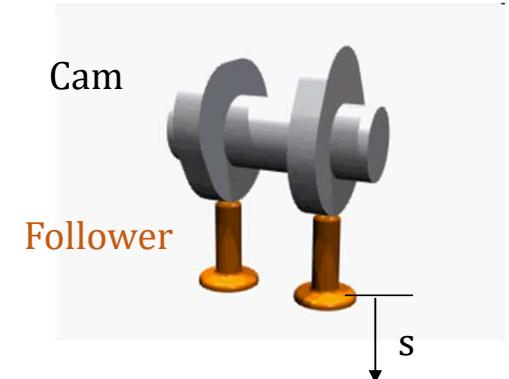


Cam Motion

Purpose:

- Transform rotational motion to translational (typically) motion
- Produce desired repetitive output motion, or displacement (s)
 - Function generator
 - Intermittent motion
- More expensive than gears
- Wears more easily, needs lubricant



https://en.wikipedia.org/wiki/Cam#/media/File:Nockenwelle_ani.gif

See these URLs for examples of motions and how to build own from paper

<http://www.robives.com/mechanisms/cams> this URL has old Adobe Flash images, but has good explanation of cam design considerations

<https://www.robives.com/mechanism/cam/> this newer URL shows multiple types of cam designs

Lecture 18

Simple Motion Control



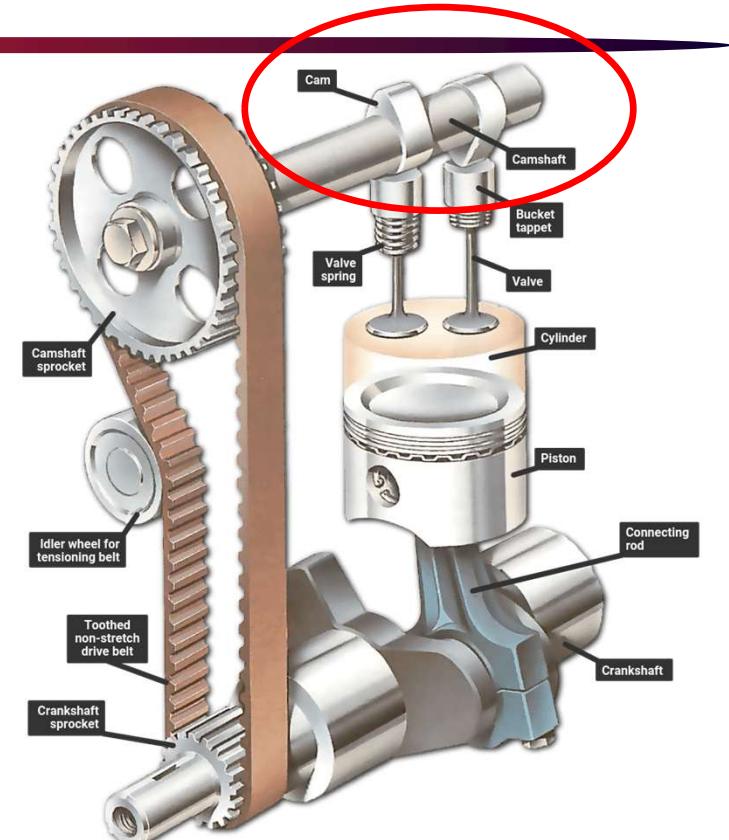
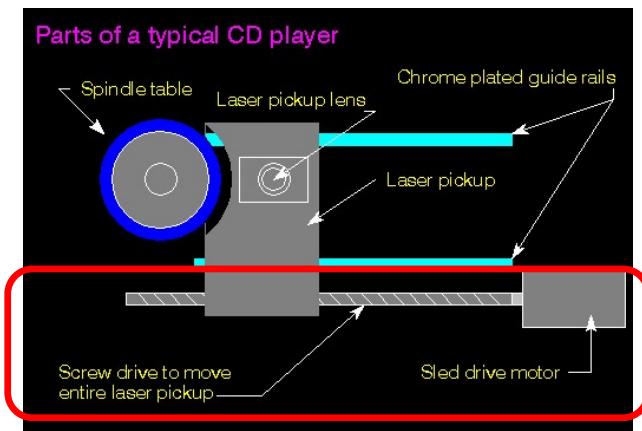
ME 370 - Mechanical Design 1

Module 6 topics: Motors, Cams and Motion Control

- **Motors**
 - DC motor principle
 - DC motor model
 - Linear Motor Model
- Constraints
- Behavior in time
- Gearboxes
- Motor Parameters
- Power and Efficiency
- Cam and Follower
 - Types of Motion
 - Types of Follower
 - Practical Considerations
- **Motion control**
 - Simple Motion Control Dwell-Rise-Dwell motions:
 - Fundamental Law of Cam (Motion) Design
 - Simple Harmonic Motion
 - Sinusoidal Acceleration (i.e., Cycloidal Displacement)
 - Advanced Motion Control
 - Additional Dwell-Rise-Dwell motions:
 - Trapezoidal acceleration
 - Modified Trapezoidal acceleration
 - Modified Sine acceleration
 - 3-4-5 Polynomial Rise Displacement
 - 4-5-6-7 Polynomial Rise Displacement
 - Rise-Fall-Dwell motions:
 - Cycloidal Motion
 - Double Harmonic
 - 3-4-5-6 Polynomial

Recall: Cam Usage

- Conventional cam design:
 - sewing machine
 - 4-stroke internal combustion engine
- New designs:
 - Replace cams with actuators
 - Pneumatic, electro-mechanical
 - Example: Laser positioning in CD drive



<https://www.howacarworks.com/basics/the-engine-how-the-valves-open-and-close>

Types of Motion Constraints

1) Critical extreme position (CEP):

- Define end positions of follower, but not path

2) Critical path motion (CPM)

- Define path (and possibly velocity and acceleration)

Dwell:

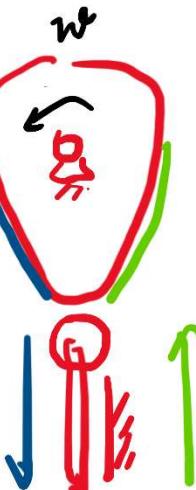
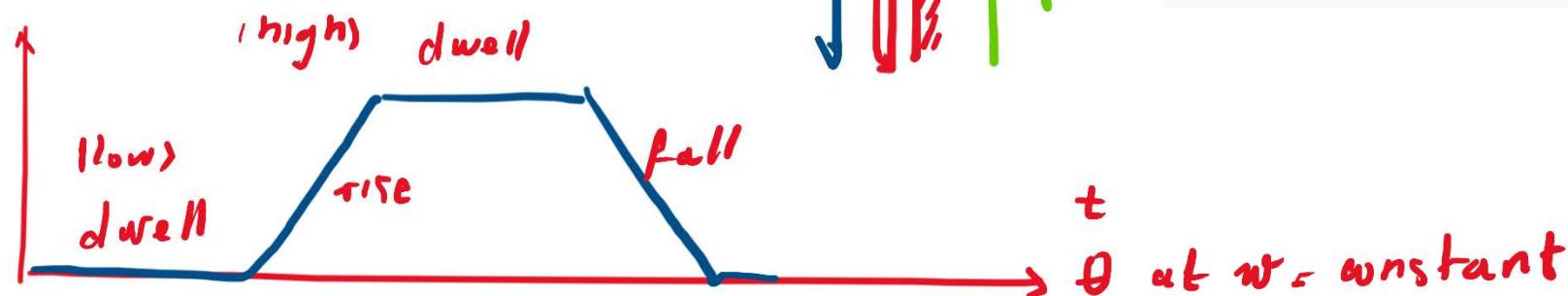
- No output motion for specified input motion



Critical Extreme Position (CEP): Use piecewise functions to produce motion program

Examples Motion Programs:

- Rise-Fall
- Rise-Fall-Dwell
- Rise-Dwell-Fall-Dwell



θ at w - constant



See these URLs for examples of motions and how to build own from paper

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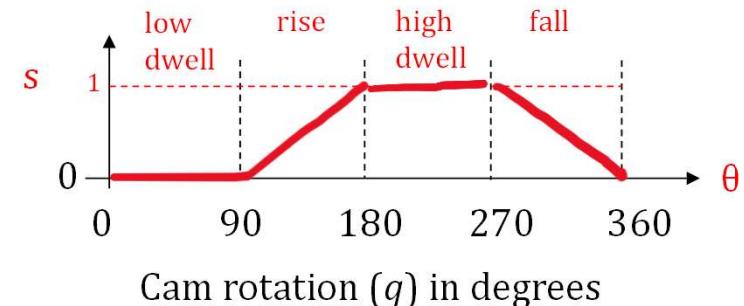
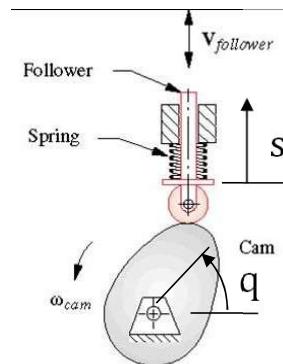
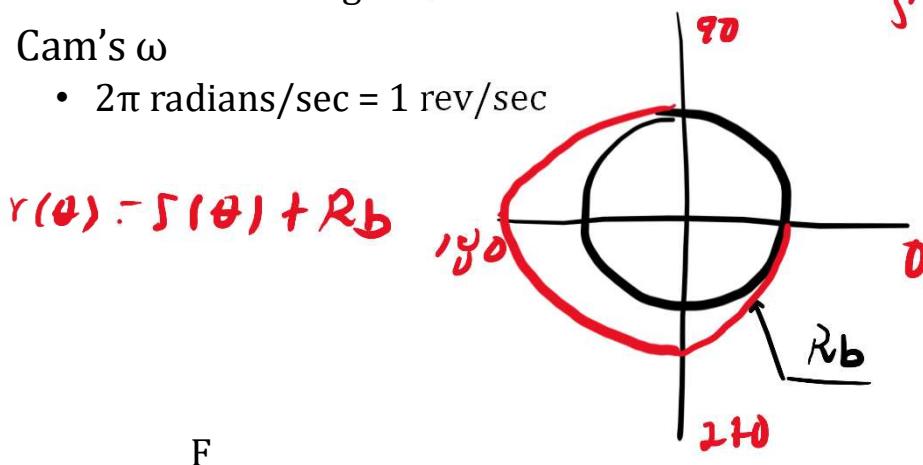
$$S = m\theta + b$$

$$\theta = 90^\circ \rightarrow S = 0$$

$$\theta = 180^\circ \rightarrow S = 1$$

Example 1: Dwell-Rise-Dwell-Fall using Linear Functions

- Dwell**
 - at zero displacement for 90 degrees (low dwell)
- Rise**
 - 1 inch in 90 degrees
- Dwell**
 - at 1 inch for 90 degrees (high dwell)
- Fall**
 - 1 inch in 90 degrees
- Cam's ω
 - 2π radians/sec = 1 rev/sec



Exercise: Write down a piecewise equation of motion.

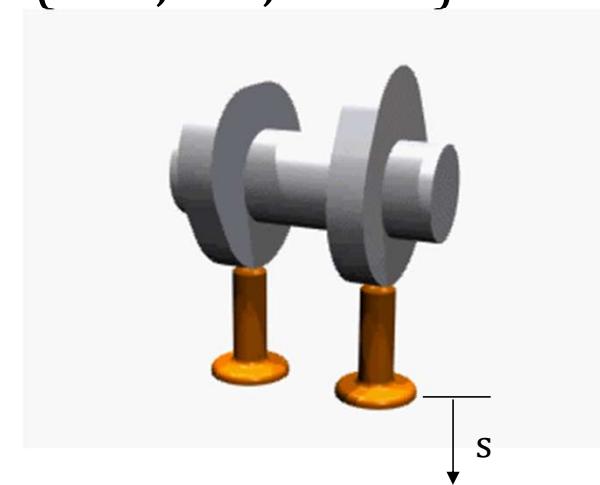
$$S(\theta) = \begin{cases} 1: 0 & 0 \leq \theta < 90^\circ \\ 2: \frac{\theta - 90}{90} & 90 \leq \theta < 180^\circ \\ 3: 1 & 180 \leq \theta < 270^\circ \\ 4: \frac{360 - \theta}{90} & 270^\circ \leq \theta \leq 360^\circ \end{cases}$$

How to Design the Motion in Cams

- The following should be considered when selecting the mathematical expressions that define each piece of motion (rise, fall, dwell) for reliable operation:

- Displacement (s)
- Velocity (v)
- Acceleration (a) → Relates to force
- Jerk (j) → Relates to force change

Excitations of harmonics



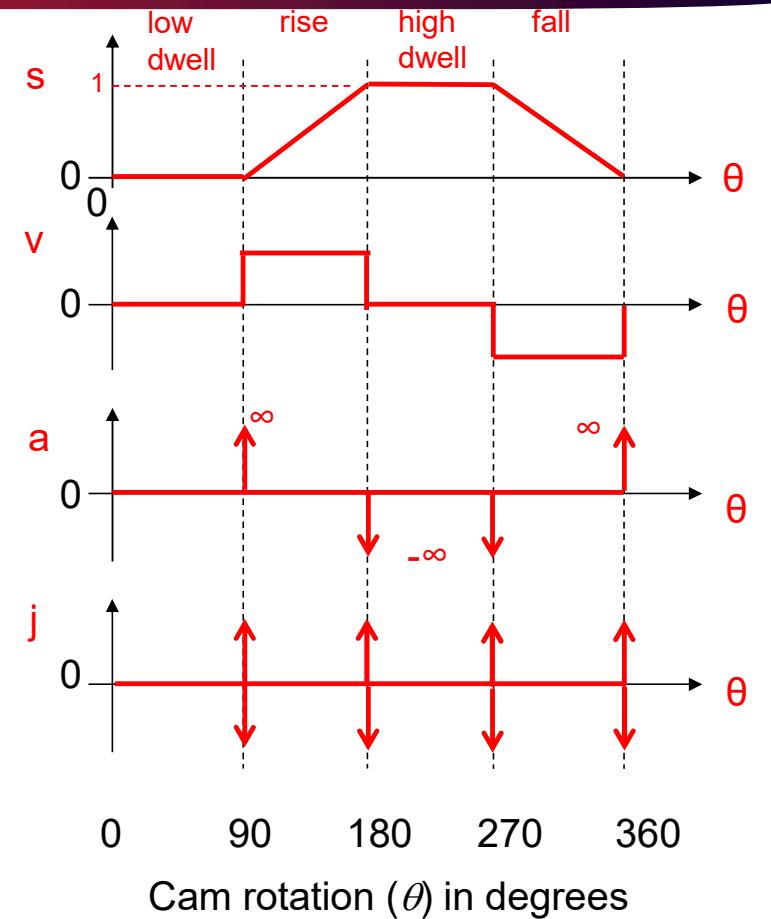
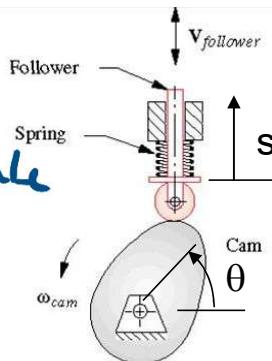
- Both the behavior of these functions within the piece of motion and in between different pieces of motion are important.

Example: Dwell-Rise-Dwell-Fall using Linear Functions

- **Dwell**
 - at zero displacement for 90 degrees (low dwell)
- **Rise**
 - 1 inch in 90 degrees
- **Dwell**
 - at 1 inch for 90 degrees (high dwell)
- **Fall**
 - 1 inch in 90 degrees
- Cam's ω
 - 2π radians/sec = 1 rev/sec

|a|

all frequencies participate
executes harmonics!



A note on angular position, θ , and time, t

We can convert between angular position θ and time t easily because ω is constant.

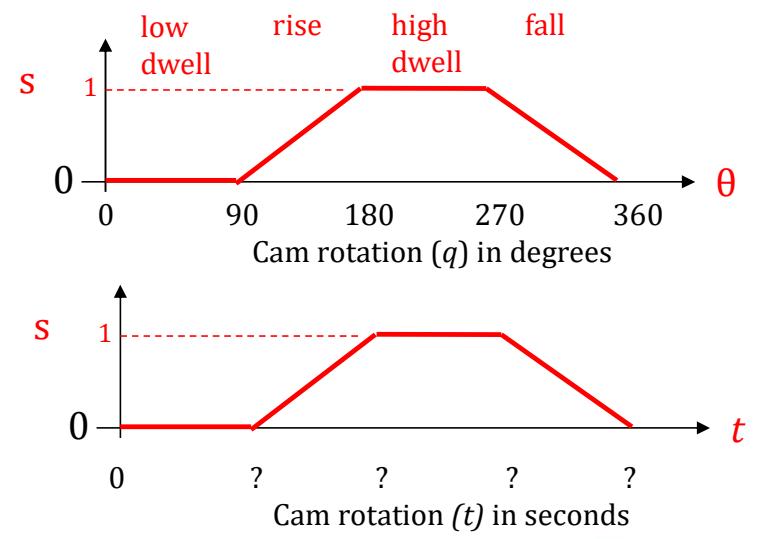
Given: $s = s(\theta)$

- Convert $s(\theta)$ to $s(t)$ by replacing θ by t and β by t' .
 - β is the angular interval of rise (or fall).
 - t' is the time interval of rise (or fall).

Ex: $\omega = 1 \text{ rpm} = 6 \text{ deg/sec.}$

- If $\beta = 120 \text{ deg}$, what is $t' = ?$

$$t' = \frac{\beta}{\omega} = \frac{120}{6} = 15 \text{ s}$$



Motion Program and Practical Considerations

- **Motion Program:**
defined by combining several separate functions (piecewise function).
- **Any discontinuity in the velocity results in infinite acceleration.**
 - $F = ma \rightarrow$ Infinite force \rightarrow Infinite stress
- **Any discontinuity in acceleration results in infinite jerk.**
 - $dF/dt = mj$, therefore, dF/dt goes to infinity.
 - Such spikes in jerk can excite harmonics in the spring-mass system of the follower \rightarrow vibration in the mechanism.
- **Points where a dwell is connected to a rise or a fall:**
 - A **dwell** always has zero velocity and acceleration.
 - Piecewise functions for rise and fall must match this zero velocity and acceleration at the points where they connect to the dwell (for avoiding discontinuity).



Fundamental Law of Cam (Motion) Design

The cam function (= follower motion) must be continuous through first and second derivatives of displacement across entire interval (360°).

∴ requires 3rd order continuity

- Displacement (C^0 continuity)
 - Velocity (C^1 continuity)
 - Acceleration (C^2 continuity)
- In other words, position, velocity and acceleration should be continuous across the entire interval (360°).
 - **Corollary:** The jerk function must be finite across the entire interval (360°).

Example 2: Simple Harmonic Motion

$$s = \frac{h}{2} \left[1 - \cos \pi \frac{\theta}{\beta} \right]$$

β : total angle of rise interval

θ : cam shaft angle

h : total rise displacement

$$s = \frac{h}{2} \left[1 - \cos \pi \frac{\theta}{\beta} \right]$$

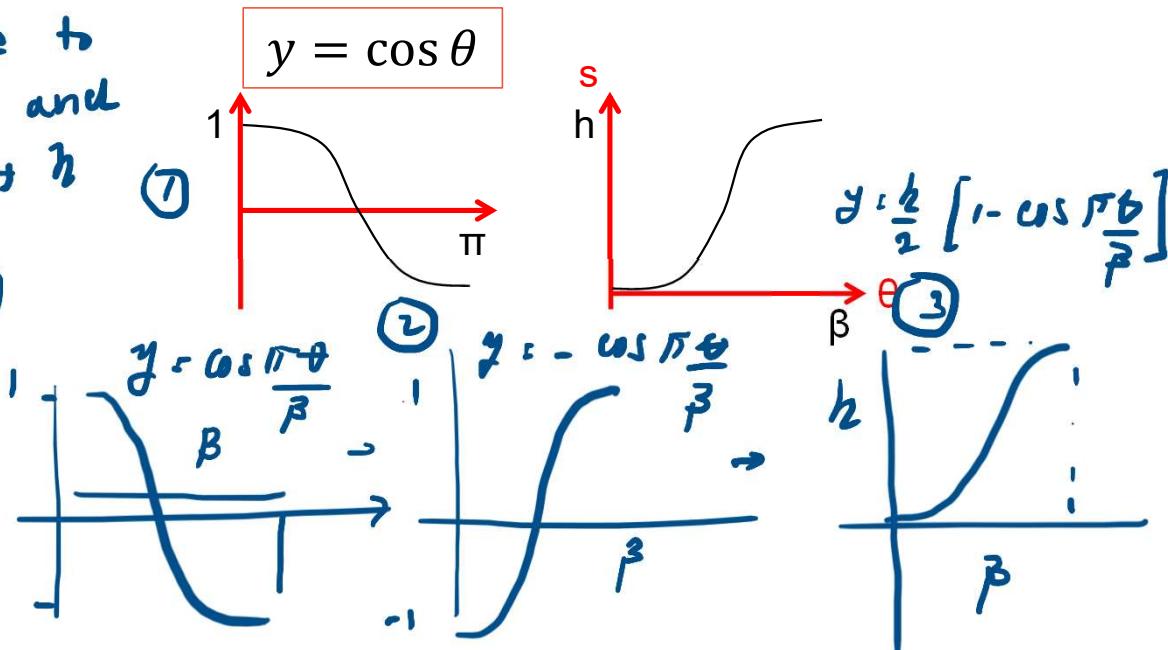
$$v = \frac{\pi h}{\beta^2} \sin \pi \frac{\theta}{\beta}$$

$$a = \frac{\pi^2 h}{\beta^3} \cos \pi \frac{\theta}{\beta}$$

$$j = \frac{\pi^3 h}{\beta^4} \sin \pi \frac{\theta}{\beta}$$

normalize to
rise of h and
interval of β

$$\begin{aligned}s &= s(\theta) \\ v &= v(\theta) \\ a &= a(\theta) \\ j &= j(\theta)\end{aligned}$$



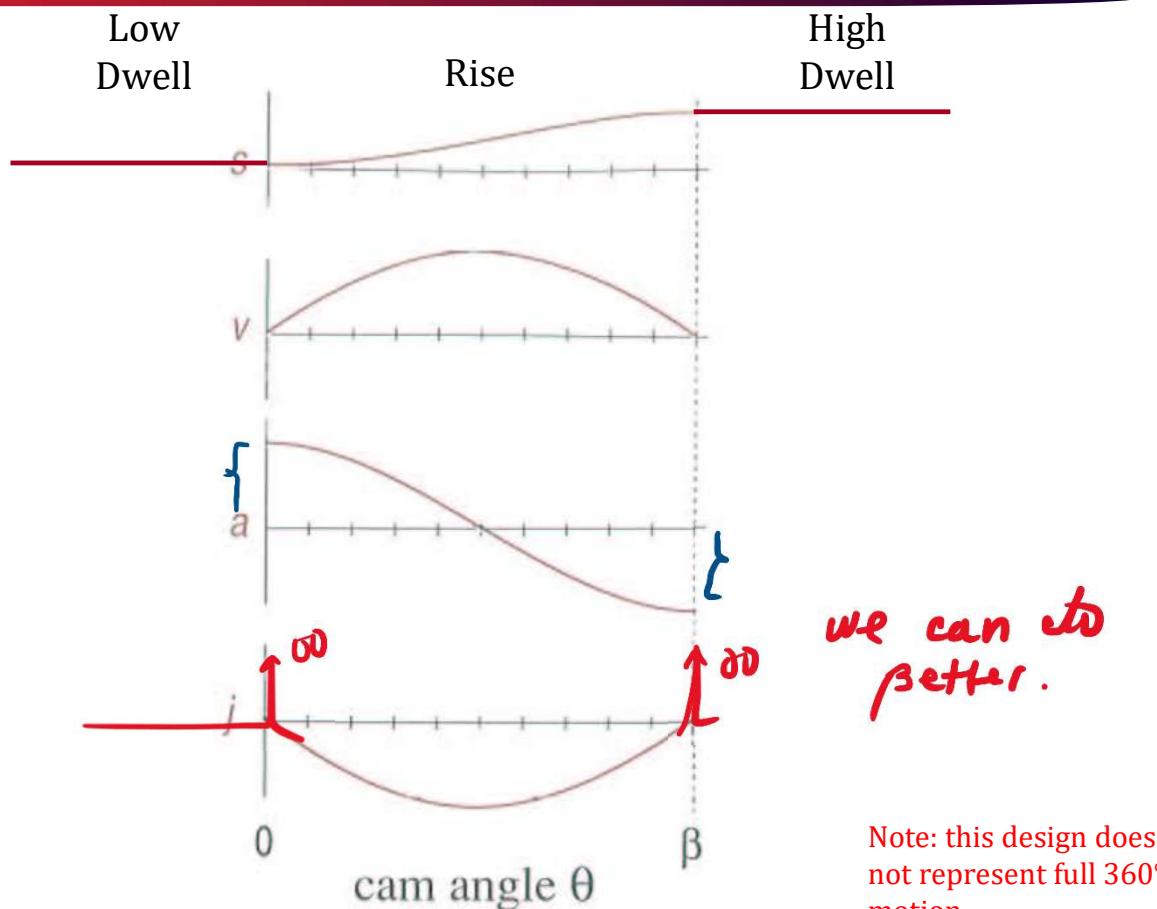
Example 2: Simple Harmonic Motion: Dwell-Rise-Dwell

$$s = \frac{h}{2} \left[1 - \cos \pi \frac{\theta}{\beta} \right]$$

$$v = \frac{\pi h}{\beta^2} \frac{1}{2} \sin \pi \frac{\theta}{\beta}$$

$$a = \frac{\pi^2 h}{\beta^3} \frac{1}{2} \cos \pi \frac{\theta}{\beta}$$

$$j = \frac{\pi^3 h}{\beta^4} \frac{1}{2} \sin \pi \frac{\theta}{\beta}$$



Note: this design does not represent full 360° motion

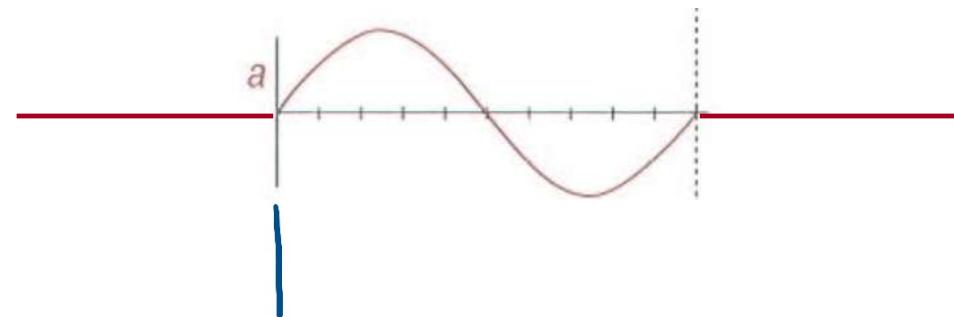
How can we get rid of spikes in jerk?

Design for acceleration function instead of displacement function

- Must have zero acceleration at ends (where it meets dwell)

Possible Solution:

- **Sinusoidal Acceleration (i.e., Cycloidal Displacement)**
 - Full-period sine function within rise (fall) interval



Cycloidal Displacement

$$\text{Let, } a = C \sin\left(2\pi \frac{\theta}{\beta}\right)$$

where C will be determined by boundary conditions.

- Equations for displacement, velocity, acceleration, and jerk will depend on boundary conditions of specific design.

Cycloidal Displacement - example

For

$$a = C \sin\left(2\pi \frac{\theta}{\beta}\right)$$

with boundary conditions of:

$$v = 0 \text{ at } \theta = 0 \text{ and } \theta = \beta$$

$$s = 0 \text{ at } \theta = 0,$$

$$s = h \text{ at } \theta = \beta$$

Get $s = h \left[\frac{\theta}{\beta} - \frac{1}{2\pi} \sin\left(2\pi \frac{\theta}{\beta}\right) \right]$

use $v = \frac{ds}{d\theta}$

$$v = \frac{h}{\beta} \left[1 - \cos\left(2\pi \frac{\theta}{\beta}\right) \right]$$

$$a = 2\pi \frac{h}{\beta^2} \sin\left(2\pi \frac{\theta}{\beta}\right)$$

$$j = 4\pi^2 \frac{h}{\beta^3} \cos\left(2\pi \frac{\theta}{\beta}\right)$$

Cycloidal displacement (Sinusoidal Acceleration)

