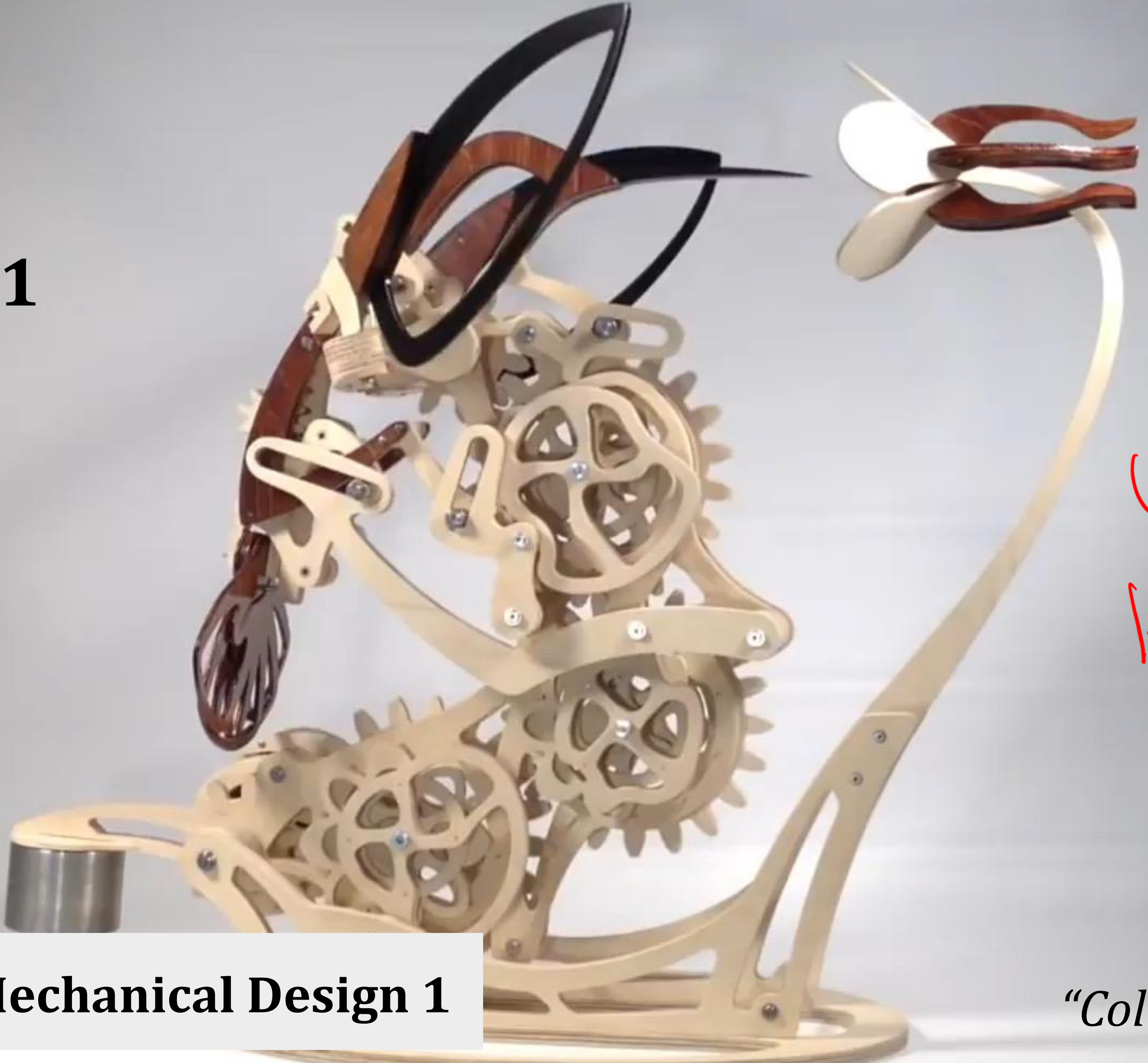
Module 5
Lecture 14
Gears – Part 1



PICK UP YOUR KIT

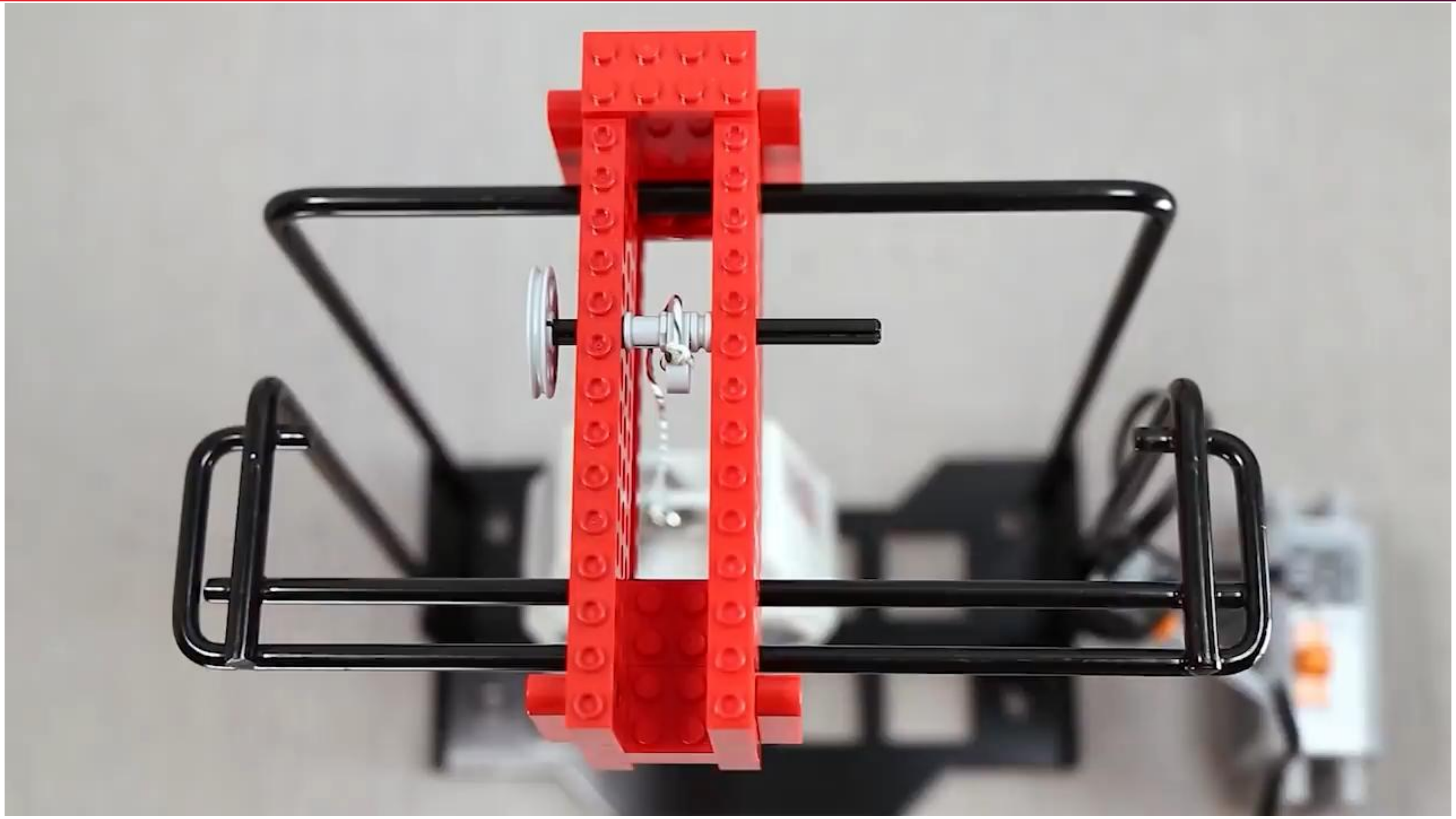
ME 370 - Mechanical Design 1

"Colibri" by Derek Hugger

** www.youtube.com/watch?v=1scj5sotD-E*

Worm gear torque, brick experiment channel

https://www.youtube.com/watch?v=8rc-gpo3auQ



Practical questions while watching video:

- Why does a worm gear have a lower efficiency than a spur gear?
- Why does lubricant lead to a higher transmission of force?

Lecture 14: Gears 1

Topics: 10/13/25 Gears – Part 1 (Norton Chap 9)

Activities & Upcoming Deadlines

- **HW:**
  - **HW 6 (PVA #2):** due Tuesday 10/14
  - **HW 7 (IC #1):** to be posted soon, due Tuesday 10/21
- **Lab 8: PVA analysis**
  - This is an **Individual Student** Lab. Pre-lab: READ all lab materials prior to lab time, submit pre-lab assignment
- **Project 1:**
  - Peer evaluation 2 in CATME (by 1 week after lab section for P1D5). This is a graded assignment. Provide comments if you give score(s) ≤ 3
- **Project 2:**
  - P2D1 (Conceptual Design Review) – during Lab 9 (in 1 week!)
    - Propose two possible designs for the legged dispensing robot (include positioning of the motor and battery holder). Also include theme images if possible
    - Presentation template to be posted soon
  - Parts kits will be distributed during Office Hours. CAD STEP files for parts are available on Pololu.

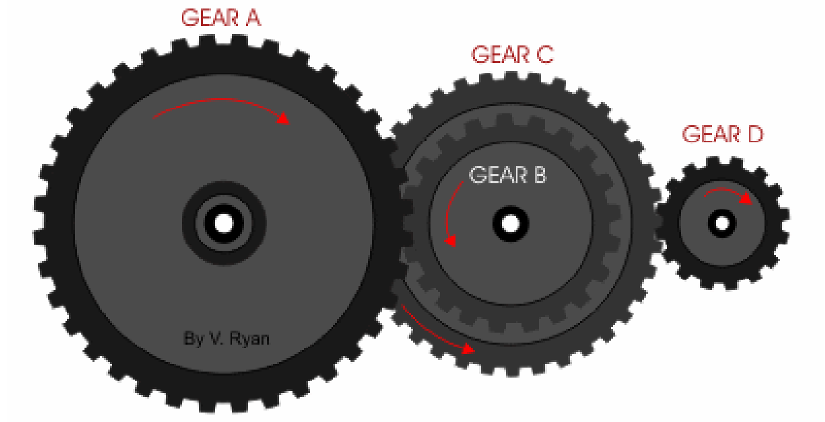
Next lecture: 10/15/25 Gears – Part 2

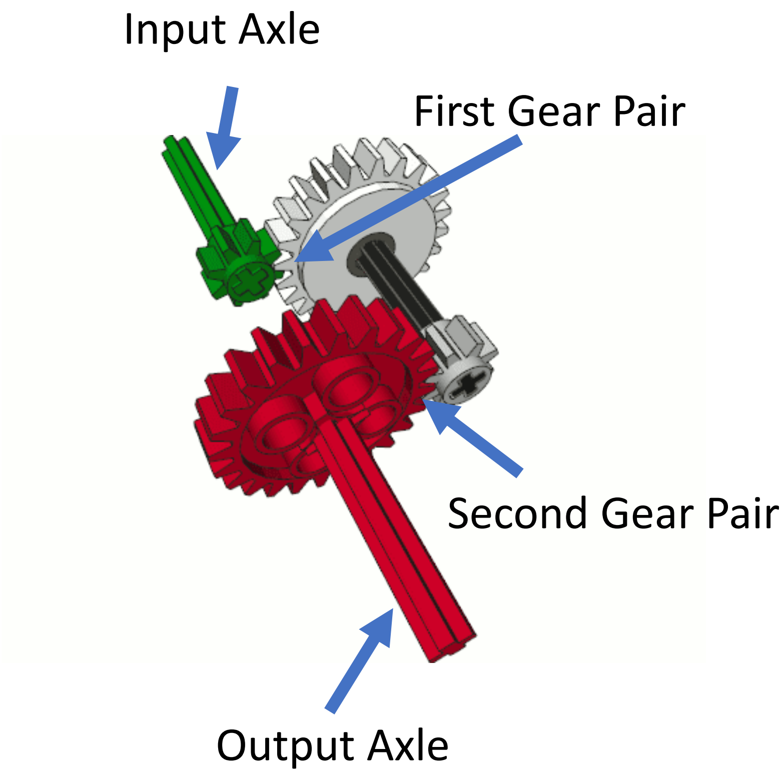
Module 5 topics: Gears

- Gear Uses, Types, Terminology
- Rolling cylinders – idealized gear set
- Velocity Ratio, Torque Ratio
- Power Transmission
- Gear Trains
    - Simple
    - Compound
- Pitch, Pressure Angle
- Gear Ratio
- Fundamental Law of Gearing
- Involute Tooth Shape and Nomenclature
- Normal forces in gears
- Epicyclic gear trains
    - Planetary
    - Differential

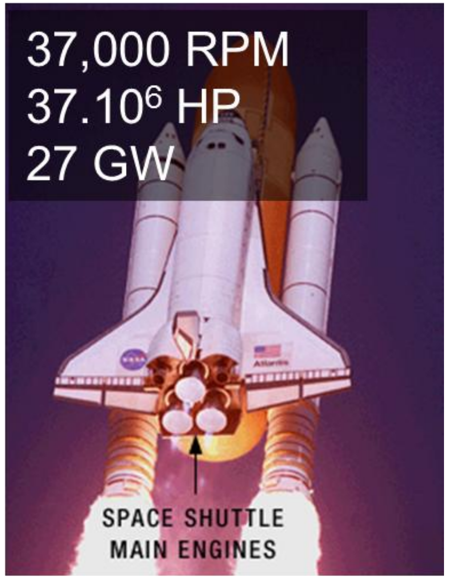
Gears are used:

1. To transmit power
   - Either increase or decrease (a) torque or (b) speed of rotation, not both
2. To move rotational motion to a different axis or direction
3. To keep the rotation of two axes synchronized



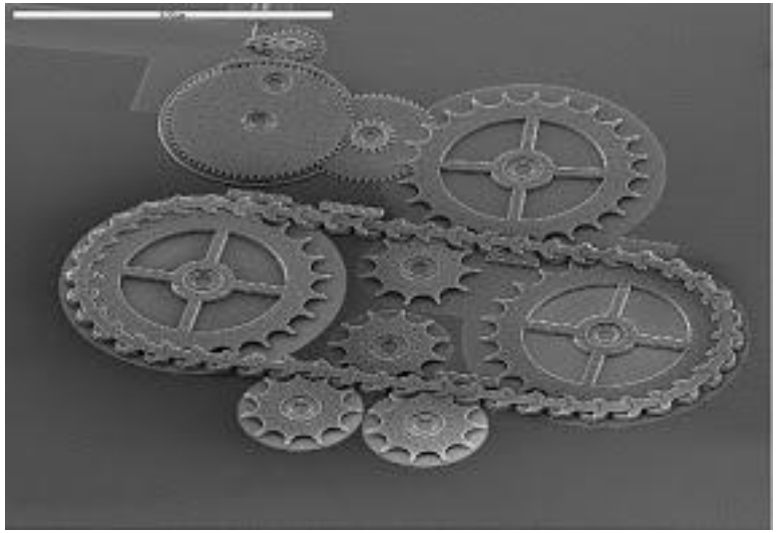


Transmission of power:



Small:

World's smallest gear chain drive by Sandia National Lab. Chain sprockets separated by ~200 microns.

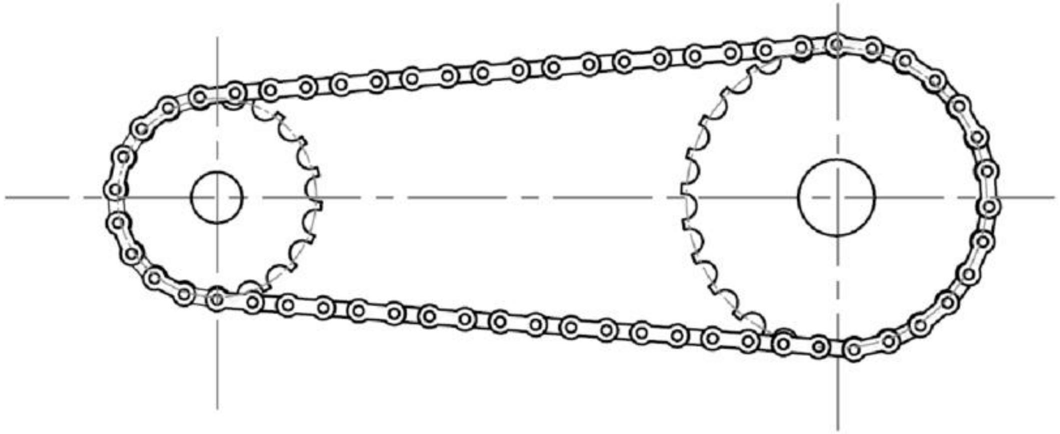


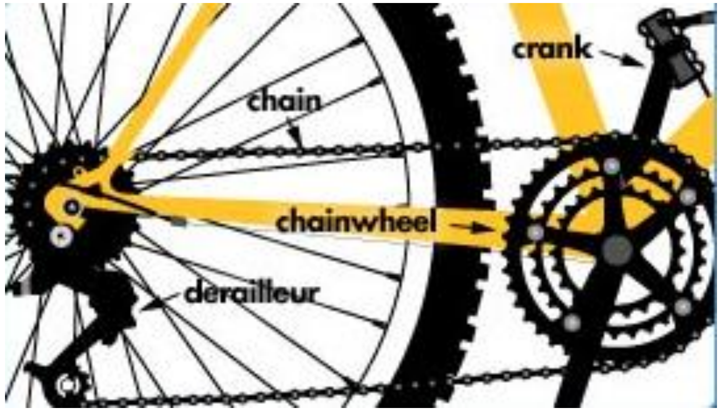
Large:

Large gears can be made to over 100 feet in diameter.

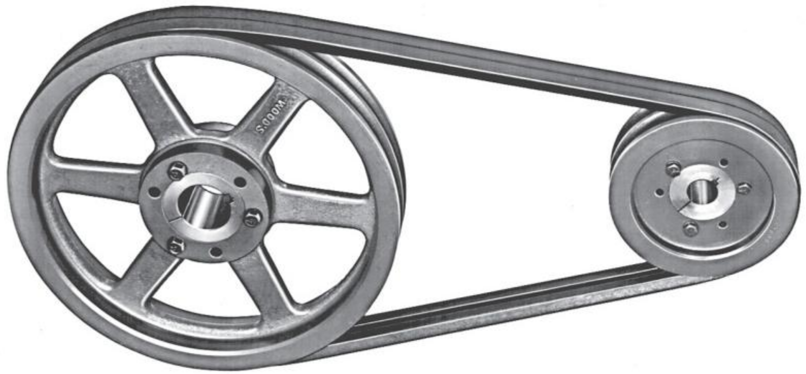
Belts and chains – when axes are separated

Chain drive:





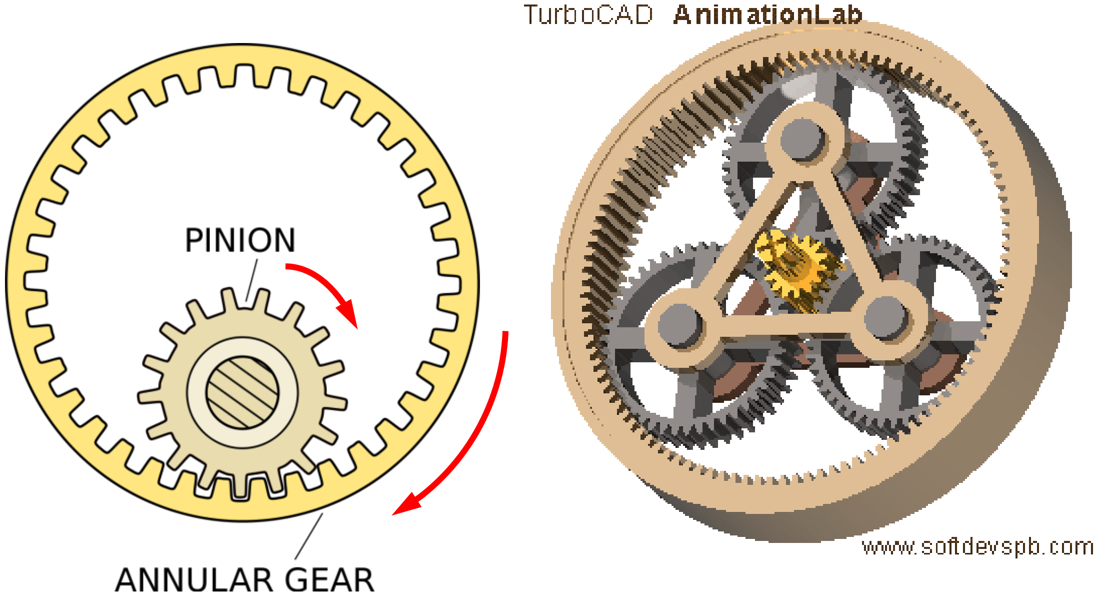
Belt drive:



External vs. internal gear trains

External gear train:
Motion of gears is opposite

Internal gear train:
Motion of gears is the same



Common Gear Types

Spur

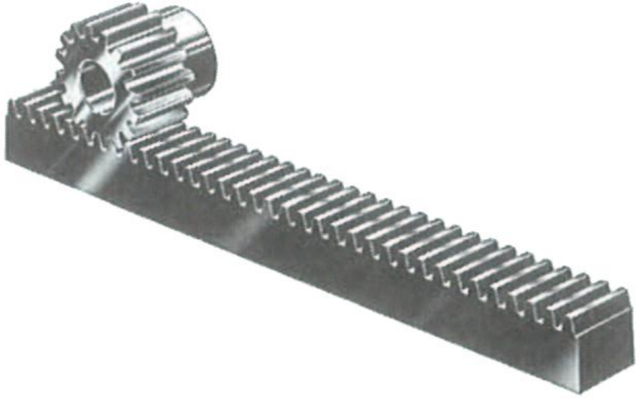
Bevel Spur

Worm

Helical

Helical Bevel

Rack-pinion

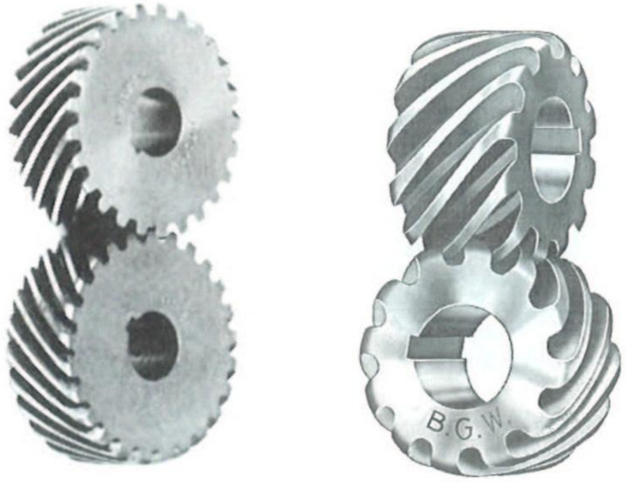


Special gears: helical and beveled

Helical Gearset

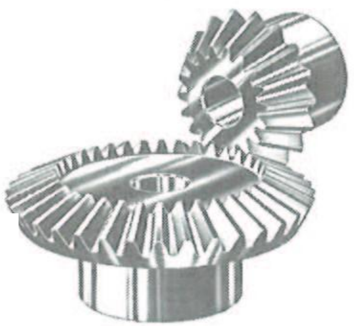
- Quieter and smoother than spur gears
- Can handle larger loads
- Less efficient than spur

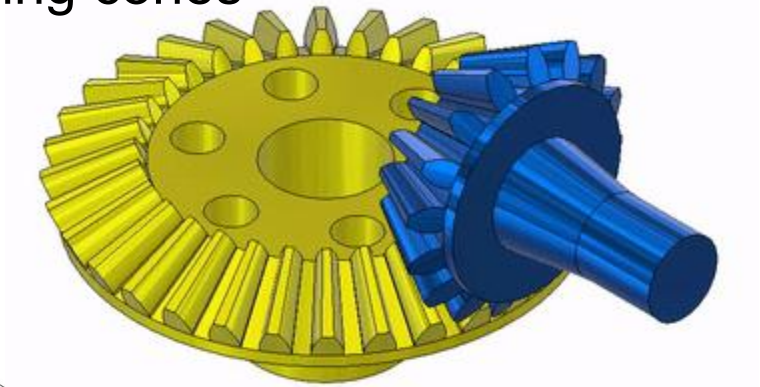
  Efficiency = Power_{out} / Power_{in}
- Can transmit non-parallel axes



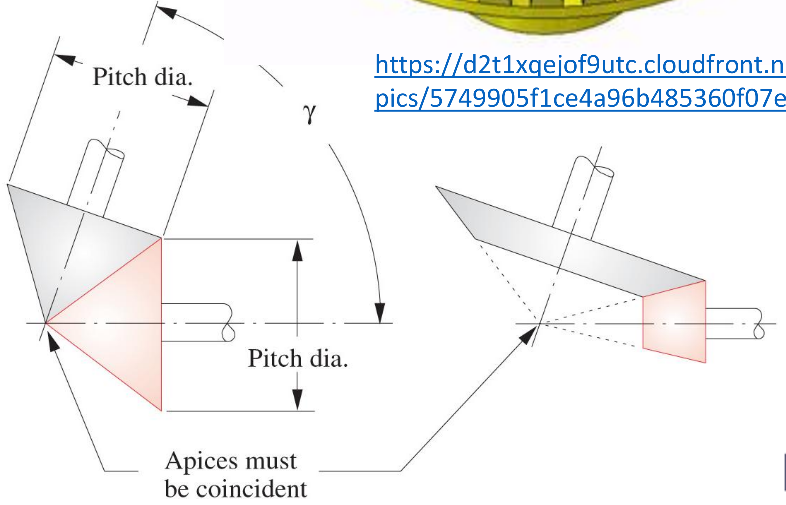
Beveled Gearset

- Transmission between non-parallel axes. Angle can be different than 90°
- Based on rolling cones





https://d2t1xqejof9utc.cloudfront.net/screenshots/pics/5749905f1ce4a96b485360f07ea063f2/large.gif



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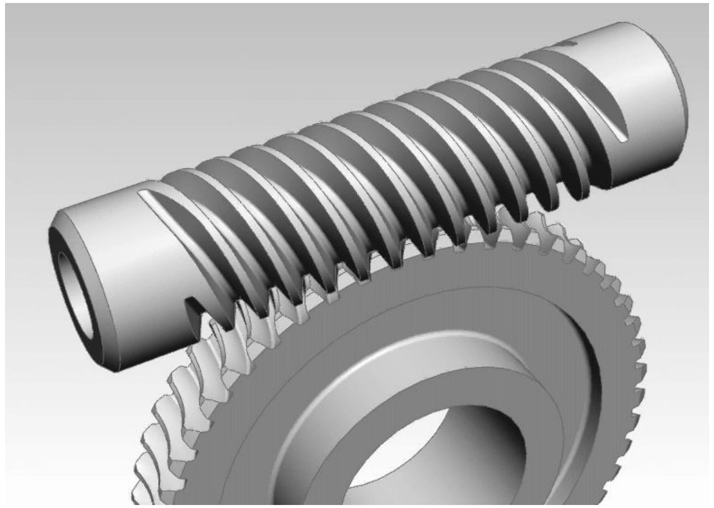
Special gears: worm and rack & pinion

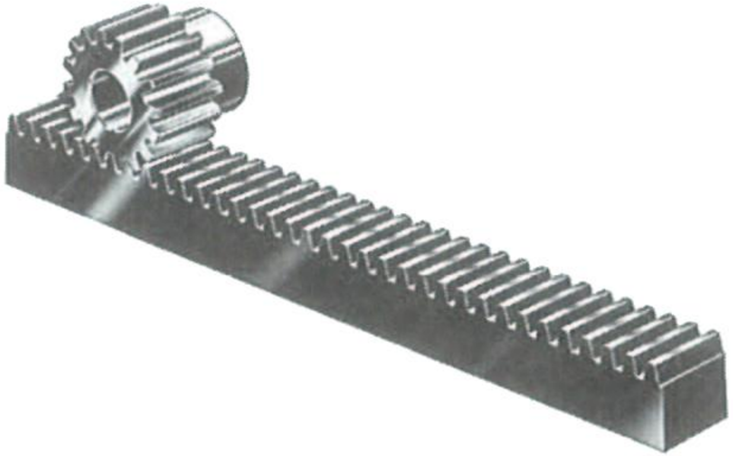
Worm gearset

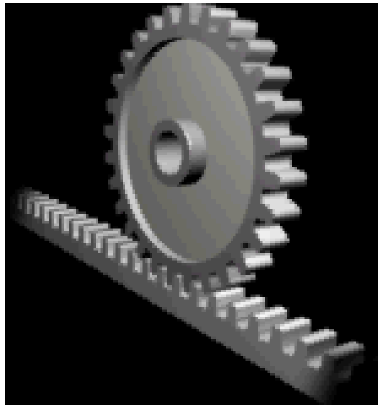
- **Worm:** on top is similar to a screw thread
- **Worm gear:** similar to a spur gear
- *A high gear ratio in small packaging*
- Self-locking (not back-drivable)

Rack and pinion

- **Rack:** infinite gear radius
- **Pinion:** spur gear
- *Converts linear to and from rotary motion*







https://en.m.wikipedia.org/wiki/Rack_and_pinion#/media/File%3ARack_and_pinion_animation.gif

https://upload.wikimedia.org/wikipedia/commons/c/c3/Worm_Gear.gif

Toothed Gears

The gear teeth have three advantages compared to friction (belt) drives:

1) Prevents slippage between the gears.

2) Makes it possible to determine exact gear ratios.

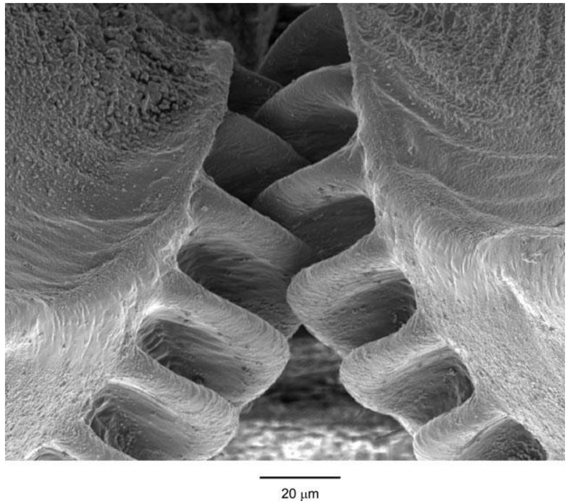
Count number of teeth and divide, N_3 / N_2.

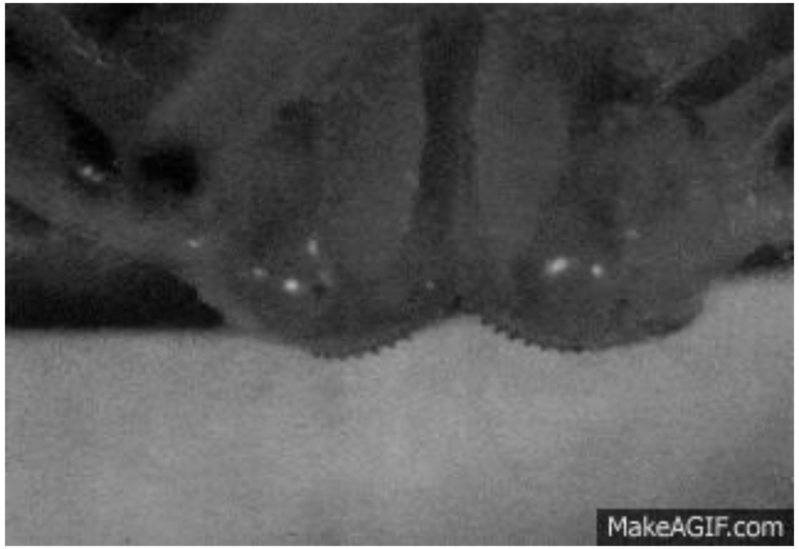
3) Slight imperfections in actual diameter and circumference of the two gears do not matter.

Gear ratio is controlled by number of teeth even if diameters are a bit off.

Toothed Gears in Nature

- *Issus* nymph: Adolescent plant hopper insect and one of the fastest accelerators in the animal kingdom
- To synchronize their legs for power jump
- Seen only in "babies" where nervous system is not ready to handle this kind of synchronization
- Absent in adults perhaps not to hinder its jumping ability if hurt





http://www.popularmechanics.com/science/animals/a9449/the-first-gear-discovered-in-nature-15916433/

Parts of a Gear Tooth

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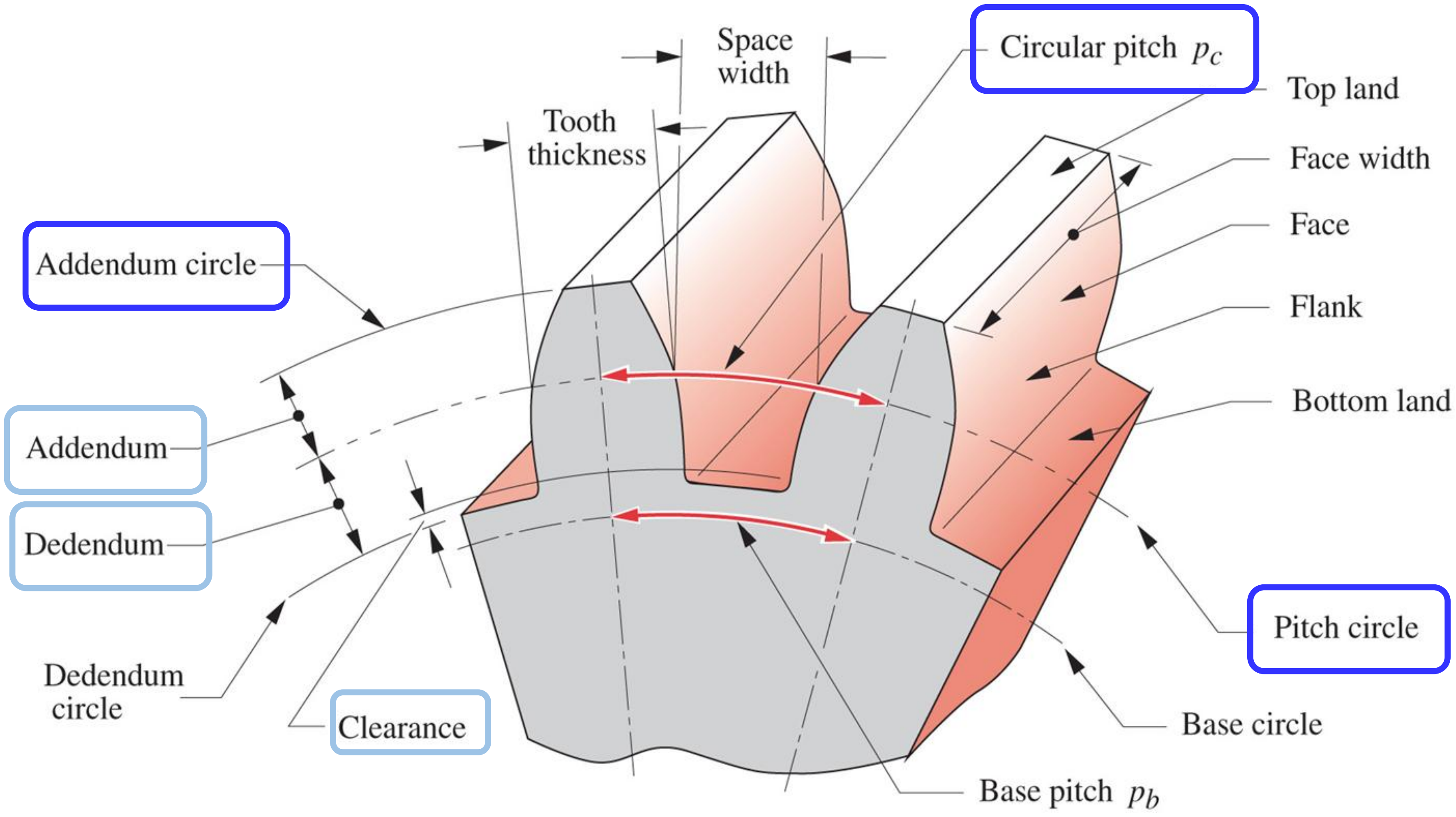
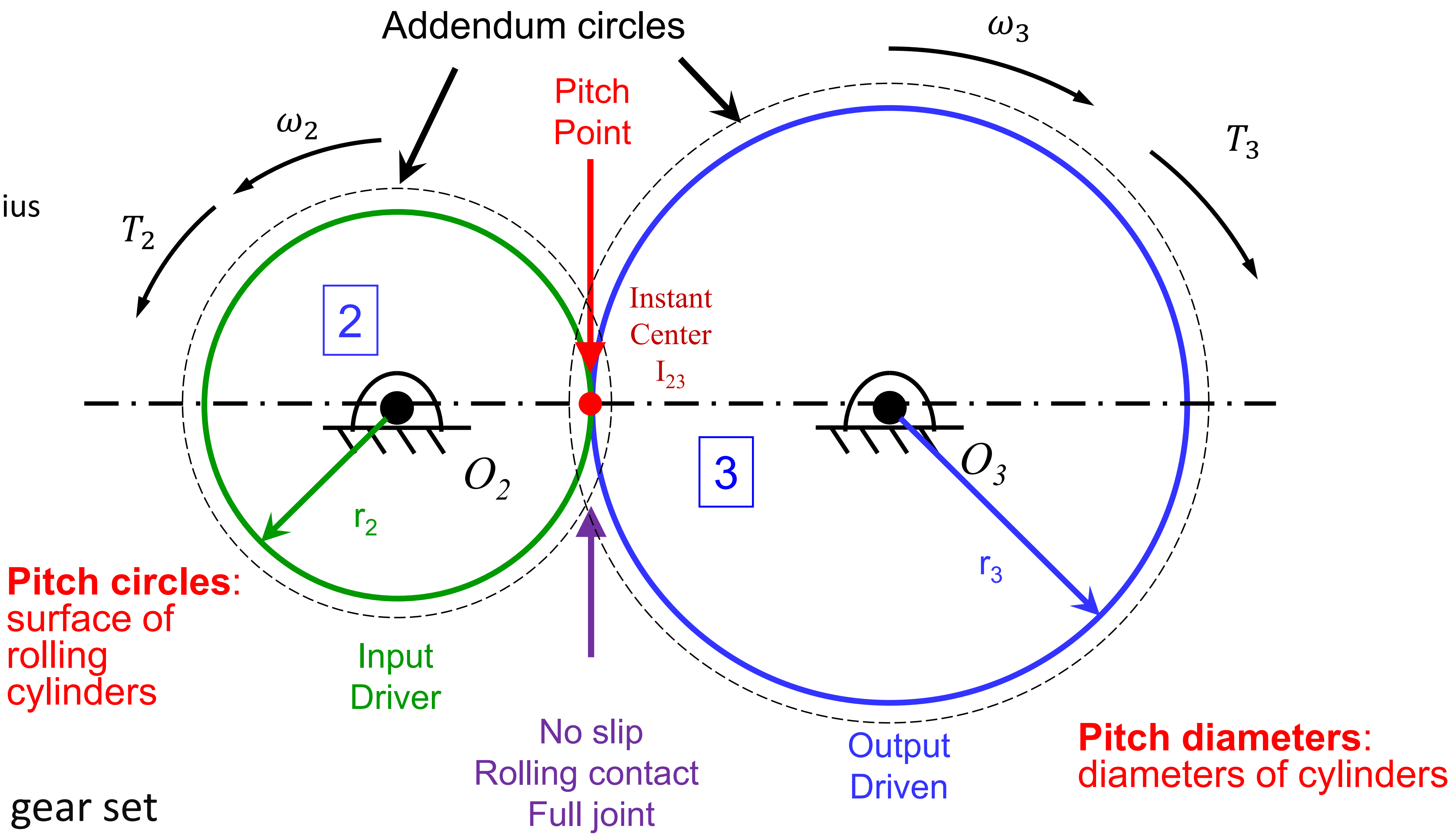


$$p_c = \frac{\pi d}{N}$$

Where d=pitch diameter
N=# teeth

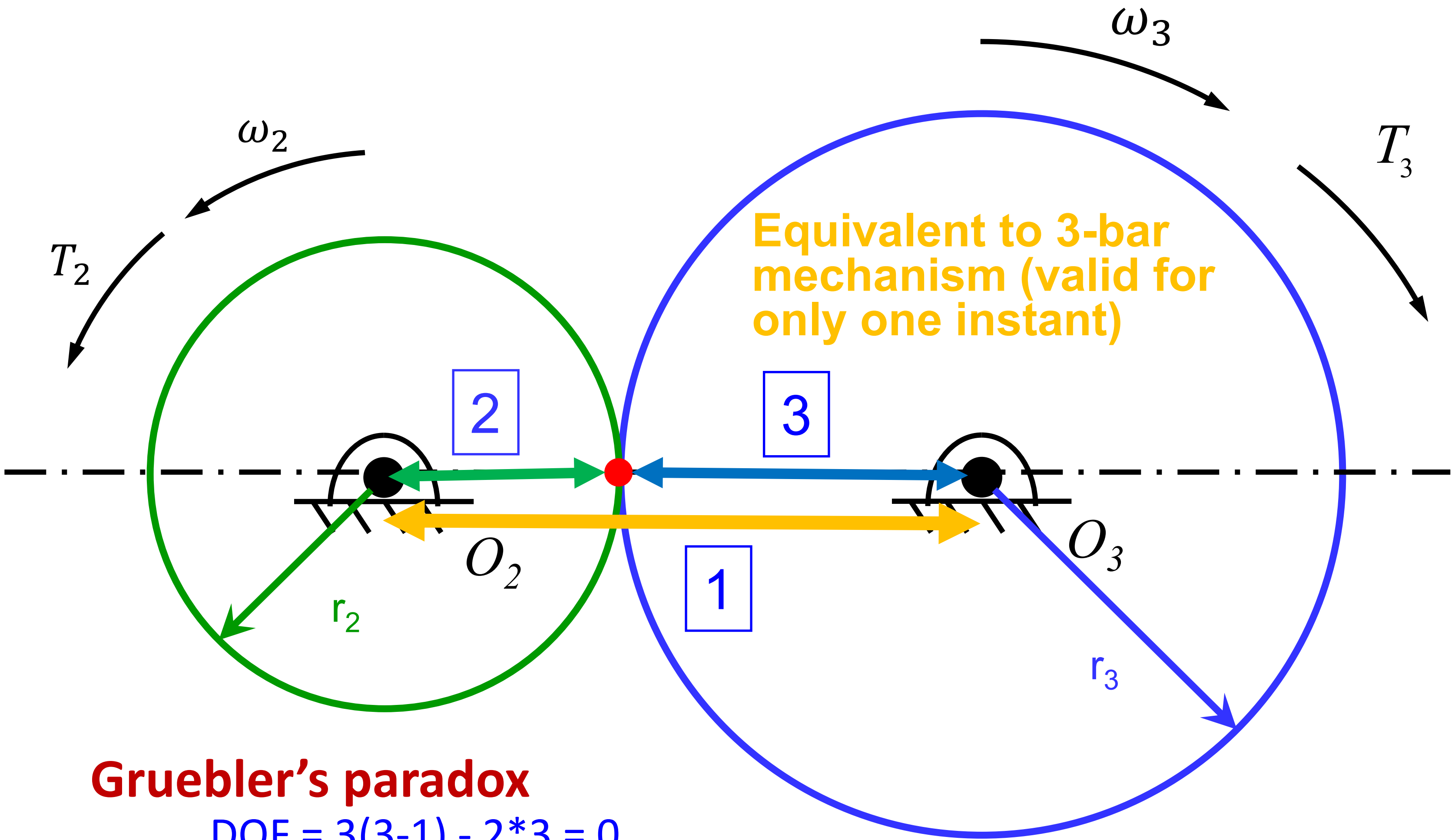
Rolling cylinders (idealized toothed gear set or friction drive)

Key terms:
Pitch circle
Pitch diameter/radius
Pitch point



An external gear set

Recall Gruebler's paradox



Gruebler's paradox

DOF = 3(3-1) - 2*3 = 0
Gruebler's predicts no motion
Ground length = sum of two radii
Gruebler's equation does not account for link size or shape.
Moral : Watch out for higher symmetry (e.g., parallel links, summed length)

An external gear set

Velocity ratio: Modeling gears as rolling cylinders:

External gear set

negative sign

Surface *velocity* at the contact point, or *pitch point*, is the same:

$V_p = V_{in} = V_{out}$; recall: $V = r\omega$

$V_p = V_{in} = r_{in}\omega_{in} = V_{out} = -\, r_{out}\omega_{out}$

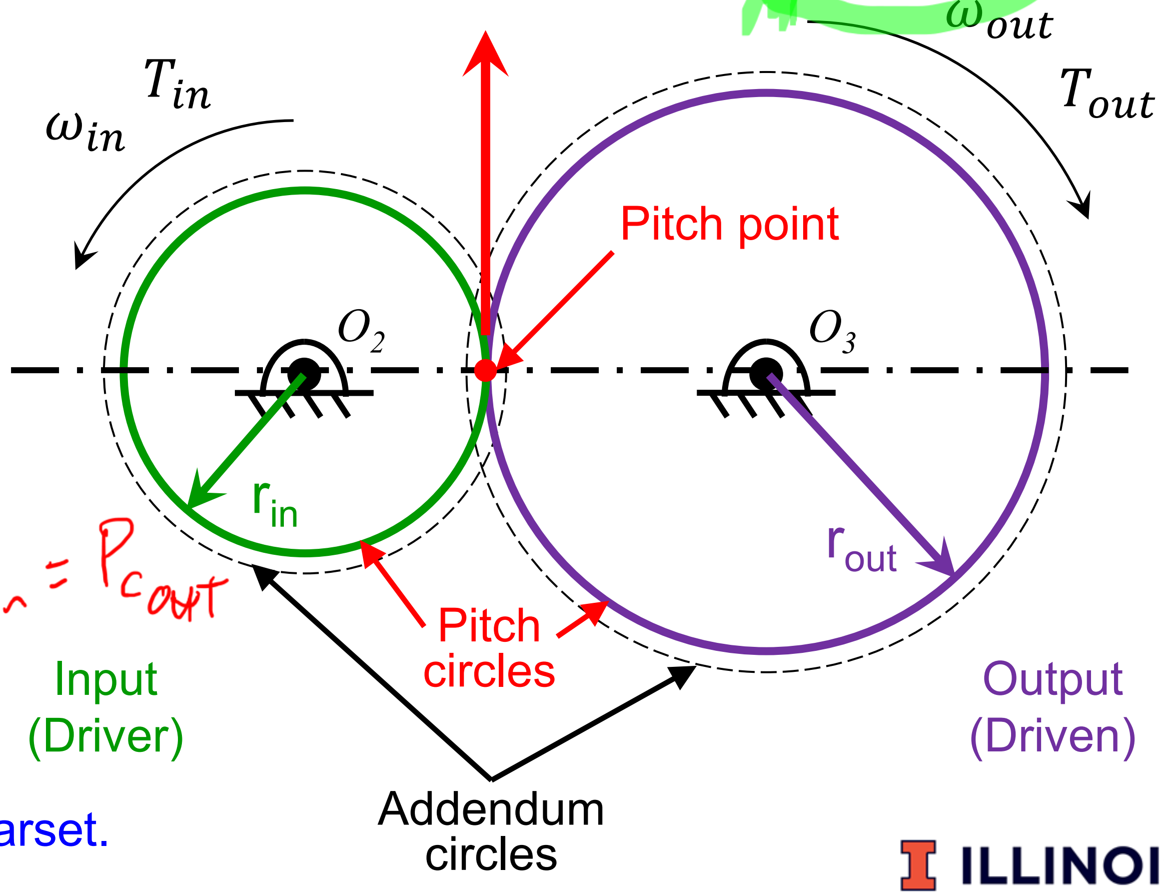
Given:

$r_{in} = 4$ cm $r_{out} = 6$ cm $\omega_{in} = 90$ rpm

$$\omega_{out} = ? \quad = \frac{-r_{in}}{r_{out}}\omega_{in} = \frac{-4}{6}\cdot 90\ rpm = \mathbf{-60\ rpm}$$

$$P_c = \frac{\pi d}{N}$$

$$P_{c_{in}} = P_{c_{out}}$$



Angular velocity ratio (m_V) :

$$\boxed{\therefore\ m_V = \pm\frac{\omega_{out}}{\omega_{in}} = \pm\frac{r_{in}}{r_{out}} = \pm\frac{d_{in}}{d_{out}} = \pm\frac{N_{in}}{N_{out}}}$$

Negative sign if external gearset, Positive sign if internal gearset.

Torque Ratio (m_T)

Tangential *force* at the contact point, or *pitch point*, is the same:

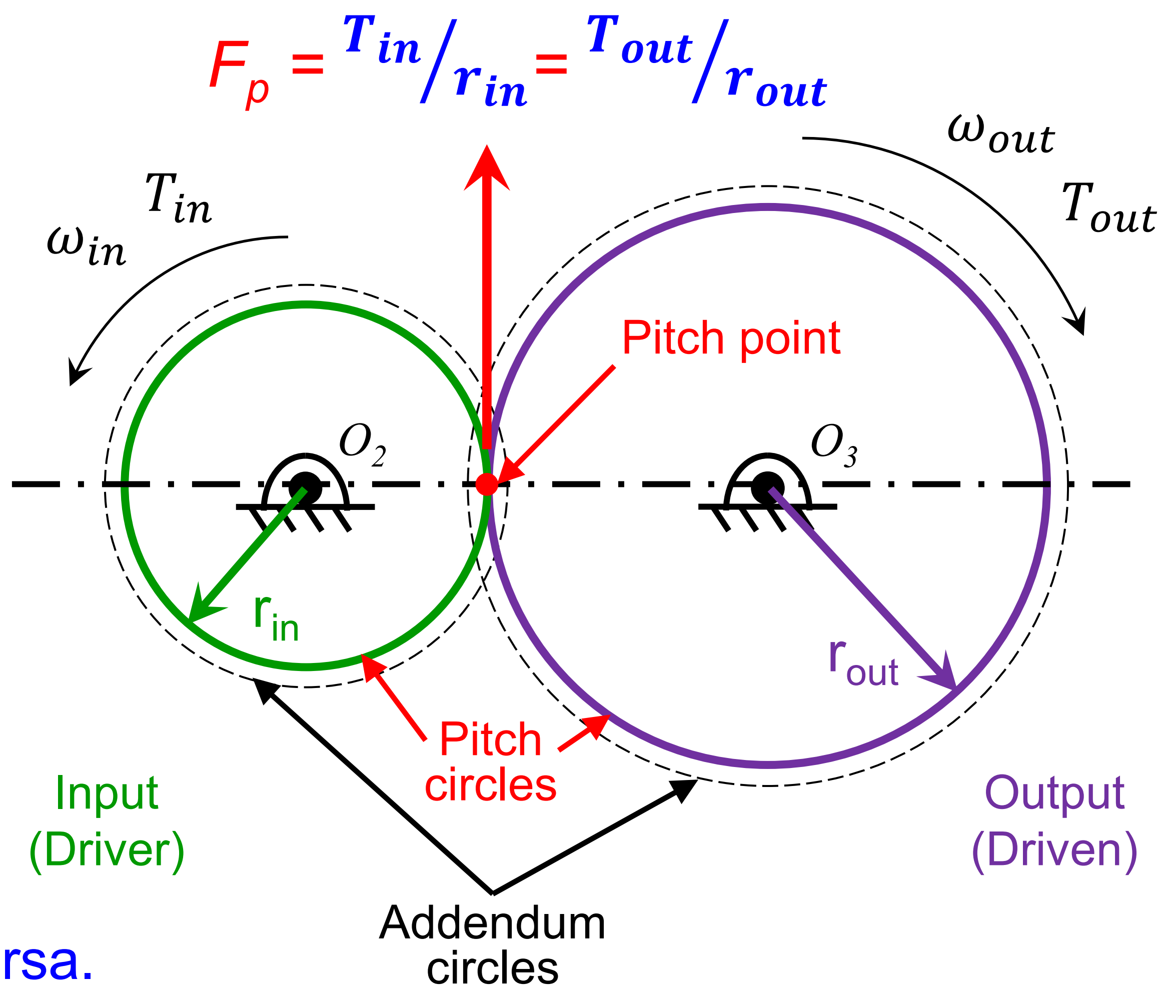
$r_{in} = 4\ cm$ $r_{out} = 6\ cm$ $T_{in} = 1\ Nm$

$T_{out} = \ ? \ = \frac{r_{out}}{r_{in}} T_{in} \ = \ \frac{6}{4} \cdot 1\ Nm =$

$\mathbf{1.5\ Nm}$

Torque ratio (m_T) :

$$\boxed{\therefore m_T = \frac{T_{out}}{T_{in}} = \frac{\omega_{in}}{\omega_{out}} = \frac{1}{m_V}}$$



Gearsets exchange torque for velocity or vice versa.

Power transmission

- Assuming a gearset is highly efficient, then $P_{out} = \sim P_{in}$

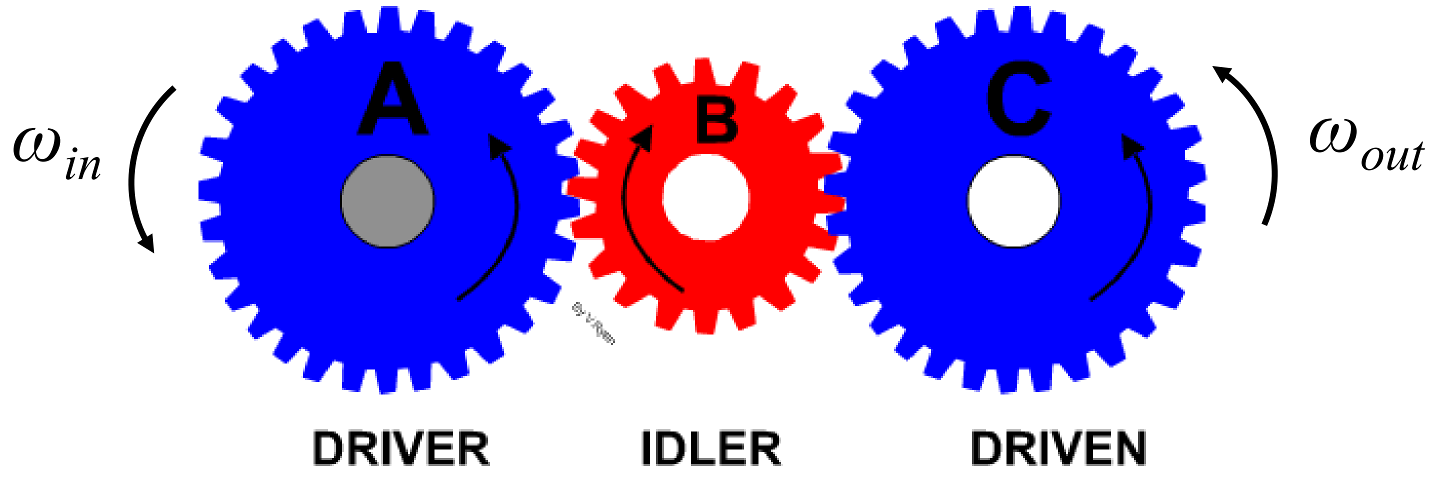
$$P = \vec{T} \cdot \vec{\omega} = \vec{T}_{in} \cdot \vec{\omega}_{in} = \sim \vec{T}_{out} \cdot \vec{\omega}_{out}$$

$$\boxed{|m_V| \equiv \frac{\omega_{out}}{\omega_{in}} = \pm \frac{r_{in}}{r_{out}} = \pm \frac{d_{in}}{d_{out}} = \pm \frac{N_{in}}{N_{out}} = \frac{1}{|m_T|} \equiv \frac{T_{in}}{T_{out}}}$$

Velocity Ratio
Gear Ratio

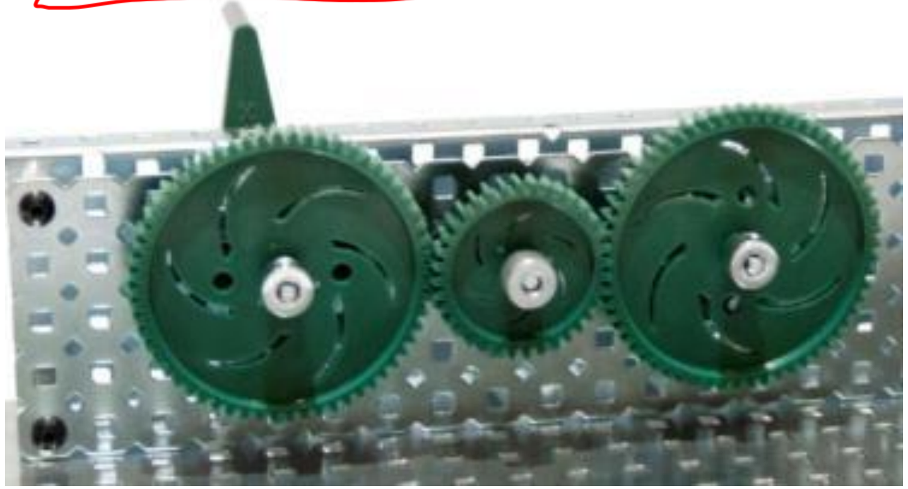
Torque Ratio

Simple gear train



https://technologystudent.com/gears1/gears2.htm

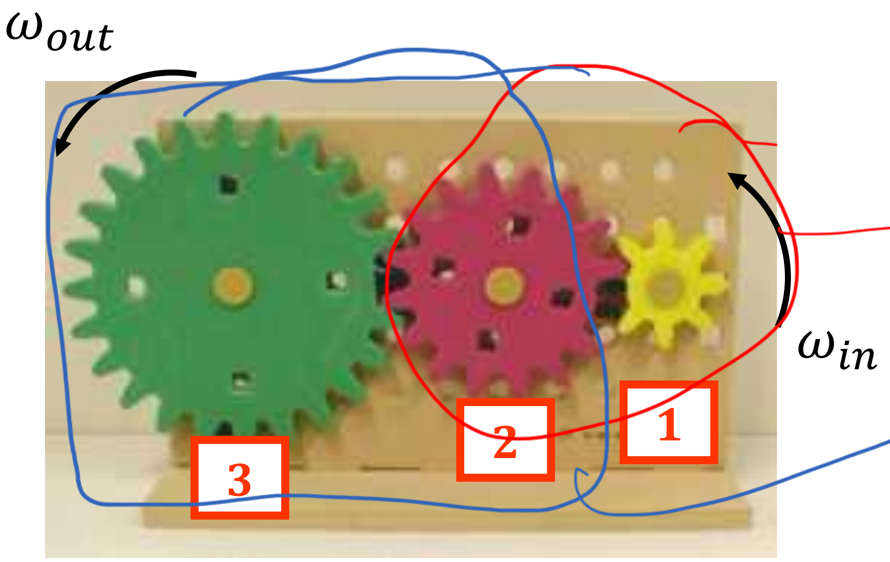
All gears on different shafts



https://quizlet.com/255781174/simple-gear-train-with-idler-diagram/

- Middle gear is called an **idler** gear.
  - Has no contribution to gear ratio.
  - Only changes direction of input to output.
  - Odd # gears: input & output same direction
  - Even # gears: input & output opposite direction
- Typically, do not use more than one idler gear. If want to cover longer distances, then use chain or belt.

Simple gear train



$N_3 = 24$ $N_2 = 16$ $N_1 = 8$

$\omega_{in} = 90$ rpm $\omega_{out} = 30$ rpm

$T_{in} = 1$ Nm $T_{out} = 3$ Nm

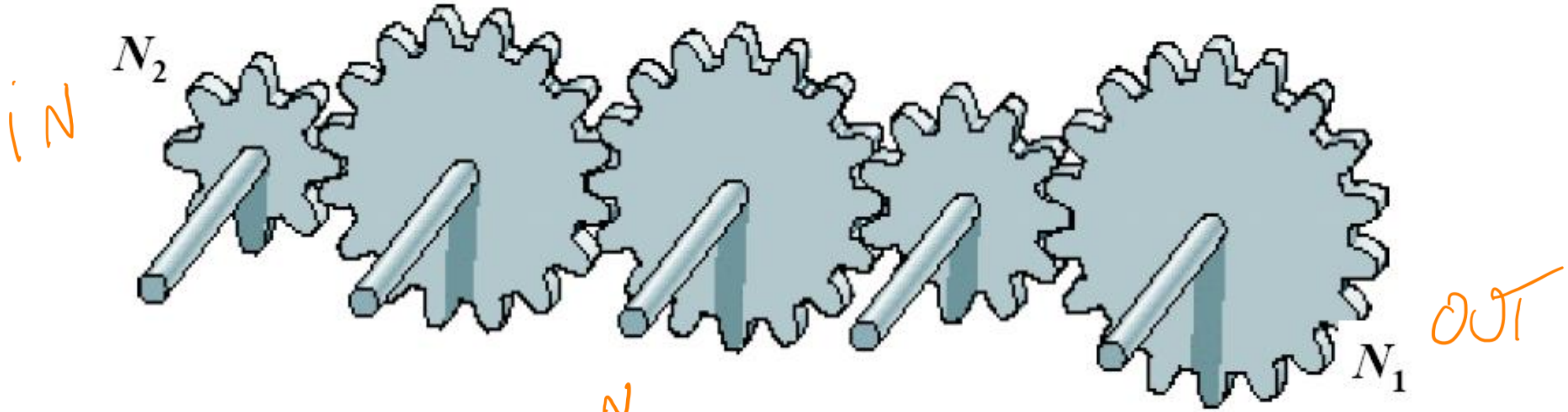
$$|m_V| \equiv \frac{\omega_{out}}{\omega_{in}} = \pm\frac{r_{in}}{r_{out}} = \pm\frac{d_{in}}{d_{out}} = \pm\frac{N_{in}}{N_{out}} = \frac{1}{|m_T|} \equiv \frac{T_{in}}{T_{out}}$$

$$m_V \equiv \frac{\omega_{out}}{\omega_{in}} = \left(-\frac{N_1}{\cancel{N_2}}\right)\left(-\frac{\cancel{N_2}}{N_3}\right) = \left(\frac{N_1}{N_3}\right) = \left(\frac{8}{24}\right) = \frac{1}{3}$$

Negative because external gear trains

$$m_T \equiv \frac{T_{out}}{T_{in}} = \frac{1}{m_V} = 3$$

Longer simple gear trains



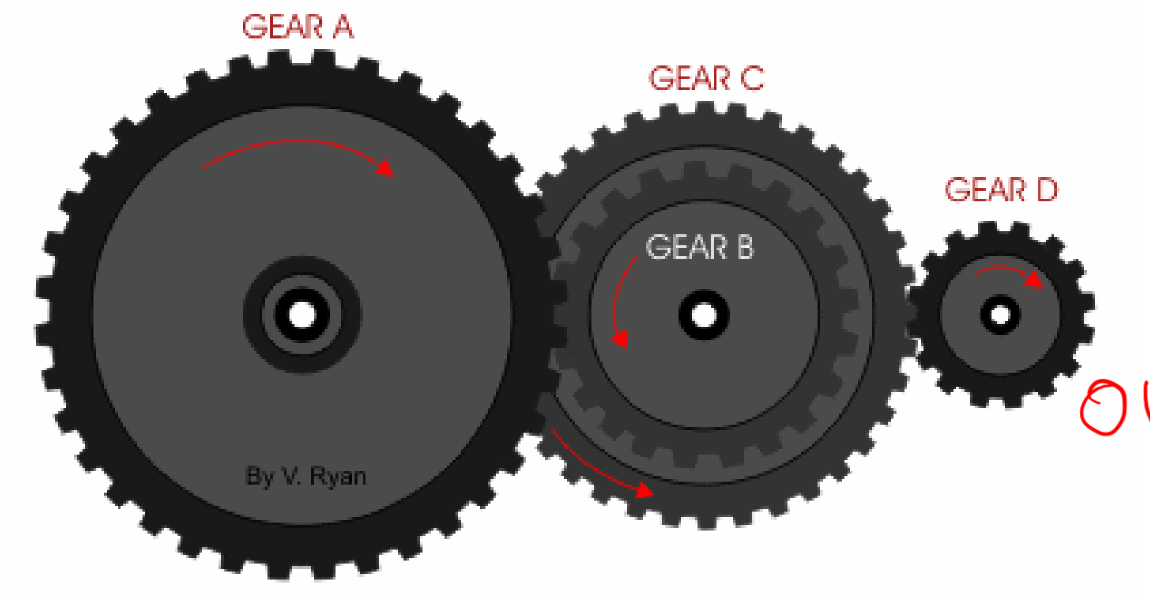
IN

$$\frac{\omega_1}{\omega_2} = -\frac{N_2}{N_1}$$ Even number of gears

OUT

$$\frac{\omega_1}{\omega_2} = \frac{N_2}{N_1}$$ Odd number of gears

Compound gear trains



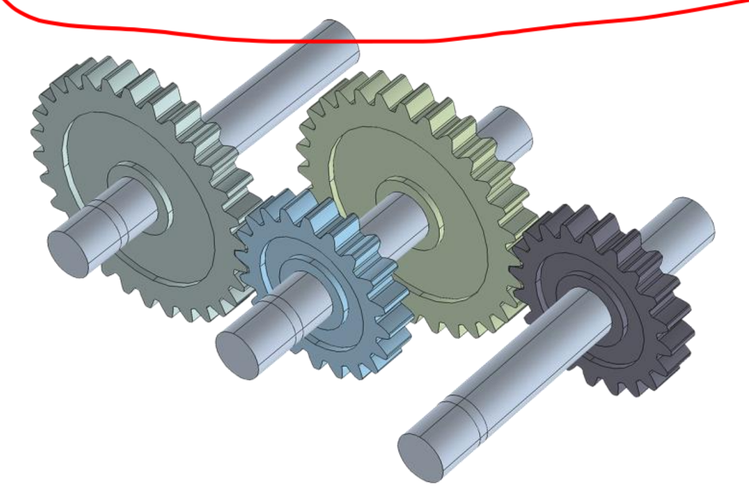
IN

OUT

http://www.technologystudent.com/gears1/gears8.htm

A & C are drivers (in)
B & D are driven (out)

Some gears share same shaft



https://images.app.goo.gl/PwfqXyj7cR33pdzF8

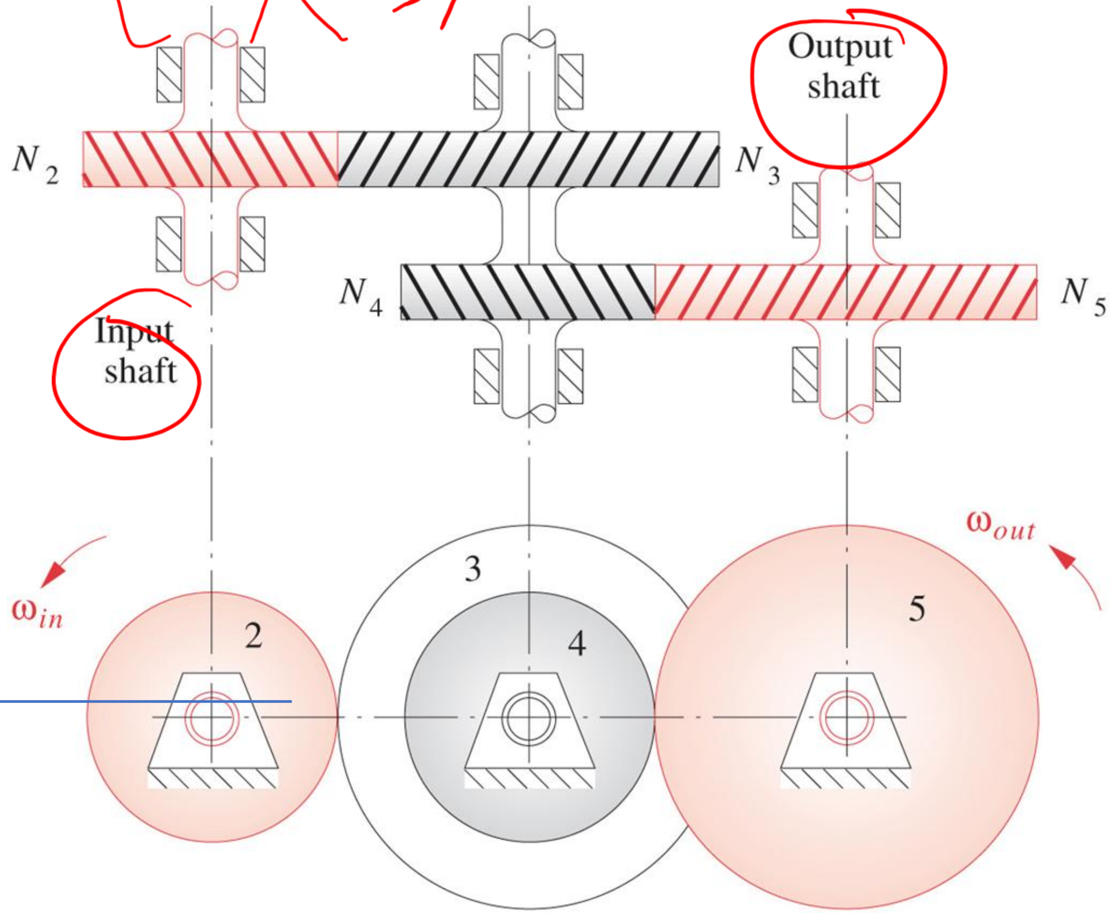
Compound gear trains

$$m_V = \pm\frac{\Pi(N_{in\ or\ "driver\ gears"})}{\Pi(N_{out\ or\ "driven\ gears"})} = -\frac{N_2}{N_3}\cdot -\frac{N_4}{N_5} = +\frac{100\times100}{150\times150} = +0.44$$

Some gears share the same shaft:

2 & 4 are drivers *(in)*
3 & 5 are driven *(out)*

$N_2 = N_4 = 100$
$N_3 = N_5 = 150$



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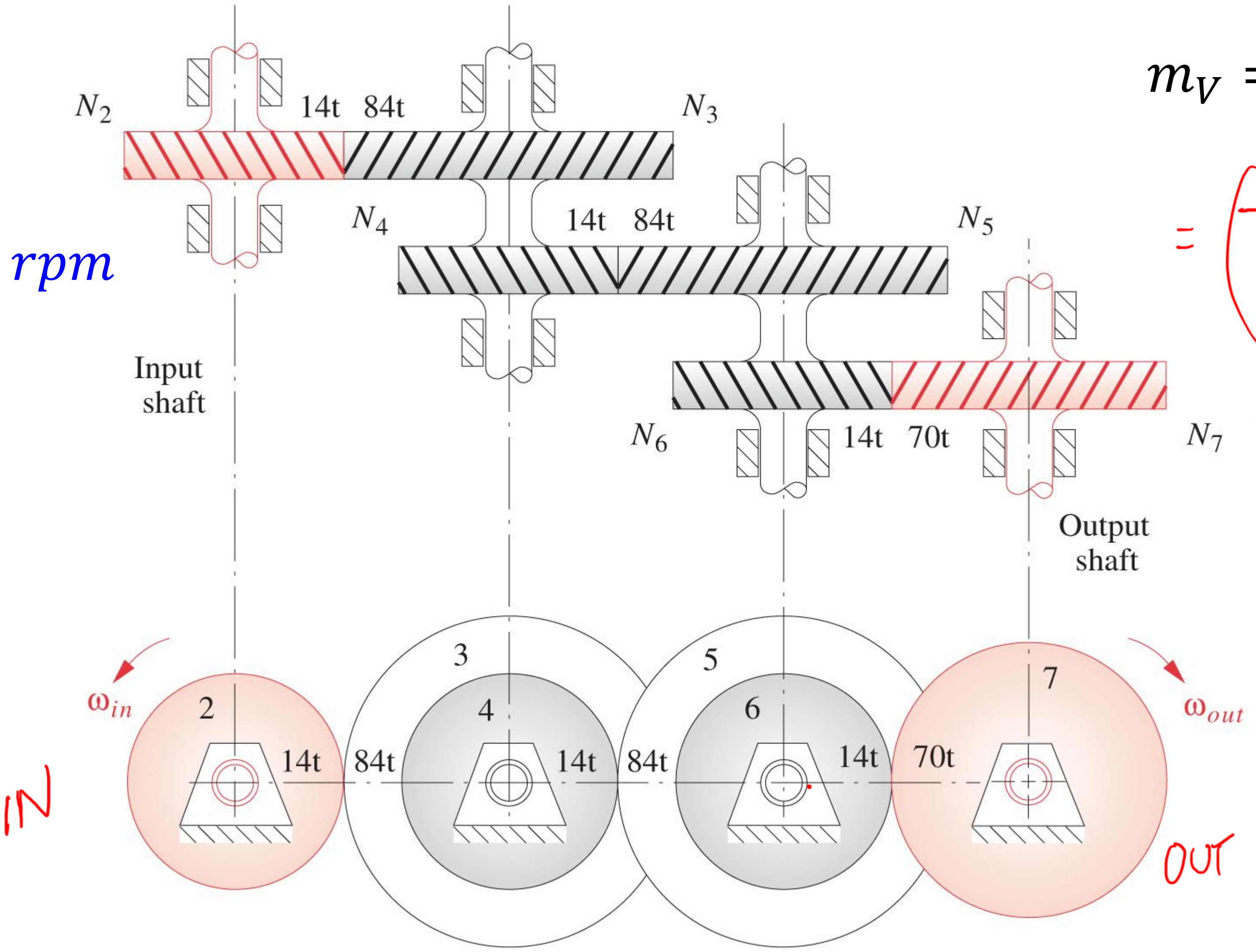
What is the output rpm?

$\omega_{in} = 180\ rpm$

$$m_V = \pm \frac{\Pi(N_{\text{driver gears}})}{\Pi(N_{\text{driven gears}})}$$

$$= \left(-\frac{N_2}{N_3}\right)\left(-\frac{N_4}{N_5}\right)\left(-\frac{N_6}{N_7}\right)$$

$$= -0.0056$$



$$m_V \equiv \frac{\omega_{out}}{\omega_{in}}$$

$$\omega_{out} = -0.0056 * 180$$

$$= -1\ rpm$$