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Homework 00

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STAT 400, Fall 2025, D. Unger

See the Homework Policy on Canvas for full instructions. Homework submissions do not have to be completed on the original assignment sheet (i.e., this file) nor does this file even need to be included. The homework submission does need to be clearly labeled and organized so that graders can match your work with each exercise.

This assignment contains exercises found in Calculus I and II. It is meant as a self-check for your own preparedness in Calculus topics. These exercises represent some of the calculations and skills that are also necessary in our course. I recommend that in addition to completing this assignment that you all find, try, and review similar exercises to these.

Your submission will count as a Participation Assignment (not a HW) and is worth 3 points.

Exercise 1 (First and Second Derivative Tests)

For each function, find the local maximum and minimum values of f.

(a) $f(x) = x^3 - 12x + 1$

(b) $f(x) = \dfrac{x^2}{x^2 + 3}$

(c) $f(x) = xe^x$

(a) $f = x^3 - 12x + 1$

$f' = 3x^2 - 12 \Rightarrow x = \pm 2$

local maximal $x = -2$

local min $x = 2$

(b) $f = \dfrac{x^2}{x^2+3} = 1 - \dfrac{3}{x^2+3}$

$f' = +(x^2+3)^{-2} \times 2x = \dfrac{2x}{(x^2+3)^2} = 0 \Rightarrow x = 0$

local min $x = 0$

(c) $f = xe^x$

$f' = (x+1)e^x = 0 \Rightarrow x = -1$

$\therefore$ local min : $x = -1$

Exercise 2 (Methods of Integration)

Evaluate the definite integral.

(a) $\int_{-1}^{0} (2x - e^x)\ dx$ $= x^2 - e^x \Big|_{-1}^{0} = 0 - 1 - \left(e^0 - e^{-1}\right) = \frac{1}{e} - 2 \approx -1.632$

(b) $\int_{0}^{1} x^2(1 + 2x^3)^5\ dx = \frac{(1+2x^3)^6}{6 \times 6}\Big|_{0}^{1} = \frac{182}{9} \approx 20.222$

(c) $\int_{1}^{2} \frac{\ln x}{x^2}\ dx$ $= \int \ln x \cdot \left(\frac{1}{x^2} dx\right) = \ln x\left(-\frac{1}{x}\right)\Big|_1^2 - \int\left(-\frac{1}{x}\right)\frac{1}{x} dx$
$= -\frac{\ln 2}{2} + \int \frac{1}{x^2} dx = \frac{-1}{x}\Big|_1^2 - \frac{\ln 2}{2} = \frac{1 - \ln 2}{2} \approx 0.153$

(d) $\int_{0}^{\infty} xe^{-5x}\ dx$ $\left(-\frac{1}{5}x - \frac{1}{25}\right)e^{-5x}\Big|_0^\infty = 0 + \frac{1}{25}e^0 = \frac{1}{25} = 0.040$

Exercise 3 (Series)

Find the sum of the series.

(a) $\sum_{k=1}^{\infty} 5\left(\frac{2}{3}\right)^{k-1}$ $= 5 \times \frac{1 - 0}{1 - \frac{2}{3}} = 5 \times 3 = 15$

(b) $\sum_{k=1}^{\infty} \frac{2^{2k+1}}{5^k}$ $= \sum_{1}^{\infty} \frac{4^k}{5^k} \times 2 = 2 \times \frac{4}{5} \times \frac{1 - 0}{1 - \frac{4}{5}} = 8$

(c) $\sum_{k=0}^{\infty} \frac{3^k}{5^k \cdot k!}$ $= \sum_{0}^{\infty} \frac{\left(\frac{3}{5}\right)^k}{k!} = e^{\frac{3}{5}} \approx 1.822$