Energy Methods (or Virtual Work Methods)

An alternative to using Newton's second law or 'Force Balance Equations' to analyze the forces or torques on links in a mechanism.

We will now use an Energy/Work balance approach

- Based on principle of virtual work
- Only for determining external forces and torques that produce work (e.g., T_{12} or F_p)
- Not suitable if we also need internal reactions.
- Requires knowledge of accelerations and velocities ← PVA
- Does not require simultaneous solution of large systems of equations

Definitions

- Work = dot product of force (or torque) and displacement

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{R}} \quad \text{or} \quad W = \vec{\mathbf{T}} \cdot \vec{\boldsymbol{\theta}}$$

- Power = dot product of force (or torque) and velocity

$$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} \quad \text{or} \quad P = \vec{\mathbf{T}} \cdot \vec{\boldsymbol{\omega}}$$

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Here will will be interested in <u>external</u> forces, torques (that do work <u>on</u> the system)

Also,

- Power = time rate of change of energy

$$P = \frac{dE}{dt}$$

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Recall definition of dot product

$$\text{If } \vec{\mathbf{B}} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

$$\vec{\mathbf{C}} = c_x\hat{\imath} + c_y\hat{\jmath} + c_z\hat{k}$$

$$\text{then } \vec{\mathbf{B}} \cdot \vec{\mathbf{C}} = b_x c_x + b_y c_y + b_z c_z$$

$$\Longrightarrow \text{scalar!}$$

For low-friction pin joints and high-speed mechanisms

- Assume negligible energy loss due to heat, friction, noise, etc.
- Assume potential energy

$$\Longrightarrow \text{Total E} = \text{KE} + \text{PE} \qquad P = \frac{dE}{dt}$$

What are expressions for KE_{trans} and KE_{rot}?

$$P = \frac{d(PE)}{dt} = \frac{d}{dt}(mgh) = mg \cdot \vec{\mathbf{v}}_{CG}$$

$$P = \frac{d(KE_{rot})}{dt} = \frac{d}{dt}\left(\frac{1}{2} I_{CG}\boldsymbol{\omega}^2\right) = I_{CG}\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{\omega}} \qquad P = \frac{d(KE_{trans})}{dt} = \frac{d}{dt}\left(\frac{1}{2} m\vec{\mathbf{v}}_{CG}{}^2\right) = m\vec{\mathbf{A}}_{CG} \cdot \vec{\mathbf{v}}_{CG}$$

$$P = m\boldsymbol{g} \cdot \vec{\mathbf{v}}_{CG}$$

Instantaneous change in potential energy of mechanism

3 $$P = m\vec{\mathbf{A}}_{CG} \cdot \vec{\mathbf{v}}_{CG}$$

4 $$P = I_{CG}\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{\omega}}$$

Instantaneous change in kinetic energy of mechanism

Virtual work

- Total power for system of n moving links:
- Assuming fixed link is link 1

Due to external forces and torques on links

Due to inertial properties of links

$$\sum_{i=2}^{n}\left(\vec{\mathbf{F}}_i \cdot \vec{\mathbf{v}}_i\right) + \sum_{i=2}^{n}\left(\vec{\mathbf{T}}_i \cdot \vec{\boldsymbol{\omega}}_i\right) = \sum_{i=2}^{n}\left(\vec{m}_i \vec{\mathbf{A}}_{CGi} \cdot \vec{\mathbf{v}}_{CGi}\right) + \sum_{i=2}^{n}\left(I_{CGi} \vec{\boldsymbol{a}}_{\boldsymbol{i}} \cdot \vec{\boldsymbol{\omega}}_{\boldsymbol{i}}\right)$$

Velocity at point of application of external force, *not* CG!

Final solution is summation of scalar entities.

Alternative way to think of energy balance