# Module 6:

# Lecture 19

# Motors, Cams, and Motion Control – Part 4

ME 370 - Mechanical Design 1

# Lecture 19: Motors, Cams, Motion Control

**Topics: 10/29/25** Module 6, Part 4: Motion Control 1 (Chapter 8).

**Activities & Upcoming Deadlines**

- **HW:**
  - **HW 9 (Gears):** due Tuesday 11/4

- **Lab 10: Motors & Gears**
  - Review Pre-lab, In-lab and Post-lab. Pre-lab and Post-lab due during usual timeline
  - Meet in Innovation Studio

- **Project 2:**
  - If applicable to your team, pickup remaining parts kit materials during Lab 10
  - Gear ratio for Pololu motor: some motors have 1:47 gear ratio, others have 1:34 gear ratio.
    - See part number stamped on motor, or determine your No-Load Speed using your phone (follow procedure similar to in-lab activity, an instructional sheet will be available shortly)
    - URL for 1:47 motor (item 3229): https://www.pololu.com/product/3229
    - URL for 1:34 motor (item 3228): https://www.pololu.com/product/3228

check specs in Proj2 Desription for batteries + motors

**Next lecture:** Start Module 7, Dynamic force analysis (DFA) (Chaps 10.1-10.8,11)

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# New class policy

- No social media during class time

# Module 6 topics: Motors, Cams and Motion Control

- **Motors**
  - DC motor principle
  - DC motor model
  - Linear Motor Model
    - Constraints
    - Behavior in time
    - Gearboxes
  - Motor Parameters
  - Power and Efficiency
- **Cam and Follower**
  - Types of Motion
  - Types of Follower
  - Practical Considerations
- **Motion control**
  - Simple Motion Control Dwell-Rise-Dwell motions:
    - Fundamental Law of Cam (Motion) Design
    - Simple Harmonic Motion
    - Sinusoidal Acceleration (i.e., Cycloidal Displacement)
  - Advanced Motion Control
    - Additional Dwell-Rise-Dwell motions:
      - Trapezoidal acceleration
      - Modified Trapezoidal acceleration
      - Modified Sine acceleration
      - 3-4-5 Polynomial Rise Displacement
      - 4-5-6-7 Polynomial Rise Displacement
    - Rise-Fall-Dwell motions:
      - Cycloidal Motion
      - Double Harmonic
      - 3-4-5-6 Polynomial

# Recall: Fundamental Law of Cam (Motion) Design

The cam function (= follower motion) must be continuous through first and second derivatives of displacement across entire interval (360°).

$\therefore$ requires 3rd order continuity

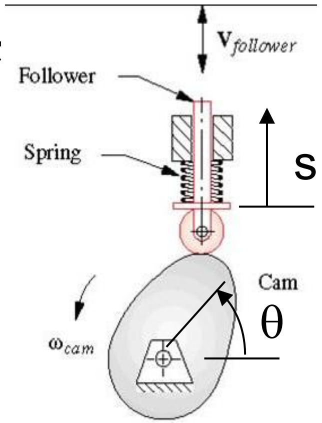
- Displacement ($C^0$ continuity)
- Velocity ($C^1$ continuity)
- Acceleration ($C^2$ continuity)

- In other words, position, velocity and acceleration should be continuous across the entire interval (360°).
- **Corollary:** The jerk function must be finite across the entire interval (360°).

# **Recall: Linear Functions** (for Rise or Fall portions)
# Dwell-Rise-Dwell-Fall Example

**Example:**

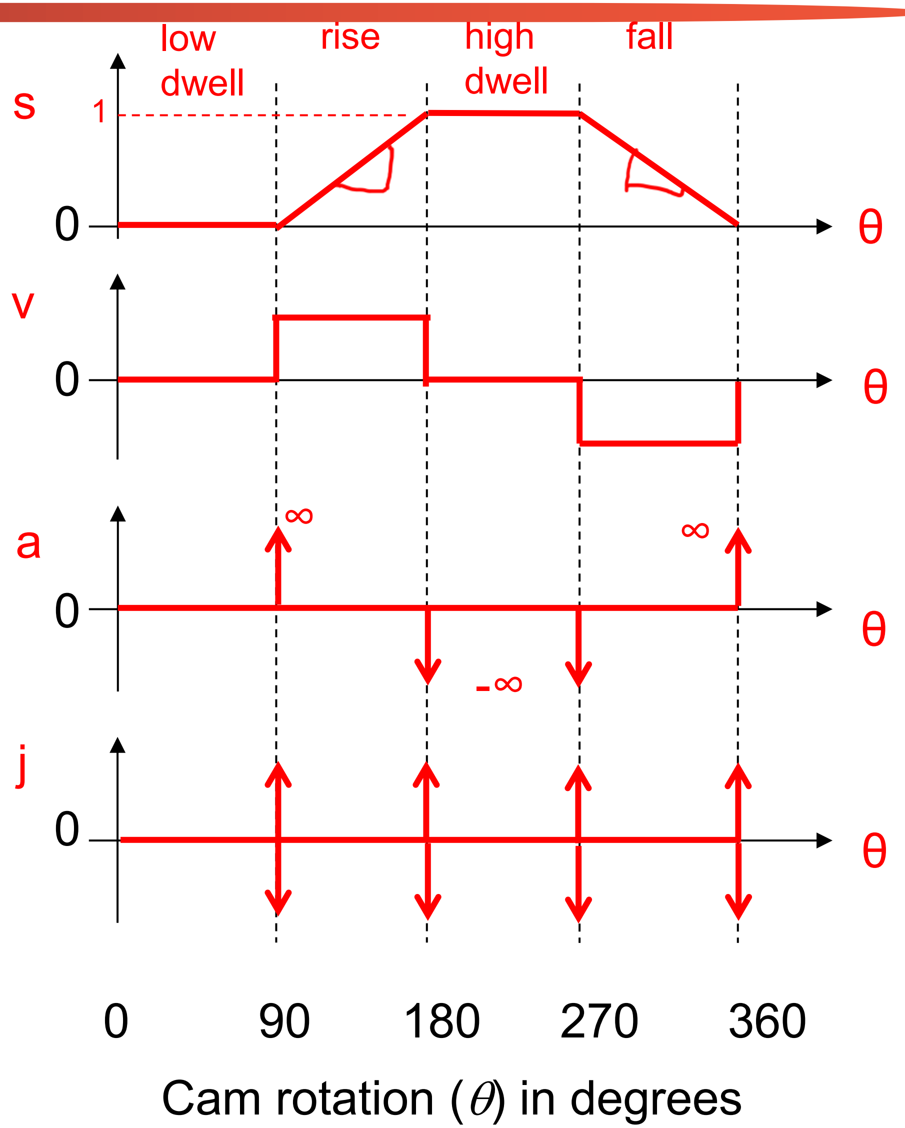
- **Dwell**
  - at zero displacement for 90 degrees (low dwell)
- **Rise**
  - 1 inch in 90 degrees
- **Dwell**
  - at 1 inch for 90 degrees (high dwell)
- **Fall**
  - 1 inch in 90 degrees
- Cam's ω
  - 2π radians/sec = 1 rev/sec




**Linear functions do NOT satisfy the Fundamental Law of Motion Design:**

**Acceleration is NOT continuous across the entire interval (360°).**

**Spikes in acceleration due to step change in velocity.**

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# **Recall: Simple Harmonic Motion** (For only Rise portion)

## Dwell -Rise-Dwell Example

When used in a **Dwell-Rise-Dwell** Motion Program:

**Simple harmonic function does NOT satisfy the Fundamental Law of Motion Design:**

**Acceleration is NOT continuous across the entire interval (360°).**

**Spikes in jerk due to discontinuity in acceleration at edges of rise (where meets dwell).**

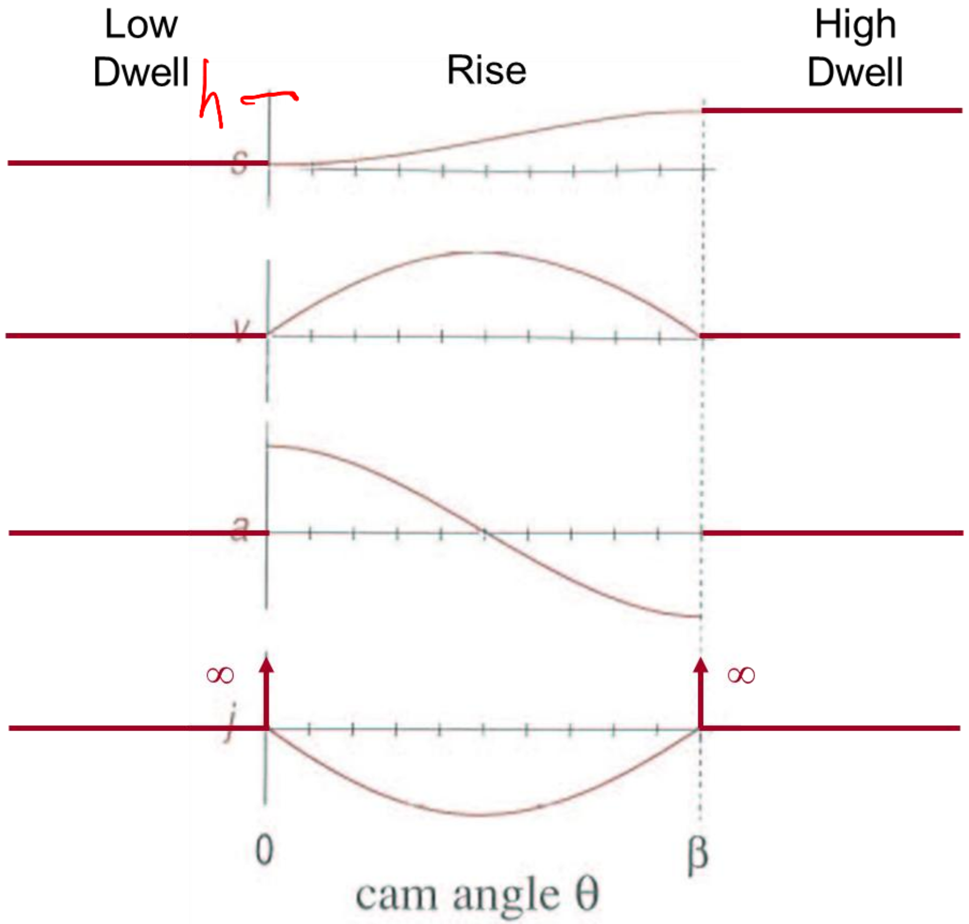
Use Simple Harmonic for Rise

$$s = \frac{h}{2}\left[1 - \cos\left(\pi\frac{\theta}{\beta}\right)\right]$$

$$v = \frac{\pi}{\beta}\frac{h}{2}\sin\left(\pi\frac{\theta}{\beta}\right)$$

$$a = \frac{\pi^2}{\beta^2}\frac{h}{2}\cos\left(\pi\frac{\theta}{\beta}\right)$$

$$j = \frac{\pi^3}{\beta^3}\frac{h}{2}\sin\left(\pi\frac{\theta}{\beta}\right)$$




Note: this design does not represent full 360° motion

# Recall: **Cycloidal displacement (Sinusoidal Acceleration)** (For only Rise portion) Dwell -Rise-Dwell Example

For a **Dwell-Rise-Dwell** Motion Program:

**Cycloidal displacement function <u>does satisfy</u> the Fundamental Law of Motion Design:**

**Velocity and Acceleration are continuous across the entire interval (360°).**

**Jerk function must be finite across entire interval (no spikes at edges of rise where meets dwell).**

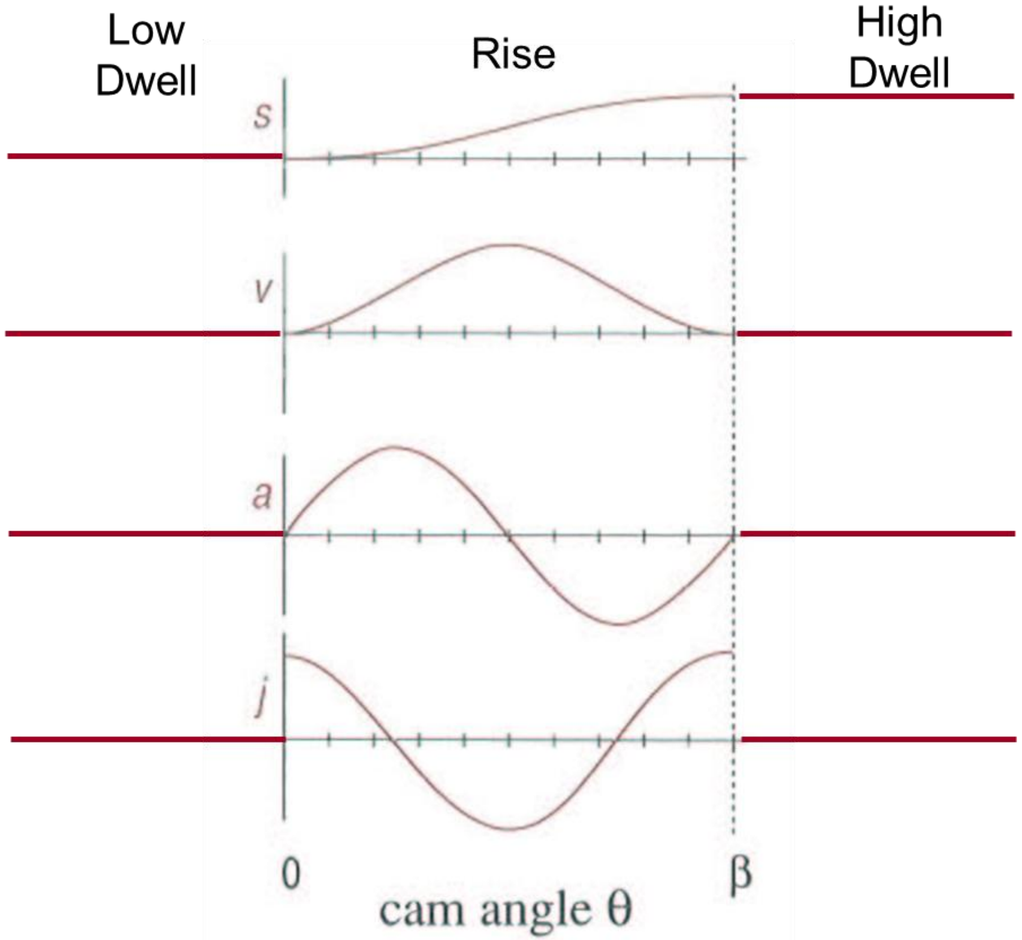
Use Cycloidal Displacement for Rise

$$s = h\left[\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\left(2\pi\frac{\theta}{\beta}\right)\right]$$

$$v = \frac{h}{\beta}\left[1 - \cos\left(2\pi\frac{\theta}{\beta}\right)\right]$$

$$a = 2\pi\frac{h}{\beta^2}\sin\left(2\pi\frac{\theta}{\beta}\right)$$

$$j = 4\pi^2\frac{h}{\beta^3}\cos\left(2\pi\frac{\theta}{\beta}\right)$$

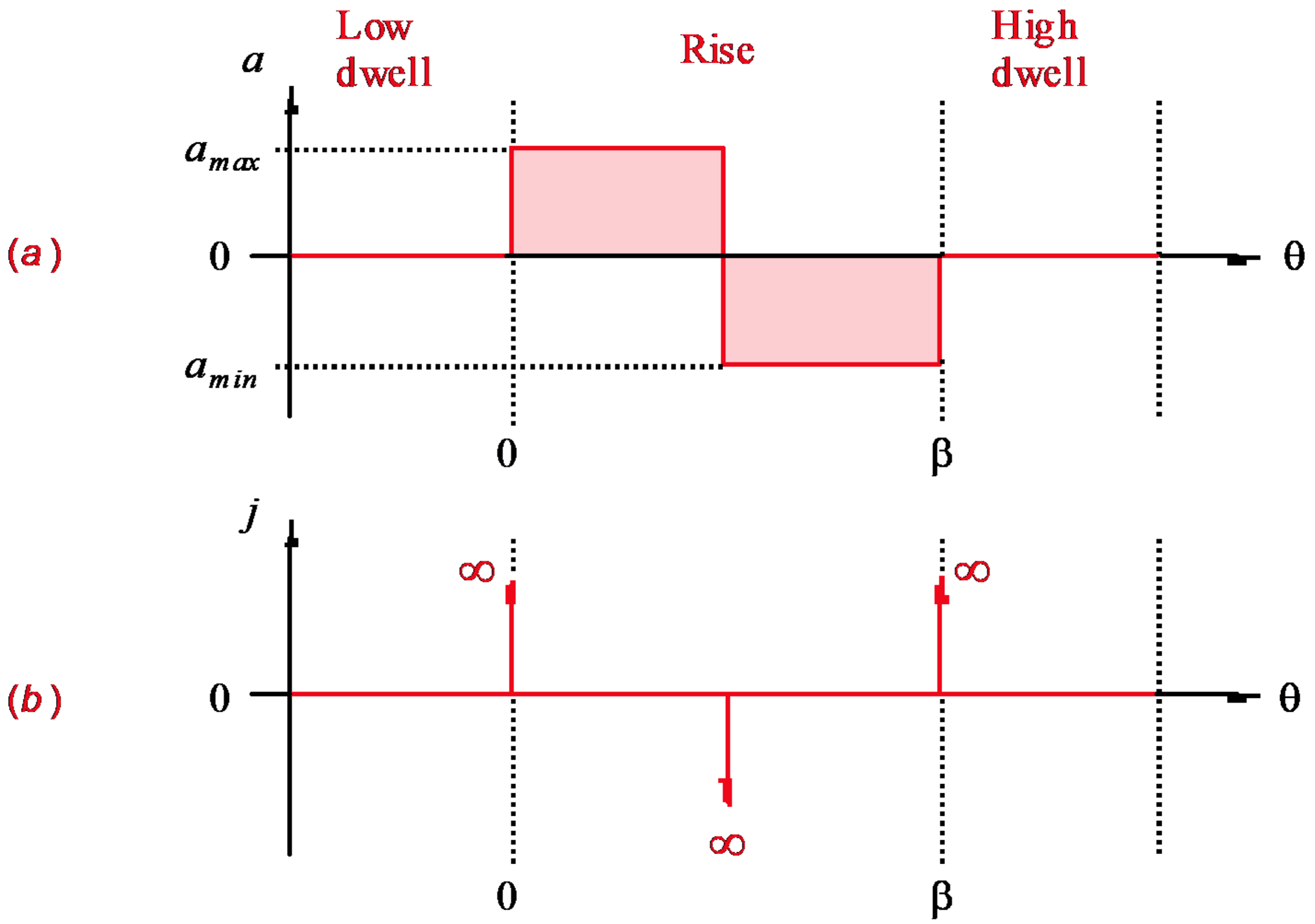



**Problem: peak acceleration can be high.**

Note: this design does not represent full 360° motion

# Problem: peak acceleration can be high. How to reduce?

Let's consider constant acceleration with smaller value:




Bad

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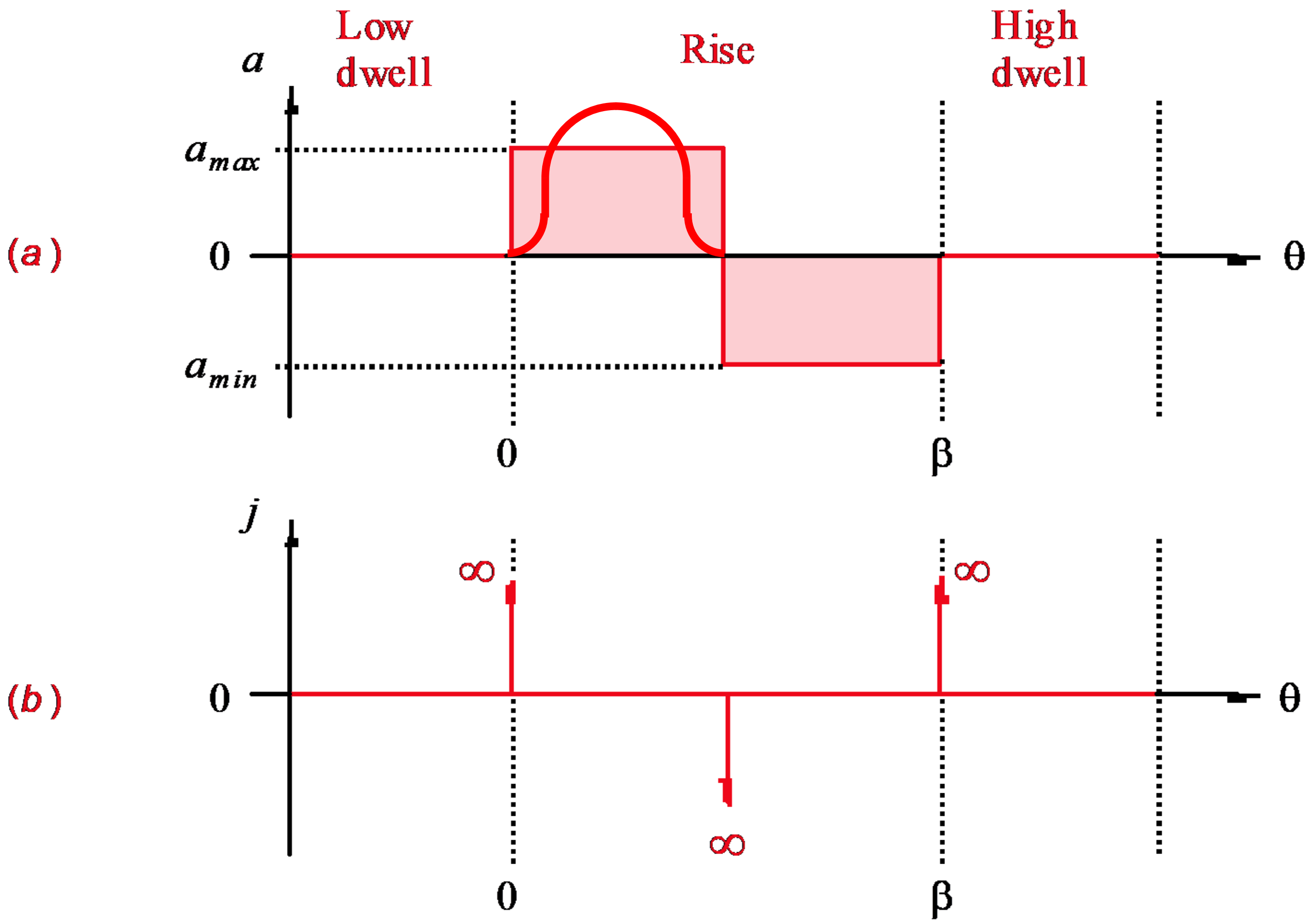
# Problem: peak acceleration can be high. How to reduce?

Adding smoother (but slightly larger peak) acceleration function could be better

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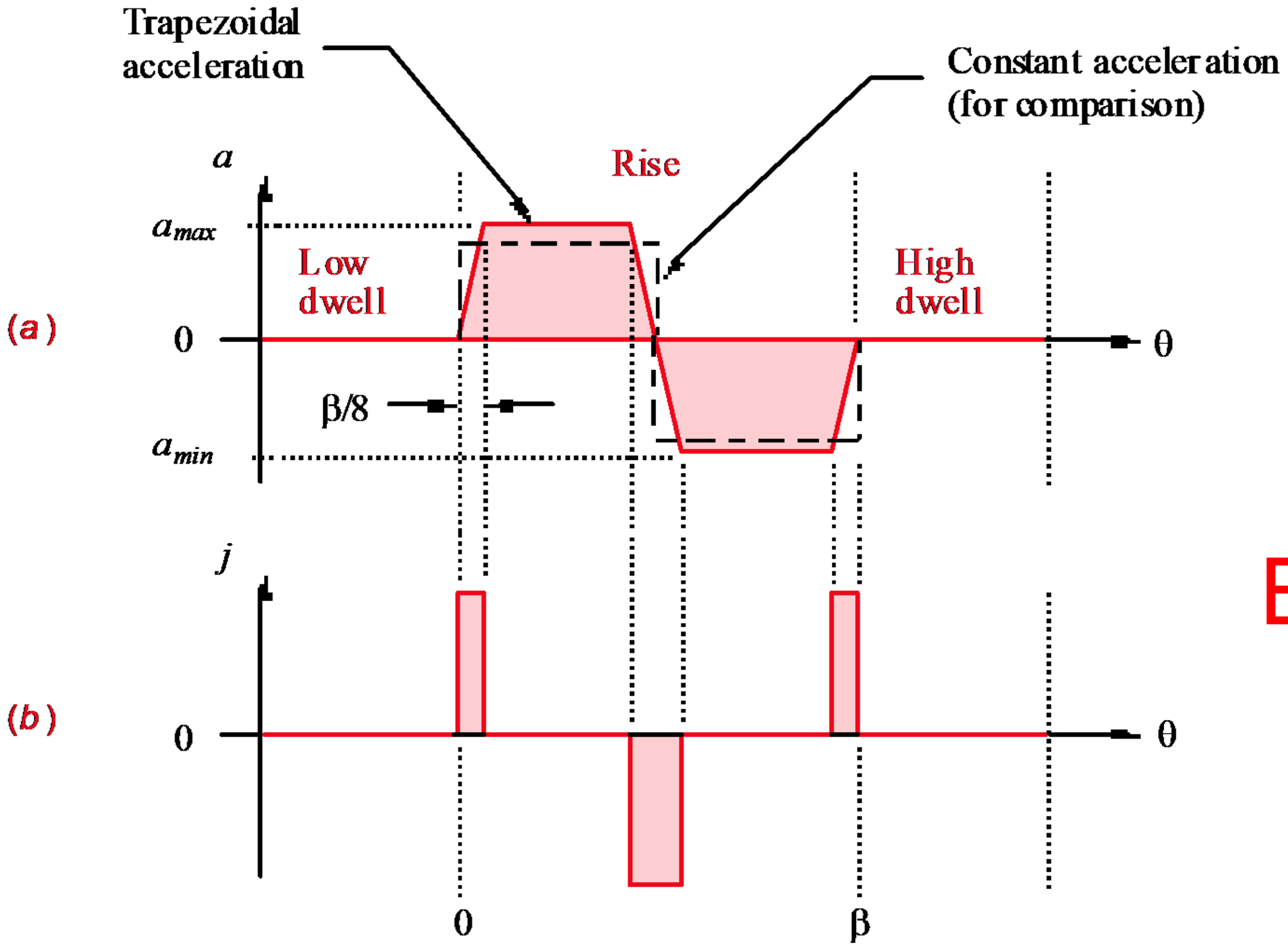
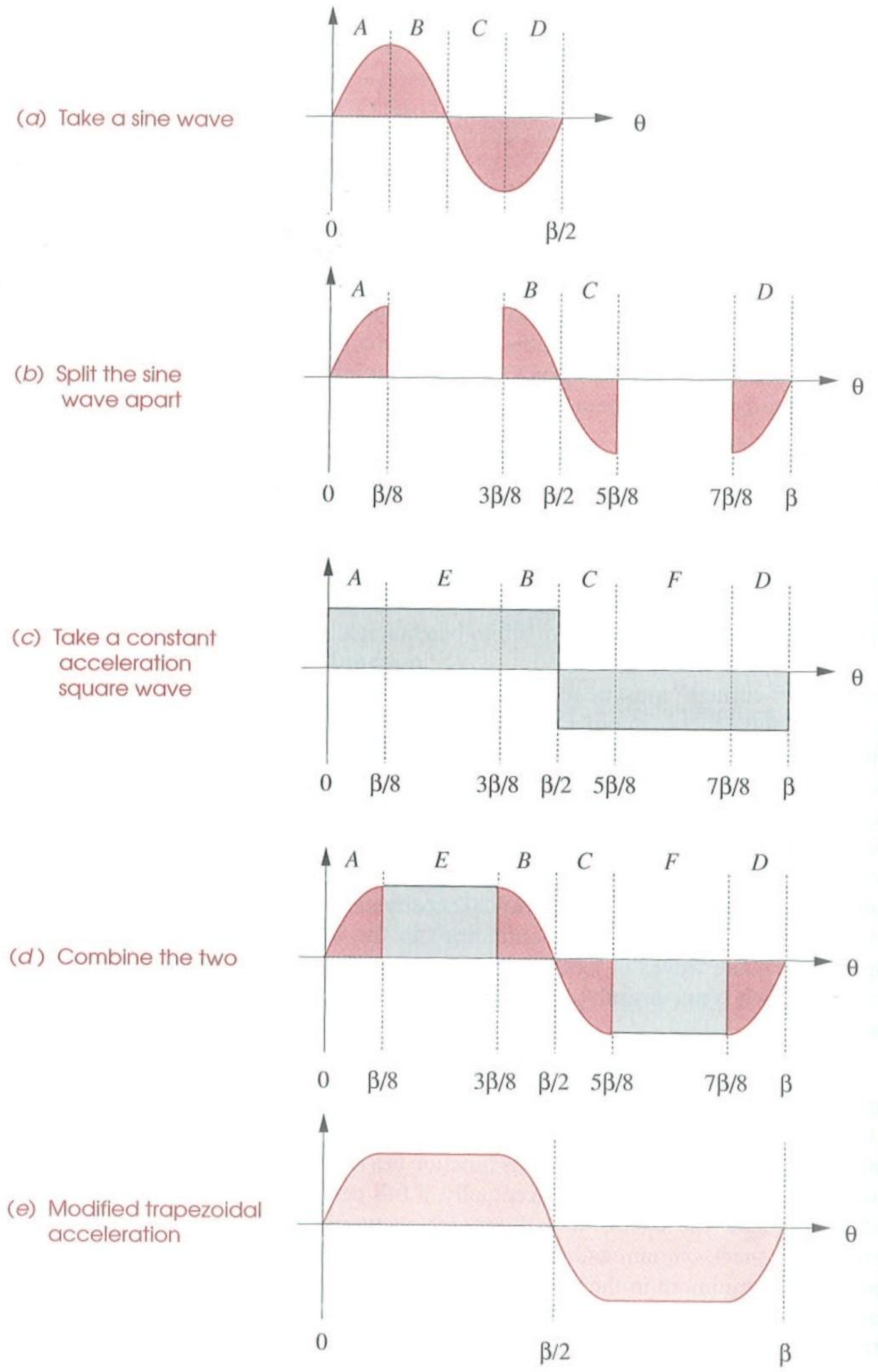
# Possible Solution: Near constant acceleration profile -- **Trapezoidal acceleration**




Better

# Modified Trapezoidal Acceleration

Much Better

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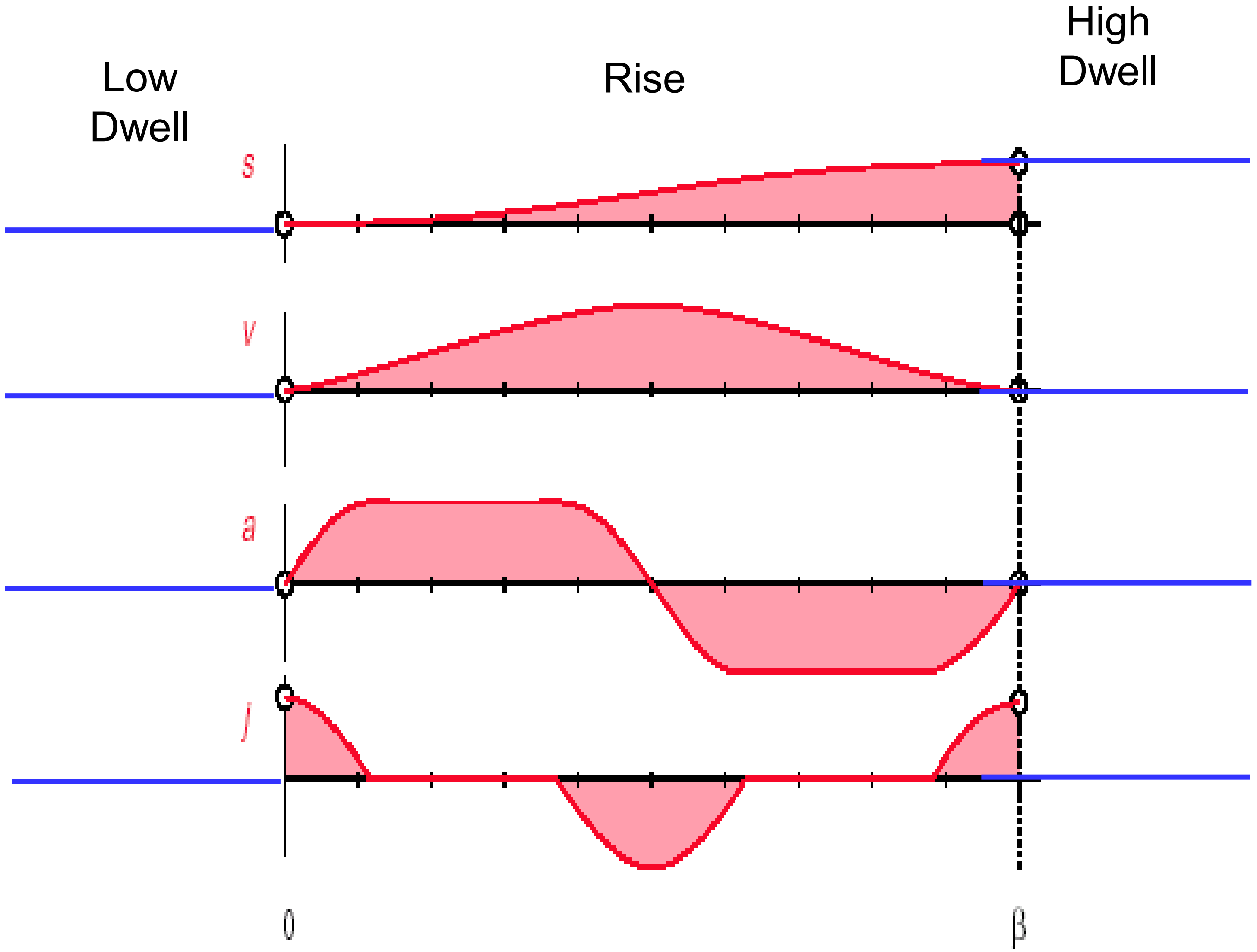
# SVAJ diagrams for Modified Trapezoidal Acceleration

**Modified Trapezoidal Acceleration function satisfies the Fundamental Law of Motion Design**

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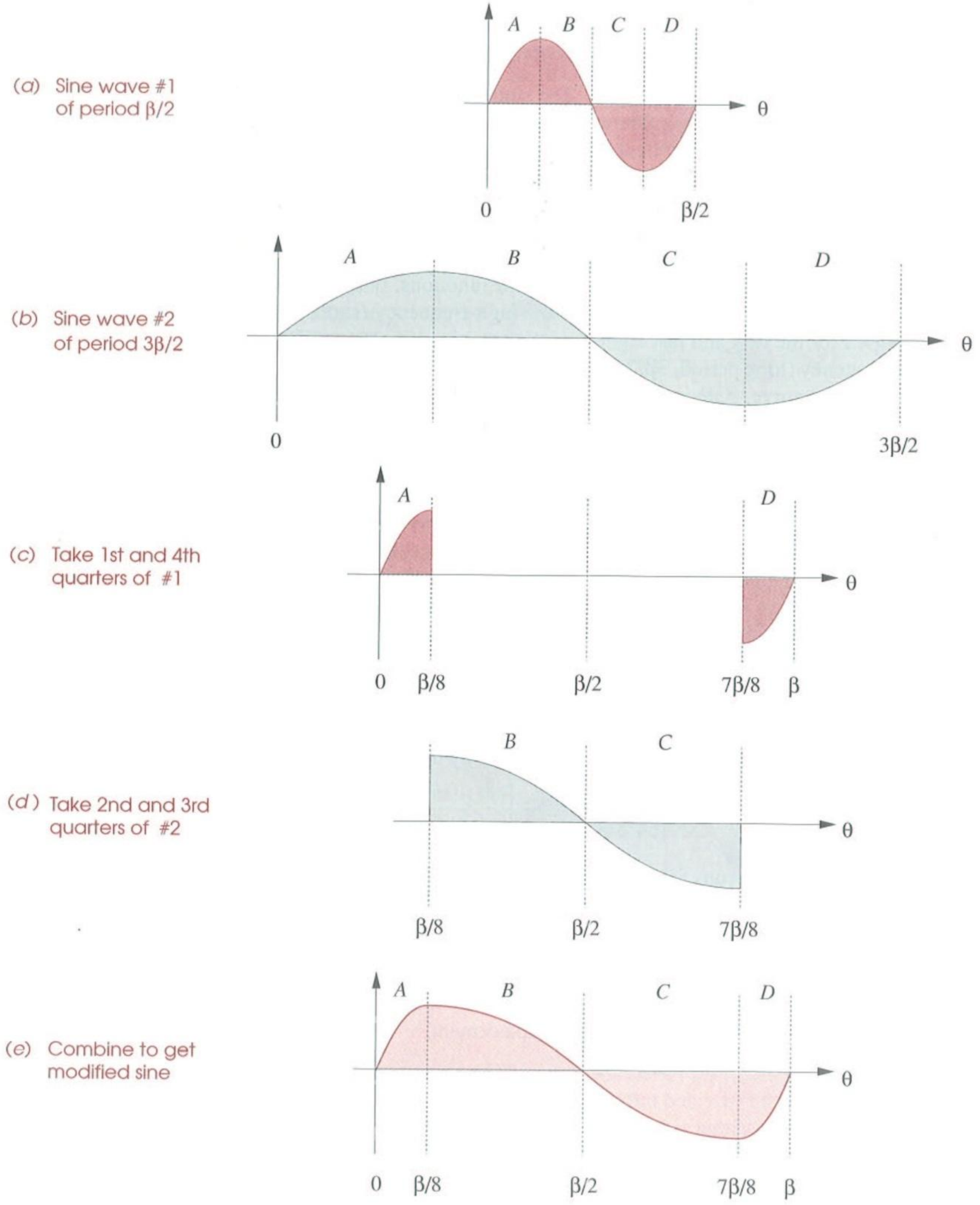
# Another alternative: **Modified Sine Acceleration**

**Modified Sine Acceleration function also satisfies the Fundamental Law of Motion Design**

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# What other functions can we use?

## Polynomial Functions

- Most versatile approach

$$s = C_o + C_1 x + C_2 x^2 + C_3 x^3 + \ldots + C_n x^n$$

where $x = \frac{\theta}{\beta}$ or $\frac{t}{t'}$ $\quad (0 \le x \le 1)$

$\theta$ = current camshaft angle

$\beta$ = total angle of rise interval

$t$ = current rise time

$t'$ = total time of rise interval

- Degree of polynomial ($n$) depends on # boundary conditions ($k$) such that $n = k - 1$.
- Solve for $C_i$ using BCs

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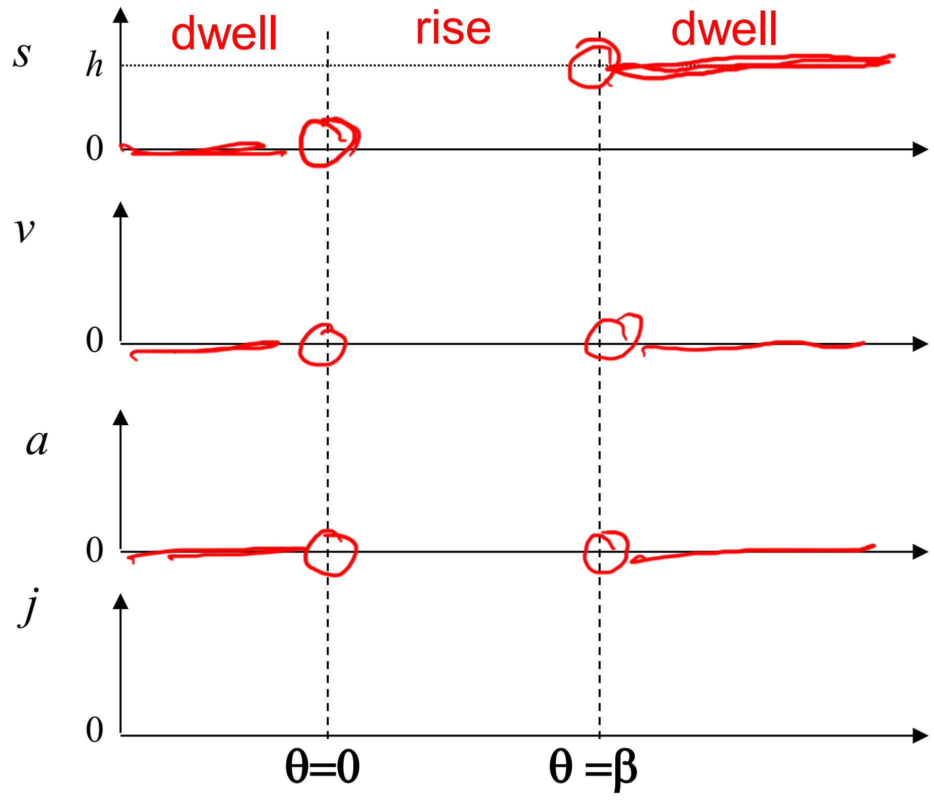
# How to determine Boundary Conditions and *k*?

- Sketch SVAJ diagram based on design requirements
- ***To satisfy Fundamental Law of Motion Design, need at least 6 boundary conditions***




BC (**<u>for Rise</u>**):

if $\theta = 0$ (or $t = 0$), then $s = 0$, $v = 0$, $a = 0$

if $\theta = \beta$ (or $t = t'$), then $s = h$, $v = 0$, $a = 0$

# For 6 BC → $n = 5$

$n = k-1$ , $k = 6$

$$s = C_o + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + C_5 x^5 \quad \textbf{Eqn 1}$$

For DWELL-RISE-DWELL, where $x = \dfrac{\theta}{\beta}$ or $\dfrac{t}{t'}$ $(0 \le x \le 1)$

BC:

if $\theta = 0$ (or $t = 0$), then $s = 0$, $v = 0$, $a = 0$

if $\theta = \beta$ (or $t = t'$), then $s = h$, $v = 0$, $a = 0$

Solve for $C_i$ by inserting BC into Eqn 1 or taking derivatives of Eqn 1.

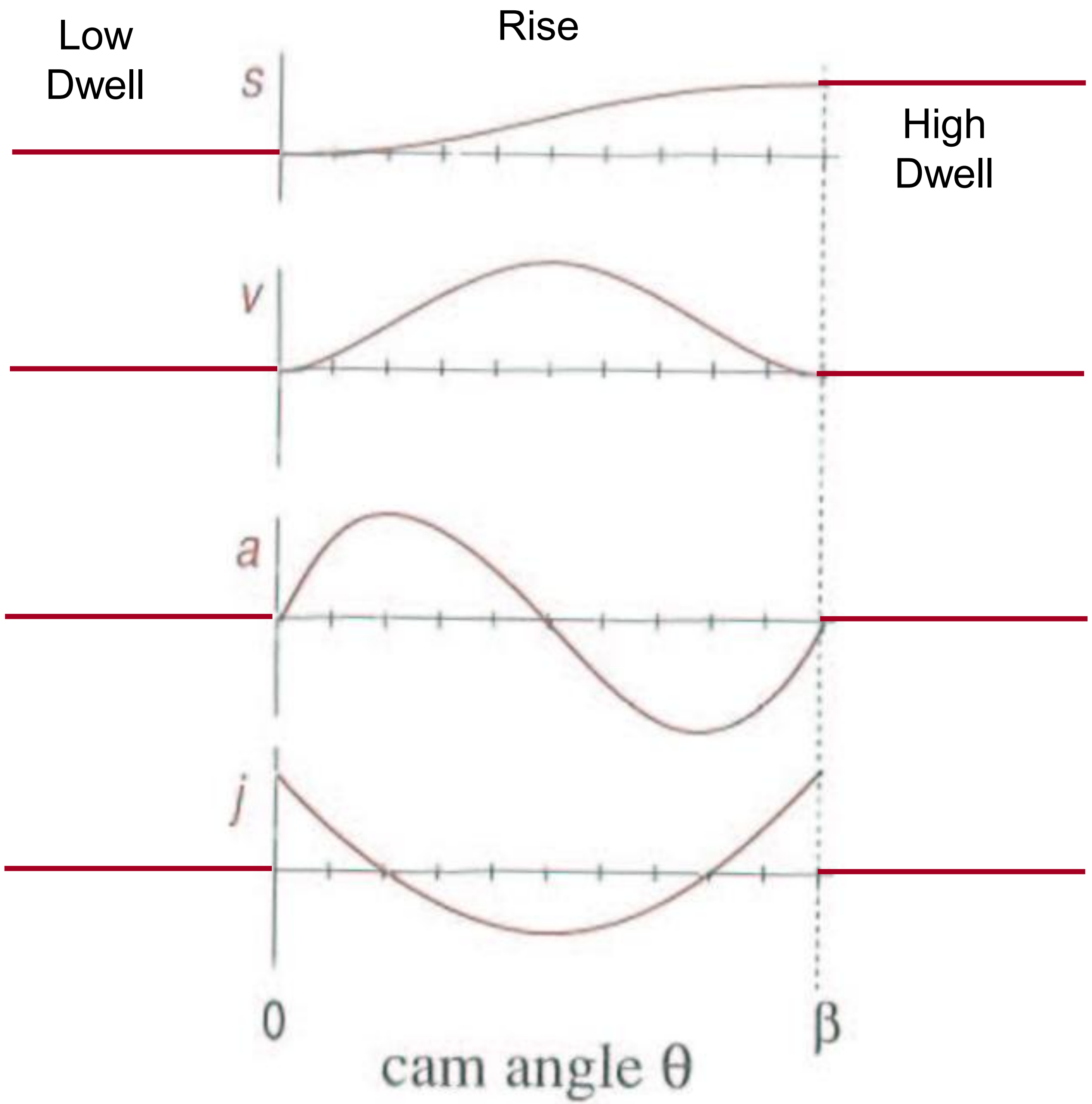
Will get

$$\boxed{s = h(10x^3 - 15x^4 + 6x^5)}$$

**This motion program is called a 3-4-5 polynomial rise function**

# 3-4-5 Polynomial Rise Displacement

$$s = h\left[10\left(\frac{\theta}{\beta}\right)^3 - 15\left(\frac{\theta}{\beta}\right)^4 + 6\left(\frac{\theta}{\beta}\right)^5\right]$$




Although this function and its derivatives satisfy the fundamental law, this function is not necessarily a great design because it causes a discontinuity in jerk.

# How to remove discontinuity in jerk?

**Add BC for jerk**

8 BC → $n = 7$

$$s = C_o + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + C_5 x^5 + C_6 x^6 + C_7 x^7$$

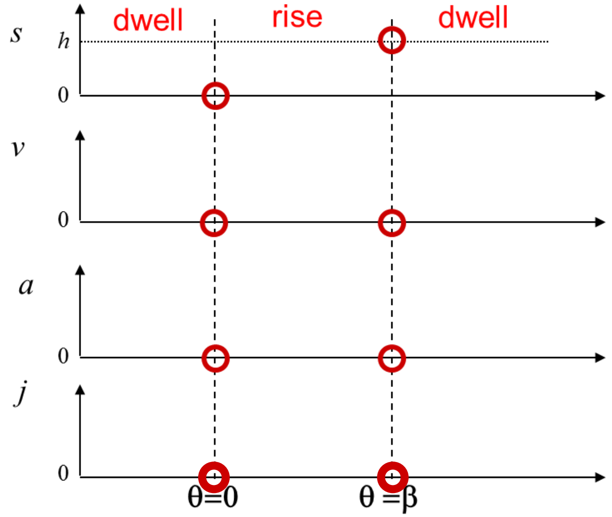
For DWELL-RISE-DWELL,

BC:

if $\theta = 0$ (or $t = 0$), then $s = 0$, $v = 0$, $a = 0$, $\boldsymbol{j = 0}$

if $\theta = \beta$ (or $t = t'$), then $s = h$, $v = 0$, $a = 0$, $\boldsymbol{j = 0}$

will get




$$s = h(35x^4 - 84x^5 + 70x^6 - 20x^7)$$

**This is called the 4-5-6-7 polynomial rise function.**

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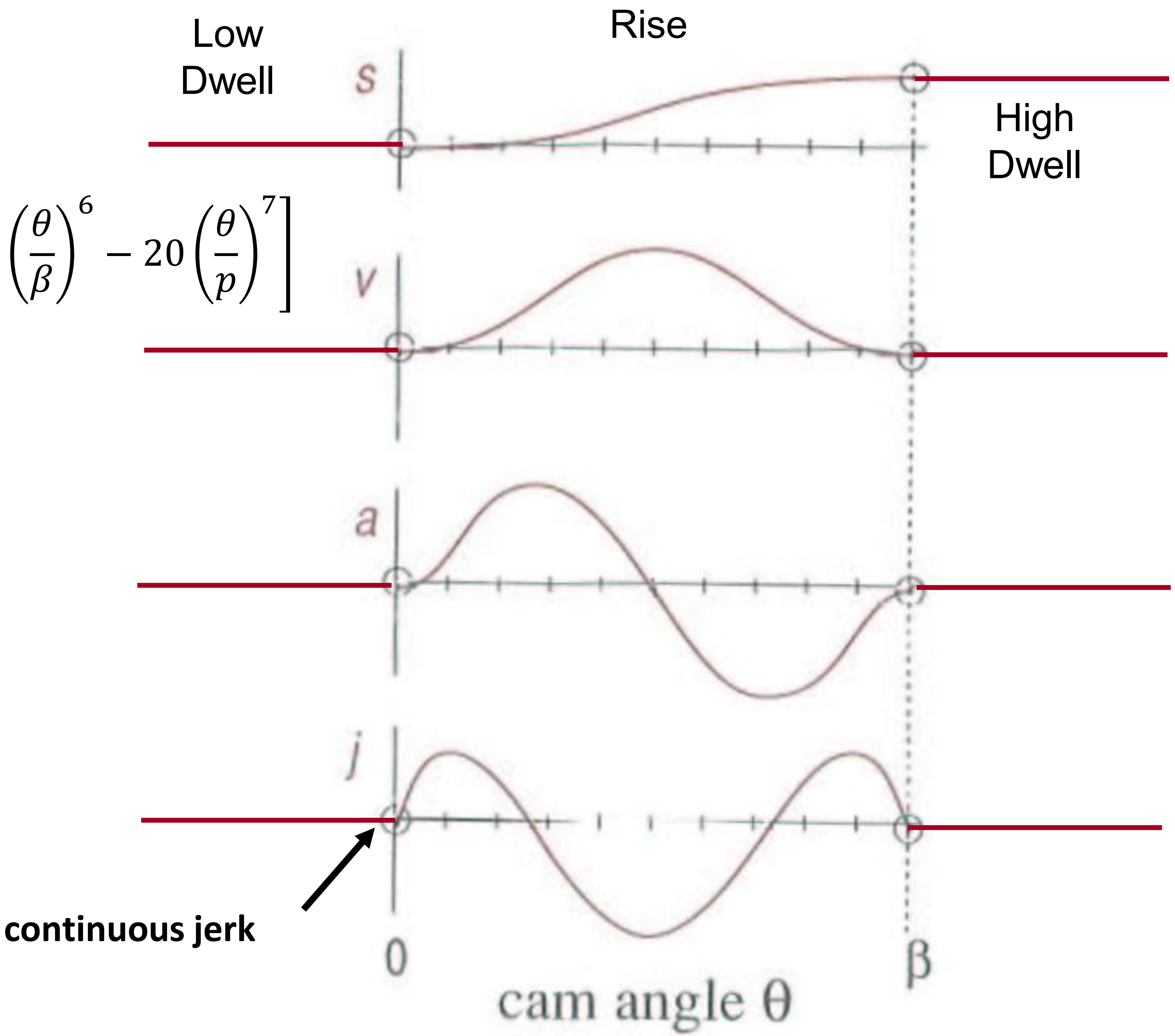
# 4-5-6-7 Polynomial Displacement

$$s = h\left[35\left(\frac{\theta}{\beta}\right)^4 - 84\left(\frac{\theta}{\beta}\right)^5 + 70\left(\frac{\theta}{\beta}\right)^6 - 20\left(\frac{\theta}{p}\right)^7\right]$$



# Other considerations

$F = m\,A$

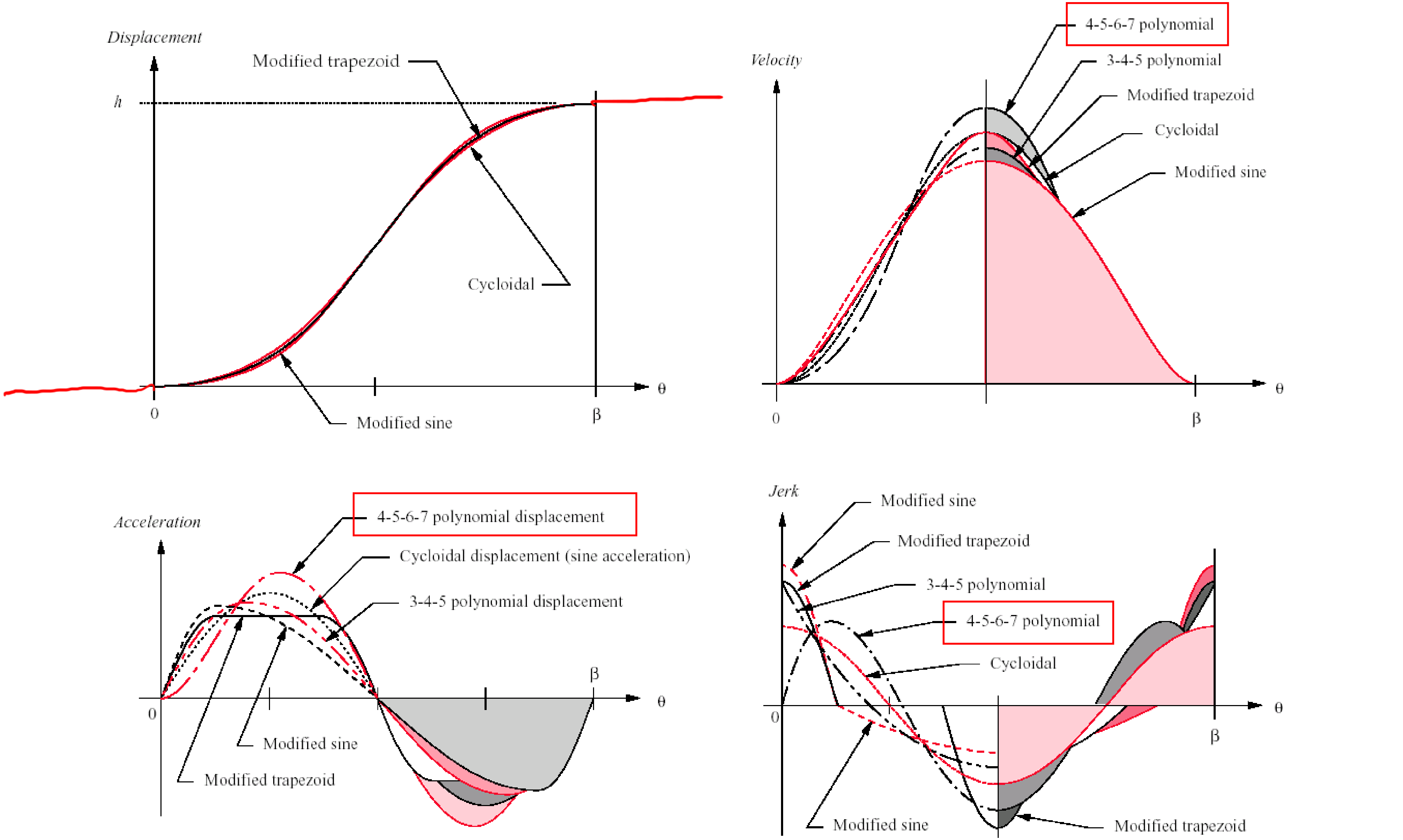
- Want to minimize dynamic forces
- Keep A small

$K = \frac{1}{2}\,m\,V^2$

- Keep $V$ small
- Want to minimize stored kinetic energy, especially if $m$ of follower train is large

*While we will be emphasizing cams, these concepts apply to motion programs in general*

# Comparison of Five Double-Dwell Functions



# Summary of motion programs *for rise displacement*

Dwell Rise Dwell

$x = \frac{\theta}{\beta} or \frac{t}{t'}$

modify Acc Profile

A. Simple Harmonic Motion: $s = \frac{h}{2}[1 - \cos(\pi x)]$

B. Cycloidal $5 = h\left[x - \frac{1}{2\pi}\sin(2\pi x)\right]$

C. Modified Trapezoid

D. Modified Sinusoid

E. 3-4-5 Polynomial $s = h[10x^3 - 15x^4 + 6x^5]$

F. 4-5-6-7 Polynomial $s = h[35x^4 - 84x^5 + 70x^6 - 20x^7]$

Note: $v(t) = \frac{\partial s}{\partial \theta}\frac{\partial \theta}{dt} = v(\theta)\omega$ , $a(t) = \frac{\partial v}{\partial \theta}\left(\frac{d\theta}{dt}\right)^2 = a(\theta)\omega^2$ ,

$j(t) = \frac{\partial a}{\partial \theta}\left(\frac{d\theta}{dt}\right)^3 = j(\theta)\omega^2$

- What are general motion behaviors of $s$, $v$, $a$, $j$ for each motion program?
- What are the advantages and disadvantages of each motion program?

- To create FALL function from RISE function:
  - $s_{Fall} = h - s_{Rise}$
  - $v_{Fall} = - v_{Rise}$
  - $a_{Fall} = - a_{Rise}$
  - $j_{Fall} = - j_{Rise}$

# Single Dwell Motions

- Previous functions were for Dwell-Rise-Dwell-Fall motions ("double dwell" motions)
- What happens if only have Rise-Fall-Dwell?

For example:

- **Rise** 1 inch in 90 degrees
- **Fall** 1 inch in 90 degrees
- **Dwell** at zero displacement for 180 degrees (low dwell)
- **Cam's ω** 2π radians/sec = 1 rev/sec

# Cycloidal Motion

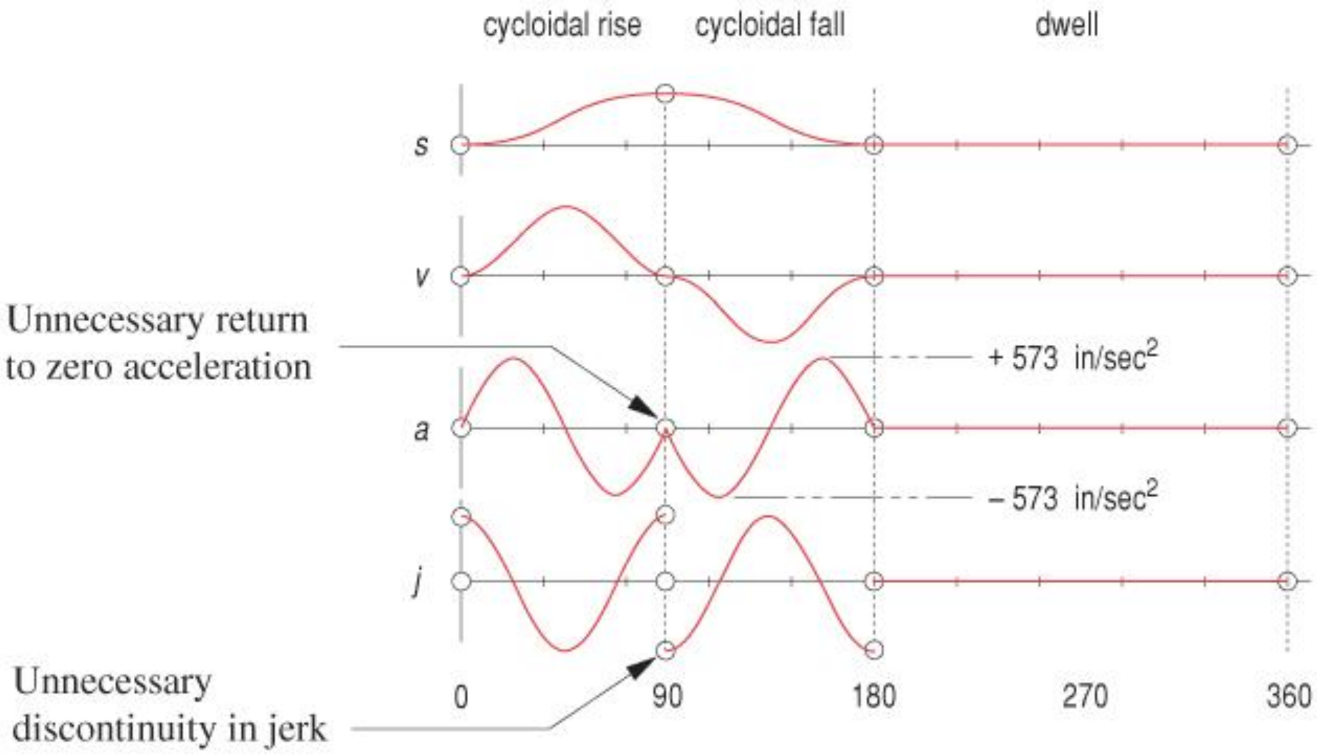



FIGURE 8-27

Cycloidal motion (or any double-dwell program) is a poor choice for the single-dwell case

- Good for doing both rise and fall portions
- Bad acceleration and jerk behaviors

# Double Harmonic

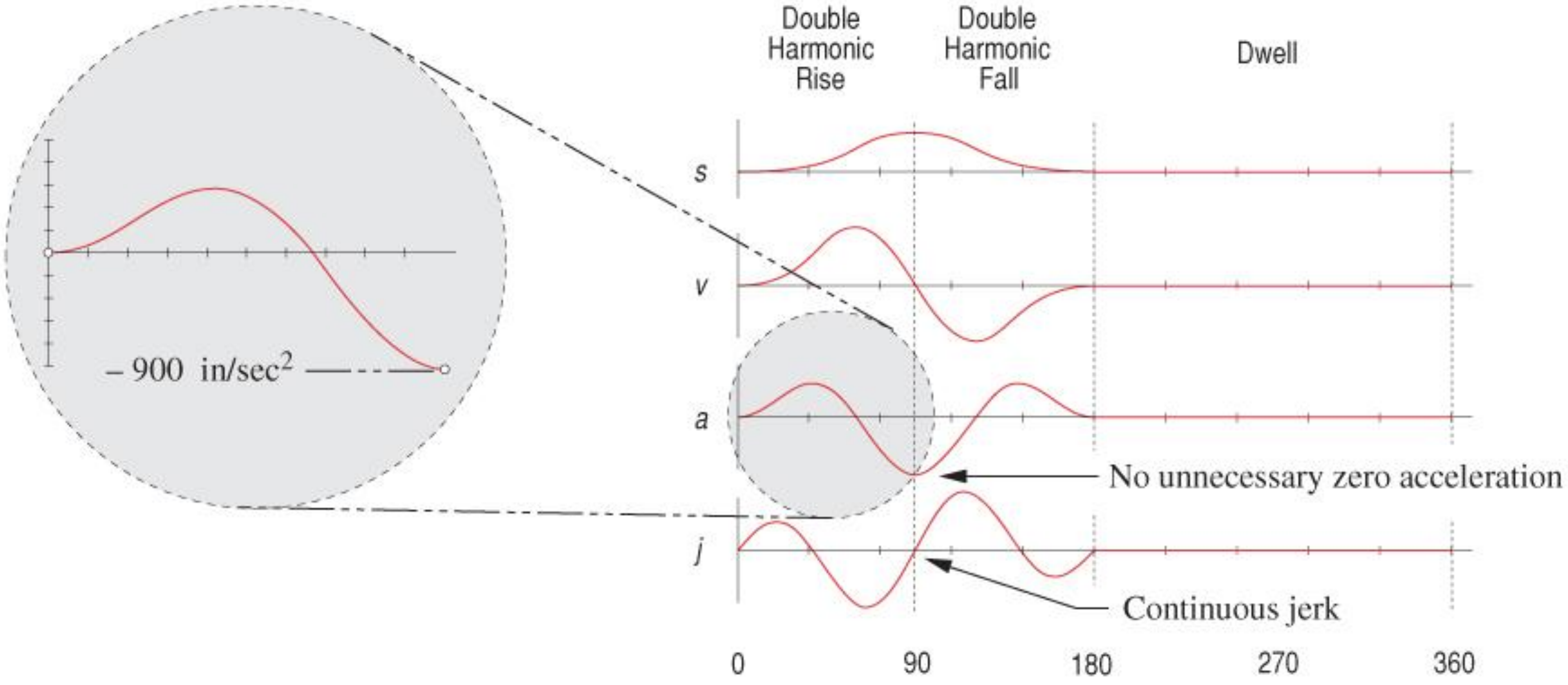



FIGURE 8-28

Double harmonic motion can be used for the single-dwell case if rise and fall durations are equal

- Good for doing both rise and fall portions
- Good acceleration and jerk behaviors

# 3-4-5-6 Polynomial

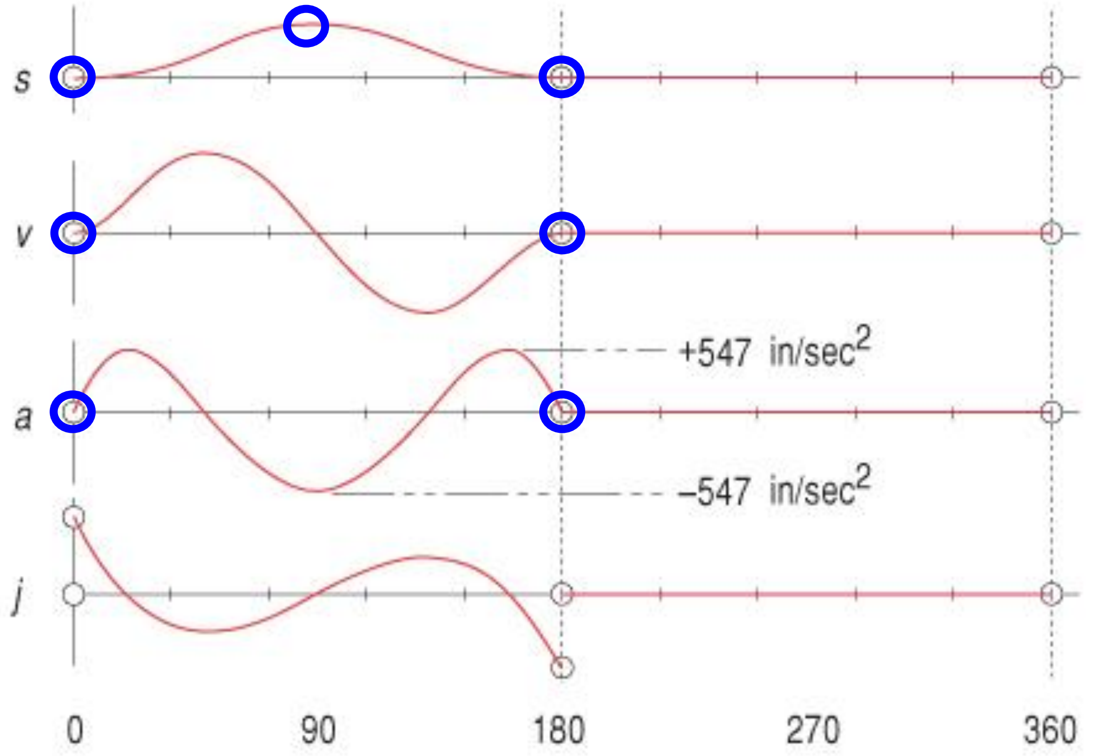



FIGURE 8-30

3-4-5-6 polynomial function for two-segment symmetrical rise-fall, single-dwell cam

- Combine both rise and fall into one segment
- Better (smaller) acceleration than double harmonic
- Need 7 BC, therefore $n$ = 6

# Class Exercise

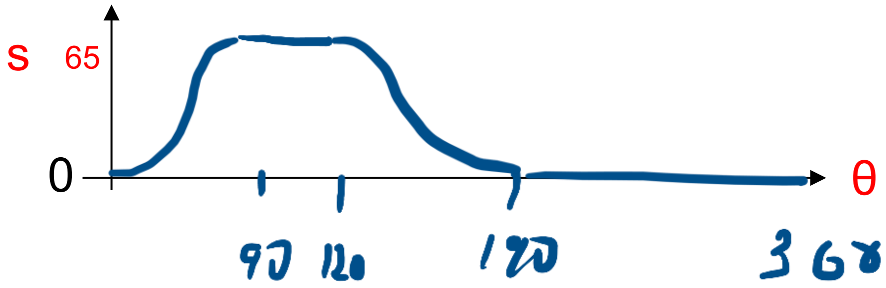
$$s = h\left[\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\left(2\pi\frac{\theta}{\beta}\right)\right]$$

- **Design Task**
  Design a cam (specify boundary conditions and piecewise functions) that moves the follower as follows:
- **Rise:** 0 → 65 mm over **90°**
- **Dwell: 30°**
- **Fall:** 65 mm → 0 over **70°**
- **Dwell:** for the remainder of the 360° cycle
- **Requirements**
- Plot the **cam displacement function** over the full cycle.
- Write the **piecewise equations** for each segment (Rise, Dwell, Fall).
- For the Rise and Fall segments, use the **Cycloidal displacement function**.
- **Recall:**
  To create a Fall function,
- $s_{Fall} = h - s_{Rise}$ , $(v,a,j)_{Fall} = -(v,a,j)_{Rise}$

# Class Exercise - solution

- **Design Task**
  Design a cam (specify boundary conditions and piecewise functions) that moves the follower as follows:
- **Rise:** 0 → 65 mm over **90°**
- **Dwell: 30°**
- **Fall:** 65 mm → 0 over **70°**
- **Dwell:** for the remainder of the 360° cycle
- **Requirements**
- Plot the **cam displacement function** over the full cycle.
- Write the **piecewise equations** for each segment (Rise, Dwell, Fall).
- For the Rise and Fall segments, use the **Cycloidal displacement function.**
- **Recall:**
  To create a Fall function,
- $s_{Fall} = h - sRise$ , $(v,a,j)_{Fall} = -(v,a,j)_{Rise}$

$$s = h\left[\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\left(2\pi\frac{\theta}{\beta}\right)\right]$$




$$s = \begin{cases} 1:\ 65mm\left[\frac{\theta}{90°} - \frac{1}{2\pi}\sin\left(2\pi\frac{\theta}{90°}\right)\right], 0 \le \theta < 90° & \beta = 90° \\ 2:\ 65, \quad 90° \le \theta < 120° \\ 3:\ 65 - 65\left[\frac{\theta-120°}{70°} - \frac{1}{2\pi}\sin\left(2\pi\frac{\theta-120°}{70°}\right)\right], 120° \le \theta < 190° & \beta = 70° \\ 4:\ 0, \quad 190° \le \theta < 360° \end{cases}$$