BASIC MATH for "DIT181: DATA STRUCTURES AND ALGORITHMS" COURSE

Below are some basic math rules and theorems that you'll need to be familiar with. We are not expecting from you to memorize them. However, during lectures there will be some math derivations that will use these rules and theorems. You can look up for these rules and theorems from this sheet so that you can follow the math derivations during the Lectures and also during the Problem Sessions.

We will also provide a cheat sheet with similar content during your Written Hall Exam.

Logarithm Rules:

- Logarithm rules are among the **most commonly used rules** in this course.

RULE NAME	RULE
Power rule	$\log_b(x^y) = y * \log_b x$
Exponent rule	$b^{\log_b x} = x$
Derivative	If f(x) $= \log_b x$, then $f'(x) = 1/(x * \ln b)$
Logarithm of base	$\log_b b = 1$
Logarithm of 1	$\log_b 1 = 0$
Quotient rule	$\log_b(x/y) = \log_b x - \log_b y$
Product rule	$\log_b(x * y) = \log_b x + \log_b y$
Base Switch rule	$\log_b c = 1/\log_c b$
Base Change rule	$\log_b x = \log_c x/\log_c b$
Logarithm of 0	$\log_b 0$ is undefined
Logarithm of infinity	$\lim_{x\to\infty} \log_b x = \infty$

Theorems for Finite Series:

- Rules for finite series are among the **most commonly used rules** in this course.

Refer to the finite series tutorial in the following website: https://calculus.nipissingu.ca/tutorials/finiteseries.html

$$\sum_{i=1}^{n} i = \frac{n * (n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n * (n+1) * (2 * n + 1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n * (n+1)}{2}\right)^2$$

Finite Geometric Series can be formulated as follows:

$$\sum_{i=0}^{n-1} a * r^k = a * \left(\frac{r^n - 1}{r - 1}\right)$$

L'Hospital's Rule (for infinity over infinity)

- We will use L'Hospital's Rule during Problem Sessions while covering the topics "Big-O Complexity" and "Recursion – Divide and Conquer Algorithms"

Assume that functions f and g are differentiable for all x larger than some fixed number.
If $\lim_{x \to a} f(x) = \infty$ and $\lim_{x \to a} g(x) = \infty$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

so long as the limit is finite, $+\infty$, or $-\infty$. Similar results hold for $x \to +\infty$ and $x \to -\infty$

Derivative Rules:

We will use some Derivative Rules during Problem Sessions while covering the topic "Big-O Complexity"

RULE NAME	FUNCTION	DERIVATIVE
Multiplication by constant	$c * f$	$c * f'$
Power rule	x^n	$n * x^{n-1}$
Sum rule	$f + g$	$f' + g'$
Difference rule	$f - g$	$f' - g'$
Product rule	$f * g$	$f * g' + f' * g$
Quotient Rule	f/g	$(f' * g + f * g')/g^2$
Reciprocal Rule	$1/f$	$-f/f^2$

Master Theorem:

- We will use Master Theorem while covering the topic "Recursion – Divide and Conquer Algorithms"

Given the following parameters:

- A, which is the number of subproblems in the problem to be solved by divide and conquer technique
- B, relative size of the subproblems (for instance $B = 2$ represents half-sized subproblems)
- k, which is the representative of the fact that overhead is $\Theta(N^k)$

The solution to the equation $T(N) = AT(N/B) + \Theta(N^k)$, where A > 1 and B >1 is:

- $T(N) = O\left(N^{\log_B A}\right)$, for $A > B^k$
- $T(N) = O(N^k logN)$, for $A = B^k$
- $T(N) = O(N^k)$, for $A < B^k$