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Efficient elliptic curve arithmetic in zero-knowledge circuits

Simon Masson

ZKNox

Joint work with	Y. El Housni	(Linea)
	T. Piellard	(Linea)
	L. Eagen	(Alpen Labs)

June 11th, 2025 – *Berlin, ZK Day*

ZKNOX team

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20^+ years experience (10^+y web3)
Security and hardware specialist
Prev. Ledger cofounder/CTO

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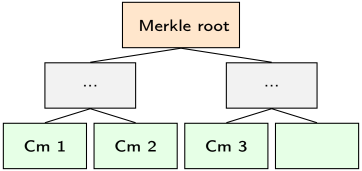
`https://zknox.eth.limo/`

`https://github.com/zknoxhq/`

Introduction: privacy in Ethereum

Alice
1 ETH



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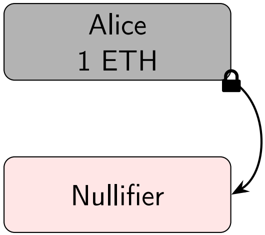

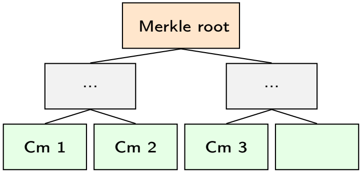

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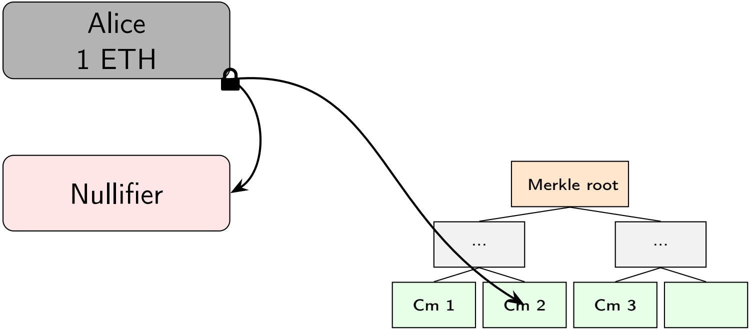

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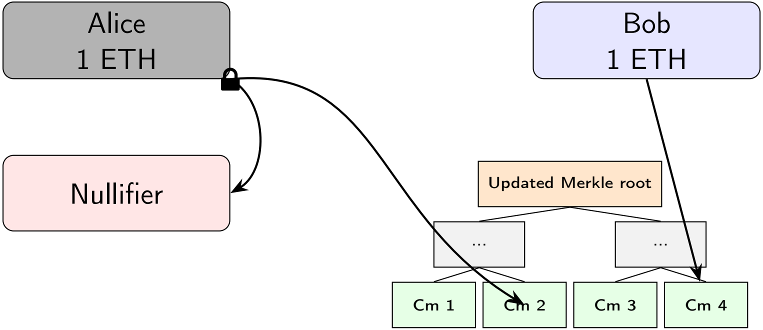

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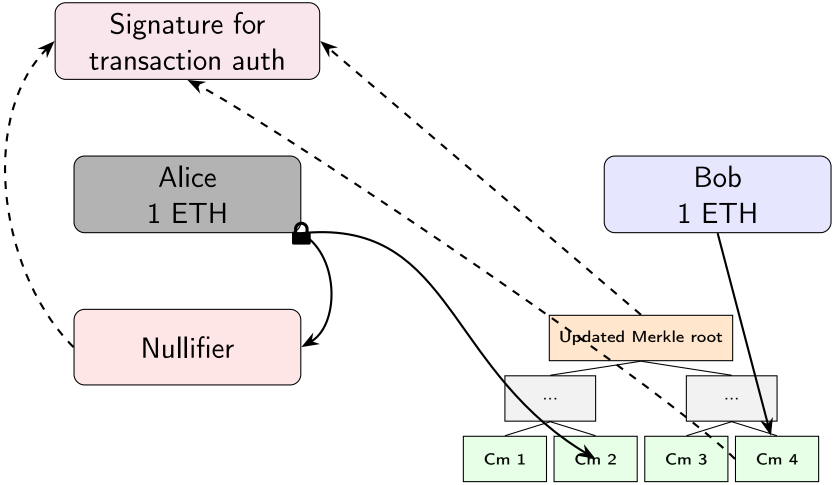

Introduction: privacy in Ethereum

A zero-knowledge proof that:

- Binds the note owner to the created nullifier,
- Proves the ownership of the note,
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  (signature verification)

The signature curve is chosen so that the authentication circuit is *compatible* with the nullifier and ownership circuits.
...but what is a circuit, exactly?

An example of circuit

Proving the knowledge of a solution ($x = 3$) of $x^3 + x + 5 = 35$:

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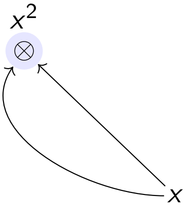

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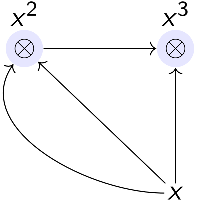

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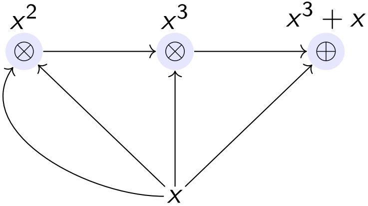

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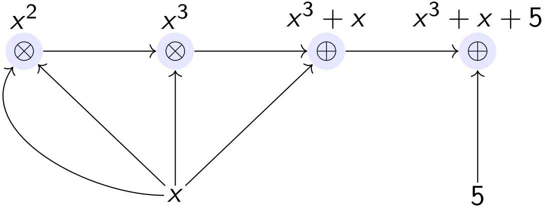

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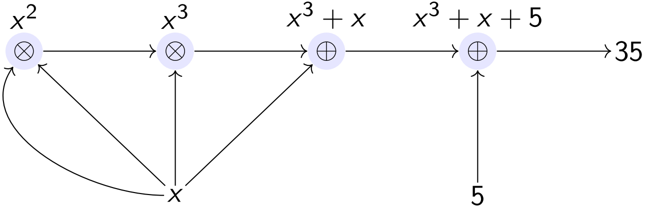

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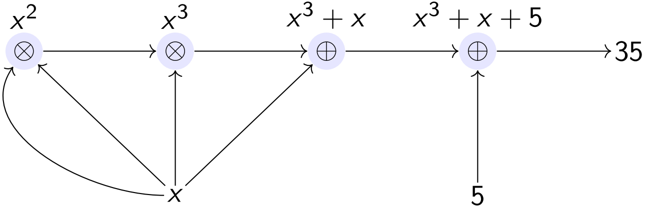

- (almost) Only additions and multiplications,
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  - Circom: BN254 scalar field,
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  - Etc.

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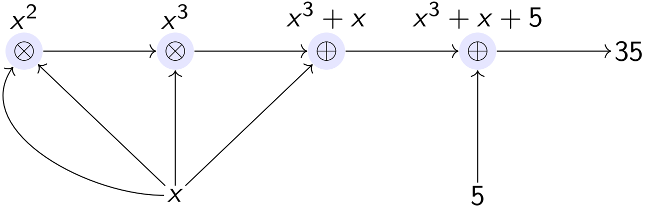

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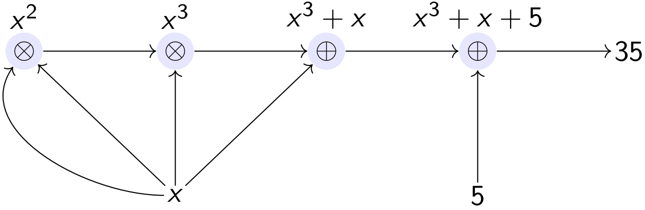

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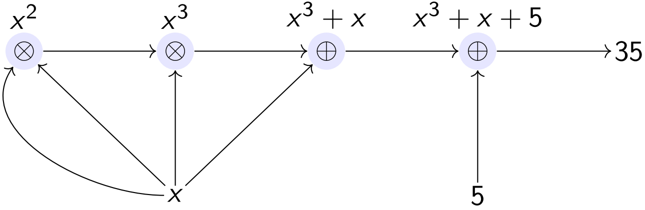

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(Railgun uses Circom; BabyJubjub defined over BN254 scalar field)

Elliptic curve in circuits

Signature verification: Elliptic curve scalar multiplications.

EdDSA in Wikipedia: $2^c SB \stackrel{?}{=} 2^c R + 2^c H(R||A||M)A$.

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In practice:

- Off-chain: very efficient (a few milliseconds),
- On-chain: non-native scalar multiplication ≈ 4 seconds.

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This work: reduce scalars using circuit hints:

- Smaller circuits for signature verification,
- Improved proof computation for circuits of elliptic curves.

Hinted scalar multiplication

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$\{x - kz = 0 \bmod r\}$ is a lattice of dimension 2:

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Expected size $\sqrt{r}$.

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$$[k]P = Q \iff [x]P - [z]Q = 0$$

- $[k]P = Q$: scalar of size 256,
- $[x]P - [z]Q = 0$: scalars of size 128. ✓

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Consider the equation $[k_1]P_1 + [k_2]P_2 = Q$.
The scalars k_1, k_2 can be simultaneously decomposed as

$$k_1 = \frac{x_1}{z} \bmod r, \qquad k_2 = \frac{x_2}{z} \bmod r.$$

Hinted double scalar multiplication

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The scalars k_1, k_2 can be simultaneously decomposed as

$$k_1 = \frac{x_1}{z} \bmod r, \qquad k_2 = \frac{x_2}{z} \bmod r.$$

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$$[k_1]P_1 + [k_2]P_2 = Q \iff [x_1]P_1 + [x_2]P_2 - [z]Q = 0$$

Triple scalar multiplication with scalars of 171 bits. ✓

GLV hinted scalar multiplication

GLV: a technique to faster scalar multiplication for specific curves:
$[k]P = [k_1]P + [k_2]\psi(P)$ where $\psi(P)$ is easy to compute, and k_1, k_2 halved size.

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Single scalar multiplication with GLV and hint

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$$[k_1]P_1 + [k_2]P_2$$
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$$[x_1]P_1 + [y_1]\psi(P_1) + [x_2]P_2 + [y_2]\psi(P_2)$$
$$-[z]Q - [t]\psi(Q) = 0$$

Sextuple 86-bit scalar multiplication.

Practical results

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Circuit	R1CS			SCS		
	Before	This work		Before	This work	
Non-native (P256)	157 685	78 940	50%	612 759	294 128	52%
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- The scalar decomposition is not optimal yet (`xgcd` vs `lll`),
- The cost is implementation-dependent,
- Out-of-circuit considerations are also important,
- Double scalar multiplication not implemented yet.

Conclusion and perspectives

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Thank you for your attention.