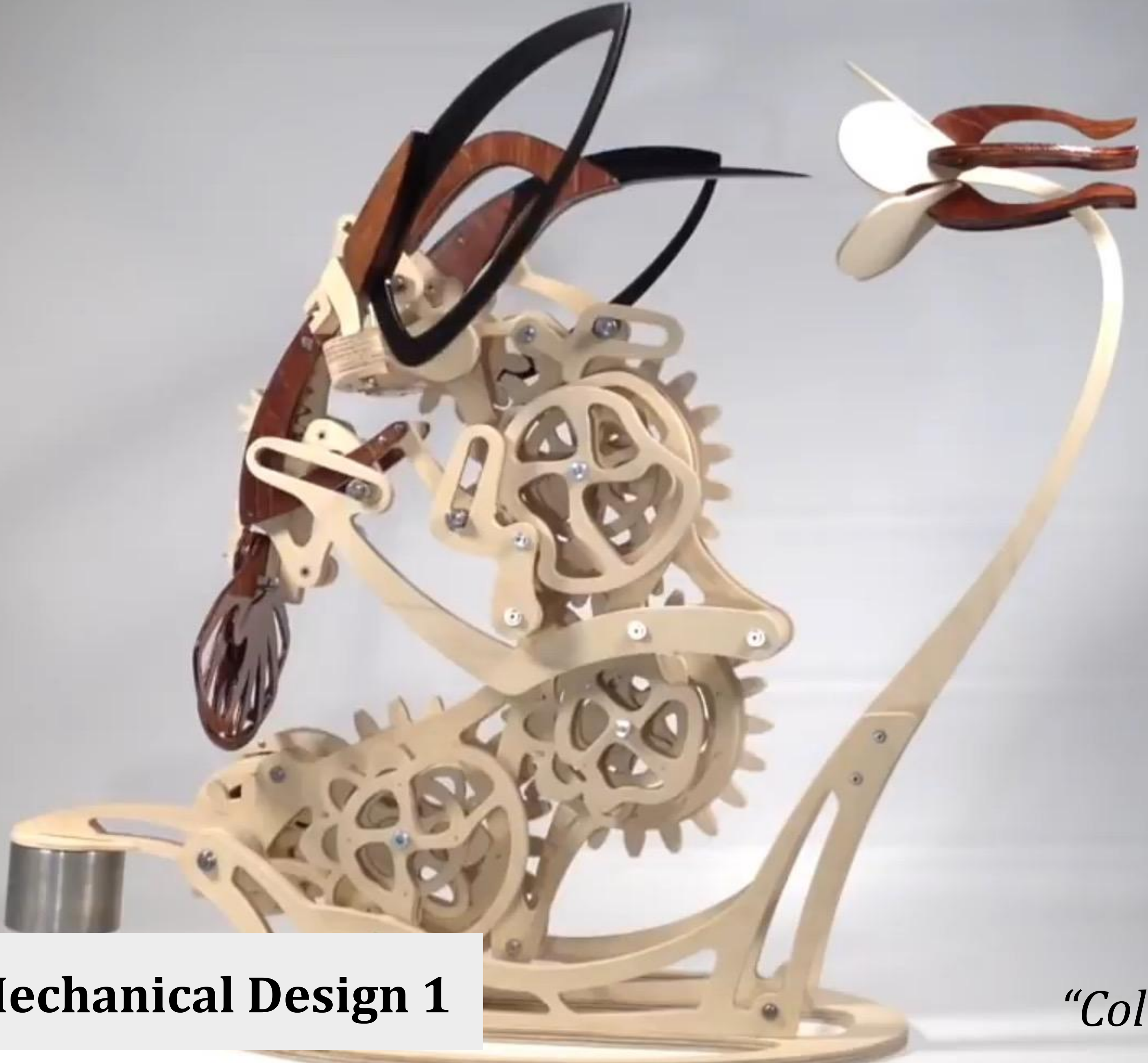
Module 9: Balancing Lectures 25



“Colibri” by Derek Hugger

* www.youtube.com/watch?v=1scj5sotD-E

ME 370 - Mechanical Design 1

Lecture 25: Balancing 1

Today Module 9, Part 1 Balancing (Reading, Norton Chapter 11.8, 11.11, 12)

Activities & Upcoming Deadlines

- **Week 14:**
  - **HW 12 (DFA) and HW 13 (Course Reflections–short assignment): BOTH** due Tuesday 12/2
  - HW 14 (VW): to be posted shortly, due Tuesday 12/9
  - **Lab 14:** Work on Project 2 (attendance expected) –teams are STRONGLY suggested to use lab time to continue working on your project
- **Project 2:**
  - **P2D3 (2-minute Video):** Due Sunday 12/7 by 11:59 pm.
  - Grading rubric has been posted
  - All students are to rank the 5 videos in your lab section based on being informative and entertaining by Tuesday Dec 9. Submit to GradeScope.
  - The top videos from across the 12 lab sections will be played during lecture on Wednesday Dec 10 - **attendance is mandatory.**
  - P2D4 (Performance Event):
    - Friday 12/12 from 7-10 pm. Attendance is mandatory.
    - Grading rubric and demonstration timeslot sign-up sheets will be posted shortly

ILLINOIS

Module 9 Topics

- Balancing of Rotating Machinery (Text 11.8, 11.11, 12)
  - Shaking force and torque (11.8)
  - Shaking moment (12.6)
  - Static balancing (12.1)
  - Dynamic balancing (12.2)
  - Balancing more complex planar mechanisms (12.3+)
    - Single piston engine
    - 4-bar linkage
  - Flywheels (11.11)
    - Torque variation
    - Motor selection
    - Sizing a flywheel

Balancing

- Walking washing machine

http://www.youtube.com/watch?v=JCq3P-ps7LU&t=01m40s

Front-loader washing machine

Vibrations of drum are minimized with spring-dampers

Balancing of wheel

Really good example of how to see unbalanced and balanced rotating device

https://youtu.be/ullnFQD4F1I?t=24

ILLINOIS

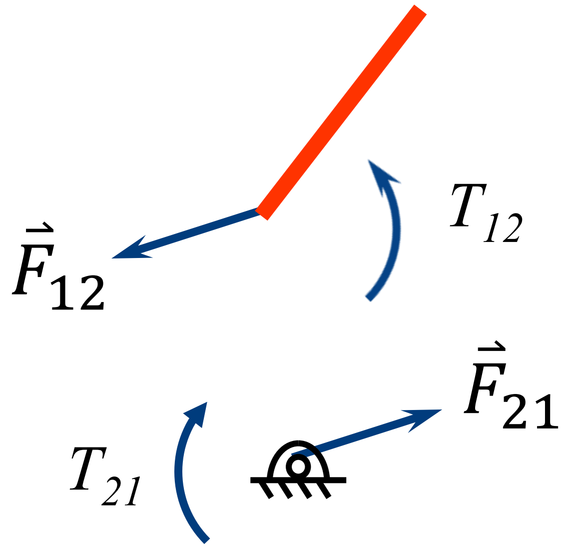
Balancing

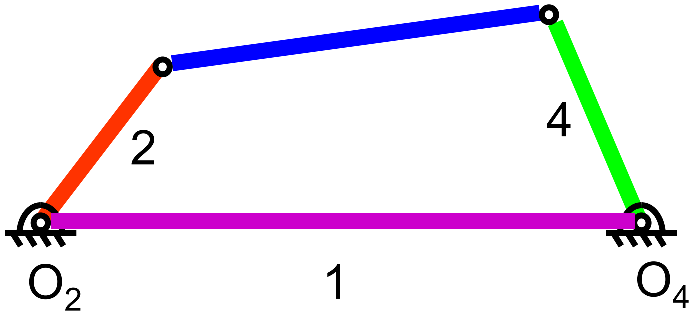
- Important for
  - Rotating machinery (gas turbines, electric motors, automobile wheels, etc.)
  - Mechanisms (piston engine, linkages, etc.)
- The goal:
  - No shaking force
  - No shaking torque (for constant velocity)

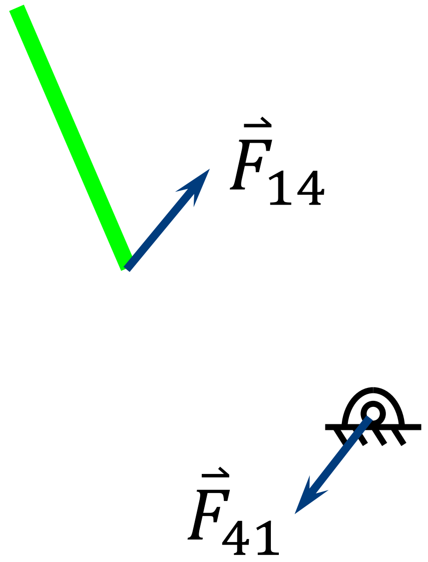
Shaking force (Norton 11.8)

- ***F_s***: The ***total*** force acting **<u>on</u>** the ground plane is defined as the ***shaking force***
  - Sum of all forces acting on ground
  - Tends to move ground plane back & forth

$$\boxed{\vec{F}_s = \vec{F}_{21} + \vec{F}_{41}}$$



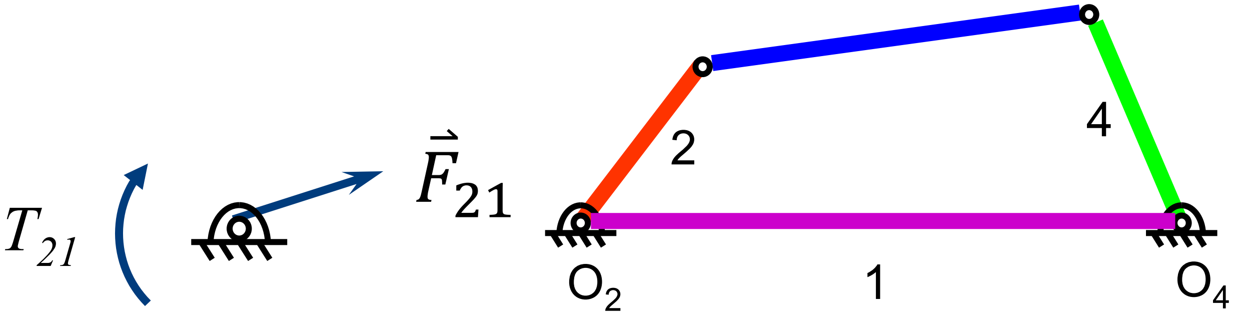




Shaking torque (Norton 11.8)

- ***T_s***: The ***reaction torque*** felt **by** the ground plane is defined as the ***shaking torque***
  - Negative of input torque on driving link from ground
  - Rocks ground plane about the crank pivot axis or driveline axis

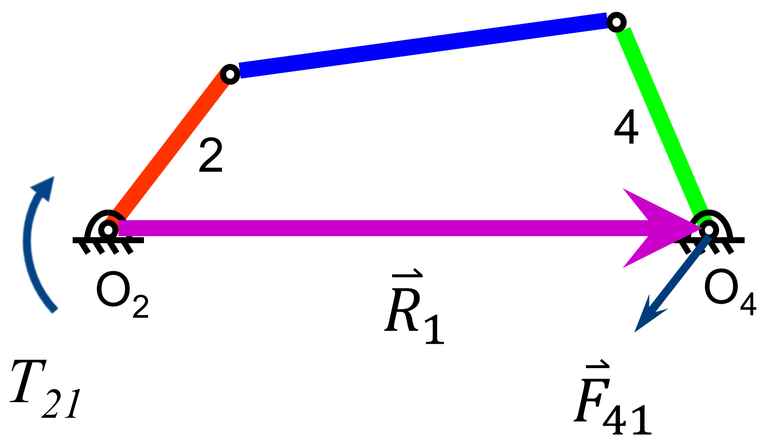
$$\boxed{T_s = T_{21} = -T_{12}}$$



Shaking moment (Norton 12.6)

- M_s: The ***total*** moment felt **<u>by</u>** the ground plane is defined as the ***shaking moment***
  - Reaction moments felt by ground
  - Tends to rocks ground about the crank pivot axis (in this case point O_2)
  - Shaking torque plus shaking couple due to reaction force about crank pivot axis

$$\boxed{\overrightarrow{M}_s = \vec{T}_{21} + (\vec{R}_1 \times \vec{F}_{41})}$$



Balancing (Chap 12)

- Involves rotating machinery
- Used to reduce shaking forces & shaking torques that are due to geometric irregularities that can cause vibration, noise, wear, failure, etc.
- Initial designs should try to eliminate potential imbalance problems; however, inaccuracies and imbalance may occur due to variability during manufacturing
- Correct imbalance with counter-weights or “balancing weights” (e.g., balancing tires/wheels on cars)

Balancing

Unbalanced forces are due to inertial forces of rotating parts (i.e., forces due to acceleration of mass)

- Not concerned with unbalanced forces due to external forces
- Assume steady-state rotary motion with constant angular velocity,
  - Acceleration is only due to centripetal (inward, normal) acceleration

$$\therefore \quad \omega = \text{constant} \Rightarrow \alpha = 0$$

$$\Longrightarrow \vec{F}_{inertia} = m\vec{A} = -m\omega^2\vec{R}$$

where $\vec{R}$ is the position vector of the mass m relative the axis of rotation

Recall:

$\vec{V}_i = j\omega_i\vec{R}_i$

$\vec{A}_i = \vec{A}^t + \vec{A}^n = j\alpha_i\vec{R}_i - \omega_i^2\vec{R}_i$

Static Balance

- 2-D, “single-plane” balance
  - masses generating inertial forces are in, or nearly in, same plane
  - axial direction << radial direction
  - ex. fan, propeller, bicycle tire
- Static balance is a subset of dynamic balance
- Goal: Move unbalanced system’s CG to a new location, preferably to a fixed pivot point, such as an axis of rotation

Static Balance

- Key is to balance forces
  - (Remember: ignore external forces)

$$\sum \vec{F} = 0$$

$$\sum \cancel{\vec{F}_{external}} - \sum \vec{F}_{inertia} = \sum \cancel{\vec{F}_{external}} - \sum m\vec{A} = 0$$

$$\boxed{\therefore \sum m_i \omega^2 \vec{R}_i = 0}$$

Recall that assume

$$m\vec{A} = -m\omega^2 \vec{R}$$

assume
const ω
$\Rightarrow \alpha = 0$

or since ω is same for all masses:

$$\boxed{\sum m_i \vec{R}_i = 0}$$

where i are original or balancing weights

Static Balancing

- Recall

$$\vec{R}_2 = ae^{i\theta_2}$$
$$\vec{V}_2 = j\omega_2 ae^{i\theta_2}$$
$$\vec{A}_2 = j^2\omega_2^2 ae^{i\theta_2} = -\vec{R}_2\omega_2^2\text{, since } \alpha = 0$$

- To balance, add a counterweight (m_b)
  - To get $\sum\vec{F} = 0$, make $\sum m\vec{A} = 0$

$$m_2\vec{A}_2 + m_b\vec{A}_b = 0$$

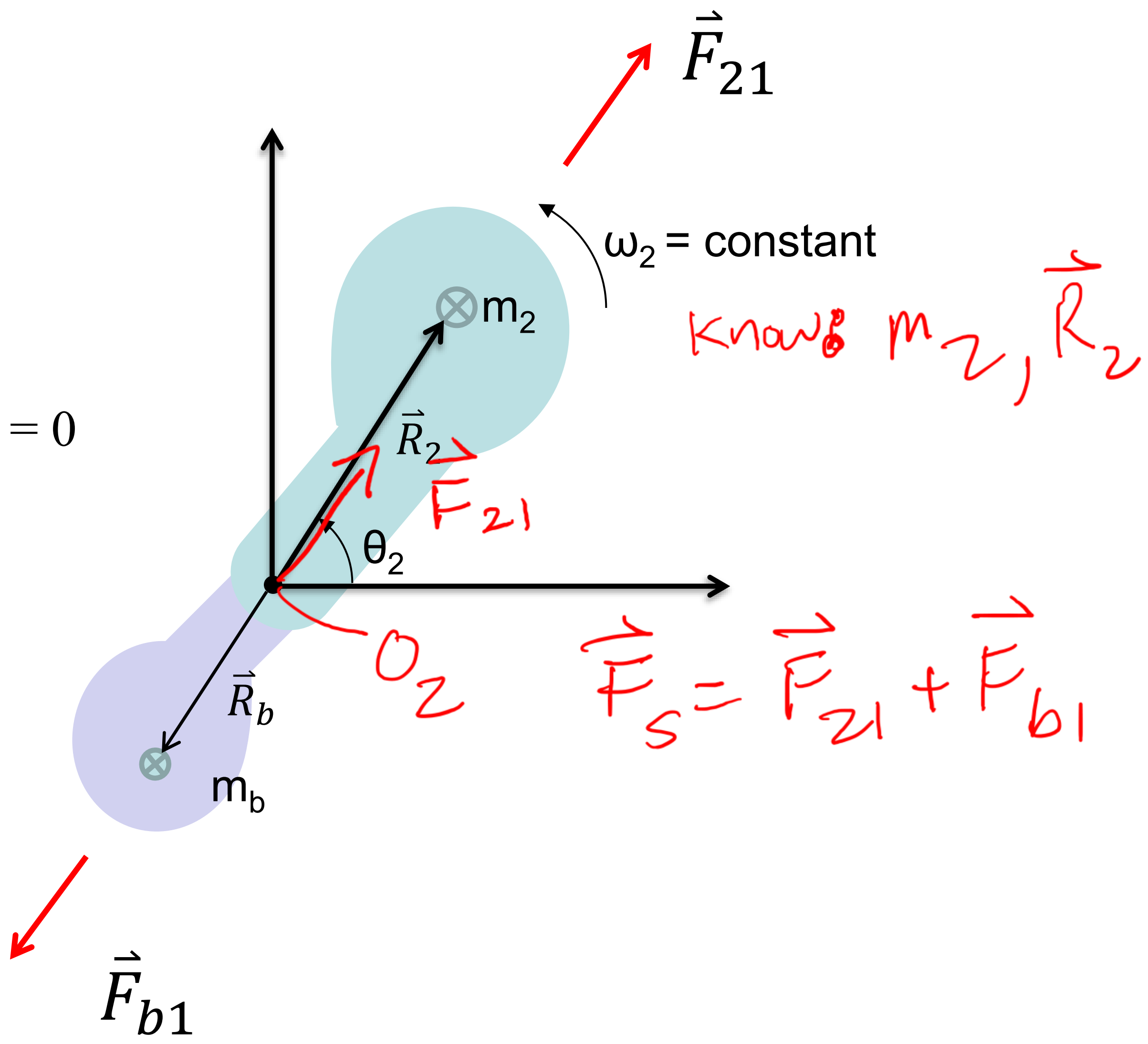
$$m_2(-\vec{R}_2\omega_2^2) + m_b(-\vec{R}_b\ \omega_b^2\ ) = 0$$

ω_2 = constant $\qquad \omega_b = \omega_2$

$$\boxed{\therefore m_2\ \vec{R}_2 + m_b\vec{R}_b = 0}$$

Convert to x & y components

$$\boxed{\begin{aligned} m_2R_{2x} + m_bR_{bx} &= 0 \\ m_2R_{2y} + m_bR_{by} &= 0 \end{aligned}}$$

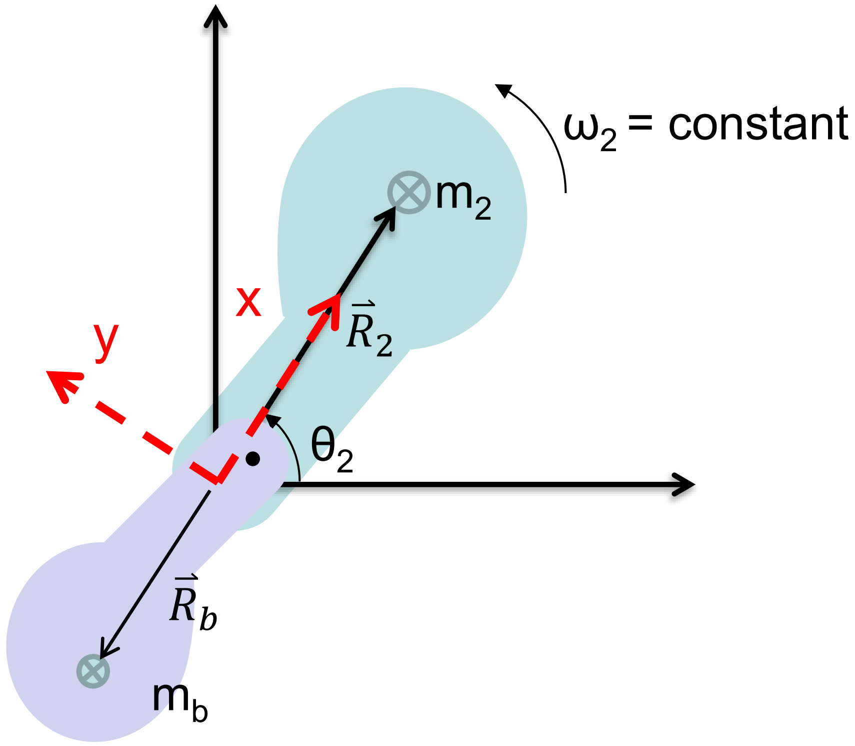


How many unknowns? 3 m_b, R_{bx}, R_{by}

Only 2 equations. How to address?

Static Balancing:
How to determine unknowns? Where to put the balancing mass?

$$m_2 R_{2x} + m_b R_{bx} = 0$$
$$m_2 R_{2y} + m_b R_{by} = 0$$



- Put x-coordinate along R_2

$$R_{2x} = \text{known}$$
$$R_{2y} = 0$$

- Then we can choose

$$R_{by} = 0$$
$$m_b R_{bx} = -m_2 R_{2x}$$

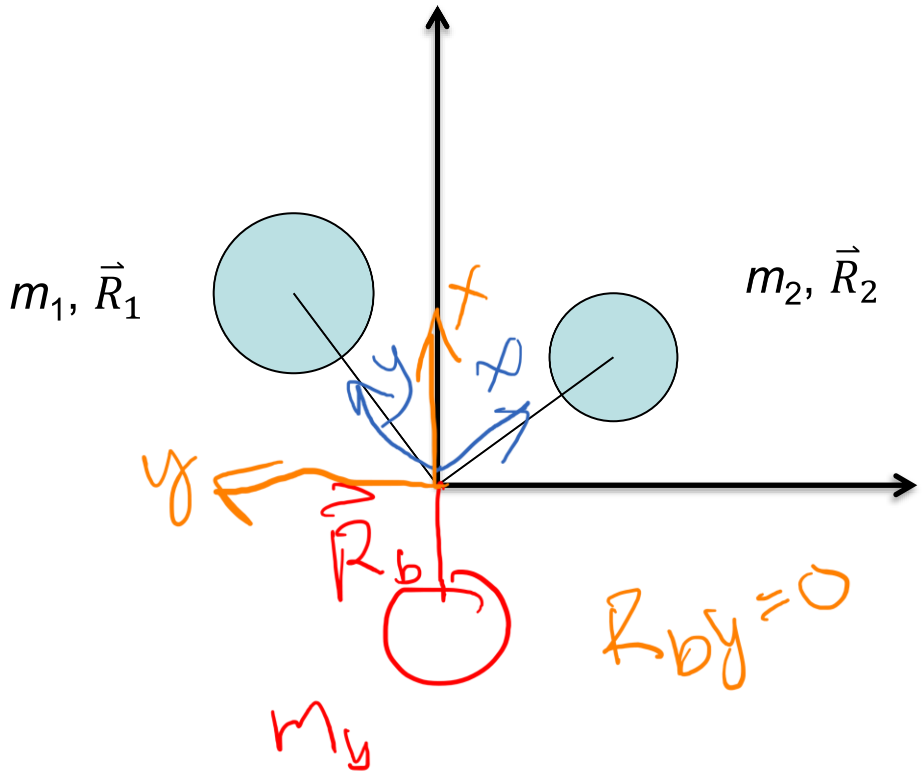
- Equivalent to putting CG on axis of rotation
- An infinity of solutions
  - Short and heavy
  - Long and light
  - What are the tradeoffs?

choose either m_b or R_{bx}

Example

Given: constant velocity mechanism and known $m_1, \vec{R}_1, m_2, \vec{R}_2$.

Determine appropriate mass and location $(m_b, \vec{R}_b)$ for a **single** balancing mass



To solve, start with this basic equation:

$$\sum m_i \omega_i^2 \vec{R}_i = 0$$

Since velocity is constant and all ω_i are the same, use:

$$\boxed{\sum m_i \vec{R}_i = 0}$$

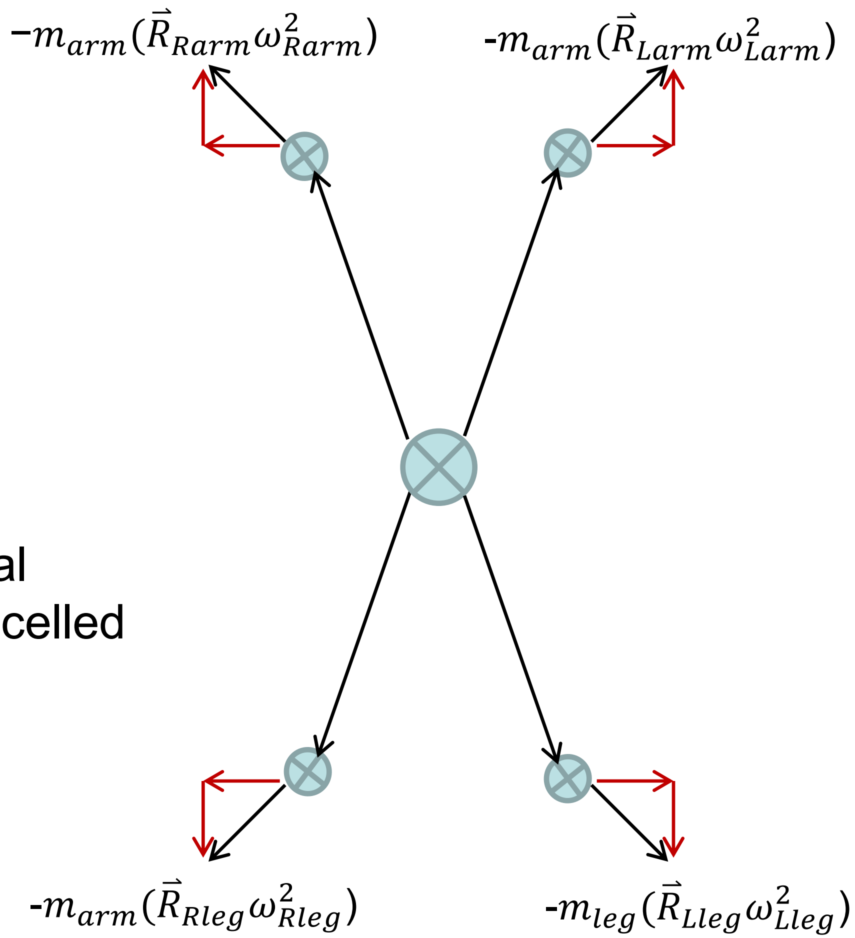
Key Static Balancing eqn

$$m_1 \vec{R}_1 + m_2 \vec{R}_2 + m_b \vec{R}_b = 0$$

$$m_1 R_{x1} + m_2 R_{x2} + m_b R_{bx} = 0$$

$$m_1 R_{y1} + m_2 R_{y2} + m_b R_{by} = 0$$

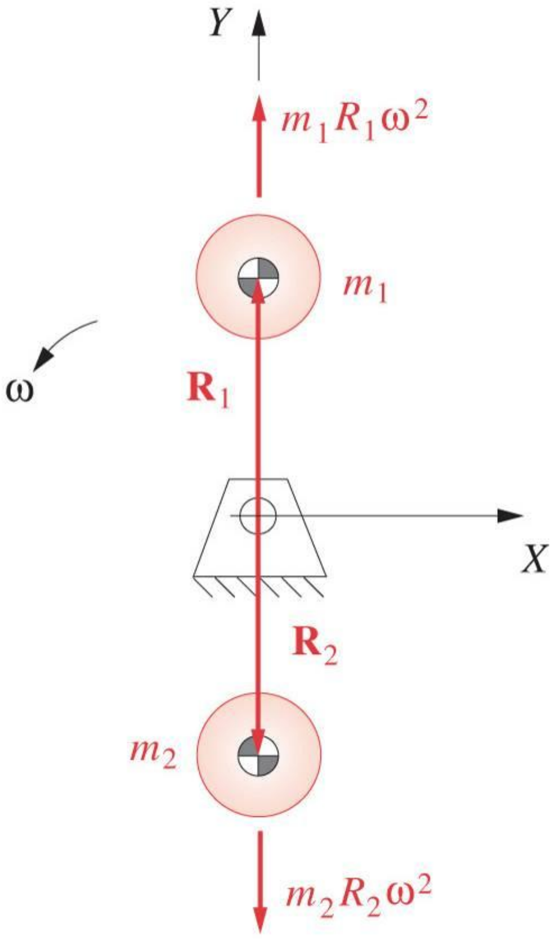
Jumping Jacks

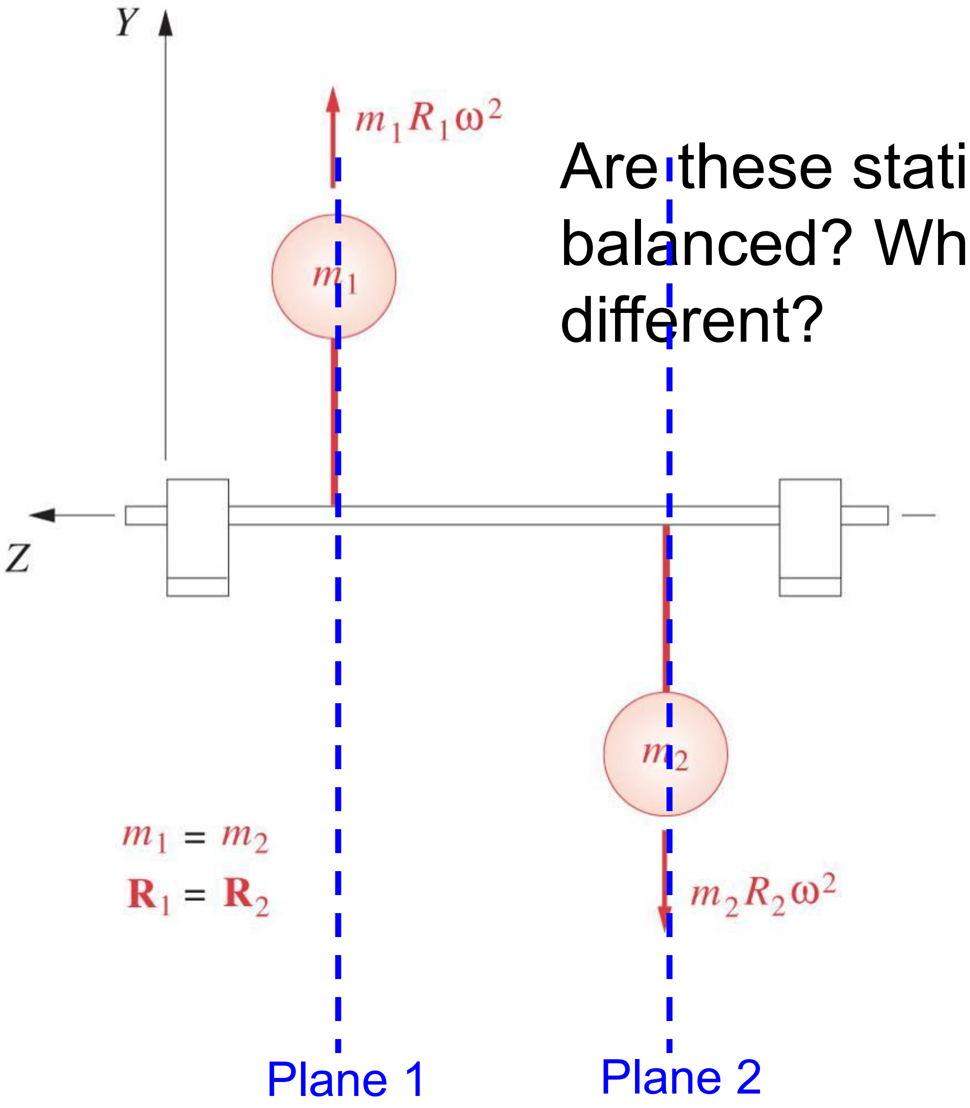


Note that the horizontal components have cancelled

- X-components cancel

Dynamic Balancing

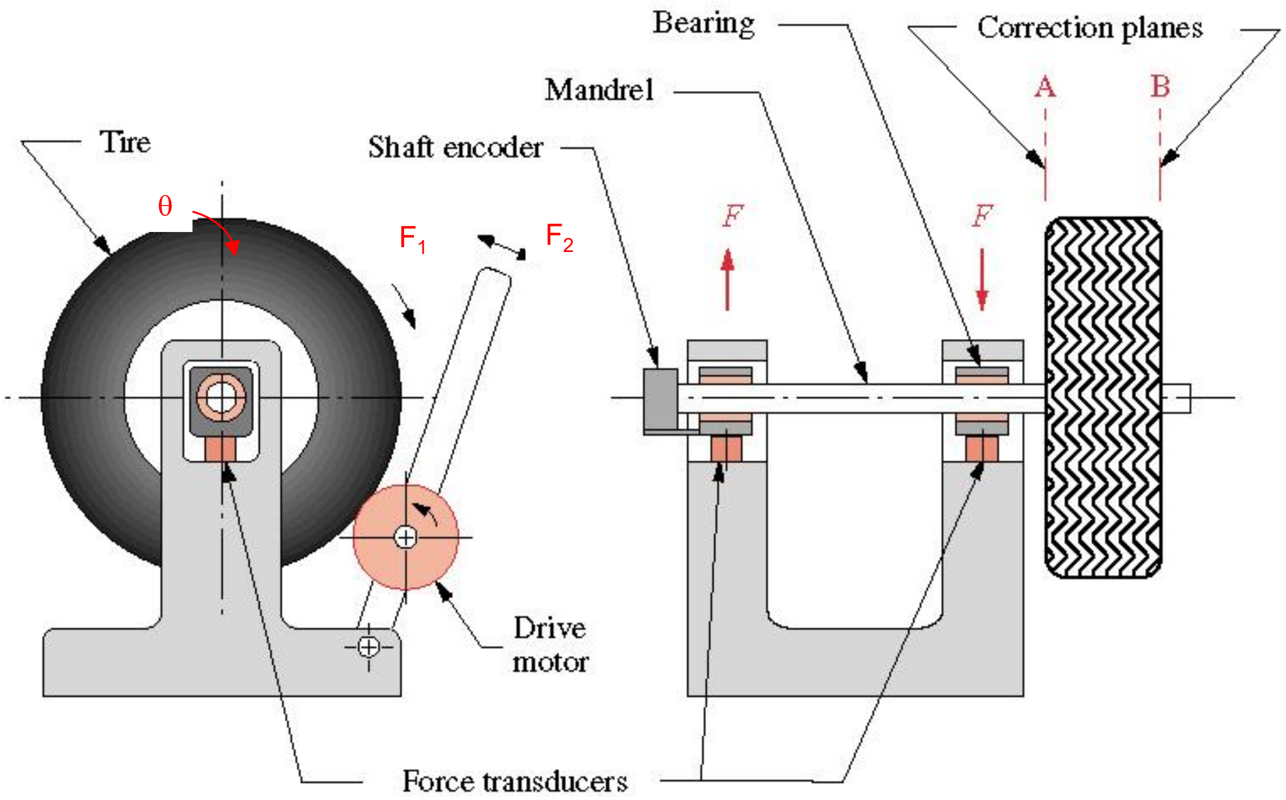




Are these statically balanced? What is different?

F

Dynamic Balancing of Car Wheels



- Spin wheel
- Measure $F_1(t)$, $F_2(t)$, $\theta(t)$
- Add weights to Plane A and/or B to get
  - Force balance
  - Moment balance

Dynamic Balance

- 3-D, “two-plane” balance
  - masses generating inertial forces are in different planes
  - long in axial direction compared to radial direction
  - applies to rotating shafts
  - ex. car tire, turbines, drive shafts, camshafts, crankshafts

Dynamic Balancing

- 2 step process
- Must satisfy both
  - Force balance $\sum \vec{F} = 0$ (balance inertial forces in radial direction)
  - Moment balance $\sum \vec{M} = 0$ (balance bending moments due to inertial forces in shaft)

- Add or remove mass from two distinct correction planes
  - Provides a counter couple to cancel out the unbalance moment
  - On a tire, you use the inner and outer edges of the rim.

Dynamic Balance: Two-step process

1. Perform Force (Static) Balance ($\sum \vec{F} = 0$) prior to Moment Balance ($\sum \vec{M} = 0$)
   - It is possible to be statically balanced but not dynamically balanced
   - Use static balance first:

$$\sum \vec{F} = 0 \rightarrow \quad \boxed{\sum m_i \vec{R}_i = 0}$$

Dynamic Balance: Two-step process

2. Key is to balance bending moments $\sum \vec{M} = 0$
   - Use two correction planes on which the balancing weights are placed
   - Pick a point on either plane about which to find the moments about (e.g., point A)

$$\sum \vec{M}_A = \sum(\vec{L}_i \times m_i \vec{A}_i) = \sum(\vec{L}_i \times m_i \omega^2 \vec{R}_i) = 0$$

where $\vec{L}_i$ is the lever arm from point A to mass m_i

- This can be written in short-hand as $\sum m_i \omega^2 \vec{R}_i l_i = 0$

  where l_i is the length of the lever arm,

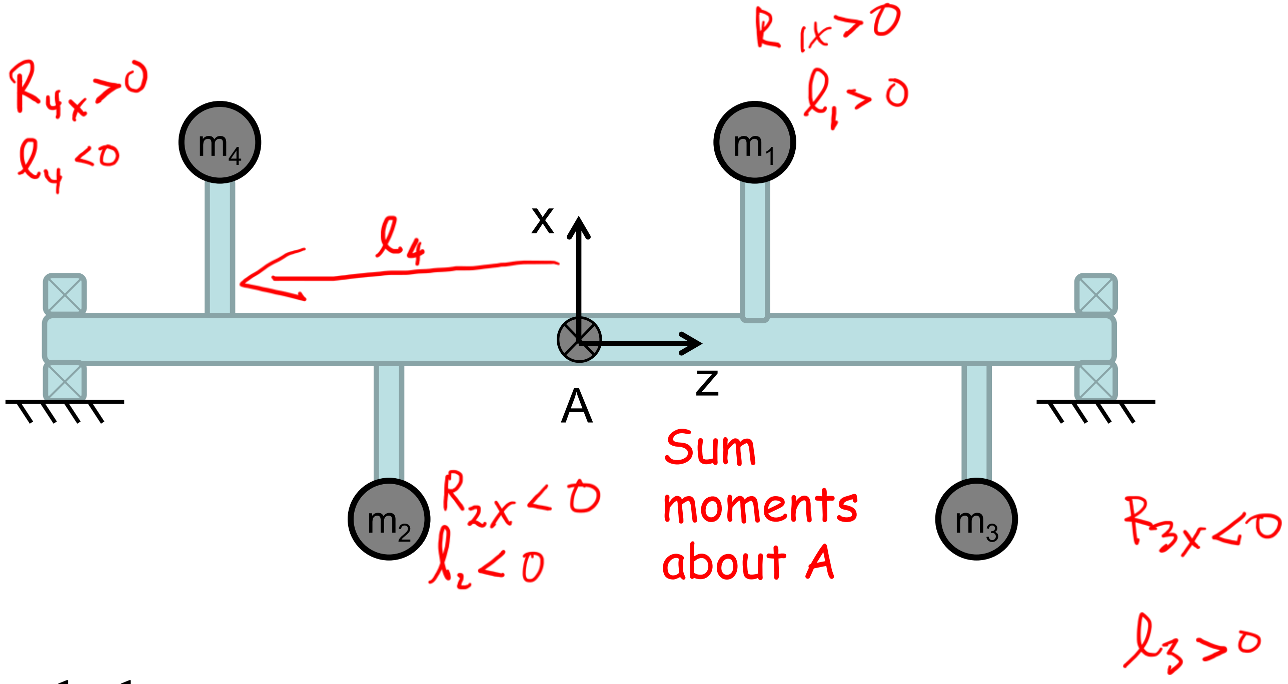
  or since ω is same for all masses: $\boxed{\sum m_i \vec{R}_i l_i = 0}$

  where i are original or balancing weights

F

ILLINOIS

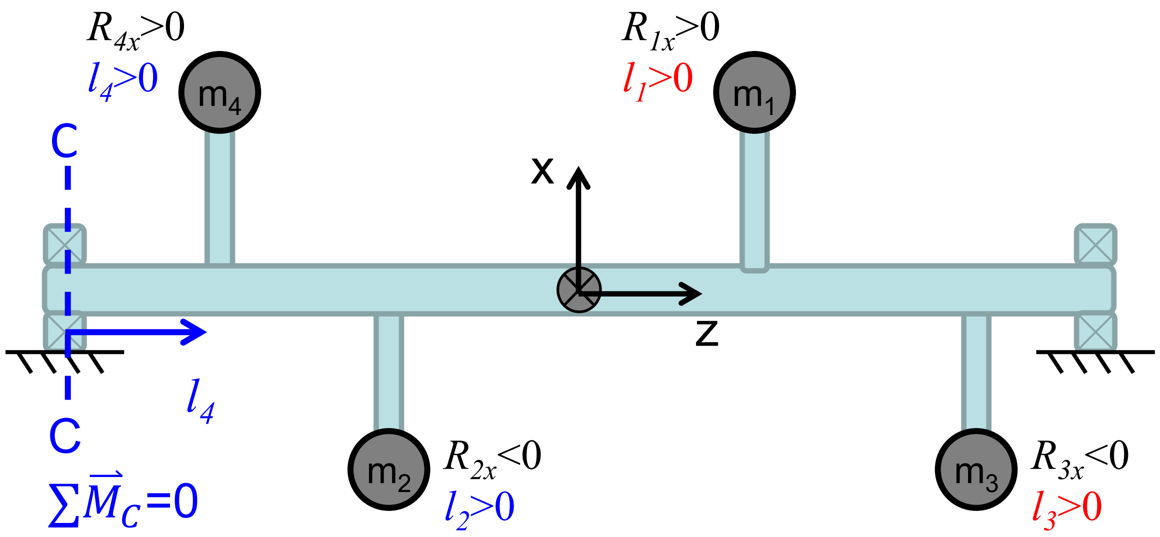
Dynamic Balancing



- Moment balance notes:
  - Measure l_i's from any convenient reference plane
  - The signs of R_{xi}, R_{yi}, l_i matter!

Dynamic Balancing

Changed starting point to left side at plane C-C. Note change in signs for l_2 and l_4



- Moment balance notes:
  - Measure l_i's from any convenient reference plane
  - The signs of R_{xi}, R_{yi}, l_i matter!