Lecture 3
Planar Mechanisms

“Colibri” by Derek Hugger

* www.youtube.com/watch?v=1scj5sotD-E

ME 370 - Mechanical Design 1

Lecture 3: Planar Linkage Kinematics

Topics: 9/3/25 Planar Mechanisms – Part 2 (Chap 2)

Announcements

- Campuswire signup https://campuswire.com/p/G3410F2ED   Access code: 4362
- Gradescope submissions: PDF format, pages for each question properly tagged. Starting Lab 2 and HW 2, lose points for improperly formatted submissions
- Project 1 description posted on Canvas

Activities & Upcoming Deadlines

- Project Teams will be identified and used starting Lab 2
- **Lab 1:** post-lab due 1 week after lab section
- **Lab 2 (Design Lab 1):** Review lab manual. Submit Pre-lab assignment to Gradescope
- **HW 1**: Due ~~9/2~~ 9/9 in Gradescope
- **HW 2**: Posted. Due 9/9 in Gradescope

Lecture 4: Monday 9/8/25 Planar Mechanisms – Part 3 (Chap 2)

Lecture 5: Wednesday 9/10/25 Module 2 (Graphical Linkage Synthesis – Part 1 (Chap 3))

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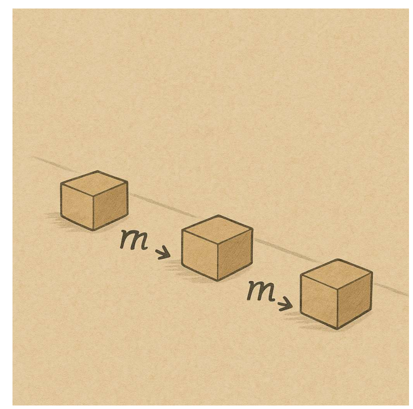
Project 1: Overview and Deliverables

- University management is seeking efficient ways to deliver small packages in dormitories, laboratories, and at McKinley Health Center.
- They want to create a scaled prototype (approximately 1/10 scale) of a legged dispensing robot that can travel down hallways and deliver items on demand.
- Project 1 will focus on the manual dispensing and timing mechanisms.



Some key system requirements

- Design and prototype a Manual Dispensing Mechanism
  - Repeated cycle with two main tasks:
    - must release exactly one object from its storage bin, allow it to drop or be discharged
    - remain in a dwell position until the next release
- The dispenser should hold at least 5 objects to be dispensed weighing 25 g or less.
- The dispenser will be hand-cranked, with no other human input other than loading objects at the beginning of the sequence.
- When mounted on legged robot in Project 2, the dispensing mechanism must release a single object every 2 meters of travel.
- Payload dispensing should be stable (no bouncing) and result in consistent orientation.
- Each team has a tracked budget of $50.00 for internal purchases from the Innovation Studio (here)
- Design must be creative, aesthetic, and human-centered.

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Project 1: Deliverables

Week	Task	Grade
9/1 – 9/5	Deliverable 1 and 2: Individual brainstorming and sketches (10%). Team brainstorming on user experience and system requirements (15%) **Lab 2 (Design Lab 1): Ideate**	Lab 2 grade
9/8 – 9/12	Deliverable 3: Initial prototyping and ideation activity **Lab 3 (Design Lab 2): Prototype**	Lab 3 grade
9/15 – 9/19	Deliverable 4: Low-fidelity prototype(s) and presentation due in lab **Lab 4: Conceptual Design Review**	40%
10/6 – 10/10	Deliverable 5: Final Prototype and Presentation **Due in Lab 7: Project 1 Final**	60%
	Peer Evaluation Multiplier – Individual final project grade will be modified based on team member peer evaluations	

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Lectures 2-4 Topics: Planar mechanism kinematics

- Planar Kinematics
- Identifying links and joints
- Degrees of Freedom
- Mechanism mobility
  - Degrees of freedom in a mechanism
- Gruebler's Equation (Gruebler-Kutzbach Equation)
- Mechanisms, structures, and preloaded structures

- Four-bar linkage mechanisms
  - Crank-rocker, double crank, double rocker, crank-slider
- Transformation Rules
- Isomers
- Gruebler's Paradoxes
- Inversions
- Grashof Condition
- Toggle Position
- Transmission angle (section 3.3)

See newly posted missing empty slides for these topics

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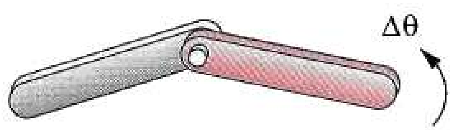
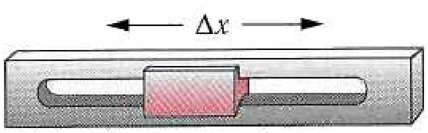
Recall- Types of joints – classified by # DOF

- Full joint: Allows 1 DOF at the joint and removes 2 DOF
- Examples:
  - *pin (rotating) joint*
  - *slider (prismatic) joint*
  - *pure rolling joint* – no slippage, e.g., gears

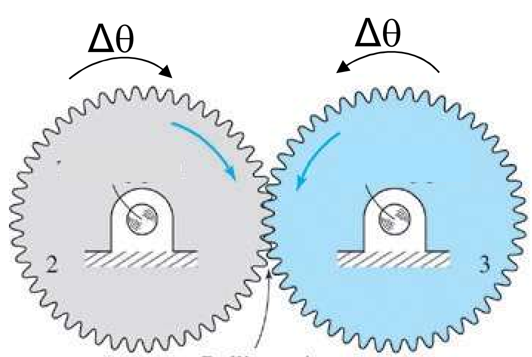
- Half joint: Allows 2 DOF at the joint, and removes 1 DOF, in planar mechanisms
- Examples
  - *pin in slot*
  - *rolling contact* – sliding



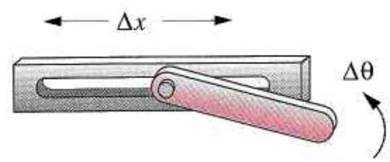
Rotating full pin (R) joint (form closed)



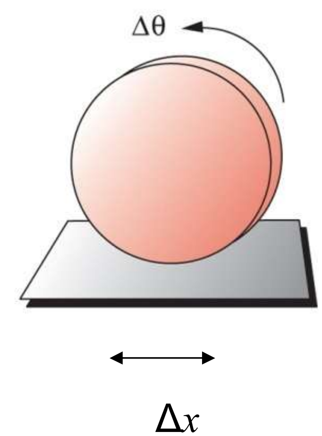
Translating full slider (P) joint (form closed)



Rolling pair
(or friction gears, no slippage)



Pin in slot (form closed)



Recall: Gruebler's Equation

- How to compute mobility or #DOF?

Lose 3 DOF by grounding one link

$$\#DOF = 3(n-1) - 2J_1 - J_2$$

Lose 1 DOF by connecting with half joint

Lose 2 DOF by connecting with full joint

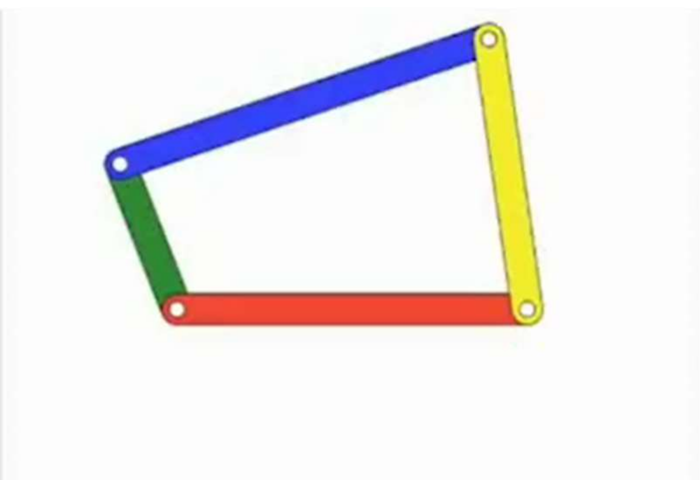
where n = # links

J_1 = # full joints

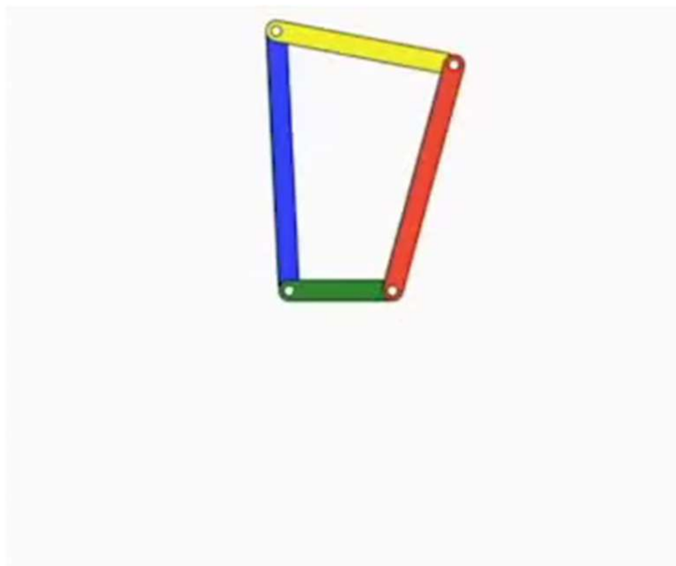
J_2 = # half joints

Four-bar linkage – basic building block mechanism

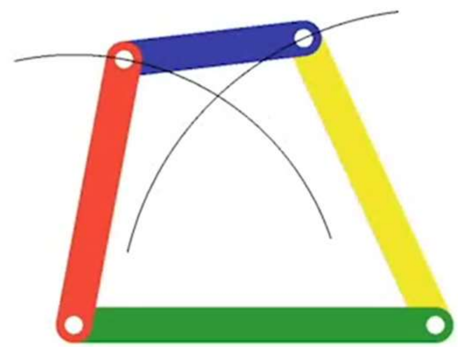
Crank-Rocker



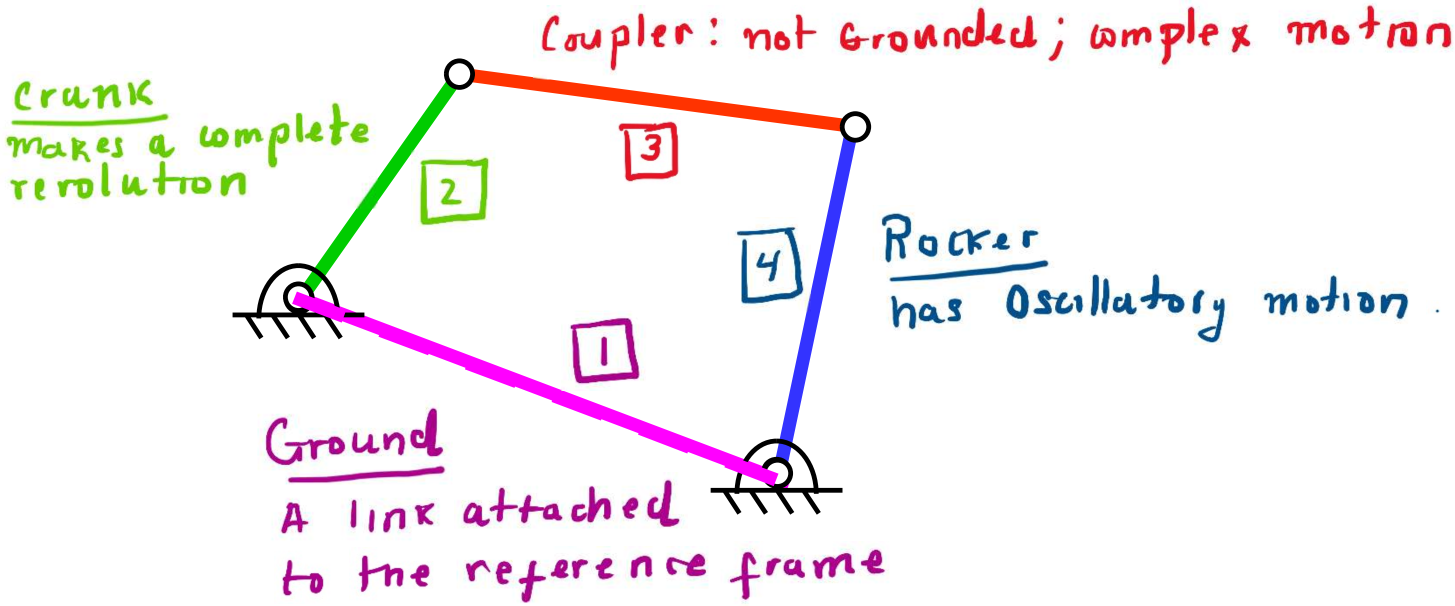
Double Crank



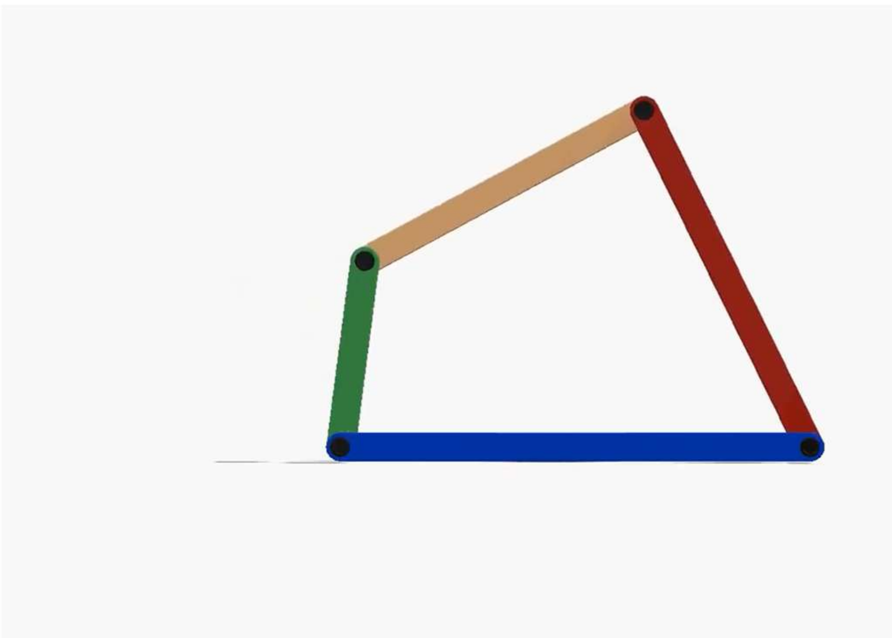
Double Rocker

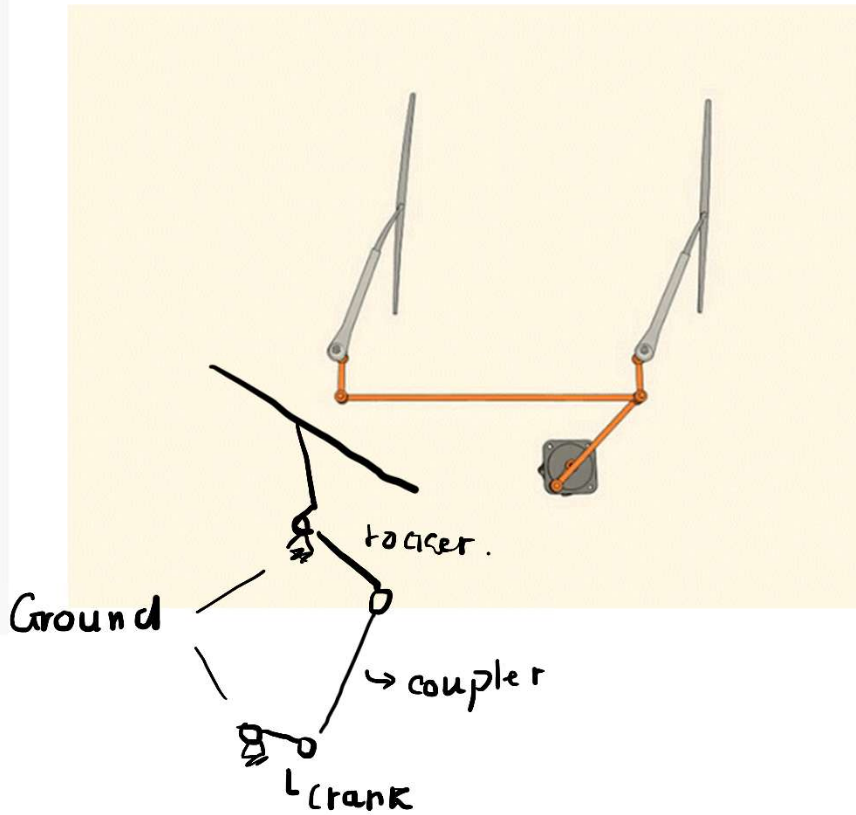


Four-bar linkage (crank-rocker)



Crank-rocker example: Windshield wiper

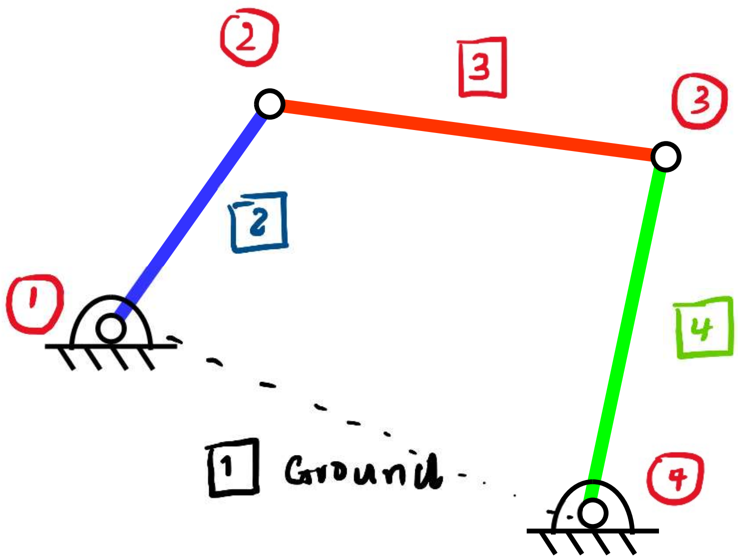




https://www.youtube.com/watch?v=UwYmWgsjtEQ

https://www.youtube.com/watch?v=D_KIOr_TpcM

Four-bar linkage: Identify links, joints and # DOF



$$\begin{aligned} dof &= 3(n-1) - 2J_1 - J_2 \\ &= 3(4-1) - 2(2) - 0 \\ &= 9 - 8 = 1 \end{aligned}$$

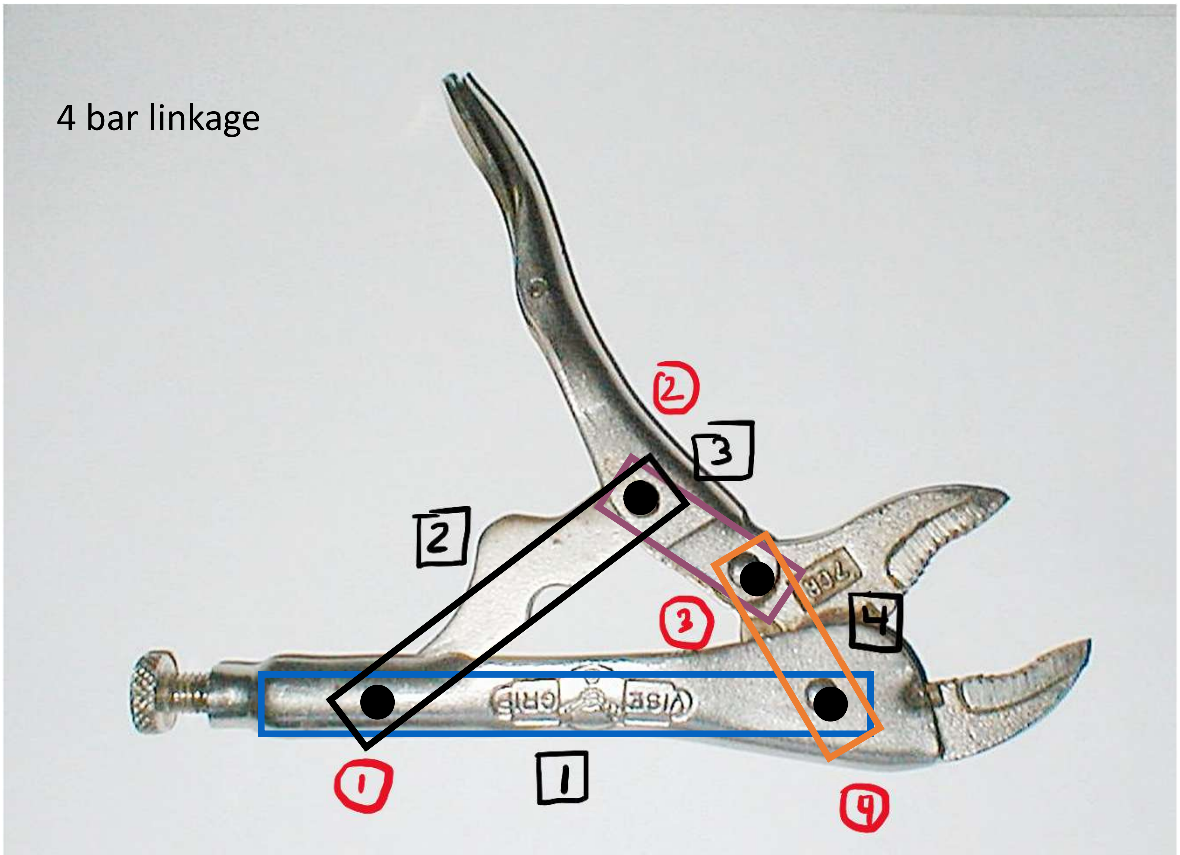
Accounting Method for determining DOF: Draw symbols on a mechanism and count up links and joints

4 = link (n)

4 = full joint (J_1)

0 = half joint (J_2)

Other 4-bars - Vise Grips - Rocker-rocker

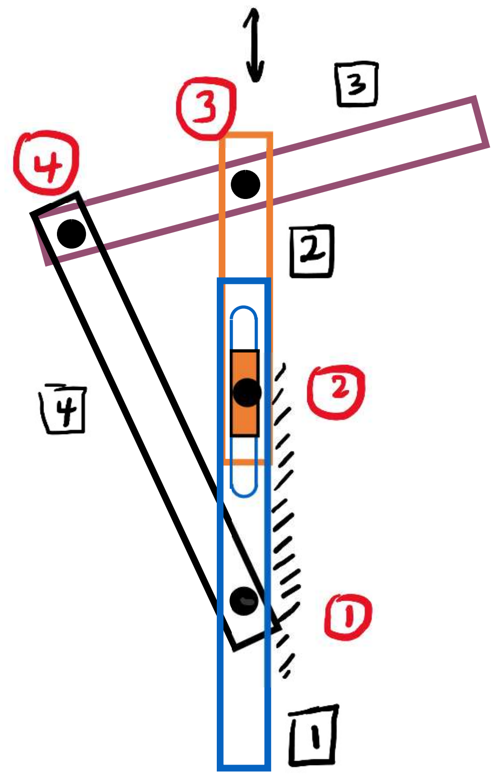


$n = 4$

$J_1 = 4$

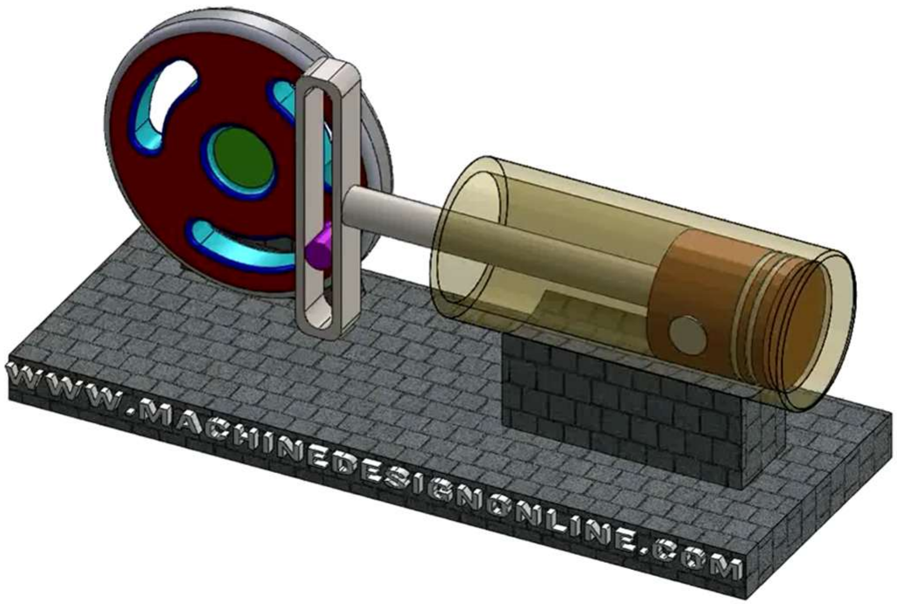
DOF = 1

Other 4-bars: Corkscrew - Rocker-slider



$n = 4$
$J_1 = 4$
$DOF = 1$

Example: Scotch yoke mechanism



Video of Scotch yoke mechanism
http://youtu.be/_K4PSV4MO70

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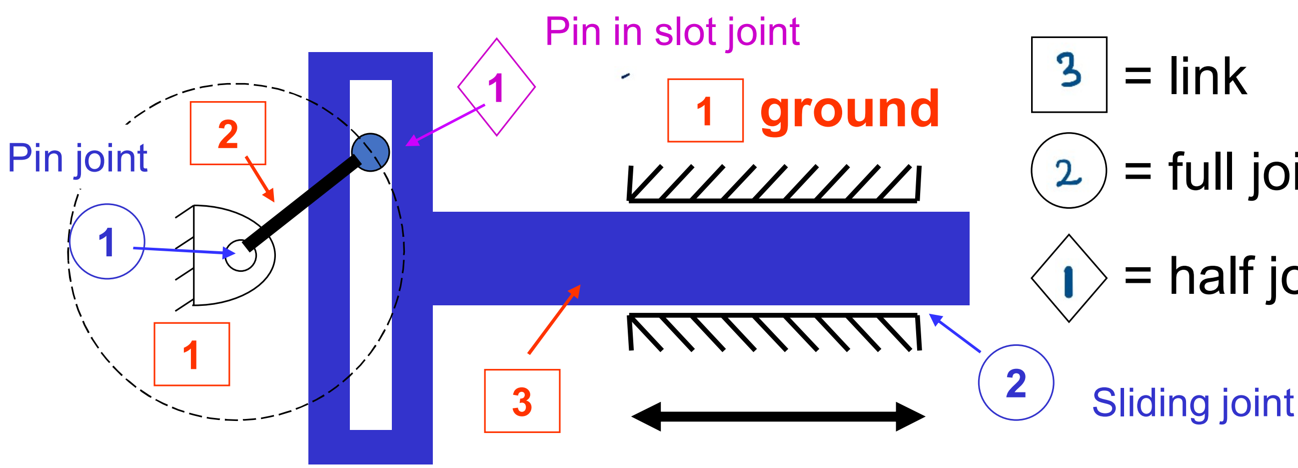
Practice: Scotch yoke

Identify links, joints and calculate # DOF by using Gruebler's equation

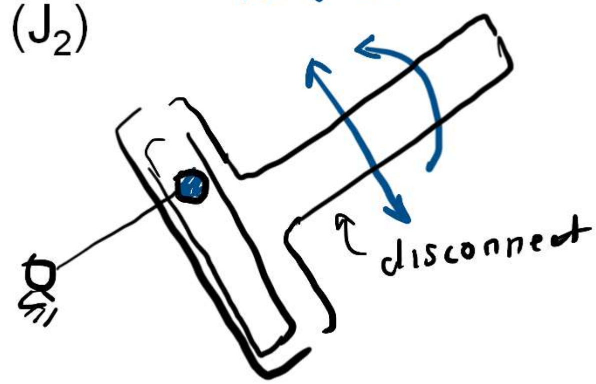


3 = link

2 = full joint (J_1)

1 = half joint (J_2)

Joint allows 2 dof → half joint

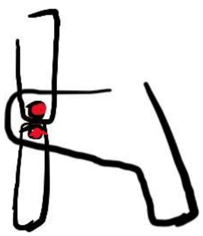


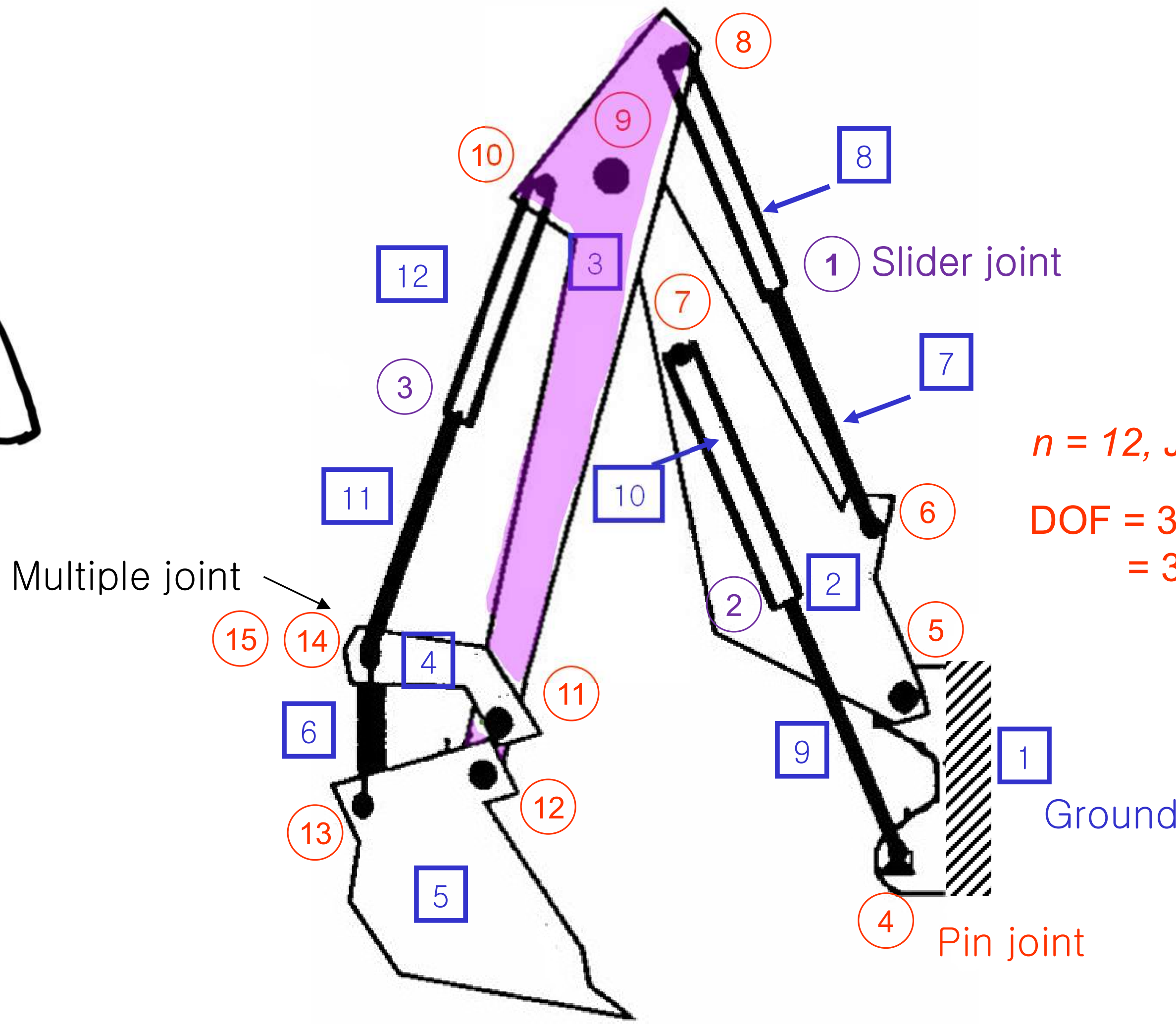
$n = 3, J_1 = 2, J_2 = 1$

$$
\begin{aligned}
\text{DOF} &= 3(n-1) - 2J_1 - J_2 \\
&= 3(3-1) - 2*2 - 1 \\
&= 1
\end{aligned}
$$

Video of Scotch yoke mechanism
http://youtu.be/_K4PSV4MO70

Class exercise #1: For backhoe, (a) Label links and joints (b) Determine mechanism DOF





n = 12, J_1 = 15, J_2 = 0

DOF = 3(12-1) - 2*15 - 0
= 3

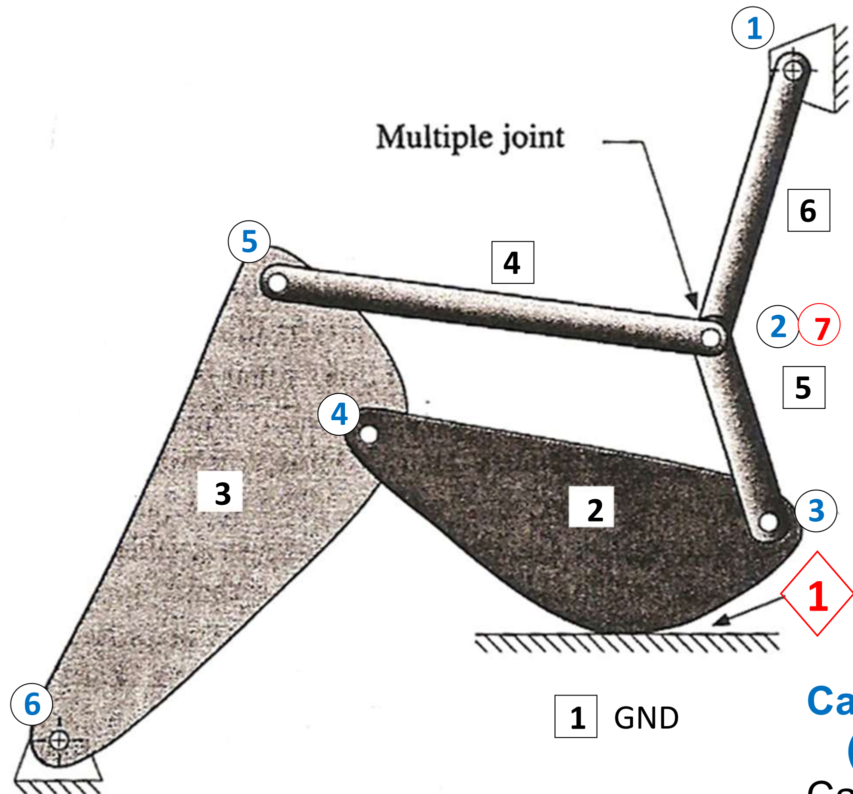
How many DOF does this mechanism have?

$\boxed{6}$ = link

(7) = full joint (J_1)

◇1 = half joint (J_2)

DOF = 3(6-1) - 2*7 - 1 = 0



Case 1: assume slipping (rolling + sliding)

~~Case 2: assume no slipping (pure rolling)~~

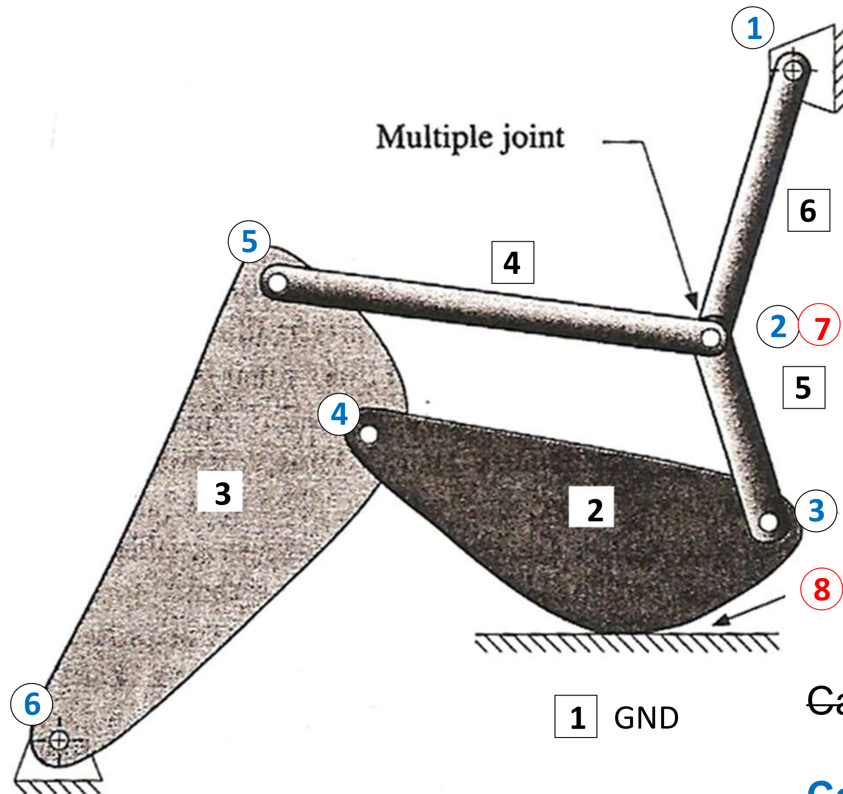
How many DOF does this mechanism have?

$\boxed{6}$ = link

(8) = full joint (J_1)

◇0 = half joint (J_2)

DOF = 3(6-1) - 2*8 - 0 = -1



~~Case 1: assume slipping (rolling + sliding)~~

Case 2: assume no slipping (pure rolling)

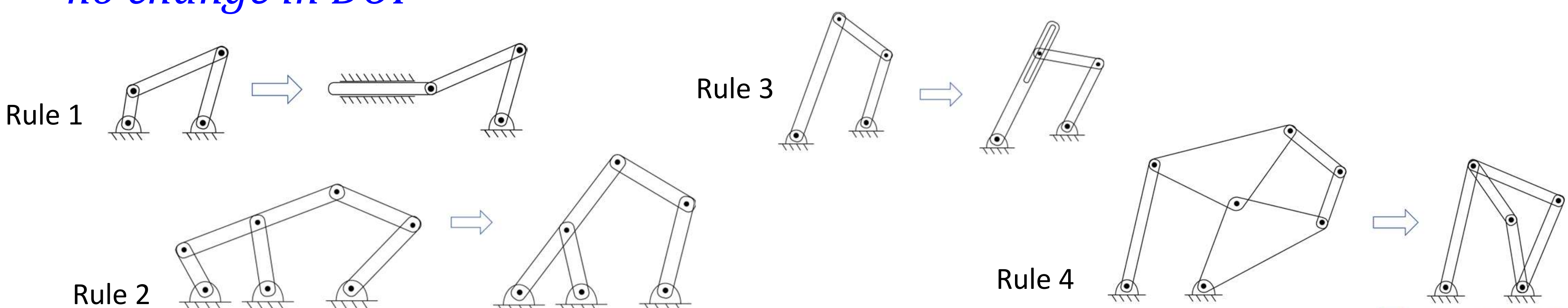
Four Transformation rules & their effect on DOF

Rule 1: replace a *pin joint* with a *sliding joint* → *no change in DOF*

Rule 2: remove a link & full-joint→ *DOF is decreased 1*

Rule 3: replace a full-joint with half-joint → *DOF increased by 1*

Rule 4: nodes can be combined to create higher order multi-joints
→ *no change in DOF*



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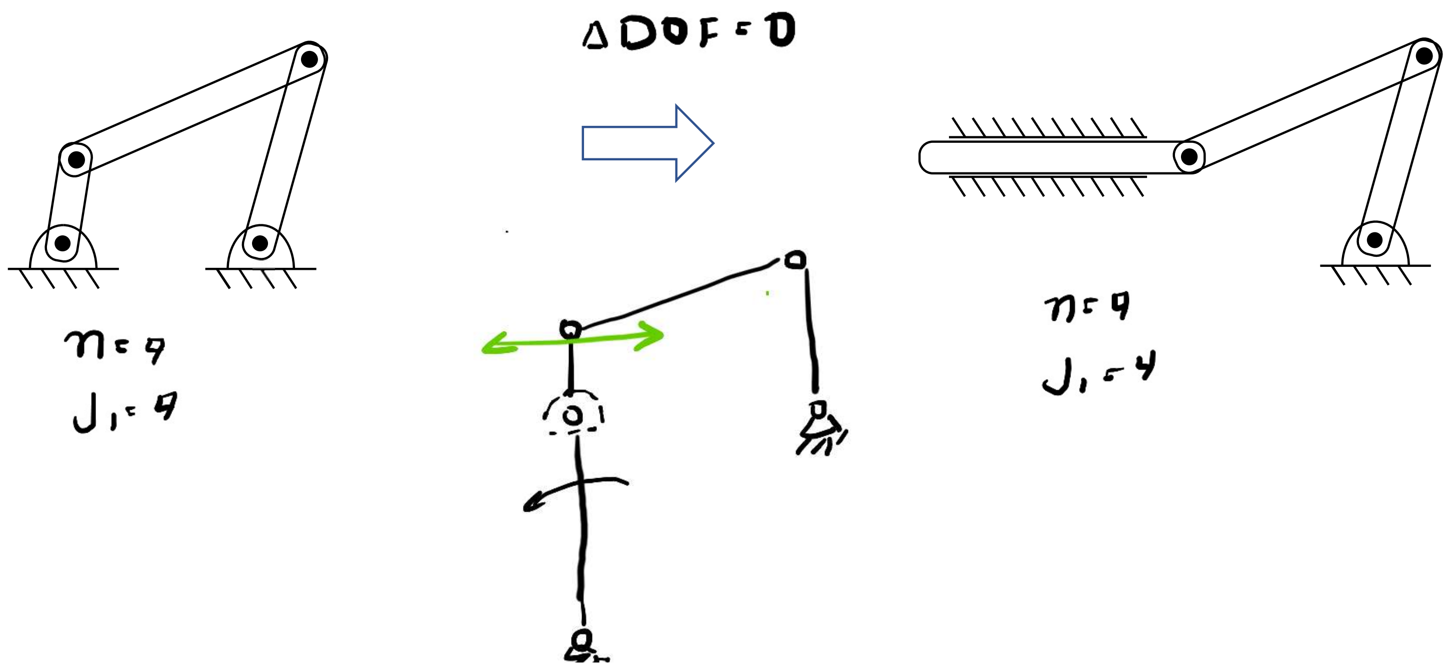
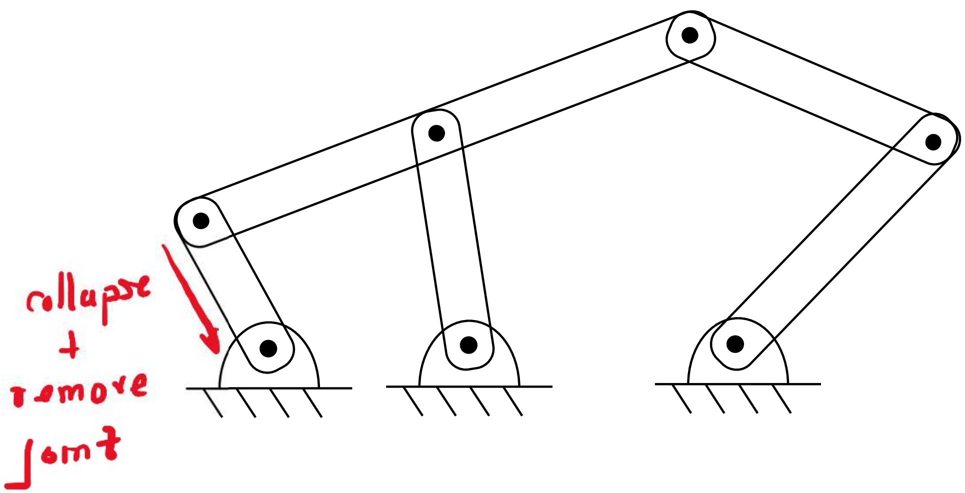
Four Transformation rules & their effect on DOF

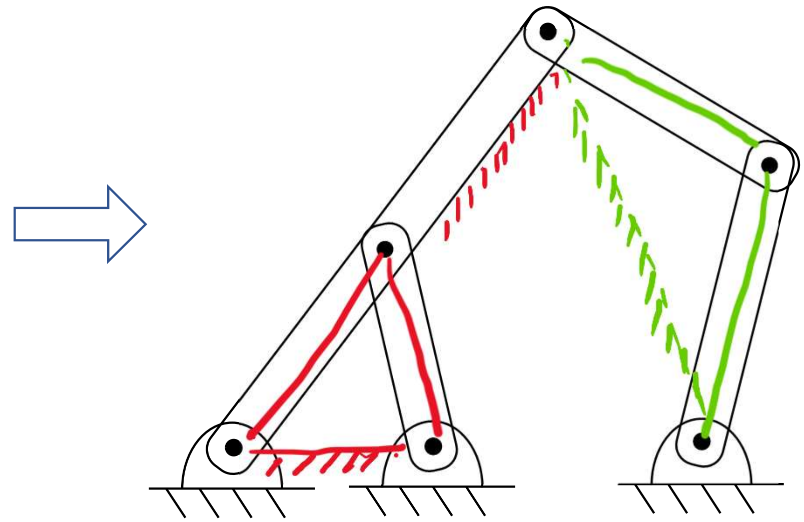
Rule 1: replace a *pin joint* with a *sliding joint* → *no change in DOF*



Rule 2: remove a link & full-joint→ *DOF is decreased 1*

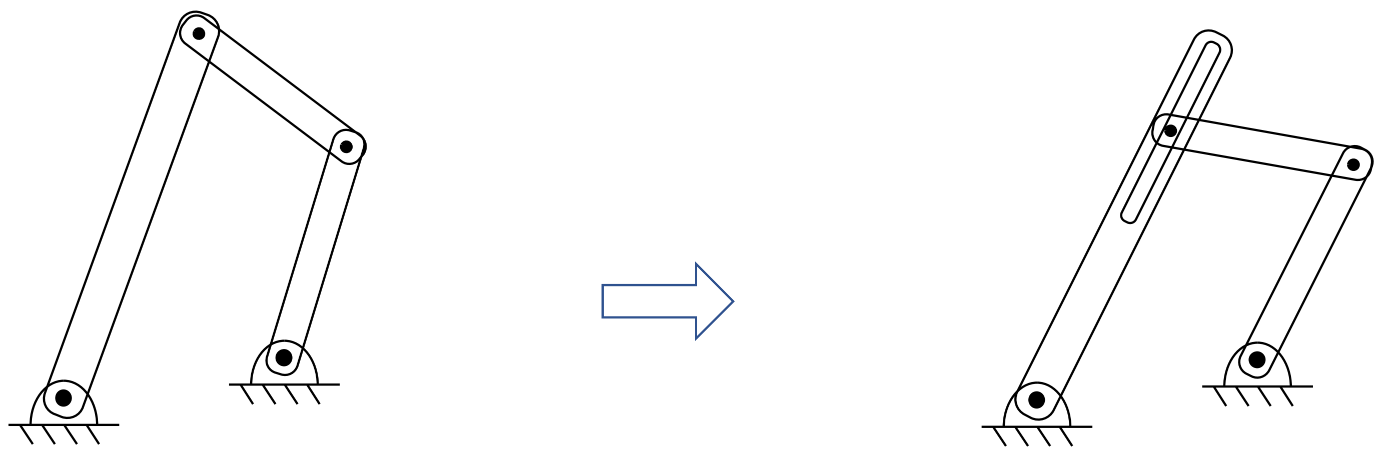


n=6
J1=7
DOF=1



n=5
J1=6
DOF=0

Rule 3: replace a full-joint with half-joint → *DOF increased by 1*



$n = 4$, $J_1 = 4$

$DOF = 1$

$n = 4$

$J_1 = 3$

$J_2 = 1$

$n = 2$

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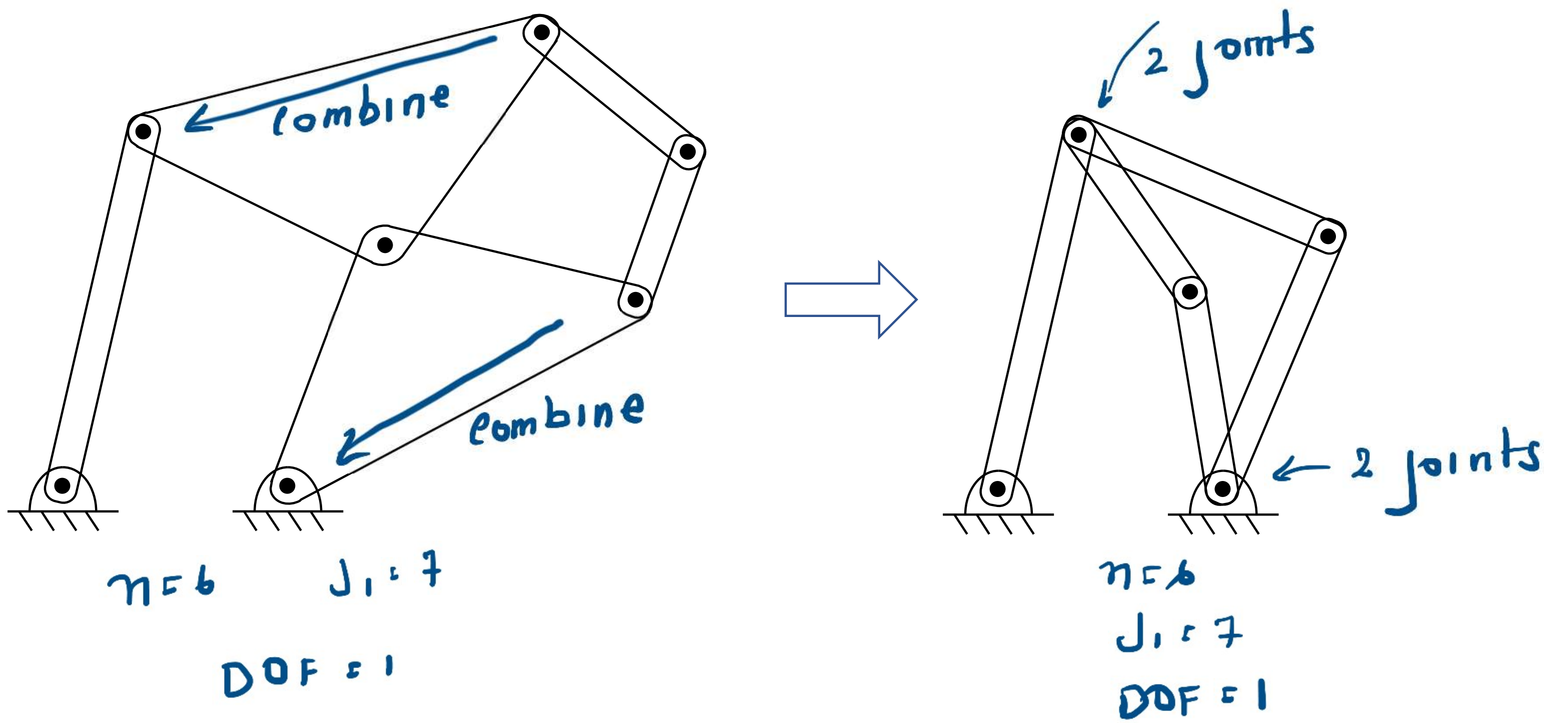
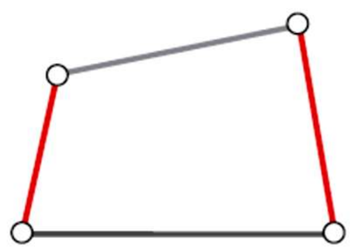
Rule 4: nodes can be combined to create higher order multi-joints
→ *no change in DOF*



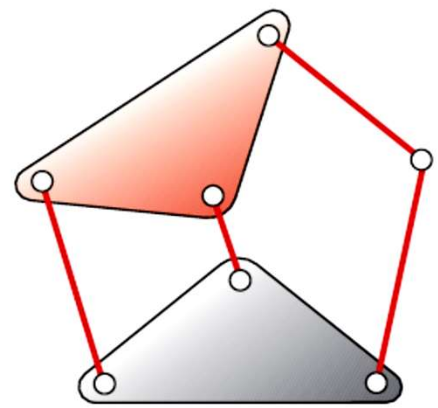
Isomers of 1-DOF mechanisms

Number of Valid Isomers

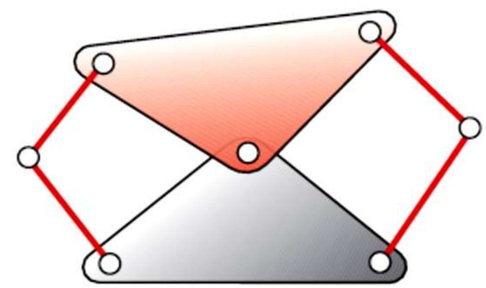
Links	Valid Isomers
4	1
6	2
8	16
10	230
12	6856



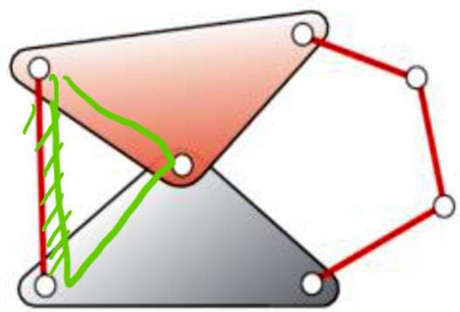
The only fourbar isomer



Stephenson's sixbar isomer



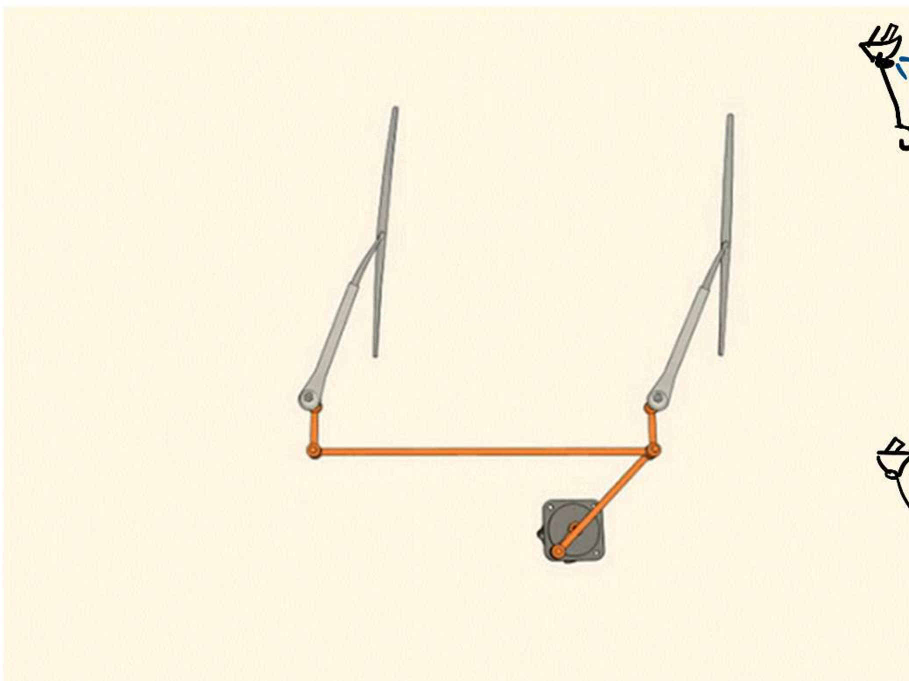
Watt's sixbar isomer

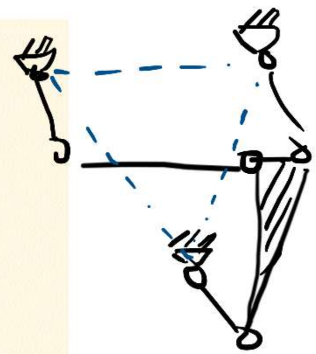


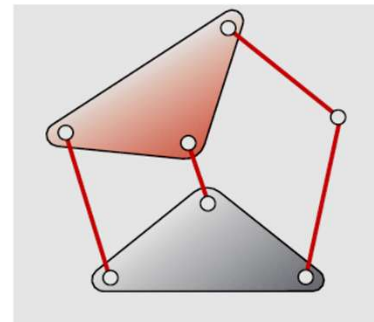
Isomers are the different forms, or arrangements, that are possible for a mechanism

Practice: Identify Sixbar Isomer

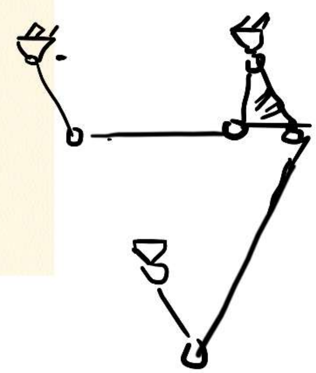
can be both (rule 4)

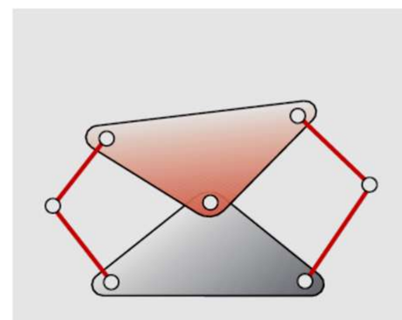






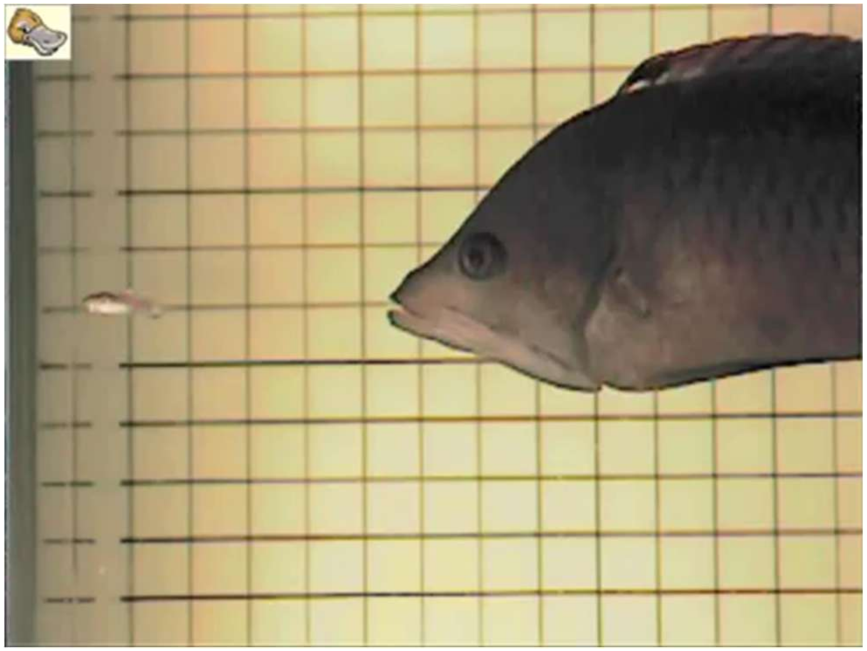
Stephenson's sixbar isomer





Watt's sixbar isomer

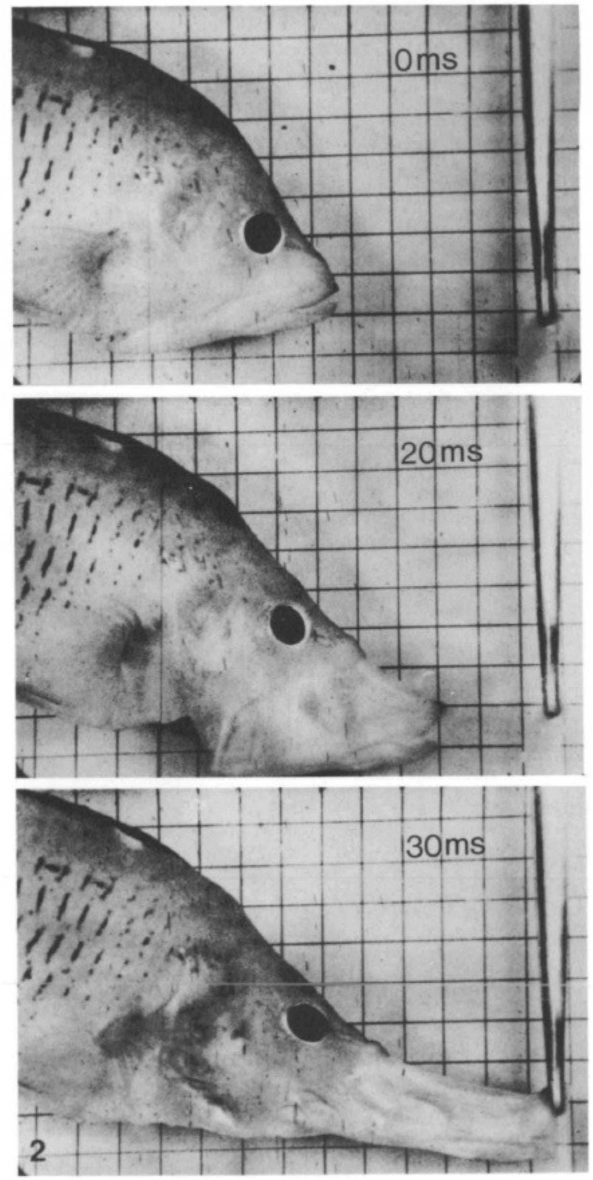
Planar mechanisms in Nature



http://www.youtube.com/watch?v=pDU4CQWXaNY

Sling Jaw Wrasse: A Unique Feeding Mechanism

- The *Epibulus insidiator* possesses the most extreme jaw protrusion ever measured in fishes
- This motion may be understood through kinematic model, combining <u>fourbar</u> and <u>sixbar</u> linkages



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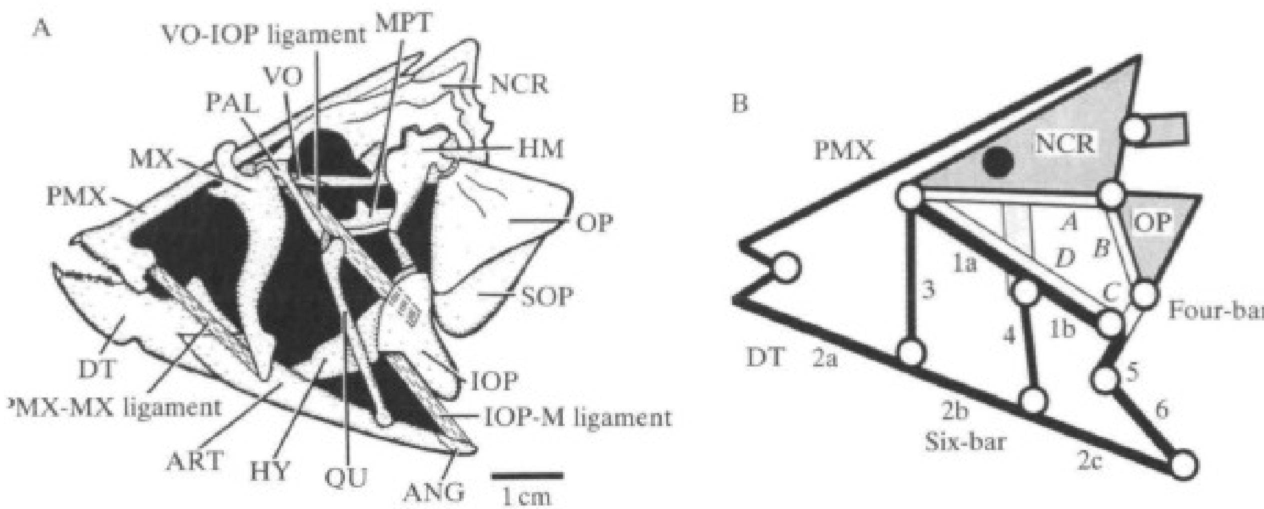
Exercise: Linkage Biomechanics

Can you find the 4 and 6 bar linkages?
The sixbar is which isomer?



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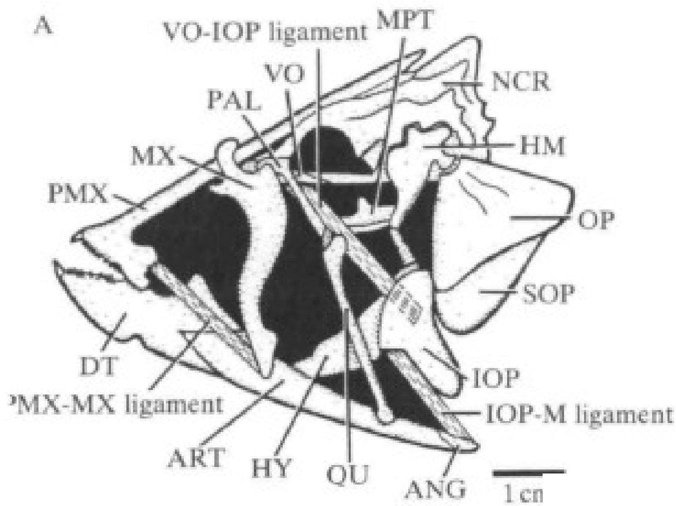
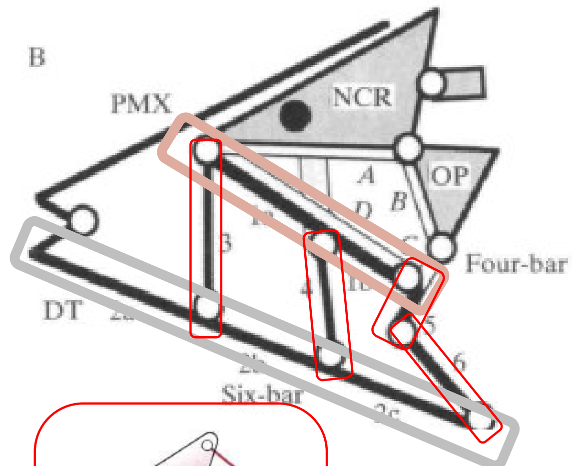
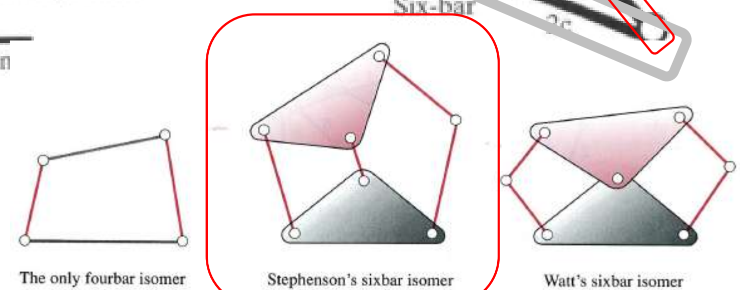
Linkage Biomechanics

The sixbar is which isomer?







(b) All valid isomers of the fourbar and sixbar linkages

More on 6-bar mechanisms
in synthesis lectures

Westneat, J. exp. Biol. **159***, 165-184 (1991))*

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