Theory of stability, velocity, and gait of mechanical walkers

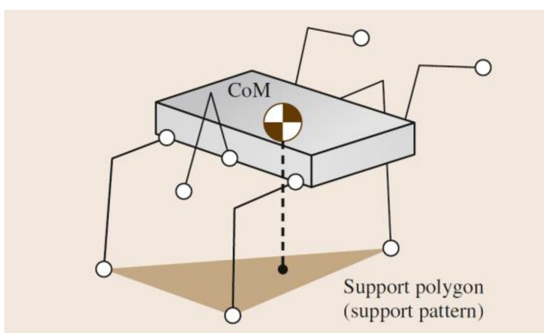
In order to understand how mechanical walkers operate, it is important to understand the underlying theory of motion depending on the number of legs and how to convert the performance criteria of the project into design criteria for the walking mechanisms. The material below has been extracted from the seminal *Handbook of Robotics* by Bruno Siciliano and Oussama Khatib (eds.), Springer, 2016. (This book is freely downloadable from the University Library website. Chapter 16 is of great relevance to your project.)

The available options in legged robots (or "mechanical walkers") are immense. Popular configurations range from 2-legged (humanoid), 4-legged (quadrupeds), 6-legged (hexapod) and 8-legged (Octopod) walkers. Note that in the ME370 project new designs become possible that are not addressed here, however, the logic can be modified to nearly any situation or locomotion concept.

Static stability and dynamic stability

Stability is fundamental to the performance of walker locomotion. Stability refers to the balancing of the walker. There are 2 types of stability: static and dynamic.

Static stability applies when there are no extra movements or forces needed to prevent the walker from falling over. The center of mass (CoM) of the walker is at all time within the support pattern (or support polygon) of the legs which have ground contact, as illustrated below.



Support polygon (support pattern) of multi-legged robot

Dynamic stability is needed when the CoM is outside or on the border of the support pattern.
When the CoM is outside the support pattern the walker will fall over when no additional forces and movements are made with the legs (balancing).

A hexapod provides better static stability than a quadruped because of the larger amount of legs
which increases the support pattern / polygon. Hexapods can perform static walking by supporting the walker body on five legs at any time, while quadrupeds can only walk

statically on a minimum of three legs. This feature makes hexapods much more stable than quadrupeds, since they can use a bigger support polygon.

Walking speeds of walkers (or legged robots)

The forward speed of a legged walker V_n (with n = the number of legs) and which performs a wave-like gait depends on the stride length L_s, the cycle time T, and the *duty factor* β, which directly depends on the number of legs. *Duty factors* (the fraction of time that a leg spends on the ground relative to the stride period) decreases to 0.5 and below with an increase in speed.

The *transfer phase* (the time that the leg is in the air in order to make the next step) is $t_T = (1-\beta)T$ and the *support phase* (the time the leg touches the ground and moves the robot forward) $t_S = \beta T$ are derived from the cycle time and the duty factor. The forward speed of the walker can then be calculated as follows:

$$V_n = \frac{L_s}{t_s} = \frac{L_s}{\beta T} = \frac{L_s}{t_T}\left(\frac{1-\beta}{\beta}\right)$$

The minimum duty factor of an n-legged robot is $\beta_n = 3/n$. Therefore, your walker's speed will be as follows:

- Quadruped: $V_4 = 0.333(L_s/t_T)$
- Hexapod: $V_6 = (L_s/t_T)$
- Octapod: $V_8 = 1.6(L_s/t_T)$

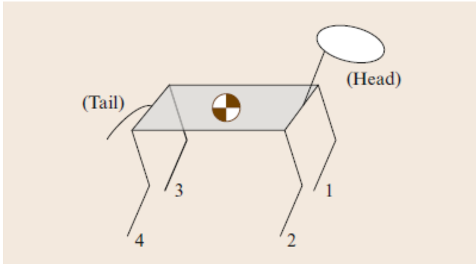
Hexapods can therefore achieve higher speeds than quadrupeds, and octopods are even faster in ideal and static stable walking gaits.

Gait analysis

After creating a walking mechanism, it is necessary to perform a leg and body motion sequence to make the mechanism walk. This sequence is known as the *gait*.

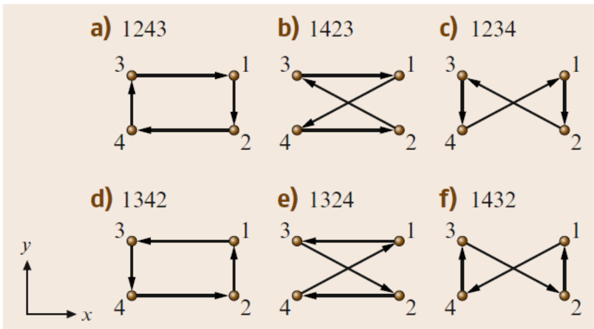
Quadrupeds

The 4-legged walker (quadruped) typically has legs at the corners of the body. To number the legs, index the left legs and the right legs by odd numbers 1, 3, … , $2n-1$ and even numbers
2, 4, ... , $2n$, respectively. Following this rule, the legs of a quadruped walker may be numbered as shown in below.



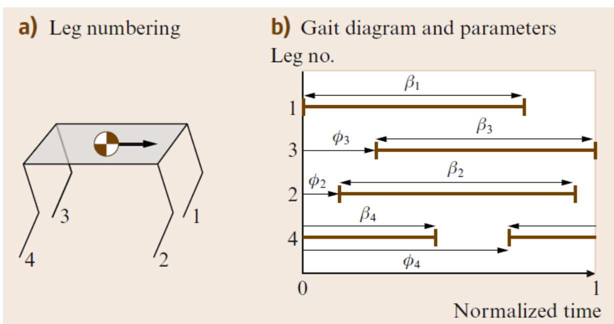
Leg labeling of a quadruped walker

A quadruped robot must lift and place only one leg at each step to achieve static stability. Such a pattern is called creeping gait. The possible creeping gaits of a quadruped robot can be expressed by a series of leg numbers to show the order of foot placing. Always choosing leg 1 as the first swing leg, we can distinguish $(4 - 1)! = 6$ different gaits as illustrated below:

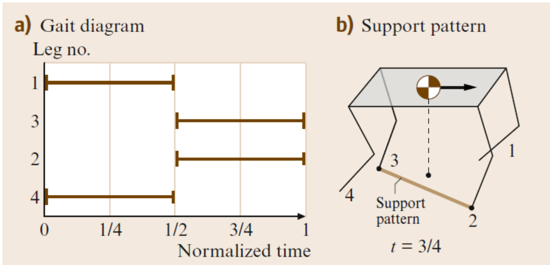


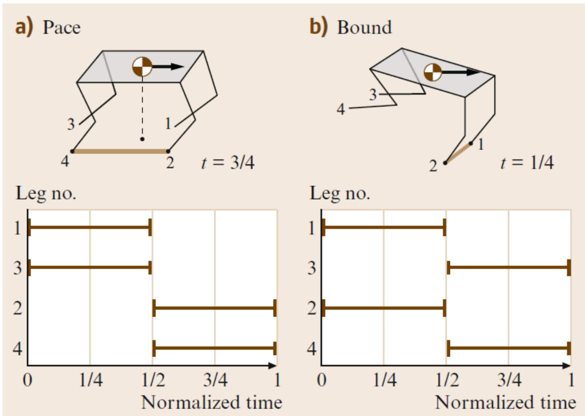
Quadruped creeping gaits that show different sequences of foot placing

The figure below shows a gait diagram to describe the gait sequence of a multi-legged walker, in this case a 4-legged walker. The horizontal axis shows the time normalized by walk cycle time *T*. A line segment associated with each leg starts from the touch-down and ends at lift-off. Therefore, the length of the line segment indicates the period of the support phase t_S.



Other walking gaits for the quadruped walker are the *trot* gait shown in the two figures below.





These trot gaits are more advanced because they move 2 pairs of legs at a time while keeping 2 on the ground. The static stability criterion is not applicable to these gaits. The support polygon is then just straight line between the two legs touching the ground. The walker should obviously feature an advanced balancing algorithm to prevent it from falling over. So, they're not recommended for your projects.

Hexapods

For walkers with 6 legs (hexapods) there are many combinations of leg placements and leg designs. For example, the legs can be configured with 3 legs on each side (rectangular), or circular around the walker body to increase leg freedom.

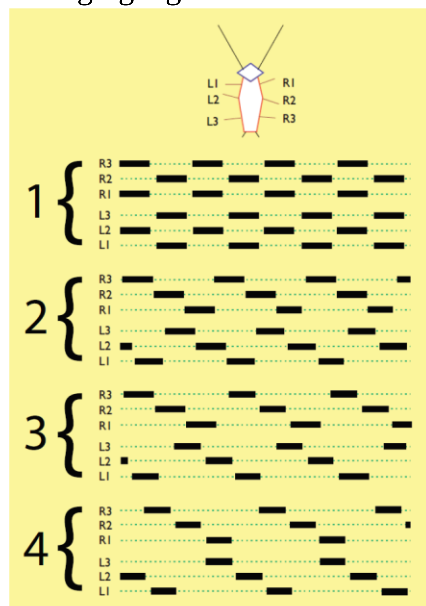
The most popular configurations give rise to a so-called wave gait of rectangle shaped robots. Rectangular designs are more natural designs, compared to the gaits of animals. This design is fast in the forward direction but less flexible in turning, moving sideways or moving backwards.

Placing the legs in a circle around the walker results in unnatural gaits but in efficient movement in all directions, at the same speed. The gaits for rectangular designs are applicable to the circular design, but not necessarily *vice versa*. A circular robot design has legs which can make the same step size forward, backwards and sideways, whereas a rectangular is commonly designed to make large steps forwards and not sideways.

The static stability between these two designs are in general equal when the rectangular is close to a square design.

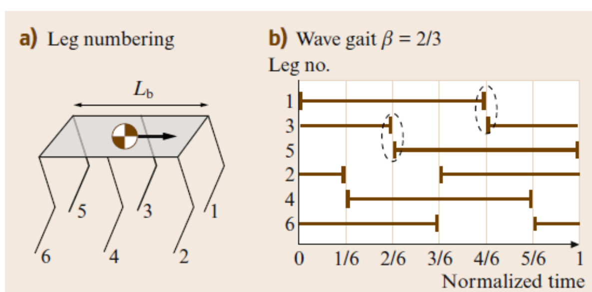
There are more wave gaits available for a 6-legged walker than for a 4-legged walker because the combination options to move a single leg or pairs of legs are larger. Insects walk with a variety of different patterns of leg movements or gaits at different speeds of locomotion. With insects, these gaits typically display static stability, that is, the polygon of support formed by connecting all the supporting legs always contains the center of mass of the body. The movement of each leg can be divided into a support phase, in which a leg provides support and propels the body, and a transfer phase, in which the leg is lifted from the ground and swings forward to begin another support phase.

Given the importance of the relative timing of swing phases (since a swinging leg provides no support to the body), the different gaits observed in insects can be described by displaying the times at which each leg swings, as shown in the figure alongside.

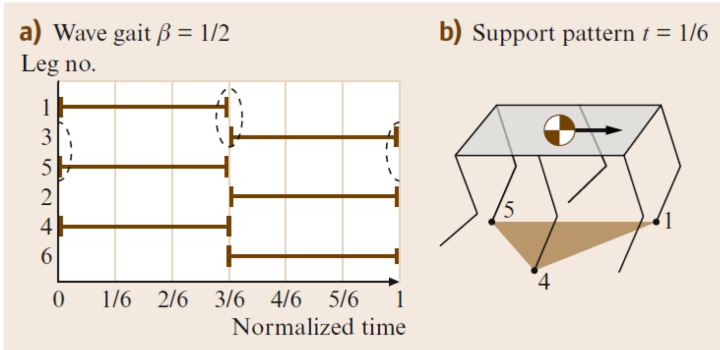


A selection of four insect-like gaits. Black bars represent the support phase of a leg and the space between bars represents its transfer phase. The leg labelling conventions are shown at the top. The gait presented at the top is the static stable tripod gait, the other three are natural gaits observed from animals like cockroaches, which are dynamic gaits. Gaits 2, 3 and 4 are unstable gaits since only 2 legs are in contact with ground at any time, the other legs are in transfer phase. The differences between 2, 3 and 4 are the leg placement sequences.

Another example of a wave gait for the hexapod robot is shown below. In this example, 4 legs are always in the support phase, leaving 2 legs in the transfer phase. This is a statically stable gait and faster than before, requiring only one leg to be lifted at a time.



The next figure shows a wave gait with β = 1/2, which is the most important for hexapod robots. This provides the smallest duty factor for hexapods and thus results in the fastest walking speed. This gait is specially called a tripod gait since a robot is supported by the three legs 1, 4, 5 and
2, 3, 6 reciprocally:



Tripod gate of a hexapod