Failure mode EM detailed process.

$$P(X,Z,T) = \prod_{l=1}^{L} P(X|T,Z)\cdot P(T|Z)\cdot P(Z)$$

① $P(Z) = \prod_{k=1}^{K} (\pi_k)^{\rho_{lk}}$

② $P(T|Z) = \prod_{k=1}^{K} \Big(\underbrace{\prod_{m=1}^{M} N(T_{lm}|\mu_{mk},\Sigma_{mk})\cdot N(T_{ly}|\mu_{yk},\Sigma_{yk})}_{P(T_{lk})} \Big)^{\rho_{lk}}$

③ $P(X|T,Z) = \prod_{k=1}^{K}\Big(\underbrace{\prod_{m=1}^{M} N(x_{lm}|\Phi_l T_{lm}, \sigma_{mk}^2 I)\cdot N(y_l|\Phi_l T_{ly}, \sigma_{yk}^2 I)}_{P(X_{lk})}\Big)^{\rho_{lk}}$

后验: $P(z_l=k|x_l) = \dfrac{P(x_l|z_l=k)P(z_l=k)}{\sum_{k=1}^{K}}$

$\hat{\rho}_{lk} \leftarrow E(\rho_{lk}) = 1\cdot P(z_l=k|x_l) + 0\cdot P(\ \ ) + 0\cdot P(\ \ ) \cdots = P(z_l=k|x_l)$ (具体略,和之前一样)

$P(T_{lm}|x_{l1}, x_{l2} \cdots x_{lm}, y_l, z_l=k)$ →需要 $x_{l1}, x_{l2} \cdots y_l$ 独立,但应该能这样

$= P(T_{lm}|x_{lm}, z_l=k) \sim N(\hat{T}_{lmk}, \hat{\Sigma}_{lmk})$ (和之前一样,换下标就行)

E step

$$P(X,Z,T) = \prod_l \Big(\prod_k \pi_k\cdot P(T_{lk})\cdot P(X_{lk})\Big)^{\rho_{lk}}$$

$$-2\ln P(X,Z,T) = \sum_l\sum_k \rho_{lk}\cdot\Big(-2\ln\pi_k + \sum_{m=1}^{M} -2\ln N_T(\ \ ) + -2\ln N_T(\ \ ) + \sum_{m=1}^{M} -2\ln N_x(\ \ ) + -2\ln N_x(\ \ )\Big)$$

以 $\mu_{mk}, \Sigma_{mk}, \sigma^2_{mk}$ 为例，去除无关项

$$= \sum_l \rho_{lk} \Big(const - 2\ln|\Sigma_{mk}|^{-\frac{1}{2}} + (T_{lmk} - \mu_{mk})^T \Sigma_{mk}^{-1} (\quad)$$
$$- 2\ln|\sigma^2_{mk} I|^{-\frac{1}{2}} + (x_{lm} - \Phi_l T_{lm})^T [\sigma^2_{mk} I]^{-1} (\quad) \Big)$$

$$= \sum_l \rho_{lk} \Big(const + \ln|\Sigma_{mk}| + (T_{lmk} - \mu_{mk})^T \Sigma_{mk}^{-1} (\quad)$$
$$+ n_l \cdot \ln \sigma^2_{mk} + \frac{1}{\sigma^2_{mk}} \cdot (x_{lm} - \Phi_l T_{lm})^T (\quad) \Big)$$

$$\frac{\partial E(\quad)}{\partial \mu_{mk}} = \sum_l E(\rho_{lk}) \cdot (-2) E(T_{lmk} - \mu_{mk})^T \Sigma_{mk}^{-1} = 0$$

$$\Rightarrow \hat{\mu}_{mk} = \frac{\sum_l \hat{\rho}_{lk} \cdot \hat{T}_{lmk}}{\sum_l \hat{\rho}_{lk}}$$

$$\frac{\partial E(\quad)}{\partial \Sigma_{mk}^{-1}} = \frac{\partial \sum_l E(\rho_{lk}) \cdot \Big(-\ln|\Sigma_{mk}^{-1}| + E\big[(T_{lmk} - \mu_{mk})^T \Sigma_{mk}^{-1} (\quad) \big] \Big)}{\partial \Sigma_{mk}^{-1}}$$

$$\text{分子} = \sum_l \hat{\rho}_{lk} \cdot \Big(-\ln|\Sigma_{mk}^{-1}| + Tr(\hat{\Sigma}_{mk} \cdot \Sigma_{mk}^{-1})$$
$$+ (\hat{T}_{lmk} - \hat{\mu}_{mk})^T \Sigma_{mk}^{-1} (\quad) \Big)$$

$$\frac{\partial \text{分子}}{\partial \Sigma_{mk}^{-1}} = \sum_l \hat{\rho}_{lk} \Big(-\Sigma_{mk} + \hat{\Sigma}_{mk} + (\hat{T}_{mk} - \hat{\mu}_{mk})(\hat{T}_{mk} - \hat{\mu}_{mk})^T \Big) = 0$$

$$\hat{\Sigma}_{mk} = \frac{\sum_l \hat{\rho}_{lk} \Big[\hat{\Sigma}_{mk} + (\hat{T}_{mk} - \hat{\mu}_{mk})(\hat{T}_{mk} - \hat{\mu}_{mk})^T \Big]}{\sum_l \hat{\rho}_{lk}}$$

$$\frac{\partial E(\quad)}{\partial \sigma_{mk}^2} = \sum_l E(\rho_{lk})\left(n_l \cdot \frac{1}{\sigma_{mk}^2} - \frac{1}{(\sigma_{mk}^2)^2} E(x_{lm} - \Phi_l T_{lmk})^T(\quad)\right)$$

$$= 0$$

两边乘 $(\sigma_{mk}^2)^2$

$$\sigma_{mk}^2 \sum_l \hat{\rho}_{lk} n_l = \sum_l \hat{\rho}_{lk} (x_{lm} - \Phi_l \hat{T}_{lm})^T(\quad) + tr(\Phi_l \hat{\Sigma}_{lmk} \Phi_l^T)$$

$$\hat{\sigma}_{mk}^2 = \frac{\sum_l \hat{\rho}_{lk}((\quad)^T(\quad) + tr(\Phi_l \hat{\Sigma} \Phi_l^T))}{\sum_l \hat{\rho}_{lk} n_l}$$