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Homework 03

Due Tuesday, September 16, 11:59pm

STAT 400, Fall 2025, D. Unger

Exercise 1

A University of Illinois student joined Orange Krush as a freshman and attended as many Illini basketball home games as they could. When the student attended the game in person, the won 82% of the time. When the student did not attend, they won 92% of the time. In total, the student attended 70% of the home games during their four years as an undergraduate.

Exercises

(a) Given that the Illini won the game, what is the probability that this student was in attendance at that game?

(b) Given that the Illini lost the game, what is the probability that this student was in attendance at that game?

Solutions

Exercise 2

At a hospital's Emergency Wing, patients are classified upon their initial diagnosis with 0.20 being critical, 0.30 being serious, and 0.50 being stable. Of those determined to be critical, 0.30 require a stay of more than 24 hours; of the serious cases, the proportion is 0.10; and of the stable cases, it is only 0.01.

Exercises

Given that a patient stays more than 24 hours, …

(a) what is the probability that the patient was initially classified as critical?

(b) what is the probability that the patient was initially classified as serious?

(c) what is the probability that the patient was initially classified as stable?

Solutions

EX1 A Illinois win the game.

B Student attend.

$P(A|B) = 0.82$ $P(B)=0.7$

$P(A|\bar{B}) = 0.92$ $P(\bar{B}) = 0.3$

(1) $P(A\cap B) = P(A|B)\times P(B) = 0.82\times 0.7$

$P(A) = P(A\cap B) + P(A\cap \bar{B})$

$= P(A|B)\times P(B) + P(A|\bar{B})\times P(\bar{B})$

$= 0.82\times 0.7 + 0.92\times 0.3$

$= 0.85$

$P(B|A) = \dfrac{P(A\cap B)}{P(A)} = \dfrac{0.82\times 0.7}{0.85} = 0.675$

(2) $P(\bar{A}) = 0.15$

$P(B\cap \bar{A}) = P(B) - P(B\cap A)$

$= 0.7 - 0.82\times 0.7 =$

$P(B|\bar{A}) = \dfrac{P(B\cap \bar{A})}{P(\bar{A})} = \dfrac{0.72 - 0.82\times 0.7}{0.15} = 0.840$

EX2 A_1 critical 0.2 A_2 serious 0.3 A_3 stable 0.5

B require a stay of more than 24 hours.

$P(B|A_1) = 0.3$ $P(B|A_2) = 0.1$ $P(B|A_3) = 0.01$

$P(B) = P(B|A_1)\times P(A_1) + P(B|A_2)\times P(A_2) + P(B|A_3)\times P(A_3)$

$= 0.3\times 0.2 + 0.3\times 0.1 + 0.5\times 0.01 = 0.095$

$P(B\cap A_1) = P(B|A_1)\times P(A_1) = 0.06$

$P(B\cap A_2) = P(B|A_2)\times P(A_2) = 0.03$

$P(B\cap A_3) = P(B|A_3)\times P(A_3) = 0.005$

(1) $P(A_1|B) = \dfrac{P(B\cap A_1)}{P(B)} = \dfrac{0.06}{0.095} = 0.632$

(2) $P(A_2|B) = \dfrac{P(B\cap A_2)}{P(B)} = \dfrac{0.03}{0.095} = 0.316$

(3) $P(A_3|B) = \dfrac{P(B\cap A_3)}{P(B)} = \dfrac{0.005}{0.095} = 0.0526$

Exercise 3

A certain screening process is used to detect the potential presence of cancer. For people with this form of cancer, there are approximately 16% *false negatives* among all screenings. That is, given a person has this form of cancer, the probability they will test negative is 0.16. For people without the cancer, there are approximately 19% *false positives* among all screenings. In the United States, about 8 in 100,000 people have this cancer.

Exercises

(a) What is the probability that a randomly selected person in the U.S. has this form of cancer if the screening returns a positive result?

(b) What is the probability that a randomly selected person in the U.S. does not have this form of cancer if the screening returns a negative result?

(c) Based on your answers to parts a and b, does this screening process seem to be an effective method for detecting this form of cancer?

Solutions

Exercise 4

Let a random experiment be the casting of a pair of fair six-sided dice, and let X equal the maximum of the two outcomes.

Exercises

(a) What is the support of X, that is, the sample space of X?

(b) Find the pmf $f(x)$ of X.

Now let Y equal the range of the two outcomes (i.e., the absolute value of the difference of the largest and smallest outcomes).

(c) What is the support of Y?

(d) Find the pmf $g(y)$ of Y.

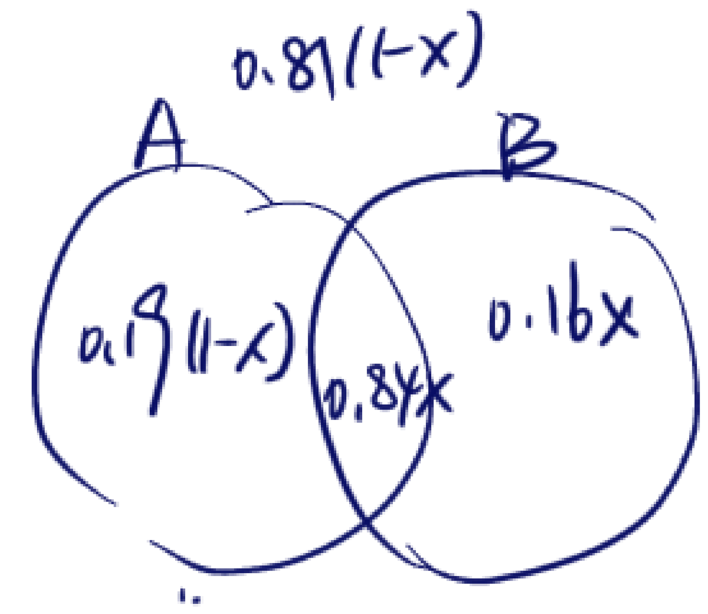
Solutions

Exercise 5

Let X denote the number of times Bobbie goes to get a calzone at DP Dough in a given week. Suppose X has the following probability distribution.

$\boxed{Ex3}$ Test positive : A

Have decease : B



$P(\bar{A}|B) = 0.16 \qquad P(A|\bar{B}) = 0.19$

let $P(B) = 0.008\% = 8\times10^{-5} \triangleq x$ $\quad P(A|B) = 0.84 \quad P(\bar{A}|\bar{B}) = 0.81$

$P(B)\times P(\bar{A}|B) = P(\bar{A}\cap B) = 0.16x$

$P(\bar{B})\times P(A|\bar{B}) = P(A\cap\bar{B}) = (1-x)\times 0.19$

$P(\bar{B})\times P(\bar{A}|\bar{B}) = (1-x)\times 0.81 = P(\bar{A}\cap\bar{B})$

$P(A\cap B) = P(A|B)\times P(B) = 0.84x$

$P(A) = 0.19(1-x) + 0.84x$

(1) $P(B|A) = \dfrac{P(A\cap B)}{P(A)} = \dfrac{0.84x}{0.19(1-x)+0.84x} = 0.000354$

(2) $P(\bar{B}|\bar{A}) = \dfrac{P(\bar{A}\cap\bar{B})}{P(\bar{A})} = \dfrac{(1-x)\times0.81}{1-[0.19(1-x)-0.84x]} = 0.99984 \approx 1.00$

(3) Yes. because it is true when 10000 test only wrong 1~3.

Or we can say that without cancer to be told is high as 0.03% than it should be no but not has cancer to to be told not is very correct. 1.00.

$\boxed{Ex4}$

(1) $S_x = \{1.2.3.4.5.6\}$.

(2) $x=1$: (1.1) $\frac{1}{36}$

$x=2$: (1.2) (2.1) (2.2) $\frac{3}{36}$

$x=3$: (1.3) (2.3) (3.3) (3.2) (3.1) $\frac{5}{36}$

$x=4$: (1.4) (2.4) (3.4) (4.4) (4.3) (4.2) (4.1) $\frac{7}{36}$

$x=5$: (1.5) (2.5) (3.5) (4.5) (5.5) (5.4) ⋯ (5.1) $\frac{9}{36}$

$x=6$: (1.6) (2.6) ⋯ (6.2) (6.1) $\frac{11}{36}$

x	1	2	3	4	5	6
$f(x)$	$\frac{1}{36}$	$\frac{1}{12}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{1}{4}$	$\frac{11}{36}$

13) $S_Y = \{0, 1, 2, 3, 4, 5\}$

$Y=0$ (1,1) (2,2) (3,3) (4,4) (5,5) (6,6) $\frac{6}{36}$

$Y=1$ (1,2)(2,3) (3,4) (4,5) (5,6) (6,5) (5,4) (4,3) (3,2) (2,1) $\frac{10}{36}$

$Y=2$ (1,3)(2,4) (3,5) (4,6) (6,4) (5,3) (4,2) (3,1) $\frac{8}{36}$

$Y=3$ (1,4) (2,5) (3,6) (6,3) (5,2) (4,1) $\frac{6}{36}$

$Y=4$ (1,5) (2,6) (6,2) (5,1) $\frac{4}{36}$

$Y=5$ (1,6) (6,1) $\frac{2}{36}$

Y	0	1	2	3	4	5
$f(Y)$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

x	0	1	2	3	4	5
$f(x)$	0.10	0.25	0.30	0.15	0.10	0.10

Exercises

(a) Verify that $f(x)$ is a valid probability mass function by verifying that the three properties of the definition of a pmf are true.

(b) Find the probability that Bobbie will get a calzone at least three times in any one week.

(c) Bobbie's friends are worried that they spend too much money on calzones. The friends recommend that Bobbie should have calzones no more than once every two weeks. What is the probability that Bobbie will follow that advice and have calzones no more than once in any two week period?

(You may assume that the number of calzone visits from one week to the next week are independent.)

(d) Bobbie's friends get extra worried and begin to spy on Bobbie. They witness Bobbie buying calzones on two different days last week. What is the probability that Bobbie actually got calzones more than three times last week?

Solutions

Exercise 6

Consider a random variable X with the probability mass function

$$f(x) = \frac{c}{(5/3)^x}, \qquad \text{for } x = 0, 1, 2, \ldots.$$

Exercises

(a) Find the value of c that will make this a valid pmf.

(b) What is the probability that $X = 2$?

(c) What is the probability that $X \geq 5$?

(d) Find $F(2)$.

Solutions

$\boxed{Ex5}$

(1). ① $\forall x\ f(x)>0$ ✓ 0.1, 0.25, 0.3, 0.15, 0.1, 0.1 ...

② $\sum f(x)=1$ ✓ $\because 0.1+0.25+0.3+0.15+0.1+0.1=1$

③ Additable. $P(X\in A)=\sum_A f(x)$

The probability of an event A is the sum of the probabilities of the individual outcomes in A

(2) $P(X\geq 3) = P(x=3)+P(x=4)+P(x=5)$

$= 0.15+0.1+0.1 = 0.35$

(3). X is first week spending on calzones times.

Y is second week ...

$P(X\cdot Y\leq 1) = P(X=1, Y=0)+P(X=0, Y=1)+P(X=0, Y=0)$

$= 0.1\times 0.25+0.25\times 0.1+0.1\times 0.1$

$= 0.06$

(4) $P(X>3\mid X\geq 2) = \dfrac{P(X>3)}{P(X\geq 2)} = \dfrac{0.2}{0.3+0.15+0.1+0.1} = 0.308$

$\boxed{Ex6}$

(1). $1=\sum f(x) = c\cdot\left[1+\frac{3}{5}+\left(\frac{3}{5}\right)^2+\cdots\right] = c\cdot\dfrac{1}{1-\frac{3}{5}} = \dfrac{5}{2}\cdot c \Rightarrow c=0.4$

(2). $P(X=2) = f(2) = \frac{2}{5}\times\left(\frac{3}{5}\right)^2 = \frac{18}{125} = 0.144$

(3) $P(X\geq 5) = f(5)+f(6)+\cdots$

$= \frac{2}{5}\times\left[\left(\frac{3}{5}\right)^5+\left(\frac{3}{5}\right)^6+\cdots\right]$

$= \frac{2}{5}\times\left(\frac{3}{5}\right)^5\times\left[1+\left(\frac{3}{5}\right)+\left(\frac{3}{5}\right)^2+\cdots\right]$

$= 0.0778$

(4) $F(2) = f(0)+f(1)+f(2) = \frac{2}{5}\times\left(1+\frac{3}{5}+\left(\frac{3}{5}\right)^2\right) \approx 0.784$