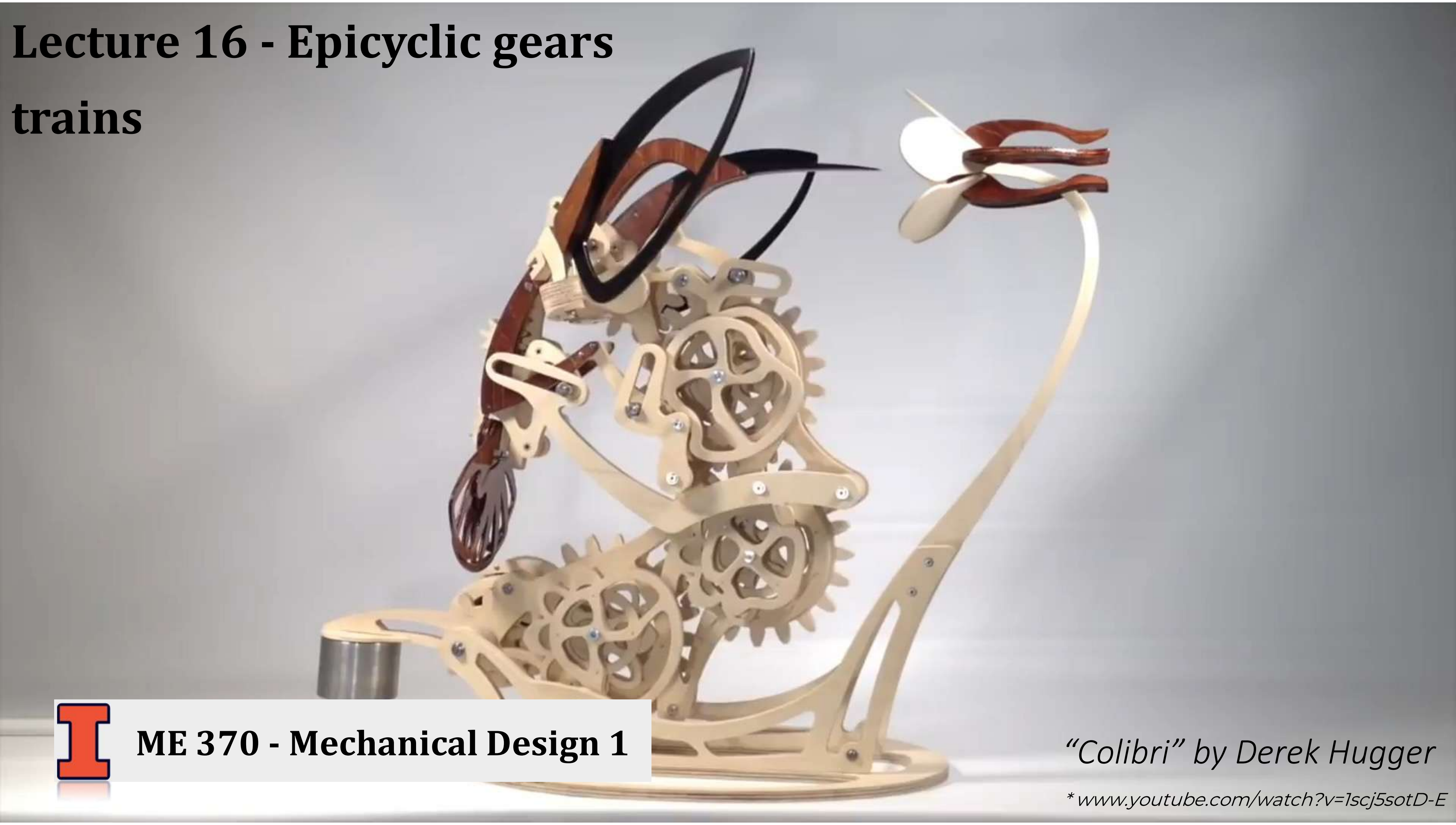
Lecture 16 - Epicyclic gears trains



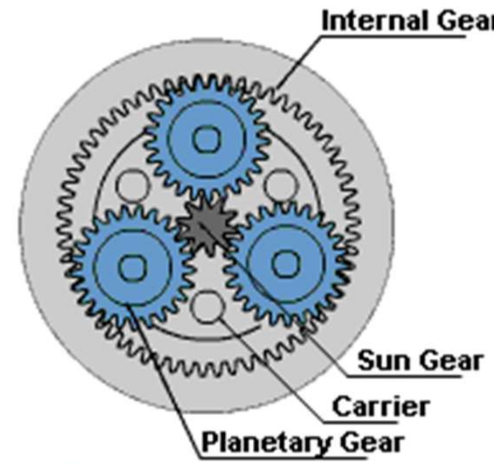
ME 370 - Mechanical Design 1

"Colibri" by Derek Hugger

** www.youtube.com/watch?v=1scj5sotD-E*

Planetary gear trains

Consist of a *sun gear* in the middle, a *ring gear* outside, and *planetary gears* in-between that are held by a *carrier or arm*.



http://www.orientalmotor.com/newsletter/PN-Geared.htm

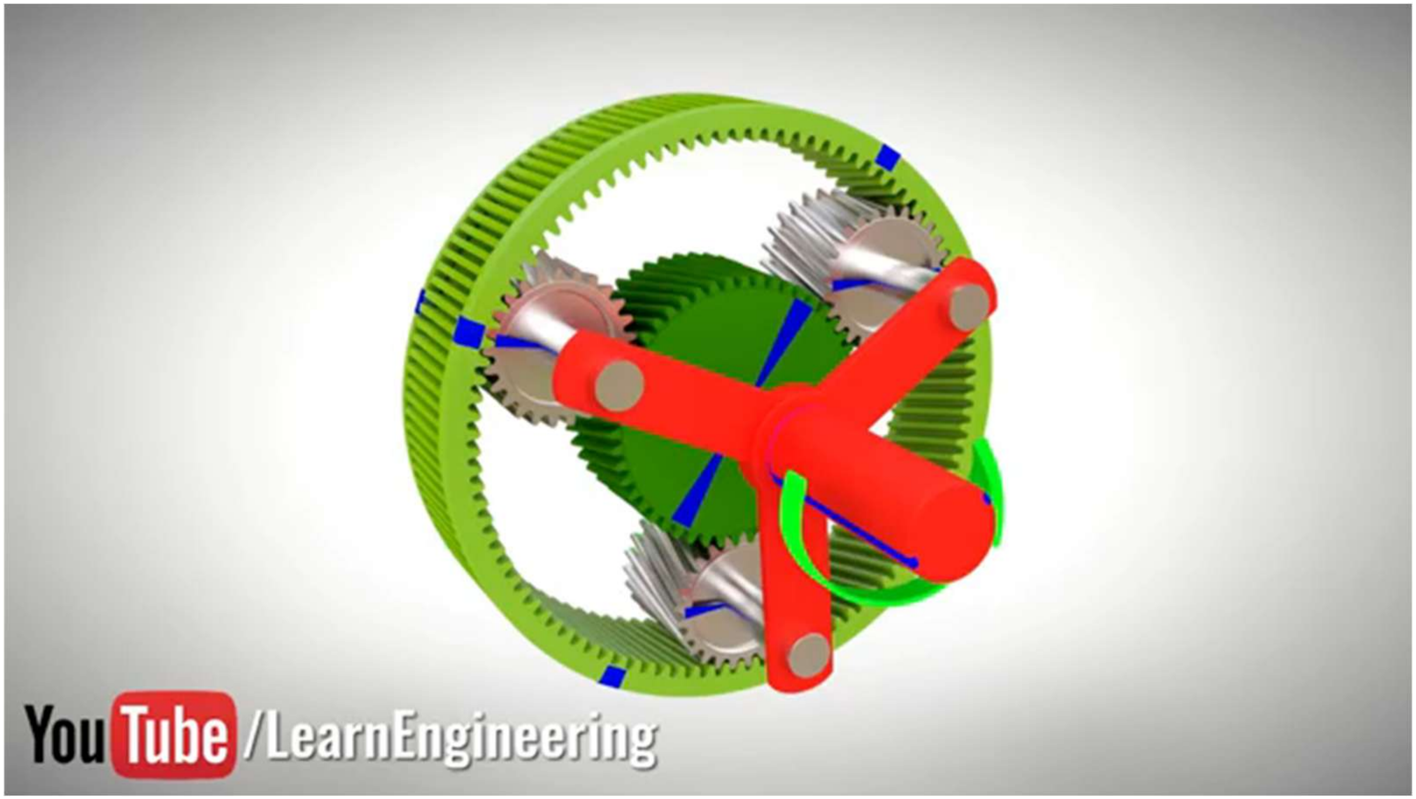
1. 2-DOF system (simple gear train DOF is 1).
2. Two inputs and one output
3. Compact design to increase gear ratio.
4. Gear train can be rugged due to multiple planets.

http://www.benchtophybrid.com/PG_Intro.html

ILLINOIS

Planetary gear motion

https://www.youtube.com/watch?v=ARd-Om2VyiE&t=4s

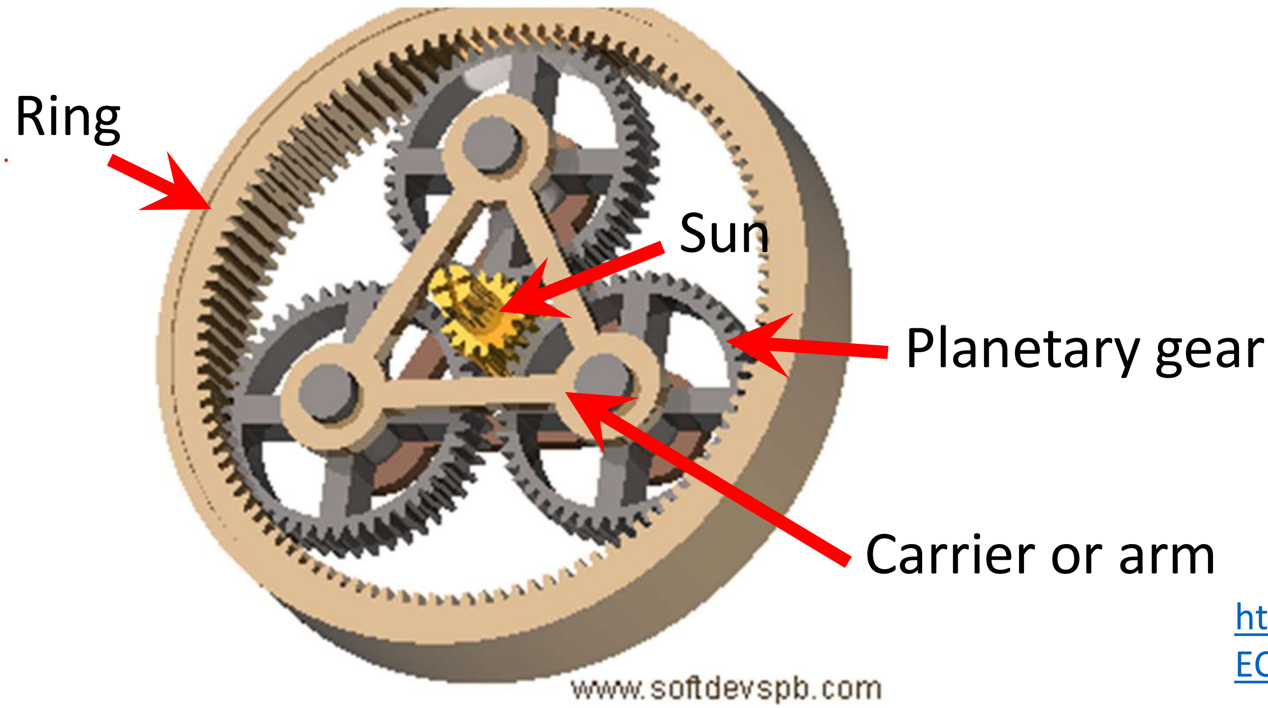


Additional resource on the use of planetary gears in automatic transmissions
https://www.youtube.com/watch?v=u_y1S8C0Hmc

ILLINOIS

Planetary Gear Trains

Four different types of motion based on which gears (sun, ring, planetary via carrier or arm) are held stationary.



https://youtu.be/EClјAo1q1RQ

Planetary Gear Trains

- The gear ratio depends on which gear is held fixed

	Input	**Output**	**Stationary**	**Calculation** $\omega_{in}/\omega_{out} = 1/m_v$	
A	Sun (**S**)	Planet Carrier (**C**)	Ring (**r**)	$1 + \frac{N_r}{N_s}$	Speed redux.
B	Planet Carrier (**C**)	Ring (**r**)	Sun (**S**)	$1/\left(1 + \frac{N_s}{N_r}\right)$	Speed increase
C	Ring (**r**)	Planet Carrier (**C**)	Sun (**S**)	$1 + \frac{N_s}{N_r}$	Speed redux
D	Sun (**S**)	Ring (**r**)	Planet Carrier (**C**)	$-\frac{N_r}{N_s}$	Speed redux (reversal)

https://en.wikipedia.org/wiki/Epicyclic_gearing

ILLINOIS

Differential Drive

How Differential Gear Trains Work by Chevrolet
https://youtu.be/K4JhruinbWc?t=532

https://www.youtube.com/watch?v=K4JhruinbWc

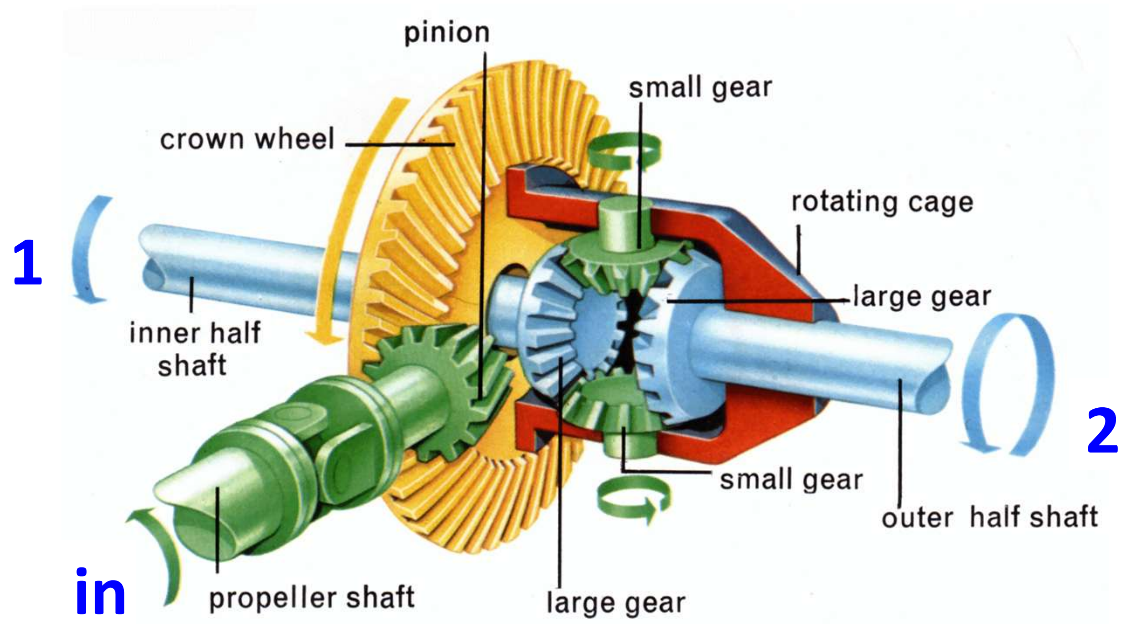
ILLINOIS

Differential Gear Trains

Green input pinion = 14 teeth

Yellow crown gear = 56 teeth

gear ratio 1:4



$$\omega_{in} = \frac{N_{input}}{N_{crown}} \cdot \underbrace{\frac{\omega_1 + \omega_2}{2}}_{\omega_{out}}$$

$$T_{in} = \frac{N_{crown}}{N_{input}} \cdot \left(\frac{2\omega_1}{\omega_1 + \omega_2} T_1 + \frac{2\omega_2}{\omega_1 + \omega_2} T_2\right)$$

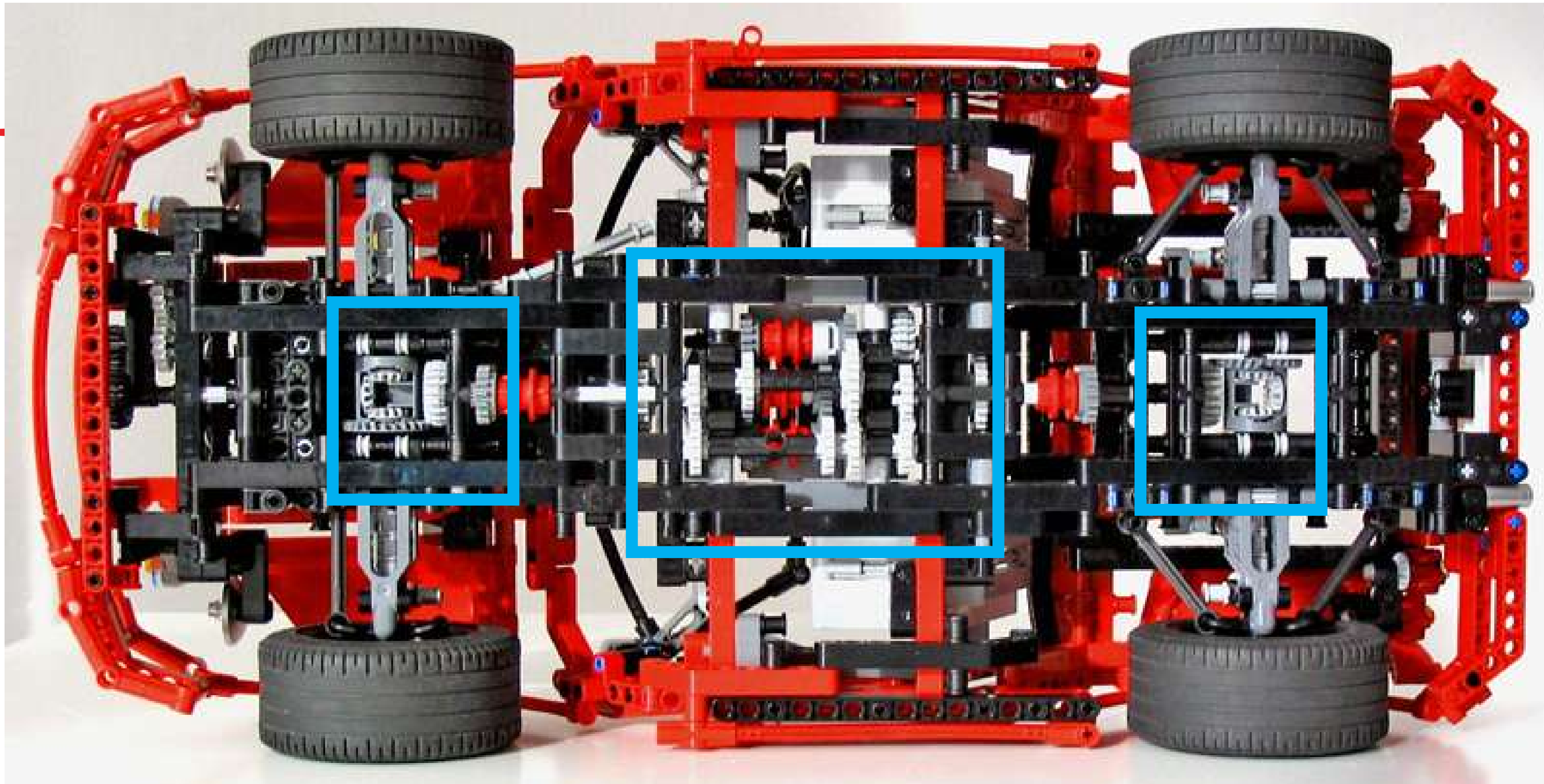
$\omega_{out} = \frac{\omega_1 + \omega_2}{2}$ let $\omega_1 = \omega_2 \rightarrow$

$\omega_{out} = \omega_1 = \omega_2$

let $\omega_2 = 0$

$\omega_1 = 2\omega_{out}$

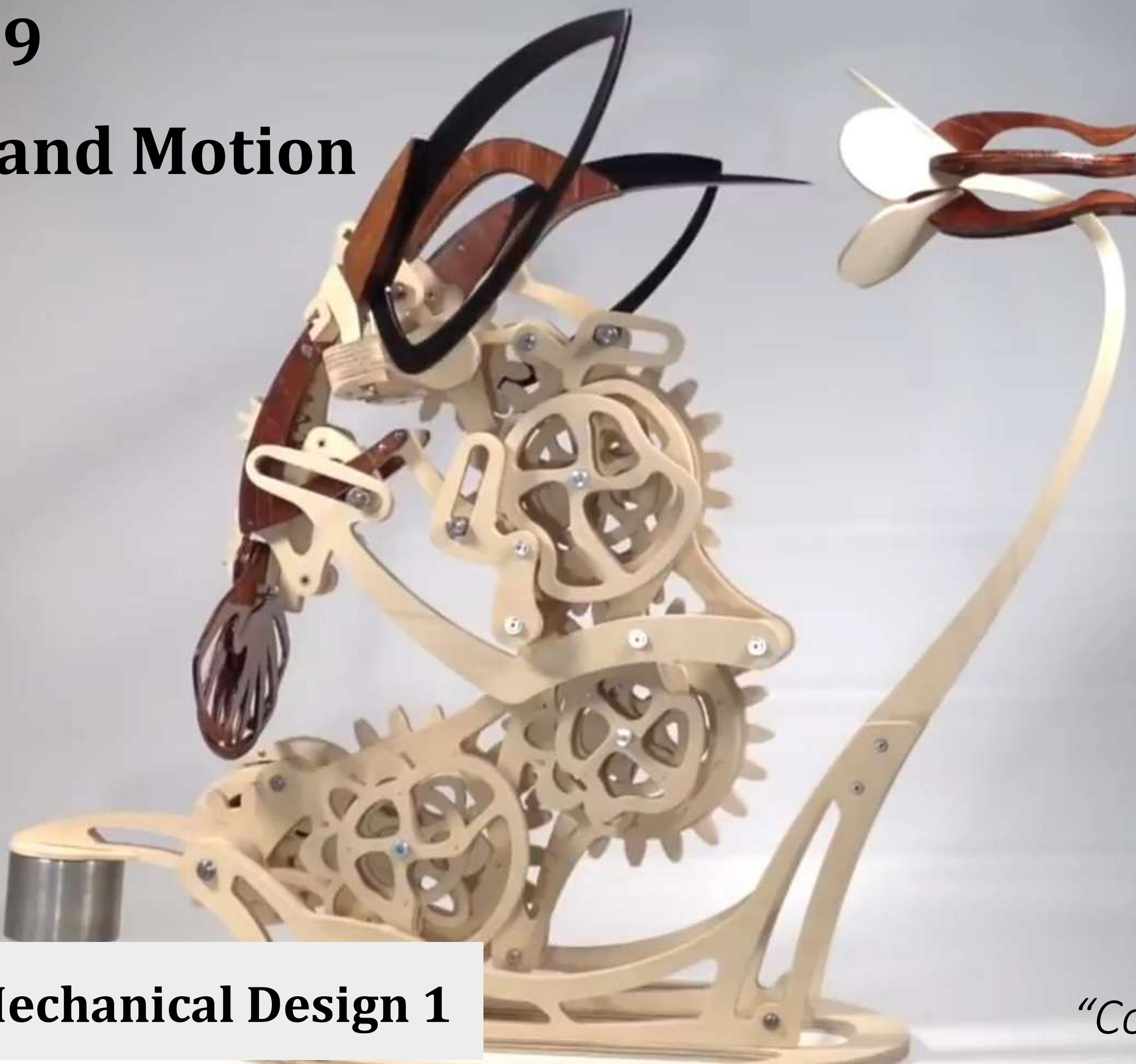
ILLINOIS



We can see several important gear elements here.

- Transmission with various gear trains (center)
- Two differential gear trains (front and rear)
- Additionally, linkage mechanisms in the suspension connected to the wheels

Lectures 16-19
Motors Cams and Motion Control



“Colibri” by Derek Hugger

* www.youtube.com/watch?v=1scj5sotD-E

ME 370 - Mechanical Design 1

Module 6 topics: Motors, Cams and Motion Control

- **Motors**
  - DC motor principle
  - DC motor model
  - Linear Motor Model
    - Constraints
    - Behavior in time
    - Gearboxes
  - Motor Parameters
  - Power and Efficiency

- **Cam and Follower**
  Types of Motion
  Types of Follower
Practical Considerations

- **Motion control**
  - Simple Motion Control Dwell-Rise-Dwell motions:
    - Fundamental Law of Cam (Motion) Design
    - Simple Harmonic Motion
    - Sinusoidal Acceleration (i.e., Cycloidal Displacement)
  - Advanced Motion Control
    - Additional Dwell-Rise-Dwell motions:
      - Trapezoidal acceleration
      - Modified Trapezoidal acceleration
      - Modified Sine acceleration
      - 3-4-5 Polynomial Rise Displacement
      - 4-5-6-7 Polynomial Rise Displacement
    - Rise-Fall-Dwell motions:
      - Cycloidal Motion
      - Double Harmonic
      - 3-4-5-6 Polynomial

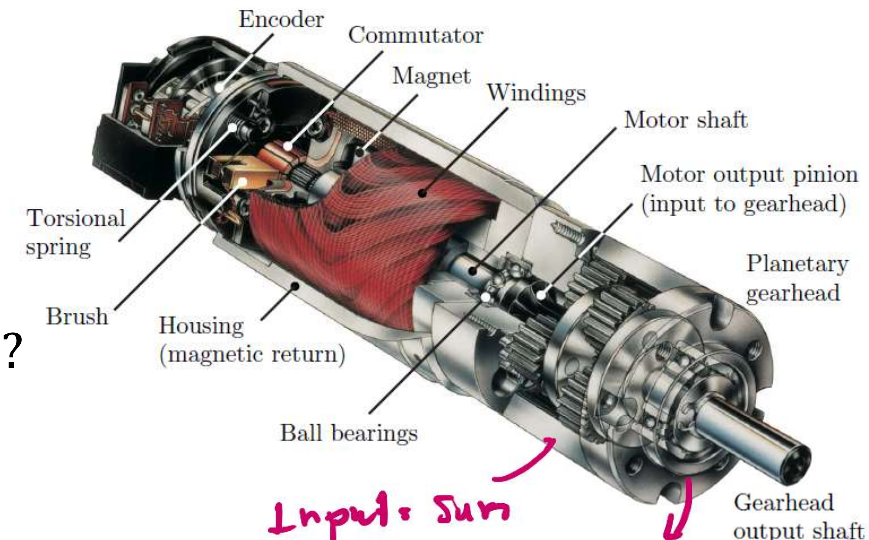
Motors: Learning Objectives

By the end of this lecture, you should be able to:

- Explain the basic operating principle of an electric motor and the meaning of key parameters
- Relate the speed and torque of a motor to predict its performance

DC Electric Motors:

- Convert electrical to mechanical power
  - Input: electrical voltage V and current I
  - Output: mechanical rotation (angular velocity ω) and torque T
- Often attached to a gearbox
  - Relatively high angular velocity and low torque
- Our objective today:
  - How do we relate V, I, ω, T?
  - How do we solve for one given the others?

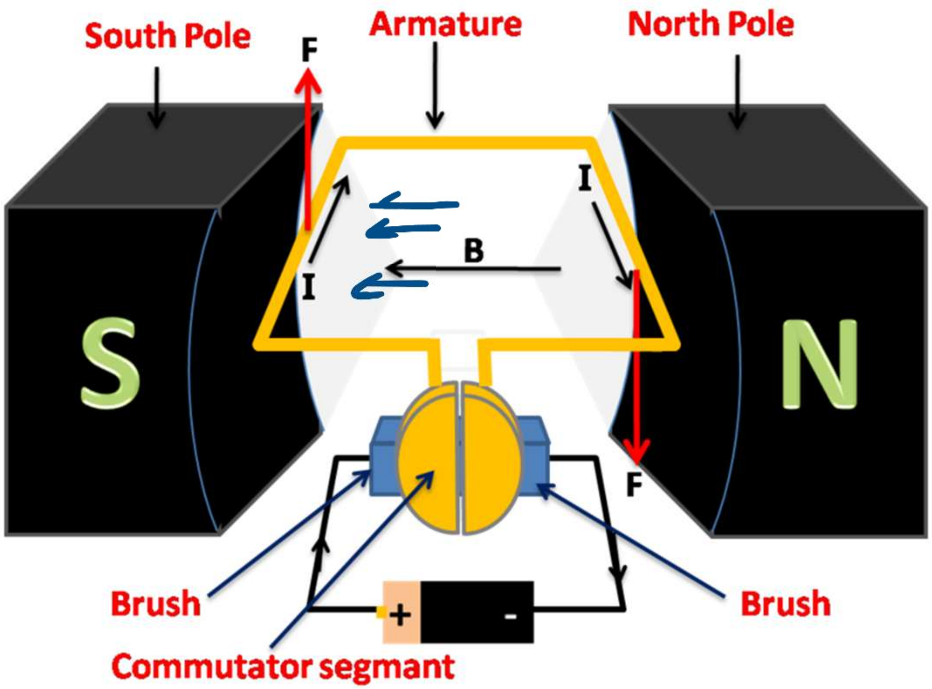


Input: sun

output carrier

Figures from *Modern Robotics*

DC motor principle



https://upload.wikimedia.org/wikipedia/commons/9/94/Electric_motor_working_process.png

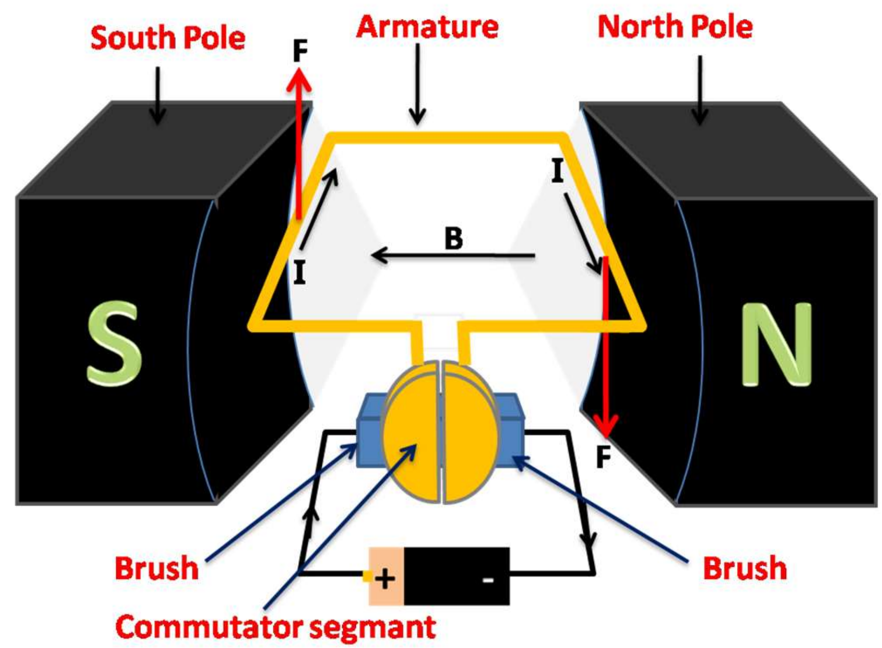
Full length video to explain DC motor operation
https://www.youtube.com/watch?v=LAtPHANEfQo

DC motor principle

- The motor velocity is controlled by changing the supply voltage
- The generated torque is proportional to the current drawn in the armature i and the magnetic flux ψ

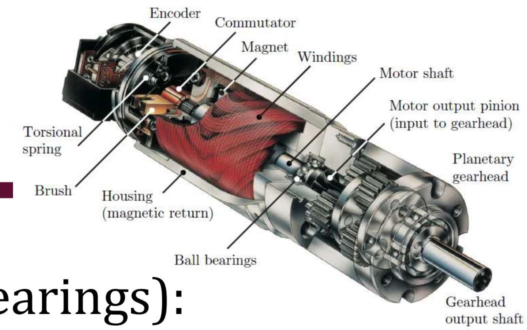
$$T \propto \psi, \qquad T \propto i$$

- Motors must commutate (swap armature current) to rotate continuously



https://upload.wikimedia.org/wikipedia/commons/9/94/Electric_motor_working_process.png

DC motor model



Electric (windings):

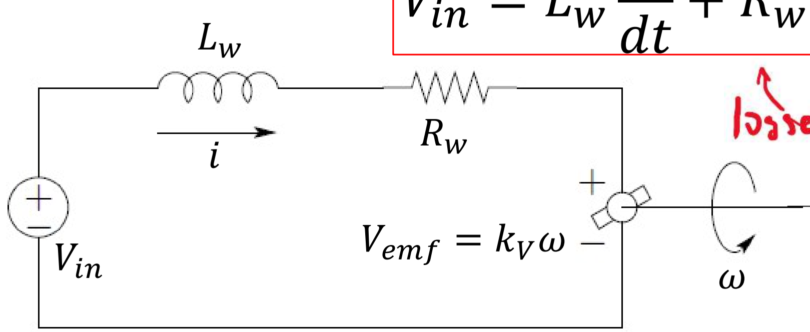
- Input voltage to the windings V_{in} (V)
- Current in the windings i (A)
- Winding inductance L_w (H)
- Winding resistance R_w (Ω)
- Torque & speed constants k_T, k_V
  (V s/rad = N m/A)

Electrical inertia

$$\boxed{V_{in} = L_w \frac{di}{dt} + R_w i + k_V \omega}$$

losses

to mech Energy



Mechanical (shaft/bearings):

- Rotor angular velocity $\omega = \dot{\theta}$ (rad/s)
- Motor torque T_m (N m)
- Load torque T_l (N m)
- Rotor inertia I_r (kg m^2)
- Bearings viscous friction b (N m s/rad)

(variables)

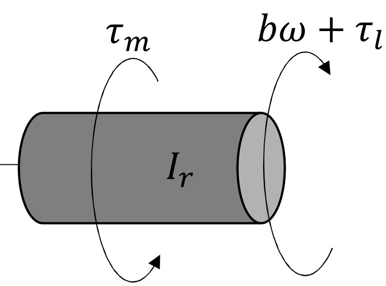
(constant parameters)

$$\boxed{T_m = k_T i = I_r \dot{\omega} + b\omega + T_l}$$

Inertia

friction

load



ILLINOIS

DC motor model

Solve current from Electric system

$$V_{in} = L_w \frac{di}{dt} + R_w i + k_V \omega$$

$$T_m = k_T i = I\dot{\omega} + b\omega + T_l$$

$$V_{in} = R_w i + k_V \omega$$

$k_T = k_v$ ideally (100%. efficiency)

$$\boxed{i = \frac{V_{in} - k_V \omega}{R_w}}$$

$\times\ k_T$

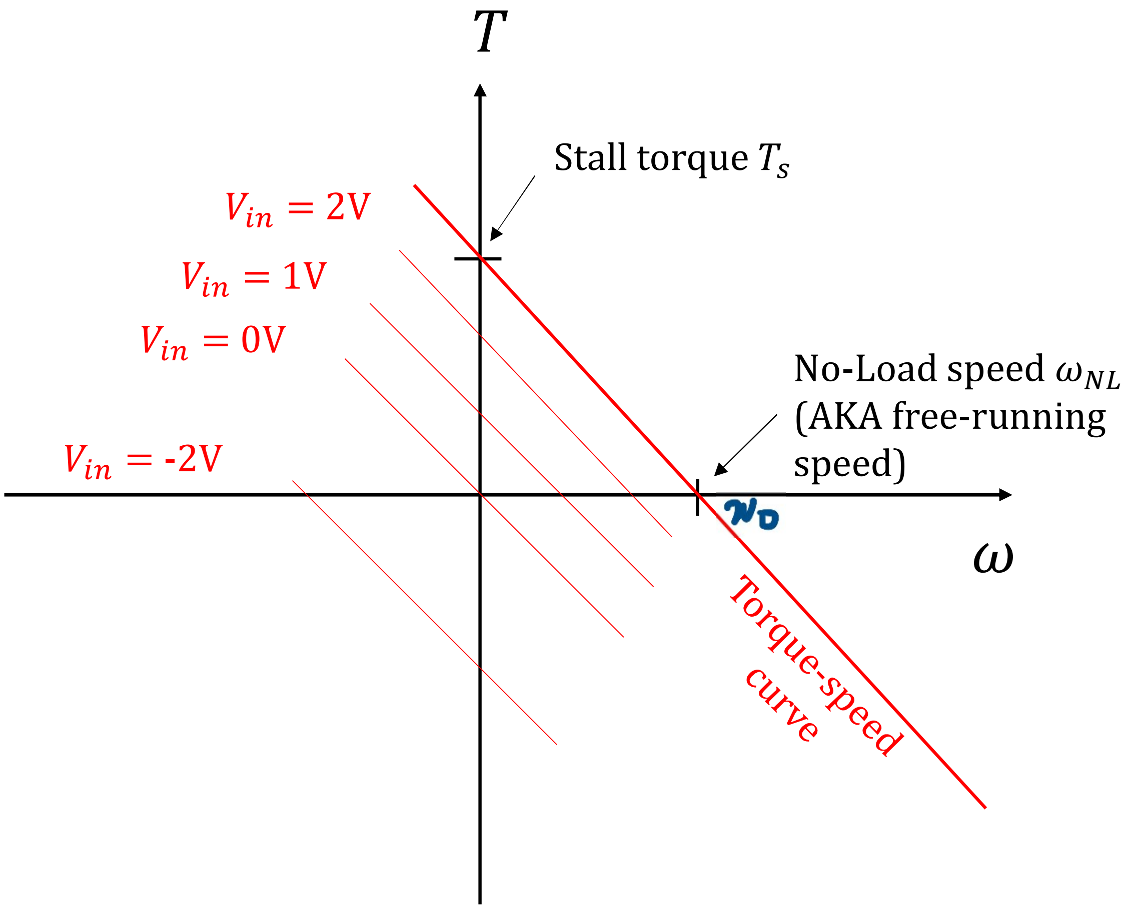
$$T = \frac{k_T}{R_w} V_{in} - \frac{k_T^2}{R_w} \omega$$

$k_T \left[\frac{Nm}{A}\right]$

$k_V \left[\frac{V}{rads/s}\right]$

equivalent

Linear Motor Model

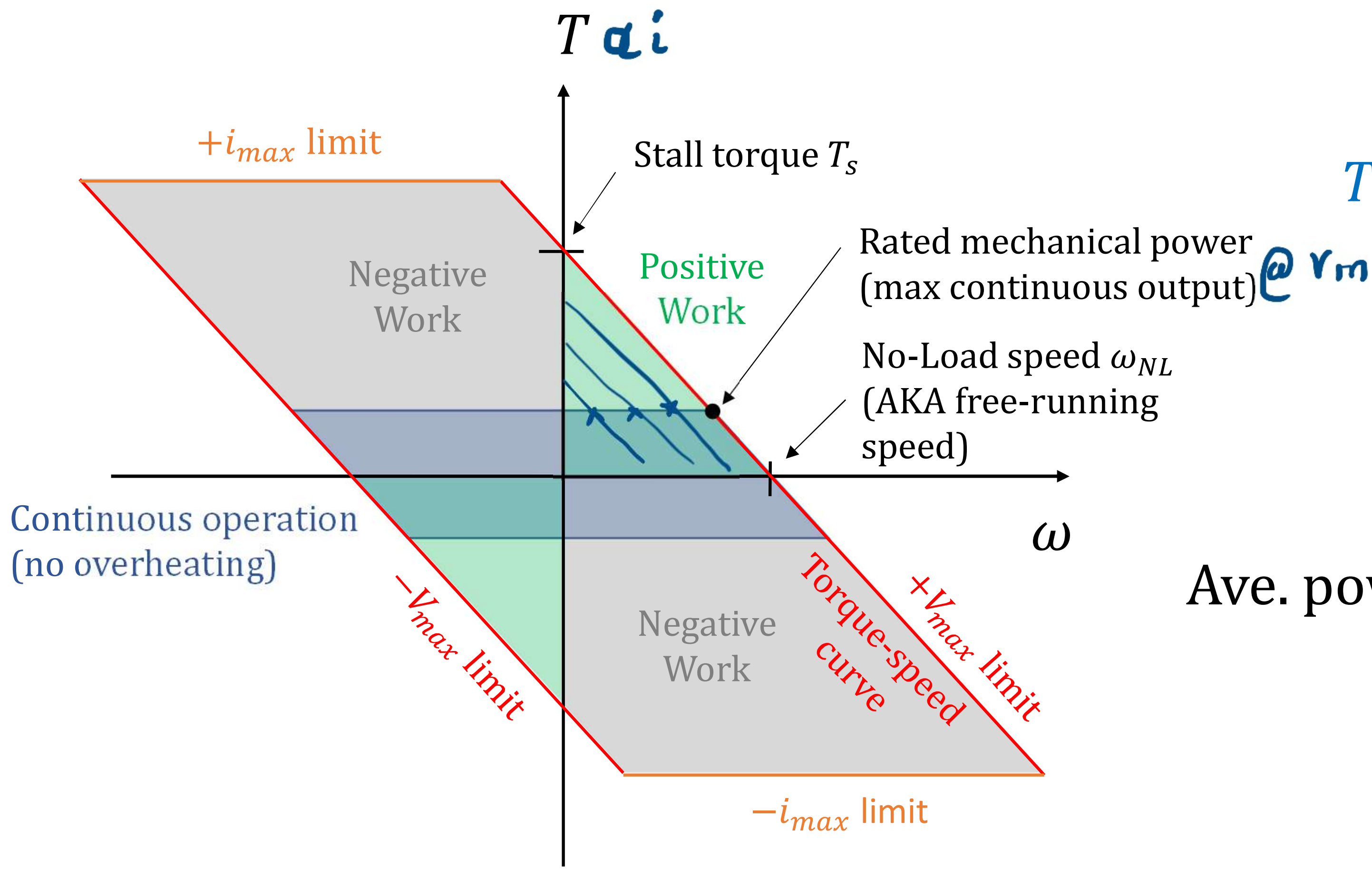


$$T = \frac{k_T}{R_w} V_{in} - \frac{k_T^2}{R_w} \omega$$

$$T = 0 : \omega_0 = \frac{V_{in}}{k_t}$$

$$\omega = 0 : T_s = \frac{k_T V_{in}}{R_w} = k_T i_s$$

Linear Motor Model: constraints

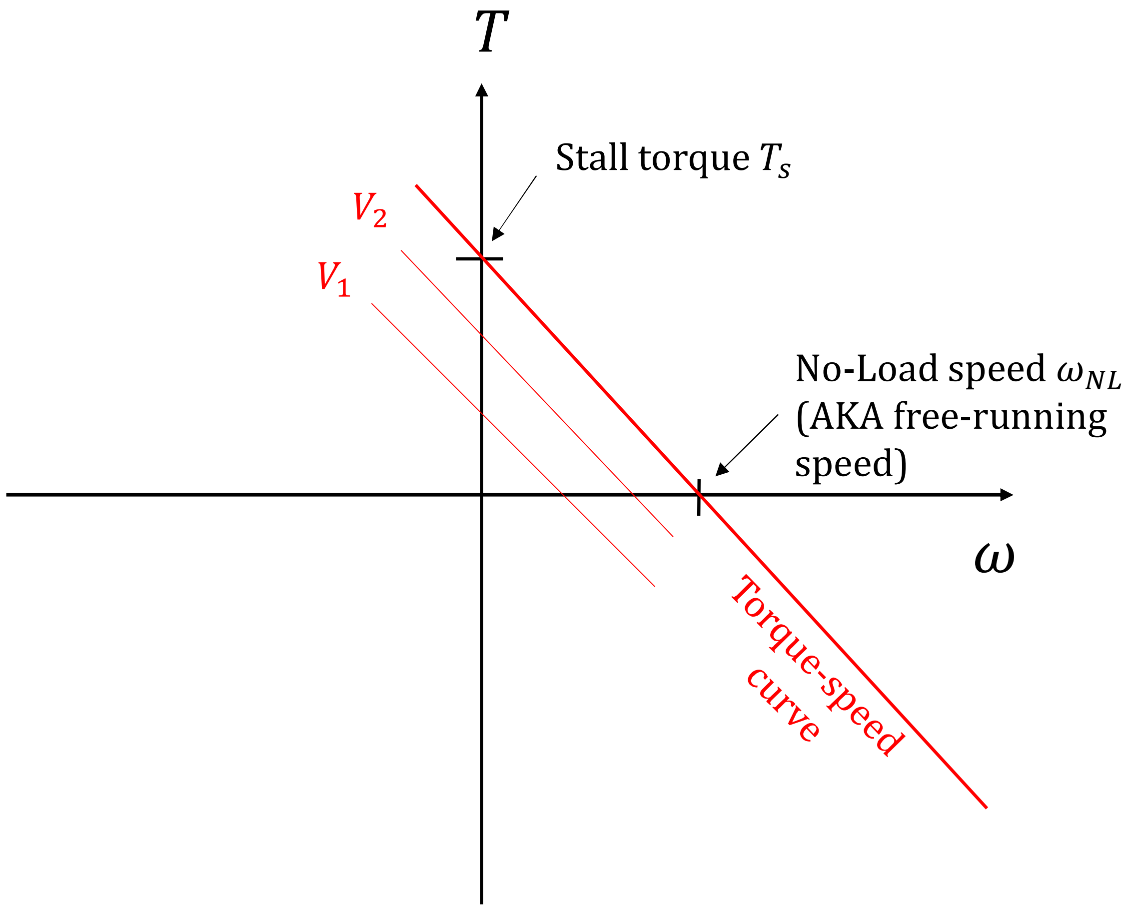


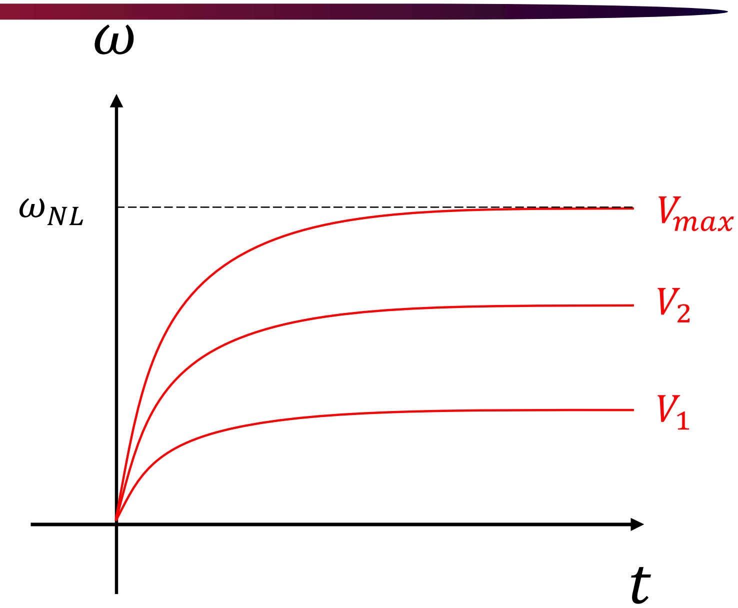
$$T = \frac{k_T}{R_w} V_{in} - \frac{k_T^2}{R_w} \omega$$

$$|V| < V_{max}$$
$$|i| < i_{max}$$

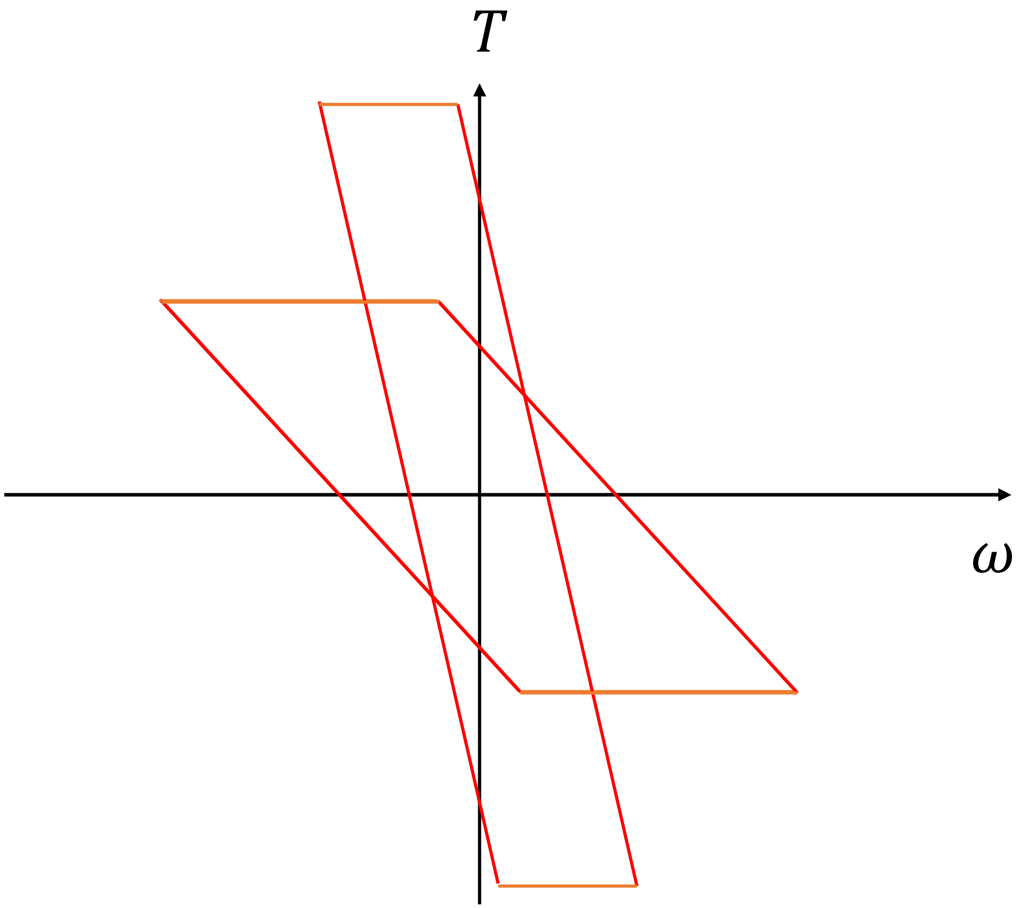
Ave. power $i^2 R$ < overheating

Linear Motor Model: behavior in time





Linear Motor Model: gearboxes



$$T = \frac{k_T}{R_w} V_{in} - \frac{k_T^2}{R_w} \omega$$

Gearboxes: velocity ratio m_v

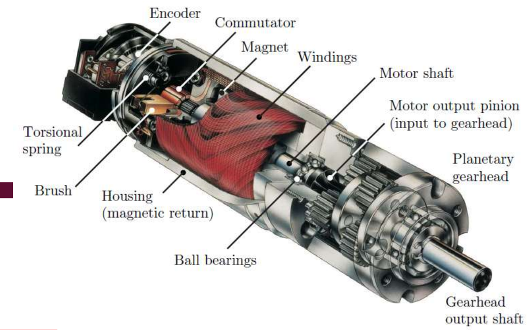
For $m_v > 1$:

Torque increases by $1/m_v$

Velocity decreases by m_v

ILLINOIS

Motor Parameters:



$$\boxed{k_V = \frac{V_{nom} - i_{NL}R_w}{\omega_{NL}}}$$

$$\boxed{k_T \approx \frac{T_{stall}}{i_{stall}}}$$

$$\boxed{R_w = \frac{V_{nom}}{i_{stall}}}$$

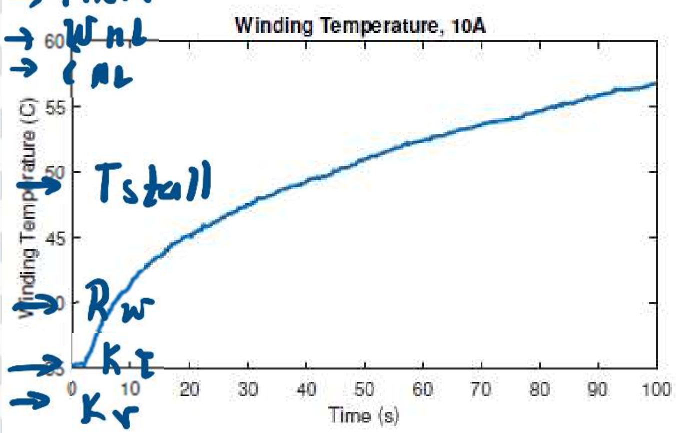
$$\boxed{P_w = R_w i^2}$$

$$V_{in} = L_w\frac{di}{dt} + R_w i + k_V\omega$$

$$k_T i = I_r\dot{\omega} + b\omega + T_l$$

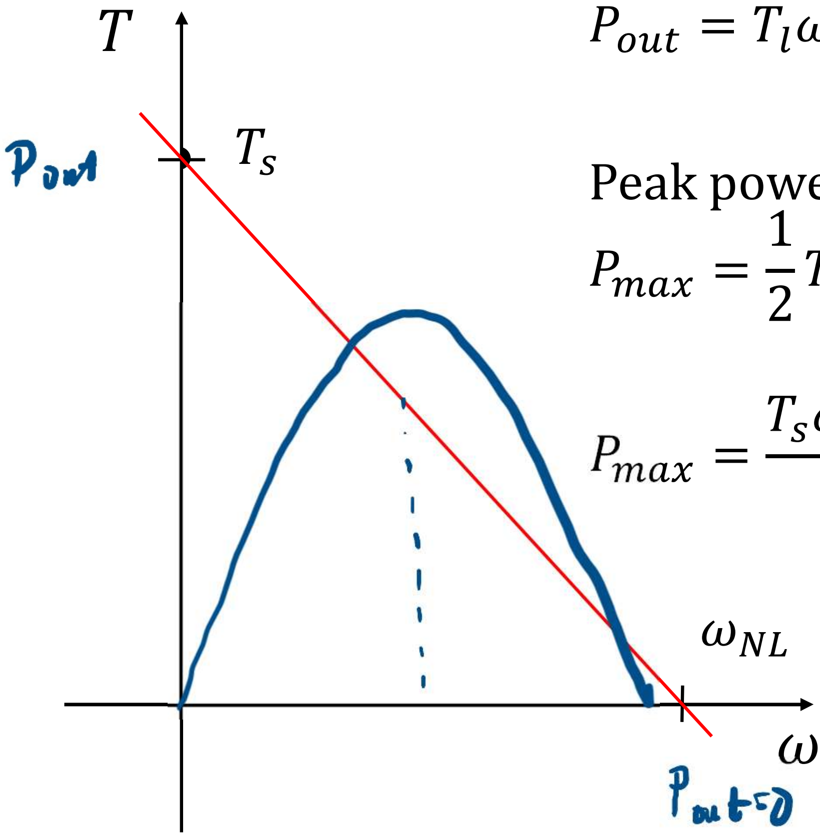
nominal = Rated (design)

Motor Data				
	Values at nominal voltage			
1	Nominal voltage	V	12	→ V_{nom}
2	No load speed	rpm	6920	→ ω_{NL}
3	No load current	mA	241	→ i_{NL}
4	Nominal speed	rpm	6380	
5	Nominal torque (max. continuous torque)	mNm	94.9	
6	Nominal current (max. continuous current)	A	6	
7	Stall torque	mNm	1720	→ T_{stall}
8	Starting current	A	105	
9	Max. efficiency	%	87	
	Characteristics			
10	Terminal resistance	Ω	0.115	→ R_w
11	Terminal inductance	mH	0.0245	
12	Torque constant	mNm/A	16.4	→ k_T
13	Speed constant	rpm/V	581	→ k_V
14	Speed / torque gradient	rpm/mNm	4.05	
15	Mechanical time constant	ms	5.89	
16	Rotor inertia	gcm²	139	



ILLINOIS

Power



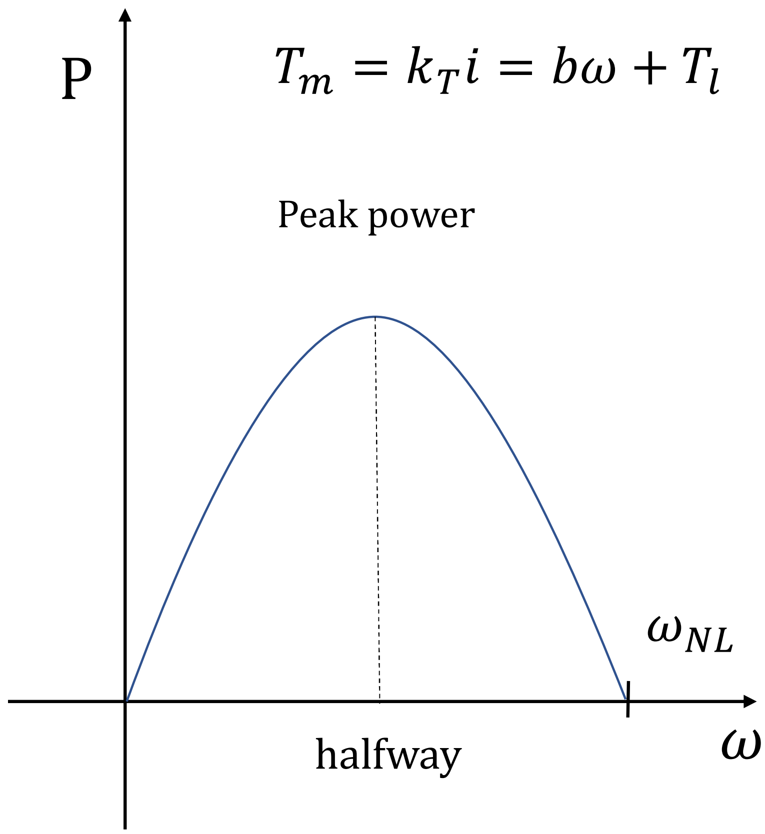
Mechanical power out:
$P_{out} = T_l \omega$

Peak power:
$$P_{max} = \frac{1}{2} T_s \frac{1}{2} \omega_{NL}$$

$$P_{max} = \frac{T_s \omega_{NL}}{4}$$

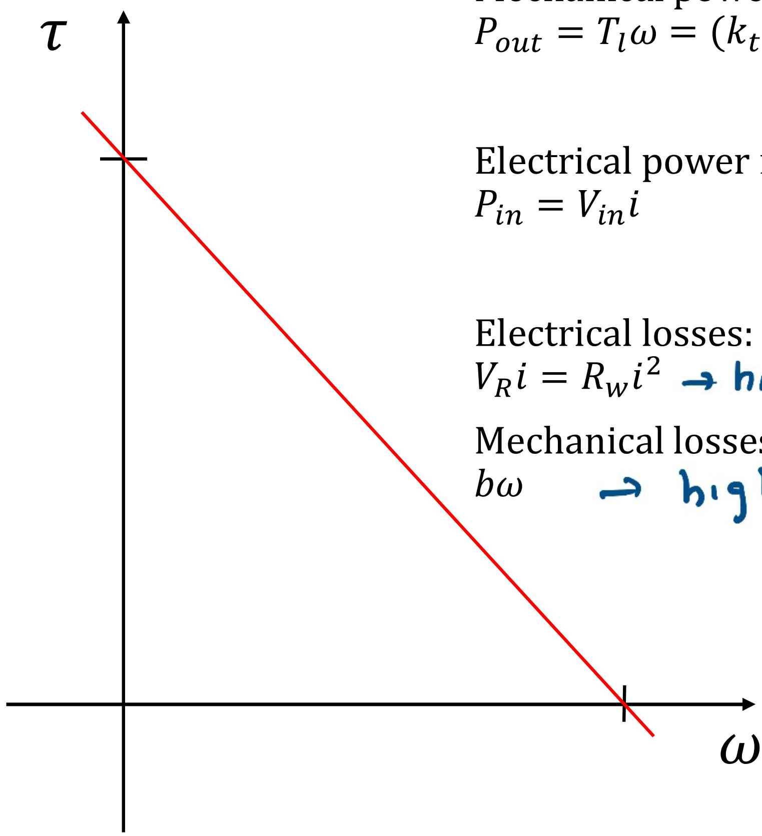
$V_{in} = R_w i + k_V \omega$

$T_m = k_T i = b\omega + T_l$



ILLINOIS

Efficiency



Mechanical power out:
$P_{out} = T_l\omega = (k_t i - b\omega)\omega$

Electrical power in:
$P_{in} = V_{in} i$

Electrical losses:
$V_R i = R_w i^2$ → high at low speeds

Mechanical losses:
$b\omega$ → high at high speeds

$$V_{in} = R_w i + k_V \omega$$

$$T_m = k_T i = b\omega + T_l$$