Module 7

Lecture 22:

Dynamic Force Analysis (DFA) - 3

“Colibri” by Derek Hugger

* www.youtube.com/watch?v=1scj5sotD-E

ME 370 - Mechanical Design 1

Lecture 22: Dynamic Force Analysis - 3

Today (Reading, Norton Ch 10.1-10.8, 11) **11/10/25**

Activities & Upcoming Deadlines

- **Week 12:**
  - **HW 10 (Motor, Cam, Motion 1):** posted and due Tuesday 11/11
  - **HW 11 (Motor, Cam, Motion 2):** posted shortly
  - **Lab 11 (Dynamic Force Analysis with Python)** – Post-lab due the night before your lab section during the week of Nov 19 (delayed by 2 weeks due to P2D2 during Lab 12).
  - **Lab 12**: Meet in 1001 MEL for P2D2 presentation

- Project 2:
  - **Project 2 Description** - Follow this document for expected deliverables for P2D2. Submit materials to Gradescope prior to lab 12 (slides + PVA appendix + CAD animation). Demonstration of prototype of entire robot (walker + dispensing mechanism together), grade will depend on level of function
  - **Complete CATME peer evaluation after finishing P2D2.**

Next lecture: start Module 8: Virtual Work

DFA topics

- Reading: Chap 10.1-10.8, 11
- Dynamics Fundamentals
  - Newton's laws
  - Mass moment ,Center of gravity
  - Mass moment of inertia, Parallel axis theorem
  - Radius of gyration
- Forward and Inverse Dynamics
- Force Analysis Procedure
  - Free body diagrams and equation development
  - Matrix format and solution
- Examples
  - Single link in rotation
  - Four bar slider crank ⇐ see lecture 21 post lecture slides for complete
  - Gear set

Recall: DFA 6 Steps

1. Draw complete system, identify knowns and unknows
2. Draw free-body diagram of each segment
3. Symbolically write out equations of motion
4. Convert to matrix format
5. Insert known values
6. Invert matrix to solve for unknown forces and torques

Gearset

Given a simple gear train of a gear (body 2) and the pinion (body 3). Consider the torque at the pinion, T_{13}, to be known. Assume that each gear has mass m_i, radius r_i, moment of inertia I_{CGi}, and the PVA analysis is already solved.

Find: internal forces between links, and driving torque T_{12}

1. *Draw complete system. Label points, dimensions, external forces & torques, kinematics.* **List known and unknown terms (kinematics, kinetics, property constants).** Categorize into property constants (e.g., properties of links that will never change), knowns (e.g., solved variables from the PVA analysis or defined input terms that may/may not change as the mechanism moves), and unknowns (e.g., output variables that need to be solved). If any variables are constrained by the design to be zero or a fixed value, identify that value.

Knowns:

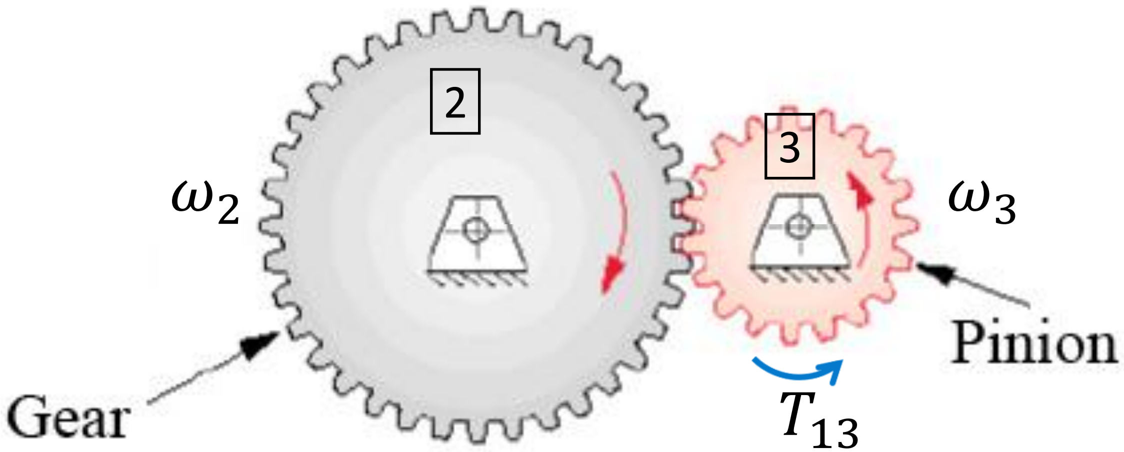
T_{13}

For $i = 2, 3$:

m_i, r_i, I_{CGi}

$\theta_i, \omega_i, \alpha_i$

$\vec{A}_{CGi} = 0$ $\quad \vec{S}_{CGi} = \vec{V}_{CGi} = 0$



Unknowns:

T_{12}

$\vec{F}_{12}, \vec{F}_{13}, \vec{F}_{23}$

Break into x,y components

$\Longrightarrow$ 7 unknowns

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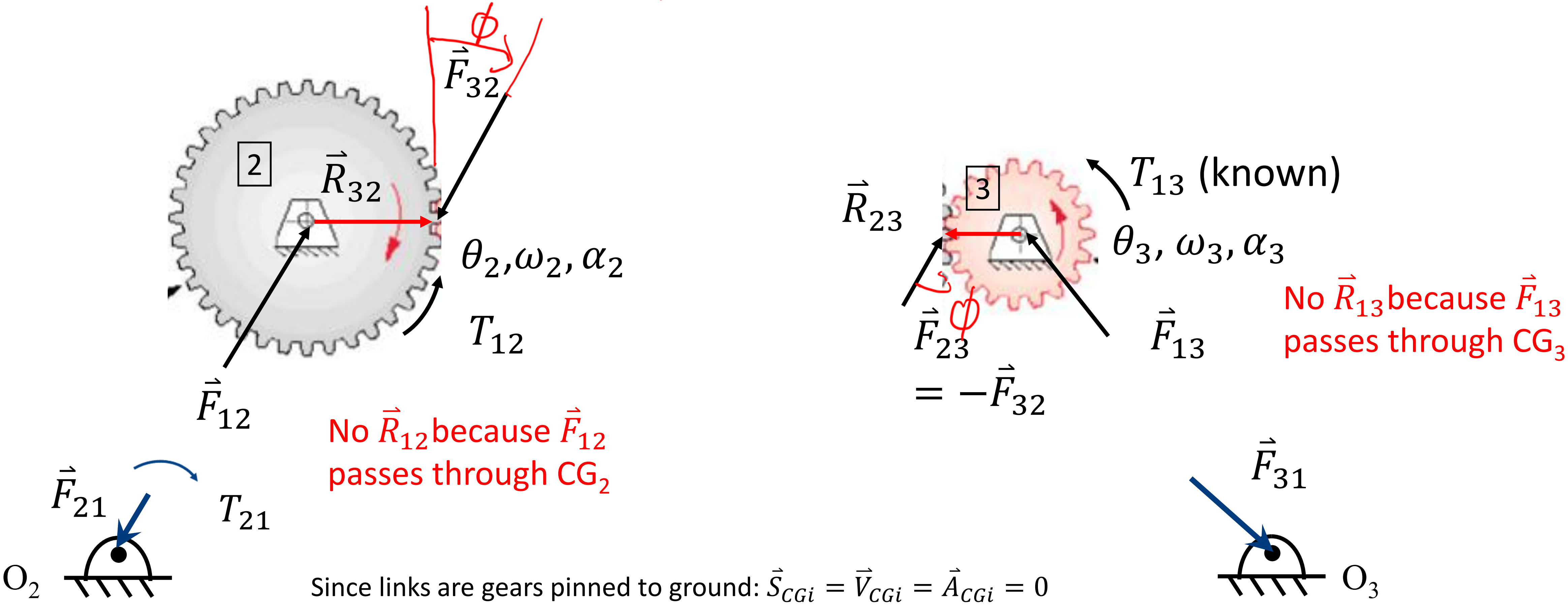
Gearset

2. Draw free body diagrams for each link. Label points, dimensions, external & internal forces & torques, kinematics. Position vectors



Since links are gears pinned to ground: $\vec{S}_{CGi} = \vec{V}_{CGi} = \vec{A}_{CGi} = 0$

Gearset

3. Write the set of equations for each of the gears. You should have 3 equations for each gear. List the unknown forces & torques.

Recall: $\sum \vec{F}_i = m_i \vec{A}_{CGi}, \sum T_{i\,z} = I_{CGi}\alpha_i$

$\vec{T} = \vec{R} \times \vec{F}$

Gear

$$\boxed{F_{12x}} + \boxed{F_{32x}} = m_2 A_{CG2x}$$

$$\boxed{F_{12y}} + \boxed{F_{32y}} = m_2 A_{CG2y}$$

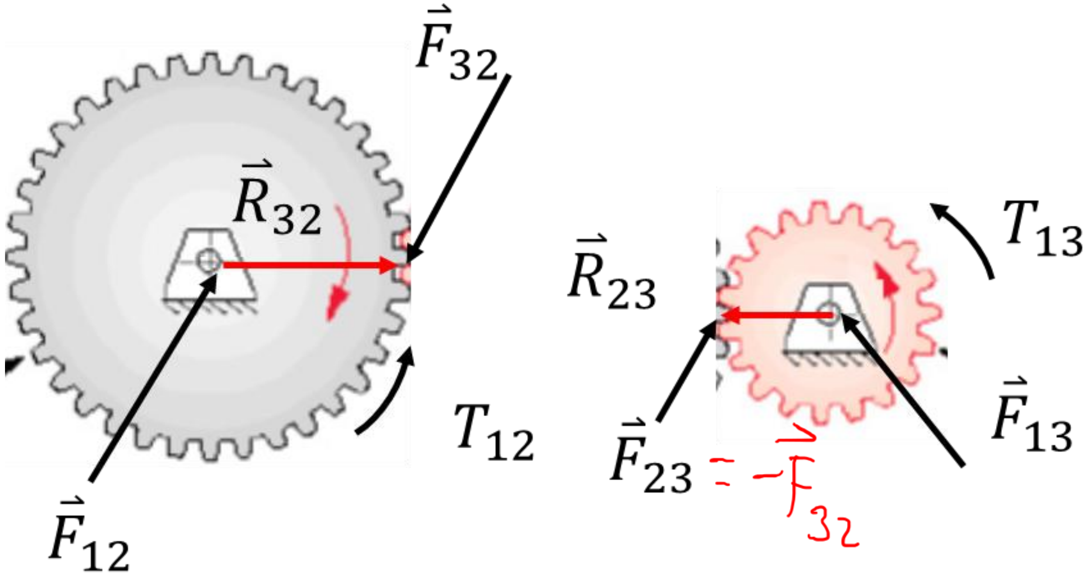
$$\boxed{T_{12}} + (R_{32x}F_{32y} - R_{32y}F_{32x}) = I_{CG2}\alpha_2$$

Pinion

$$\boxed{F_{13x}} - F_{32x} = m_3 A_{CG3x}$$

$$\boxed{F_{13y}} - F_{32y} = m_3 A_{CG3y}$$

$$T_{13} - (R_{23x}F_{32y} - R_{23y}F_{32x}) = I_{CG3}\alpha_3$$



7 unknowns, but only 6 equations

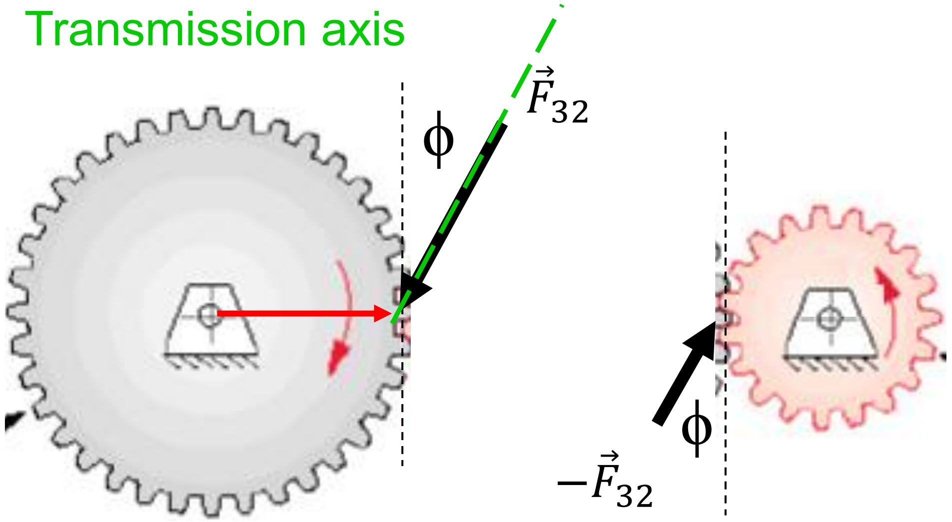
Gearset – identifying constraints

Too many unknowns, need an additional constraint

Fixed pressure angle adds constraint for gear set:

$$\begin{aligned} F_{32x} &= \tan(\phi)\left|F_{32y}\right| \\ &= \tan(\phi)\,\mathrm{sign}\,(F_{32y})F_{32y} \\ &= cF_{32y} \end{aligned}$$

$$\boxed{\begin{aligned} F_{32x} &= cF_{32y} \\ \Longrightarrow c &= \tan(\phi)\,\mathrm{sign}\big(F_{32y}\big) \end{aligned}}$$



where ϕ is the pressure angle from gear tooth geometry

Gearset

4) Convert to matrix format **[A]** **{B}** = **{C}**,

$$(1)\ F_{12x} + F_{32x} = mA_{CG2x} \qquad (4)\ F_{13x} - F_{32x} = mA_{CG3x} \qquad (7)\ F_{32x} = cF_{32y}$$

$$(2)\ F_{12y} + F_{32y} = mA_{CG2y} \qquad (5)\ F_{13y} - F_{32y} = mA_{CG3y} \qquad c = \tan(\phi)\,\mathrm{sign}(F_{32y})$$

$$(3)\ T_{12} + (R_{32x}F_{32y} - R_{32y}F_{32x}) = I_{CG2}\alpha_2 \qquad (6)\ T_{13} - (R_{23x}F_{32y} - R_{23y}F_{32x}) = I_{CG3}\alpha_3$$

Matrix solves for 6 unknowns. Use equation 7 to solve for F_{32x}

Write out unknowns above [A] for bookkeeping

$$\begin{array}{c} \begin{array}{cccccc} F_{12x} & F_{12y} & F_{32y} & F_{13x} & F_{13y} & T_{12} \end{array} \\ \begin{bmatrix} 1 & 0 & c & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & X & 0 & 0 & 1 \\ 0 & 0 & -c & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & Y & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{Bmatrix} F_{12x} \\ F_{12y} \\ F_{32y} \\ F_{13x} \\ F_{13y} \\ T_{12} \end{Bmatrix} = \begin{Bmatrix} m_2\cancel{A_{CG2x}} \\ m_2\cancel{A_{CG2y}} \\ I_{CG2}\alpha_2 \\ m_3\cancel{A_{CG3x}} \\ m_3\cancel{A_{CG3y}} \\ I_{CG3}\alpha_3 - T_{13} \end{Bmatrix} \qquad \vec{A}_{CGi} = 0$$

(5) Insert known/given values for variables in **[A]** *&* **{C}**.

$$X = R_{32x} - cR_{32y} \qquad Y = cR_{23y} - R_{23x}$$

(6) Solve for unknown forces and torques in **{B}** *(typically internal joint forces and torques) using* **{B}** = **[A]**$^{-1}$ **{C}**.

If want to solve for $T_{12,}$ need to solve Eq (3), which requires solving matrix.

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Gearset

How many rows in unknown {B} for the gearset problem?

5 1%

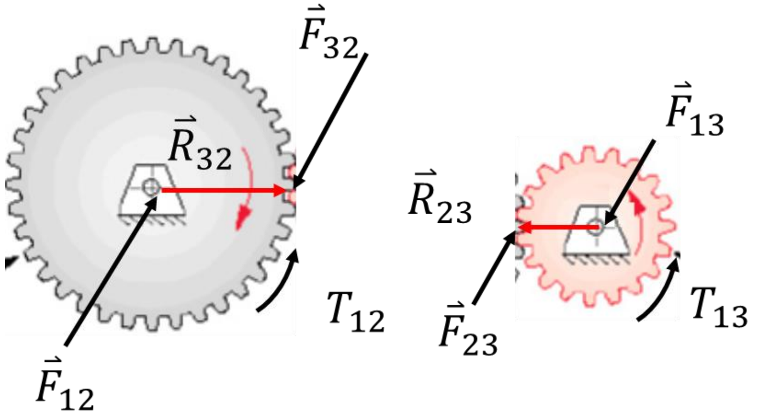
6 93%

7 6%

8

Join Code: **370**

Summary: Gearset example: solve for unknown internal and external (applied) forces and/or torques



Knowns:

T_{13}

For $i = 2, 3$:

$m_i, r_i, I_{CGi},$

$\theta_i, \omega_i, \alpha_i$

$\vec{A}_{CGi} = 0$

Unknowns:

T_{12}

$\vec{F}_{12}, \vec{F}_{13}, \vec{F}_{23}$

For each MOVING link:

$$\sum \vec{F}_i = m_i \vec{A}_{CGi} \qquad \sum T_{iz} = I_{CGi}\alpha_i$$

Link 2

(1) $F_{12x} + F_{32x} = mA_{CG2x}$

(2) $F_{12y} + F_{32y} = mA_{CG2y}$

(3) $T_{12} + \left(R_{32x}F_{32y} - R_{32y}F_{32x}\right) = I_{CG2}\alpha_2$

Link 3

(4) $F_{13x} - F_{32x} = mA_{CG3x}$

(5) $F_{13y} - F_{32y} = mA_{CG3y}$

(6) $T_{13} - \left(R_{23x}F_{32y} - R_{23y}F_{32x}\right) = I_{CG3}\alpha_3$

Due to gears

(7) $F_{32x} = cF_{32y},\ c = \tan(\phi)\,\mathrm{sign}\left(F_{32y}\right)$

$$\begin{bmatrix} 1 & 0 & c & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & X & 0 & 0 & 1 \\ 0 & 0 & -c & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & Y & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} F_{12x} \\ F_{12y} \\ F_{32y} \\ F_{13x} \\ F_{13y} \\ T_{12} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ I_{CG2}\alpha_2 \\ 0 \\ 0 \\ I_{CG3}\alpha_3 - T_3 \end{Bmatrix}$$

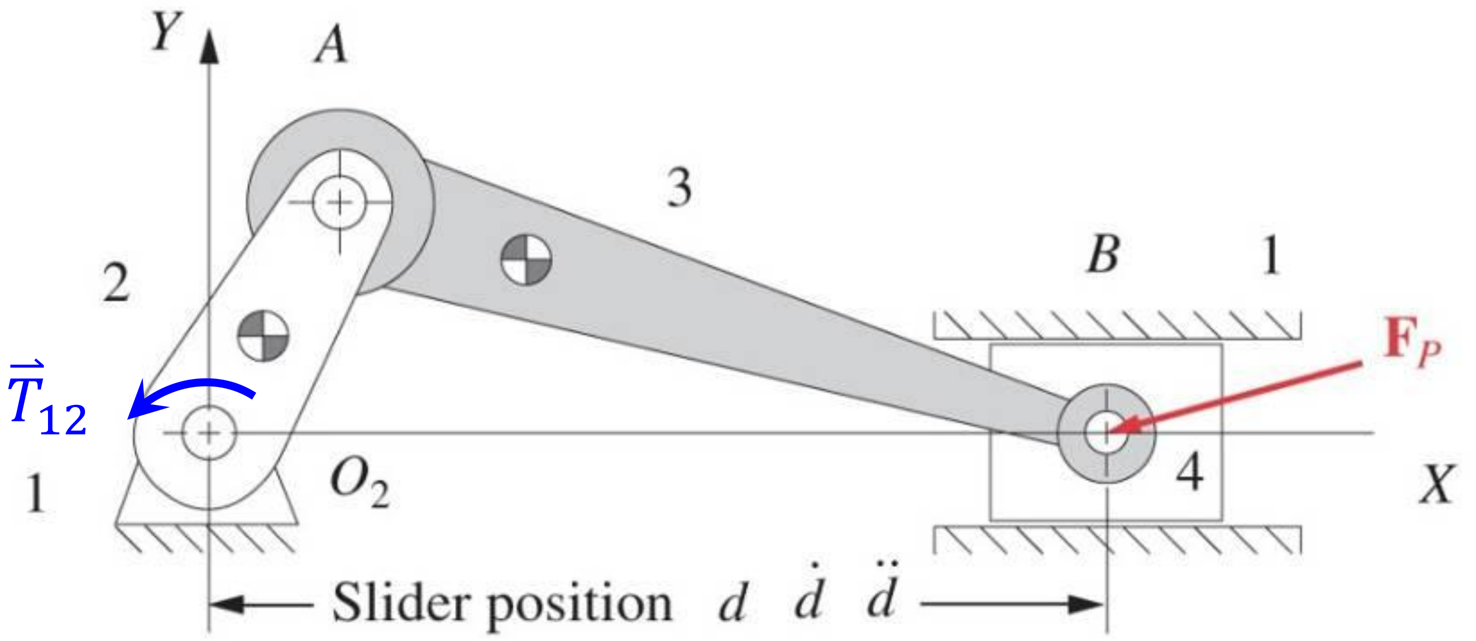
Where $X = R_{32x} - cR_{32y}$ and $Y = cR_{23y} - R_{23x}$

If want to solve for $T_{12,}$ need to solve Eq (3), which requires solving matrix.

Class Exercise 1: Piston (slider-crank)

For a given time-varying force applied to link 4 ($\vec{F}_P$(t)), we want to know the output torque with respect to time ($\vec{T}_{12}$(t)). Assume (a) each link has mass m_i and length l_i, (b) the PVA analysis for this mechanism is already solved, including piston kinematics ($d, \dot{d}, \ddot{d}$), and (c) there is friction at link 4, so: $F_{14x} = \pm\mu F_{14y}$

Solve for output torque $\vec{T}_{12}$ via Dynamic Force Analysis (perform steps 1-3)



Class Exercise 1: Piston (slider-crank)

DFA Step 1:

A) On the figure, label each link's kinematic terms (rotational and translational).

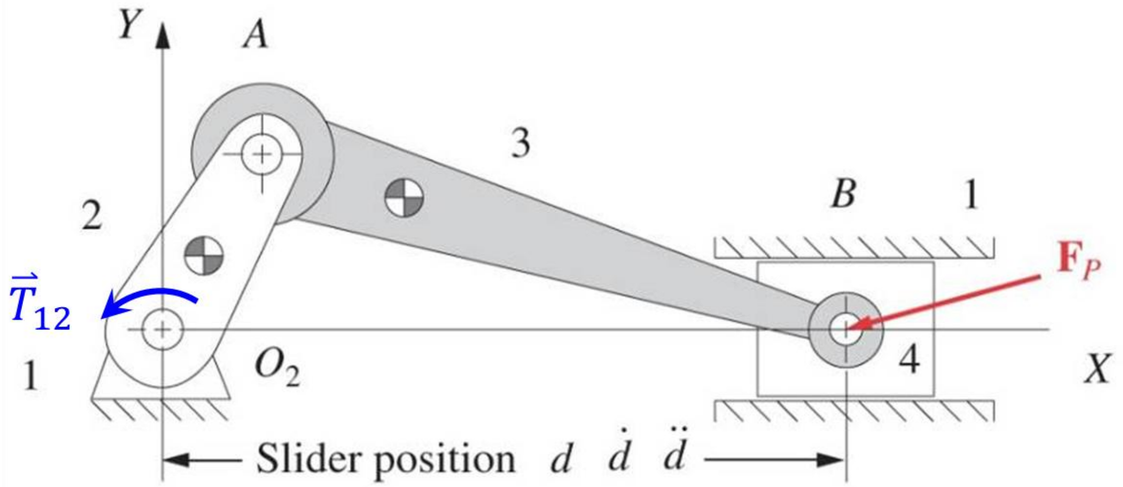
B) **List all known and unknown terms (kinematics, kinetics, property constants)**

Known:

Unknown:

DFA Step 2: Draw a free-body diagram of each segment. On each link: label local coordinate systems, kinematics, external and internal forces & torques, and position vectors. Label forces such that force vector $\vec{F}_{ij}$ represents the force of link i on link j and is applied at the common joint on link j.

DFA Step 3: Symbolically write out equations of motion for each moving link. On each equation, indicate which terms are unknown and need to be solved. How do we solve for output torque $\vec{T}_{12}$?



Step 1: label each link's kinematic terms, identify knowns and unknowns

Knowns:

$\vec{F}_P$

$\theta_i, \omega_i, \alpha_i$

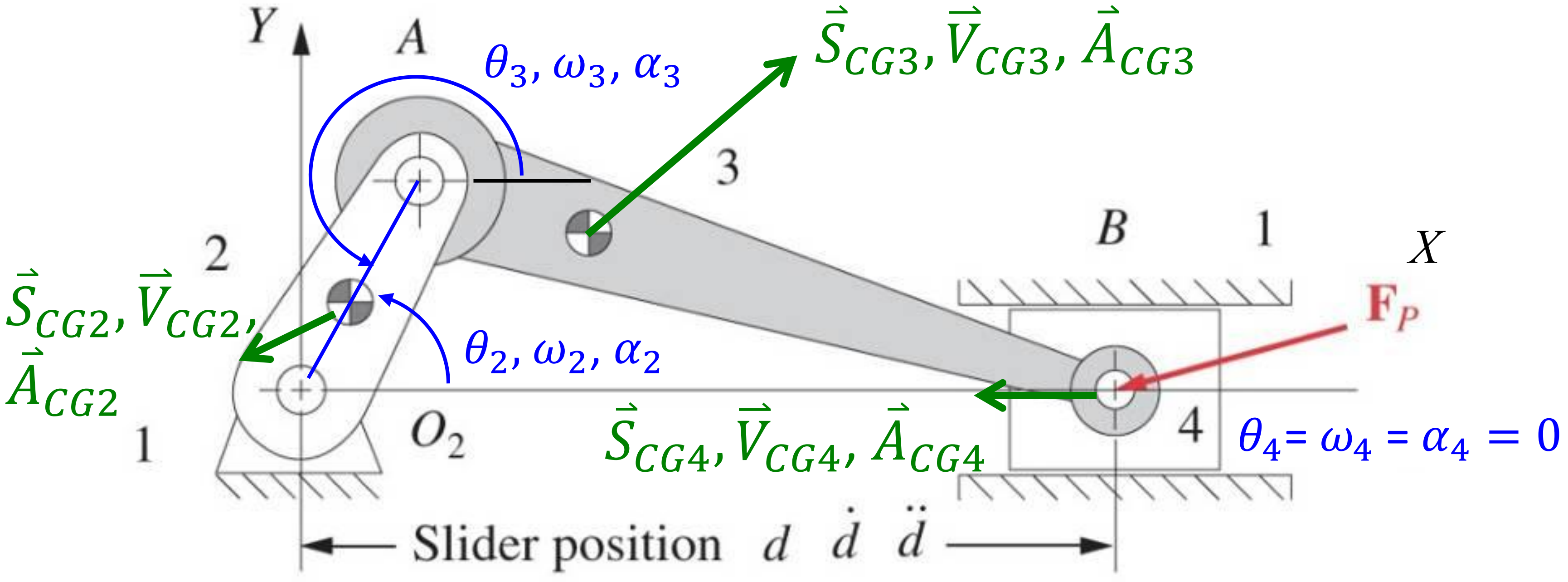
$\vec{V}_{CGi}, \vec{A}_{CGi}$

I_{CGi}, m_i

l_i

where i = (1:4)

$d, \dot{d}, \ddot{d}$



Unknowns:

T_{12}

DFA only:

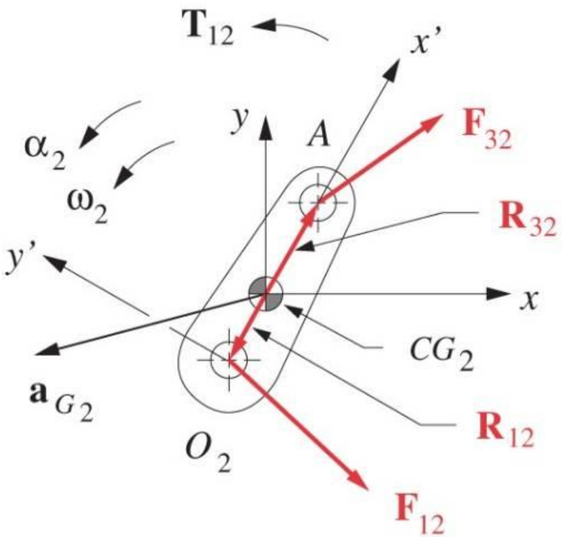
$\vec{F}_{12} = F_{12x}, F_{12y}$

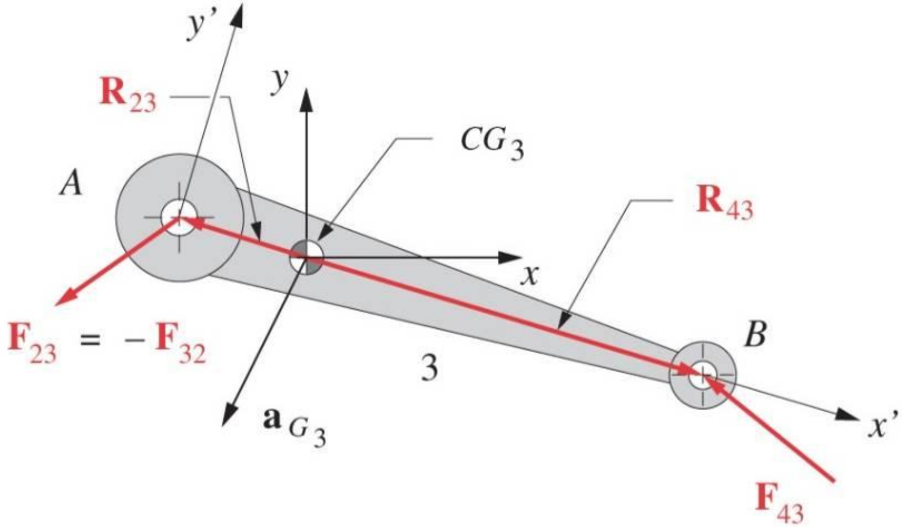
$\vec{F}_{32} = F_{32x}, F_{32y}$

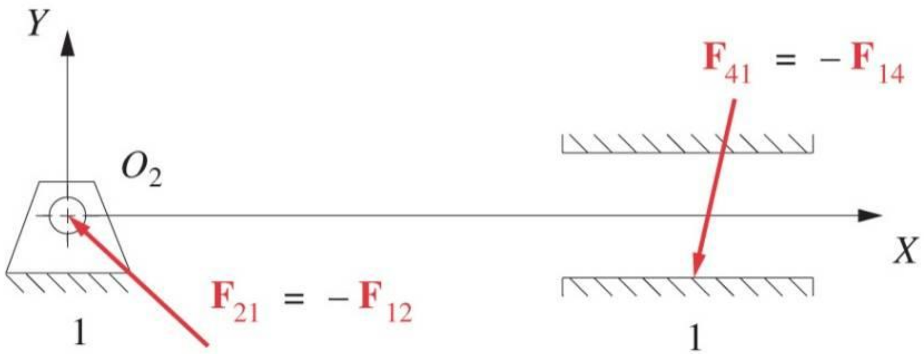
$\vec{F}_{43} = F_{43x}, F_{43y}$

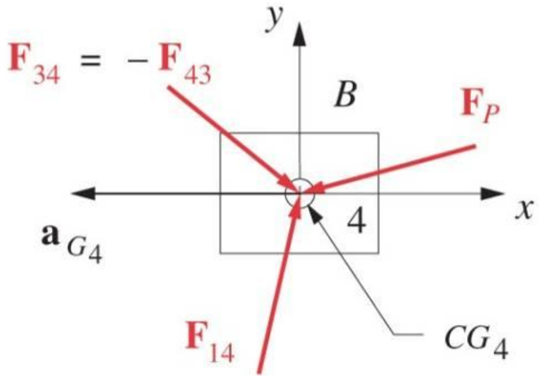
$\vec{F}_{14} = F_{14x}, F_{14y}$

Step 2: draw free body diagrams





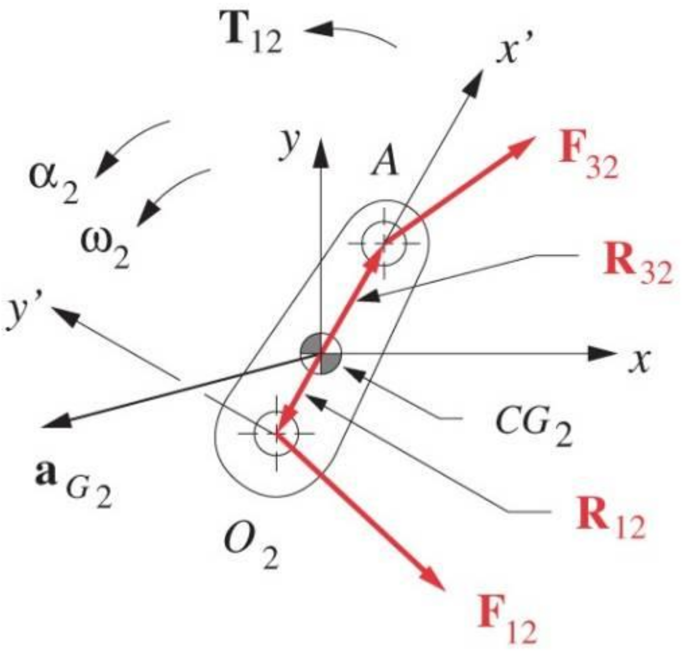




Step 3: Write out EOMs for each link

Write EOM for Link 2

On each equation, indicate which terms are unknown
and need to be solved.



$$\boxed{F_{12x}} + \boxed{F_{32x}} = m_2 a_{CG2x}$$

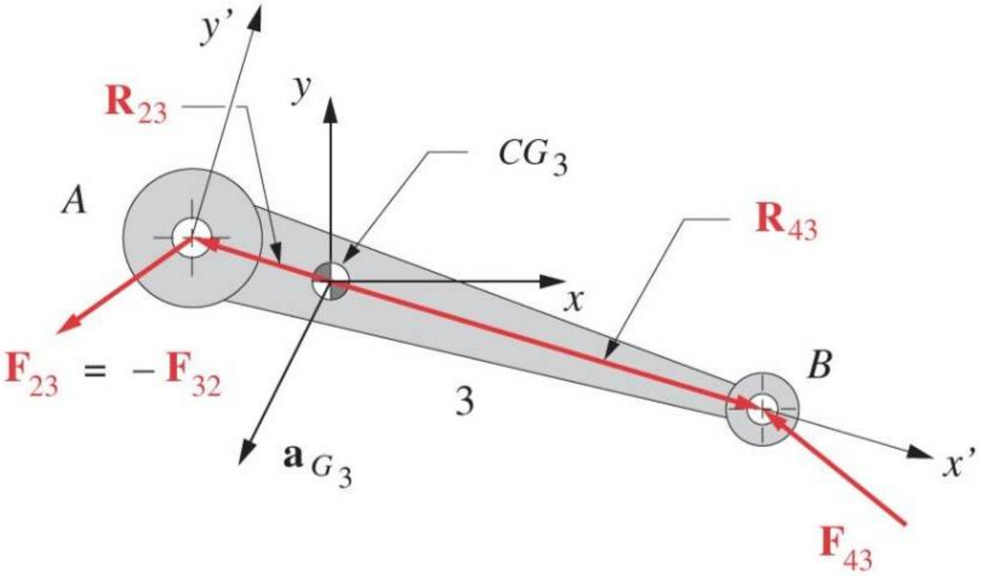
$$\boxed{F_{12y}} + \boxed{F_{32y}} = m_2 a_{CG2y}$$

$$\boxed{T_{21}} + (R_{12x}F_{12y} - R_{12y}F_{12x}) + (R_{32x}F_{32y} - R_{32y}F_{32x}) = I_{CG2}\alpha_2$$

Step 3: Write out EOMs for each link (con't)

Write EOM for Link 3

On each equation, indicate which terms are unknown and need to be solved.



$$\boxed{F_{43x}} - \boxed{F_{32x}} = m_3 a_{CG3x}$$

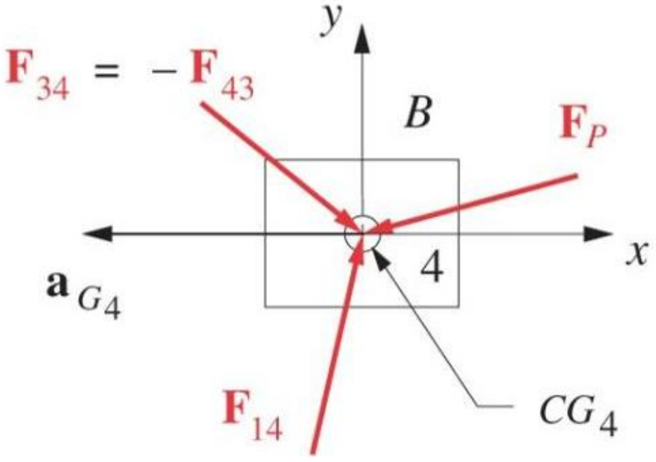
$$\boxed{F_{43y}} - \boxed{F_{32y}} = m_3 a_{CG3y}$$

$$(R_{43x}F_{43y} - R_{43y}F_{43x}) - (R_{32x}F_{32y} - R_{32y}F_{32x}) = I_{CG3}\alpha_3$$

Step 3: Write out EOMs for each link (con't)

Write EOM for Link 4

On each equation, indicate which terms are unknown and need to be solved.



$$F_{14x} - \boxed{F_{43x}} = m_4 a_{CG4x}$$
$$\boxed{F_{14y}} - \boxed{F_{43y}} = m_4 a_{CG4y} = 0$$
$$\boxed{F_{14x}} = \pm \mu F_{14y}$$

Step 4: Convert to matrix format [A] {B} = {C}

Link 2:

$$\boxed{F_{12x}} + \boxed{F_{32x}} = m_2 a_{CG2x}$$

$$\boxed{F_{12y}} + \boxed{F_{32y}} = m_2 a_{CG2y}$$

$$\boxed{T_{21}} + (R_{12x}F_{12y} - R_{12y}F_{12x}) + (R_{32x}F_{32y} - R_{32y}F_{32x}) = I_{CG2}\alpha_2$$

Link 3:

$$\boxed{F_{43x}} - F_{32x} = m_3 a_{CG3x}$$

$$\boxed{F_{43y}} - F_{32y} = m_3 a_{CG3y}$$

$$(R_{43x}F_{43y} - R_{43y}F_{43x}) - (R_{32x}F_{32y} - R_{32y}F_{32x}) = I_{CG3}\alpha_3$$

Link 4:

$$F_{14x} - F_{43x} + F_{px} = m_4 a_{CG4x}$$

$$\boxed{F_{14y}} - F_{43y} + F_{px} = m_4 a_{CG4y} = 0$$

$$\boxed{F_{14x}} = \pm\mu F_{14y}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -R_{12_y} & R_{12_x} & -R_{32_y} & R_{32_x} & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & R_{23_y} & -R_{23_x} & -R_{43_y} & R_{43_x} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -\mu SGN\left(\dot{d}\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} F_{12_x} \\ F_{12_y} \\ F_{32_x} \\ F_{32_y} \\ F_{43_x} \\ F_{43_y} \\ F_{14_y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G_{2x}} \\ m_2 a_{G_{2y}} \\ I_{G_2}\alpha_2 \\ m_3 a_{G_{3x}} \\ m_3 a_{G_{3y}} \\ I_{G_3}\alpha_3 \\ m_4 a_{G_{4x}} - F_{P_x} \\ -F_{P_y} \end{bmatrix}$$