Module 3:

PVA Position, Velocity, and Acceleration Analysis (Topics 1-3)

"Colibri" by Derek Hugger

** www.youtube.com/watch?v=1scj5sotD-E*

ME 370 - Mechanical Design 1

Observations from P1D4

- We are now allowing the dispensing “wheel” or conveyor belt ideas, but require that the package must be dispensed after at least one complete turn of the hand crank
- The dispensing mechanism will need to attach to the single motor of the walker
  - Where to position the hand crank for ease of conversion? Will you need to resize/redesign the dispensing mechanism? Compact size, light weight, and ease of turning the crank are key features needed in P2. Design proactively to reduce need for major redesign
- Ensure that shafts are secured in at least two planes to maintain orientation
  - Gears want to push apart due to contact forces – secure all shafts
- Longer slider links will increase smooth sliding and reduce the likelihood of jamming or rotating in the slot
- No glue or tape allowed for structural supports in final design
- Ensure that user theme aligns with specifications in the Project Description

Module 3. PVA Topics: Reading - Norton Ch 4, 6, 7

1. Vector notation (Complex and Compact)
2. Analytical analysis method
   a. **P**osition analysis
   b. **V**elocity analysis
   c. **A**cceleration analysis
3. PVA analysis of a moving point
4. PVA analysis of a four-bar linkage
   a. Vector loop equation
5. PVA analysis of other four-bar mechanisms
   a. Offset slider-crank
   b. Inverted offset slider-crank
6. PVA analysis of mechanisms > four links

Covered in this slide deck

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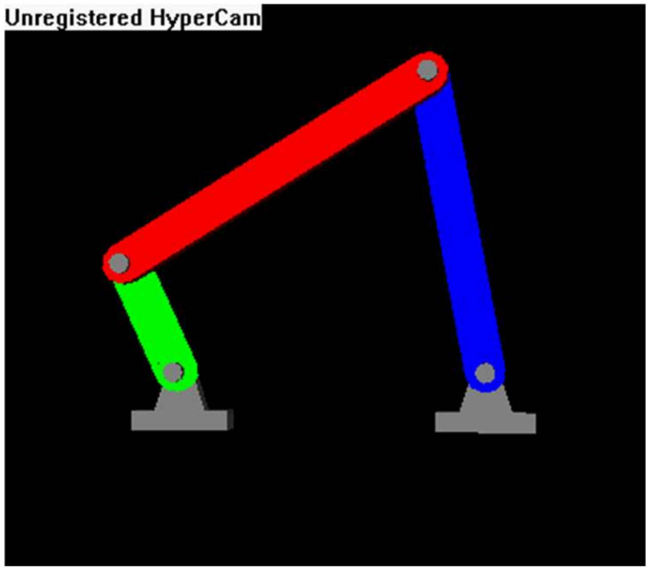
What do we know so far?

- Planar mechanisms Synthesis
  - Mobility
  - Quality of Motion
  - Graphically generate a desired motion or path
  - Visually (or with very little math) determine instantaneous velocity
- What is next?
  - Planar mechanisms Analysis using Position, Velocity and Acceleration

Position, Velocity, Acceleration (PVA) Analysis

- Now we want to analyze this mechanism
  - WHY?
    - Predict output
    - Prevent any failure
  - HOW?
    - Force & torques (F, T)
      - ▶ Acceleration (a, α)
        - ▶ Velocity (v, ω)
          - ▶ Position (r, θ)



Goal of PVA

Given:

- pivot locations
- link lengths
- Input (e.g., $\theta_2(t), \omega_2(t)$)

how many DOF do you need

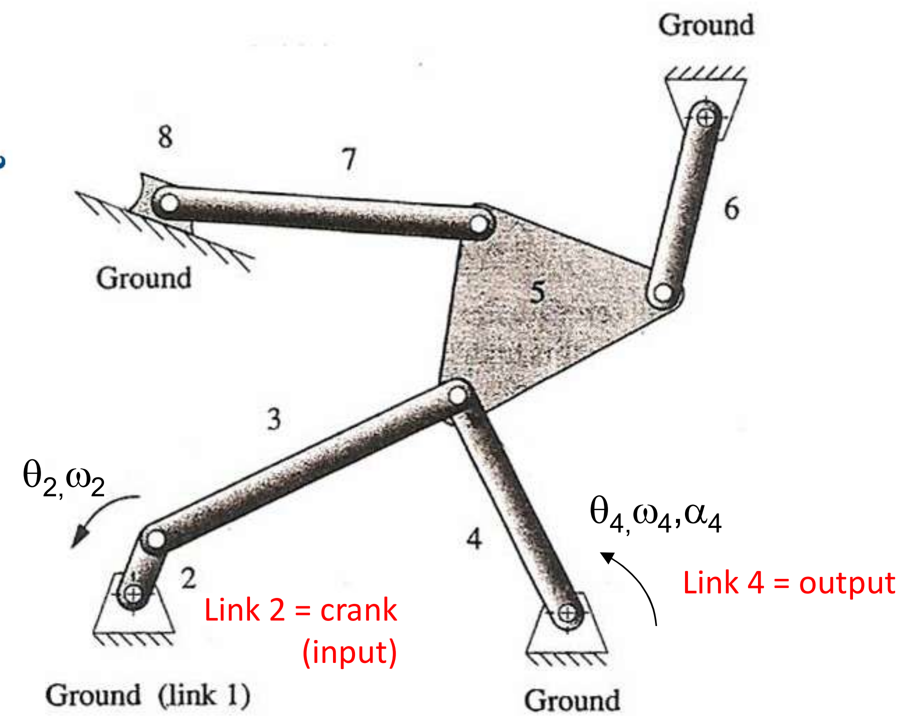
Find:

- Position $\theta_4(t)$
- Velocity $\omega_4(t)$
- Acceleration $\alpha_4(t)$

and any other

as a function of input (e.g., $\theta_2(t), \omega_2(t)$)

For a 1 DOF mechanism, these PVA values will dictate, as function of time, the exact orientation of the entire mechanism and the force & torques in the links and joints



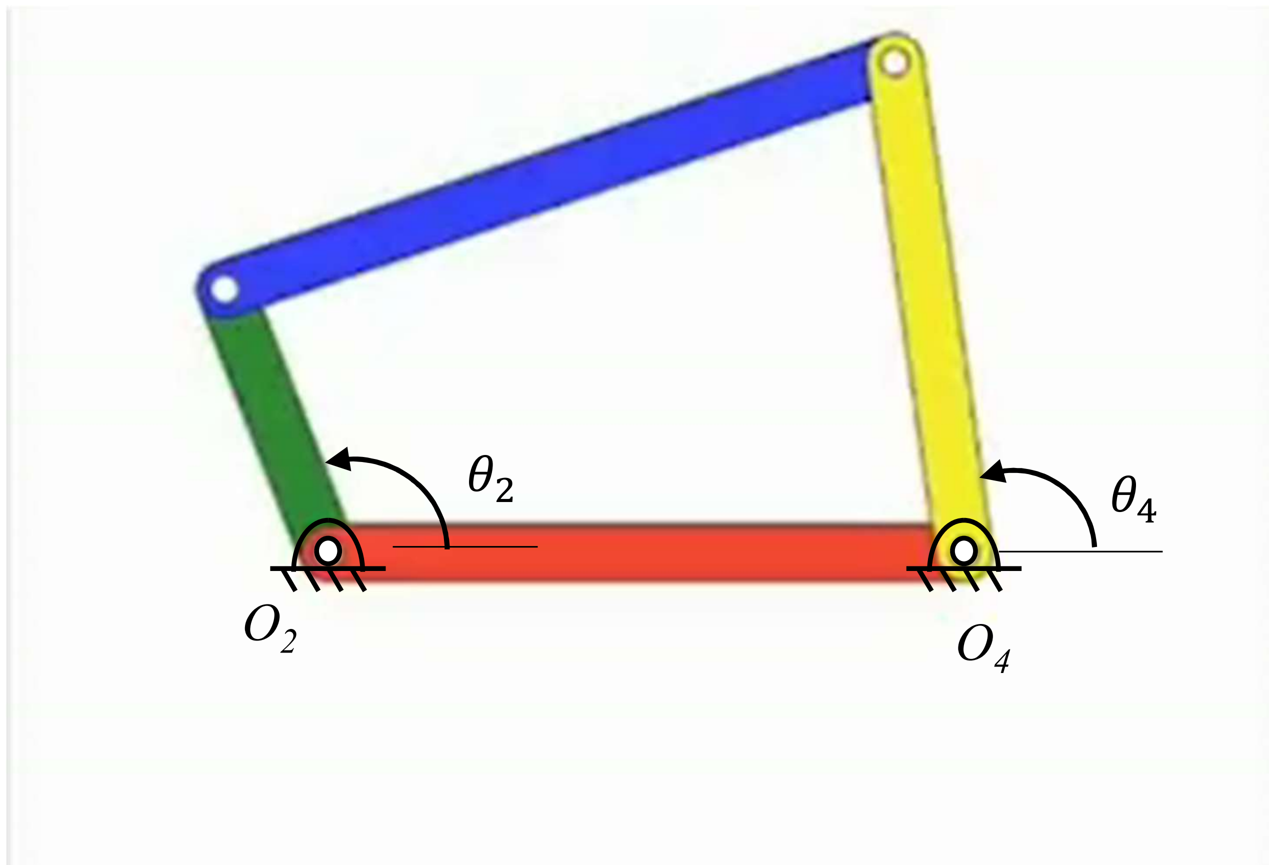
n = 8, J_1 = 10, J_2 = 0, DOF = 1

Two Techniques for PVA

1. Previous: Graphical (geometric)
    - Section 4.4
    - Only useful for specific position
    - Need to recalculate for different position
    - Quick first order approximation
2. Now: Analytical (algebraic)
    - Section 4.5+
    - Useful for multiple position
    - Based on equations
    - Can be solved by computer software (MATLAB, Python)
    - Use vector notation (compact notation)

PVA analysis of 4-bar linkage

- Given: a, b, c, d, and θ_2
- Solve for θ_3, θ_4



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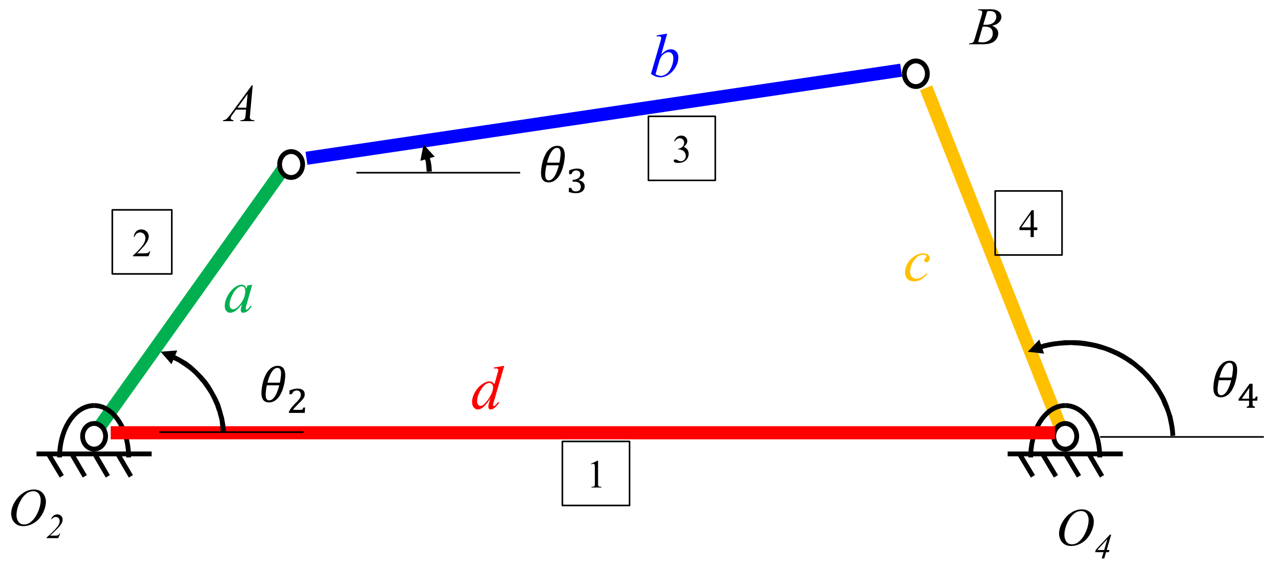

What is the value of θ_1(t)?

- uses distances and directions → we want vectors
- vector convention should be: - compact
  - easy to write for position
  - easy to compute velocity

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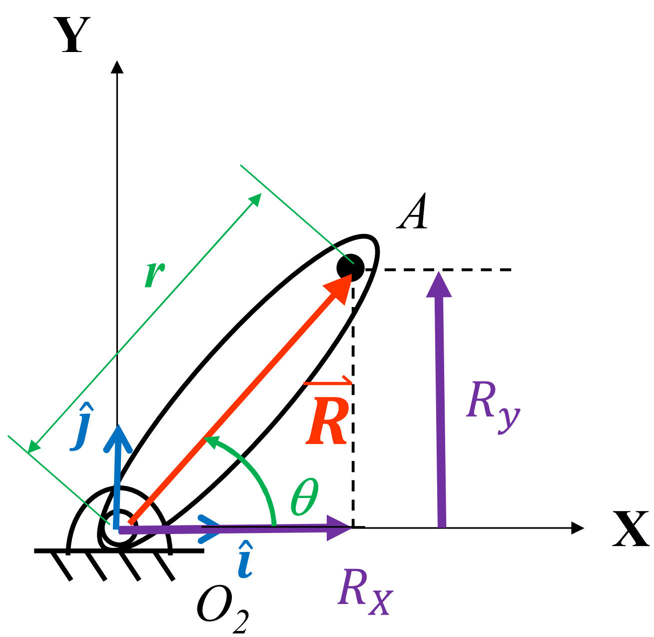
Vector notation

- Use for position, velocity and acceleration analysis
- Coordinate systems and position vectors
- Three vector notations
    1. Cartesian
    2. Radial-transverse
    3. Complex number
        - *j* operator
        - Euler Identity
        - Derivatives
        - Compact notation

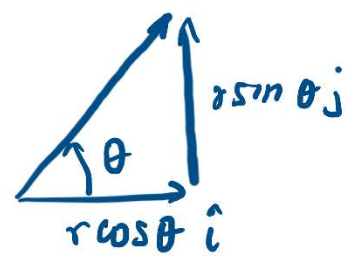
Cartesian vector

$\vec{R}$ is a vector pointing from O_2 to A.
How to represent in vector notations?



Cartesian $\vec{R} = R_X\hat{\imath} + R_Y\hat{\jmath}$

$$\vec{R} = r\cos\theta\,\hat{\imath} + r\sin\theta\,\hat{\jmath}$$

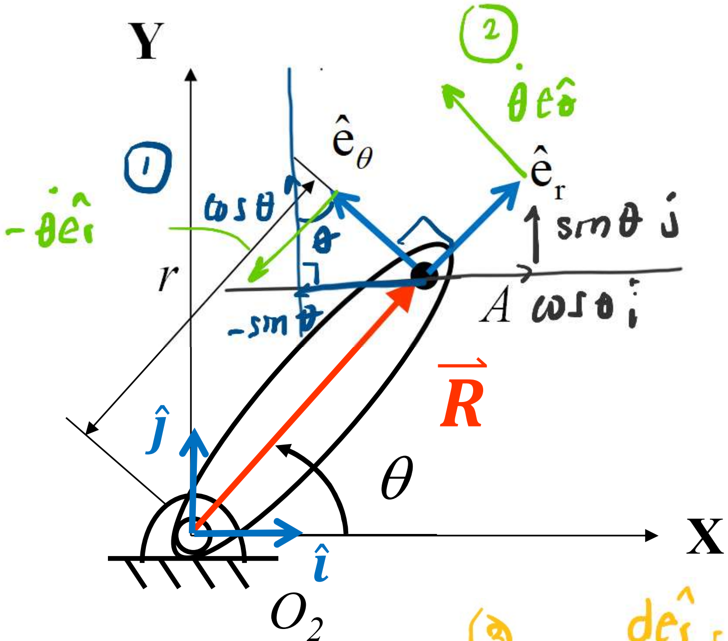


$\hat{i}, \hat{j}$ constant.

$$\frac{d\vec{R}}{dt} = -r\sin\theta\,\hat{i} + r\cos\theta\,\hat{j}$$

Radial-transverse vector

Use radial-transverse basis unit vectors: $\hat{e}_r, \hat{e}_\theta$



Radial-transverse $\vec{R} = r\hat{e}_r$

where ① $\hat{e}_r = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$

$\hat{e}_\theta = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$

Note Derivatives: ② $\dfrac{d\hat{e}_r}{dt} = \dot{\theta}\hat{e}_\theta$

$\dfrac{d\hat{e}_\theta}{dt} = -\dot{\theta}\hat{e}_r$

③ $\dfrac{d\hat{e}_r}{dt} = \dfrac{d\hat{e}_r}{d\theta}\dfrac{d\theta}{dt} = \hat{e}_\theta\dot{\theta}$

$\dfrac{d\hat{e}_\theta}{dt} = \dfrac{d\hat{e}_\theta}{d\theta}\dfrac{d\theta}{dt} = -\hat{e}_r\dot{\theta}$

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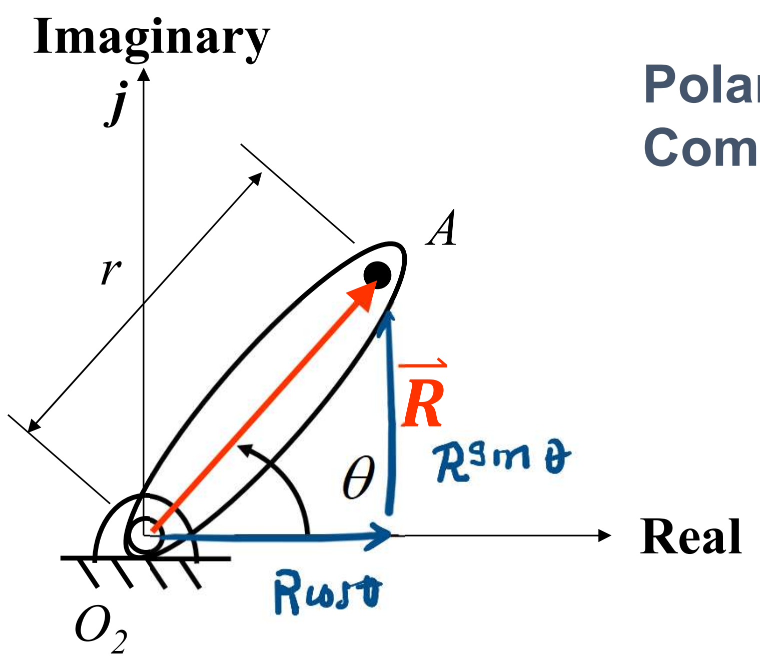
Complex and Compact number vector forms



Complex number $\vec{R} = \underbrace{r\cos\theta}_{\text{real}} + \underbrace{jr\sin\theta}_{\text{imaginary.}}$

euler formula

Polar form
Compact notation $\vec{R} = re^{j\theta}$ compact notation.

Note
Derivatives: $\dfrac{de^{j\theta}}{d\theta} = je^{j\theta}$

w.r.t θ

$$\frac{de^{j\theta}}{dt} = \underbrace{je^{j\theta}}_{\frac{de^{j\theta}}{d\theta}}\frac{d\theta}{dt} = j\omega e^{j\theta}$$

w.r.t time

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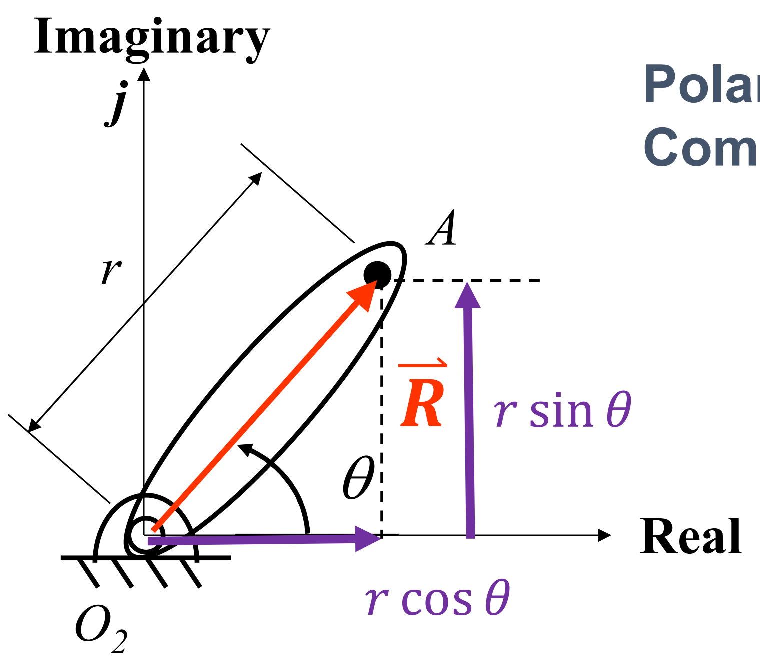
Complex and Compact number vector forms

Complex number $\vec{R} = \underbrace{r\cos\theta}_{\text{Real}} + j\underbrace{r\sin\theta}_{\text{Imaginary}}$



Polar form
Compact notation

$$\vec{R} = re^{j\theta}$$

Note
Derivatives:

$$\frac{de^{j\theta}}{d\theta} = je^{j\theta}$$

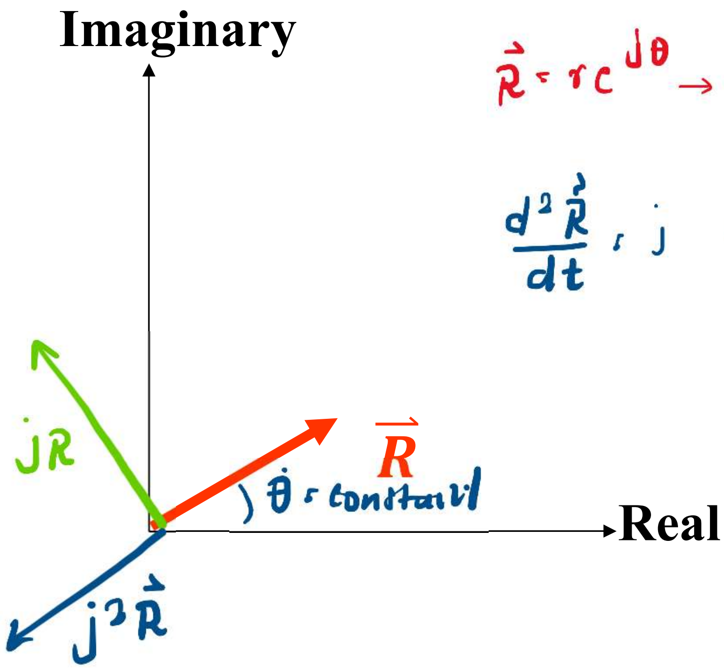
$$\frac{de^{j\theta}}{dt} = \frac{de^{j\theta}}{d\theta}\frac{d\theta}{dt} = je^{j\theta}\frac{d\theta}{dt}$$

$$\Longrightarrow \frac{de^{j\theta}}{dt} = j\omega e^{j\theta}$$

j operator $\quad j = \pm\sqrt{-1}$

Rotates vector by 90° (use right hand rule)

What do $j\vec{R}$ and $j^2\vec{R}$ look like?



$$\vec{R} = r e^{j\theta} \rightarrow \frac{d\vec{R}}{dt} = j\omega r e^{j\theta} = j\dot{\theta}\vec{R} \sim j\vec{R}$$

$$\frac{d^2\vec{R}}{dt} = j\frac{d\vec{R}}{dt} = j\dot{\theta}(j\dot{\theta})\vec{R} = -\dot{\theta}^2\vec{R} \sim j^2 R$$

j operator $j = \sqrt{-1}$

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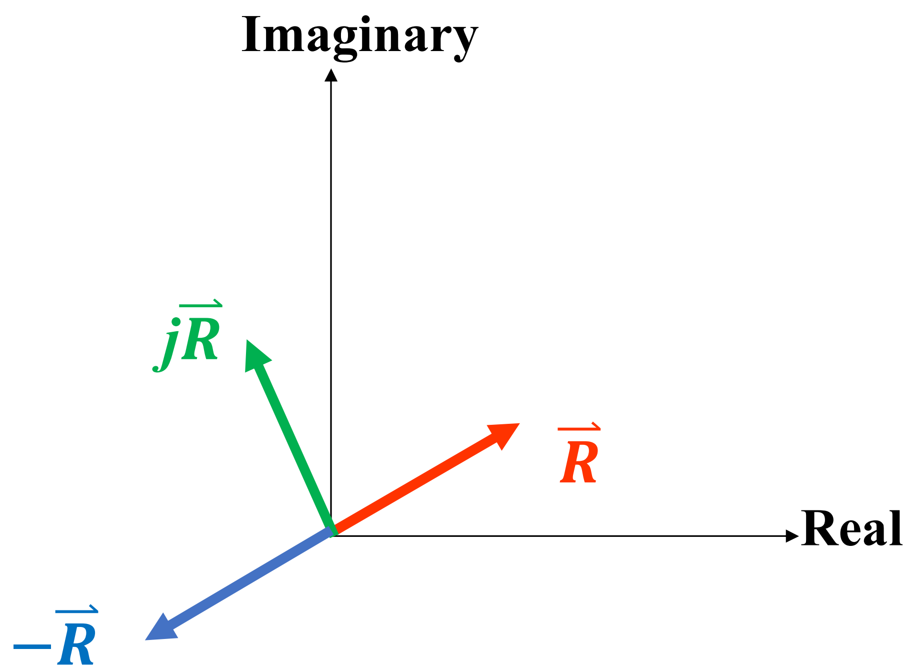

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Euler's identity & Euler's formula

Euler Identity	calculus, log natural base Geometry $e^{j\pi} + 1 = 0$ → null operation. complex numbers natural numbers
Euler's Formula	$e^{\pm j\theta} = cos\theta + jsin\theta$

$\theta = \pi$.

$e^{j\pi} = \cos\pi + j\cancel{\sin\pi}^{\to 0}$

$e^{j\pi} = -1$ or $e^{j\pi} + 1 = 0$

Position analysis (Chap 4)

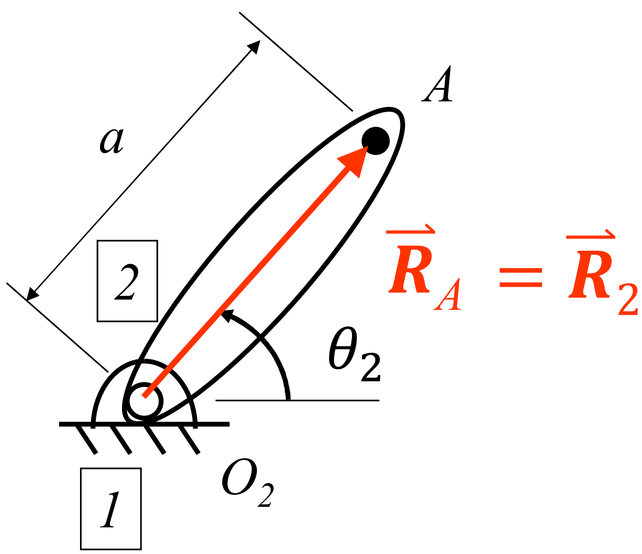
- Vector notation can be used to identify position of specified point on linkage or mechanism

Position analysis: use compact notation to define vectors

Given:

- $\vec{R}_A$ position vector of point A relative to ground point O_2.
- Point A is distance a from ground point O_2
- The orientation of link 2 is defined by $\theta_2(t)$

Let $\vec{R}_A = \vec{R}_2$ position vector of link 2.



Use compact notation:

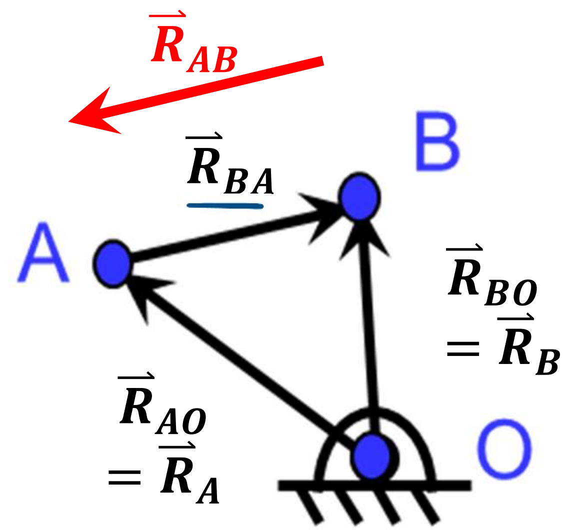
$$\vec{R}_2 = ae^{j\theta_2}$$

In this case, a = constant, and $\vec{R}_2(t)$ and $\theta_2(t)$ will vary with time

Notes on vector subscript notation

- $\vec{R}_{BA}$ represents the position vector of point B relative to point A, and it points from point A to point B. (This is the notation used by the Norton textbook.)
- Note that $\vec{R}_{AB}$ would point the opposite direction, from B to A.
- Often if the vector is pointing from a ground point, we frequently drop the subscript for the ground point, e.g., $\vec{R}_B$ instead of $\vec{R}_{BO}$
- Further you can also think about how the subscripts cancel when adding vectors together,

  e.g., $\vec{R}_{BO} = \vec{R}_{BA} + \vec{R}_{AO}$, note how the A's cancel to get BO.



or $\vec{R}_A + \vec{R}_{BA} = \vec{R}_B$

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Velocity analysis (Chap 5)

- Uses:
  - 2nd step when performing acceleration analysis
  - To determine points of zero and peak velocity
  - To compute kinetic energy: $KE = 1/2\ mv^2$

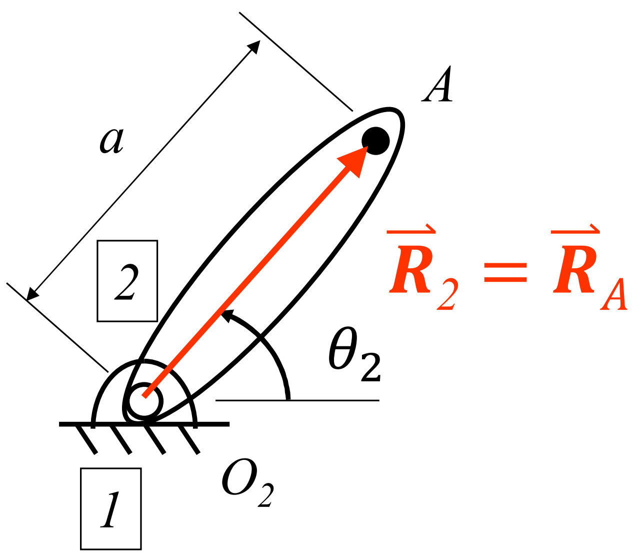
Velocity analysis: use compact notation to find velocity

Given:

- $\vec{R}_2$ determine $\vec{V}_A$

$$\vec{R}_2 = \vec{R}_{AO_2} = \vec{R}_A = ae^{j\theta_2}$$

$$\vec{V}_A = ?$$



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Velocity analysis: use compact notation to find velocity

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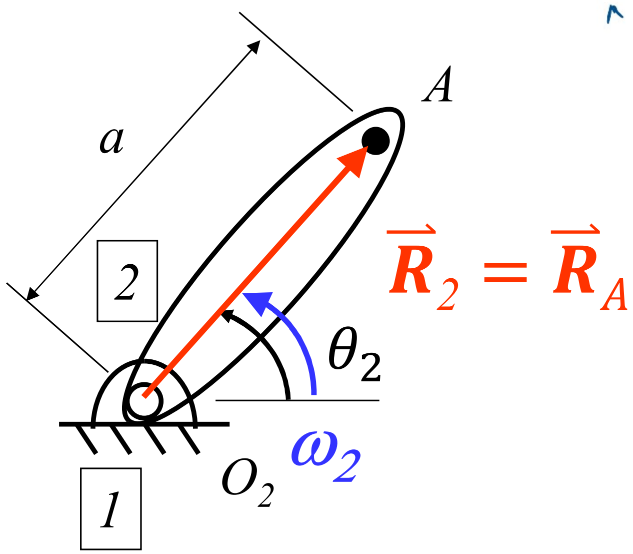


$j\theta_2 e^{j\theta_2}$.

$\frac{de^{j\theta_2}}{dt} = \cancel{\frac{de^{j\theta_2}}{d\theta}} \times \frac{d\theta_2}{dt}$

$$\vec{R}_2 = \vec{R}_{AO_2} = \vec{R}_A = ae^{j\theta_2}$$

$$\vec{V}_A = ?$$

$$\vec{V}_A = \frac{d\vec{R}_2}{dt}$$

Product Rule

w.r.t. θ

$$= \frac{d}{dt}\left(ae^{j\theta_2}\right) = aje^{j\theta_2}\frac{d\theta_2}{dt}$$

w.r.t time.

$$= j\omega_2 ae^{j\theta_2}$$

Recall: $\omega = \frac{d\theta}{dt}$

$$= j\omega_2\vec{R}_2$$

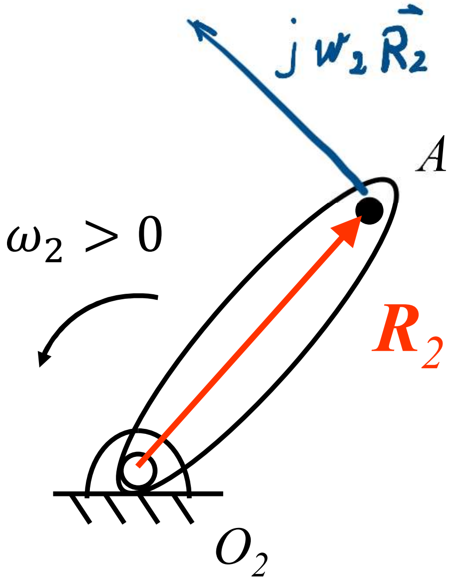
What does V_A look like?

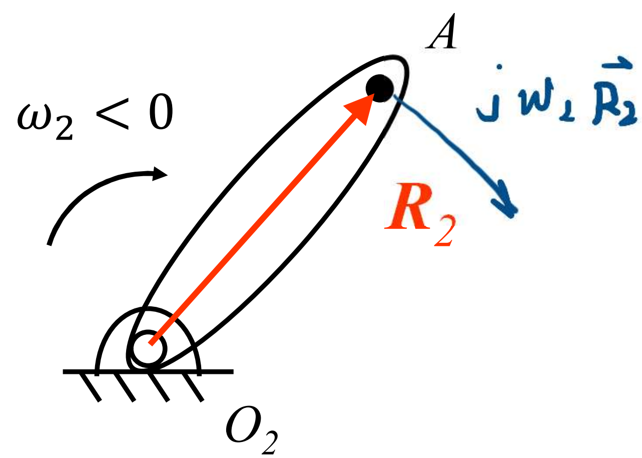
$$\vec{V}_A = j\omega_2\vec{R}_2$$

What does *j* do to a vector?

Direction of $\vec{V}_A$ depends on sign of ω_2

What direction does $\vec{V}_A$ point for each case of ω_2?





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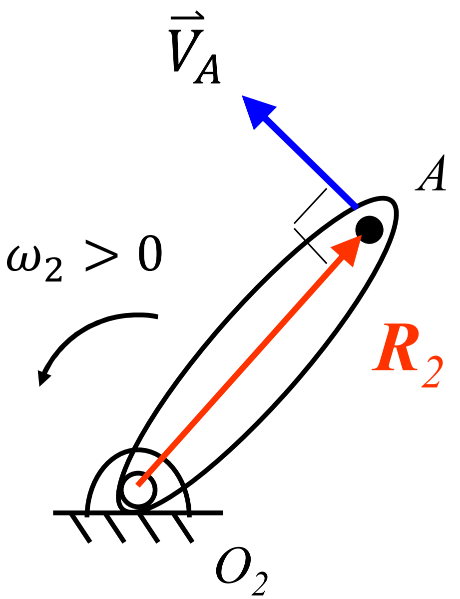
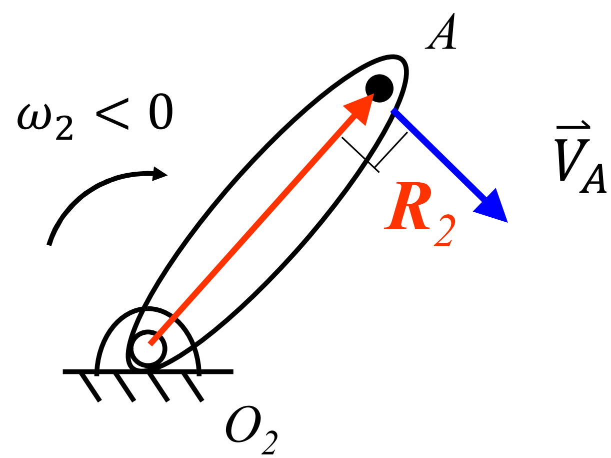




Acceleration analysis (Chap 7)

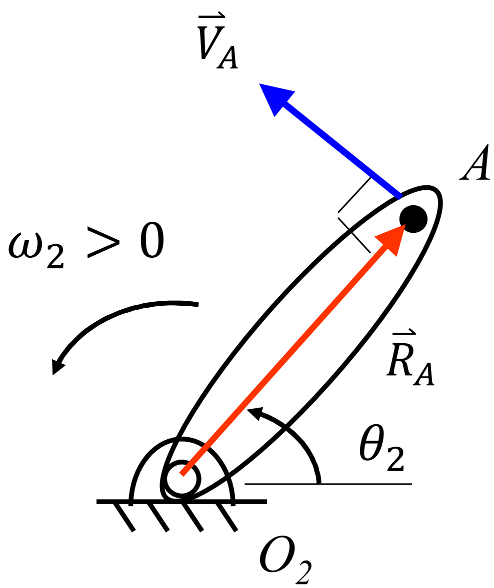
- Uses:
  - To compute forces and moments:

    $$F = ma,\ T = I\alpha$$

  - To determine points of zero and peak acceleration

Acceleration analysis: use compact notation to find acceleration

$$\vec{A}_A = ?$$



$$\vec{A}_A = \frac{d\vec{V}_A}{dt} = \frac{d}{dt}\left(j\omega_2 a e^{j\theta_2}\right)$$

$$= j\frac{d}{dt}\left(\omega_2(t)\, a\, e^{j\theta_2(t)}\right)$$

$$= j\frac{d\omega_2}{dt} a e^{j\theta_2} + j\omega_2 a j e^{j\theta_2}\omega_2$$

$$j \cdot j = -1$$

$$= j\alpha_2 \underbrace{a e^{j\theta_2}}_{R_2} - \omega_2^2 a e^{j\theta_2}$$

$$= j\alpha_2 \vec{R}_2 - \omega_2^2 \vec{R}_2$$

tangential normal.

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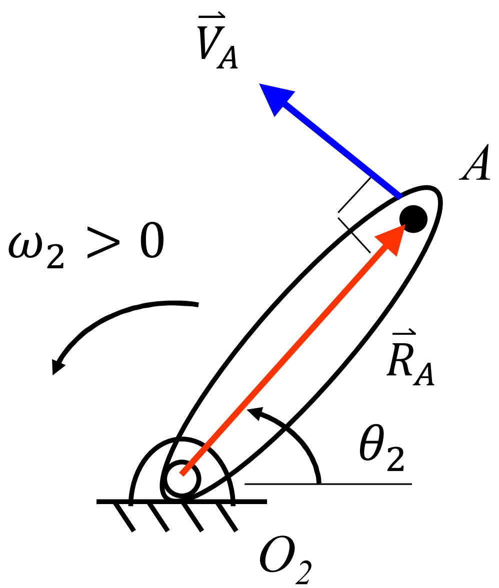
Acceleration analysis: use compact notation to find acceleration

$\vec{A}_A = ?$



$$\vec{A}_A = \frac{d\vec{V}_A}{dt} = \frac{d}{dt}\left(j\omega_2 a e^{j\theta_2}\right)$$

$$= \frac{d\omega_2}{dt}\left[jae^{j\theta_2}\right] + j\omega_2 \frac{d}{dt}\left(ae^{j\theta_2}\right)$$

$$= \frac{d\omega_2}{dt}\left[jae^{j\theta_2}\right] + j\omega_2 j\omega_2 ae^{j\theta_2}$$

$$= \alpha_2 jae^{j\theta_2} + (-1)\omega_2^2 ae^{j\theta_2}$$

$$= j\alpha_2 \vec{R}_2 - \omega_2^2 \vec{R}_2$$

$$= \vec{A}_A^t + \vec{A}_A^n$$

Tangential Normal

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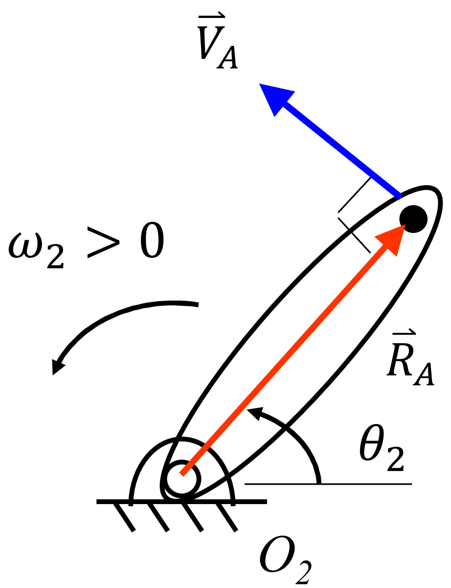
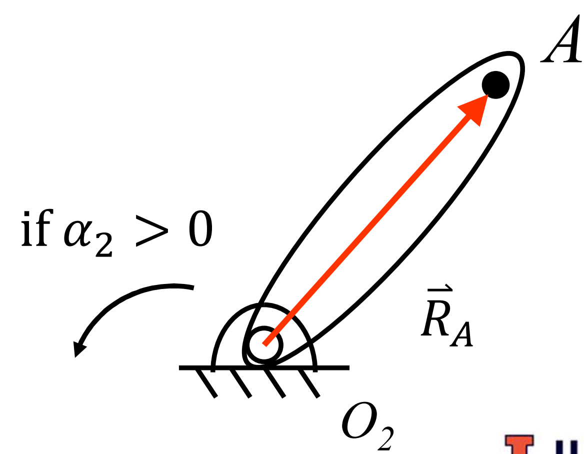
Which direction do $A^t{}_A$, $A^n{}_A$ point?

$$\vec{A}_A = j\alpha_2\vec{R}_2 - \omega_2^2\vec{R}_2$$
$$= \vec{A}_A^t + \vec{A}_A^n$$



$\vec{A}_A^t$ depends on sign of α_2
$\vec{A}_A^n$ is always opposite of $\vec{R}$



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Which direction do $A^t{}_A$, $A^n{}_A$ point?

$$\vec{A}_A = j\alpha_2\vec{R}_2 - \omega_2^2\vec{R}_2$$
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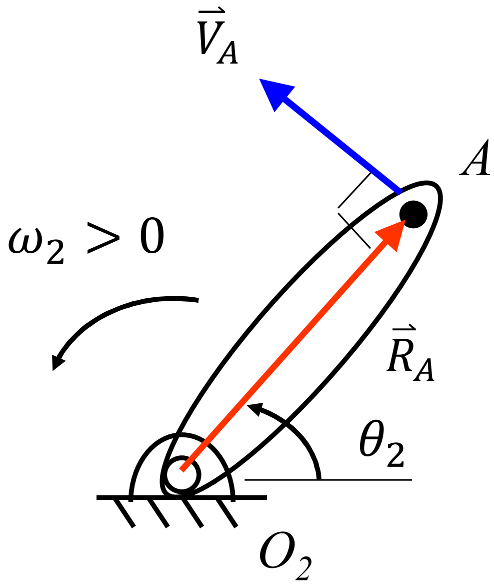

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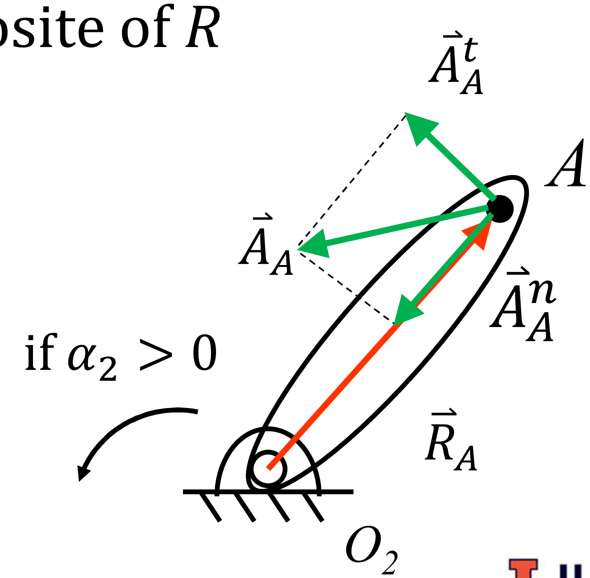

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