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Homework 02

Due Tuesday, September 9, 11:59pm

STAT 400, Fall 2025, D. Unger

Exercise 1

If P(A) = 0.83, P(B) = 0.47, and P(A' ∩ B') = 0.12, then find the following. Cite the definitions, theorems, properties, etc. that you are using.

(a) P(A ∪ B)

(b) P(A ∩ B)

(c) P(A | B)

(d) P(B | A)

Exercise 2

It is known that 30% of all the students at Faber College live off campus. Suppose that 43% of all the students are females. Among all female students, 25% live off campus.

(a) What is the probability that a randomly selected student is a female and lives off campus?

(b) What is the probability that a randomly selected student either is a female or lives off campus, or both?

(c) Given a student lives off-campus, what is the probability they are female?

Exercise 3

During Spring Break at Faber College, students are invited to participate in a service trip to rebuild homes in Haiti affected by tropical storms. Of the students going on the trip, 60% percent are seniors; 25% are juniors; 10% are sophomores; and the rest are freshmen. Experience with manual labor for home building such as carpentry and painting is not required to go on the service, but several students have such experience. Ninety percent of the seniors have building experience, as do 80% of the juniors, 50% of the sophomores, and 20% of the freshmen.

(a) If a student on the trip has building experience, what is the probability that this student is a senior?

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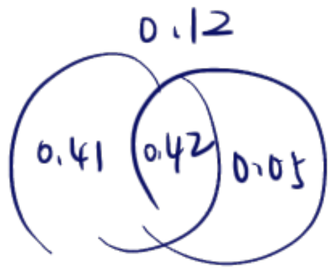
Ex1



(a) $P(A \cup B) = 1 - P(A' \cap B') = 0.88$

(b) $P(A \cap B) = -P(A \cup B) + P(A) + P(B) = 0.42$

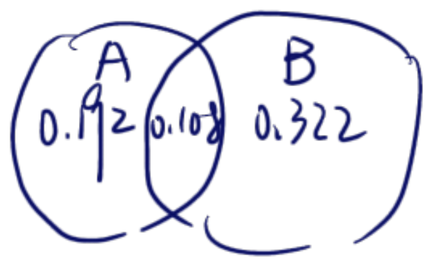
(c) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.42}{0.47} = 0.894$

(d) $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.42}{0.83} = 0.506$

Ex2 A: live in campus = 30%

B: female = 43%

$P(A|B) = 25\%$



(a) $P(A \cap B) = P(B) \times P(A|B) = 25\% \times 43\% \approx 0.108$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.73 - 0.1075 = 0.623$

(c) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.1075}{0.3} = 0.358$

(b) If a student on the trip has no building experience, what is the probability that this student is not a senior?

(c) If a student on the trip is not a senior, what is the probability that this student has no building experience?

(d) What proportion of the underclassmen (freshmen and sophomores) attending the service trip have building experience?

(e) The college administration encourages students to go on the trip. The Chancellor's Office will recognize seniors and those with building experience with a special service leadership award. Find the proportion of the students on the trip who either are seniors, or have building experience, or both.

(f) Are the events "a student on the trip is a senior" and "a student has building experience" independent? Justify your answer for full credit. (No credit will be given if there is only a yes/no response with no explanation.)

(g) Are events "a student on the trip is a junior" and "a student has building experience" independent? Justify your answer for full credit. (No credit will be given if there is only a yes/no response with no explanation.)

(h) Are events "a student on the trip is a junior" and "a student has building experience" disjoint? Justify your answer for full credit. (No credit will be given if there is only a yes/no response with no explanation.)

Exercise 4

Amazon Prime delivers a package to the wrong address with probability 0.02 on any delivery. Suppose that each delivery is independent of all the others.

(a) One driver has many packages to deliver. What is the probability that the fifth package they deliver is the first one to go to a wrong address.

(b) In their second hour, a driver had 7 packages to deliver. What is the probability that at least one package was delivered to the wrong address?

Exercise 5

In a statistics class, Harry, Ron, Hermione, and Neville are working independently on an in-class quiz. The quiz is challenging enough that Hermione has an 85% probability of passing it successfully. Harry has only a 60% probability of successfully passing the quiz. Ron's probability is 40%. Neville constantly feels intimidated by the instructor, Professor Snape, and as a result has only a 20% chance of successfully passing the quiz.

Ex3.

A_1: seniors A_2: juniors A_3: sophemore A_4: freshman.

0.6 0.25 0.1 0.05

B with building experience.

$P(B|A_1)=0.9$ $P(B|A_2)=0.8$ $P(B|A_3)=0.5$ $P(B|A_4)=0.2$

(a) $P(B)=\sum_{i=1}^{4} P(B|A_i)\times P(A_i) = 0.9\times 0.6+0.25\times 0.8+0.1\times 0.5+0.05\times 0.2 = 0.8$

$P(A_1\cap B)=P(B|A_1)\times P(A_1)=0.54$

$\therefore P(A_1|B)=\frac{P(A_1\cap B)}{P(B)}=\frac{0.54}{0.8}=0.675$

(b) $P(B')=0.2$ $P(A_1\cap B')=P(B'|A_1)\times P(A_1)=0.1\times 0.6=0.06$

$\therefore P(A_1|B')=\frac{P(A_1\cap B')}{P(B')}=\frac{0.06}{0.2}=0.3$ $P(\bar{A}_1|B')=1-P(A_1|B')=0.7$

(c) $P(\bar{B}\cap\bar{A}_1)=P(\bar{A}_2)\times P(\bar{B}\cap A_2)+P(A_3)\times P(\bar{B}|A_3)+P(A_4)\times P(\bar{B}|A_4)=0.04+0.05+0.05=0.14$

$P(\bar{B}|\bar{A}_1)=\frac{P(\bar{B}\cap\bar{A}_1)}{P(\bar{A}_1)}=\frac{0.14}{0.4}=0.35$

(d) $P(A_3\cap B)+P(A_4\cap B)=P(B|A_3)\times P(A_3)+P(B|A_4)\times P(A_4)=0.1\times 0.5+0.05\times 0.2=0.06$

$P(B|A_3\cap A_4)=\frac{P(A_3\cap B)+P(A_4\cap B)}{P(A_3)+P(A_4)}=\frac{0.06}{0.15}=0.4$

(e) $P(A_1\cup B)=P(A_1)+P(B)-P(A_1\cap B)=0.6+0.8-0.6\times 0.9=0.86$

(f) Not independent

$\because P(A_1\cap B)=0.54 \neq P(A_1)\times P(B)=0.48$

(g) Independent

$P(A_2\cap B)=0.25\times 0.8=0.2=P(A_2)\times P(B)$

$=P(B|A_2)\times P(A_2)$

(h) Not disjoint

Because there are much people who is $A_2\cap B$ so it must be joint.

Ex 4

all four right five right.

(a) $p(\text{five is first wrong}) = (0.98)^4 \times 0.02$

$= 0.0184$

(b) $p(\text{at least one wrong}) = 1 - p(\text{none wrong})$

$= 1 - (0.98)^7$

$= 0.132$

Ex 5

Harry A_1 | Hermione A_2 | Ron A_3 | Neville A_4

$p(A_1) = 0.6$ | $p(A_2) = 0.85$ | $p(A_3) = 0.4$ | $p(A_4) = 0.2$

$p(\bar{A}_1) = 0.4$ | $p(\bar{A}_2) = 0.15$ | $p(\bar{A}_3) = 0.6$ | $p(\bar{A}_4) = 0.8$

(a) $p(\text{at least one pass}) = 1 - p(\text{none pass}) = 1 - p(\bar{A}_1) \times p(\bar{A}_2) \times p(\bar{A}_3) \times p(\bar{A}_4)$

$= 1 - 0.4 \times 0.15 \times 0.6 \times 0.8$

$= 0.971$

(b) $p(\text{at most one}) = p(\text{none}) + p(\text{one})$

$= p(\bar{A}_1) \times p(\bar{A}_2) \times p(\bar{A}_3) \times p(\bar{A}_4) + \sum_{sys} p(\bar{A}_1) \times p(\bar{A}_2) \times p(A_3) \times p(A_4)$

$= 0.4 \times 0.15 \times 0.6 \times 0.8 + 0.4 \times 0.15 \times 0.6 \times 0.2 + 0.6 \times 0.15 \times 0.6 \times 0.8 +$

$0.85 \times 0.4 \times 0.6 \times 0.8 + 0.4 \times 0.4 \times 0.15 \times 0.8$

$= 0.262$

(a) Find the probability that at least one of the four students will be able to successfully pass the quiz.

(b) Find the probability that at most one of the four students will be able to successfully pass the quiz.

Exercise 6

Maria and Sophie challenge each other to a chess match. They will play games of chess again and again until one of them wins two games in a row. Maria will get the first move in the first game. Sophie gets the first move in the second game. After that, the first move continues to alternate between them. Maria has a 60% chance of winning a game when she moves first, but only a 30% chance of winning a game if Sophie moves first. Assume that all games are independent. Also assume that every individual game ends with one of the two players winning (i.e., there are no draws/stalemates).

Find the probability that Maria wins the overall match. That is, find the probability that Maria is the first to win two games in a row.

Ex6

A means Maria first and win $= 0.6$

B means Sophia first and win $= 0.7$

Maria win $\begin{cases} A\bar{B} \\ ABA\bar{B} \\ ABABA\bar{B} \\ \cdots \end{cases}$ $P(A\bar{B}) + P(ABA\bar{B}) + P(ABABA\bar{B}) + \cdots$

$= 0.6 \times 0.3 \times (1 + 0.18 \times 0.7 + (0.18 \times 0.7)^2 + \cdots)$

$= 0.18 \times \frac{1}{1-0.42} = 0.3103$

$\begin{cases} \bar{A}\bar{B}A \\ \bar{A}\bar{B}A\bar{B}A \\ \bar{A}\bar{B}A\bar{B}A\bar{B}A \\ \cdots \end{cases}$ $P(\bar{A}\bar{B}A) + P(\bar{A}\bar{B}\bar{A}\bar{B}A) + \cdots$

$= 0.4 \times 0.3 \times 0.6 \times (1 + 0.4 \times 0.3 + (0.4 \times 0.3)^2 + \cdots)$

$= 0.072 \times \frac{1}{1-0.4 \times 0.3} = 0.0818$

$\therefore P(\text{Total Maria win}) = 0.392$