Week 3 - Discussion Exercises

STAT 400, D. Unger

Table of Contents

Exercise 1 (Bayes Theorem)

A certain die-hard football fan watched as many Chicago Bears games as they could over the last ten years. When the fan watched the Bears, they lost 90% of the time. When the fan did not watch them play, they won 70% of the time. In total, the fan watched about 80% of the games over this span.

(a) Find the probability that the Bears won any (randomly chosen) game.

(b) Given that the Bears lost a game, what is the probability that the die-hard fan watched it?

Exercise 2 (Bayes Theorem)

A doctor is concerned about the relationship between blood pressure and irregular heartbeats. Among her patients, she classifies blood pressures as high, normal, or low and heartbeats as regular or irregular and finds that

   a. 16% have high blood pressure;

   b. 19% have low blood pressure;

   c. 17% have an irregular heartbeat;

   d. of those with an irregular heartbeat, 35% have high blood pressure; and

   e. of those with normal blood pressure, 11% have an irregular heartbeat.

What percentage of her patients have a regular heartbeat and low blood pressure?

Exercise 3 (Discrete Random Variables)

A fair coin is tossed three times. Let X be the difference between the number of heads and the number of tails in the three tosses.

(a) What is the sample space of X?

(b) What is the pmf of X?

(c) What is the CDF of X?

Exercise 4 (Discrete Random Variables)

Let a random experiment be the casting of a pair of fair six-sided dice and let X equal the minimum of the two outcomes.

(a) What is the support of X, that is, the sample space of X?

(b) Find the pmf $f(x)$ of X.

(c) Find the CDF $F(x)$ of X.

Exercise 5 (Discrete Random Variables)

A pocket contains five billiard balls numbered 1 to 5. Jake reaches in and pulls out two of them randomly.

(a) How many different subsets of two billiards are there in this pocket?

(b) Let X be the larger of the two numbers drawn. What is the pmf of X?

Exercise 6 (Discrete Random Variables)

A roulette wheel used in a U.S. casino has 38 slots numbered 1 to 36 with two others numbered 0 and 00. Of the slots number 1 to 36, 18 are red and 18 are black. The 0 and 00 are both green. A ball is rolled around the wheel and ends up in one of the slots with equal probability. In each of the following scenarios, the player bets $1.

- If the player wins their bet, they win the amount given as well as have their $1 bet returned to them.
- If the player loses their bet, they lose their $1 bet.

(a) Suppose that a player bets on Red, and let X be an indicator of whether or not the ball lands on Red. What is the support of X? What is the pmf of X?

(b) If the ball lands in Red, the player wins $1. If not, they lose $1. Let Y be the net winnings in dollars for a single bet on Red. What is the support of Y? What is the pmf of Y?

(c) Suppose that a player bets on the Range from 25-36. If the ball lands on any number from 25 to 36, the player wins $2. If not, they lose $1. Let W be the net winnings in dollars for a single bet on the Range from 25-36. What is the support of W? What is the pmf of W?