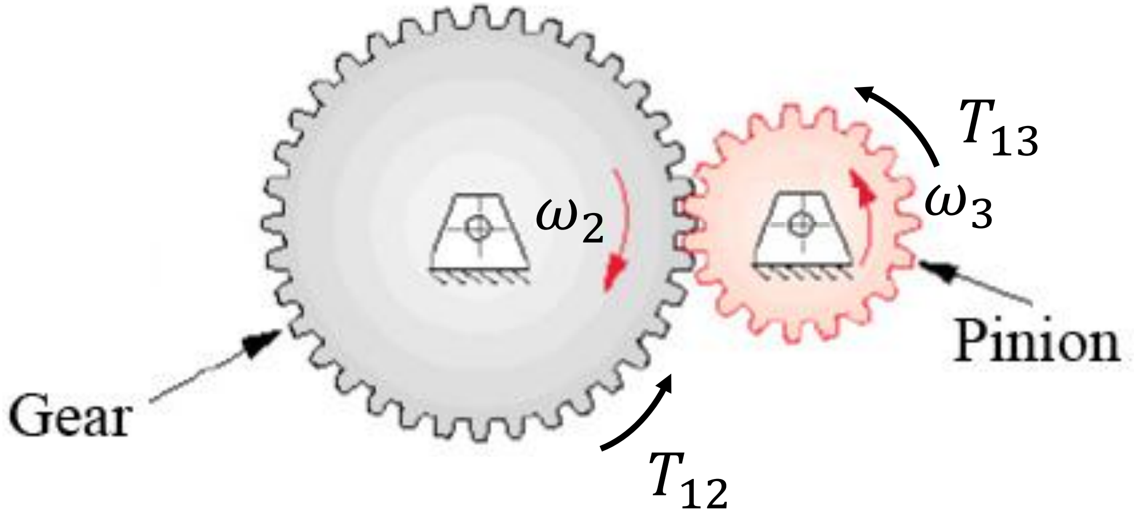
Alternative solution: using virtual work

Solve for T_{12}



total # moving links

$$\sum_{i=2}^{n}(\vec{\mathbf{F}}_i \cdot \vec{\mathbf{v}}_i) + \sum_{i=2}^{n}(\vec{\mathbf{T}}_i \cdot \vec{\boldsymbol{\omega}}_i) = \sum_{i=2}^{n}(m_i\vec{\mathbf{A}}_{CG_i} \cdot \vec{\mathbf{v}}_{CG_i}) + \sum_{i=2}^{n}(I_{CG_i}\vec{\boldsymbol{\alpha}}_i \cdot \vec{\boldsymbol{\omega}}_i)$$

0 0 0

- $\vec{F}_{12}, \vec{F}_{13}, \vec{F}_{32}, \vec{F}_{23}$ are internal forces and do no work on our system
- No externally applied forces, $\therefore \vec{F}_i = 0$
- $\vec{V}_{CG2}, \vec{V}_{CG3}, \vec{A}_{CG2},$ and $\vec{A}_{CG2}$ are zero since grounded

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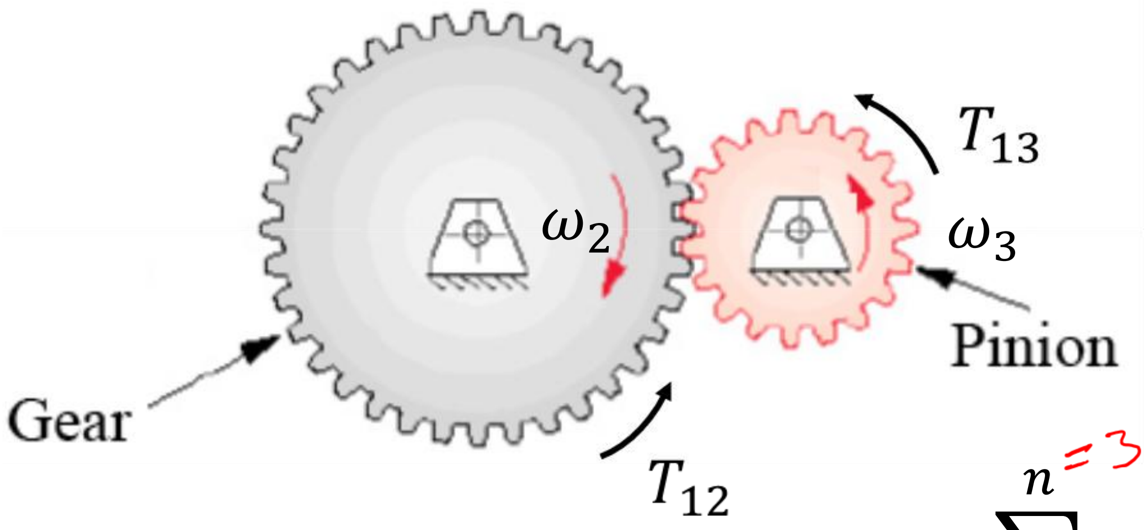
Alternative solution: using virtual work

Solve for T_{12}



$$\sum_{i=2}^{n=3}\left(\vec{\mathbf{T}}_i \cdot \vec{\boldsymbol{\omega}}_i\right) = \sum_{i=2}^{n=3}\left(I_{CG_i}\vec{\boldsymbol{\alpha}}_i \cdot \vec{\boldsymbol{\omega}}_i\right)$$

$$T_{12}\omega_2 + T_{13}\omega_3 = I_{CG_2}\alpha_2\omega_2 + I_{CG_3}\alpha_3\omega_3$$

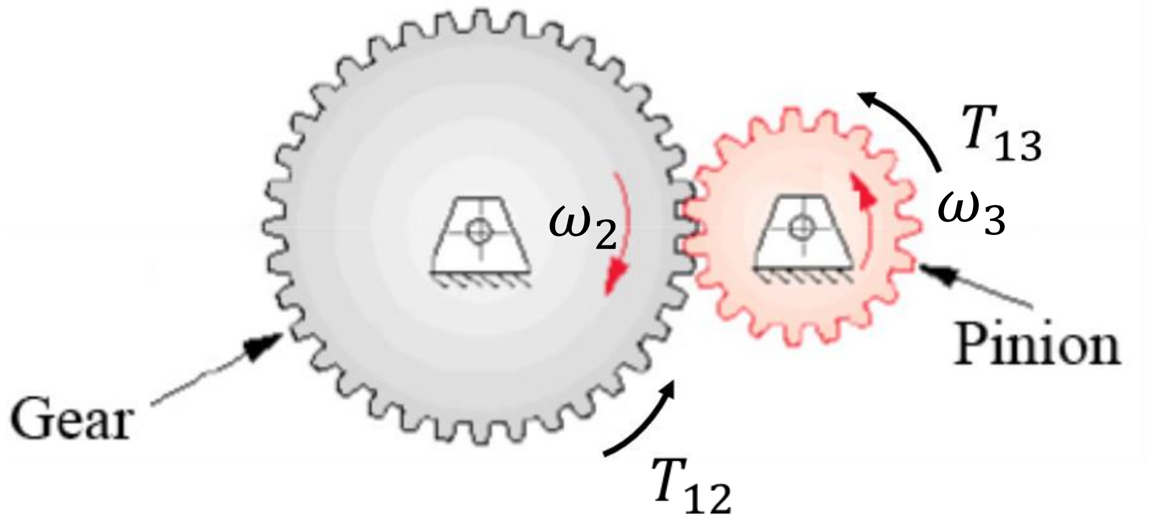
$$T_{12} = -T_{13}\left(\frac{\omega_3}{\omega_2}\right) + I_{CG_2}\alpha_2 + I_{CG_3}\alpha_3\left(\frac{\omega_3}{\omega_2}\right) \leftarrow \text{Apply } (A)$$

$$\frac{\omega_3}{\omega_2} = -\frac{N_2}{N_3}$$

$$\boxed{T_{12} = \left(\frac{N_2}{N_3}\right)T_{13} + I_{CG_2}\alpha_2 - \left(\frac{N_2}{N_3}\right)I_{CG_3}\alpha_3} \quad (2)$$

F

Gearset – Virtual Work



$$T_{12} = \left(\frac{N_2}{N_3}\right) T_{13} + I_{CG2}\alpha_2 - \left(\frac{N_2}{N_3}\right) I_{CG3}\alpha_3$$

Compare equations (1) and (2):

- Same result from two methods (DFA vs. VW)!
- One equation!
- What is missing? Can’t solve for internal reaction forces

Project 2: D3 Video

Due Sunday Dec 7. Students from each lab section will rate the videos from the 5 teams in their lab section – due Tuesday Dec 9. The top video from each of the 12 lab sections will receive a bonus. Additionally, these 12 videos will be played during the last lecture classes on Wednesday December 10. The top video will receive an additional bonus. Attendance is mandatory at the last lecture.

What is your preference for the type of bonus that the winners of the lab sections and our lecture section will receive as a small bonus?

A: some extra points 60%

B: some small award 40%

Join Code: **370**

No class on Wednesday!
Have a great Break!