Commitment schemes

Distributed Lab

August 20, 2024

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Commitments Overview

Commitment Definition

Definition

A cryptographic commitment scheme allows one party to commit to a chosen statement without revealing the statement itself. The commitment can be revealed in full or in part at a later time, ensuring the integrity and secrecy of the original statement until the moment of disclosure.

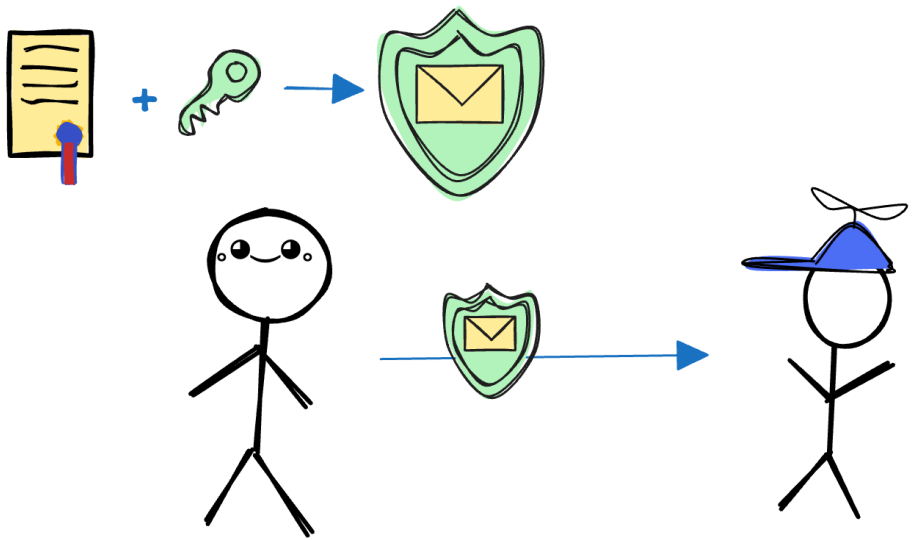


Figure: Overview of a commitment scheme

Commitment Definition

Definition

Commitment Scheme $\Pi_{\text{commitment}}$ is a tuple of three algorithms: $\Pi_{\text{commitment}} = (\text{Setup}, \text{Commit}, \text{Verify})$.

1. Setup (1^λ): returns public parameter pp for both comitter and verifier;
2. Commit (pp, m, r): returns a commitment c to the message m using public parameters pp and, optionally, a secret opening hit r;
3. Open (pp, c, m, r): verifies the opening of the commitment to the message m with an opening hit r.

Commitment Scheme Properties

Definition

1. *Hiding:* verifier should not learn any additional information about the message given only the commitment c.
   1. *Perfect hiding*: adversary with any computation capability tries even forever cannot understand what you have hidden.
   2. *Computationally hiding*: we assume that the adversary have limited computational resources and cannot try forever to recover hidden value.
2. *Binding:* prover could not find another message m_1 and open the commitment c without revealing the commited message m.
   1. *Perfect binding*: adversary with any computation capability tries even forever cannot find another m_1 that would result to the same c.
   2. *Computationally binding*: we assume that the adversary have limited computational resources and cannot try forever.

Note

Perfect hiding and perfect binding cannot be achived at the same time

Hash-based Commitments

Hash-based commitments

As the name implies, we are using a cryptographic hash function H in such scheme.

Definition

1. Prover selects a message m from a message space $\mathcal{M}$ which he wants to commit to: $m \leftarrow \mathcal{M}$
2. Prover samples random value r (usually called blinding factor) from a challange space $\mathcal{C} \subset \mathbb{Z}$: $r \xleftarrow{R} \mathcal{C}$
3. Both values will be concatenated and hashed with the hash function H to produce the commitment: $c = H(m \parallel r)$

Vector Commitments

Merkle Tree commitments

A naive approach for a vector commitment would be hash the whole vector. More sophisticated scheme uses divide-and-conquer approach by building a binary tree out of vector elements.

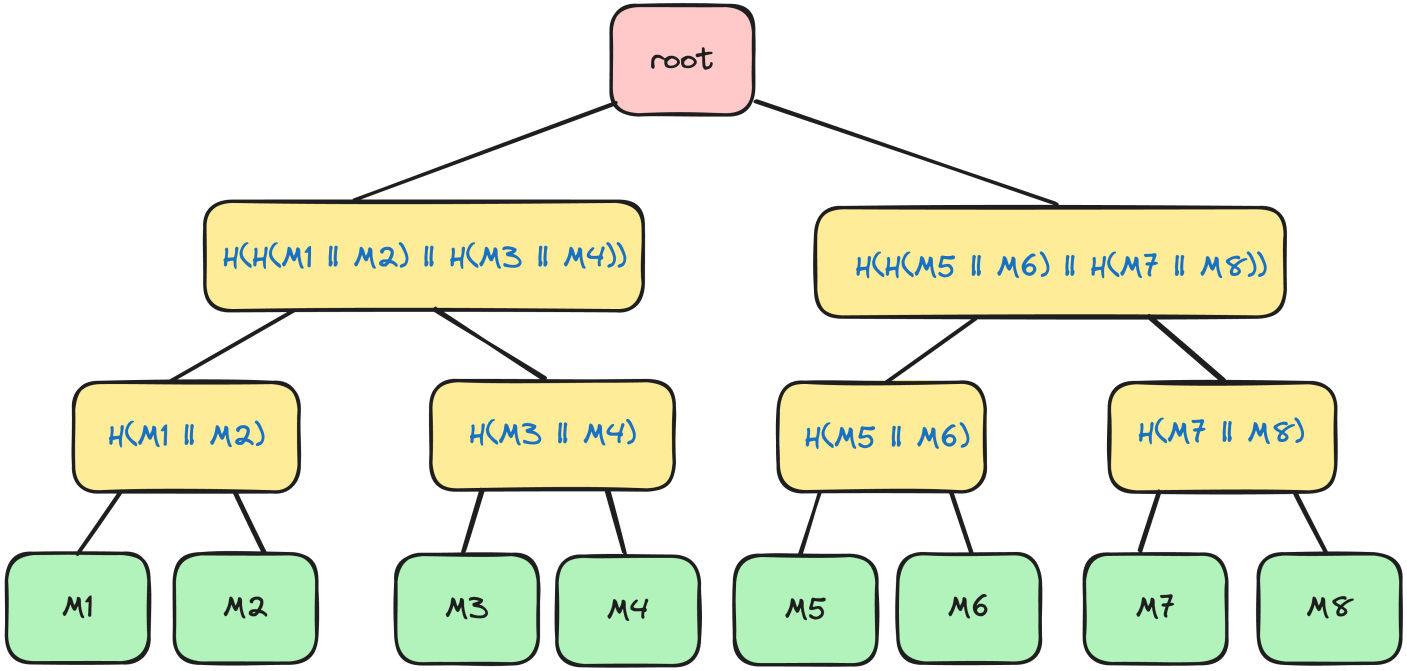


Figure: Merkle Tree structure

Merkle Tree Proof (MTP)

To prove the inclusion of element into the tree, a corresponding Merkle Branch is used. It allows to perform selective disclosure of the elements without revealing all of them at once.

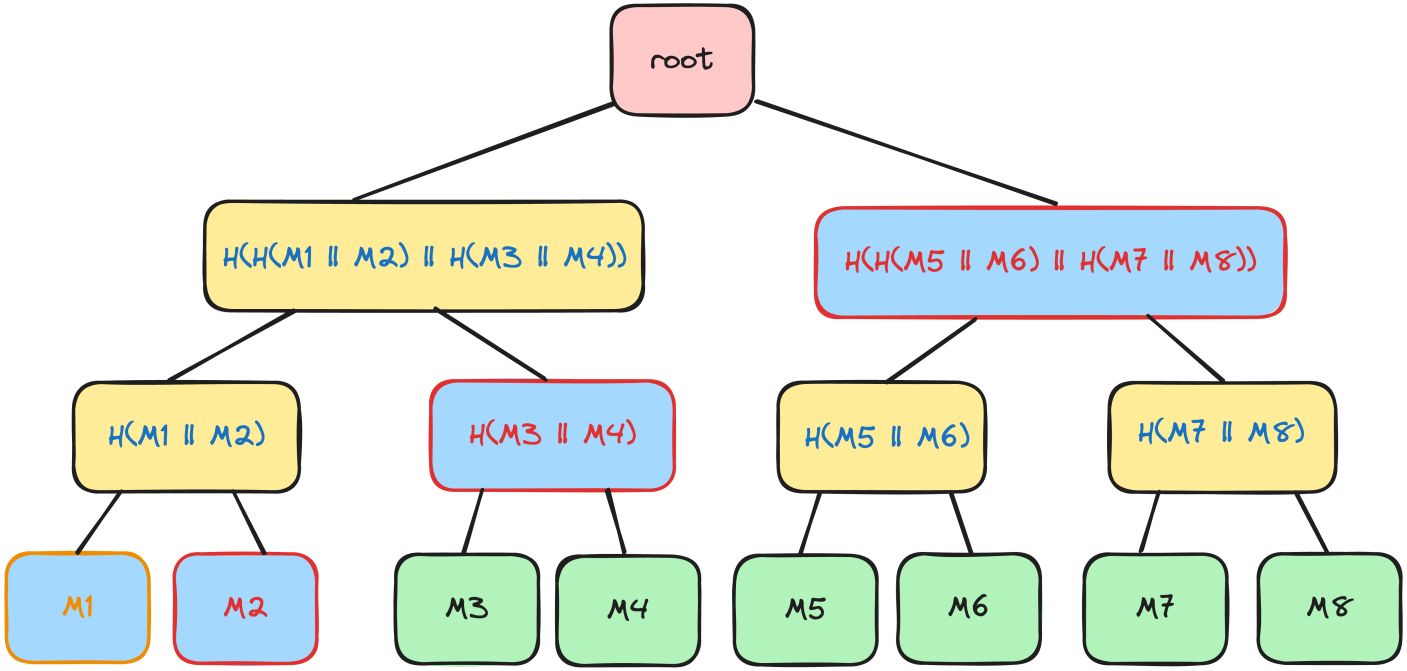


Figure: Merkle Tree inclusion proof branch

Pedersen Commitment

Pedersen commitments allow us to represent arbitrarily large vectors with a single elliptic curve point. Pedersen commitment uses a public group $\mathbb{G}$ of order q and two random public generators G and U: $U = [u]G$. Secret parameter u should be unknown to anyone, otherwise the *Binding* property of the commitment scheme will be violated.

Note: Transparent random points generation

User can pick the publicly known number (like x coordinate of group generator G), calculate $x_i = H(x \parallel i)$ and corresponding y_i. Check whether (x_i, y_i) is in the elliptic curve group. Repeat the process for sequential $i = 1, 2 \ldots$ until point (x_i, y_i) is in the elliptic curve group.

Pedersen Commitment

Definition

Pedersen commitment scheme algorithm:

1. Prover and Verifier agrees on G and U points in a elliptic curve point group $\mathbb{G}$, q is the order of the group.
2. Prover selects a value m to commit and a blinder factor r: $m \leftarrow \mathbb{Z}_q$, $r \xleftarrow{R} \mathbb{Z}_q$
3. Prover generates a commitment and sends it to the Verifier: $c \leftarrow [m]G + [r]U$

During the opening stage, prover reveals (m, r) to the verifier.
To check the commitment, verifier computes: $c_1 = [m]G + [r]U$.
If $c_1 = c$, prover has revealed the correct pair (m, r).

Pedersen Commitment

In case the discrete logarithm of U is leaked, the *binding* property can be violated by the *Prover*:

$$c = [m]G + [r]U = [m]G + [r \cdot u]G = [m + r \cdot u]G$$

For example, $(m + u, r - 1)$ will have the same commitment value:

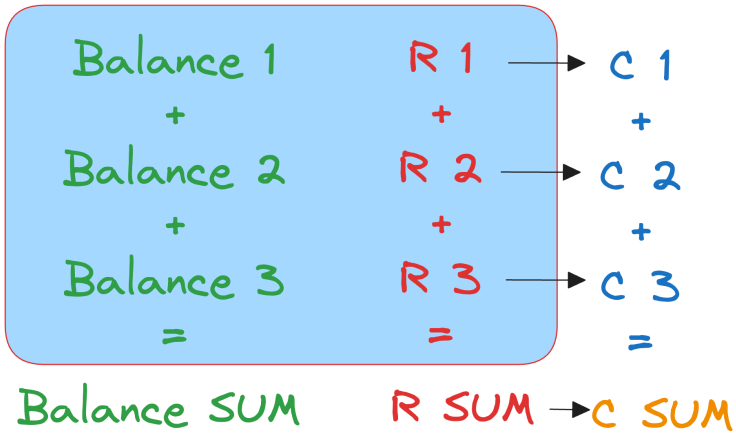
$$[m + u + (r - 1) \cdot u]G = [m + u - u + r \cdot u]G = [m + r \cdot u]G$$

Pedersen Commitment Aggregation

Pedersen commitment have some advantages compared to hash-based commitments. Additively homomorphic property allows to accumulate multiple commitments into one. Consider two pairs: $(m_1, r_1), (m_2, r_2)$.

$$
\begin{aligned}
c_2 &= [m_1]G + [r_1]U, \\
c_2 &= [m_2]G + [r_2]U, \\
c_a = c_1 + c_2 &= [m_1 + m_2]G + [r_1 + r_2]U
\end{aligned}
$$

This works for any number of commitments, so we can encode as many points as we like in a single one.



Pedersen Vector Commitment

Suppose we have a set of random elliptic curve points $(G_1, \ldots, G_n)$ of cyclic group $\mathbb{G}$ (that nobody knows the discrete logarithm of), a vector $(m_1, m_2 \ldots m_n)$ and a random value r. We can do the following:

$$c = m_1 \cdot [G_1] + m_2 \cdot [G_2] \ldots + m_n \cdot [G_n] + r \cdot [Q]$$

Since the *Prover* does not know the discrete logarithm of the generators, so he can only reveal $(v_1, \ldots, v_n)$ to produce $[C]$ later, they cannot produce another vector.
Prover can later open the commitment by revealing the vector $(m_1, m_2 \ldots m_n)$ and a blinding term r.

Polynomial commitment

Polynomial Commitment

Definition

Polynomial commitment can be used to prove that the commited polynomial satisfies certain properties (passes through a certain point (x, y)), without revealing what the polynomial is. The commitment is generally succint, which means that it is much smaller than the polynomial it represents.

KZG Commitment. Simplified example

Given the polynomial: $P(x) = x^3 - 15x^2 + 71x - 103$

Prove that $P(3) = 2$

$P(3) = 2 \rightarrow 3$ is a root of polynomial $P(x) - 2$

Proof: $Q(x) = \dfrac{P(x) - 2}{x - 3} = \dfrac{(x^3 - 15x^2 + 71x - 103) - 2}{x - 3} = x^2 - 12x + 35$

Verify: $Q(x) \cdot (x - 3) = P(x) - 2$

KZG Commitment

The KZG (Kate-Zaverucha-Goldberg) is a polynomial commitment scheme:
One-time "Powers-of-tau" trusted setup stage. During trusted setup a set of elliptic curve points is generated. Let G be a generator point of some pairing-friendly elliptic curve group $\mathbb{G}$, s some random value in the order of the G point and d be the maximum degree of the polynomials we want to commit to.

$$[\tau^0]G, [\tau^1]G, \ldots, [\tau^d]G$$

Parameter τ should be deleted after the ceremony. If it is revealed, the commitment scheme can be broken. This parameter is usually called the *toxic waste*.

KZG Commitment

Commit to polynomial. Given the polynomial $p(x) = \sum_{i=0}^{d} p_i x^i$, compute the commitment $c = [p(\tau)]G$ using the trusted setup. Although the committer cannot compute $[p(\tau)]G$ directly since the value of τ is unknown, he can compute it using values $([\tau^0]G, [\tau^1]G, \ldots, [\tau^d]G)$.

KZGCommitment

Prove an evaluation. Given evaluation $p(x_0) = y_0$ compute proof $q(\tau)$, where $q(x) = \frac{p(x)-y_0}{x-x_0}$.
Polynomial q is called "quotient polynomial" and only exists if and only if $p(x_0) = y_0$. The existance of this quotient polynomial serves as a proof of the evaluation.

KZG Commitment

Verify the proof. Given a commitment $c = [p(\tau)]G$, an evaluation $p(x_0) = y_0$ and a proof $[q(\tau)G]$, we need to ensure that $q(\tau) \cdot (\tau - x_0) = p(\tau) - y_0$. This can be done using trusted setup without knowledge of τ using bilinear mapping:

$$e(q(\tau), [\tau]G_2 - [x_0]G_2) = e(c - [y_0]G_1, G_2)$$

Polynomial commiment schemes such as KZG are used in zero knowledge proof system to encode circuit constraints as a polynomial, so that verifier could check random points to ensure that the constraints are met.

Thanks for your attention!