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## 计算几何

二维计算几何基础

凸包

# 数据结构

## 树状数组

### 普通

```
template<typename T>
struct Fenwick {
    int n;
    std::vector<T> a;

    Fenwick(int n_ = 0) {
        init(n_);
    }

    void init(int n_) {
        n = n_;
        a.assign(n + 1, T{});
    }

    void add(int x, const T &v) {
        if (x <= 0 || x > n) return;
        for (int i = x; i <= n; i += i & -i) {
            a[i] = a[i] + v;
        }
    }

    T Query(int x) {
        if (x <= 0) return T{};
        if (x > n) x = n;
        T ans{};
        for (int i = x; i != 0; i -= i & -i) {
            ans = ans + a[i];
        }
    }
};
```

```

    }
    return ans;
}

T range_Query(int l, int r) {
    if (l > r) return 0;
    return Query(r) - Query(l - 1);
}

int kth(const T &k) {
    int x = 0;
    T cur{};
    for (int i = 1 << std::__lg(n); i; i /= 2) {
        if (x + i <= n && cur + a[x + i] < k) {
            x += i;
            cur = cur + a[x];
        }
    }
    return x + 1;
}
};

```

## 区间加

```

template<typename T>
struct Range_Fenwick {
    int n;
    Fenwick <T> a, b;

    Range_Fenwick (int _n = 0) {
        init (_n);
    }

    void init (int _n) {
        n = _n;
        a.init(n); b.init(n);
    }

    void range_Change (int l, int r, const T& k) {
        a.add(l, k); a.add(r + 1, -k);
        b.add(l, k * l); b.add(r + 1, -k * (r + 1));
    }

    T range_Query (int l, int r) {
        return (r + 1) * a.Query(r) - l * a.Query(l - 1) - b.range_Query(l, r);
    }

    int kth(const T &k) {
        int x = 0;
        T cur0{}, cur1{};
        for (int i = 1 << std::__lg(n); i; i /= 2) {
            if (x + i <= n && (cur0 + a.a[x + i]) * (x + i + 1) - (cur1 + b.a[x +
i])) < k) {
                x += i;
                cur0 = cur0 + a.a[x];
            }
        }
        return x + 1;
    }
};

```

```

        cur1 = cur1 + b.a[x];
    }
}
return x + 1;
}
};

```

## 二维

```

template<typename T>
struct Two_dimensional_Fenwick {
    struct Base_Fenwick {
        int n, m;
        std::vector<std::vector<T>> s;

        Base_Fenwick(int _n = 0, int _m = 0) {
            init(_n, _m);
        }

        void init(int _n, int _m) {
            n = _n, m = _m;
            s.assign(n + 1, std::vector<T>(m + 1, T()));
        }

        void change(int x, int y, const T &v) {
            if (x <= 0 || y <= 0) return;
            if (x > n) x = n;
            if (y > m) y = m;
            for (int i = x; i <= n; i += i & (-i))
                for (int j = y; j <= m; j += j & (-j))
                    s[i][j] += v;
        }

        T Query(int x, int y) {
            if (x <= 0 || y <= 0) return T();
            if (x > n) x = n;
            if (y > m) y = m;
            T ans = 0;
            for (int i = x; i != 0; i -= i & (-i))
                for (int j = y; j != 0; j -= j & (-j))
                    ans += s[i][j];
            return ans;
        }
    };

    int n, m;
    Base_Fenwick A, B, C, D;

    Two_dimensional_Fenwick(int _n = 0, int _m = 0) {
        init(_n, _m);
    }

    void init(int _n, int _m) {
        n = _n, m = _m;
        A.init(n, m);
    }

```

```

        B.init(n, m);
        C.init(n, m);
        D.init(n, m);
    }

    void Base_add(int x, int y, int v) {
        A.change(x, y, v);
        B.change(x, y, v * x);
        C.change(x, y, v * y);
        D.change(x, y, v * x * y);
    }

    T Base_Query(int x, int y) {
        return A.Query(x, y) * (x * y + x + y + 1)
            - B.Query(x, y) * (y + 1)
            - C.Query(x, y) * (x + 1)
            + D.Query(x, y);
    }

    void add(int x0, int y0, int x1, int y1, int v) {
        Base_add(x0, y0, v);
        Base_add(x0, y1 + 1, -v);
        Base_add(x1 + 1, y0, -v);
        Base_add(x1 + 1, y1 + 1, v);
    }

    T Query(int x0, int y0, int x1, int y1) {
        return Base_Query(x1, y1) - Base_Query(x0 - 1, y1)
            - Base_Query(x1, y0 - 1) + Base_Query(x0 - 1, y0 - 1);
    }
};

```

## RMQ

```

/**
 * author:jiangly
 * pretreatment:O(n)
 * Inquire:O(1)
 */
template<class T,
        class Cmp = std::less<T>>
struct RMQ {
    const Cmp cmp = Cmp();
    static constexpr unsigned B = 64;
    using u64 = unsigned long long;
    int n;
    std::vector<std::vector<T>>> a;
    std::vector<T> pre, suf, ini;
    std::vector<u64> stk;
    RMQ() {}
    RMQ(const std::vector<T> &v) {
        init(v);
    }
    void init(const std::vector<T> &v) {
        n = v.size();
    }
};

```

```

pre = suf = ini = v;
stk.resize(n);
if (!n) {
    return;
}
const int M = (n - 1) / B + 1;
const int lg = std::__lg(M);
a.assign(lg + 1, std::vector<T>(M));
for (int i = 0; i < M; i++) {
    a[0][i] = v[i * B];
    for (int j = 1; j < B && i * B + j < n; j++) {
        a[0][i] = std::min(a[0][i], v[i * B + j], cmp);
    }
}
for (int i = 1; i < n; i++) {
    if (i % B) {
        pre[i] = std::min(pre[i], pre[i - 1], cmp);
    }
}
for (int i = n - 2; i >= 0; i--) {
    if (i % B != B - 1) {
        suf[i] = std::min(suf[i], suf[i + 1], cmp);
    }
}
for (int j = 0; j < lg; j++) {
    for (int i = 0; i + (2 << j) <= M; i++) {
        a[j + 1][i] = std::min(a[j][i], a[j][i + (1 << j)], cmp);
    }
}
for (int i = 0; i < M; i++) {
    const int l = i * B;
    const int r = std::min(1U * n, l + B);
    u64 s = 0;
    for (int j = 1; j < r; j++) {
        while (s && cmp(v[j], v[std::__lg(s) + 1])) {
            s ^= 1ULL << std::__lg(s);
        }
        s |= 1ULL << (j - 1);
        stk[j] = s;
    }
}
}

T operator()(int l, int r) {
    if (l / B != (r - 1) / B) {
        T ans = std::min(suf[l], pre[r - 1], cmp);
        l = l / B + 1;
        r = r / B;
        if (l < r) {
            int k = std::__lg(r - l);
            ans = std::min({ans, a[k][l], a[k][r - (1 << k)]}, cmp);
        }
        return ans;
    } else {
        int x = B * (l / B);
        return ini[__builtin_ctzll(stk[r - 1] >> (l - x)) + 1];
    }
}

```

```
}  
};
```

## 线段树

### 单点

```
template<class Info>  
struct SegmentTree {  
    int n;  
    std::vector<Info> info;  
    SegmentTree() : n(0) {}  
    SegmentTree(int n_, Info v_ = Info()) {  
        init(n_, v_);  
    }  
    template<class T>  
    SegmentTree(std::vector<T> init_) {  
        init(init_);  
    }  
    void init(int n_, Info v_ = Info()) {  
        init(std::vector(n_, v_));  
    }  
    template<class T>  
    void init(std::vector<T> init_) {  
        n = init_.size();  
        info.assign(4 << std::lg(n), Info());  
        std::function<void(int, int, int)> build = [&](int p, int l, int r) {  
            if (r - l == 1) {  
                info[p] = init_[l];  
                return;  
            }  
            int m = (l + r) / 2;  
            build(2 * p, l, m);  
            build(2 * p + 1, m, r);  
            pull(p, l, m, r);  
        };  
        build(1, 0, n);  
    }  
    void pull(int p, int l, int m, int r) {  
        info[p].update(info[2 * p], info[2 * p + 1], l, m, r);  
    }  
    void modify(int p, int l, int r, int x, const Info &v) {  
        if (r - l == 1) {  
            info[p].apply(v, l, r);  
            return;  
        }  
        int m = (l + r) / 2;  
        if (x < m) {  
            modify(2 * p, l, m, x, v);  
        } else {  
            modify(2 * p + 1, m, r, x, v);  
        }  
        pull(p, l, m, r);  
    }  
};
```

```

}
void modify(int p, const Info &v) {
    if(p >= n) return;
    modify(1, 0, n, p, v);
}
Info rangeQuery(int p, int l, int r, int x, int y) {
    if (l >= x && r <= y) {
        return info[p];
    }
    int m = (l + r) / 2;
    if (m >= y) {
        return rangeQuery(2 * p, l, m, x, y);
    } else if (m <= x) {
        return rangeQuery(2 * p + 1, m, r, x, y);
    } else {
        return Info::merge(rangeQuery(2 * p, l, m, x, y), rangeQuery(2 * p +
1, m, r, x, y), std::max(l, x), m, std::min(r, y));
    }
}
Info rangeQuery(int l, int r) {
    if (l >= r) return Info();
    return rangeQuery(1, 0, n, l, r);
}
// int BS(int p, int l, int r, i64 k) {
//     // debug(l, r, k, info[p]);
//     if (info[p] < k) return -1;
//     if (r - l == 1) return l;
//     int m = (l + r) / 2;
//     if (info[p * 2].sum >= k)
//         return BS(p * 2, l, m, k);
//     else
//         return BS(p * 2 + 1, m, r, k - info[p * 2].sum);
// };
// int BS(i64 k) {
//     // debug(k);
//     return BS(1, 0, n, k);
// }

template<class F>
int findFirst(int p, int l, int r, int x, int y, F pred) {
    if (l >= y || r <= x || !pred(info[p])) {
        return -1;
    }
    if (r - l == 1) {
        return l;
    }
    int m = (l + r) / 2;
    int res = findFirst(2 * p, l, m, x, y, pred);
    if (res == -1) {
        res = findFirst(2 * p + 1, m, r, x, y, pred);
    }
    return res;
}
template<class F>
int findFirst(int l, int r, F pred) {
    return findFirst(1, 0, n, l, r, pred);
}

```



```

}
template<class F>
int findLast(int p, int l, int r, int x, int y, F pred) {
    if (l >= y || r <= x || !pred(info[p])) {
        return -1;
    }
    if (r - l == 1) {
        return l;
    }
    int m = (l + r) / 2;
    int res = findLast(2 * p + 1, m, r, x, y, pred);
    if (res == -1) {
        res = findLast(2 * p, l, m, x, y, pred);
    }
    return res;
}

template<class F>
int findLast(int l, int r, F pred) {
    return findLast(1, 0, n, l, r, pred);
}

void show(int p, int l, int r, int x, int y, int dep = 0) {
    if (l >= y || r <= x) return;
    int m = (l + r) >> 1;
    if (r - l > 1)
        show(p * 2, l, m, x, y, dep + 1);
    for (int i = 0; i < dep; i += 1) {
        cerr << '\t';
    }
    cerr << l << ' ' << r << ' '; info[p].show();
    cerr << '\n';
    if (r - l > 1)
        show(p * 2 + 1, m, r, x, y, dep + 1);
}

void show(int l, int r) {
    show(1, 0, n, l, r);
}

};

struct Info {
    void apply(const Info &rhs, int l, int r) {}
    void update(const Info &lhs, const Info &rhs, int l, int m, int r) {}
    static Info merge(const Info &lhs, const Info &rhs, int l, int m, int r) {
        Info info = Info();
        info.update(lhs, rhs, l, m, r);
        return info;
    }
    void show() const {
        cerr << "info: ";
    }
};

using Tree = SegmentTree<Info>;

```

## 区间

```
template<class Info, class Tag>
struct LazySegmentTree {
    int n;
    std::vector<Info> info;
    std::vector<Tag> tag;
    LazySegmentTree() : n(0) {}
    LazySegmentTree(int n_, Info v_ = Info()) {
        init(n_, v_);
    }
    template<class T>
    LazySegmentTree(std::vector<T> init_) {
        init(init_);
    }
    void init(int n_, Info v_ = Info()) {
        init(std::vector(n_, v_));
    }
    template<class T>
    void init(std::vector<T> init_) {
        n = init_.size();
        info.assign(n * 4, Info());
        tag.assign(n * 4, Tag());
        std::function<void(int, int, int)> build = [&](int p, int l, int r) {
            if (r - l == 1) {
                info[p] = init_[l];
                return;
            }
            int m = (l + r) / 2;
            build(2 * p, l, m);
            build(2 * p + 1, m, r);
            pull(p, l, m, r);
        };
        build(1, 0, n);
    }
    void pull(int p, int l, int m, int r) {
        info[p].update(info[2 * p], info[2 * p + 1], l, m, r);
    }
    void apply(int p, const Tag &v, int l, int r) {
        info[p].apply(v, l, r);
        tag[p].apply(v);
    }
    void push(int p, int l, int m, int r) {
        if (bool(tag[p])) {
            apply(2 * p, tag[p], l, m);
            apply(2 * p + 1, tag[p], m, r);
            tag[p] = Tag();
        }
    }
    void modify(int p, int l, int r, int x, const Info &v) {
        if (r - l == 1) {
            info[p] = v;
            return;
        }
        int m = (l + r) / 2;
```

```

    push(p, l, m, r);
    if (x < m) {
        modify(2 * p, l, m, x, v);
    } else {
        modify(2 * p + 1, m, r, x, v);
    }
    pull(p, l, m, r);
}

void modify(int p, const Info &v) {
    modify(1, 0, n, p, v);
}

Info rangeQuery(int p, int l, int r, int x, int y) {
    if (l >= x && r <= y) {
        return info[p];
    }
    int m = (l + r) / 2;
    push(p, l, m, r);
    if (m >= y) {
        return rangeQuery(2 * p, l, m, x, y);
    } else if (m <= x) {
        return rangeQuery(2 * p + 1, m, r, x, y);
    } else {
        return Info::merge(rangeQuery(2 * p, l, m, x, y), rangeQuery(2 * p +
1, m, r, x, y), l, m, r);
    }
}

Info rangeQuery(int l, int r) {
    if (l >= r) return Info();
    return rangeQuery(1, 0, n, l, r);
}

void rangeApply(int p, int l, int r, int x, int y, const Tag &v) {
    if (l >= y || r <= x) {
        return;
    }
    int m = (l + r) / 2;
    if (l >= x && r <= y) {
        apply(p, v, l, r);
        return;
    }
    push(p, l, m, r);
    rangeApply(2 * p, l, m, x, y, v);
    rangeApply(2 * p + 1, m, r, x, y, v);
    pull(p, l, m, r);
}

void rangeApply(int l, int r, const Tag &v) {
    return rangeApply(1, 0, n, l, r, v);
}

template<class F>
int findFirst(int p, int l, int r, int x, int y, F pred) {
    if (l >= y || r <= x || !pred(info[p])) {
        return -1;
    }
    if (r - l == 1) {
        return l;
    }
    int m = (l + r) / 2;

```

```

    push(p, l, m, r);
    int res = findFirst(2 * p, l, m, x, y, pred);
    if (res == -1) {
        res = findFirst(2 * p + 1, m, r, x, y, pred);
    }
    return res;
}

template<class F>
int findFirst(int l, int r, F pred) {
    return findFirst(1, 0, n, l, r, pred);
}

template<class F>
int findLast(int p, int l, int r, int x, int y, F pred) {
    if (l >= y || r <= x || !pred(info[p])) {
        return -1;
    }
    if (r - l == 1) {
        return l;
    }
    int m = (l + r) / 2;
    push(p, l, m, r);
    int res = findLast(2 * p + 1, m, r, x, y, pred);
    if (res == -1) {
        res = findLast(2 * p, l, m, x, y, pred);
    }
    return res;
}

template<class F>
int findLast(int l, int r, F pred) {
    return findLast(1, 0, n, l, r, pred);
}

void show(int p, int l, int r, int x, int y, int dep = 0) {
    if (l >= y || r <= x) return;
    int m = (l + r) >> 1;
    if (r - l > 1)
        show(p * 2, l, m, x, y, dep + 1);
    for (int i = 0; i < dep; i += 1) {
        cerr << '\t';
    }
    cerr << l << ' ' << r << ' '; info[p].show(), tag[p].show();
    cerr << '\n';
    if (r - l > 1)
        show(p * 2 + 1, m, r, x, y, dep + 1);
}

void show(int l, int r) {
    show(1, 0, n, l, r);
}

};

constexpr i64 inf = 1e18;

struct Tag {
    i64 d = 0;
    void apply(Tag t) {
        d += t.d;
    }
}

```

```

operator bool() {
    return d != 0;
}

void show() const {
#ifdef LOCAL
    cerr << "tag: " << d << ";";
#endif
}

};

constexpr int N = 20;

struct Info {
    array<double, 2> val{0, 1};
    void apply(const Tag &t, int l, int r) {
        tie(val[0], val[1])
            = make_tuple(val[0] * cos(t.d) + val[1] * sin(t.d),
                          val[1] * cos(t.d) - val[0] * sin(t.d));
    }
    void update(const Info &lhs, const Info &rhs, int l, int m, int r) {
        for (auto i : {0, 1}) {
            val[i] = lhs.val[i] + rhs.val[i];
        }
    }
    static Info merge(const Info &lhs, const Info &rhs, int l, int m, int r) {
        Info info = Info();
        info.update(lhs, rhs, l, m, r);
        return info;
    }
    void show() {
#ifdef LOCAL
        cerr << "info: " << val << "; ";
#endif
    }
};

using lazySegmentTree = LazySegmentTree<Info, Tag>;

```

## tourist zkw 线段树 (精简版) 区间最大值

```

struct SegmTree {
    vector<int> T; int n;
    SegmTree(int n) : T(2 * n, (int)-2e9), n(n) {}

    void Update(int pos, int val) {
        for (T[pos += n] = val; pos > 1; pos /= 2)
            T[pos / 2] = max(T[pos], T[pos ^ 1]);
    }

    int Query(int b, int e) {
        int res = -2e9;
        for (b += n, e += n; b < e; b /= 2, e /= 2) {
            if (b % 2) res = max(res, T[b++]);
            if (e % 2) res = max(res, T[--e]);
        }
    }
};

```

```

        return res;
    }
};

```

## 动态开点线段树

```

/**
 * 262144000
 */
constexpr int max_size = 262144000;
uint8_t buf[max_size];
uint8_t *head = buf;

using Tp = long long;
template<typename Info, typename Tag>
struct segment_tree {
    int n;
    struct node {
        Info info;
        Tag tag;
        array<int, 2> _ch;
        node(): info(), tag(), _ch{} {}
        node *ch(int x) const {
            return (node *)(_ch[x] + buf);
        }
        void clear() {
            *this = node();
        }
    };
};

using p_Tp = node *;
int root{0};
int _new(Tp l, Tp r) {
    int cur = (head += sizeof(node)) - buf;
    p_Tp p = p_Tp(buf + cur);
    // p->info = Info::merge(l, r);
    assert(cur < max_size);
    return cur;
}

void apply(int &cur, const Tag &v, Tp l, Tp r) {
    if (!cur) {
        cur = _new(l, r);
    }
    p_Tp p = p_Tp(buf + cur);
    p->info.apply(v, l, r);
    p->tag.apply(v);
}

void push(int &cur, Tp l, Tp m, Tp r) {
    p_Tp p = p_Tp(buf + cur);
    // assert(l < r);
    if (!bool(p->tag))
        return;
    apply(p->_ch[0], p->tag, l, m);
    apply(p->_ch[1], p->tag, m, r);
    p->tag.clear();
}

```

```

void pull(int &cur, Tp l, Tp m, Tp r) {
    p_Tp p = p_Tp(buf + cur);
    p->info.update(p->ch(0)->info, p->ch(1)->info, l, m, r);
}
Tp floor, ceil;
segment_tree(Tp floor, Tp ceil) : floor(floor) , ceil(ceil) {}
void modify(int &cur, const Tag &v, Tp l, Tp r, Tp x) {
    if (!cur)
        cur = _new(l, r);
    p_Tp p = p_Tp(buf + cur);
    Tp m = (l + r) >> 1;
    if (r - l == 1) {
        p->info.apply(v, l, r);
        return;
    }
    // push(cur, l, m, r);
    if (m > x)
        modify(p->_ch[0], v, l, m, x);
    else
        modify(p->_ch[1], v, m, r, x);
    pull(cur, l, m, r);
}
void modify(Tp x, const Tag &v) {
    modify(root, v, floor, ceil, x);
}
void rangeApply(int &cur, const Tag &v, Tp l, Tp r, Tp x, Tp y) {
    if (!cur)
        cur = _new(l, r);
    p_Tp p = p_Tp(buf + cur);
    Tp m = (l + r) >> 1;
    if (x <= l && r <= y) {
        apply(cur, v, l, r);
        return;
    }
    push(cur, l, m, r);
    if (m > x)
        rangeApply(p->_ch[0], v, l, m, x, y);
    if (m < y)
        rangeApply(p->_ch[1], v, m, r, x, y);
    pull(cur, l, m, r);
}
void rangeApply(Tp x, Tp y, const Tag &v) {
    if (x >= y) return;
    rangeApply(root, v, floor, ceil, x, y);
}
Info Query(int &cur, Tp l, Tp r, Tp x) {
    if (!cur)
        return Info::merge(l, r);
    p_Tp p = p_Tp(buf + cur);
    Tp m = (l + r) >> 1;
    if (r - l == 1) {
        return p->info;
    }
    // push(cur, l, m, r);
    if (m > x)
        return Query(p->_ch[0], l, m, x);

```

```

        else
            return Query(p->_ch[1], m, r, x);
    }
    Info Query(Tp x) {
        return Query(root, floor, ceil, x);
    }
    Info rangeQuery(int &cur, Tp l, Tp r, Tp x, Tp y) {
        if (!cur)
            return Info::merge(l, r);
        p_Tp p = p_Tp(buf + cur);
        Tp m = (l + r) >> 1;
        if (x <= l && r <= y) {
            return p->info;
        }
        push(cur, l, m, r);
        if (m >= y) {
            return rangeQuery(p->_ch[0], l, m, x, y);
        } else if (m <= x) {
            return rangeQuery(p->_ch[1], m, r, x, y);
        } else {
            return Info::merge(rangeQuery(p->_ch[0], l, m, x, y), rangeQuery(p->_ch[1], m, r, x, y), l, m, r);
        }
    }
    Info rangeQuery(Tp x, Tp y) {
        return rangeQuery(root, floor, ceil, x, y);
    }
    double BS(int &cur, Tp l, Tp r, i64 k) {
        if (!cur) cur = _new(l, r);

        p_Tp p = p_Tp(buf + cur);

        // debug(l, r, k, p->info);

        if (r - l == 1) {
            assert(p->info != 0);
            // if (p->info == 0) exit(0);
            return l + 1. * k / p->info;
        }

        Tp m = (l + r) >> 1;
        push(cur, l, m, r);

        if (p->ch(0)->info >= k)
            return BS(p->_ch[0], l, m, k);
        else
            return BS(p->_ch[1], m, r, k - p->ch(0)->info);
    }
    double BS(i64 k) {
        return BS(root, floor, ceil, k);
    }
    void show(int &cur, Tp l, Tp r, Tp x, Tp y, int dep = 0) {
        if (l >= y || r <= x || !cur) return;
        p_Tp p = p_Tp(buf + cur);
        Tp m = (l + r) >> 1;
        if (r - l > 1)

```



```

        show(p->_ch[0], l, m, x, y, dep + 1);
        for (int i = 0; i < dep; i += 1) cerr << '\t';
        cerr << l << ' ' << r << ' '; p->info.show(), p->tag.show();
        cerr << '\n';
        if (r - l > 1)
            show(p->_ch[1], m, r, x, y, dep + 1);
    }
    void show(Tp x, Tp y) {
        show(root, floor, ceil, x, y);
    }
    p_Tp p_Tp_root() { return p_Tp(buf + root); }
};

struct Tag {
    int x = 0;
    void apply(const Tag &rhs) {
        x += rhs.x;
    }
    operator bool() {
        return x != 0;
    }
    void clear() {
        x = 0;
    }
    void show() const {
#ifdef LOCAL
        cerr << "Tag: " << x;
#endif
    }
};

struct Info {
    i64 x = 0;
    operator i64() {
        return x;
    }

    void apply(const Tag &rhs, Tp l, Tp r) {
        x += rhs.x * (r - l);
    }
    void update(const Info &lhs, const Info &rhs, Tp l, Tp m, Tp r) {
        x = lhs.x + rhs.x;
    }
    static Info merge(const Info &lhs, const Info &rhs, Tp l, Tp m, Tp r) {
        Info info = Info();
        info.update(lhs, rhs, l, m, r);
        return info;
    }
    static Info merge(Tp l, Tp r) {
        return {0};
    }
    void show() const {
#ifdef LOCAL
        cerr << "Info: " << x << ' ';
#endif
    }
}

```

```
};
```

```
using SegmentTree = segment_tree<Info, Tag>;
```

## 线段树分治

```
template<class Info>
struct SegmentTree {
    int n;
    std::vector<Info> info;
    SegmentTree() : n(0) {}
    SegmentTree(int n_, Info v_ = Info()) {
        init(n_, v_);
    }
    template<class T>
    SegmentTree(std::vector<T> init_) {
        init(init_);
    }
    void init(int n_, Info v_ = Info()) {
        init(std::vector(n_, v_));
    }
    template<class T>
    void init(std::vector<T> init_) {
        n = init_.size();
        info.assign(4 << std::lg(n), Info());
        std::function<void(int, int, int)> build = [&](int p, int l, int r) {
            if (r - l == 1) {
                info[p] = init_[l];
                return;
            }
            int m = (l + r) / 2;
            build(2 * p, l, m);
            build(2 * p + 1, m, r);
        };
        build(1, 0, n);
    }
    void rangeChange(int x, int y, const Info &tag) {
        std::function<void(int, int, int, int, int, const Info&)>
            rangeChange = [&](int p, int l, int r, int x, int y, const Info
&tag) {
            if (l >= y || r <= x) {
                return;
            }
            if (l >= x && r <= y) {
                info[p].apply(tag);
                return;
            }
            int m = (l + r) / 2;
            rangeChange(p << 1, l, m, x, y, tag);
            rangeChange(p << 1 | 1, m, r, x, y, tag);
        };
        rangeChange(1, 0, n, x, y, tag);
    }
};
```

```

struct Info {
    vector<array<ll, 2>> x;
    void apply(const Info& tag) {
        for (auto u : tag.x) {
            x.push_back(u);
        }
    }
};

using Segmenttree = SegmentTree<Info>;

```

## 可持久化线段树

```

constexpr int max_size = 262144000;
uint8_t _buf[max_size];
uint8_t *head = _buf;

template<typename Info>
struct persistent_segment_tree {
    int n;
    struct node {
        Info m_info;
        int ls, rs;
        node () : m_info(), ls(), rs() {}
        void reset () {
            *this = node();
        }
    };
    using pointer = node *;
    int _new() {
        assert(head < _buf + max_size);
        return (head += sizeof(node)) - _buf;
    }
    vector<int> root;
    persistent_segment_tree(): n(0) {}
    persistent_segment_tree(int _n, Info _v = Info()) {
        _init(std::vector(_n, _v));
    }
    template<typename T>
    persistent_segment_tree(std::vector<T> _init) {
        _init(_init);
    }
    void _pull(int cur1) {
        pointer p1 = pointer(_buf + cur1);
        pointer lc = pointer(_buf + p1->ls);
        pointer rc = pointer(_buf + p1->rs);
        p1->m_info.set(Info::op(lc->m_info, rc->m_info));
    }
    template<typename T>
    void _init(std::vector<T> _init) {
        n = _init.size();
        root.push_back(_new());
        std::function<void(int, int, int)>
        build = [&] (int cur, int l, int r) {
            pointer p = pointer(_buf + cur);

```

```

        if (r - l == 1) {
            p->m_info = _init[l];
            return;
        }
        int m = (l + r) / 2;
        p->ls = _new(), p->rs = _new();
        build(p->ls, l, m), build(p->rs, m, r);
        _pull(cur);
    };
    build(root.back(), 0, n);
}

template<typename Tag>
void _modify(int cur0, int cur1, const Tag &v, int l, int r, int x) {
    pointer p0 = pointer(_buf + cur0), p1 = pointer(_buf + cur1);
    if (r - l == 1) {
        p1->m_info = p0->m_info;
        p1->m_info.apply(v);
        return;
    }
    int m = (l + r) >> 1;
    if (m > x) {
        p1->ls = _new();
        p1->rs = p0->rs;
        _modify(p0->ls, p1->ls, v, l, m, x);
    } else {
        p1->ls = p0->ls;
        p1->rs = _new();
        _modify(p0->rs, p1->rs, v, m, r, x);
    }
    _pull(cur1);
}

template<typename Tag>
void modify(int x, const Tag &v, int from = -1) {
    int cur0 = (from == -1 ? root.back() : root[from]);
    int cur1 = _new();
    root.push_back(cur1);
    _modify(cur0, cur1, v, 0, n, x);
}

typename Info::op_t _range_query(int cur0, int cur1, int l, int r, int x, int
y) {
    pointer p0 = pointer(_buf + cur0), p1 = pointer(_buf + cur1);
    if (x <= l && r <= y) {
        return Info::del(p1->m_info, p0->m_info);
    }
    int m = (l + r) >> 1;
    if (m >= y) {
        return _range_query(p0->ls, p1->ls, l, m, x, y);
    } else if (m <= x) {
        return _range_query(p0->rs, p1->rs, m, r, x, y);
    } else {
        return Info::op(_range_query(p0->ls, p1->ls, l, m, x, y),
_range_query(p0->rs, p1->rs, m, r, x, y));
    }
}

typename Info::op_t range_query(int from, int to, int x, int y) {
    return _range_query(root[from], root[to], 0, n, x, y);
}

```

```

}
typename Info::op1_t _kth(int cur0, int cur1, int l, int r, i64 k) {
    pointer p0 = pointer(_buf + cur0), p1 = pointer(_buf + cur1);
    pointer ls0 = pointer(_buf + p0->ls), ls1 = pointer(_buf + p1->ls);
    if (r - l == 1) {
        return Info::op1(l, Info::op1(k));
    }
    int m = (l + r) >> 1;
    typename Info::op1_t lhs = Info::del1(ls1->m_info, ls0->m_info);
    if (int(lhs) >= k) {
        return _kth(p0->ls, p1->ls, l, m, k);
    } else {
        return Info::op1(lhs, _kth(p0->rs, p1->rs, m, r, k - int(lhs)));
    }
}

typename Info::op1_t kth(int from, int to, i64 k) {
    return _kth(root[from], root[to], 0, n, k);
}

void _show(int cur, int l, int r) {
    pointer p = pointer(_buf + cur);
    if (r - l == 1) {
        p->m_info.show();
        return;
    }
    int m = (l + r) >> 1;
    _show(p->ls, l, m);
    _show(p->rs, m, r);
}

void show(int time) {
    _show(root[time], 0, n);
}

};

struct Info {
    i64 cnt = 0;
    using op_t = int;
    using op1_t = int;
    operator op_t() {
        return cnt;
    }
    void set(op_t rhs) {
        cnt = rhs;
    }
    static op_t op(op_t lhs, op_t rhs) {
        return lhs + rhs;
    }
    static op_t del(op_t lhs, op_t rhs) {
        return lhs - rhs;
    }
    static array<ll, 1> op1 (i64 k) {
        return array<ll, 1>{0};
    }
    static op1_t op1(int x, array<ll, 1> mul) {
        return x;
    }
    static op1_t op1(op1_t lhs, op1_t rhs) {

```

```

        return rhs;
    }
    static op1_t del1(op1_t lhs, op1_t rhs) {
        return lhs - rhs;
    }
    void apply(Info x) {
        cnt += x.cnt;
    }
    void show() {
        cerr << cnt << ' ';
    }
};

using SegmentTree = persistent_segment_tree<Info>;

```

## 李超线段树

```

template<typename T, class Line, class Cmp>
struct Li_Chao_SegmentTree {
    int n;
    std::vector<int> id;
    std::vector<T> real;
    std::vector<Line> line;
    Cmp cmp;
    Li_Chao_SegmentTree() {}
    Li_Chao_SegmentTree(int _n) {
        init(_n);
    }
    Li_Chao_SegmentTree(const std::vector<T> &_init) {
        init(_init);
    }
    void init(int _n) {
        std::vector<int> _init(_n);
        iota(_init.begin(), _init.end(), 0);
        init(_init);
    }
    void init(const std::vector<T> &_init) {
        n = _init.size();
        id.assign(4 << std::__lg(n), 0);
        line.push_back(Line());
        real = _init;
        sort(real.begin(), real.end());
        real.erase(std::unique(real.begin(), real.end()), real.end());
        real.push_back(real.back() + 1);
    }
    void rangeChange (int x, int y, Line add) {
        int u = line.size();
        line.push_back(add);
        std::function<void(int, int, int, int)>
        range_change = [&] (int l, int r, int p, int u) {
            int &v = id[p], m = (l + r) / 2;
            if (cmp(line, u, v, real[m])) {
                swap(u, v);
            }
            if (cmp(line, u, v, real[l])) {

```

```

        range_Change(l, m, p * 2, u);
    }
    if (cmp(line, u, v, real[r - 1])) {
        range_Change(m, r, p * 2 + 1, u);
    }
};

std::function<void(int, int, int)>
range_find = [&] (int l, int r, int p) {
    if (real[l] >= y || real[r] <= x) {
        return;
    }
    if (x <= real[l] && real[r] <= y) {
        range_Change(l, r, p, u);
        return;
    }
    int m = (l + r) / 2;
    range_find(l, m, p * 2);
    range_find(m, r, p * 2 + 1);
};

range_find(0, n, 1);
}

void insert(Line add) {
    rangeChange(real[0], real.back(), add);
}

int Query(int x) {
    std::function<int(int, int, int)>
    Query = [&] (int l, int r, int p) {
        if (r - l == 1) {
            return id[p];
        }
        int m = (l + r) / 2;
        int u = id[p], v = -1;
        if (x < real[m]) {
            v = Query(l, m, p * 2);
        } else {
            v = Query(m, r, p * 2 + 1);
        }
        return cmp(line, u, v, x) ? u : v;
    };
    return Query(0, n, 1);
}

T slope_dp_Query(int x) {
    return line[Query(x)](x);
}

};

template<typename T>
struct Line {
    T k, b;
    Line(T k = 0, T b = 0) : k(k), b(b){}
    T operator()(T x) {
        return __int128(k) * x + b;
    }
};

template<>
struct Line<double> {

```

```

double k, b;
Line(double k = 0, double b = 0) : k(k), b(b){}
template<typename T>
Line(T x0, T y0, T x1, T y1) {
    if (x0 == x1) {
        k = 0;
        b = std::max(y0, y1);
    } else {
        k = (y0 - y1) / (0. + x0 - x1);
        b = y0 - k * x0;
    }
}
double operator()(double x) {
    return k * x + b;
}
};

template<typename T>
struct Cmp {
    bool operator() (vector<Line<T>> &line, int u, int v, T x) {
        return line[u](x) < line[v](x) || (line[u](x) == line[v](x) && u < v);
    }
};

template<>
struct Cmp<double> {
    bool operator() (vector<Line<double>> &line, int u, int v, double x) {
        constexpr double exp = 1e-9;
        return line[u](x) - line[v](x) > exp || (std::abs(line[u](x) - line[v]
(x)) <= exp && u < v);
    }
};

template<typename T, typename T1 = int>
using SegmentTree =
    Li_Chao_SegmentTree<T1, Line<T>, Cmp<T>>;

```

## 扫描线

```

struct ScanLine {
    int n;
    struct Line {
        int x1, x2, y;
        int type;
        bool operator<(Line another) const {
            return y < another.y;
        }
    };
};

struct Info {
    int l, r;
    int len = 0, cnt = 0;
};
vector<Info> info;
vector<Line> line;
vector<int> X;

```



```

void add(int x1, int y1, int x2, int y2) {
    line.push_back({x1, x2, y1, 1});
    line.push_back({x1, x2, y2, -1});
    x.push_back(x1);
    x.push_back(x2);
}

int work(int n) {
    sort(line.begin(), line.end());
    sort(x.begin(), x.end());
    int tot = unique(x.begin(), x.end()) - x.begin();
    vector<Info> init_;
    for (int i = 0; i < tot - 1; i++) {
        init_.push_back({i + 1, i + 1, 0, 0});
    }
    init(init_);
    int ans = 0;
    for (int i = 0; i < 2 * n - 1; i++) {
        modify(1, line[i].x1, line[i].x2, line[i].type);
        ans += info[1].len * (line[i + 1].y - line[i].y);
    }
    return ans;
}

ScanLine() : n(0) {};

void init(const vector<Info> &_init) {
    n = (int)_init.size();
    info.assign(n * 8, Info());
    function<void(int, int, int)> build = [&](int p, int l, int r) {
        info[p].l = l;
        info[p].r = r;
        if (l == r) {
            info[p] = _init[l - 1];
            return;
        }
        int m = (l + r) / 2;
        build(2 * p, l, m);
        build(2 * p + 1, m + 1, r);
        pull(p);
    };
    build(1, 1, n);
}

void pull(int p) {
    if (info[p].cnt) {
        info[p].len = x[info[p].r] - x[info[p].l - 1];
    } else {
        info[p].len = info[2 * p].len + info[2 * p + 1].len;
    }
}

void modify(int p, int L, int R, int val) {
    int l = info[p].l;
    int r = info[p].r;

```

```

        if (X[r] <= L || R <= X[l - 1]) {
            return;
        }
        if (L <= X[l - 1] && X[r] <= R) {
            info[p].cnt += val;
            pull(p);
            return;
        }
        modify(2 * p, L, R, val);
        modify(2 * p + 1, L, R, val);
        pull(p);
    }
};

```

## link-cut-tree

### 单点

```

template<class Info>
struct LinkCutTree {
    struct node {
        int s[2], p, tag;
        Info mval;
    };
    int n;
    vector<node> tree;

    int &fa(int x) { return tree[x].p; }
    int &lc(int x) { return tree[x].s[0]; }
    int &rc(int x) { return tree[x].s[1]; }
    bool notroot(int x) {
        return tree[tree[x].p].s[0] == x || tree[tree[x].p].s[1] == x;
    }
    // 不能以0开头
    LinkCutTree(int n) : n(n) { tree.resize(n + 1); tree[0].mval.defaultclear(); }

private:
    void pull(int x) {
        tree[x].mval.update(tree[lc(x)].mval, tree[rc(x)].mval);
    }

    void push(int x) {
        if (tree[x].tag) {
            swap(lc(x), rc(x));
            tree[lc(x)].mval.reverse();
            tree[rc(x)].mval.reverse();
            tree[rc(x)].tag ^= 1;
            tree[lc(x)].tag ^= 1;
            tree[x].tag = 0;
        }
    }

    void maintain(int x) {

```

```

        if (notroot(x)) maintain(fa(x));
        push(x);
    }

    void rotate(int x) {
        int y = fa(x), z = fa(y);
        int k = rc(y) == x;
        if (notroot(y))
            tree[z].s[rc(z) == y] = x;
        fa(x) = z;
        tree[y].s[k] = tree[x].s[k ^ 1];
        fa(tree[x].s[k ^ 1]) = y;
        tree[x].s[k ^ 1] = y;
        fa(y) = x;
        pull(y);
    }

public:
    void splay(int x) {
        maintain(x);
        while (notroot(x)) {
            int y = fa(x), z = fa(y);
            if (notroot(y))
                ((rc(z) == y) ^ (rc(y) == x))
                ? rotate(x) : rotate(y);
            rotate(x);
        }
        pull(x);
    }

    void access(int x) {
        for (int y = 0; x;) {
            splay(x);
            rc(x) = y;
            pull(x);
            y = x;
            x = fa(x);
        }
    }

    void makeroot(int x) {
        access(x);
        splay(x);
        tree[x].tag ^= 1;
    }

    //y变成原树和辅助树的根
    const Info &split(int x, int y) {
        makeroot(x);
        access(y);
        splay(y);
        return tree[y].mval;
    }

    int findroot(int x) {
        access(x);
    }

```

```

        splay(x);
        while (lc(x))
            push(x, x = lc(x));
        splay(x);
        return x;
    }

    void link(int x, int y) {
        makeroot(x);
        if (findroot(y) != x) fa(x) = y;
    }

    void cut(int x, int y) {
        makeroot(x);
        if (findroot(y) == x
            && fa(y) == x && !lc(y)) {
            rc(x) = fa(y) = 0;
            pull(x);
        }
    }

    void modify(int x, const Info &val) {
        splay(x);
        tree[x].mval.modify(val);
        pull(x);
    }

    bool same(int x, int y) {
        makeroot(x);
        return findroot(y) == x;
    }
    node &operator[](int x) {
        return tree[x];
    }
};

struct Info {
    int v = 1; int id = -1; int sum = 0; int max = 0;
    void reverse() {}
    void modify(const Info& rhs) {
        v = rhs.v;
    }
    void update(const Info &lhs, const Info &rhs) {
        sum = lhs.sum + v + rhs.sum;
        max = std::max({lhs.max, id, rhs.max});
    }
    void defaultclear() {
        v = 0;
    }
};

using Tree = LinkCutTree<Info>;

```

## 区间

```
template<class Info, class Tag>
struct LazyLinkCutTree {
    struct node {
        int s[2], p, tag;
        Info mval;
        Tag mtag;
    };
    int n;
    vector<node> tree;

    int &fa(int x) { return tree[x].p; }
    int &lc(int x) { return tree[x].s[0]; }
    int &rc(int x) { return tree[x].s[1]; }
    bool notroot(int x) {
        return tree[tree[x].p].s[0] == x || tree[tree[x].p].s[1] == x;
    }
    // 不能以0开头
    LazyLinkCutTree(int n) : n(n) {
        tree.resize(n + 1);
        tree[0].mtag.default_clear();
        tree[0].mval.default_clear();
    }

private:
    void pull(int x) {
        tree[x].mval.update(tree[lc(x)].mval, tree[rc(x)].mval);
    }

    void apply(int x, const Tag &rhs) {
        if (x) {
            tree[x].mval.apply(rhs);
            tree[x].mtag.apply(rhs);
        }
    }

    void push(int x) {
        if (tree[x].tag) {
            swap(lc(x), rc(x));
            tree[lc(x)].mval.reverse();
            tree[rc(x)].mval.reverse();
            tree[rc(x)].tag ^= 1;
            tree[lc(x)].tag ^= 1;
            tree[x].tag = 0;
        }
        if (bool(tree[x].mtag)) {
            apply(lc(x), tree[x].mtag);
            apply(rc(x), tree[x].mtag);
            tree[x].mtag.clear();
        }
    }

    void maintain(int x) {
        if (notroot(x)) maintain(fa(x));
```

```

    push(x);
}

void rotate(int x) {
    int y = fa(x), z = fa(y);
    int k = rc(y) == x;
    if (notroot(y))
        tree[z].s[rc(z) == y] = x;
    fa(x) = z;
    tree[y].s[k] = tree[x].s[k ^ 1];
    fa(tree[x].s[k ^ 1]) = y;
    tree[x].s[k ^ 1] = y;
    fa(y) = x;
    pull(y);
}

public:
void splay(int x) {
    maintain(x);
    while (notroot(x)) {
        int y = fa(x), z = fa(y);
        if (notroot(y))
            ((rc(z) == y) ^ (rc(y) == x))
            ? rotate(x) : rotate(y);
        rotate(x);
    }
    pull(x);
}

void access(int x) {
    for (int y = 0; x;) {
        splay(x);
        rc(x) = y;
        pull(x);
        y = x;
        x = fa(x);
    }
}

void makeroot(int x) {
    access(x);
    splay(x);
    tree[x].tag ^= 1;
}

//y变成原树和辅助树的根
const Info &split(int x, int y) {
    makeroot(x);
    access(y);
    splay(y);
    return tree[y].mval;
}

int findroot(int x) {
    access(x);
    splay(x);
}

```

```

        while (lc(x))
            push(x), x = lc(x);
        splay(x);
        return x;
    }

    void link(int x, int y) {
        makeroot(x);
        if (findroot(y) != x) fa(x) = y;
    }

    void cut(int x, int y) {
        makeroot(x);
        if (findroot(y) == x
            && fa(y) == x && !lc(y)) {
            rc(x) = fa(y) = 0;
            pull(x);
        }
    }

    void modify(int x, const Info &val) {
        splay(x);
        tree[x].mval.modify(val);
        pull(x);
    }

    void line_modify(int u, int v, const Tag &rhs) {
        split(u, v);
        apply(v, rhs);
    }

    bool same(int x, int y) {
        makeroot(x);
        return findroot(y) == x;
    }

    node &operator[](int x) {
        return tree[x];
    }
};

struct Tag {
    int set = 0;
    void apply(const Tag &rhs) {
        set = rhs.set;
    }
    void clear() {
        set = 0;
    }
    operator bool() {
        return set != 0;
    }
    void default_clear() {}
};

struct Info {

```

```

int c = 0; int sum = 0, l = 0, r = 0, id = 0;
void reverse() {
    swap(l, r);
}
void modify(const Info& rhs) {
    l = r = c = rhs.c;
}
void update(const Info &lhs, const Info &rhs) {
    sum = lhs.sum + (c != lhs.r && lhs.r != 0) + (c != rhs.l && rhs.l != 0) +
rhs.sum;
    l = (lhs.r == 0 ? c : lhs.l);
    r = (rhs.l == 0 ? c : rhs.r);
}
void apply(const Tag &rhs) {
    l = r = c = rhs.set; sum = 0;
}
void show() const {
    debug(id);
    cerr << l << ' ' << c << ' ' << r << ' ' << sum << endl;
}
void default_clear() {}
};

using Tree = LazyLinkCutTree<Info, Tag>;

```

## 并查集

### 普通

```

struct DSU {
    std::vector<int> f, siz;

    DSU() {}
    DSU(int n) {
        init(n);
    }

    void init(int n) {
        f.resize(n);
        std::iota(f.begin(), f.end(), 0);
        siz.assign(n, 1);
    }

    int find(int x) {
        while (x != f[x]) {
            x = f[x] = f[f[x]];
        }
        return x;
    }

    bool same(int x, int y) {
        return find(x) == find(y);
    }
}

```



```

bool merge(int x, int y) {
    x = find(x);
    y = find(y);
    if (x == y) {
        return false;
    }
    siz[x] += siz[y];
    f[y] = x;
    return true;
}

int size(int x) {
    return siz[find(x)];
}
};

```

## 可撤回

```

struct DSU {
    std::vector<int> fa, size_;
    std::stack<pair<int &, int>> history_size;
    std::stack<pair<int &, int>> history_fa;

    DSU() {}

    DSU(int n) {
        init(n);
    }

    void init(int n) {
        fa.resize(n);
        std::iota(fa.begin(), fa.end(), 0);
        size_.assign(n, 1);
    }

    int find(int x) {
        while (x != fa[x]) x = fa[x];
        return x;
    }

    int size(int x) {
        return size_[find(x)];
    }

    bool same(int u, int v) {
        return find(u) == find(v);
    }

    void merge(int u, int v) {
        int x = find(u);
        int y = find(v);
        if (x == y) return;
        if (size_[x] < size_[y]) std::swap(x, y);
        history_size.emplace(size_[x], size_[x]);
        size_[x] = size_[x] + size_[y];
    }
};

```

```

        history_fa.emplace(fa[y], fa[y]);
        fa[y] = x;
    }

    int history() {
        return history_fa.size();
    }

    void roll(int h) {
        while (history_fa.size() > h) {
            history_fa.top().first = history_fa.top().second;
            history_fa.pop();
            history_size.top().first = history_size.top().second;
            history_size.pop();
        }
    }
};

```

## 小波树

```

struct BitRank {
    // block 管理一行一行的bit
    std::vector<unsigned long long> block;
    std::vector<unsigned int> count;
    BitRank() {}
    // 位向量长度
    void resize(const unsigned int num) {
        block.resize(((num + 1) >> 6) + 1, 0);
        count.resize(block.size(), 0);
    }
    // 设置i位bit
    void set(const unsigned int i, const unsigned long long val) {
        block[i >> 6] |= (val << (i & 63));
    }
    void build() {
        for (unsigned int i = 1; i < block.size(); i++) {
            count[i] = count[i - 1] + __builtin_popcountll(block[i - 1]);
        }
    }
    // [0, i) 1的个数
    unsigned int rank1(const unsigned int i) const {
        return count[i >> 6] +
            __builtin_popcountll(block[i >> 6] & ((1ULL << (i & 63)) - 1ULL));
    }
    // [i, j) 1的个数
    unsigned int rank1(const unsigned int i, const unsigned int j) const {
        return rank1(j) - rank1(i);
    }
    // [0, i) 0的个数
    unsigned int rank0(const unsigned int i) const { return i - rank1(i); }
    // [i, j) 0的个数
    unsigned int rank0(const unsigned int i, const unsigned int j) const {
        return rank0(j) - rank0(i);
    }
};

```

```

class WaveletMatrix {
private:
    unsigned int height;
    std::vector<BitRank> B;
    std::vector<int> pos;

public:
    WaveletMatrix() {}
    WaveletMatrix(std::vector<int> vec)
        : WaveletMatrix(vec, *std::max_element(vec.begin(), vec.end()) + 1) {}
    // sigma: 字母表大小(字符串的话), 数字序列的话是数的种类
    WaveletMatrix(std::vector<int> vec, const unsigned int sigma) {
        init(vec, sigma);
    }
    void init(std::vector<int>& vec, const unsigned int sigma) {
        height = (sigma == 1) ? 1 : (64 - __builtin_clz11(sigma - 1));
        B.resize(height), pos.resize(height);
        for (unsigned int i = 0; i < height; ++i) {
            B[i].resize(vec.size());
            for (unsigned int j = 0; j < vec.size(); ++j) {
                B[i].set(j, get(vec[j], height - i - 1));
            }
            B[i].build();
            auto it = stable_partition(vec.begin(), vec.end(), [&](int c) {
                return !get(c, height - i - 1);
            });
            pos[i] = it - vec.begin();
        }
    }

    int get(const int val, const int i) { return val >> i & 1; }
    // [l, r) 中val出现的频率

    int rank(const int val, const int l, const int r) {
        return rank(val, r) - rank(val, l);
    }
    // [0, i) 中val出现的频率
    int rank(int val, int i) {
        int p = 0;
        for (unsigned int j = 0; j < height; ++j) {
            if (get(val, height - j - 1)) {
                p = pos[j] + B[j].rank1(p);
                i = pos[j] + B[j].rank1(i);
            } else {
                p = B[j].rank0(p);
                i = B[j].rank0(i);
            }
        }
        return i - p;
    }
    // [l, r) 中k小
    int quantile(int k, int l, int r) {
        int res = 0;
        for (unsigned int i = 0; i < height; ++i) {

```

```

        const int j = B[i].rank0(l, r);
        if (j > k) {
            l = B[i].rank0(l);
            r = B[i].rank0(r);
        } else {
            l = pos[i] + B[i].rank1(l);
            r = pos[i] + B[i].rank1(r);
            k -= j;
            res |= (1 << (height - i - 1));
        }
    }
    return res;
}

int rangefreq(const int i, const int j, const int a, const int b, const int l,
              const int r, const int x) {
    if (i == j || r <= a || b <= l) return 0;
    const int mid = (l + r) >> 1;
    if (a <= l && r <= b) {
        return j - i;
    } else {
        const int left =
            rangefreq(B[x].rank0(i), B[x].rank0(j), a, b, l, mid, x + 1);
        const int right = rangefreq(pos[x] + B[x].rank1(i),
                                     pos[x] + B[x].rank1(j), a, b, mid, r, x + 1);
        return left + right;
    }
}

// [l,r) 在[a, b) 值域的数字个数
int rangefreq(const int l, const int r, const int a, const int b) {
    return rangefreq(l, r, a, b, 0, 1 << height, 0);
}

int rangemin(const int i, const int j, const int a, const int b, const int l,
              const int r, const int x, const int val) {
    if (i == j || r <= a || b <= l) return -1;
    if (r - l == 1) return val;
    const int mid = (l + r) >> 1;
    const int res =
        rangemin(B[x].rank0(i), B[x].rank0(j), a, b, l, mid, x + 1, val);
    if (res < 0)
        return rangemin(pos[x] + B[x].rank1(i), pos[x] + B[x].rank1(j), a, b, mid,
                        r, x + 1, val + (1 << (height - x - 1)));
    else
        return res;
}

// [l,r) 在[a,b) 值域内存在的最小值是什么, 不存在返回-1
int rangemin(int l, int r, int a, int b) {
    return rangemin(l, r, a, b, 0, 1 << height, 0, 0);
}
};

```

## two\_sat

```
struct TwoSat {
    int n;
    std::vector<std::vector<int>>> e;
    std::vector<bool> ans;
    TwoSat(int n) : n(n), e(2 * n), ans(n) {}
    void addClause(int u, bool f, int v, bool g) {
        e[2 * u + !f].push_back(2 * v + g);
        e[2 * v + !g].push_back(2 * u + f);
    }
    void notClause(int u, bool f, int v, bool g) {
        addClause(u, !f, v, !g);
    }
    bool satisfiable() {
        std::vector<int> id(2 * n, -1), dfn(2 * n, -1), low(2 * n, -1);
        std::vector<int> stk;
        int now = 0, cnt = 0;
        std::function<void(int)> tarjan = [&](int u) {
            stk.push_back(u);
            dfn[u] = low[u] = now++;
            for (auto v : e[u]) {
                if (dfn[v] == -1) {
                    tarjan(v);
                    low[u] = std::min(low[u], low[v]);
                } else if (id[v] == -1) {
                    low[u] = std::min(low[u], dfn[v]);
                }
            }
            if (dfn[u] == low[u]) {
                int v;
                do {
                    v = stk.back();
                    stk.pop_back();
                    id[v] = cnt;
                } while (v != u);
                ++cnt;
            }
        };
        for (int i = 0; i < 2 * n; ++i) if (dfn[i] == -1) tarjan(i);
        for (int i = 0; i < n; ++i) {
            if (id[2 * i] == id[2 * i + 1]) return false;
            ans[i] = id[2 * i] > id[2 * i + 1];
        }
        return true;
    }
    std::vector<bool> answer() { return ans; }
};
```

## 小矩阵

```
struct Matrix {
    Z a[4];
    Matrix() :a{} {}
    friend Matrix operator*(const Matrix &lhs, const Matrix &rhs) {
        Matrix res;
        res.a[0] = (lhs.a[0] * rhs.a[0] + lhs.a[1] * rhs.a[2]) ;
        res.a[1] = (lhs.a[0] * rhs.a[1] + lhs.a[1] * rhs.a[3]) ;
        res.a[2] = (lhs.a[2] * rhs.a[0] + lhs.a[3] * rhs.a[2]) ;
        res.a[3] = (lhs.a[2] * rhs.a[1] + lhs.a[3] * rhs.a[3]) ;
        return res;
    }
};

struct Matrix {
    Z a[9];

    Matrix() : a{} {}

    friend Matrix operator*(const Matrix &lhs, const Matrix &rhs) {
        Matrix res;
        res.a[0] = lhs.a[0] * rhs.a[0] + lhs.a[1] * rhs.a[3] + lhs.a[2] *
rhs.a[6];
        res.a[1] = lhs.a[0] * rhs.a[1] + lhs.a[1] * rhs.a[4] + lhs.a[2] *
rhs.a[7];
        res.a[2] = lhs.a[0] * rhs.a[2] + lhs.a[1] * rhs.a[5] + lhs.a[2] *
rhs.a[8];

        res.a[3] = lhs.a[3] * rhs.a[0] + lhs.a[4] * rhs.a[3] + lhs.a[5] *
rhs.a[6];
        res.a[4] = lhs.a[3] * rhs.a[1] + lhs.a[4] * rhs.a[4] + lhs.a[5] *
rhs.a[7];
        res.a[5] = lhs.a[3] * rhs.a[2] + lhs.a[4] * rhs.a[5] + lhs.a[5] *
rhs.a[8];

        res.a[6] = lhs.a[6] * rhs.a[0] + lhs.a[7] * rhs.a[3] + lhs.a[8] *
rhs.a[6];
        res.a[7] = lhs.a[6] * rhs.a[1] + lhs.a[7] * rhs.a[4] + lhs.a[8] *
rhs.a[7];
        res.a[8] = lhs.a[6] * rhs.a[2] + lhs.a[7] * rhs.a[5] + lhs.a[8] *
rhs.a[8];

        return res;
    }
};
```

# 平衡树

## splay

```
constexpr int max_size = 262144000;
uint8_t buf[max_size];
uint8_t *head = buf;

using u32 = uint32_t;

template <class T>
struct u32_p {
    u32 x;
    u32_p(u32 x = 0) : x(x) {}
    T *operator->() {
        return (T *) (buf + x);
    }
    operator bool() {
        return x;
    }
    operator u32() {
        return x;
    }
    bool operator==(u32_p rhs) const {
        return x == rhs.x;
    }
    static u32_p __new() {
        // assert(x < max_size);
        return (head += sizeof(T)) - buf;
    }
};

template<class Info, class Tag>
struct Balance_Tree {
    struct Tree {
        Info info;
        Tag tag;
        bool rev;
        using Tp = u32_p<Tree>;
        Tp ch[2], p;
    };
    using Tp = u32_p<Tree>;

    Balance_Tree() {
        Tp()->info.Null();
    }
    Tp __new () {
        return Tp::__new();
    }

    Tp build (int l, int r) {
        if (l > r) return 0;
        int m = l + r >> 1;
        Tp p = __new();
```

```

    p->ch[0] = build(l, m - 1);
    if (p->ch[0]) p->ch[0]->p = p;
    {
        // fun
    }
    p->ch[1] = build(m + 1, r);
    if (p->ch[1]) p->ch[1]->p = p;
    pull(p);
    return p;
}

template<typename F>
Tp build (int l, int r, F fun) {
    if (l > r) return 0;
    int m = l + r >> 1;
    Tp p = __new();
    p->ch[0] = build(l, m - 1, fun);
    if (p->ch[0]) p->ch[0]->p = p;
    fun(p, m);
    p->ch[1] = build(m + 1, r, fun);
    if (p->ch[1]) p->ch[1]->p = p;
    pull(p);
    return p;
}

bool pos(Tp t) {
    return t->p->ch[1] == t;
}

void apply(Tp t, const Tag &v) {
    if (t) {
        t->info.apply(v);
        t->tag.apply(v);
    }
}

void push(Tp t) {
    if (t->rev) {
        t->ch[0]->rev ^= 1;
        t->ch[1]->rev ^= 1;
        swap(t->ch[0], t->ch[1]);
        t->rev = 0;
    }
    if (t->tag) {
        apply(t->ch[0], t->tag);
        apply(t->ch[1], t->tag);
        t->tag = Tag();
    }
}

void pull(Tp t) {
    t->info.up(t->ch[0]->info, t->ch[1]->info);
}

void rotate(Tp t) {
    Tp q = t->p;
    int x = !pos(t);

```



```

    q->ch[!x] = t->ch[x];
    if (t->ch[x]) t->ch[x]->p = q;
    t->p = q->p;
    if (q->p) q->p->ch[pos(q)] = t;
    t->ch[x] = q;
    q->p = t;
    pull(q);
}

void pushall(Tp t) {
    if (t->p) pushall(t->p);
    push(t);
}

void splay(Tp t, Tp top = 0) {
    pushall(t);
    while (t->p != top) {
        if (t->p->p != top)
            rotate(pos(t) ^ pos(t->p) ? t : t->p);
        rotate(t);
    }
    pull(t);
}

void insert(Tp &t, Tp x) {
    Tp p = 0;

    while (t && t->info.x != x->info.x) {
        push(t);
        p = t;
        t = t->ch[x->info.x > t->info.x];
    }

    if (!t) {
        t = x;
        t->p = p;
        if (p) p->ch[t->info.x > p->info.x] = t;
    } else {
        t->info.apply(x->info);
    }
    splay(t);
}

Tp rank(Tp &t, int k) {
    while (true) {
        push(t);
        if (k > t->ch[0]->info.sz + t->info.cnt) {
            k -= t->ch[0]->info.sz + t->info.cnt;
            t = t->ch[1];
        } else if (k <= t->ch[0]->info.sz) {
            t = t->ch[0];
        } else break;
    }
    splay(t);
    return t;
}

```

```

Tp shrink(Tp &t, int l, int r) {
    if (r == t->info.sz && l == 1) {
        return t;
    } else if (r == t->info.sz) {
        rank(t, l - 1);
        return t->ch[1];
    } else if (l == 1) {
        rank(t, r + 1);
        return t->ch[0];
    } else {
        Tp lhs = rank(t, l - 1);
        rank(t, r + 1);
        splay(lhs, t);
        return lhs->ch[1];
    }
}

void find(Tp &t, const Info &rhs) {
    // if (!t) {
    //     return;
    // }
    while (t->info.x != rhs.x && t->ch[rhs.x > t->info.x]) {
        t = t->ch[rhs.x > t->info.x];
    }
    splay(t);
}

Tp prev(Tp &t, const Info &rhs) {
    find(t, rhs);
    if (t->info.x >= rhs.x) {
        t = t->ch[0];
        while (t->ch[1]) {
            t = t->ch[1];
        }
    }
    splay(t);
    return t;
}

Tp next(Tp &t, const Info &rhs) {
    find(t, rhs);
    if (rhs.x >= t->info.x) {
        t = t->ch[1];
        while (t->ch[0]) {
            t = t->ch[0];
        }
    }
    splay(t);
    return t;
}

void erase(Tp &t, const Info &rhs) {
    find(t, rhs);
    if (t->info == rhs && t->info.erase()) {
        Tp lhs = t->ch[0], rhs = t->ch[1];
    }
}

```

```

        lhs->p = 0, rhs->p = 0;
        t = merge(lhs, rhs);
    }
}

void dfs(Tp t, int dep = 0) {
    if (!t) {
        return;
    }
    push(t);
    dfs(t->ch[0], dep + 1);
    cout << t->info.x << ' ';
    // for (int i = 0; i < dep; i += 1) cerr << '\t';
    // std::cerr << t->info << "\n";
    dfs(t->ch[1], dep + 1);
}

std::pair<Tp, Tp> split_by_val(Tp t, int x) {
    if (!t) {
        return {t, t};
    }
    Tp v = 0;
    Tp j = t;
    for (Tp i = t; i; ) {
        push(i);
        j = i;
        if (i->info >= x) {
            v = i;
            i = i->ch[0];
        } else {
            i = i->ch[1];
        }
    }

    splay(j);
    if (!v) {
        return {j, 0};
    }

    splay(v);

    Tp u = v->ch[0];
    if (u) {
        v->ch[0] = u->p = 0;
        pull(v);
    }
    return {u, v};
}

std::pair<Tp, Tp> split_by_rank(Tp t, int x) {
    if (t->info.sz < x) {
        return {t, 0};
    }

    rank(t, x);

```

```

    Tp u = t->ch[0];
    if (u) {
        t->ch[0] = u->p = 0;
        pull(t);
    }
    return {u, t};
}

Tp merge(Tp l, Tp r) {
    if (l.x * r.x == 0) {
        return l.x | r.x;
    }
    Tp i = l;
    push(i);
    for (; i->ch[1]; i = i->ch[1], push(i));
    splay(i);
    i->ch[1] = r;
    r->p = i;
    pull(i);
    return i;
}
};

struct Tag {
    constexpr operator bool() {
        return false;
    }
    void apply(const Tag &t) {}
};

struct Info {
    int x = 0, cnt = 1, sz = 1;
    void up(const Info &lhs, const Info &rhs) {
        sz = lhs.sz + cnt + rhs.sz;
    }
    bool erase() {
        return !(-- cnt);
    }
    void apply(const Tag &t) {}
    void apply(const Info &t) {
        cnt += 1; sz += 1;
    }
    friend ostream &operator<<(ostream &cout, Info rhs) {
        return cout << rhs.x << ' ' << rhs.cnt << ' ' << rhs.sz;
    }
    void Null() {}
};

using BT = Balance_Tree<Info, Tag>;
using Tp = BT::Tp;
BT tree;

```

```
constexpr int inf = 1e9;
```

```
constexpr int max_size = 262144000;
```

```

uint8_t buf[max_size];
uint8_t *head = buf;

using u32 = uint32_t;

template <class T>
struct u32_p {
    u32 x;
    u32_p(u32 x = 0) : x(x) {}
    T *operator->() {
        return (T *) (buf + x);
    }
    operator bool() {
        return x;
    }
    operator u32() {
        return x;
    }
    bool operator==(u32_p rhs) const {
        return x == rhs.x;
    }
    static u32_p __new() {
        // assert(x < max_size);
        return (head += sizeof(T)) - buf;
    }
};

struct Tree {
    int add = 0;
    int val = 0;
    int id = 0;
    u32_p<Tree> ch[2], p;
};

using Tp = u32_p<Tree>;

Tp __new() {
    return Tp::__new();
}

int pos(Tp t) {
    return t->p->ch[1] == t;
}

void add(Tp t, int v) {
    t->val += v;
    t->add += v;
}

void push(Tp t) {
    if (t->ch[0]) {
        add(t->ch[0], t->add);
    }
    if (t->ch[1]) {
        add(t->ch[1], t->add);
    }
}

```

```

    t->add = 0;
}

void rotate(Tp t) {
    Tp q = t->p;
    int x = !pos(t);
    q->ch[!x] = t->ch[x];
    if (t->ch[x]) t->ch[x]->p = q;
    t->p = q->p;
    if (q->p) q->p->ch[pos(q)] = t;
    t->ch[x] = q;
    q->p = t;
}

void splay(Tp t) {
    std::vector<Tp> s;
    for (Tp i = t; i->p; i = i->p) s.push_back(i->p);
    while (!s.empty()) {
        push(s.back());
        s.pop_back();
    }
    push(t);
    while (t->p) {
        if (t->p->p) {
            if (pos(t) == pos(t->p)) rotate(t->p);
            else rotate(t);
        }
        rotate(t);
    }
}

void insert(Tp &t, Tp x, Tp p = 0) {
    if (!t) {
        t = x;
        x->p = p;
        return;
    }

    push(t);
    if (x->val < t->val) {
        insert(t->ch[0], x, t);
    } else {
        insert(t->ch[1], x, t);
    }
}

void dfs(Tp t) {
    if (!t) {
        return;
    }
    push(t);
    dfs(t->ch[0]);
    std::cerr << t->val << " ";
    dfs(t->ch[1]);
}

```

```

std::pair<Tp , Tp > split(Tp t, int x) {
    if (!t) {
        return {t, t};
    }
    Tp v = 0;
    Tp j = t;
    for (Tp i = t; i; ) {
        push(i);
        j = i;
        if (i->val >= x) {
            v = i;
            i = i->ch[0];
        } else {
            i = i->ch[1];
        }
    }

    splay(j);
    if (!v) {
        return {j, 0};
    }

    splay(v);

    Tp u = v->ch[0];
    if (u) {
        v->ch[0] = u->p = 0;
    }
    return {u, v};
}

Tp merge(Tp l, Tp r) {
    if (!l) {
        return r;
    }
    if (!r) {
        return l;
    }
    Tp i = l;
    while (i->ch[1]) {
        i = i->ch[1];
    }
    splay(i);
    i->ch[1] = r;
    r->p = i;
    return i;
}

```

# 数学

## 取模类

```
using i64 = long long;
template<class T>
constexpr T power(T a, i64 b) {
    T res = 1;
    for (; b; b /= 2, a *= a) {
        if (b % 2) {
            res *= a;
        }
    }
    return res;
}

constexpr i64 mul(i64 a, i64 b, i64 p) {
    i64 res = a * b - i64(1.L * a * b / p) * p;
    res %= p;
    if (res < 0) {
        res += p;
    }
    return res;
}

template<i64 P>
struct MLong {
    i64 x;
    constexpr MLong() : x{} {}
    constexpr MLong(i64 x) : x{norm(x % getMod())} {}

    static i64 Mod;
    constexpr static i64 getMod() {
        if (P > 0) {
            return P;
        } else {
            return Mod;
        }
    }
}

constexpr static void setMod(i64 Mod_) {
    Mod = Mod_;
}

constexpr i64 norm(i64 x) const {
    if (x < 0) {
        x += getMod();
    }
    if (x >= getMod()) {
        x -= getMod();
    }
    return x;
}

constexpr i64 val() const {
    return x;
}

explicit constexpr operator i64() const {
```



```

        return x;
    }
    constexpr MLong operator-() const {
        MLong res;
        res.x = norm(getMod() - x);
        return res;
    }
    constexpr MLong inv() const {
        assert(x != 0);
        return power(*this, getMod() - 2);
    }
    constexpr MLong &operator*=(MLong rhs) & {
        x = mul(x, rhs.x, getMod());
        return *this;
    }
    constexpr MLong &operator+=(MLong rhs) & {
        x = norm(x + rhs.x);
        return *this;
    }
    constexpr MLong &operator-=(MLong rhs) & {
        x = norm(x - rhs.x);
        return *this;
    }
    constexpr MLong &operator/=(MLong rhs) & {
        return *this *= rhs.inv();
    }
    friend constexpr MLong operator*(MLong lhs, MLong rhs) {
        MLong res = lhs;
        res *= rhs;
        return res;
    }
    friend constexpr MLong operator+(MLong lhs, MLong rhs) {
        MLong res = lhs;
        res += rhs;
        return res;
    }
    friend constexpr MLong operator-(MLong lhs, MLong rhs) {
        MLong res = lhs;
        res -= rhs;
        return res;
    }
    friend constexpr MLong operator/(MLong lhs, MLong rhs) {
        MLong res = lhs;
        res /= rhs;
        return res;
    }
    friend constexpr std::istream &operator>>(std::istream &is, MLong &a) {
        i64 v;
        is >> v;
        a = MLong(v);
        return is;
    }
    friend constexpr std::ostream &operator<<(std::ostream &os, const MLong &a) {
        return os << a.val();
    }
    friend constexpr bool operator==(MLong lhs, MLong rhs) {

```

```

        return lhs.val() == rhs.val();
    }
    friend constexpr bool operator!=(MLong lhs, MLong rhs) {
        return lhs.val() != rhs.val();
    }
};

template<>
i64 MLong<0LL>::Mod = i64(1E18) + 9;

template<int P>
struct MInt {
    int x;
    constexpr MInt() : x{} {}
    constexpr MInt(i64 x) : x{norm(x % getMod())} {}

    static int Mod;
    constexpr static int getMod() {
        if (P > 0) {
            return P;
        } else {
            return Mod;
        }
    }
}

constexpr static void setMod(int Mod_) {
    Mod = Mod_;
}

constexpr int norm(int x) const {
    if (x < 0) {
        x += getMod();
    }
    if (x >= getMod()) {
        x -= getMod();
    }
    return x;
}

constexpr int val() const {
    return x;
}

explicit constexpr operator int() const {
    return x;
}

explicit constexpr operator i64() const {
    return x;
}

constexpr MInt operator-() const {
    MInt res;
    res.x = norm(getMod() - x);
    return res;
}

constexpr MInt inv() const {
    assert(x != 0);
    return power(*this, getMod() - 2);
}

constexpr MInt &operator*=(MInt rhs) & {
    x = 1LL * x * rhs.x % getMod();
}

```

```

        return *this;
    }
    constexpr MInt &operator+=(MInt rhs) & {
        x = norm(x + rhs.x);
        return *this;
    }
    constexpr MInt &operator-=(MInt rhs) & {
        x = norm(x - rhs.x);
        return *this;
    }
    constexpr MInt &operator/=(MInt rhs) & {
        return *this *= rhs.inv();
    }
    friend constexpr MInt operator*(MInt lhs, MInt rhs) {
        MInt res = lhs;
        res *= rhs;
        return res;
    }
    friend constexpr MInt operator+(MInt lhs, MInt rhs) {
        MInt res = lhs;
        res += rhs;
        return res;
    }
    friend constexpr MInt operator-(MInt lhs, MInt rhs) {
        MInt res = lhs;
        res -= rhs;
        return res;
    }
    friend constexpr MInt operator/(MInt lhs, MInt rhs) {
        MInt res = lhs;
        res /= rhs;
        return res;
    }
    friend constexpr std::istream &operator>>(std::istream &is, MInt &a) {
        i64 v;
        is >> v;
        a = MInt(v);
        return is;
    }
    friend constexpr std::ostream &operator<<(std::ostream &os, const MInt &a) {
        return os << a.val();
    }
    friend constexpr bool operator==(MInt lhs, MInt rhs) {
        return lhs.val() == rhs.val();
    }
    friend constexpr bool operator!=(MInt lhs, MInt rhs) {
        return lhs.val() != rhs.val();
    }
};

template<>
int MInt<0>::Mod = 998244353;

template<int V, int P>
constexpr MInt<P> CInv = MInt<P>(V).inv();

```

```
constexpr int P = 998244353;
using Z = MInt<P>;
```

## 多项式

```
std::vector<int> rev;
template<int P>
std::vector<MInt<P>> roots{0, 1};

template<int P>
constexpr MInt<P> findPrimitiveRoot() {
    MInt<P> i = 2;
    int k = __builtin_ctz(P - 1);
    while (true) {
        if (power(i, (P - 1) / 2) != 1) {
            break;
        }
        i += 1;
    }
    return power(i, (P - 1) >> k);
}

template<int P>
constexpr MInt<P> primitiveRoot = findPrimitiveRoot<P>();

template<>
constexpr MInt<998244353> primitiveRoot<998244353> {31};

template<int P>
constexpr void dft(std::vector<MInt<P>> &a) {
    int n = a.size();

    if (int(rev.size()) != n) {
        int k = __builtin_ctz(n) - 1;
        rev.resize(n);
        for (int i = 0; i < n; i++) {
            rev[i] = rev[i >> 1] >> 1 | (i & 1) << k;
        }
    }

    for (int i = 0; i < n; i++) {
        if (rev[i] < i) {
            std::swap(a[i], a[rev[i]]);
        }
    }

    if (roots<P>.size() < n) {
        int k = __builtin_ctz(roots<P>.size());
        roots<P>.resize(n);
        while ((1 << k) < n) {
            auto e = power(primitiveRoot<P>, 1 << (__builtin_ctz(P - 1) - k - 1));

            for (int i = 1 << (k - 1); i < (1 << k); i++) {
                roots<P>[2 * i] = roots<P>[i];
                roots<P>[2 * i + 1] = roots<P>[i] * e;
            }
        }
    }
}
```

```

        k++;
    }
}
for (int k = 1; k < n; k *= 2) {
    for (int i = 0; i < n; i += 2 * k) {
        for (int j = 0; j < k; j++) {
            MInt<P> u = a[i + j];
            MInt<P> v = a[i + j + k] * roots<P>[k + j];
            a[i + j] = u + v;
            a[i + j + k] = u - v;
        }
    }
}

template<int P>
constexpr void idft(std::vector<MInt<P>> &a) {
    int n = a.size();
    std::reverse(a.begin() + 1, a.end());
    dft(a);
    MInt<P> inv = (1 - P) / n;
    for (int i = 0; i < n; i++) {
        a[i] *= inv;
    }
}

template<int P = ::P>
struct Poly : public std::vector<MInt<P>> {
    using Value = MInt<P>;

    Poly() : std::vector<Value>() {}
    explicit constexpr Poly(int n) : std::vector<Value>(n) {}

    explicit constexpr Poly(const std::vector<Value> &a) : std::vector<Value>(a)
    {}
    constexpr Poly(const std::initializer_list<Value> &a) : std::vector<Value>(a)
    {}

    template<class InputIt, class = std::_RequireInputIter<InputIt>>
    explicit constexpr Poly(InputIt first, InputIt last) : std::vector<Value>
    (first, last) {}

    template<class F>
    explicit constexpr Poly(int n, F f) : std::vector<Value>(n) {
        for (int i = 0; i < n; i++) {
            (*this)[i] = f(i);
        }
    }

    constexpr Poly shift(int k) const {
        if (k >= 0) {
            auto b = *this;
            b.insert(b.begin(), k, 0);
            return b;
        } else if (this->size() <= -k) {
            return Poly();
        }
    }
}

```

```

    } else {
        return Poly(this->begin() + (-k), this->end());
    }
}

constexpr Poly trunc(int k) const {
    Poly f = *this;
    f.resize(k);
    return f;
}

constexpr friend Poly operator+(const Poly &a, const Poly &b) {
    Poly res(std::max(a.size(), b.size()));
    for (int i = 0; i < a.size(); i++) {
        res[i] += a[i];
    }
    for (int i = 0; i < b.size(); i++) {
        res[i] += b[i];
    }
    return res;
}

constexpr friend Poly operator-(const Poly &a, const Poly &b) {
    Poly res(std::max(a.size(), b.size()));
    for (int i = 0; i < a.size(); i++) {
        res[i] += a[i];
    }
    for (int i = 0; i < b.size(); i++) {
        res[i] -= b[i];
    }
    return res;
}

constexpr friend Poly operator-(const Poly &a) {
    std::vector<Value> res(a.size());
    for (int i = 0; i < int(res.size()); i++) {
        res[i] = -a[i];
    }
    return Poly(res);
}

constexpr friend Poly operator*(Poly a, Poly b) {
    if (a.size() == 0 || b.size() == 0) {
        return Poly();
    }
    if (a.size() < b.size()) {
        std::swap(a, b);
    }
    int n = 1, tot = a.size() + b.size() - 1;
    while (n < tot) {
        n *= 2;
    }
    if (((P - 1) & (n - 1)) != 0 || b.size() < 128) {

        Poly c(a.size() + b.size() - 1);
        for (int i = 0; i < a.size(); i++) {
            for (int j = 0; j < b.size(); j++) {
                c[i + j] += a[i] * b[j];
            }
        }
        return c;
    }
}

```

```

    }
    a.resize(n);
    b.resize(n);
    dft(a);
    dft(b);
    for (int i = 0; i < n; ++i) {
        a[i] *= b[i];
    }
    idft(a);
    a.resize(tot);
    return a;
}

constexpr friend Poly operator*(Value a, Poly b) {
    for (int i = 0; i < int(b.size()); i++) {
        b[i] *= a;
    }
    return b;
}

constexpr friend Poly operator*(Poly a, Value b) {
    for (int i = 0; i < int(a.size()); i++) {
        a[i] *= b;
    }
    return a;
}

constexpr friend Poly operator/(Poly a, Value b) {
    for (int i = 0; i < int(a.size()); i++) {
        a[i] /= b;
    }
    return a;
}

constexpr Poly &operator+=(Poly b) {
    return (*this) = (*this) + b;
}

constexpr Poly &operator-=(Poly b) {
    return (*this) = (*this) - b;
}

constexpr Poly &operator*=(Poly b) {
    return (*this) = (*this) * b;
}

constexpr Poly &operator*=(Value b) {
    return (*this) = (*this) * b;
}

constexpr Poly &operator/=(Value b) {
    return (*this) = (*this) / b;
}

template <class T>
constexpr Value operator() ( T x ) {
    Value ans = 0 ;
    Value cnt = 1 ;
    for ( int i = 0 ; i < this->size () ; ++ i ) {
        ans += (* this) [ i ] * cnt ;
        cnt *= x ;
    }
    return ans ;
}

constexpr Poly deriv() const {

```

```

    if (this->empty()) {
        return Poly();
    }
    assert (this->size() != 0);
    Poly res(this->size() - 1);
    for (int i = 0; i < this->size() - 1; ++i) {
        res[i] = (i + 1) * (*this)[i + 1];
    }
    return res;
}

constexpr Poly integr() const {
    Poly res(this->size() + 1);
    for (int i = 0; i < this->size(); ++i) {
        res[i + 1] = (*this)[i] / (i + 1);
    }
    return res;
}

constexpr Poly inv(int m) const {
    Poly x((*this)[0].inv());
    int k = 1;
    while (k < m) {
        k *= 2;
        x = (x * (Poly{2} - trunc(k) * x)).trunc(k);
    }
    return x.trunc(m);
}

constexpr Poly log(int m) const {
    return (deriv() * inv(m)).integr().trunc(m);
}

constexpr Poly exp(int m) const {
    Poly x{1};
    int k = 1;
    while (k < m) {
        k *= 2;
        x = (x * (Poly{1} - x.log(k) + trunc(k))).trunc(k);
    }
    return x.trunc(m);
}

constexpr Poly pow(int k, int m) const {
    int i = 0;
    while (i < this->size() && (*this)[i] == 0) {
        i++;
    }
    if (i == this->size() || 1LL * i * k >= m) {
        return Poly(m);
    }
    value v = (*this)[i];
    auto f = shift(-i) * v.inv();
    return (f.log(m - i * k) * k).exp(m - i * k).shift(i * k) * power(v, k);
}

constexpr Poly pow(int k, int m, int k2) const {
    int i = 0;
    while (i < this->size() && (*this)[i] == 0) {
        i++;
    }
    if (i == this->size() || 1LL * i * k >= m) {

```



```

        return Poly(m);
    }
    value v = (*this)[i];
    auto f = shift(-i) * v.inv();
    return (f.log(m - i * k) * k).exp(m - i * k).shift(i * k) * power(v, k2);
}
constexpr Poly sqrt(int m) const {
    Poly x{1};
    int k = 1;
    while (k < m) {
        k *= 2;
        x = (x + (trunc(k) * x.inv(k)).trunc(k)) * CInv<2, P>;
    }
    return x.trunc(m);
}
constexpr Poly inv() const {
    return move (inv(this->size ())) ;
}
constexpr Poly log() const {
    return move(log(this->size ()));
}
constexpr Poly exp() const {
    return move(exp(this->size ()));
}
constexpr Poly pow(i64 b) const {
    Poly<> res (vector<Z> { 1 }) ;
    auto a = * this ;
    for (; b; b /= 2, a *= a) {
        if (b % 2) {
            res *= a;
        }
    }
    return res;
}
constexpr Poly sqrt() const {
    return move(sqrt(this->size()));
}
constexpr Poly mult(Poly b) const {
    if (b.size() == 0) {
        return Poly();
    }
    int n = b.size();
    std::reverse(b.begin(), b.end());
    return ((*this) * b).shift(-(n - 1));
}
constexpr std::vector<Value> eval(std::vector<Value> x) const {
    if (this->size() == 0) {
        return std::vector<Value>(x.size(), 0);
    }
    const int n = std::max(x.size(), this->size());
    std::vector<Poly> q(4 * n);
    std::vector<Value> ans(x.size());
    x.resize(n);
    std::function<void(int, int, int)> build = [&](int p, int l, int r) {
        if (r - l == 1) {
            q[p] = Poly{1, -x[l]};

```

```

        } else {
            int m = (l + r) / 2;
            build(2 * p, l, m);
            build(2 * p + 1, m, r);
            q[p] = q[2 * p] * q[2 * p + 1];
        }
    };
    build(1, 0, n);
    std::function<void(int, int, int, const Poly &)> work = [&](int p, int l,
int r, const Poly &num) {
        if (r - l == 1) {
            if (l < int(ans.size())) {
                ans[l] = num[0];
            }
        } else {
            int m = (l + r) / 2;
            auto need = move(num.mulT(q[2 * p + 1]));
            need.resize ( m - l );
            work(2 * p, l, m, need);
            need = move(num.mulT(q[2 * p]));
            need.resize ( r - m );
            work(2 * p + 1, m, r, need);
        }
    };
    work(1, 0, n, mulT(q[1].inv(n)));
    return ans;
}

};

template<int P = ::P>
Poly<P> berlekampMassey(const Poly<P> &s) {
    Poly<P> c;
    Poly<P> oldC;
    int f = -1;
    for (int i = 0; i < s.size(); i++) {
        auto delta = s[i];
        for (int j = 1; j <= c.size(); j++) {
            delta -= c[j - 1] * s[i - j];
        }
        if (delta == 0) {
            continue;
        }
        if (f == -1) {
            c.resize(i + 1);
            f = i;
        } else {
            auto d = oldC;
            d *= -1;
            d.insert(d.begin(), 1);
            MInt<P> df1 = 0;
            for (int j = 1; j <= d.size(); j++) {
                df1 += d[j - 1] * s[f + 1 - j];
            }
            assert(df1 != 0);
            auto coef = delta / df1;
            d *= coef;

```

```

        Poly<P> zeros(i - f - 1);
        zeros.insert(zeros.end(), d.begin(), d.end());
        d = zeros;
        auto temp = c;
        c += d;
        if (i - temp.size() > f - oldC.size()) {
            oldC = temp;
            f = i;
        }
    }
}
c *= -1;
c.insert(c.begin(), 1);
return c;
}

template<int P = ::P>
MInt<P> linearRecurrence(Poly<P> p, Poly<P> q, i64 n) {
    int m = q.size() - 1;
    while (n > 0) {
        auto newq = q;
        for (int i = 1; i <= m; i += 2) {
            newq[i] *= -1;
        }
        auto newp = p * newq;
        newq = q * newq;
        for (int i = 0; i < m; i++) {
            p[i] = newp[i * 2 + n % 2];
        }
        for (int i = 0; i <= m; i++) {
            q[i] = newq[i * 2];
        }
        n /= 2;
    }
    return p[0] / q[0];
}

```

## 多项式扩展包

```

/**
 * 多项式扩展包
 */
namespace ExPoly {
    template<int P = ::P, class T1, class T2>
    constexpr static Poly <P> Lagrange(T1 x, T2 y) {
        int n = x.size();
        vector <Poly<>> M(4 * n);
        std::function<void(int, int, int)> build = [&](int p, int l, int r) {
            if (r - l == 1) {
                M[p] = Poly{(int) -x[l], 1};
            } else {
                int m = (l + r) / 2;
                build(2 * p, l, m);
                build(2 * p + 1, m, r);
            }
        };
    };
}

```

```

        M[p] = M[2 * p] * M[2 * p + 1];
    }
};
build(1, 0, n);
auto M_ = M[1].deriv().eval(x);
for (int i = 0; i < n; ++i) {
    M_[i] = y[i] * M_[i].inv();
}
vector<Poly<>> f(4 * n);
std::function<void(int, int, int)> work = [&](int p, int l, int r) ->
void {
    if (r - l == 1) {
        if (l < n) {
            f[p] = Poly{(int) M_[l]};
        }
    } else {
        int m = (l + r) / 2;
        work(2 * p, l, m);
        work(2 * p + 1, m, r);
        f[p] = f[2 * p] * M[2 * p + 1] + f[2 * p + 1] * M[2 * p];
    }
};
work(1, 0, n);
return f[1];
}

```

/\*\*

\*作用：对多项式进行平移操作

\*时间复杂度 $O(n\log(n))$

\*/

```

template<int P = ::P>
constexpr static Poly<P> Polynomial_translation(Poly<P> f, int k) {
    i64 n = (i64) f.size() - 1;
    Poly<P> g(n + 1);
    Z res = 1;
    for (int i = 0; i <= n; ++i) {
        g[n - i] = res * comb.invfac(i);
        res *= k;
        f[i] *= comb.fac(i);
    }
    Poly<P> here = g * f;
    here = here.shift(-n);
    for (int i = 0; i <= n; ++i) {
        here[i] *= comb.invfac(i);
    }
    return here;
}

```

/\*\*

\*作用：对相同的 $n$ 对 $i \in (0, n)$  求出将 $n$ 个不同的元素划分为 $i$ 个非空集的方案数

\*第二类Stirling数

\*时间复杂度 $O(n\log(n))$

\*/

```

template<int P = ::P>
constexpr static Poly<P> Second_Stirling_Same_N(int n) {
    Poly<P> f(n + 1), g(n + 1);
}

```

```

        for (int i = 0; i <= n; ++i) {
            g[i] = (i & 1 ? (Z) - 1 : Z(1)) * comb.invfac(i);
            f[i] = power((Z) i, n) * comb.invfac(i);
        }
        f *= g;
        f.resize(n + 1);
        return f;
    }

/**
 *作用：对相同的k对不同n 求出将n个不同的元素划分为k个非空集的方案数
 *第二类Stirling数
 *时间复杂度O(nlog(n))
 */
template<int P = ::P>
constexpr static Poly <P> Second_Stirling_Same_K(int Max_n, int k) {
    comb.init(Max_n + 1);
    Poly <P> f(vector<Z>(comb._invfac.begin(), comb._invfac.begin() + Max_n +
1));
    f[0] = 0;
    f = f.pow(k, Max_n + 1);
    for (int i = 0; i <= Max_n; ++i) {
        f[i] = f[i] * comb.fac(i) * comb.invfac(k);
    }
    return f;
}

/**
 *作用：对相同的n对i \in ( 0 , n ) 求出将n个不同的元素划分为i个非空轮换的方案数
 *第一类Stirling数
 *时间复杂度O(nlog(n))
 */
template<int P = ::P>
constexpr static Poly <P> First_Stirling_Same_N(int n) {
    ll len = __lg(n);
    Poly <P> f = {1};
    ll cnt = 0;
    for (int i = len; i >= 0; --i) {
        f *= Polynomial_translation(f, cnt);
        cnt <= 1;
        if (n >> i & 1) f *= Poly{cnt, 1}, cnt += 1;
    }
    return f;
}

/**
 *作用：对相同的k对不同n 求出将n个不同的元素划分为k个非轮换的方案数
 *第一类Stirling数
 *时间复杂度O(nlog(n))
 */
template<int P = ::P>
constexpr static Poly <P> First_Stirling_Same_K(int Max_n, int k) {
    comb.init(Max_n + 1);
    Poly <P> f(comb._inv.begin(), comb._inv.begin() + Max_n + 1);
    f = f.pow(k, Max_n + 1);
    for (int i = 0; i <= Max_n; ++i) {

```

```

        f[i] *= comb.fac(i) * comb.invfac(k);
    }
    return f;
}
};

```

## 矩阵

```

namespace matrix {
    using i64 = long long;

    template<typename T>
    struct Matrix : public std::vector<std::vector<T>> {
        using std::vector<std::vector<T>>::vector;

        Matrix(int x) : std::vector<std::vector<T>>(x, std::vector<T>(x)) {};
        Matrix(int x, int y) : std::vector<std::vector<T>>(x, std::vector<T>(y))
    };

        Matrix(int x, int y, T c) : std::vector<std::vector<T>>(x, std::vector<T>
(y, c)) {};

        constexpr Matrix operator+(Matrix a);
        constexpr Matrix operator-(Matrix a);
        constexpr Matrix operator*(Matrix a);

        template <typename T1, typename T2>
        friend constexpr Matrix<T1> operator*(Matrix<T1> x, T2 a);

        constexpr Matrix& operator+=(Matrix a);
        constexpr Matrix& operator-=(Matrix a);
        constexpr Matrix& operator*=(Matrix a);

        template <typename T1, typename T2>
        friend constexpr Matrix<T1>& operator*=(Matrix<T1>& x, T2 a);

        constexpr Matrix pow(i64 b);
        constexpr Matrix Transpose();
        constexpr Matrix inv();
    };

    template <typename T>
    constexpr Matrix<T> Matrix<T>::operator+(Matrix<T> a) {
        auto it = *this;
        int n = (int)a.size();
        int m = (int)a.back().size();
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < m; ++j)
                it[i][j] += a[i][j];
        return it;
    }

    template <typename T>
    constexpr Matrix<T> Matrix<T>::operator-(Matrix<T> a) {
        auto it = *this;
        int n = (int)a.size();

```

```

        int m = (int)a.back().size();
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < m; ++j)
                it[i][j] -= a[i][j];
        return it;
    }

    template <typename T>
    constexpr Matrix<T> Matrix<T>::operator*(Matrix<T> a) {
        int n = (int)this->size();
        int mid = (int)a.size();
        int m = (int)a.back().size();
        Matrix<T> it(n, m);
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < m; ++j)
                for (int k = 0; k < mid; ++k)
                    it[i][j] += (*this)[i][k] * a[k][j];
        return it;
    }

    template <typename T1, typename T2>
    constexpr Matrix<T1> operator*(Matrix<T1> x, T2 a) {
        int n = (int)x.size();
        int m = (int)x.back().size();
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < m; ++j)
                x[i][j] *= a;
        return x;
    }

    template <typename T>
    constexpr Matrix<T>& Matrix<T>::operator+=(Matrix<T> a) {
        return *this = *this + a;
    }

    template <typename T>
    constexpr Matrix<T>& Matrix<T>::operator-=(Matrix<T> a) {
        return *this = *this - a;
    }

    template <typename T>
    constexpr Matrix<T>& Matrix<T>::operator*=(Matrix<T> a) {
        return *this = *this * a;
    }

    template <typename T1, typename T2>
    constexpr Matrix<T1>& operator*=(Matrix<T1>& x, T2 a) {
        return x = x * a;
    }

    template <typename T>
    constexpr Matrix<T> Matrix<T>::pow(int b) {
        auto res = Matrix<T>(this->size(), this->size());
        for (int i = 0; i < (int)this->size(); ++i)
            res[i][i] = 1;
        auto a = *this;

```

```

    for (; b; b /= 2, a *= a)
        if (b % 2) res *= a;
    return res;
}

template <typename T>
constexpr Matrix<T> Matrix<T>::Transpose() {
    int n = this->back().size(), m = this->size();
    auto it = Matrix(n, m);
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < m; ++j)
            it[i][j] = (*this)[j][i];
    return it;
}

template <typename T>
constexpr Matrix<T> Matrix<T>::inv() {
    int n = this->size();
    Matrix<T> it(n, 2 * n);
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
            it[i][j] = (*this)[i][j];
    for (int i = 0; i < n; ++i)
        it[i][i + n] = 1;
    for (int i = 0; i < n; ++i) {
        int r = i;
        for (int k = i; k < n; ++k)
            if ((i64)it[k][i]) { r = k; break; }
        if (r != i)
            swap(it[r], it[i]);
        if (!(i64)it[i][i])
            return Matrix<T>();

        T x = (T) 1 / it[i][i];
        for (int k = 0; k < n; ++k) {
            if (k == i)
                continue;
            T t = it[k][i] * x;
            for (int j = i; j < 2 * n; ++j)
                it[k][j] -= t * it[i][j];
        }
        for (int j = 0; j < 2 * n; ++j)
            it[i][j] *= x;
    }
    Matrix<T> ans(n, n);
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
            ans[i][j] = it[i][j + n];
    return ans;
}
// namespace Matrix
};

```



# 数学类

```
/**
 * 数学工具箱
 */

namespace Math {
    using i64 = long long;
    using Int = __int128;
    using ui64 = unsigned long long;
    std::mt19937
rng(std::chrono::system_clock::now().time_since_epoch().count());

    struct math {

/**
 * @brief 带模乘
 * @return (a ^ b)% m
 */
        i64 static mul(i64 a, i64 b, i64 m);

/**
 * @brief 快速幂
 */
        template<class T>
        constexpr static T power(T a, i64 b);

        i64 static power(i64 a, i64 b, i64 m);

/**
 * @brief 求和
 */
        template<typename T>
        constexpr static T __sum1(T it);

        template<typename T>
        constexpr static T __sum2(T it);

/**
 * 欧几里得算法相关
 */

/**
 * 算法: 扩展欧几里得算法
 * 作用: 求解  $ax + by = \gcd(a, b)$ 
 * 返回:  $\gcd, x, y$ 
 */
        template<typename T = i64>
        constexpr array<T, 3> static Exgcd(T a, T b);

/**
 * 算法: 扩展欧几里得算法
 * 作用: 求解  $ax + by = \text{res}$ 
 * 限制:  $\gcd(a, b) \mid \text{res}$ 
 */
    };
}
```

```

*/
    template<typename T = i64>
    constexpr array<T, 3> static __Exgcd(T a, T b, T res);

/**
 * 算法: 线性同余方程
 * 作用: 求解  $ax \equiv b \pmod{n}$ 
 *         的最小整数解
 * 要求:  $\gcd(a, n) \mid b$ 
 */
    template<typename T = i64>
    constexpr T static Linear_congruence_equation(i64 a, i64 b, i64 mod);

/**
 * 算法: 扩展欧几里得算法求逆元
 * 作用: 求解  $ax \equiv 1 \pmod{n}$  的最小整数解
 * 要求: a 与 n 互质
 */
    template<typename T = i64>
    constexpr T static inv(i64 a, i64 mod);

/**
 * 扩展欧几里得结束
 */

/**
 * 算法: Miller_Rabin_Test
 * 作用: 在 long long 范围内快速判断质数
 * 时间复杂度:  $O(\log^3(n))$ 
 */
    constexpr static bool Miller_Rabin_Test(i64 n);

/**
 * 算法: Pollard_Rho
 * 作用: 能快速找到大整数的一个非1、非自身的因子的算法
 * 时间复杂度:  $O(n^{\{1/4\}} \log(n))$ 
 */
    static i64 Pollard_Rho(i64 N);

/**
 * 算法: 使用 Pollard_Rho 进行质因数分解
 * 返回: 顺序所有质因子(重复)
 */
    std::vector<i64> static factorize(i64 n);

/**
 * 算法: 中国剩余定理
 * 作用: 求解一元线性同余方程 ( $x \equiv a \pmod{m}$ ) 在模 n (所有的模积) 的解
 * 限制: 所有模互质
 */
    template<typename T = i64>
    constexpr static T Chinese_remainder_theorem
        (vector<i64> &a, vector<i64> &m);

/**
 * 算法: 扩展中国剩余定理

```

```

* 作用：求解一元线性同余方程（ $x \equiv a \pmod{m}$ ）在模 $n$ （所有模的最小公倍数）的解
* 无限制：所有模互质
*/

template<typename T = i64>
constexpr static T Extend_the_Chinese_remainder_theorem
    (vector<i64> &a, vector<i64> &m);

/**
* 算法：欧拉函数
* 作用：求欧拉函数
* 时间复杂度： $O(\sqrt{n})$ 
*/

template<typename T = i64>
constexpr static T Euler_phi(T n);

/**
* 算法：扩展欧拉定理（欧拉降幂）
* 作用：大指数快速幂
* 时间复杂度： $O(\sqrt{m})$ 
*/

static i64 Extending_Euler_theorem(i64 a, string b, i64 m);

/**
* 算法：求最小原根
* 要求：请自行保证这个数有原根(2,4,p^q,2*p^q)
* 时间复杂度： $O(\sqrt{n})$ 
*/

static i64 min_primitive_root(i64 m);

/**
* 求一个数的所有原根
* 注意提前使用质数筛，名称为s，开到n，并筛出欧拉函数
* 需要Linear_sieves_max、s
* 时间复杂度： $O(\sqrt{m})$ 
*/
#ifdef _Linear_sieves
    std::vector<i64> static primitive_root(i64 n);
#endif

/**
* 算法：扩展BSGS
* 作用：求解  $a^x = b \pmod{m}$ 
* 无要求： $a$ 与 $m$ 互质
* 返回：问题的最小非负x，无解返回-1
* 建议使用自定义Hash
*/

constexpr i64 static exBSGS(i64 a, i64 b, i64 m, i64 k = 1);

/**
* 算法：n次剩余
* 作用：求解  $x^a = b \pmod{m}$ 
* 要求： $m$ 是质数
* 返回： $x$ ，无解返回-1e15
* 建议使用自定义Hash

```

```

*/
    static std::vector<i64> n_times_remaining(i64 a, i64 b, i64 m);

/**
 * 算法: 扩展lucas
 * 作用: 在p为非质数情况下, 大数组合数C(n,m)
 * 必要情况下, 预处理降低复杂度
 */

    static i64 Exlucas(i64 n, i64 m, i64 P);

    //struct math
};

i64 math::mul(i64 a, i64 b, i64 m) {
    return static_cast<__int128>(a) * b % m;
}

template<class T>
constexpr T math::power(T a, i64 b) {
    T res = 1;
    for (; b; b /= 2, a *= a)
        if (b % 2) res *= a;
    return res;
}

i64 math::power(i64 a, i64 b, i64 m) {
    i64 res = 1 % m;
    for (; b; b >>= 1, a = mul(a, a, m))
        if (b & 1)
            res = mul(res, a, m);
    return res;
}

template<typename T>
constexpr T math::__sum1(T it) { return (it * (it + 1)) / ((T) 2); }

template<typename T>
constexpr T math::__sum2(T it) { return it * (it + 1) * (2 * it + 1) / ((T)
6); }

template<typename T>
constexpr array<T, 3> math::Exgcd(T a, T b) {
    T x1 = 1, x2 = 0, x3 = 0, x4 = 1;
    while (b != 0) {
        T c = a / b;
        std::tie(x1, x2, x3, x4, a, b) =
            std::make_tuple(x3, x4, x1 - x3 * c, x2 - x4 * c, b, a - b *
c);
    }
    return {a, x1, x2}; //x = x1, y = x2;
}

```

```

}

template<typename T>
constexpr array<T, 3> math::__Exgcd(T a, T b, T res) {
    assert(res % __gcd(a, b) == 0);
    auto [gcd, x, y] = Exgcd(a, b);
    return {gcd, res / gcd * x, res / gcd * y};
}

template<typename T>
constexpr T math::Linear_congruence_equation(i64 a, i64 b, i64 mod) {
    auto [gcd, x, k] = __Exgcd<T>((T) a, (T) mod, (T) b);
    T t = mod / gcd;
    return (x % t + t) % t;
}

template<typename T>
constexpr T math::inv(i64 a, i64 mod) {
    auto [gcd, x, k] = Exgcd<T>((T) a, (T) mod);
    return (x % mod + mod) % mod;
}

constexpr bool math::Miller_Rabin_Test(i64 n) {
    if (n < 3 || n % 2 == 0) return n == 2; // 特判
    i64 u = n - 1, t = 0;
    while (u % 2 == 0) u /= 2, ++t;
    constexpr std::array<i64, 7> ud = {2, 325, 9375, 28178, 450775, 9780504,
1795265022};
    for (i64 a: ud) {
        i64 v = power(a, u, n);
        if (v == 1 || v == n - 1 || v == 0) continue;
        for (int j = 1; j <= t; j++) {
            v = mul(v, v, n);
            if (v == n - 1 && j != t) {
                v = 1;
                break;
            } // 出现一个n-1, 后面都是1, 直接跳出
            if (v == 1) return 0; // 这里代表前面没有出现n-1这个解, 二次检验失败
        }
        if (v != 1) return 0; // Fermat检验
    }
    return 1;
}

i64 math::Pollard_Rho(i64 N) {
    if (N == 4) // 特判4
        return 2;
    if (Miller_Rabin_Test(N)) // 特判质数
        return N;
    auto randint = [&](i64 l, i64 r) {
        return l + rng() % (r - l + 1);
    };
    while (true) {
        i64 c = randint(1, N - 1); // 生成随机的c

```

```

        auto f = [=](i64 x) { return ((Int) x * x + c) % N; }; // Int表示
__int128, 防溢出
        i64 t = f(0), r = f(f(0));
        while (t != r) {
            i64 d = gcd(abs(t - r), N);
            if (d > 1)
                return d;
            t = f(t), r = f(f(r));
        }
    }
}

std::vector<i64> math::factorize(i64 n) {
    std::vector<i64> p;
    std::function<void(i64)> f = [&](i64 n) {
        if (n <= 10000) {
            for (int i = 2; i * i <= n; ++i)
                for (; n % i == 0; n /= i)
                    p.push_back(i);
            if (n > 1)
                p.push_back(n);
            return;
        }
        if (Miller_Rabin_Test(n)) {
            p.push_back(n);
            return;
        }
        auto g = [&](i64 x) {
            return (mul(x, x, n) + 1) % n;
        };
        i64 x0 = 2;
        while (true) {
            i64 x = x0;
            i64 y = x0;
            i64 d = 1;
            i64 power = 1, lam = 0;
            i64 v = 1;
            while (d == 1) {
                y = g(y);
                ++lam;
                v = mul(v, std::abs(x - y), n);
                if (lam % 127 == 0) {
                    d = std::gcd(v, n);
                    v = 1;
                }
                if (power == lam) {
                    x = y;
                    power *= 2;
                    lam = 0;
                    d = std::gcd(v, n);
                    v = 1;
                }
            }
            if (d != n) {
                f(d);
                f(n / d);
            }
        }
    }
}

```

```

        return;
    }
    ++x0;
}
};
f(n);
std::sort(p.begin(), p.end());
return p;
}

template<typename T>
constexpr T math::Chinese_remainder_theorem
    (vector<i64> &a, vector<i64> &m) {
    T n = accumulate(m.begin(), m.end(), (T) 1, multiplies<T>()), ans = 0;

    for (int i = 0; i < (i64) a.size(); ++i) {
        T m1 = n / m[i], b;
        b = inv(m1, m[i]);
        ans = (ans + a[i] * m1 * b % n) % n;
    }
    return (ans % n + n) % n;
}

template<typename T>
constexpr T math::Extend_the_Chinese_remainder_theorem
    (vector<i64> &a, vector<i64> &m) {
    T m1 = m[0], a1 = a[0];
    for (int i = 1; i < (i64) a.size(); ++i) {
        T m2 = m[i], a2 = a[i];
        auto [gcd, p, q] = __Exgcd(m1, m2, a2 - a1);
        a1 = m1 * p + a1;
        m1 = m1 * m2 / gcd;
        a1 = (a1 % m1 + m1) % m1;
    }
    return a1;
}

template<typename T>
constexpr T math::Euler_phi(T n) {
    T ans = n;
    for (i64 i = 2; i * i <= n; i++)
        if (n % i == 0) {
            ans = ans / i * (i - 1);
            while (n % i == 0) n /= i;
        }
    if (n > 1) ans = ans / n * (n - 1);
    return ans;
}

i64 math::Extending_Euler_theorem(i64 a, string b, i64 m) {
    i64 gcd = __gcd(a, m);
    i64 phi = Euler_phi(m);
    i64 res = 0;
    bool flag = 0;
    for (auto u: b) {
        res = res * 10 + u - '0';

```

```

        while (res >= phi) {
            res -= phi;
            if (!flag) flag = 1;
        }
    }
    if (gcd != 1 && flag) res += phi;
    return power(a, res, m);
}

```

```

i64 math::min_primitive_root(i64 m) {
    i64 phi = math::Euler_phi(m);
    auto div = [&](i64 x) {
        vector<i64> f;
        for (i64 i = 2; i * i <= x; ++i) {
            if (x % i != 0) continue;
            f.push_back(i);
            while (x % i == 0) x /= i;
        }
        if (x != 1 && x != phi) f.push_back(x);
        return f;
    };
    auto d = div(phi);
    i64 root = -1;
    auto check = [&](i64 x) {
        for (auto u: d)
            if (math::power(x, u, m) == 1)
                return false;
        root = x;
        return true;
    };
    for (i64 i = 1;; ++i) {
        if (__gcd(i, m) != 1)
            continue;
        if (check(i)) break;
    }
    return root;
}

```

```

# ifdef _Linear_sieves

```

```

std::vector<i64> math::primitive_root(i64 n) {
    static vector<bool> exist(Linear_sieves_max + 1);
    auto __exist = [&]() {
        static bool __existed = 0;
        if (__existed) return;
        __existed = 1;
        exist[2] = 1;
        exist[4] = 1;
        for (ll p: s.Prime) {
            if ((p & 1) == 0) continue;
            for (ll now = p; now <= (ll) exist.size() - 1; now *= p) {
                exist[now] = 1;
                if (now * 2 <= (ll) exist.size() - 1)
                    exist[now * 2] = 1;
            }
        }
    };
};

```



```

__exist();
if (!exist[n]) return vector<i64>();
vector<ll> f;
ll phi = s.eu(n);
ll pphi = s.eu(phi);
ll m = phi;
for (int i = 2; i * i <= m; ++i) {
    if (m % i == 0) {
        f.push_back(i);
        while (m % i)
            m /= i;
    }
}
if (m != 1) f.push_back(m);
// Debug ( f );
ll root = -1;
auto check = [&](ll x) {
    for (auto u: f)
        if (power(x, phi / u, n) == 1)
            return false;
    root = x;
    return true;
};
for (i64 i = 1; ++i) {
    if (__gcd(i, n) != 1) continue;
    if (check(i)) break;
}
vector<ll> ans;
for (i64 now = root, i = 1; i <= phi; ++i) {
    if (__gcd(phi, i) == 1)
        ans.push_back(now);
    now = (now * root) % n;
}
sort(ans.begin(), ans.end());
return ans;
}

# endif

constexpr i64 math::exBSGS(i64 a, i64 b, i64 m, i64 k) {
    constexpr i64 inf = 1e15;
    auto BSGS = [&](i64 a, i64 b, i64 m, i64 k = 1) {
# ifdef _Hash
        unordered_map<ui64, ui64, Hash> map;
# else
        std::map<ui64, ui64> map;
# endif

        i64 cur = 1, t = sqrt(m) + 1;
        for (i64 B = 1; B <= t; ++B) {
            (cur *= a) %= m;
            map[b * cur % m] = B;
        }
        ll now = cur * k % m;
        for (i64 A = 1; A <= t; ++A) {
            auto it = map.find(now);
            if (it != map.end())

```

```

        return A * t - (i64) it->second;
    }
    (now *= cur) %= m;
}
return -inf; // 无解
};

i64 A = a %= m, B = b %= m, M = m;
if (b == 1) return 0;
i64 cur = 1 % m;
for (int i = 0;; i++) {
    if (cur == B) return i;
    cur = cur * A % M;
    i64 d = __gcd(a, m);
    if (b % d) return -inf;
    if (d == 1) {
        auto ans = BSGS(a, b, m, k * a % m);
        if (ans == -inf) return -1;
        else return ans + i + 1;
    }
    k = k * a / d % m, b /= d, m /= d;
}
}

std::vector<i64> math::n_times_remaining(i64 a, i64 b, i64 m) {
    b %= m;
    vector<array<i64, 3>> fs;
    [&] (i64 m) {
        for (i64 i = 2; i * i <= m; i += 1) {
            if (m % i == 0) {
                array<i64, 3> f{i, 1, 0};
                while(m % i == 0) m /= i, f[1] *= i, f[2] += 1;
                fs.push_back(f);
            }
        }
        if (m > 1) fs.push_back({m, m, 1});
    } (m);
    auto get_step = [&] (i64 a, i64 n, i64 mod) { // 求阶
        i64 ans = n;
        for (i64 i = 2; i * i <= n; i++)
            if (n % i == 0) {
                while (ans % i == 0 && power(a, ans / i, mod) == 1) ans /= i;
                for (; n % i == 0; n /= i);
            }
        if (power(a, ans / n, mod) == 1) ans /= n;
        return ans;
    };

    i64 ans = 1;
    auto cntor = [&] (i64 A, i64 B, i64 m, i64 phi) {
        i64 c = get_step(B, phi, m), y = phi / c, G = __gcd(A, phi);
        if (y % G) ans = 0; ans *= G;
    };
    for (auto [p, pt, t] : fs) {
        if (!ans) break;
        if (b % pt == 0) ans *= power(p, t - (t + a - 1) / a, 1e9);
        else {
            i64 z = 0, b0 = b;

```

```

        for (; b0 % p == 0; Z ++, pt /= p, t--, b0 /= p);
        if (Z % a) ans = 0;
        else {
            cntor(a, b0, pt, pt - pt / p);
            ans *= power(p, Z - Z / a, 1e9);
        }
    }
}

return std::vector<i64>{ans};
}

i64 math::Exlucas(i64 n, i64 m, i64 P) {
    std::vector<i64> p, a;
    function<i64(i64, i64, i64)> calc = [&](i64 n, i64 x, i64 P) mutable ->
i64 {
        if (!n) return 1;
        i64 s = 1;
        for (i64 i = 1; i <= P; ++i) //求阶乘, 可预处理降低复杂度
            if (i % x != 0) s = math::mul(s, i, P);
        s = math::power(s, n / P, P);
        for (i64 i = n / P * P + 1; i <= n; ++i)
            if (i % x != 0) s = math::mul(i, s, P);
        return math::mul(s, calc(n / x, x, P), P);
    };
    function<i64(i64, i64, i64, i64)> multilucas = [&](i64 n, i64 m, i64 x,
i64 P) -> i64 {
        i64 cnt = 0;
        for (i64 i = n; i != 0; i /= x) cnt += i / x;
        for (i64 i = m; i != 0; i /= x) cnt -= i / x;
        for (i64 i = n - m; i != 0; i /= x) cnt -= i / x;
        return static_cast<__int128>(1) * math::power(x, cnt, P) % P *
calc(n, x, P) % P
            * math::inv(calc(m, x, P), P) % P * math::inv(calc(n - m, x,
P), P) % P;
    };
    for (i64 i = 2; i * i <= P; ++i) {
        if (P % i == 0) {
            p.emplace_back(1);
            while (P % i == 0) p.back() *= i, P /= i;
            a.emplace_back(multilucas(n, m, i, p.back()));
        }
    }
    if (P > 1) p.emplace_back(P), a.emplace_back(multilucas(n, m, P, P));
    return math::Extend_the_Chinese_remainder_theorem(a, p);
}

// namespace Math
}

using namespace Math;

```

## 线性基

```
struct Linear_Base {
    int siz;
    vector<int> a;
    Linear_Base(int _siz = 61) {
        siz = _siz;
        a.resize(siz + 1);
    }
    void insert(int x) { // 插入
        for (int i = siz; i >= 0; i--) if (x & (1ll << i)) {
            if (!a[i]) { a[i] = x; return; }
            else x ^= a[i];
        }
    }
    bool check(int x) { // 查询x是否能被异或出来
        for (int i = siz; i >= 0; i--) if (x & (1ll << i)) {
            if (!a[i]) break;
            x ^= a[i];
        }
        return x == 0;
    }
    int querymax(int res) { // 查询最大异或和
        for (int i = siz; i >= 0; i--) if ((res ^ a[i]) > res) res ^= a[i];
        return res;
    }
    int querymin(int res) { // 查询最小
        for (int i = siz; i >= 0; i--) if (res & (1ll << i)) res ^= a[i];
        return res;
    }
    int querykth(int k) { // 查询第k大的异或和
        vector<int> tmp(siz + 10);
        int res = 0, cnt = 0;
        for (int i = 0; i <= siz; i++) {
            for (int j = i - 1; j >= 0; j--) if (a[i] & (1ll << j)) a[i] ^= a[j];
            if(a[i]) tmp[cnt++] = a[i];
        }
        for (int i = 0; i < cnt; i++) if (k & (1ll << i)) res ^= tmp[i];
        return res;
    }
    void merge(const Linear_Base& other) { // 合并
        for (int i = 0; i <= other.siz; i++) insert(other.a[i]);
    }
};
```

## 线性筛

```
struct Linear_sieves {
    # define _Linear_sieves
    int n;
    vector<int> Prime, Euler, Morbius, Approximate, Approximate_cnt;
    vector<bool> notprime;
```

```

vector<array<i64, 2>> div;

Linear_sieves() {};

Linear_sieves(int _n) { init(_n); };

void init(int _n) {
    n = _n;
    Prime_work();
}

void Prime_work() {
    notprime.assign(n + 1, 0);
    notprime[0] = 1;
    notprime[1] = 1;
    for (i64 i = 2; i <= n; ++i) {
        if (notprime[i] == 0) {
            Prime.push_back(i);
        }
        for (i64 j = 0; i * Prime[j] <= n; ++j) {
            notprime[i * Prime[j]] = 1;

            if (i % Prime[j] == 0) break;
        }
    }
}

void Euler_work() {
    Euler.assign(n + 1, 0);
    Euler[1] = 1;
    for (i64 i = 2; i <= n; ++i) {
        if (notprime[i] == 0) Euler[i] = i - 1;
        for (i64 j = 0; i * Prime[j] <= n; ++j) {
            i64 now = i * Prime[j];
            if (i % Prime[j] != 0) {
                Euler[now] = (Prime[j] - 1) * Euler[i];
            } else {
                Euler[now] = Prime[j] * Euler[i];
                break;
            }
        }
    }
}

void Morbius_work() {
    Morbius.assign(n + 1, 0);
    Morbius[1] = 1;
    for (i64 i = 2; i <= n; ++i) {
        if (notprime[i] == 0) Morbius[i] = -1;
        for (i64 j = 0; i * Prime[j] <= n; ++j) {
            i64 now = i * Prime[j];
            if (i % Prime[j] != 0) {
                Morbius[now] = -Morbius[i];
            } else break;
        }
    }
}

```

```

}

void Div_work() {
    div.resize(n + 1);
    div[0] = {1, 1};
    div[1] = {1, 1};
    for (i64 i = 2; i <= n; ++i) {
        if (notprime[i] == 0) {
            div[i] = {1, i};
        }
        for (i64 j = 0; i * Prime[j] <= n; ++j) {
            div[i * Prime[j]] = {Prime[j], i};
            if (i % Prime[j] == 0) break;
        }
    }
}

/**
 * 求约数个数
 */
void Approximate_work() {
    Approximate.assign(n + 1, 0);
    Approximate_cnt.assign(n + 1, 0);
    Approximate[1] = 1;
    Approximate_cnt[1] = 0;
    for (i64 i = 2; i <= n; ++i) {
        if (notprime[i] == 0) {
            Approximate[i] = 2;
            Approximate_cnt[i] = 1;
        }
        for (i64 j = 0; i * Prime[j] <= n; ++j) {
            i64 now = i * Prime[j];
            if (i % Prime[j] != 0) {
                Approximate_cnt[now] = 1;
                Approximate[now] = Approximate[i] * 2;
            } else {
                Approximate_cnt[now] = Approximate_cnt[i] + 1;
                Approximate[now] = Approximate[i] / Approximate_cnt[now] *
(Approximate_cnt[now] + 1);
                break;
            }
        }
    }
}

std::vector<i64> get_frac(i64 x) {
    vector<i64> f;
    for (; x > 1; f.push_back(div[x][0]), x = div[x][1]);
    return f;
}

i64 size() { return (i64) Prime.size(); }

bool isprime(int n) { return !notprime[n]; }

i64 eu(int n) { return Euler[n]; }

```

```
i64 mo(int n) { return Morbius[n]; }  
};
```

## 组合数学

```
template<class T>  
struct Comb {  
    int n;  
    std::vector<T> _fac;  
    std::vector<T> _invfac;  
    std::vector<T> _inv;  
  
    Comb() : n{0}, _fac{1}, _invfac{1}, _inv{0} {}  
  
    Comb(int n) : Comb() {  
        init(n);  
    }  
  
    void init(int m) {  
        m = std::min(m, T::getMod() - 1);  
        if (m <= n) return;  
        _fac.resize(m + 1);  
        _invfac.resize(m + 1);  
        _inv.resize(m + 1);  
  
        for (int i = n + 1; i <= m; i++) {  
            _fac[i] = _fac[i - 1] * i;  
        }  
        _invfac[m] = _fac[m].inv();  
        for (int i = m; i > n; i--) {  
            _invfac[i - 1] = _invfac[i] * i;  
            _inv[i] = _invfac[i] * _fac[i - 1];  
        }  
        n = m;  
    }  
  
    T fac(int m) {  
        if (m > n) init(2 * m);  
        return _fac[m];  
    }  
  
    T invfac(int m) {  
        if (m > n) init(2 * m);  
        return _invfac[m];  
    }  
  
    T inv(int m) {  
        if (m > n) init(2 * m);  
        return _inv[m];  
    }  
  
    T binom(int n, int m) {  
        if (n < m || m < 0) return 0;  
        return fac(n) * invfac(m) * invfac(n - m);  
    }  
};
```

```

    }

/**
 * 第二类斯特林数
 * 时间复杂度 :  $O(m * \log(m))$ 
 */
T Stirling(int n, int m) {
    T ans = 0;
    for (int i = 0; i <= m; ++i) {
        ans += (((m - i) & 1) == 1 ? -1 : 1) * power((T) i, n) * invfac(i) *
invfac(m - i);
    }
    return ans;
}

T Catalan(int n) {
    return binom(2 * n, n) * inv(n + 1);
}

/**
 * 算法: 卢卡斯定理
 * 作用: 大数组合数
 * 注意在p较小时使用p
 * p为Z的质数
 * 时间复杂度为 $O(\log p)$ 
 */
T lucas(i64 n, i64 m) {
    if (m == 0) return T(1);
    return binom(n % T::getMod(), m % T::getMod()) * lucas(n / T::getMod(), m
/ T::getMod());
}
};

Comb<Z> comb;

```

# 图论

## SCC

### 一般

```

struct SCC {
    int n, cnt = 0, tot = -1;
    vector<vector<int>> map;
    vector<int> d, id, stack, tag;
    vector<bool> instack;

    SCC(int n): n(n), map(n), d(n, -1), id(n), tag(n, -1), instack(n, 0) {}

private:
    void _scc(int now) {
        d[now] = id[now] = ++tot;
        stack.push_back(now);
        instack[now] = 1;
    }
};

```



```

        for (auto u : map[now]) {
            if (!~d[u]) {
                _scc(u);
                id[now] = min(id[now], id[u]);
            } else if (instack[u]) {
                id[now] = min(id[now], id[u]);
            }
        }
        if (d[now] == id[now]) {
            ++cnt;
            do {
                instack[stack.back()] = 0;
                tag[stack.back()] = cnt;
                stack.pop_back();
            } while (instack[now]);
        }
    }

public:
    void addedge(int u, int v) {
        map[u].push_back(v);
    }
    void scc(int now) {
        --cnt;
        _scc(now);
        ++cnt;
    }
};

```

## 割边

```

struct CutEdge {
    int n, tot = -1;
    vector<pair<int, int>> edge;
    vector<vector<int>> map;
    vector<int> d, id, ans;

    CutEdge(int n) : n(n), d(n, -1), id(n, -1), map(n) {};

private :
    void _cutedge(int now, int _edge) {
        d[now] = id[now] = ++tot;
        for (auto tag: map[now]) {
            auto &here = edge[tag].second;
            if (!~d[here]) {
                _cutedge(here, tag);
                id[now] = min(id[now], id[here]);
                if (id[here] > d[now]) {
                    ans.push_back(tag);
                }
            } else if (tag != (_edge ^ 1)) {
                id[now] = min(id[here], id[now]);
            }
        }
    }
}

```

```

public:
    void addedge(int u, int v) {
        edge.push_back({u, v});
        map[u].push_back(int(edge.size()) - 1);
    }

    void cutedge(int u, int _edge) {
        _cutedge(u, _edge);
    }
};

```

## 割点

```

struct SCC {
    int n, cnt = 0, tot = -1;
    vector<vector<int>> map;
    vector<int> d, id, stack, tag;
    vector<bool> instack;

    SCC(int n): n(n), map(n), d(n, -1), id(n), tag(n, -1), instack(n, 0) {}

private:
    void _scc(int now) {
        d[now] = id[now] = ++tot;
        stack.push_back(now);
        instack[now] = 1;
        for (auto u : map[now]) {
            if (!~d[u]) {
                _scc(u);
                id[now] = min(id[now], id[u]);
            } else if (instack[u]) {
                id[now] = min(id[now], id[u]);
            }
        }
        if (d[now] == id[now]) {
            ++cnt;
            do {
                instack[stack.back()] = 0;
                tag[stack.back()] = cnt;
                stack.pop_back();
            } while (instack[now]);
        }
    }

public:
    void addedge(int u, int v) {
        map[u].push_back(v);
    }

    void scc(int now) {
        --cnt;
        _scc(now);
        ++cnt;
    }
}

```

```
};
```

## Lca、dfn、虚树

```
template<class T,
        class Cmp = less<T>>
struct RMQ {
    const Cmp cmp = Cmp();
    static constexpr unsigned B = 64;
    using u64 = unsigned long long;
    int n;
    vector<vector<T>> a;
    vector<T> pre, suf, ini;
    vector<u64> stk;
    RMQ() {}
    RMQ(const vector<T> &v) {
        init(v);
    }
    void init(const vector<T> &v) {
        n = v.size();
        pre = suf = ini = v;
        stk.resize(n);
        if (!n) {
            return;
        }
        const int M = (n - 1) / B + 1;
        const int lg = __lg(M);
        a.assign(lg + 1, vector<T>(M));
        for (int i = 0; i < M; i++) {
            a[0][i] = v[i * B];
            for (int j = 1; j < B && i * B + j < n; j++) {
                a[0][i] = min(a[0][i], v[i * B + j], cmp);
            }
        }
        for (int i = 1; i < n; i++) {
            if (i % B) {
                pre[i] = min(pre[i], pre[i - 1], cmp);
            }
        }
        for (int i = n - 2; i >= 0; i--) {
            if (i % B != B - 1) {
                suf[i] = min(suf[i], suf[i + 1], cmp);
            }
        }
        for (int j = 0; j < lg; j++) {
            for (int i = 0; i + (2 << j) <= M; i++) {
                a[j + 1][i] = min(a[j][i], a[j][i + (1 << j)], cmp);
            }
        }
        for (int i = 0; i < M; i++) {
            const int l = i * B;
            const int r = min(1U * n, l + B);
            u64 s = 0;
            for (int j = l; j < r; j++) {
                while (s && cmp(v[j], v[__lg(s) + 1])) {
```

```

        s ^= 1ULL << __lg(s);
    }
    s |= 1ULL << (j - 1);
    stk[j] = s;
}
}
}
T operator()(int l, int r) {
    if (l / B != (r - 1) / B) {
        T ans = min(suf[l], pre[r - 1], cmp);
        l = l / B + 1;
        r = r / B;
        if (l < r) {
            int k = __lg(r - l);
            ans = min({ans, a[k][l], a[k][r - (1 << k)]}, cmp);
        }
        return ans;
    } else {
        int x = B * (l / B);
        return ini[__builtin_ctzll(stk[r - 1] >> (l - x)) + 1];
    }
}
};

```

```

struct DFN {
    int n;
    vector<int> dfn, dep, sz, fa;
    RMQ<array<int, 2>> rmq;
    DFN() = default;
    DFN(const vector<vector<int>> &adj, int root = 0) {
        init(adj, root);
    }
    void init(const vector<vector<int>> &adj, int root = 0) {
        n = adj.size();
        dfn.assign(n, 0);
        dep.assign(n, 0);
        sz.assign(n, 0);
        fa.assign(n, 0);
        virtual_tree.assign(n, {});
        vector<array<int, 2>> inrmq(n);
        int tot = 0;
        auto &pa = fa;
        dep[root] = -1;
        auto dfs = [&](auto&&dfs, int now, int fa) -> void {
            dfn[now] = tot++;
            dep[now] = dep[fa] + 1;
            pa[now] = fa;
            for (auto here : adj[now]) {
                if (here == fa) continue;
                dfs(dfs, here, now);
                sz[now] += sz[here];
            }
            sz[now] += 1;
        };
        dfs(dfs, root, root);
        for (int i = 0; i < n; i += 1) {

```

```

        inrmq[dfn[i]] = {dep[i], i};
    }
    rmq.init(inrmq);
}
int lca (int lhs, int rhs) {
    if (lhs == rhs) return lhs;
    if (dfn[lhs] > dfn[rhs]) swap(lhs, rhs);
    return fa[rmq[dfn[lhs] + 1, dfn[rhs] + 1][1]];
}
vector<vector<int>> virtual_tree;
vector<int> real_key;
template<class T>
vector<vector<int>> &build_virtual_tree(vector<T> key) {
    for (auto u : real_key) {
        virtual_tree[u].clear();
    }
    real_key.clear();
    sort(key.begin(), key.end(), [&] (T x, T y) {return dfn[x] < dfn[y];});
    for (int i = 0; i < int(key.size()) - 1; i += 1) {
        real_key.push_back(key[i]);
        real_key.push_back(lca(key[i], key[i + 1]));
    }
    real_key.push_back(key.back());
    sort(real_key.begin(), real_key.end(), [&] (T x, T y) {return dfn[x] <
dfn[y];});
    real_key.erase(unique(real_key.begin(), real_key.end(), real_key.end()));
    for (int i = 0; i < int(real_key.size()) - 1; i += 1) {
        int Lca = lca(real_key[i], real_key[i + 1]);
        virtual_tree[Lca].push_back(real_key[i + 1]);
        virtual_tree[real_key[i + 1]].push_back(Lca);
    }
    return virtual_tree;
}
};

```

## 重链剖分

```

struct HLD {
    int n;
    std::vector<int> siz, top, dep, parent, in, out, seq;
    std::vector<std::vector<int>> adj;
    int cur;

    HLD() {}
    HLD(int n) {
        init(n);
    }
    void init(int n) {
        this->n = n;
        siz.resize(n);
        top.resize(n);
        dep.resize(n);
        parent.resize(n);
        in.resize(n);
        out.resize(n);
    }
};

```

```

    seq.resize(n);
    cur = 0;
    adj.assign(n, {});
}

void addEdge(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
}

void work(int root = 0) {
    top[root] = root;
    dep[root] = 0;
    parent[root] = -1;
    dfs1(root);
    dfs2(root);
}

void dfs1(int u) {
    if (parent[u] != -1) {
        adj[u].erase(std::find(adj[u].begin(), adj[u].end(), parent[u]));
    }

    siz[u] = 1;
    for (auto &v : adj[u]) {
        parent[v] = u;
        dep[v] = dep[u] + 1;
        dfs1(v);
        siz[u] += siz[v];
        if (siz[v] > siz[adj[u][0]]) {
            std::swap(v, adj[u][0]);
        }
    }
}

void dfs2(int u) {
    in[u] = cur++;
    seq[in[u]] = u;
    for (auto v : adj[u]) {
        top[v] = v == adj[u][0] ? top[u] : v;
        dfs2(v);
    }
    out[u] = cur;
}

int lca(int u, int v) {
    while (top[u] != top[v]) {
        if (dep[top[u]] > dep[top[v]]) {
            u = parent[top[u]];
        } else {
            v = parent[top[v]];
        }
    }
    return dep[u] < dep[v] ? u : v;
}

int dist(int u, int v) {
    return dep[u] + dep[v] - 2 * dep[lca(u, v)];
}

int jump(int u, int k) {

```

```

    if (dep[u] < k) {
        return -1;
    }

    int d = dep[u] - k;

    while (dep[top[u]] > d) {
        u = parent[top[u]];
    }

    return seq[in[u] - dep[u] + d];
}

bool isAncestor(int u, int v) {
    return in[u] <= in[v] && in[v] < out[u];
}

int rootedParent(int u, int v) {
    std::swap(u, v);
    if (u == v) {
        return u;
    }
    if (!isAncestor(u, v)) {
        return parent[u];
    }
    auto it = std::upper_bound(adj[u].begin(), adj[u].end(), v, [&](int x,
int y) {
        return in[x] < in[y];
    }) - 1;
    return *it;
}

int rootedSize(int u, int v) {
    if (u == v) {
        return n;
    }
    if (!isAncestor(v, u)) {
        return siz[v];
    }
    return n - siz[rootedParent(u, v)];
}

int rootedLca(int a, int b, int c) {
    return lca(a, b) ^ lca(b, c) ^ lca(c, a);
}
};

```

# 流

## 网络流

```
constexpr int inf = 1E9;
template<class T>
struct MaxFlow {
    struct _Edge {
        int to;
        T cap;
        _Edge(int to, T cap) : to(to), cap(cap) {}
    };

    int n;
    std::vector<_Edge> e;
    std::vector<std::vector<int>>> g;
    std::vector<int> cur, h;

    MaxFlow() {}
    MaxFlow(int n) {
        init(n);
    }

    void init(int n) {
        this->n = n;
        e.clear();
        g.assign(n, {});
        cur.resize(n);
        h.resize(n);
    }

    bool bfs(int s, int t) {
        h.assign(n, -1);
        std::queue<int> que;
        h[s] = 0;
        que.push(s);
        while (!que.empty()) {
            const int u = que.front();
            que.pop();
            for (int i : g[u]) {
                auto [v, c] = e[i];
                if (c > 0 && h[v] == -1) {
                    h[v] = h[u] + 1;
                    if (v == t) {
                        return true;
                    }
                    que.push(v);
                }
            }
        }
        return false;
    }

    T dfs(int u, int t, T f) {
        if (u == t) {
```



```

        return f;
    }
    auto r = f;
    for (int &i = cur[u]; i < int(g[u].size()); ++i) {
        const int j = g[u][i];
        auto [v, c] = e[j];
        if (c > 0 && h[v] == h[u] + 1) {
            auto a = dfs(v, t, std::min(r, c));
            e[j].cap -= a;
            e[j ^ 1].cap += a;
            r -= a;
            if (r == 0) {
                return f;
            }
        }
    }
    return f - r;
}

void addEdge(int u, int v, T c) {
    g[u].push_back(e.size());
    e.emplace_back(v, c);
    g[v].push_back(e.size());
    e.emplace_back(u, 0);
}

T flow(int s, int t) {
    T ans = 0;
    while (bfs(s, t)) {
        cur.assign(n, 0);
        ans += dfs(s, t, std::numeric_limits<T>::max());
    }
    return ans;
}

std::vector<bool> minCut() {
    std::vector<bool> c(n);
    for (int i = 0; i < n; i++) {
        c[i] = (h[i] != -1);
    }
    return c;
}

struct Edge {
    int from;
    int to;
    T cap;
    T flow;
};

std::vector<Edge> edges() {
    std::vector<Edge> a;
    for (int i = 0; i < e.size(); i += 2) {
        Edge x;
        x.from = e[i + 1].to;
        x.to = e[i].to;
        x.cap = e[i].cap + e[i + 1].cap;
        x.flow = e[i + 1].cap;
        a.push_back(x);
    }
}

```

```

    }
    return a;
}
};

```

## 费用流

```

struct MCFGraph {
    struct Edge {
        int v, c, f;
        Edge(int v, int c, int f) : v(v), c(c), f(f) {}
    };
    const int n;
    std::vector<Edge> e;
    std::vector<std::vector<int>>> g;
    std::vector<i64> h, dis;
    std::vector<int> pre;
    bool dijkstra(int s, int t) {
        dis.assign(n, std::numeric_limits<i64>::max());
        pre.assign(n, -1);
        std::priority_queue<std::pair<i64, int>, std::vector<std::pair<i64,
int>>, std::greater<std::pair<i64, int>>>> que;
        dis[s] = 0;
        que.emplace(0, s);
        while (!que.empty()) {
            i64 d = que.top().first;
            int u = que.top().second;
            que.pop();
            if (dis[u] < d) continue;
            for (int i : g[u]) {
                int v = e[i].v;
                int c = e[i].c;
                int f = e[i].f;
                if (c > 0 && dis[v] > d + h[u] - h[v] + f) {
                    dis[v] = d + h[u] - h[v] + f;
                    pre[v] = i;
                    que.emplace(dis[v], v);
                }
            }
        }
        return dis[t] != std::numeric_limits<i64>::max();
    }
    MCFGraph(int n) : n(n), g(n) {}
    void addEdge(int u, int v, int c, int f) {
        g[u].push_back(e.size());
        e.emplace_back(v, c, f);
        g[v].push_back(e.size());
        e.emplace_back(u, 0, -f);
    }
    std::pair<int, i64> flow(int s, int t) {
        int flow = 0;
        i64 cost = 0;
        h.assign(n, 0);

```

```

        while (dijkstra(s, t)) {
            for (int i = 0; i < n; ++i) h[i] += dis[i];
            int aug = std::numeric_limits<int>::max();
            for (int i = t; i != s; i = e[pre[i] ^ 1].v) aug = std::min(aug,
e[pre[i]].c);
            for (int i = t; i != s; i = e[pre[i] ^ 1].v) {
                e[pre[i]].c -= aug;
                e[pre[i] ^ 1].c += aug;
            }
            flow += aug;
            cost += i64(aug) * h[t];
        }
        return std::make_pair(flow, cost);
    }
};

```

## 笛卡尔树

```

template<class T>
struct Descartes {
    int n;
    vector<T> v;
    vector<int> ls, rs;

    Descartes(int n) : ls(n, -1), rs(n, -1), v(n) {}

    Descartes(vector<T> &v) : n((ll) v.size()), ls(n, -1), rs(n, -1), v(v) {}

    int build() /* return root */ {
        vector<int> s(n);
        int top = 0;
        int root = -1;
        for (int i = 0; i < n; ++i) {
            int realtop = top;
            while (top != 0 && v[s[top]] > v[i]) { --top; }
            if (top < realtop) ls[i] = s[top + 1];
            if (top != 0) rs[s[top]] = i;
            s[++top] = i;
        }
        root = s[1];
        assert(!s.empty());
        return root;
    }
};

```

# 杂

## 初始

```
# include <bits/stdc++.h>
using namespace std;
# ifdef LOCAL
    # include "C:\Users\Kevin\Desktop\demo\save\debug.h"
# else
# define debug(...) 114514
# define ps 114514
# endif

using ll = long long;
using i64 = long long;

void solve() {

}

int main () {
# ifndef cin
    std::ios::sync_with_stdio (false);
    std::cin.tie (nullptr) ;
# endif
    // __fin("C:\\Users\\Kevin\\Desktop\\cpp\\in.in") ;

    i64 _ = 1 ;
    // cin >> _ ;
    while (_ --) {
        solve () ;
# ifdef LOCAL
# ifndef cin
        cout.flush();
# endif
# endif
    }
    return 0 ;
}
```

## 对拍

- 一共 4 个文件:
  - baoli.cpp
  - std.cpp
  - data.cpp
    - 关键

```

std::mt19937
rng(std::chrono::steady_clock::now().time_since_epoch().count());

auto rnd = (i64 l, i64 r) [&] {
    uniform_int_distribution<long long> _rnd(l, r);
    return _rnd(rng);
};

shuffle(v.begin(), v.end(), rng);

```

```

#include <bits/stdc++.h>
using namespace std;

using ll = long long;
using i64 = long long;

int main () {
    # ifndef cin
        std::ios::sync_with_stdio (false);
        std::cin.tie (nullptr) ;
    # endif
    std::mt19937
    rng(std::chrono::steady_clock::now().time_since_epoch().count());
    auto rnd = (i64 l, i64 r) [&] {
        uniform_int_distribution<long long> _rnd(l, r);
        return _rnd(rng);
    };
    vector<int> a;
    shuffle(a.begin(), a.end(), rng);

    return 0 ;
}

```

o 对拍.cpp

```

#include <iostream>
#include <cstdio>
#include <windows.h>
#include <cstdlib>
#include <ctime>

using namespace std;

int main() {
    int ok = 0;
    int n = 50;
    for (int i = 1;; ++i) {
        system("data.exe > in.txt");
        system("std.exe < in.txt > std.txt");
        double begin = clock();
        system("baoli.exe < in.txt > baoli.txt");
        double end = clock();

        double t = (end - begin);
        if (system("fc std.txt baoli.txt")) {

```

```

        printf("测试点#%d Wrong Answer\n", i);
        system("pause");
    } else if (t > 2000) {
        printf("测试点#%d Time Limited Exceeded 用时 %.01fms\n",
i, t);
    } else {
        printf("测试点#%d Accepted 用时%.01fms\n", i, t);
        ok++; //AC数量+1
    }
}
printf("\n");
double res = 100.0 * ok / n;
printf("共 %d 组测试数据, AC数据 %d 组。 得分%.11f。", n, ok, res);
}

```

## 简易版取模类

```

template<typename T>
T power(T x, long long b) {
    T res = 1;
    while (b) {
        if (b & 1) res *= x;
        x *= x;
        b >>= 1;
    }
    return res;
}

template<int P>
struct mod_int {
    int x;
    static int mod;
    mod_int() : x{} {}
    mod_int(long long x) : x(norm(x % getMod())) {}

    int norm(int x) {
        if (x >= P) x -= P;
        if (x < 0) x += P;
        return x;
    }

    static void setMod(int x) {
        mod = x;
    }

    static int getMod() {
        return (P > 0 ? P : mod);
    }

    mod_int operator-() {
        return -x;
    }

    mod_int &operator+=(mod_int rhs) {
        x = norm(x + rhs.x);
        return *this;
    }
}

```

```

mod_int &operator--(mod_int rhs) {
    x = norm(x - rhs.x);
    return *this;
}

mod_int &operator*=(mod_int rhs) {
    x = 111 * x * rhs.x % getMod();
    return *this;
}

mod_int inv() {
    return power(*this, P - 2);
}

mod_int &operator/=(mod_int rhs) {
    x = 111 * x * rhs.inv().x % getMod();
    return *this;
}

friend mod_int operator+(mod_int lhs, mod_int rhs) {
    return lhs += rhs;
}

friend mod_int operator-(mod_int lhs, mod_int rhs) {
    return lhs -= rhs;
}

friend mod_int operator*(mod_int lhs, mod_int rhs) {
    return lhs *= rhs;
}

friend mod_int operator/(mod_int lhs, mod_int rhs) {
    return lhs /= rhs;
}

friend bool operator==(mod_int lhs, mod_int rhs) {
    return lhs.x == rhs.x;
}

friend bool operator!=(mod_int lhs, mod_int rhs) {
    return lhs.x != rhs.x;
}

template<class istream>
friend istream &operator>>(istream &input, mod_int &rhs) {
    long long x;
    input >> x;
    rhs = x;
    return input;
}

template<class ostream>
friend ostream &operator<<(ostream &output, mod_int rhs) {
    return output << rhs.x;
}

};

template<>
int mod_int<0>::mod = 998244353;

constexpr int P = 1e9 + 7;
using Z = mod_int<P>;

```

## debug.h

```
template<typename A, typename B> ostream& operator<<(ostream &os, const pair<A, B> &p) { return os << '(' << p.first << ", " << p.second << ')'; }
template<typename T_container, typename T = typename
enable_if<!is_same<T_container, string>::value, typename
T_container::value_type>::type> ostream& operator<<(ostream &os, const
T_container &v) { os << '{'; string sep; for (const T &x : v) os << sep << x, sep
= ", "; return os << '}'; }

void debug_out() { cerr << endl; }
template<typename Head, typename... Tail> void debug_out(Head H, Tail... T) {
cerr << ' ' << H; debug_out(T...); }

# define ps cerr << "YES" << endl
# define debug(...) cerr << "(" << #__VA_ARGS__ << "):" << endl,
debug_out(__VA_ARGS__)
```

## hash

```
struct Hash {
# define _Hash
static uint64_t splitmix64(uint64_t x) {
    x += 0x9e3779b97f4a7c15;
    x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
    x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
    return x ^ (x >> 31);
}

size_t operator()(uint64_t x) const {
    static const uint64_t FIXED_RANDOM =
        chrono::steady_clock::now().time_since_epoch().count();
    return splitmix64(x + FIXED_RANDOM);
}

// 针对 std::pair<int, int> 作为主键类型的哈希函数
size_t operator()(pair<uint64_t, uint64_t> x) const {
    static const uint64_t FIXED_RANDOM =
        chrono::steady_clock::now().time_since_epoch().count();
    return splitmix64(x.first + FIXED_RANDOM) ^
        (splitmix64(x.second + FIXED_RANDOM) >> 1);
}
};
```

## O2优化

```
#pragma GCC optimize("Ofast")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4,popcnt,abm,mmx,avx,avx2,fma")
#pragma GCC optimize("unroll-loops")
```



```
struct Input {
    using Long = long long ;
    # define cin input
    Input () {}
    # define MAXSIZE (1 << 20)
    # define isdigit(x) (x >= '0' && x <= '9')

    char buf[MAXSIZE], *p1 = buf, *p2 = buf;
    #define gc()
\
    (p1 == p2 && (p2 = (p1 = buf) + fread(buf, 1, MAXSIZE, stdin), p1 == p2)
\
    ? EOF
\
    : *p1++)

    bool blank(char ch) {
        return ch == ' ' || ch == '\n'
            || ch == '\r' || ch == '\t' || ch == EOF;
    }
    template <typename T>
    Input &operator>> (T &x) {
        x = 0 ;
        bool sign = 0;
        char ch = gc();
        for (; !isdigit(ch); ch = gc())
            if (ch == '-') sign = 1;
        for (; isdigit(ch); ch = gc())
            x = (x << 3) + (x << 1) + ch - '0';
        if (sign)x = -x;
        return *this ;
    }
    Input &operator>> (char &x) {
        x = ' ';
        for (; blank(x); x = gc ());
        return *this ;
    }
    Input &operator>> (double &x) {
        x = 0 ;
        double tmp = 1;
        bool sign = 0;
        char ch = gc();
        for (; !isdigit(ch); ch = gc())
            if (ch == '-') sign = 1;
        for (; isdigit(ch); ch = gc())
            x = x * 10 + ch - '0';
        if (ch == '.')
            for (ch = gc(); isdigit(ch); ch = gc())
                tmp /= 10.0, x += tmp * (ch - '0');
        if (sign) x = -x;
        return *this ;
    }
    Input &operator>>(string &s) {
```

```

        s.clear() ;
        char ch = gc();
        for (; blank(ch); ch = gc());
        for (; !blank(ch); ch = gc()) {
            s += ch ;
        }
        return *this ;
    }
}input;
struct __setprecision {
    int precision ;
};
__setprecision setprecision (int x) { return __setprecision {x};}
struct Output {
    # define MAXSIZE (1 << 20)
    # define cout output
    char pbuf[MAXSIZE], *pp = pbuf ;
    void push(const char &c) {
        // putchar (c);
        if (pp - pbuf == MAXSIZE)
            fwrite(pbuf, 1, MAXSIZE, stdout), pp = pbuf;
        *pp++ = c;
    }
    int precision ;
    Output () { precision = 6 ;}
    ~Output () { fwrite(pbuf, 1, pp - pbuf, stdout);}
    template<class T>
    Output &operator<< (const T &x) {
        char stack[35]; int top = 0 ;
        T tmp = x ;
        bool _ = tmp < 0 ;
        if (_) tmp *= -1;
        while (tmp) stack[++ top] = '0' + tmp % 10, tmp /= 10;
        if (_) stack[++ top] = '-';
        while (top) push (stack [top]), -- top;
        if (x == 0)push ('0');
        return *this ;
    }
    Output &operator<< (const string &x) {
        for (auto&u : x) push (u);
        return *this;
    }
    template<size_t N>
    Output &operator<< (const char(&x)[N]) {
        *this << string (x) ;
        return *this;
    }
    Output &operator<< (const char* const &x) {
        for (const char* ptr = x; *ptr != '\0'; ++ptr)
            push(*ptr);
        return *this;
    }
    Output &operator<< (const char &x) {
        push (x);
        return *this;
    }
}

```

```

Output &operator<< (const bool &x) {
    push (x ? '1' : '0');
    return *this;
}
Output &operator<<(const double &x) {
    int intPart = static_cast<int>(x);
    *this << intPart; // Output the integer part

    push('.'); // Decimal point

    double decimalPart = x - intPart;
    for (int i = 0; i < precision; ++i) {
        decimalPart *= 10;
        int digit = static_cast<int>(decimalPart);
        *this << char('0' + digit);
        decimalPart -= digit;
    }
    return *this;
}
Output &operator<< (const __setprecision &x) {
    precision = x.precision ;
    return *this ;
}
}output;

```

## u32 指针

```

@ -0,0 +1,27 @@
constexpr int max_size = 262144000;
uint8_t buf[max_size];
uint8_t *head = buf;

using u32 = uint32_t;

template <class T>
struct u32_p {
    u32 x;
    u32_p(u32 x = 0) : x(x) {}
    T *operator->() {
        return (T *) (buf + x);
    }
    operator bool() {
        return x;
    }
    operator u32() {
        return x;
    }
    bool operator==(u32_p rhs) const {
        return x == rhs.x;
    }
    static u32_p __new() {
        // assert(x < max_size);
        return (head += sizeof(T)) - buf;
    }
}

```

```
};
```

# 字符串

## Ac自动机

```
struct AhoCorasick {
    static constexpr int ALPHABET = 26;
    struct Node {
        int len;
        int link;
        int top;
        int val;
        int d;
        std::array<int, ALPHABET> next;
        Node() : len{}, link{}, next{}, top{}, val {-1}, d{} {}
    };

    std::vector<Node> t;

    AhoCorasick() {
        init();
    }

    void init() {
        t.assign(2, Node());
        t[0].next.fill(1);
        t[0].len = -1;
    }

    int newNode() {
        t.emplace_back();
        return t.size() - 1;
    }

    int add(const std::vector<int> &a) {
        int p = 1;
        for (auto x : a) {
            if (t[p].next[x] == 0) {
                t[p].next[x] = newNode();
                t[t[p].next[x]].len = t[p].len + 1;
            }
            p = t[p].next[x];
        }
        apply (t[p].val);
        return p;
    }

    int add(const std::string &a, char offset = 'a') {
        std::vector<int> b(a.size());
        for (int i = 0; i < a.size(); i++) {
            b[i] = a[i] - offset;
        }
        return add(b);
    }
};
```

```

}

void work() {
    std::queue<int> q;
    q.push(1);

    while (!q.empty()) {
        int x = q.front();
        q.pop();

        t[x].top = t[link(x)].val >= 0 ? link(x) : top(link(x));

        for (int i = 0; i < ALPHABET; i++) {
            if (t[x].next[i] == 0) {
                t[x].next[i] = t[t[x].link].next[i];
            } else {
                t[t[x].next[i]].link = t[t[x].link].next[i];
                t[t[t[x].link].next[i]].d += 1;
                q.push(t[x].next[i]);
            }
        }
    }
}

int next(int p, int x) {
    return t[p].next[x];
}

int next(int p, char c, char offset = 'a') {
    return next(p, c - 'a');
}

int link(int p) {
    return t[p].link;
}

int len(int p) {
    return t[p].len;
}

int& val(int p) {
    return t[p].val;
}

int top (int p) {
    return t[p].top;
}

int size() {
    return t.size();
}

int& d ( int p ) {
    return t[p].d;
}

```

```

void apply (auto& val) {
    val = 0 ;
}
};

```

## 字符串哈希

```

std::mt19937 rng(std::chrono::steady_clock::now().time_since_epoch().count());

bool isprime(int n) {
    if (n <= 1) return false;
    for (int i = 2; i * i <= n; i++)
        if (n % i == 0)
            return false;
    return true;
}

int findPrime(int n) {
    while (!isprime(n))
        n++;
    return n;
}

template<int N>
struct StringHash {
    static array<int, N> mod;
    static array<int, N> base;
    vector<array<int, N>> p, h;
    StringHash() = default;
    StringHash(const string& s) {
        int n = s.size();
        p.resize(n);
        h.resize(n);
        fill(p[0].begin(), p[0].end(), 1);
        for (int i = 0; i < n; i++)
            for (int j = 0; j < N; j++) {
                p[i][j] = 111 * (i == 0 ? 111 : p[i - 1][j]) * base[j] % mod[j];
                h[i][j] = (111 * (i == 0 ? 011 : h[i - 1][j]) * base[j] + s[i]) %
mod[j];
            }
    }

    array<int, N> query(int l, int r) {
        assert(r >= l - 1);
        array<int, N> ans{};
        if (l > r) return {0, 0};
        for (int i = 0; i < N; i++) {
            ans[i] = (h[r][i] - 111 * (l == 0 ? 011 : h[l - 1][i]) * (r - l + 1
== 0 ? 111 : p[r - 1][i]) % mod[i] + mod[i]) % mod[i];
        }
        return ans;
    }
};

constexpr int HN = 2;
template<>
array<int, 2> StringHash<HN>::mod =

```

```

        {findPrime(rng() % 900000000 + 100000000),
         findPrime(rng() % 900000000 + 100000000)};
template<>
array<int, 2> StringHash<HN>::base {13331, 131};
using Hashing = StringHash<HN>;

```

## 后缀数组

```

using i64 = long long;
struct SuffixArray {
    int n;
    std::vector<int> sa, rk, lc;
    SuffixArray(const std::string &s) {
        n = s.length();
        sa.resize(n);
        lc.resize(n - 1);
        rk.resize(n);
        std::iota(sa.begin(), sa.end(), 0);
        std::sort(sa.begin(), sa.end(), [&](int a, int b) {return s[a] < s[b];});
        rk[sa[0]] = 0;
        for (int i = 1; i < n; ++i)
            rk[sa[i]] = rk[sa[i - 1]] + (s[sa[i]] != s[sa[i - 1]]);
        int k = 1;
        std::vector<int> tmp, cnt(n);
        tmp.reserve(n);
        while (rk[sa[n - 1]] < n - 1) {
            tmp.clear();
            for (int i = 0; i < k; ++i)
                tmp.push_back(n - k + i);
            for (auto i : sa)
                if (i >= k)
                    tmp.push_back(i - k);
            std::fill(cnt.begin(), cnt.end(), 0);
            for (int i = 0; i < n; ++i)
                ++cnt[rk[i]];
            for (int i = 1; i < n; ++i)
                cnt[i] += cnt[i - 1];
            for (int i = n - 1; i >= 0; --i)
                sa[--cnt[rk[tmp[i]]]] = tmp[i];
            std::swap(rk, tmp);
            rk[sa[0]] = 0;
            for (int i = 1; i < n; ++i)
                rk[sa[i]] = rk[sa[i - 1]] + (tmp[sa[i - 1]] < tmp[sa[i]] || sa[i - 1] + k == n || tmp[sa[i - 1] + k] < tmp[sa[i] + k]);
            k *= 2;
        }
        for (int i = 0, j = 0; i < n; ++i) {
            if (rk[i] == 0) {
                j = 0;
            } else {
                for (j -= j > 0; i + j < n && sa[rk[i] - 1] + j < n && s[i + j] == s[sa[rk[i] - 1] + j]; )
                    ++j;
                lc[rk[i] - 1] = j;
            }
        }
    }
};

```

```

    }
}
};

```

## KMP

```

struct KMP{
    int n;
    std::vector<int> pi;
    std::vector<vector<int>> aut;

    KMP(const std::string &s) {
        n = (int)s.length();
        prefix_function(s);
        compute_automaton(s);
    }

    void prefix_function(string s) {
        pi.resize(n);
        for (int i = 1; i < n; i++) {
            int j = pi[i - 1];
            while (j > 0 && s[i] != s[j]) j = pi[j - 1];
            if (s[i] == s[j]) j++;
            pi[i] = j;
        }
    }

    void compute_automaton(string s) {
        aut.resize(n, vector<int>(26));
        for (int i = 0; i < n; i++) {
            for (int c = 0; c < 26; c++) {
                if (i > 0 && 'a' + c != s[i])
                    aut[i][c] = aut[pi[i - 1]][c];
                else
                    aut[i][c] = i + ('a' + c == s[i]);
            }
        }
    }
};

```

## Trie

```

constexpr int max_size = 262144000;
uint8_t buf[max_size];
uint8_t *head = buf;

using u32 = uint32_t;

template <class T>
struct u32_p {
    u32 x;

```



```

u32_p(u32 x = 0) : x(x) {}
T *operator->() {
    return (T *) (buf + x);
}
operator bool() {
    return x;
}
operator u32() {
    return x;
}
bool operator==(u32_p rhs) const {
    return x == rhs.x;
}
static u32_p __new() {
    // assert(x < max_size);
    return (head += sizeof(T)) - buf;
}
};

constexpr int N = 2e5;

struct node;
using Trie = u32_p<node>;

struct node {
    array<Trie, 2> ch{};
    int x; int sum;
};

```

## Manacher

```

std::vector<int> manacher(std::string s) {
    std::string t = "#";
    for (auto c : s) {
        t += c;
        t += '#';
    }
    int n = t.size();
    std::vector<int> r(n);
    for (int i = 0, j = 0; i < n; i++) {
        if (2 * j - i >= 0 && j + r[j] > i) {
            r[i] = std::min(r[2 * j - i], j + r[j] - i);
        }
        while (i - r[i] >= 0 && i + r[i] < n && t[i - r[i]] == t[i + r[i]]) {
            r[i] += 1;
        }
        if (i + r[i] > j + r[j]) {
            j = i;
        }
    }
    return r;
}

```

## Z函数

```
std::vector<int> zFunction(std::string s) {
    int n = s.size();
    std::vector<int> z(n + 1);
    z[0] = n;
    for (int i = 1, j = 1; i < n; i++) {
        z[i] = std::max(0ll, std::min(j + z[j] - i, z[i - j]));
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
            z[i]++;
        }
        if (i + z[i] > j + z[j]) {
            j = i;
        }
    }
    return z;
}
```

## 动态规划

### dp优化

### 斜率优化

板子：x 单调, k 单调

```
// k层dp, 每层n位
int n, k;
vector<ll> f(n + 1), g(n + 1); // 滚动数组

// 斜率优化, 点(X, Y), 斜率k,
auto X = [&](int i) { return 1; }; //
auto Y = [&](int i) { return 1; }; //
auto K = [&](int i) { return 1; }; //

// 计算斜率
auto slope = [&](int i, int j) -> long double{
    if(X(j) == X(i)) return (Y(j) >= Y(i) ? 1e18 : -1e18);
    else {
        return (long double)(Y(j) - Y(i)) / (X(j) - X(i));
    }
};

// 队列存凸包
vector<int> q(n + 3);

for(int i = 1; i <= n; ++i) {
    // g[i] = ... ;
    // 初始化k = 1, 一般可以直接计算
}
```

```

// 下凸包为例
for(int c = 2; c <= k; ++c) {
    int head = 1, tail = 0;
    q[++tail] = 0;
    for(int i = 1; i <= n; ++i) {
        while(head < tail && slope(q[head], q[head + 1]) <= K(i)) ++head;
        ll B = Y(q[head]) - K(i) * X(q[head]);
        // f[i] = B + ...; f[i] 与 B 之间的式子
        while(head < tail && i != n && slope(q[tail - 1], q[tail]) >=
slope(q[tail], i)) --tail;
        q[++tail] = i;
    }
    std::swap(f, g);
}
cout << g[n];

```

## 板子：x 单调, k 不单调

```

int n;
vector<ll> dp(n + 1, 1e18);
dp[0] = 0;

// 斜率优化, 点(X, Y), 斜率K,
auto X = [&](int i) { return 1; }; //
auto Y = [&](int i) { return 1; }; //
auto K = [&](int i) { return 1; }; //

// 计算斜率
auto slope = [&](int i, int j) -> long double{
    if(X(j) == X(i)) return (Y(j) >= Y(i) ? 1e18 : -1e18);
    else {
        return (long double)(Y(j) - Y(i)) / (X(j) - X(i));
    }
};

// 队列维护凸包
vector<int> q(n + 5);
int head = 1, tail = 0;
q[++tail] = 0;

// 二分最优策略点, 下凸包为例
auto ask = [&](ll k) {
    int l = head, r = tail;
    while(l < r) {
        int mid = (l + r) >> 1;
        if(slope(q[mid], q[mid + 1]) >= k) r = mid;
        else l = mid + 1;
    }
    return q[l];
};

// 下凸包为例
for(int i = 1; i <= n; ++i) {
    int j = ask(K(i));
    ll B = Y(j) - K(i) * X(j);

```

```

        // dp[i] = B + ... ;
        while(head < tail && i != n && slope(q[tail - 1], q[tail]) >= slope(q[tail],
i)) --tail;
        q[++tail] = i;
    }

    cout << dp[n];

```

## 板子：x 不单调, k 不单调

```

// CDQ板子，以下凸包为例
using ll = long long;
const int maxn = 1e5 + 5;

struct node {
    int id;
    ll x, y, k;
};

// a表示原数组，b为归并辅助数组
vector<node> a(maxn), b(maxn);

ll X(int i) { return a[i].x; }
ll Y(int i) { return a[i].y; }
ll K(int i) { return a[i].k; }
long double slope(int i, int j) {
    if(X(j) == X(i)) return (Y(j) >= Y(i) ? 1e20 : -1e20);
    else {
        return (long double)(Y(j) - Y(i)) / (X(j) - X(i));
    }
};

// dp数组，切记f[]的初始化
vector<ll> f(maxn, 1e18);

// 按照x进行归并
void merge(int L, int mid, int R)
{
    int p1 = L, p2 = mid + 1;
    int tp = L;
    while(p1 <= mid && p2 <= R) {
        if(a[p1].x <= a[p2].x) b[tp++] = a[p1++];
        else b[tp++] = a[p2++];
    }
    while(p1 <= mid) b[tp++] = a[p1++];
    while(p2 <= R) b[tp++] = a[p2++];
    for(int i = L; i <= R; ++i) a[i] = b[i];
}

void cdq(int L, int R) {
    if(L == R) {
        int pos = a[L].id;
        // f[pos] = ...; //视情况而修改，有些求解为前缀最优，则在此处修改。
        // 例f[pos] = max(f[pos], f[pos - 1]);
        // a[L].x = ;
        // a[L].y = ;
    }
}

```

```

        return ;
    }

    int mid = (L + R) >> 1;
    // 分为左右两边
    int p1 = L, p2 = mid + 1;
    for(int i = L; i <= R; ++i) {
        if(a[i].id <= mid) b[p1++] = a[i];
        else b[p2++] = a[i];
    }
    for(int i = L; i <= R; ++i) a[i] = b[i];

    cdq(L, mid);

    // 下凸包, 上凸包则需要改成 slope() <= slope()
    vector<int> q(R - L + 3);
    int head = 1, tail = 0;
    for(int i = L; i <= mid; ++i) {
        while(head < tail && slope(q[tail - 1], q[tail]) >= slope(q[tail], i)) --
tail;
        q[++tail] = i;
    }

    // 下凸包, 上凸包则需要改成 slope() >= K(), 同时f[pos] = max(f[pos], B ...)
    for(int i = mid + 1; i <= R; ++i) {
        while(head < tail && slope(q[head], q[head + 1]) <= K(i)) ++head;
        ll B = Y(q[head]) - K(i) * X(q[head]);
        int pos = a[i].id;
        // f[pos] = min(f[pos], B ...);
    }

    cdq(mid + 1, R);

    merge(L, mid, R);
}

void solve() {
    int n;
    cin >> n;
    for(int i = 1; i <= n; ++i) {
        // a[i].id = i;
        // a[i].k = 2 * h[i];
    }

    // 下凸包, 上凸包修改为 x.k > y.k;
    sort(a.begin() + 1, a.begin() + n + 1, [&](node &x, node &y){
        return x.k < y.k;
    });

    f[1] = 0; // 视情况而初始化
    cdq(1, n);
    cout << f[n];
}

```

# 计算几何

## 二维计算几何基础

```
//#include <bits/stdc++.h>
//
//using namespace std;
//#define IOS ios::sync_with_stdio(false),cin.tie(nullptr),cout.tie(nullptr);
//#define int long long
//
//
template<class T>
struct Point {
    T x;
    T y;

    Point(T x_ = 0, T y_ = 0) : x(x_), y(y_) {}

    template<class U>
    operator Point<U>() {
        return Point<U>(U(x), U(y));
    }

    Point &operator+=(Point p) &{
        x += p.x;
        y += p.y;
        return *this;
    }

    Point &operator-=(Point p) &{
        x -= p.x;
        y -= p.y;
        return *this;
    }

    Point &operator*=(T v) &{
        x *= v;
        y *= v;
        return *this;
    }

    Point operator-() const {
        return Point(-x, -y);
    }

    friend Point operator+(Point a, Point b) {
        return a += b;
    }

    friend Point operator-(Point a, Point b) {
        return a -= b;
    }

    friend Point operator*(Point a, T b) {
```

```

        return a *= b;
    }

    friend Point operator*(T a, Point b) {
        return b *= a;
    }

    friend bool operator==(Point a, Point b) {
        return a.x == b.x && a.y == b.y;
    }

    friend std::istream &operator>>(std::istream &is, Point &p) {
        return is >> p.x >> p.y;
    }

    friend std::ostream &operator<<(std::ostream &os, Point p) {
        return os << "(" << p.x << ", " << p.y << ")";
    }
};

//点乘
template<class T>
T dot(Point<T> a, Point<T> b) {
    return a.x * b.x + a.y * b.y;
}

//叉乘
template<class T>
T cross(Point<T> a, Point<T> b) {
    return a.x * b.y - a.y * b.x;
}

//template<class T>
//////ca 与 cb 叉乘
//T cross(Point<T> a, Point<T> b, Point<T> c) {
//    Point<T> pa = {b.x - a.x, b.y - a.y};
//    Point<T> pb = {c.x - a.x, c.y - b.y};
//    return cross(pa, pb);
//}

//点到原点距离的平方
template<class T>
T square(Point<T> p) {
    return dot(p, p);
}

//点到原点距离
template<class T>
double length(Point<T> p) {
    return std::sqrt(double(square(p)));
}

long double length(Point<long double> p) {
    return std::sqrt(square(p));
}

```

```

}

//斜率
template<class T>
double slope(Point<T> p) {
    return (double) p.y / (double) p.x;
}

long double slope(Point<long double> p) {
    return (double) p.y / (double) p.x;
}

template<class T>
Point<T> rotate(Point<T> a) {
    return Point(-a.y, a.x);
} // 逆时针旋转90°

template<class T>
int sgn(Point<T> a) {
    return a.y > 0 || (a.y == 0 && a.x > 0) ? 1 : -1;
}

template<class T>
int Quadrant(Point<T> a) {
    //象限排序，注意包含四个坐标轴
    if (a.x > 0 && a.y >= 0) return 1;
    if (a.x <= 0 && a.y > 0) return 2;
    if (a.x < 0 && a.y <= 0) return 3;
    if (a.x >= 0 && a.y < 0) return 4;
}

//极角序
template<class T>
bool cmp(Point<T> a, Point<T> b) {
    Point<T> c(0, 0); //原点
    if (cross(c, a, b) == 0) //计算叉积，函数在上面有介绍，如果叉积相等，按照x从小到大排序
        return a.x < b.x;
    else return cross(c, a, b) > 0;
}

template<class T>
struct Line {
    Point<T> a;
    Point<T> b;

    Line(Point<T> a_ = Point<T>(), Point<T> b_ = Point<T>()) : a(a_), b(b_) {}
};

template<class T>
Point<T> getprojection(Line<T> l, Point<T> c) {
    auto a = l.a;
    auto b = l.b;
    if (a == b) {

```



```

        return a;
    }
    long double x1 = a.x, x2 = b.x, x0 = c.x, y1 = a.y, y2 = b.y, y0 = c.y;
    long double k = -((x1 - x0) * (x2 - x1) + (y1 - y0) * (y2 - y1)) / ((x1 - x2)
* (x1 - x2) + (y1 - y2) * (y1 - y2));
    long double xf = k * (x2 - x1) + x1;
    long double yf = k * (y2 - y1) + y1;
    return Point<T>(xf, yf);
}

template<class T>
Point<T> getreflection(Line<T> l, Point<T> c) {
    auto pf = getprojection(l, c);
    long double xf = pf.x;
    long double yf = pf.y;
    return Point<T>(2 * xf - c.x, 2 * yf - c.y);
}

template<class T>
Point<T> lineIntersection(Line<T> l1, Line<T> l2) {
    return l1.a + (l1.b - l1.a) * (cross(l2.b - l2.a, l1.a - l2.a) / cross(l2.b -
l2.a, l1.a - l1.b));
}

template<class T>
bool pointOnSegment(Point<T> p, Line<T> l) {
    return cross(p - l.a, l.b - l.a) == 0 && std::min(l.a.x, l.b.x) <= p.x && p.x
<= std::max(l.a.x, l.b.x)
        && std::min(l.a.y, l.b.y) <= p.y && p.y <= std::max(l.a.y, l.b.y);
}

template<class T>
bool pointInPolygon(Point<T> a, std::vector<Point<T>> p) {
    int n = p.size();
    for (int i = 0; i < n; i++) {
        if (pointOnSegment(a, Line(p[i], p[(i + 1) % n]))) {
            return true;
        }
    }
    //先检查是否边上

    int t = 0;
    for (int i = 0; i < n; i++) {
        auto u = p[i];
        auto v = p[(i + 1) % n];
        if (u.x < a.x && v.x >= a.x && pointOnLineLeft(a, Line(v, u))) {
            t ^= 1;
        }
        if (u.x >= a.x && v.x < a.x && pointOnLineLeft(a, Line(u, v))) {
            t ^= 1;
        }
    }

    return t == 1;
}

```

```

// 0 : not intersect
// 1 : strictly intersect
// 2 : overlap
// 3 : intersect at endpoint
template<class T>
std::tuple<int, Point<T>, Point<T>> segmentIntersection(Line<T> l1, Line<T> l2) {
    if (std::max(l1.a.x, l1.b.x) < std::min(l2.a.x, l2.b.x)) {
        return {0, Point<T>(), Point<T>()};
    }
    if (std::min(l1.a.x, l1.b.x) > std::max(l2.a.x, l2.b.x)) {
        return {0, Point<T>(), Point<T>()};
    }
    if (std::max(l1.a.y, l1.b.y) < std::min(l2.a.y, l2.b.y)) {
        return {0, Point<T>(), Point<T>()};
    }
    if (std::min(l1.a.y, l1.b.y) > std::max(l2.a.y, l2.b.y)) {
        return {0, Point<T>(), Point<T>()};
    }
    if (cross(l1.b - l1.a, l2.b - l2.a) == 0) {
        if (cross(l1.b - l1.a, l2.a - l1.a) != 0) {
            return {0, Point<T>(), Point<T>()};
        } else {
            auto maxx1 = std::max(l1.a.x, l1.b.x);
            auto minx1 = std::min(l1.a.x, l1.b.x);
            auto maxy1 = std::max(l1.a.y, l1.b.y);
            auto miny1 = std::min(l1.a.y, l1.b.y);
            auto maxx2 = std::max(l2.a.x, l2.b.x);
            auto minx2 = std::min(l2.a.x, l2.b.x);
            auto maxy2 = std::max(l2.a.y, l2.b.y);
            auto miny2 = std::min(l2.a.y, l2.b.y);
            Point<T> p1(std::max(minx1, minx2), std::max(miny1, miny2));
            Point<T> p2(std::min(maxx1, maxx2), std::min(maxy1, maxy2));
            if (!pointOnSegment(p1, l1)) {
                std::swap(p1.y, p2.y);
            }
            if (p1 == p2) {
                return {3, p1, p2};
            } else {
                return {2, p1, p2};
            }
        }
    }
    auto cp1 = cross(l2.a - l1.a, l2.b - l1.a);
    auto cp2 = cross(l2.a - l1.b, l2.b - l1.b);
    auto cp3 = cross(l1.a - l2.a, l1.b - l2.a);
    auto cp4 = cross(l1.a - l2.b, l1.b - l2.b);

    if ((cp1 > 0 && cp2 > 0) || (cp1 < 0 && cp2 < 0) || (cp3 > 0 && cp4 > 0) ||
        (cp3 < 0 && cp4 < 0)) {
        return {0, Point<T>(), Point<T>()};
    }

    Point p = lineIntersection(l1, l2);
    if (cp1 != 0 && cp2 != 0 && cp3 != 0 && cp4 != 0) {
        return {1, p, p};
    }
}

```

```

    } else {
        return {3, p, p};
    }
}

template<class T>
bool segmentInPolygon(Line<T> l, std::vector<Point<T>> p) {
    int n = p.size();
    if (!pointInPolygon(l.a, p)) {
        return false;
    }
    if (!pointInPolygon(l.b, p)) {
        return false;
    }
    for (int i = 0; i < n; i++) {
        auto u = p[i];
        auto v = p[(i + 1) % n];
        auto w = p[(i + 2) % n];
        auto [t, p1, p2] = segmentIntersection(l, Line(u, v));

        if (t == 1) {
            return false;
        }
        if (t == 0) {
            continue;
        }
        if (t == 2) {
            if (pointOnSegment(v, l) && v != l.a && v != l.b) {
                if (cross(v - u, w - v) > 0) {
                    return false;
                }
            }
        }
    }
    } else {
        if (p1 != u && p1 != v) {
            if (pointOnLineLeft(l.a, Line(v, u))
                || pointOnLineLeft(l.b, Line(v, u))) {
                return false;
            }
        }
        } else if (p1 == v) {
            if (l.a == v) {
                if (pointOnLineLeft(u, l)) {
                    if (pointOnLineLeft(w, l)
                        && pointOnLineLeft(w, Line(u, v))) {
                        return false;
                    }
                }
            } else {
                if (pointOnLineLeft(w, l)
                    || pointOnLineLeft(w, Line(u, v))) {
                    return false;
                }
            }
        }
        } else if (l.b == v) {
            if (pointOnLineLeft(u, Line(l.b, l.a))) {
                if (pointOnLineLeft(w, Line(l.b, l.a))
                    && pointOnLineLeft(w, Line(u, v))) {
                    return false;
                }
            }
        }
    }
}

```

```

        }
    } else {
        if (pointOnLineLeft(w, Line(l.b, l.a))
            || pointOnLineLeft(w, Line(u, v))) {
            return false;
        }
    }
} else {
    if (pointOnLineLeft(u, l)) {
        if (pointOnLineLeft(w, Line(l.b, l.a))
            || pointOnLineLeft(w, Line(u, v))) {
            return false;
        }
    } else {
        if (pointOnLineLeft(w, l)
            || pointOnLineLeft(w, Line(u, v))) {
            return false;
        }
    }
}
}
}
}
}
return true;
}

```

```

using Vec = Point<int>; //注意类型
//using Vec = Point<double>

```

```

//
//template<class T>
/////半平面交
//std::vector<Point<T>> hp(std::vector<Line<T>> lines) {
//    std::sort(lines.begin(), lines.end(), [&](auto l1, auto l2) {
//        auto d1 = l1.b - l1.a;
//        auto d2 = l2.b - l2.a;
//
//        if (sgn(d1) != sgn(d2)) {
//            return sgn(d1) == 1;
//        }
//
//        return cross(d1, d2) > 0;
//    });
//
//    std::deque<Line<T>> ls;
//    std::deque<Point<T>> ps;
//    for (auto l: lines) {
//        if (ls.empty()) {
//            ls.push_back(l);
//            continue;
//        }
//
//        while (!ps.empty() && !pointOnLineLeft(ps.back(), l)) {
//            ps.pop_back();
//            ls.pop_back();
//        }
//    }
//}

```

```

//      }
//
//      while (!ps.empty() && !pointOnLineLeft(ps[0], l)) {
//          ps.pop_front();
//          ls.pop_front();
//      }
//
//      if (cross(l.b - l.a, ls.back().b - ls.back().a) == 0) {
//          if (dot(l.b - l.a, ls.back().b - ls.back().a) > 0) {
//
//              if (!pointOnLineLeft(ls.back().a, l)) {
//                  assert(ls.size() == 1);
//                  ls[0] = l;
//              }
//              continue;
//          }
//          return {};
//      }
//
//      ps.push_back(lineIntersection(ls.back(), l));
//      ls.push_back(l);
//  }
//
//  while (!ps.empty() && !pointOnLineLeft(ps.back(), ls[0])) {
//      ps.pop_back();
//      ls.pop_back();
//  }
//  if (ls.size() <= 2) {
//      return {};
//  }
//  ps.push_back(lineIntersection(ls[0], ls.back()));
//
//  return std::vector(ps.begin(), ps.end());
//}
template<class T>
struct Frac {
    T num;
    T den;
    Frac(T num_, T den_) : num(num_), den(den_) {
        if (den < 0) {
            den = -den;
            num = -num;
        }
    }
    Frac() : Frac(0, 1) {}
    Frac(T num_) : Frac(num_, 1) {}
    explicit operator double() const {
        return 1. * num / den;
    }
    Frac &operator+=(const Frac &rhs) {
        num = num * rhs.den + rhs.num * den;
        den *= rhs.den;
        return *this;
    }
    Frac &operator-=(const Frac &rhs) {
        num = num * rhs.den - rhs.num * den;

```

```

        den *= rhs.den;
        return *this;
    }
    Frac &operator*=(const Frac &rhs) {
        num *= rhs.num;
        den *= rhs.den;
        return *this;
    }
    Frac &operator/=(const Frac &rhs) {
        num *= rhs.den;
        den *= rhs.num;
        if (den < 0) {
            num = -num;
            den = -den;
        }
        return *this;
    }
    friend Frac operator+(Frac lhs, const Frac &rhs) {
        return lhs += rhs;
    }
    friend Frac operator-(Frac lhs, const Frac &rhs) {
        return lhs -= rhs;
    }
    friend Frac operator*(Frac lhs, const Frac &rhs) {
        return lhs *= rhs;
    }
    friend Frac operator/(Frac lhs, const Frac &rhs) {
        return lhs /= rhs;
    }
    friend Frac operator-(const Frac &a) {
        return Frac(-a.num, a.den);
    }
    friend bool operator==(const Frac &lhs, const Frac &rhs) {
        return lhs.num * rhs.den == rhs.num * lhs.den;
    }
    friend bool operator!=(const Frac &lhs, const Frac &rhs) {
        return lhs.num * rhs.den != rhs.num * lhs.den;
    }
    friend bool operator<(const Frac &lhs, const Frac &rhs) {
        return lhs.num * rhs.den < rhs.num * lhs.den;
    }
    friend bool operator>(const Frac &lhs, const Frac &rhs) {
        return lhs.num * rhs.den > rhs.num * lhs.den;
    }
    friend bool operator<=(const Frac &lhs, const Frac &rhs) {
        return lhs.num * rhs.den <= rhs.num * lhs.den;
    }
    friend bool operator>=(const Frac &lhs, const Frac &rhs) {
        return lhs.num * rhs.den >= rhs.num * lhs.den;
    }
    friend std::ostream &operator<<(std::ostream &os, Frac x) {
        T g = std::gcd(x.num, x.den);
        if (x.den == g) {
            return os << x.num / g;
        } else {
            return os << x.num / g << "/" << x.den / g;
        }
    }

```

```

    }
}
};

```

## 凸包

```

struct Point {
    i64 x;
    i64 y;
    Point(i64 x = 0, i64 y = 0) : x(x), y(y) {}
};

bool operator==(const Point &a, const Point &b) {
    return a.x == b.x && a.y == b.y;
}

Point operator+(const Point &a, const Point &b) {
    return Point(a.x + b.x, a.y + b.y);
}

Point operator-(const Point &a, const Point &b) {
    return Point(a.x - b.x, a.y - b.y);
}

i64 dot(const Point &a, const Point &b) {
    return a.x * b.x + a.y * b.y;
}

i64 cross(const Point &a, const Point &b) {
    return a.x * b.y - a.y * b.x;
}

void norm(std::vector<Point> &h) {
    int i = 0;
    for (int j = 0; j < int(h.size()); j++) {
        if (h[j].y < h[i].y || (h[j].y == h[i].y && h[j].x < h[i].x)) {
            i = j;
        }
    }
    std::rotate(h.begin(), h.begin() + i, h.end());
}

int sgn(const Point &a) {
    return a.y > 0 || (a.y == 0 && a.x > 0) ? 0 : 1;
}

std::vector<Point> getHull(std::vector<Point> p) {
    std::vector<Point> h, l;
    std::sort(p.begin(), p.end(), [&](auto a, auto b) {
        if (a.x != b.x) {
            return a.x < b.x;
        } else {
            return a.y < b.y;
        }
    });
}

```

```

});
p.erase(std::unique(p.begin(), p.end()), p.end());
if (p.size() <= 1) {
    return p;
}

for (auto a : p) {
    while (h.size() > 1 && cross(a - h.back(), a - h[h.size() - 2]) <= 0) {
        h.pop_back();
    }
    while (l.size() > 1 && cross(a - l.back(), a - l[l.size() - 2]) >= 0) {
        l.pop_back();
    }
    l.push_back(a);
    h.push_back(a);
}

l.pop_back();
std::reverse(h.begin(), h.end());
h.pop_back();
l.insert(l.end(), h.begin(), h.end());
return l;
}

```





