Lecture 7:
Path Synthesis

“Colibri” by Derek Hugger

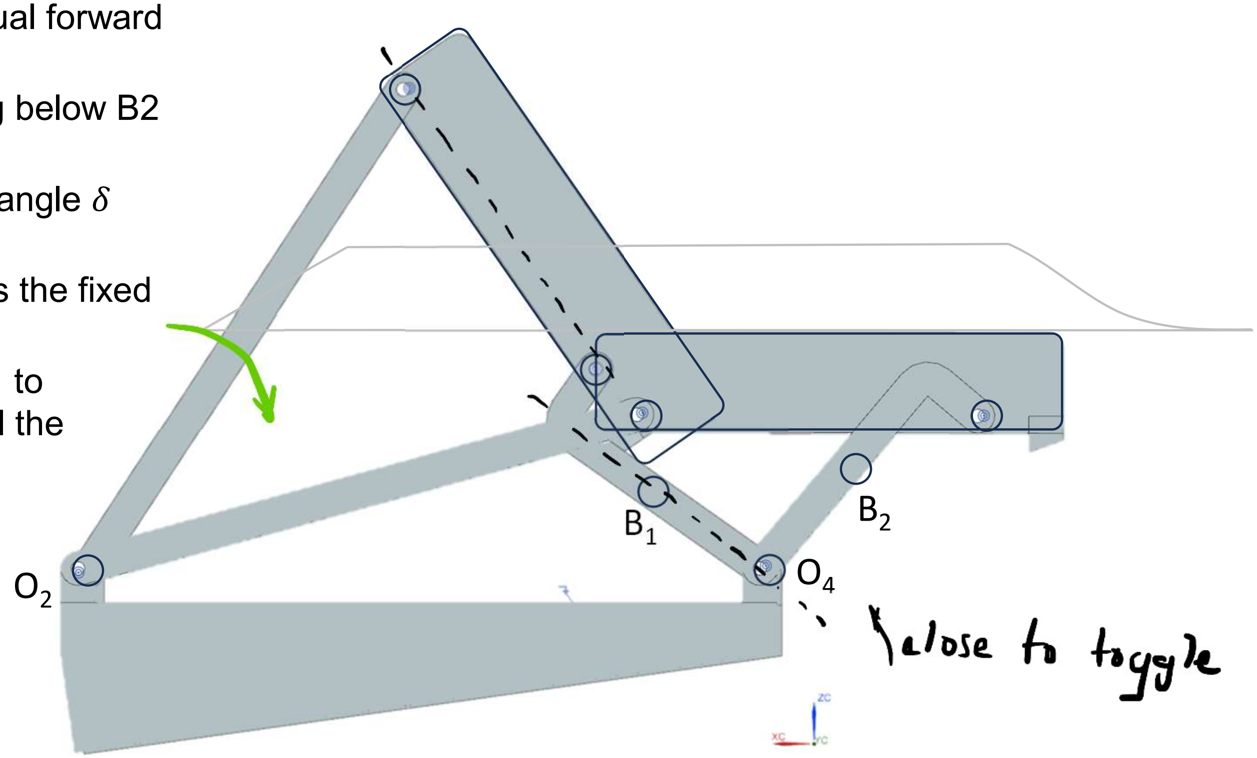
* www.youtube.com/watch?v=1scj5sotD-E

ME 370 - Mechanical Design 1

The MechSE Solar Car team designed a four-bar hinge to lift the canopy. They now want to automate it using a dyad driver that moves the crank O4B1 from position B1 to B2. Design a quick-return mechanism with a time ratio ≈ 0.7 ($\delta \approx 30°$) that drives the crank from B1 to B2 and returns it to B1.

Synthesis steps:

1. Draw Ref. Line 1 through B1 and B2. (Locating the fixed point on this line produces equal forward and return times.)
2. Draw Ref. Line 2 through B1, extending below B2 at a convenient angle (for quick return).
3. Draw Ref. Line 3 through B1, offset by angle δ from Ref. Line 2.
4. The intersection of Ref. Lines 2 and 3 is the fixed point O6 of the dyad driver.
5. Draw an arc centered at O6 through B1 to intersect the extended line O6B2. Label the intersection B1′.
6. The crank length is 0.5×B2B1′.



Design property of MechSE Solar Car Team, UIUC.

Synthesis steps:

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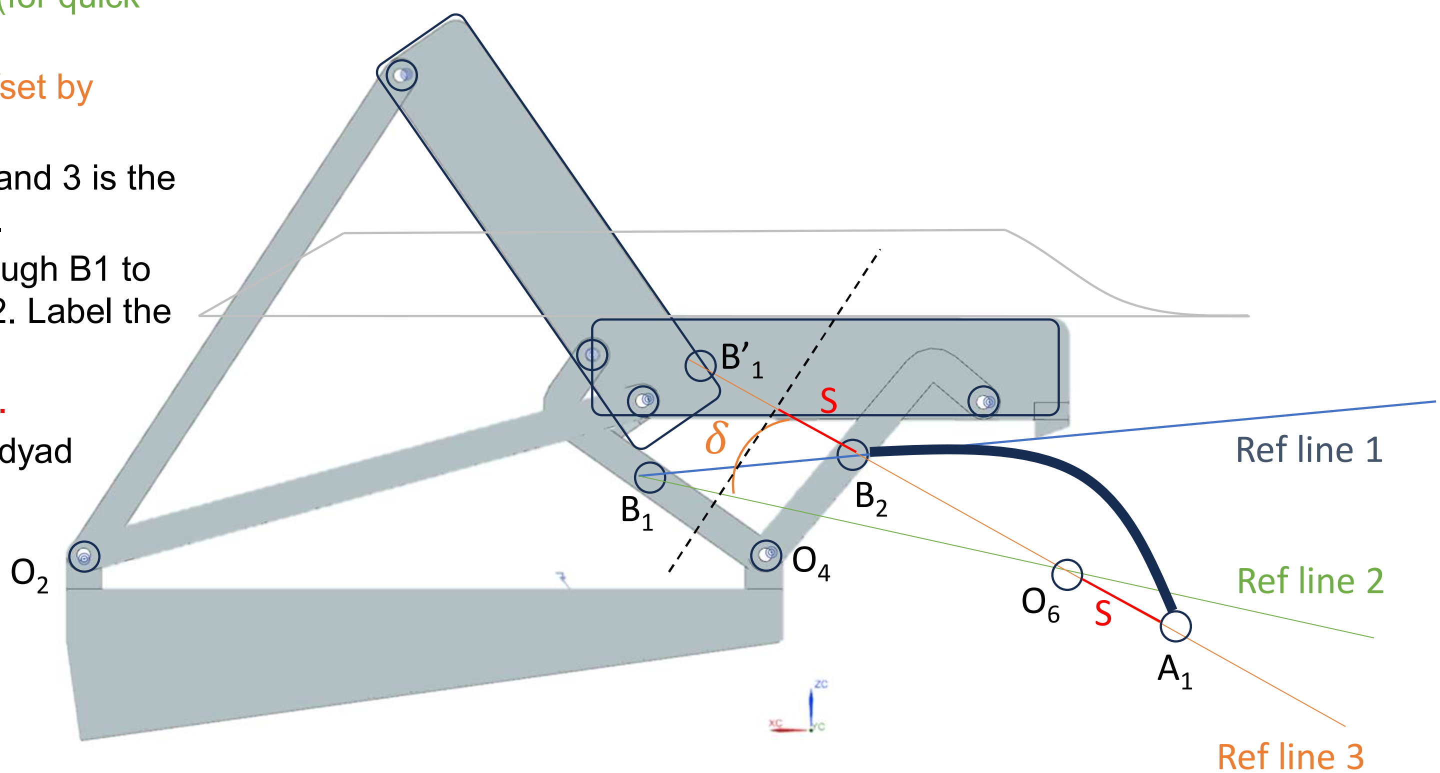
4. The intersection of Ref. Lines 2 and 3 is the fixed point O6 of the dyad driver.
5. Draw an arc centered at O6 through B1 to intersect the extended line O6B2. Label the intersection B1′.
6. The crank length S is 0.5×B2B1′.
7. Connect A1 to B2 to finalize the dyad construction



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Topic 2: Graphical Linkage Synthesis

- Graphical synthesis types and general strategies
  - Generation types and strategies
- Two-position synthesis
  - Rocker output
  - Coupler output
  - Rotopoles
- Dyad drivers
- Quick return mechanisms
- Three-position synthesis
  - Specified moving points
  - Alternate moving points
  - Specified moving points & fixed points (video lecture with Jupiter notebook)
- Path Synthesis
  - Atlas of curves

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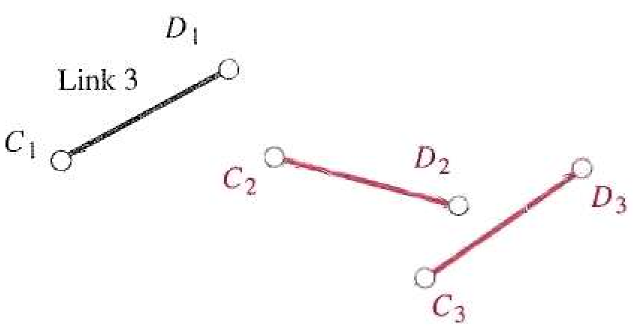
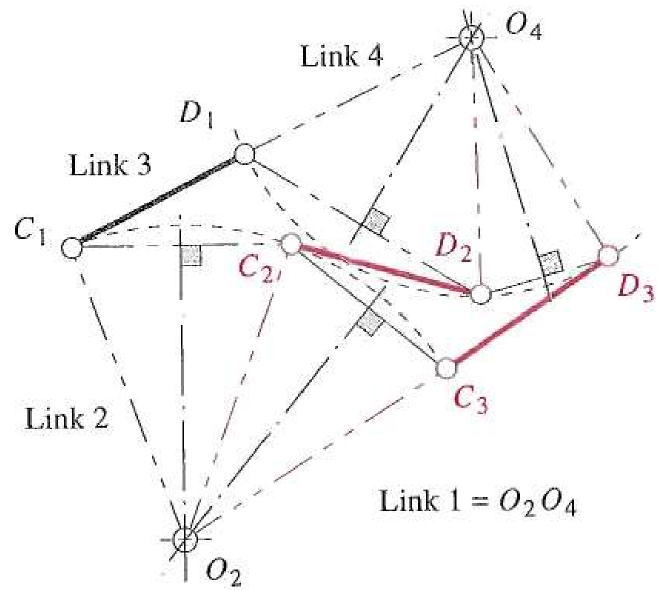
Three-position synthesis: Specified moving points

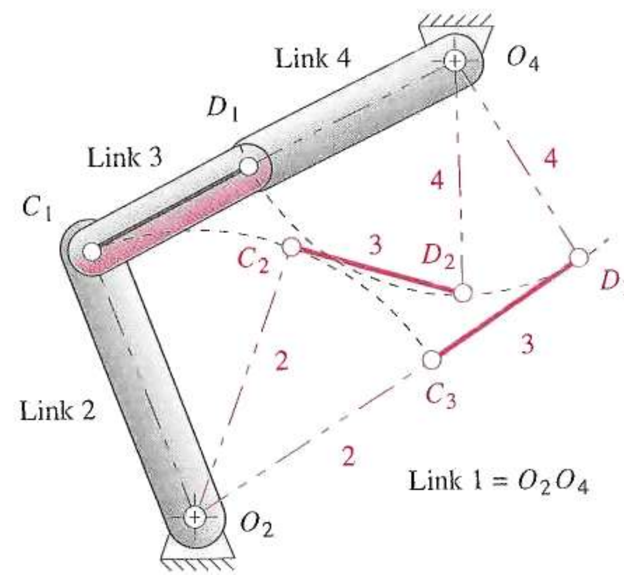
- To find ground point O_2:
  - Find intersection of bisections of C_1C_2 and C_2C_3
- Repeat for O_4 using point D
- Example 3-5 in Norton 3.4

problem



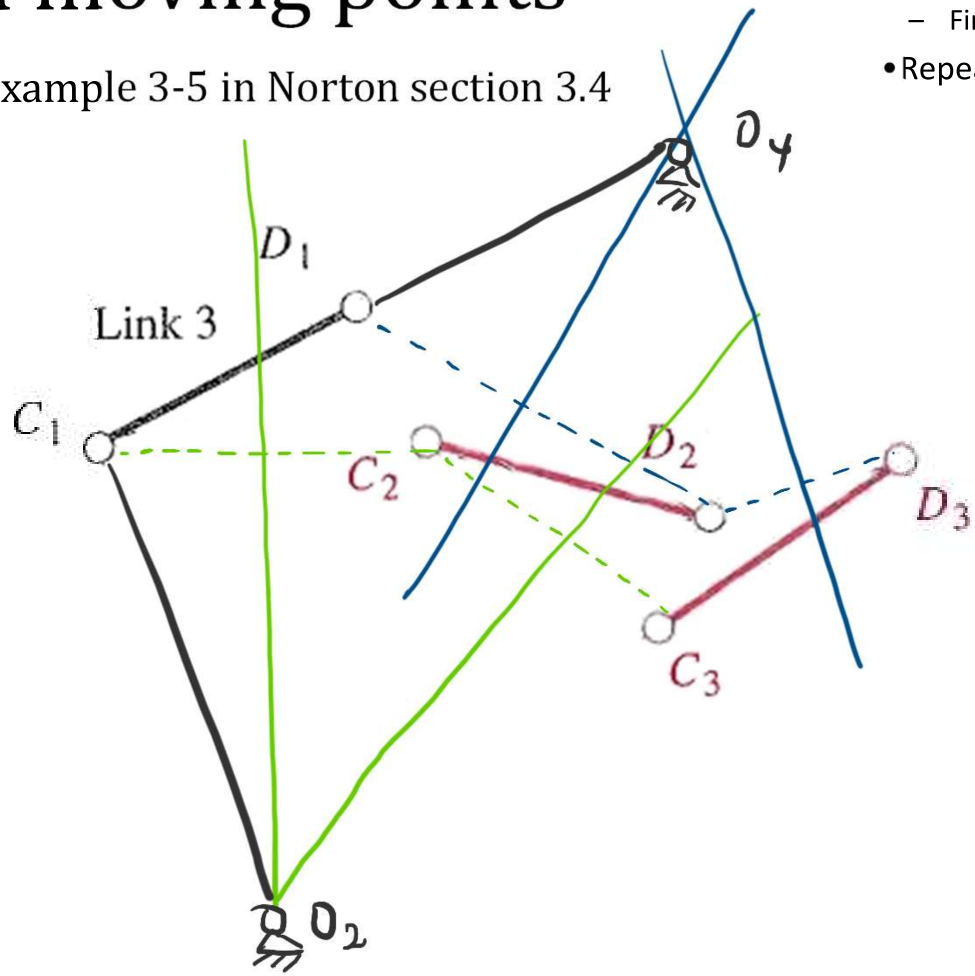
solution





Specified moving points

- Example 3-5 in Norton section 3.4



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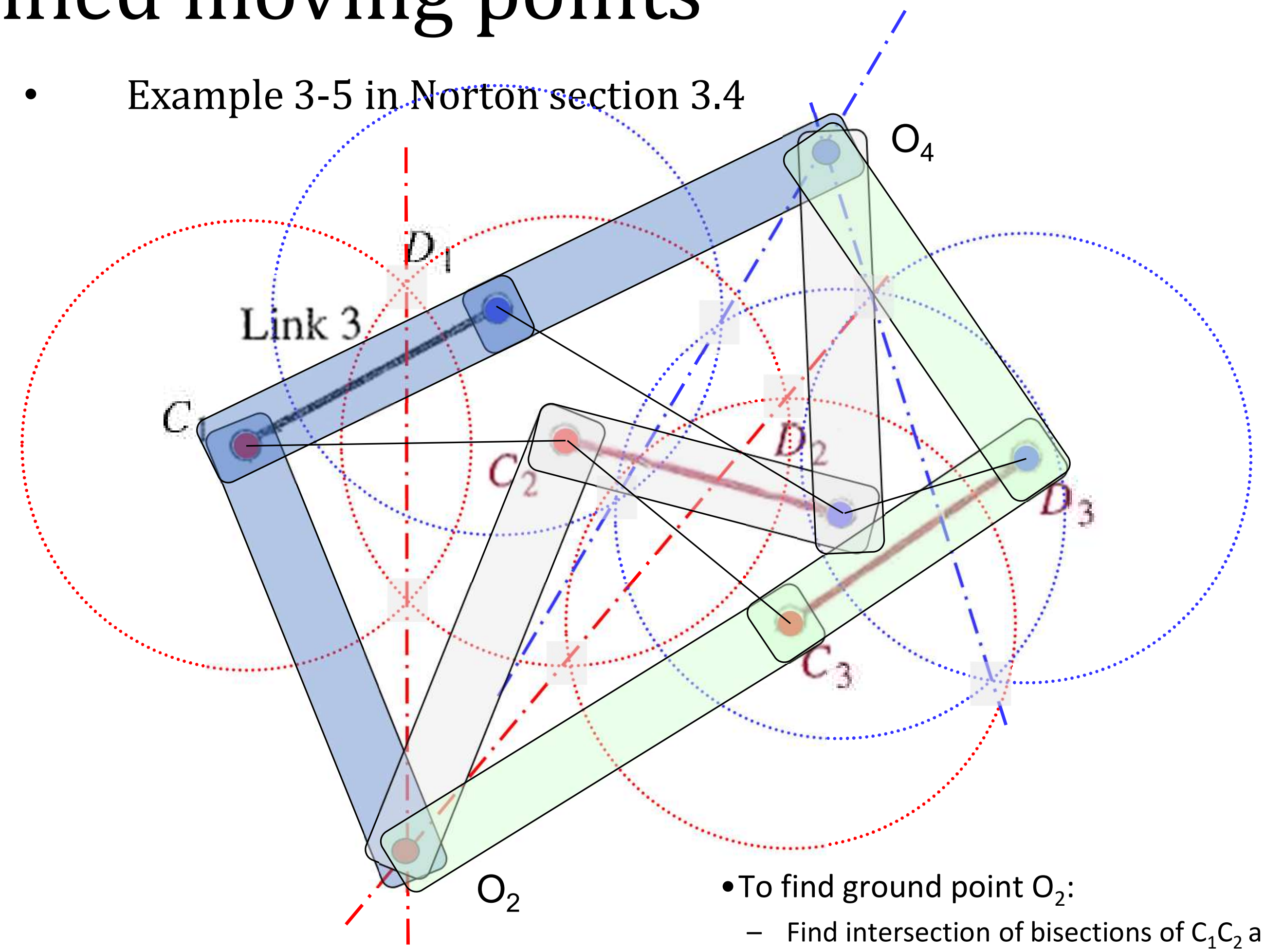
Specified moving points

- Example 3-5 in Norton section 3.4



- To find ground point O_2:
  - Find intersection of bisections of C_1C_2 and C_2C_3
- Repeat for O_4 using point D

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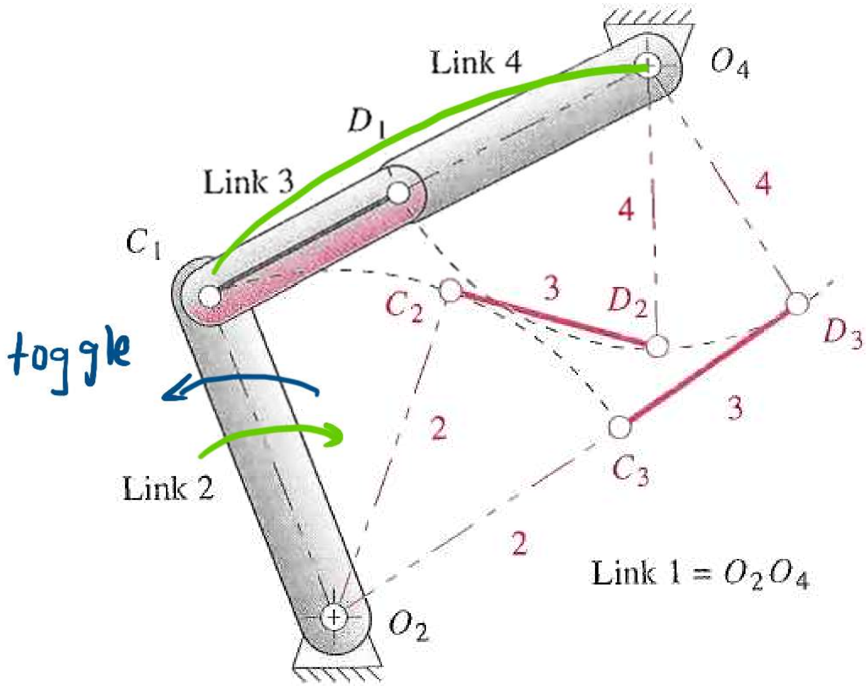
Specified moving points – final design



Note toggle positions in positions 1 & 3

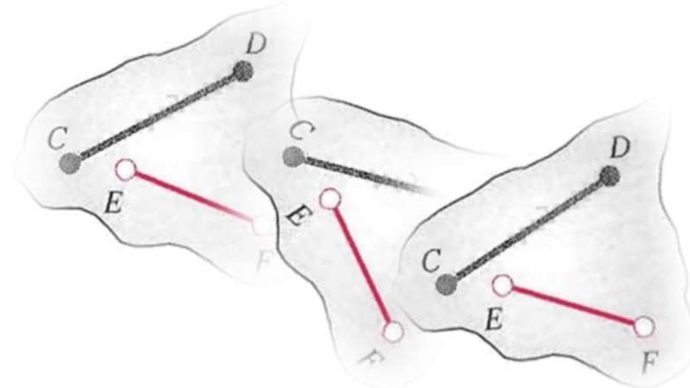
- Can we use a dyad to drive out of the toggle positions?

no

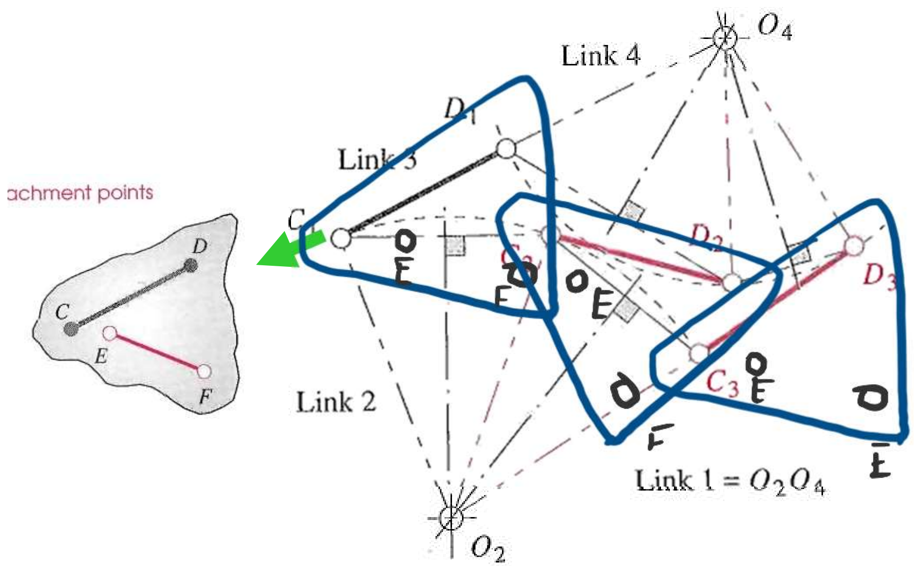
- What happens if the previous technique places the fixed points (O_2 and O_4) in undesirable locations?

Solution: Alternative moving points

Change the *link size* & *node locations* → must change the *node path*



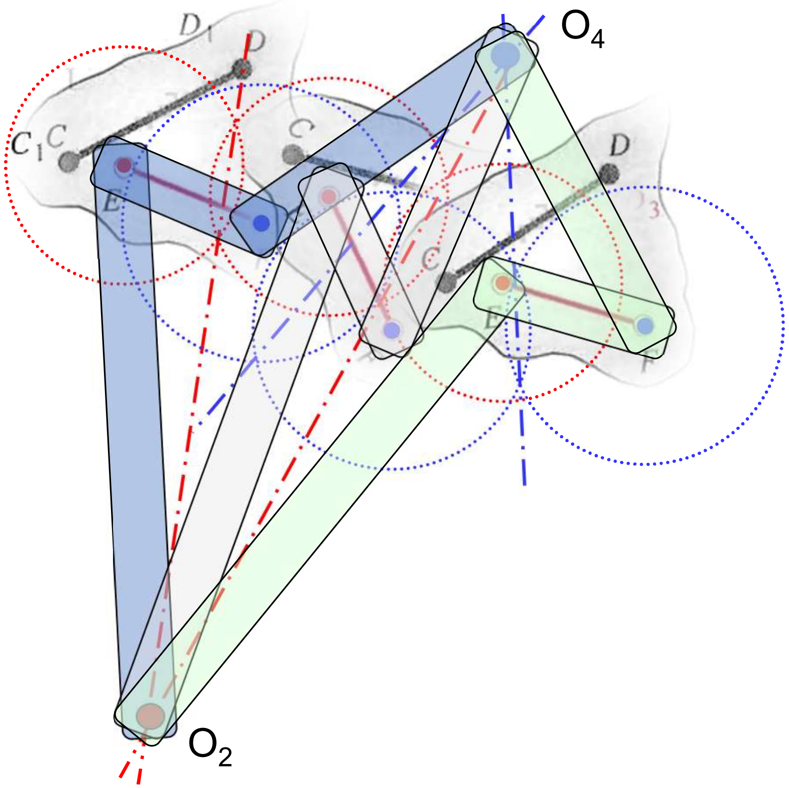
problem



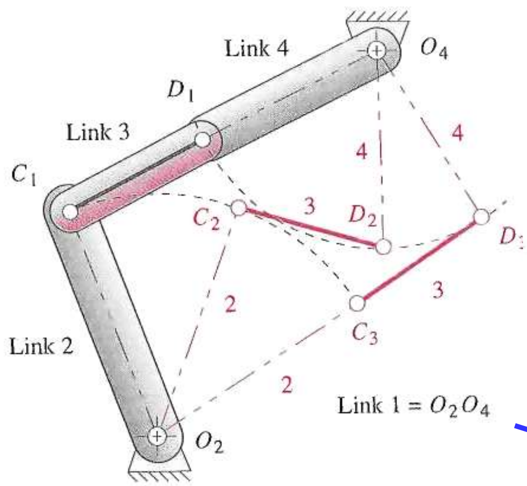
Follow same procedure as before, but use new nodes EF to define locations of O_2 and O_4

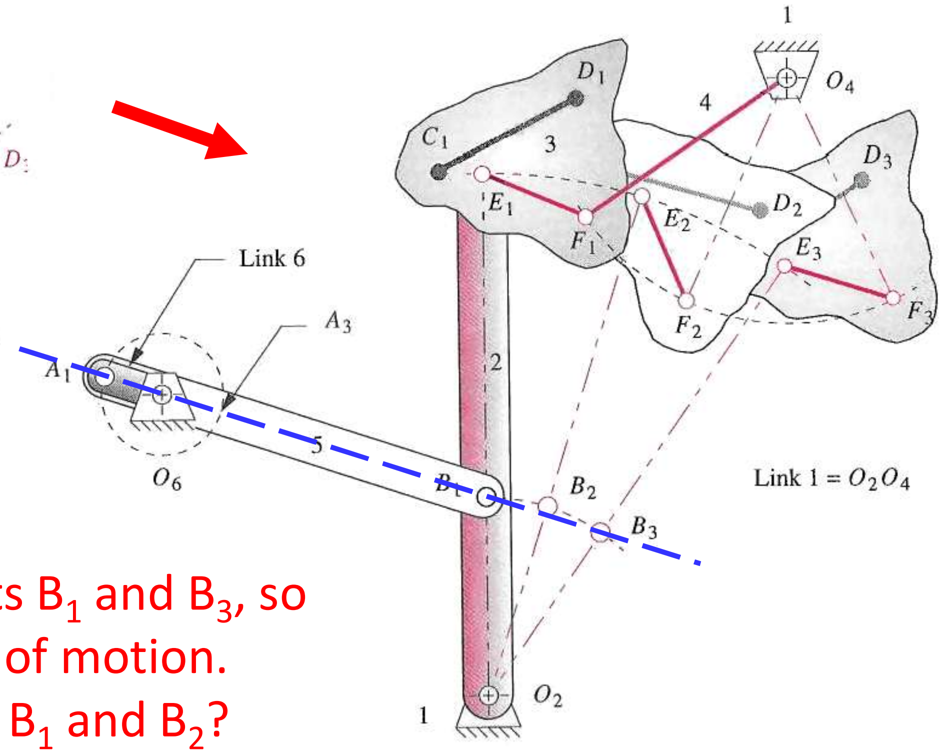
Example 3-6 in Norton section 3.4

Alternate moving points - construction



Alternate moving points – final design





Note: dyad driver connects points B_1 and B_3, so that can drive through full range of motion. What would happen if only used B_1 and B_2?

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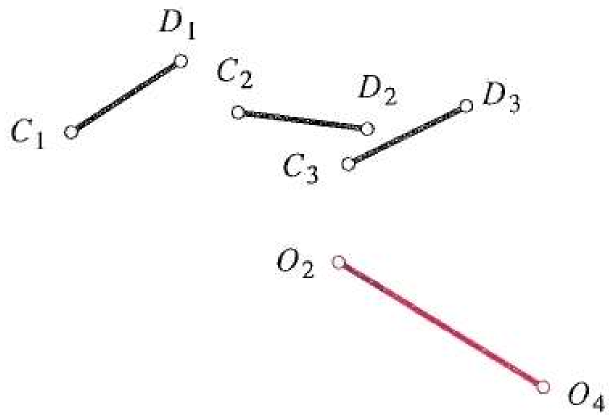
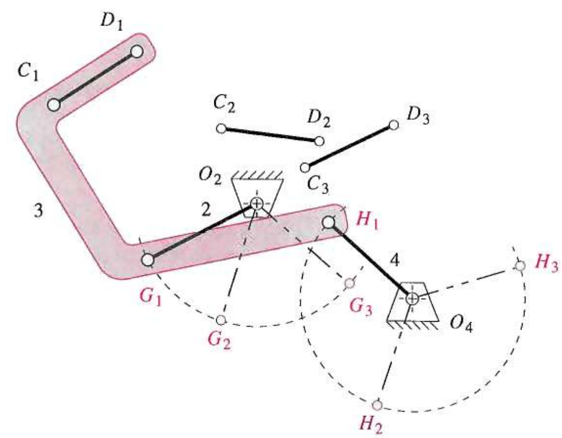
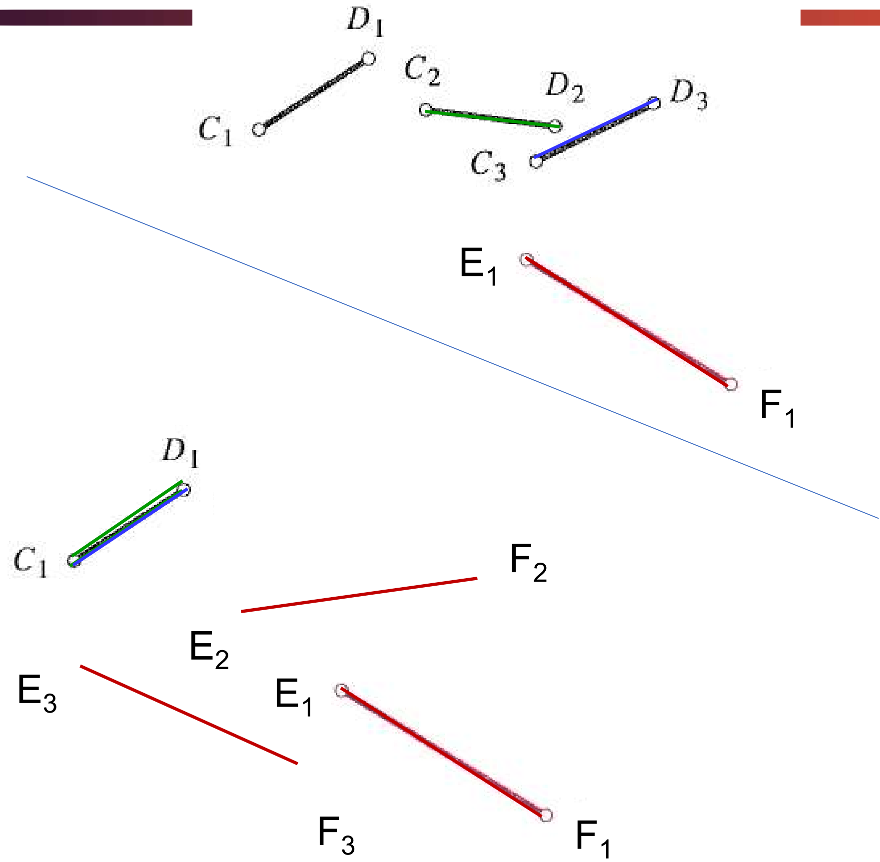
Specified moving & fixed points

- Given 3 coupler positions **AND** specified fixed points (ground joint locations)
- Use inversions and work backward
- Example 3-7 and 3-8 in Norton 3.4





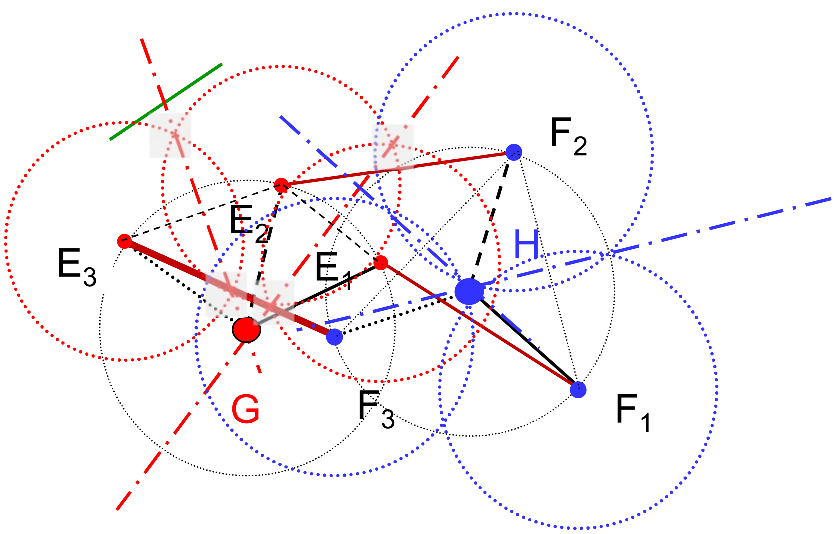
Specified moving & fixed points



1. Invert the linkage
   - A. Label O_2, O_4 as E_1,F_1
   - B. Find the position of the ground plane relative to position #2 (green line)
   - C. Transfer the second ground plane position to reference the first position
     - label as E_2F_2
   - D. Find the position of the ground plane relative to position #3 (blue line)
   - E. Transfer the third ground plane position to reference the first position
     - label as E_3F_3

Specified moving & fixed points

2. Perform 3 position synthesis
   - use E_1F_1, E_2F_2 and E_3F_3
   - develop the "rotopoles" G and H.



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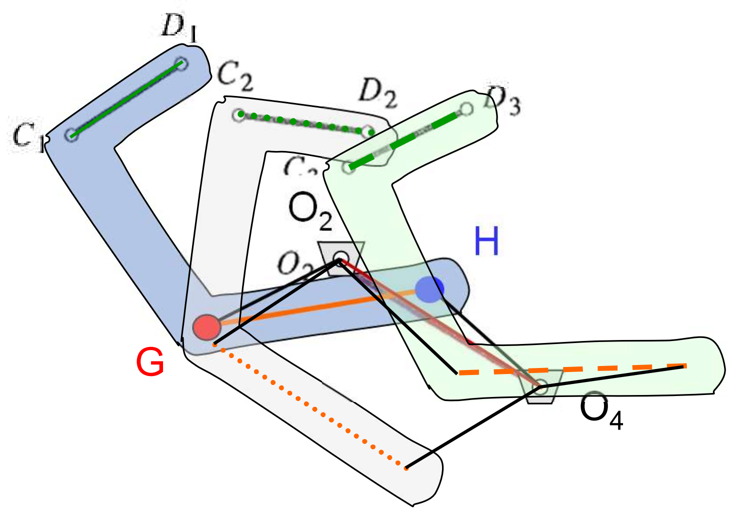
Specified moving & fixed points



3. Re-invert the linkage
   - mark location of E_1 and label as O_2
   - mark location of F_1 and label as O_4
   - mark G & H; connect them to form coupler
   - connect G to O_2 and H to O_4
   - line up O_2 and O_4, and trace the position of C_1D_1
   - draw a link that encompasses C_1, D_1, G & H; this is the coupler.

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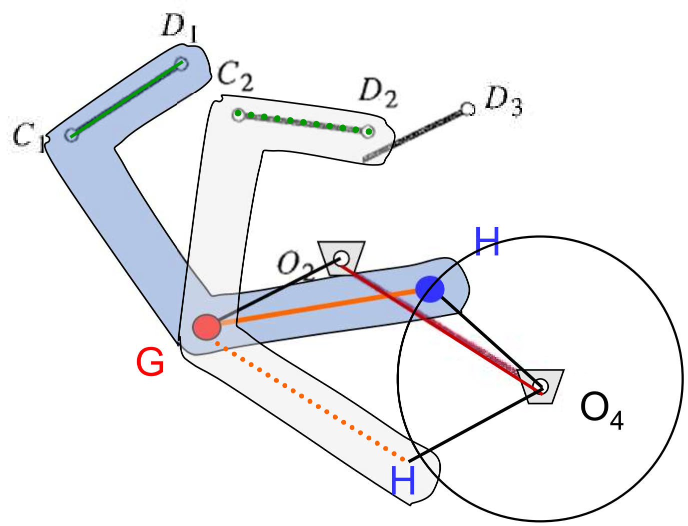
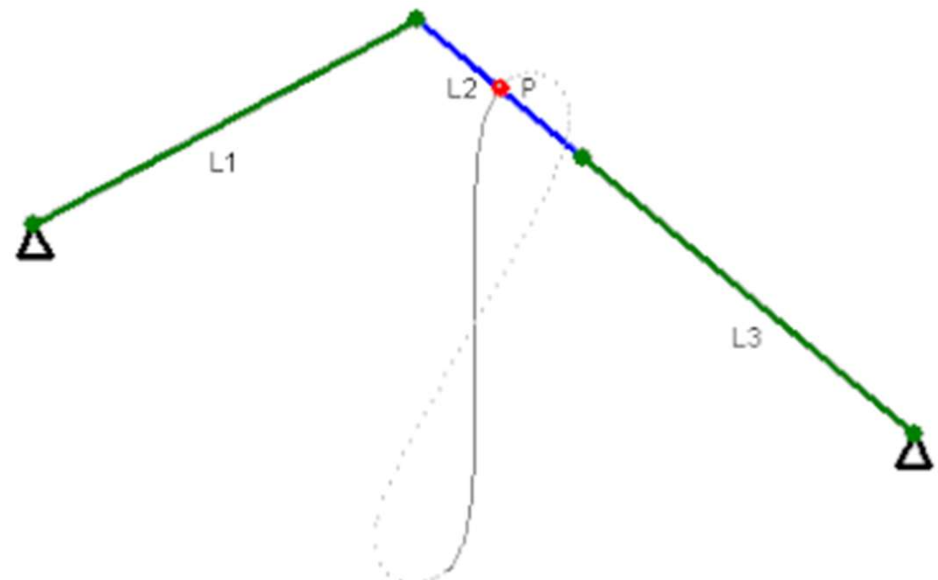
Specified moving & fixed points



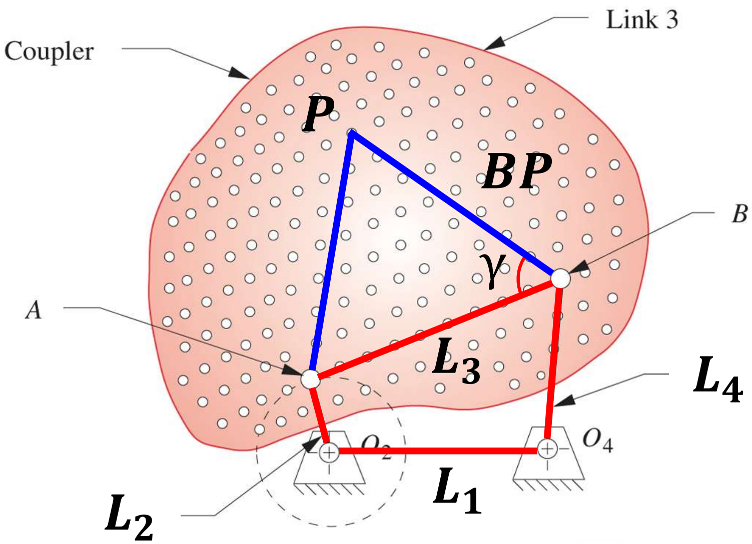
Crank O2G is at same angle when rocker O4H
is at two different poses.

Path Synthesis or Path Generation

Path Synthesis/Generation: Designing a mechanism to make a point follow a prescribed path



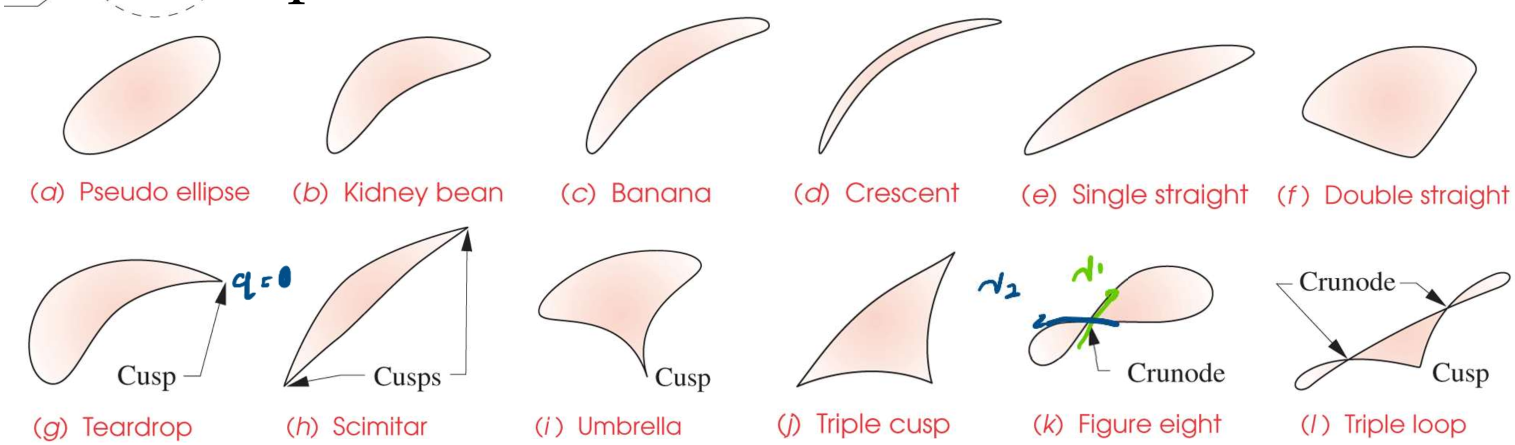
Four-bar mechanism – coupler curves…



- Put a pen in any hole, *such as "P,"* in link-3.
- As link-2 rotates it creates an "output drawing."
- As the *length* of *link-2* or *link-4*, or the "*location*" in *link-3* changes, you will get "families" of different output
  - Visualizing Paths as a function of 4-bar geometry

http://dynref.engr.illinois.edu/aml.html

https://demonstrations.wolfram.com/CouplerCurvesOfAFourBarLinkage/

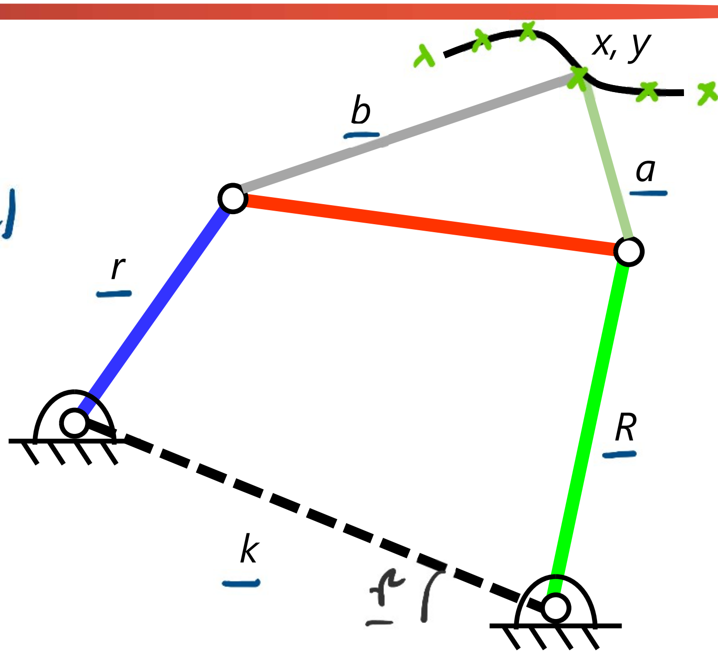


Analytical Path Generation of coupler points

- The equation describing a point on the coupler curve is of 6th order.
- It is complex and can be written in many forms. (implicit, Explicit, parametric)
- Such complex path generation is usually solved graphically, or by numerical trial-and-error.

using lookup tables

- The Beyer form is written as:



$$a^2\left[(x-k)^2+y^2\right]\left(x^2+y^2+b^2-r^2\right)^2$$
$$-2ab\left[\left(x^2+y^2-kx\right)\cos\gamma+ky\sin\gamma\right]\left(x^2+y^2++b^2-r^2\right)\left[(x-k)^2+y^2+a^2-R^2\right]$$
$$+b^2\left(x^2+y^2\right)\left[(x-k)^2+y^2+a^2-R^2\right]^2$$
$$-4a^2b^2\left[\left(x^2+y^2-kx\right)\sin\gamma-ky\cos\gamma\right]^2=0$$

Path Generation: Coupler Points

Determining independent parameters to define the size, orientation, and shape of a coupler curve

To determine mechanism: L_1, L_2, L_3, L_4

To determine path position P: $R, \emptyset,$ OR BP, γ

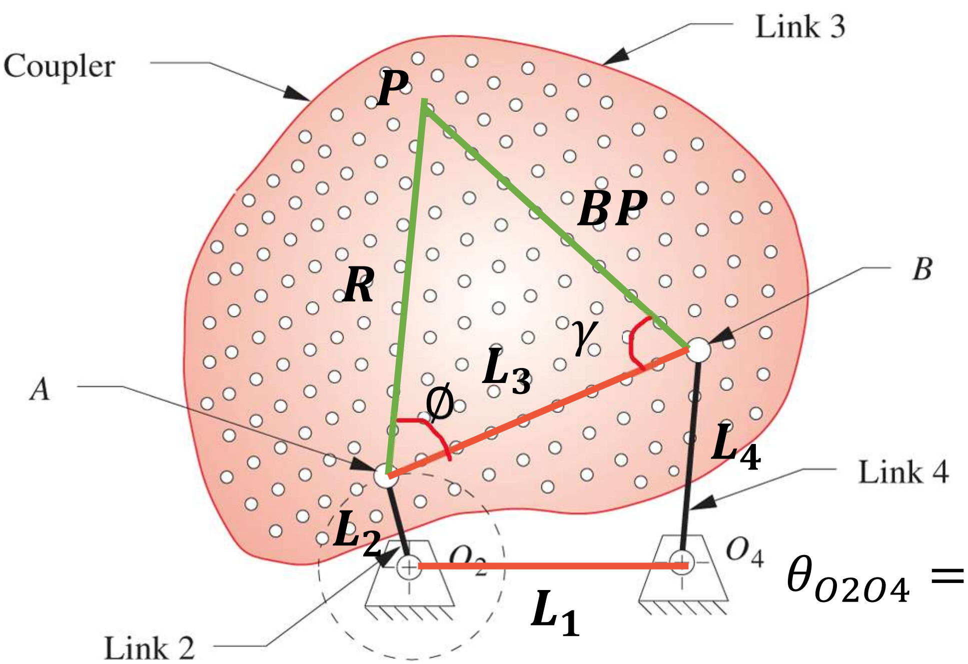
Independent parameters:

Size: 1 Orientation: 1

L_2 θ_{O2O4}

Independent parameters determining <u>shape</u>: 5

$$\frac{L_1}{L_2}, \frac{L_3}{L_2}, \frac{L_4}{L_2}, \frac{R\ or\ BP}{L_2}, \emptyset\ or\ \gamma$$



$\theta_{O2O4} = 0°$

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Path Generation: Coupler Points

Determining independent parameters to define the size, orientation, and shape of a coupler curve

To determine mechanism: $L_1, L_2, L_3, L_4, \hat{\theta}_{O_2O_4}$ (5)

To determine path position P: R, ϕ or BP, γ (2)

Independent parameters: • curve form changes with proportion, not size.

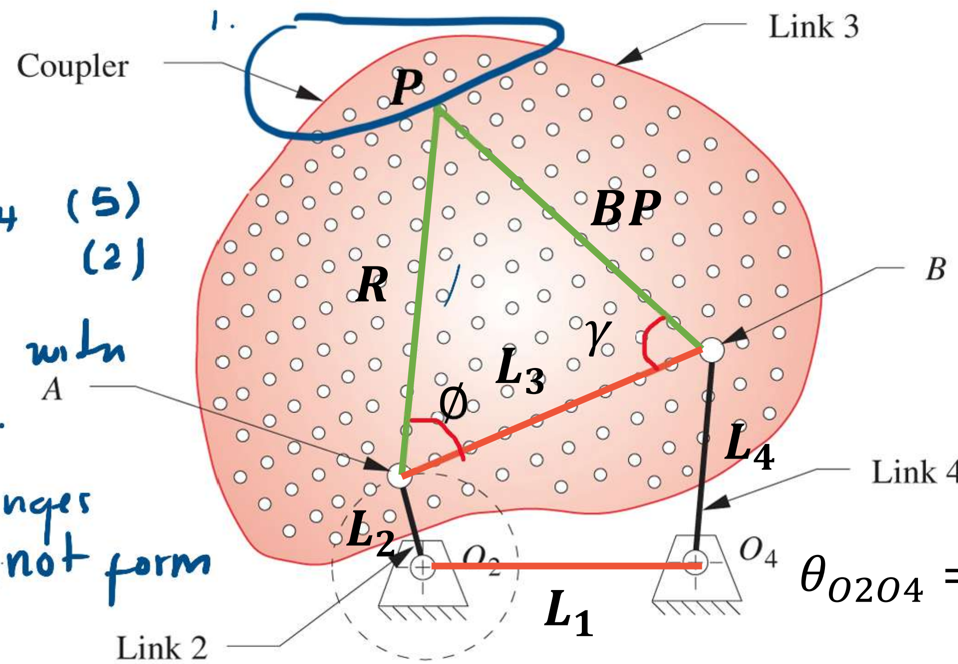
Scale : L_2

Orientation : $\theta_{O_2O_4}$

orientation θ_{2O4} changes orientation of curve, not form

5 independent params

$\frac{L_1}{L_2}, \frac{L_3}{L_2}, \frac{L_4}{L_2}$ R, ϕ



$\theta_{O2O4} = 0°$

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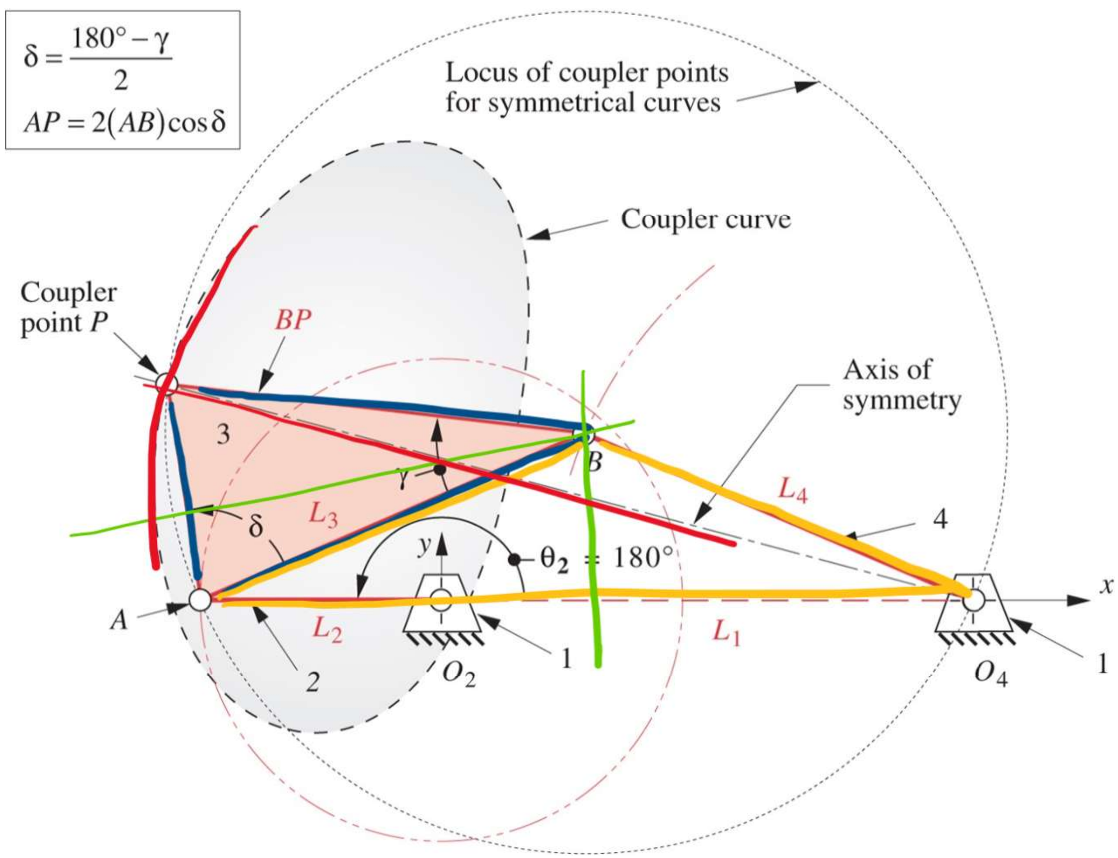
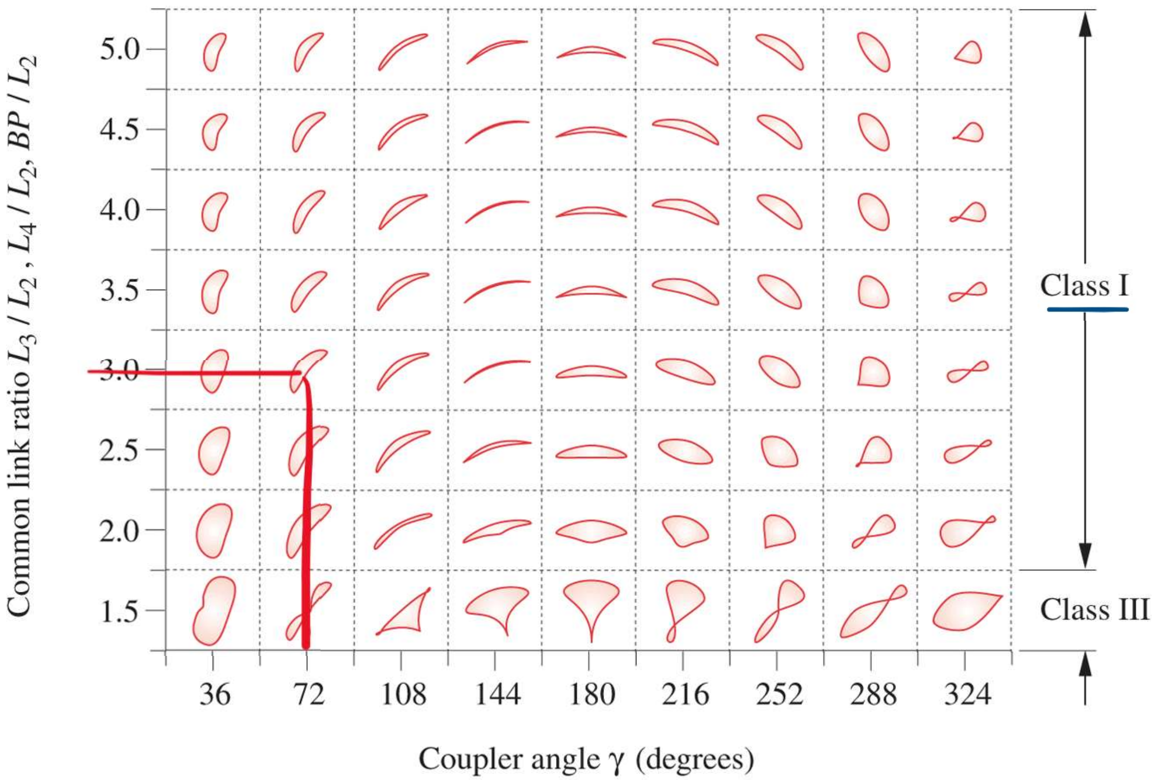
Five input parameter of coupler curve tables



1. Ground link ratio L_1/L_2
2. Coupler link ratio L_3/L_2
3. Output link ratio L_4/L_2
4. Offset link ratio BP/L_2
5. Offset angle γ

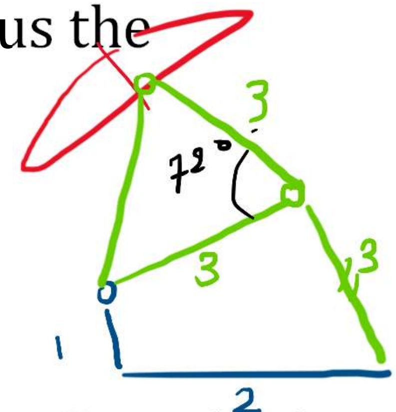
$L_3 = L_4 = BP \rightarrow$
curves with axis of symetry.

Coupler curves for symmetric 4-bar linkages



(*a*) Variation of coupler curve shape with common link ratio and coupler angle for a ground link ratio $L_1 / L_2 = 2.0$

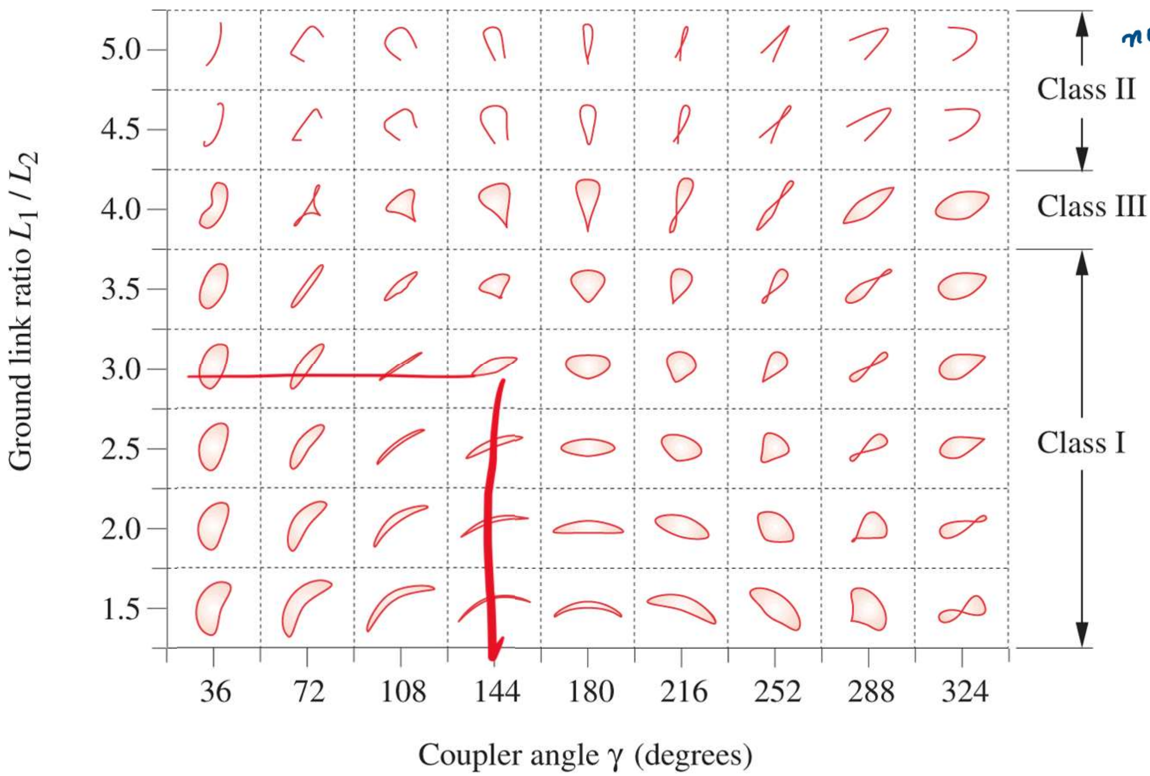
- We can use lookup tables and atlases to find the mechanism parameters that will give us the desired curve



${L_1}/{L_2} = 2$ for all graphs shown

Coupler curves for symmetric 4-bar linkages

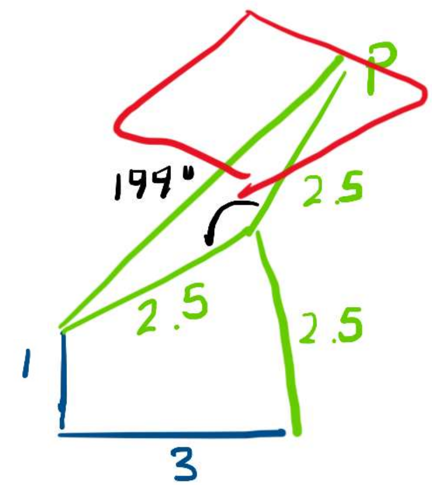
non grashof.



(*b*) Variation of coupler curve shape with ground link ratio and coupler angle for a common link ratio $L_3 / L_2 = L_4 / L_2 = BP / L_2 = 2.5$

$$L_3/L_2 = L_4/L_2 = BP/L_2 = 2.5$$

for all graphs



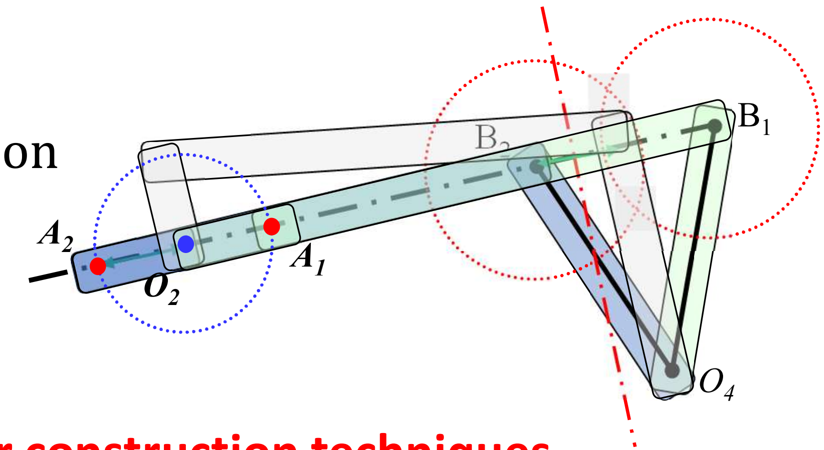
$\frac{L_1}{L_2} = 3$

Using lookup tables for path synthesis

1. Define the motion you want
2. Use the look up table to find the motion that most closely resembles the motion you desire. Read off the mechanism parameters that give that motion and use them as a starting point for your design
3. Examine how the path changes by varying the different mechanism parameters along each axis.
4. Adjust the motion by varying the mechanism parameters and test using simulation or prototyping until desired motion is achieved.

Precision is important in synthesis

- Poor precision means:
  - Designed mechanism might not give desired motion
  - Assembly may not be possible
  - Might have incorrect timing
- Two major mistakes during GLS:



To small/big of mechanism

Poor construction techniques