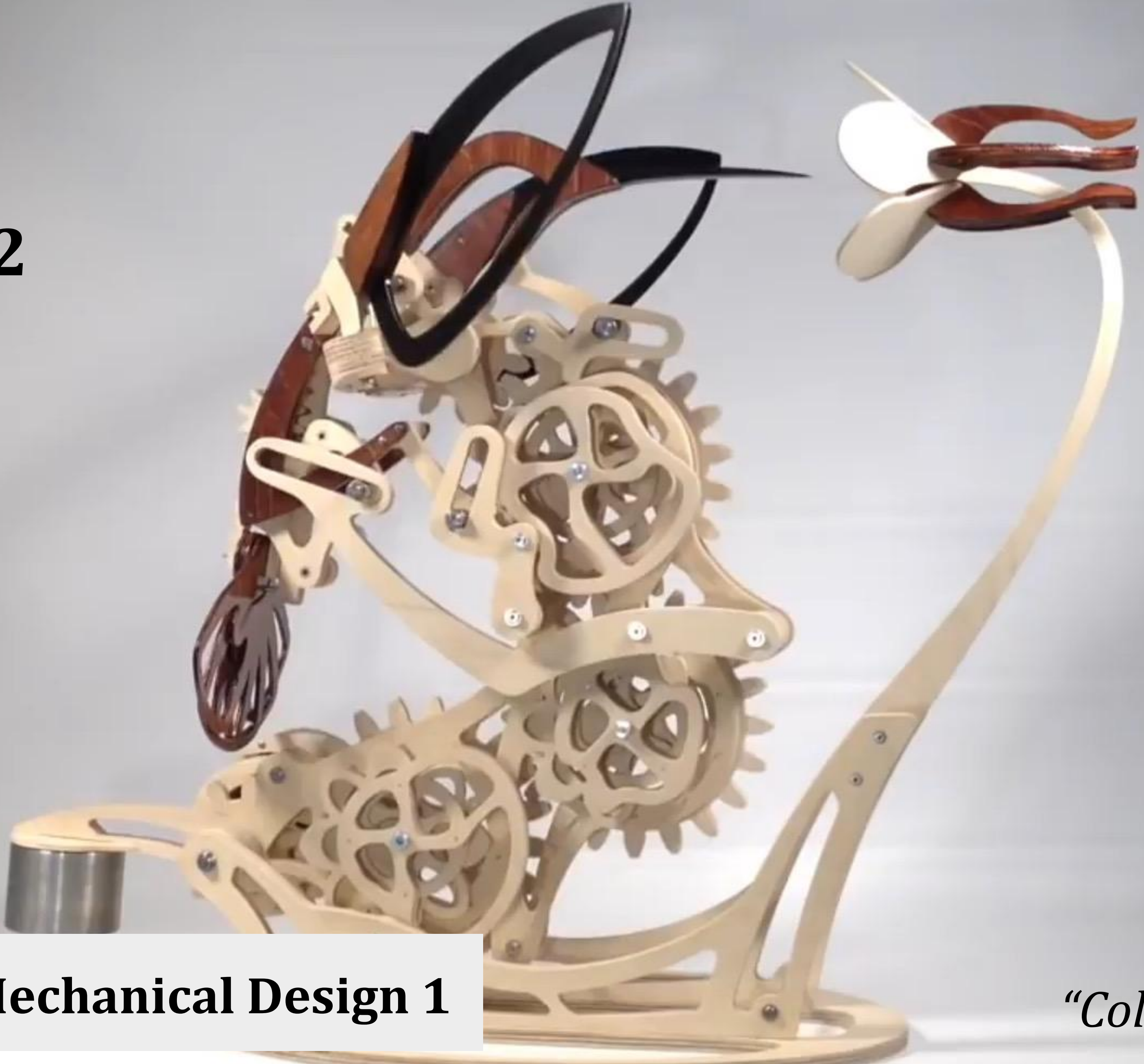
Module 9:
Lecture 26
Balancing - 2



ME 370 - Mechanical Design 1

"Colibri" by Derek Hugger

* www.youtube.com/watch?v=1scj5sotD-E

Lecture 26: Balancing 2

Today Module 9, Part 2 Balancing (Reading, Norton Chapter 11.8, 11.11, 12)

Activities & Upcoming Deadlines

- **General:**
  - **Innovation Studio** anonymous feedback form, responses may result in more space to work on projects for future classes.
- **Week 14:**
  - HW 14 (VW): due Tuesday 12/9
  - Lab 14: Work on Project 2 (attendance expected) –use lab time to continue working on your project
- **Project 2:**
  - **P2D3 (2-minute Video):** Due Sunday 12/7 by 11:59 pm.
    - See sample videos (2 in P2 Project Description PDF, **new video in Project 2 Module – see Deliverable 3**)
    - All students are to rank the 5 videos in your lab section by Tuesday Dec 9. Submit to GradeScope.
    - The top videos from across the 12 lab sections will be played during lecture on Wednesday Dec 10 - **attendance is mandatory.**
  - **P2D4 (Performance Event):**
    - Friday 12/12 from 7-10 pm. **Attendance is mandatory**.
    - Must walk forward within 2m width over 10m distance
    - Grading rubric and demonstration timeslot sign-up sheets will be posted shortly

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Module 9 Topics

- Balancing of Rotating Machinery (Text 11.8, 11.11, 12)
  - Shaking force and torque (11.8)
  - Shaking moment (12.6)
  - Static balancing (12.1)
  - Dynamic balancing (12.2)
  - Balancing more complex planar mechanisms (12.3+)
    - Single piston engine
    - 4-bar linkage
  - Flywheels (11.11)
    - Torque variation
    - Motor selection
    - Sizing a flywheel

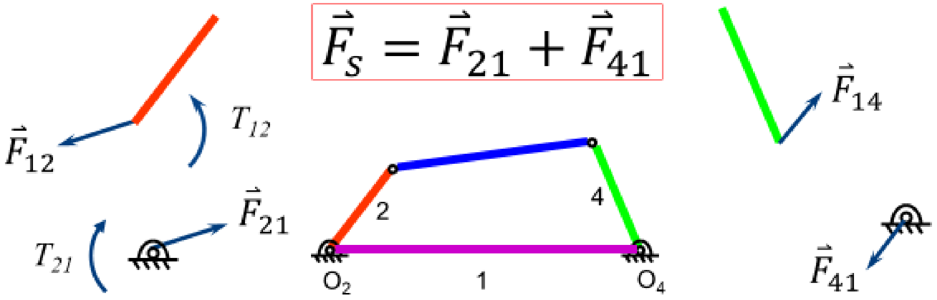
Covered in lecture 25

Recall: Shaking force, torque, and moment

Shaking force (Norton 11.8)

- F_s: The ***total*** force acting **<u>on</u>** the ground plane is defined as the ***shaking force***
  - Sum of all forces acting on ground
  - Tends to move ground plane back & forth

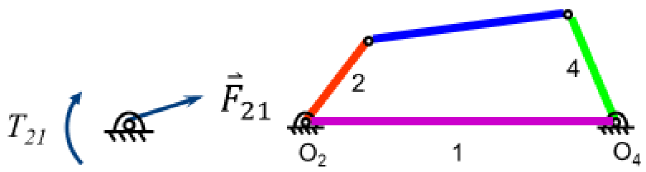
$$\boxed{\vec{F}_s = \vec{F}_{21} + \vec{F}_{41}}$$



Shaking torque (Norton 11.8)

- T_s: The ***reaction torque*** felt **<u>by</u>** the ground plane is defined as the ***shaking torque***
  - Negative of input torque on driving link from ground
  - Rocks ground plane about the crank pivot axis or driveline axis

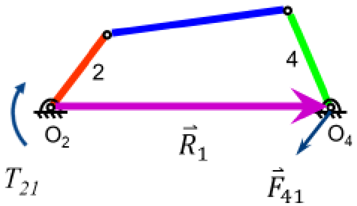
$$\boxed{T_s = T_{21} = -T_{12}}$$



Shaking moment (Norton 12.6)

- M_s: The ***total*** moment felt **<u>by</u>** the ground plane is defined as the ***shaking moment***
  - Reaction moments felt by ground
  - Tends to rocks ground about the crank pivot axis (in this case point O_2)
  - Shaking torque plus shaking couple due to reaction force about crank pivot axis

$$\boxed{\vec{M}_s = \vec{T}_{21} + (\vec{R}_1 \times \vec{F}_{41})}$$



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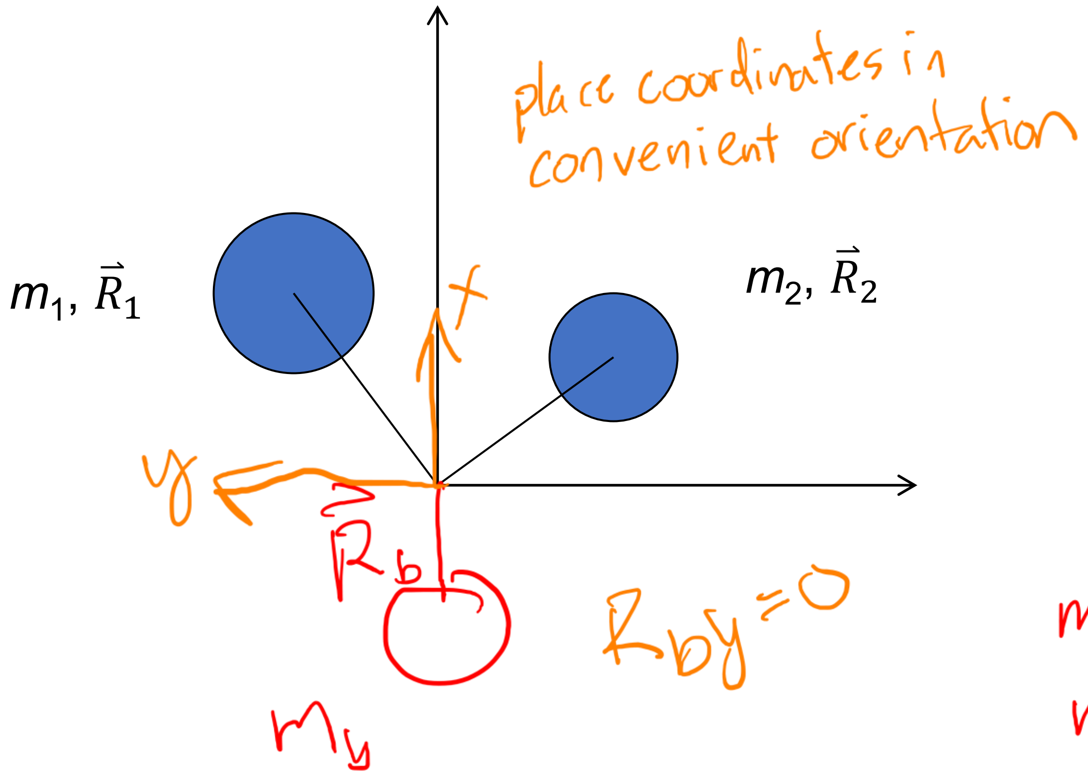
Recall: Example

Given: constant velocity mechanism and known m_1, $\vec{R}_1$, m_2, $\vec{R}_2$.

Determine appropriate mass and location (m_b, $\vec{R}_b$) for a **single** balancing mass



To solve, start with this basic equation:

$$\sum m_i \omega_i^2 \vec{R}_i = 0$$

Since velocity is constant and all ω_i are the same, use:

$$\boxed{\sum m_i \vec{R}_i = 0}$$

Key Static Balancing eqn

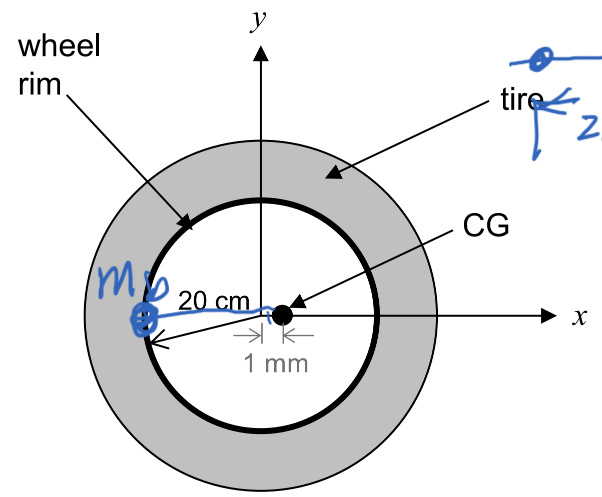
$$m_1 \vec{R}_1 + m_2 \vec{R}_2 + m_b \vec{R}_b = 0$$

$$m_1 R_{x1} + m_2 R_{x2} + m_b R_{bx} = 0$$

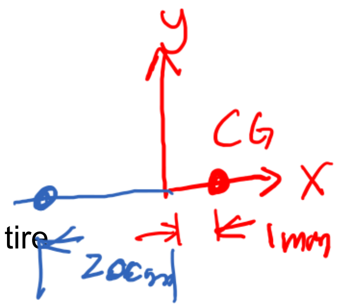
$$m_1 R_{y1} + m_2 R_{y2} + m_b R_{by} = 0$$

Class exercise 1: Static balancing of a tire

An automobile wheel and tire weigh 10 kg (10,000 g). The outer rim of the wheel has a radius of 20 cm. The center of gravity of the wheel/tire combination is 1 mm away from the axis of rotation (i.e., z-axis). The tire can be balanced by adding a single balancing weight to the outer rim of the wheel. **How much weight (in grams) must be added to statically balance the wheel? Mark where the weight should be located**. Hint: for convenience, align the unbalanced tire so that its CG is on the *x*-axis



drawing not to scale



$$\sum m_i \vec{R}_i = 0$$

$$m_1 \vec{R}_1 + m_b \vec{R}_b = 0$$

$$m_1 R_{1x} + m_b R_{bx} = 0$$

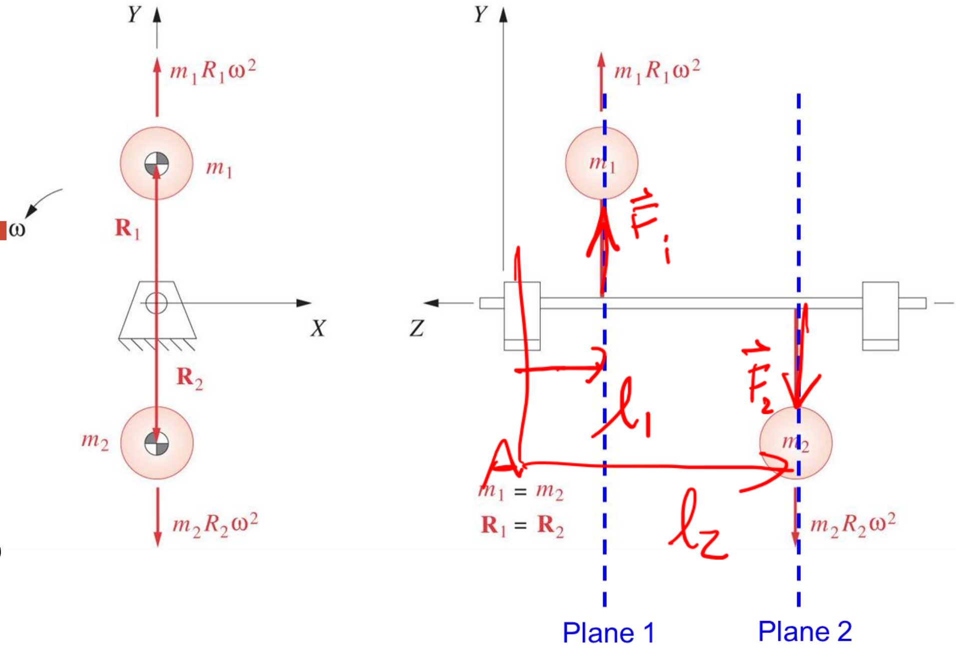
$$m_1 \cancel{R_{1y}}^{0} + m_b \cancel{R_{by}}^{0} = 0$$

$$m_b = -\frac{m_1 R_{1x}}{R_{bx}}$$

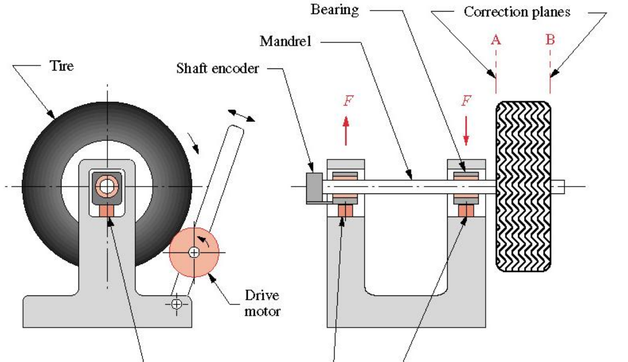
$$\Rightarrow \boxed{m_b = 50g}$$

Recall: Dynamic Balancing

- 3-D, “two-plane” balance
  - long in axial direction compared to radial direction
  - ex. car tire, turbines, drive shafts, camshafts, crankshafts



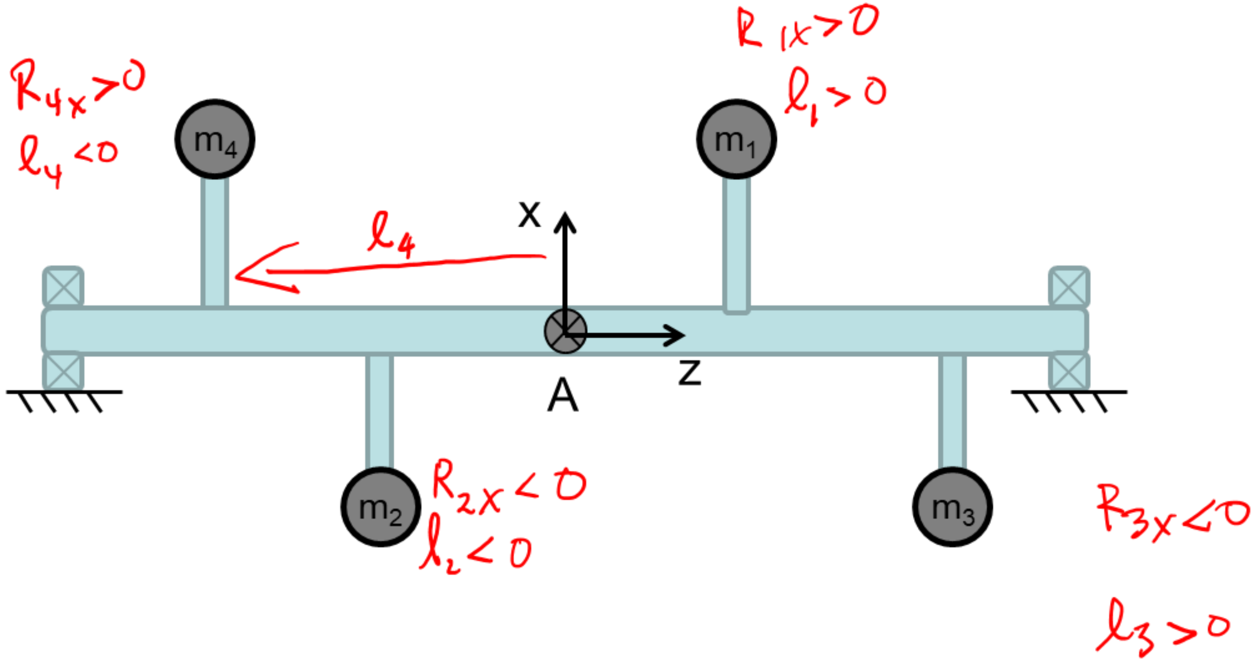
- 2 step process: Must satisfy both
  1. Force balance $\sum\vec{F} = 0$ (balance inertial forces in radial direction)
     - $\sum m_i\vec{R}_i = 0$ Static balancing
  2. Moment balance $\sum\vec{M} = 0$ (balance bending moments due to inertial forces in shaft)
     - $\sum m_i l_i\vec{R}_i = 0$ Dynamic balancing



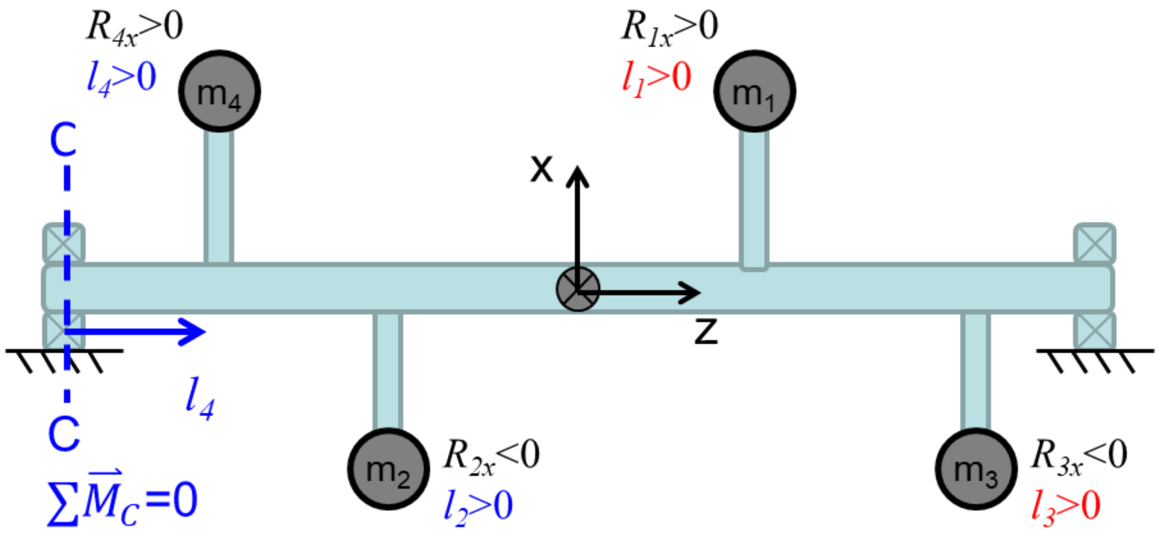
Recall: Dynamic Balancing

- Moment balance notes:
  - Measure l_i's from any convenient reference plane
  - The signs of R_{xi}, R_{yi}, l_i matter!

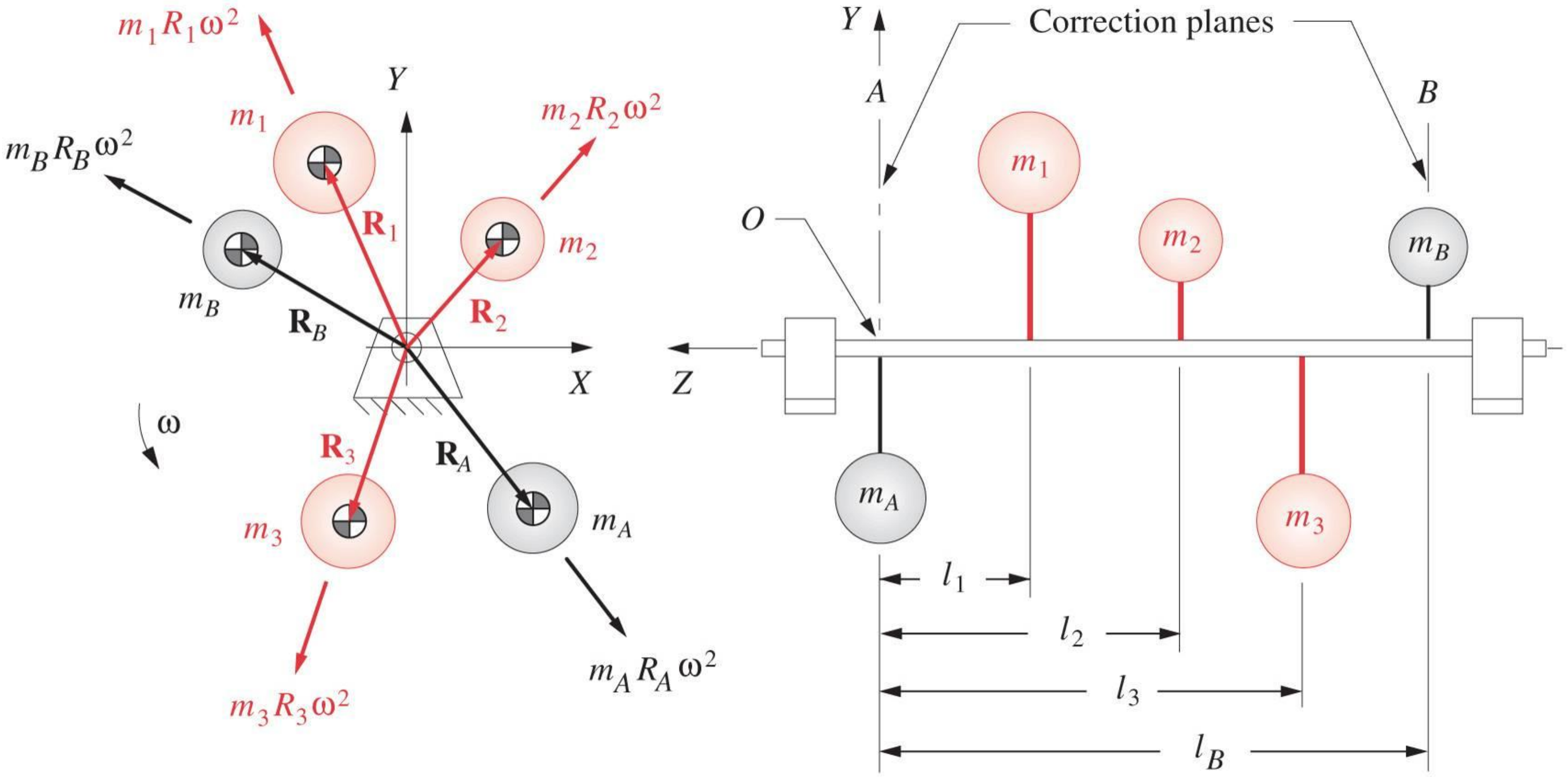
Sum moments about A



Changed starting point to left side at plane C-C.
Note change in signs for l_2 and l_4



Dynamic Balancing Example



How do we dynamically balance this configuration?

Dynamic Balancing Example

Norton Example 12.2

- Sum the forces (static balancing)

Constant velocity

$$\sum m_i \vec{R}_i = 0$$

$$m_1\vec{R}_1 + m_2\vec{R}_2 + m_3\vec{R}_3 + m_A\vec{R}_A + m_B R_B = 0$$

Convert to x & y components

(1) $m_1 R_{1x} + m_2 R_{2x} + m_3 R_{3x} + m_A R_{Ax} + m_B R_{Bx} = 0$

(2) $m_1 R_{1y} + m_2 R_{2y} + m_3 R_{3y} + m_A R_{Ay} + m_B R_{By} = 0$

- Sum the moments (about plane A)

$$\sum m_i \vec{R}_i \, l_i = 0$$

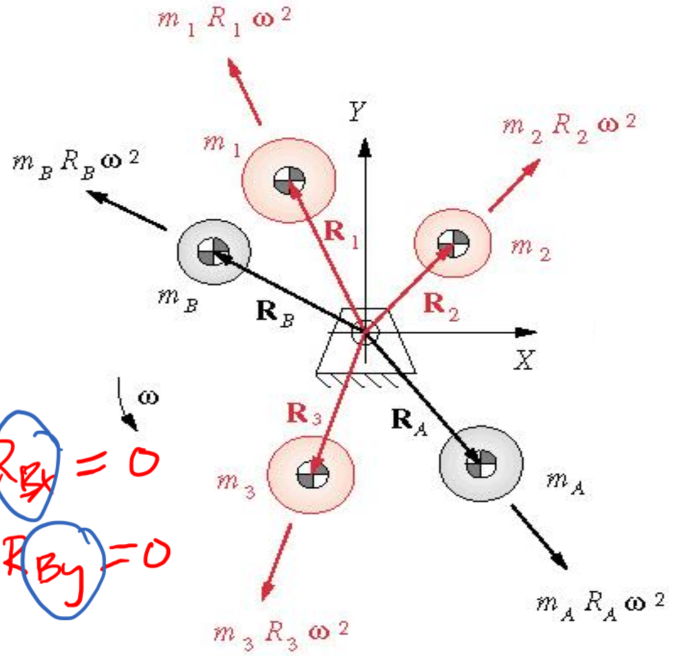
$$-m_1\vec{R}_1 l_1 - m_2\vec{R}_2 l_2 - m_3 R_3 l_3 - \cancel{m_A\vec{R}_A l_A} - m_B R_B l_B = 0$$

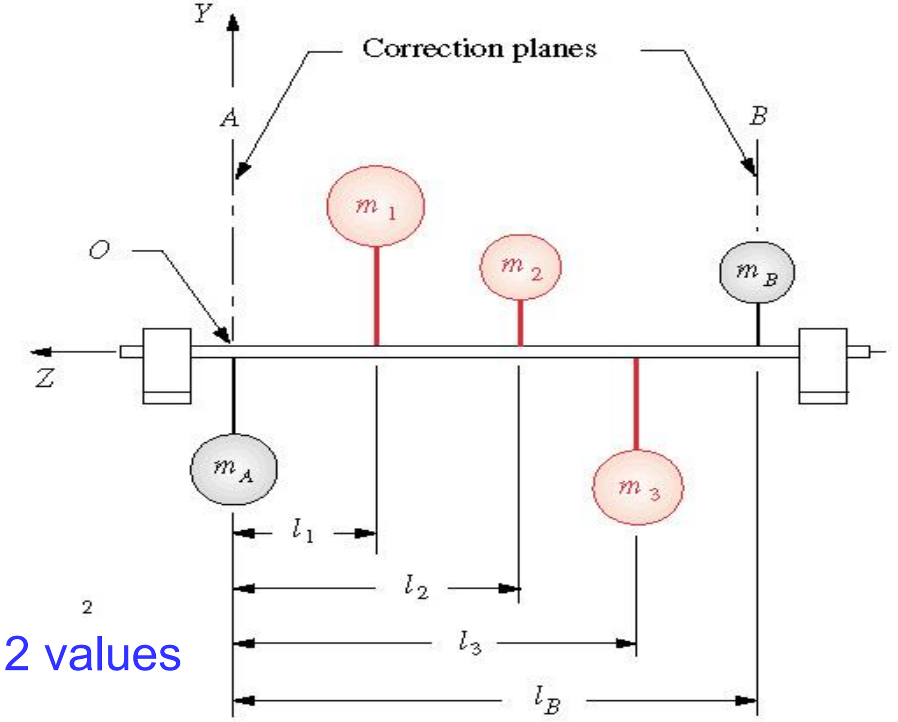
ΣM_A

Convert to x & y components

(3) $m_B R_{Bx} = \dfrac{-m_1 R_{1x} l_1 - m_2 R_{2x} l_2 - m_3 R_{3x} l_3}{l_B}$

(4) $m_B R_{By} = \dfrac{-m_1 R_{1y} l_1 - m_2 R_{2y} l_2 - m_3 R_{3y} l_3}{l_B}$





2

F

4 equations, 6 unknowns (masses, positions). Designer must pick 2 values

Example

Since ω_i is constant

$$\boxed{\sum m_i \vec{R}_i = 0} \qquad \boxed{\sum m_i \vec{R}_i l_i = 0}$$

- Given: $m_1, \vec{R}_1, m_2, \vec{R}_2$
- Determine appropriate masses and locations $(m_{b1}, \vec{R}_{b1}; m_{b2}, \vec{R}_{b2})$ for two balancing masses

1st static balance

$\sum m_i \vec{R}_i = 0$

$m_1\vec{R}_1 + m_2\vec{R}_2 + m_{b1}\vec{R}_{b1} + m_{b2}\vec{R}_{b2} = 0$

write as x & y components

⇒ get 2 eqns

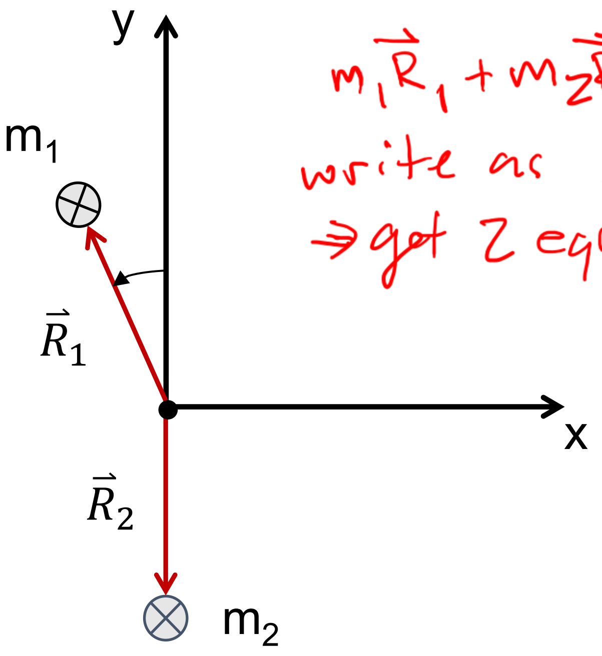
2nd sum of moments $\sum m_i \vec{R}_i l_i = 0$

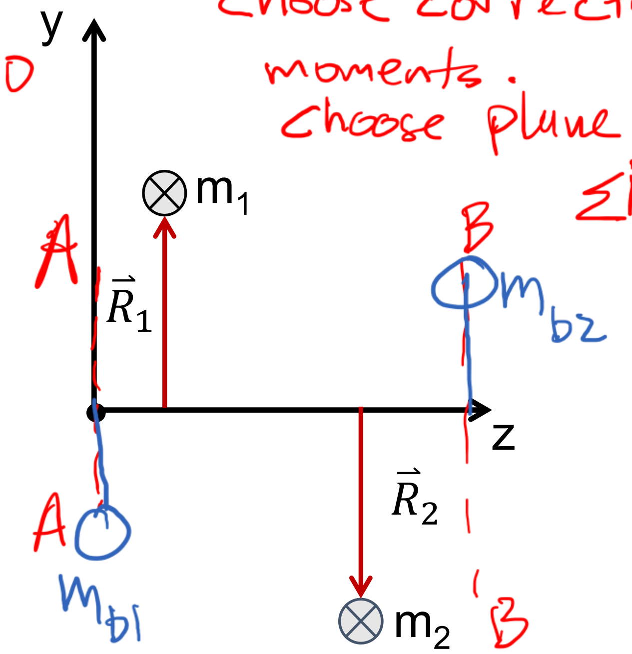
choose correction plane to sum moments.

choose plane A-A

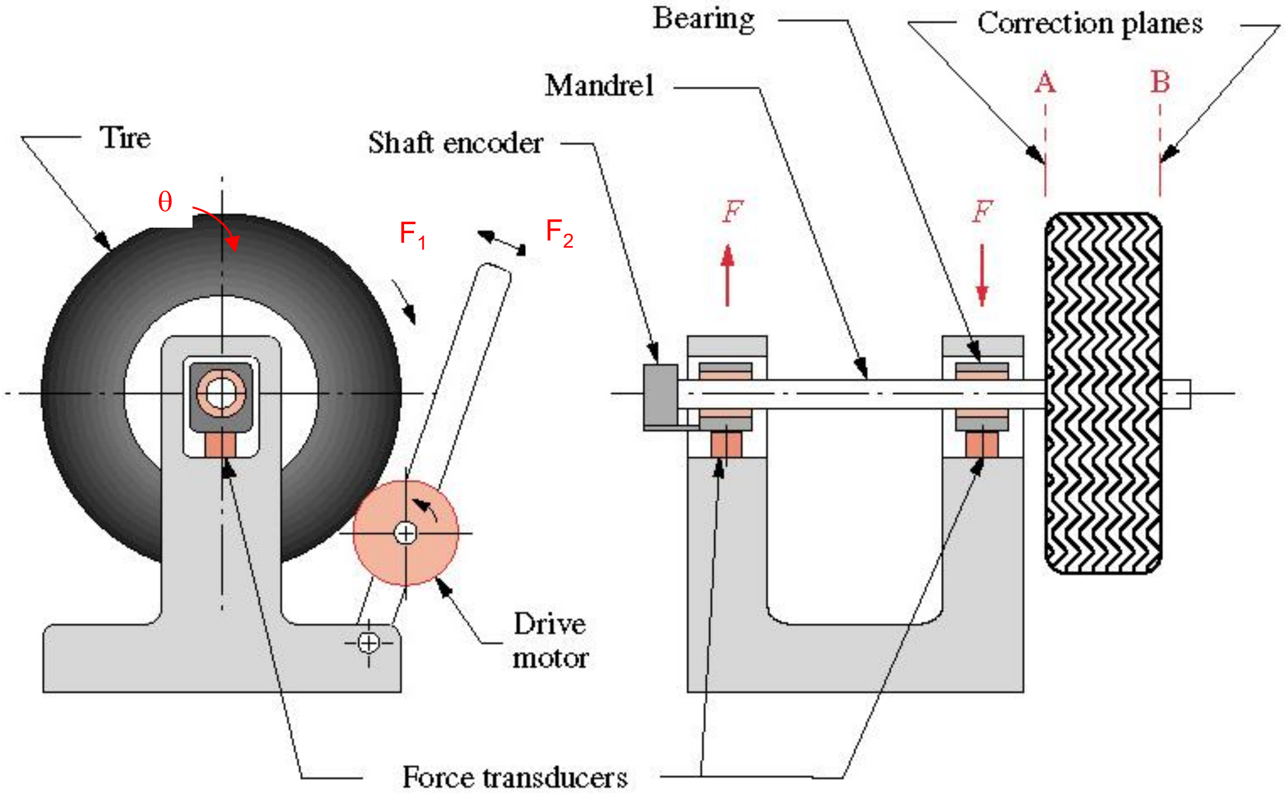
$\sum \vec{M}_A = m_1\vec{R}_1 l_1 + m_2\vec{R}_2 l_2 + m_{b2}\vec{R}_{b2} l_b = 0$

write out x & y components





Dynamic Balancing of Car Wheels



- Spin wheel
- Measure $F_1(t)$, $F_2(t)$, $\theta(t)$
- Add weights to Plane A and/or B to get
  - Force balance
  - Moment balance

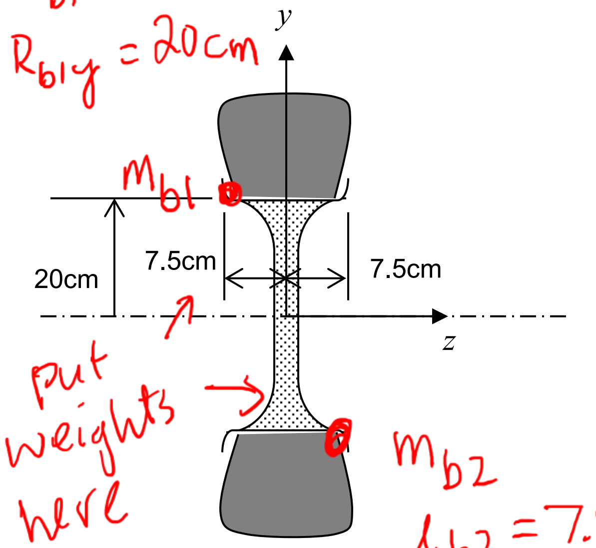
Class exercise 2: Dynamic balancing of a tire

An automobile wheel and tire system is statically balanced, but the mass distribution is such that $\sum m_i R_{iy} l_i = 10{,}000$ g cm^2. Add two balancing weights to complete the dynamic balancing of the wheel/tire system. Assume that the balancing weights are the same weight. Place the weights on the rim, at a radius of 20 cm and on planes that are located 7.5 cm on either side of the center of gravity of the wheel. How much weight (in grams) must be added to dynamically balance the wheel? Mark where the weights should be added on the sketch below.

Hint: to balance the system, we want $\sum m_i R_{iy} l_i = 0$, therefore what sign should the product of $R_{by} l_b$ have for each balancing weight?

$l_{b1} = -7.5$ cm

$R_{b1y} = 20$cm



put weights here

m_{b2}

$l_{b2} = 7.5$cm

$R_{b2y} = -20$cm

given: $\sum m_i R_{iy} l_i = 10{,}000$ g cm^2

use sum of moments $\sum m_i \vec{R}_i l_i = 0$

$$\therefore 10{,}000 \text{ gcm}^2 + \underbrace{m_{b1} R_{b1y} l_{b1} + m_{b2} R_{b2y} l_{b2}}_{-10{,}000} = 0 \Rightarrow (R_y l)_b < 0$$

$\Rightarrow$ must place masses on opposite sides

$m_{b1} = m_{b2} = m_b$

$$\Rightarrow m_b = \frac{10{,}000 \text{ gcm}^2}{2(20\text{cm})(7.5\text{cm})}$$

$$\boxed{m_b = 33\text{g}}$$