ID5130 - Parallel Scientific Computing

Assignment - 1

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Q1.

$$3x_1 - x_2 = 2 \quad - (1)$$
$$-x_1 + 3x_2 - x_3 = 1 \quad - (2)$$
$$-x_2 + 3x_3 - x_4 = 1 \quad - (3)$$
$$-x_3 + 3x_4 = 2 \quad - (4)$$

$$\Longleftrightarrow \begin{bmatrix} 3 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{Bmatrix}$$

Using recursive doubling:

Iteration 1:

Eq^n 1: $3x_1 - x_2 = 2$

From eq^n 2: $-x_1 + 3x_2 - x_3 = 1$

$$\Rightarrow x_2 = \frac{1}{3}(1 + x_1 + x_3)$$

Substituting this, we have:

$$3x_1 - \frac{1}{3}(1 + x_1 + x_3) = 2$$

$$\Rightarrow \frac{8}{3}x_1 - \frac{1}{3}x_3 = \frac{7}{3} \quad - (5)$$

Eq^n 2: $-x_1 + 3x_2 - x_3 = 1$

From eq^n 1: $3x_1 - x_2 = 2$

$$x_1 = \frac{1}{3}(2 + x_2)$$

From eq^n 3: $x_3 = \frac{1}{3}(1 + x_2 + x_4)$

$$\therefore -\frac{1}{3}(2 + x_2) + 3x_2 - \frac{1}{3}(1 + x_2 + x_4) = 1$$

$$\Rightarrow \frac{7}{3}x_2 - \frac{1}{3}x_4 = 2$$

$Eq^n\ 3: \quad -x_2 + 3x_3 - x_4 = 1$

From $Eq^n\ 2: \ x_2 = \frac{1}{3}(1 + x_1 + x_3)$

From $Eq^n\ 4: \ x_4 = \frac{1}{3}(2 + x_3)$

$$\therefore \quad -\frac{1}{3}(1 + x_1 + x_3) + 3x_3 - \frac{1}{3}(2 + x_3) = 1$$

$$\Rightarrow \quad -\frac{1}{3}x_1 + \frac{7}{3}x_3 = 2 \quad - (7)$$

$Eq^n\ 4:$

$$-x_3 + 3x_4 = 2$$

From $Eq^n\ 3: \ x_3 = \frac{1}{3}(1 + x_2 + x_4)$

$$\therefore \quad -\frac{1}{3}(1 + x_2 + x_4) + 3x_4 = 2$$

$$\Rightarrow \quad -\frac{1}{3}x_2 + \frac{8}{3}x_4 = \frac{7}{3} \quad - (8)$$

$$\begin{aligned} \frac{8}{3}x_1 - \frac{1}{3}x_3 &= \frac{7}{3} \\ \frac{7}{3}x_2 - \frac{1}{3}x_4 &= 2 \\ -\frac{1}{3}x_1 + \frac{7}{3}x_3 &= 2 \\ -\frac{1}{3}x_2 + \frac{8}{3}x_4 &= \frac{7}{3} \end{aligned} \iff \begin{bmatrix} \frac{8}{3} & 0 & -\frac{1}{3} & 0 \\ 0 & \frac{7}{3} & 0 & -1/3 \\ -\frac{1}{3} & 0 & \frac{7}{3} & 0 \\ 0 & -\frac{1}{3} & 0 & 8/3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} \frac{7}{3} \\ 2 \\ 2 \\ \frac{7}{3} \end{Bmatrix}$$

Iteration 2:

$Eq^n\ 5:$

$$\frac{8}{3}x_1 - \frac{1}{3}x_3 = \frac{7}{3}$$

From $Eq^n\ 7: \ x_3 = \frac{3}{7}\left(2 + \frac{1}{3}x_1\right)$

$$\therefore \quad \frac{8x_1}{3} - \frac{1}{3}\cdot\frac{3}{7}\left(2 + \frac{x_1}{3}\right) = \frac{7}{3}$$

$$\Rightarrow \quad \frac{55}{21}x_1 = \frac{55}{21} \quad - (9)$$

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Eq^n 6: $\frac{7}{3}x_2 - \frac{1}{3}x_4 = 2$

From Eq^n 8: $x_4 = \frac{3}{8}\left(\frac{7}{3} + \frac{x_2}{3}\right)$

$\therefore \frac{7}{3}x_2 - \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{8}\left(\frac{7+x_2}{3}\right) = 2$

$\Rightarrow \frac{55}{24}x_2 = \frac{55}{24}$ — (10)

Eq^n 7: $-\frac{1}{3}x_1 + \frac{7}{3}x_3 = 2$

From eq^n 5: $x_1 = \frac{3}{8}\left(\frac{7}{3} + \frac{x_3}{3}\right)$

$\therefore -\frac{1}{3} \cdot \frac{3}{8}\left(\frac{7+x_3}{3}\right) + \frac{7}{3}x_3 = 2$

$\Rightarrow \frac{55}{24}x_3 = \frac{55}{24}$ — (11)

Eq^n 8: $-\frac{1}{3}x_2 + \frac{8}{3}x_4 = \frac{7}{3}$

From Eq^n 6: $x_2 = \frac{3}{7}\left(2 + \frac{1}{3}x_4\right)$

$\therefore -\frac{1}{3} \cdot \frac{3}{7}\left(2 + \frac{1}{3}x_4\right) + \frac{8}{3}x_4 = \frac{7}{3}$

$\Rightarrow \frac{55}{21}x_4 = \frac{55}{21}$ — (12)

From Eq^{n}s 9, 10, 11, 12:

Back-Substituting:

$$\begin{bmatrix} \frac{55}{21} & 0 & 0 & 0 \\ 0 & \frac{55}{24} & 0 & 0 \\ 0 & 0 & \frac{55}{24} & 0 \\ 0 & 0 & 0 & \frac{55}{21} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} \frac{55}{21} \\ \frac{55}{24} \\ 55/24 \\ 55/21 \end{Bmatrix} \Leftrightarrow \boxed{\begin{matrix} x_1 = 1 \\ x_2 = 1 \\ x_3 = 1 \\ x_4 = 1 \end{matrix}}$$

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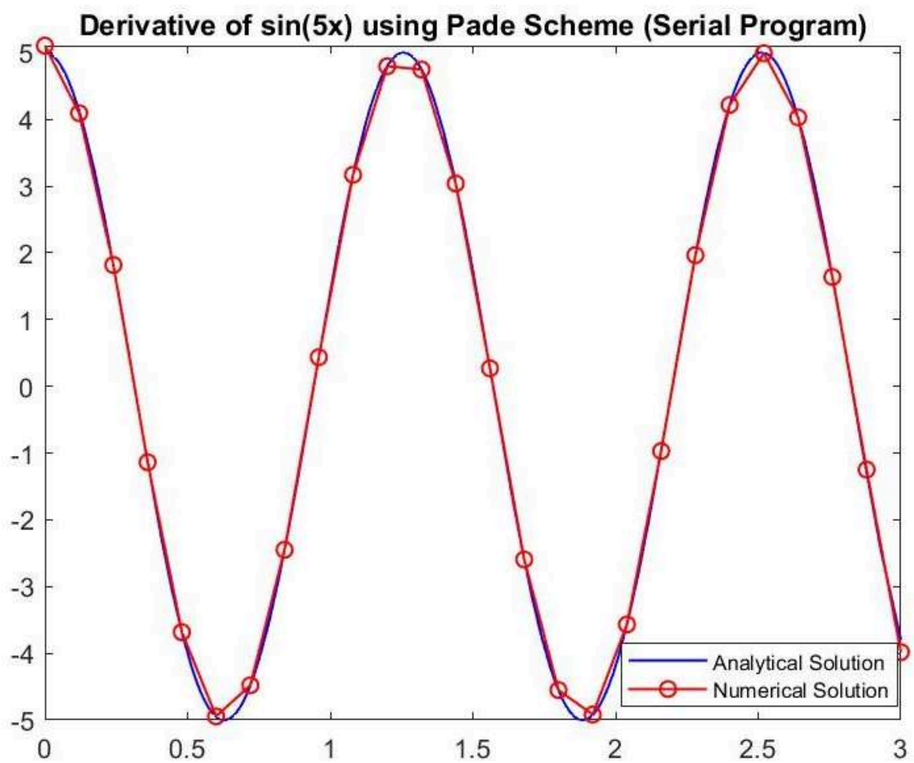
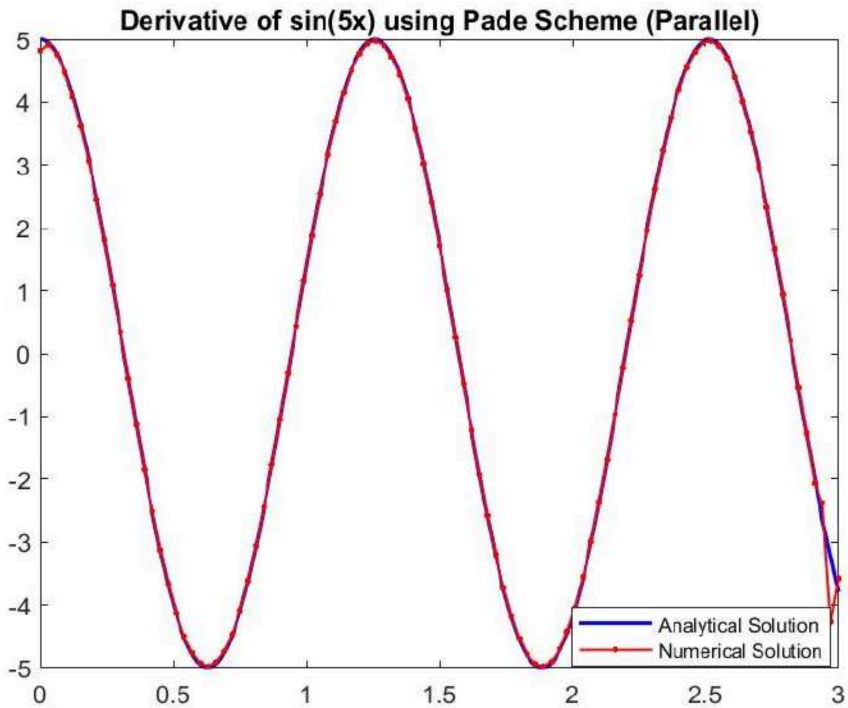
Q2. Calculating the derivative of $f(x) = \sin(5x)$ using fourth-order accurate Padé scheme for the interior and third-order accurate one-sided Padé scheme near the boundaries.

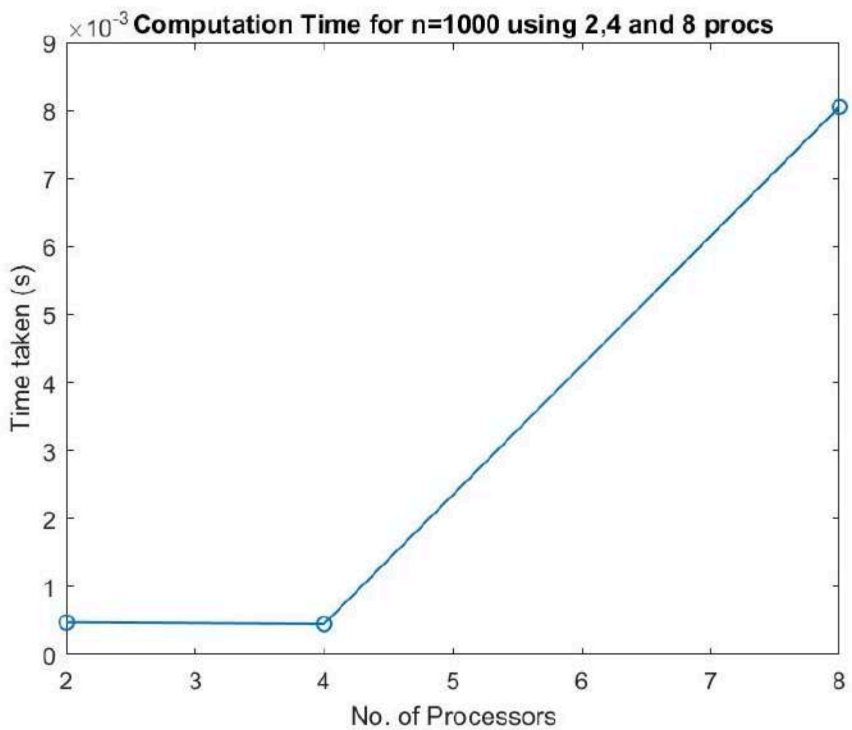
a)



The above was calculated using LU Decomposition with n=25 (n - grid points) b)

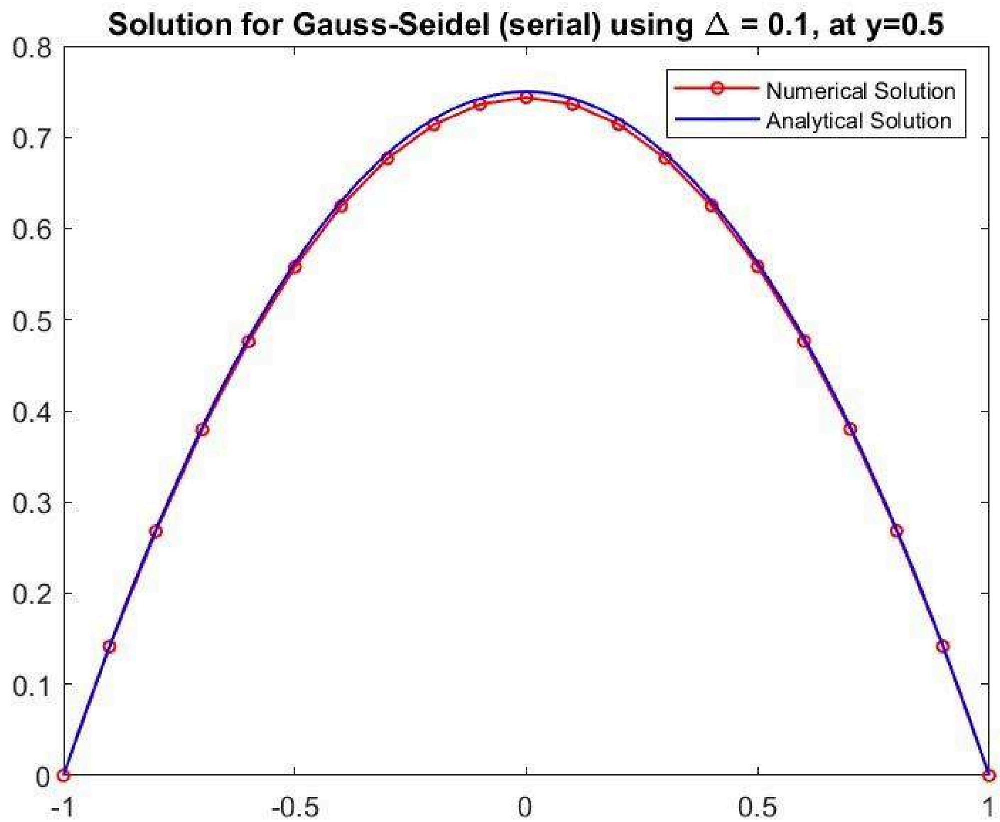


The above was calculated using recursive-doubling algorithm with n=100
(p = number of threads used = 2)



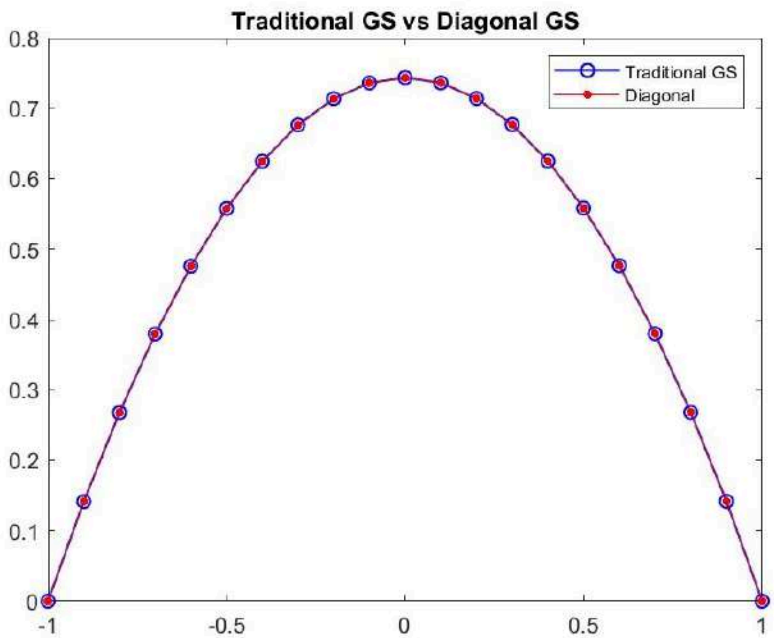
Q3

a)

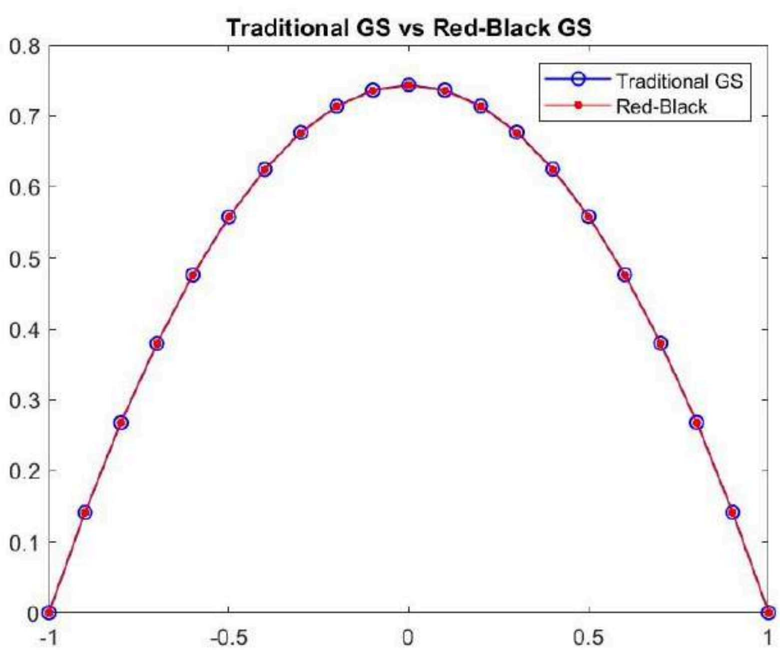


Solution for traditional Gauss-Seidel using $\Delta = \Delta x = \Delta y = 0.1$ – ϕ vs x at y=0.5
The method takes 191 iterations to converge within 1% error.

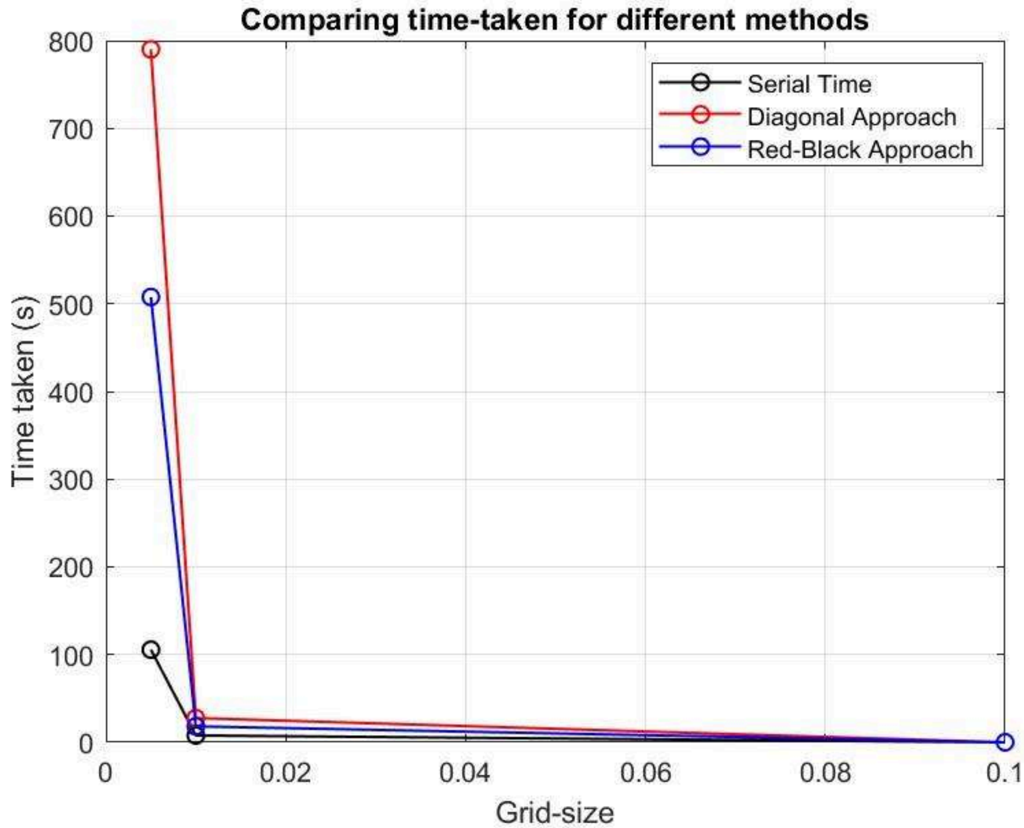
c)



Comparing Traditional Gauss-Seidel and the Diagonal approach
Using Δ = Δx = Δy = 0.1 – plotting ϕ vs x at y=0.5



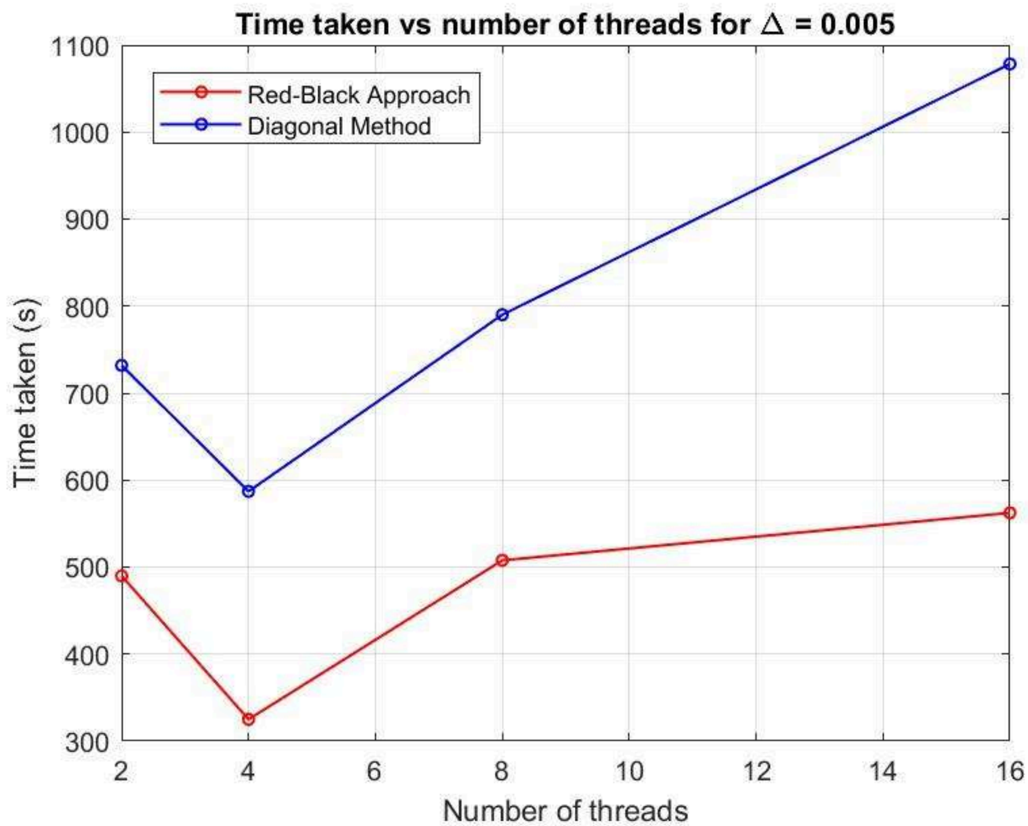
Comparing Traditional Gauss-Seidel and the Red-Black coloring approach
Using Δ = Δx = Δy = 0.1 – plotting ϕ vs x at y=0.5



On running the methods in the local system, no improvements were observed in moving from serial to parallel.

Among the parallel methods, the red-black approach to solution took lesser time and is observed to be a better approach in this case.

d)



Based on the above observation, we can see that the red-black method has a better performance on different numbers of threads.