Computer Vision
---*Lighting and photometric stereo II*

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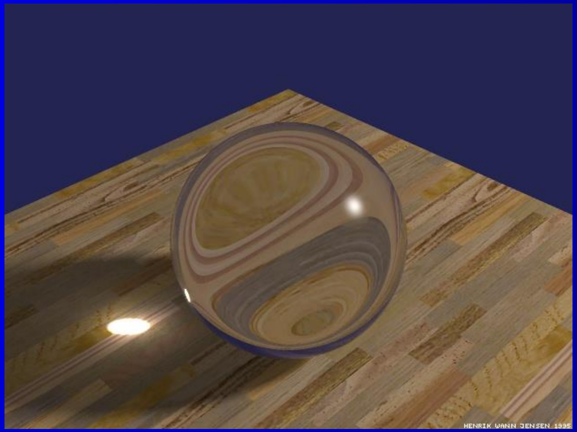
Lighting in Vision

- Complex Appearances





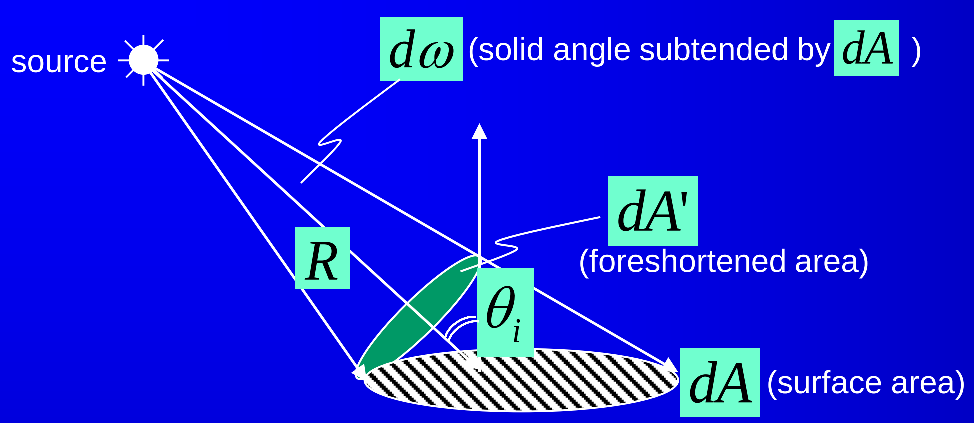


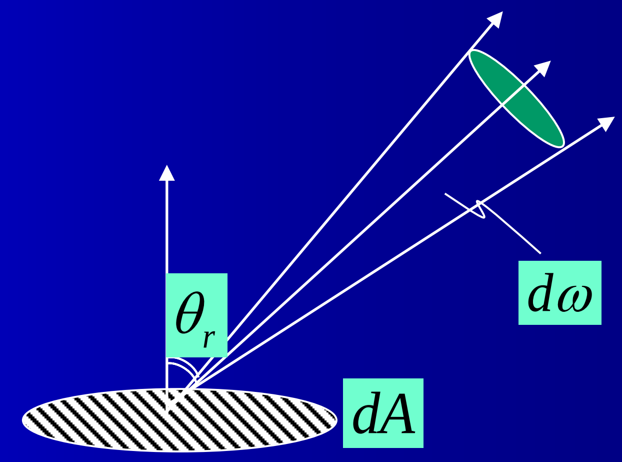






Radiometric concepts – boring…but, important!





(1) Solid Angle : $d\omega = \frac{dA'}{R^2} = \frac{dA\cos\theta_i}{R^2}$ (steradian)

$$d\omega = \frac{dA}{r^2} = \sin\theta\, d\theta\, d\phi$$

(2) Radiant Intensity of Source : $J = \frac{d\Phi}{d\omega}$ (watts / steradian)

Light Flux (power) emitted per unit solid angle

(3) Surface Irradiance : $E = \frac{d\Phi}{dA}$ (watts / m)

Light Flux (power) incident per unit surface area.

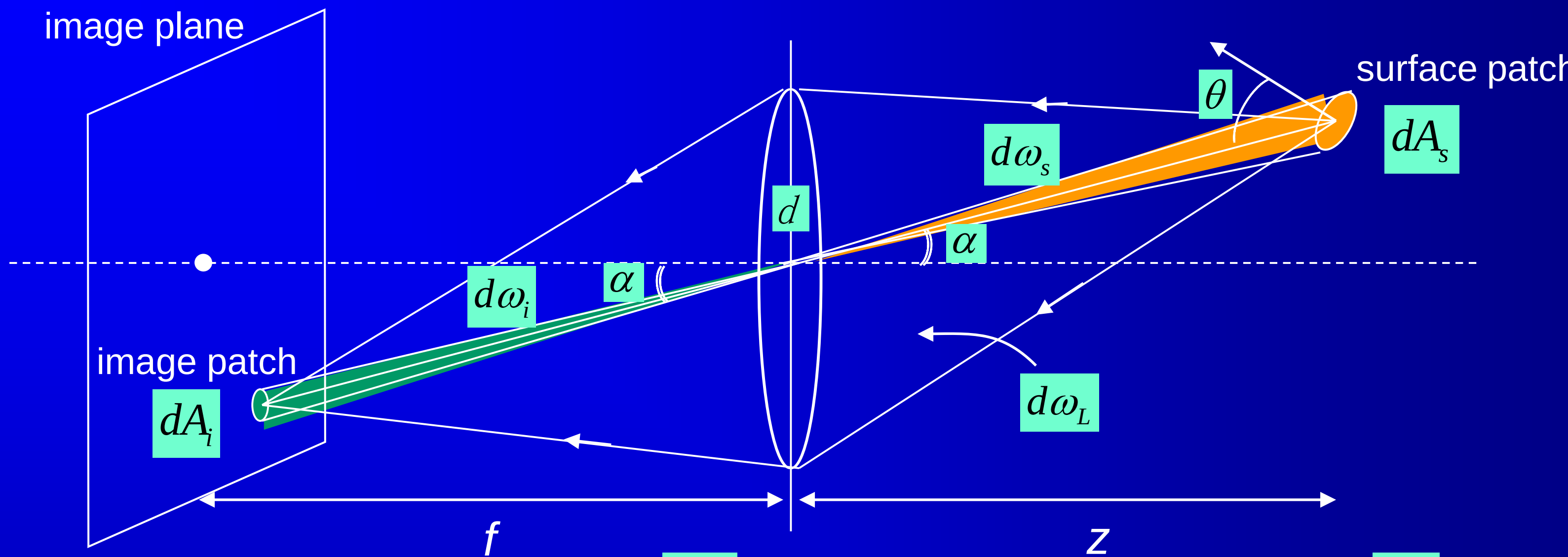
Does not depend on where the light is coming from!

(4) Surface Radiance (tricky) :

$$L = \frac{d^2\Phi}{(dA\cos\theta_r)\ d\omega}$$ (watts / m^2 steradian)

- Flux emitted per unit foreshortened area per unit solid angle.
- L depends on direction θ_r
- Surface can radiate into whole hemisphere.
- L depends on reflectance properties of surface.

Relationship between Scene and Image Brightness



- Flux received by lens from dA_s = Flux projected onto image dA_i

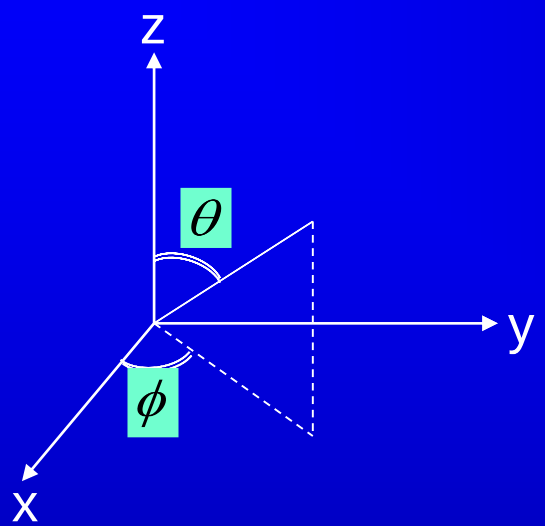
$$L\,(dA_s \cos\theta)\,d\omega_L = E\,dA_i \longrightarrow (3)$$

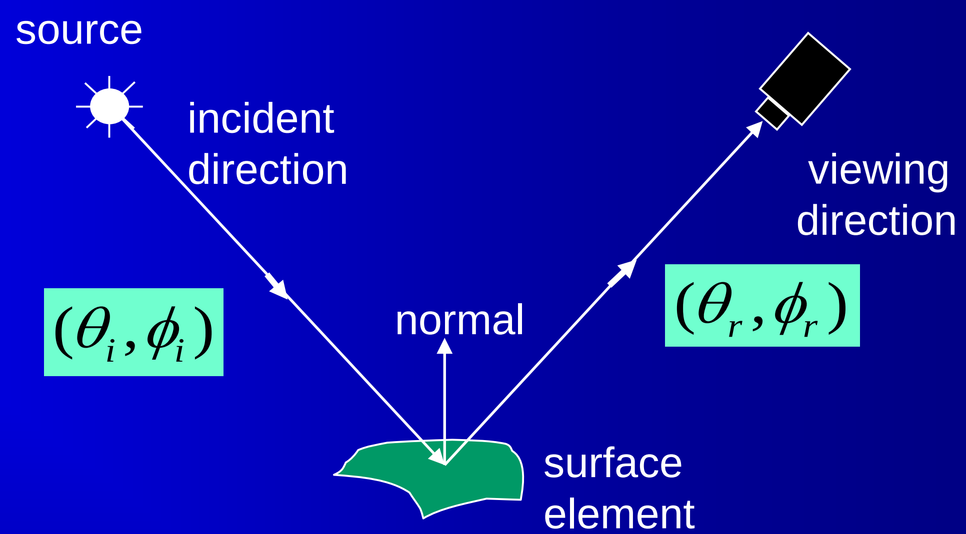
- From (1), (2), and (3):

$$\boxed{E = L\,\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos\alpha^4}$$

  - Image irradiance is proportional to Scene Radiance!
  - Small field of view → Effects of 4th power of cosine are small.

BRDF: Bidirectional Reflectance Distribution Function



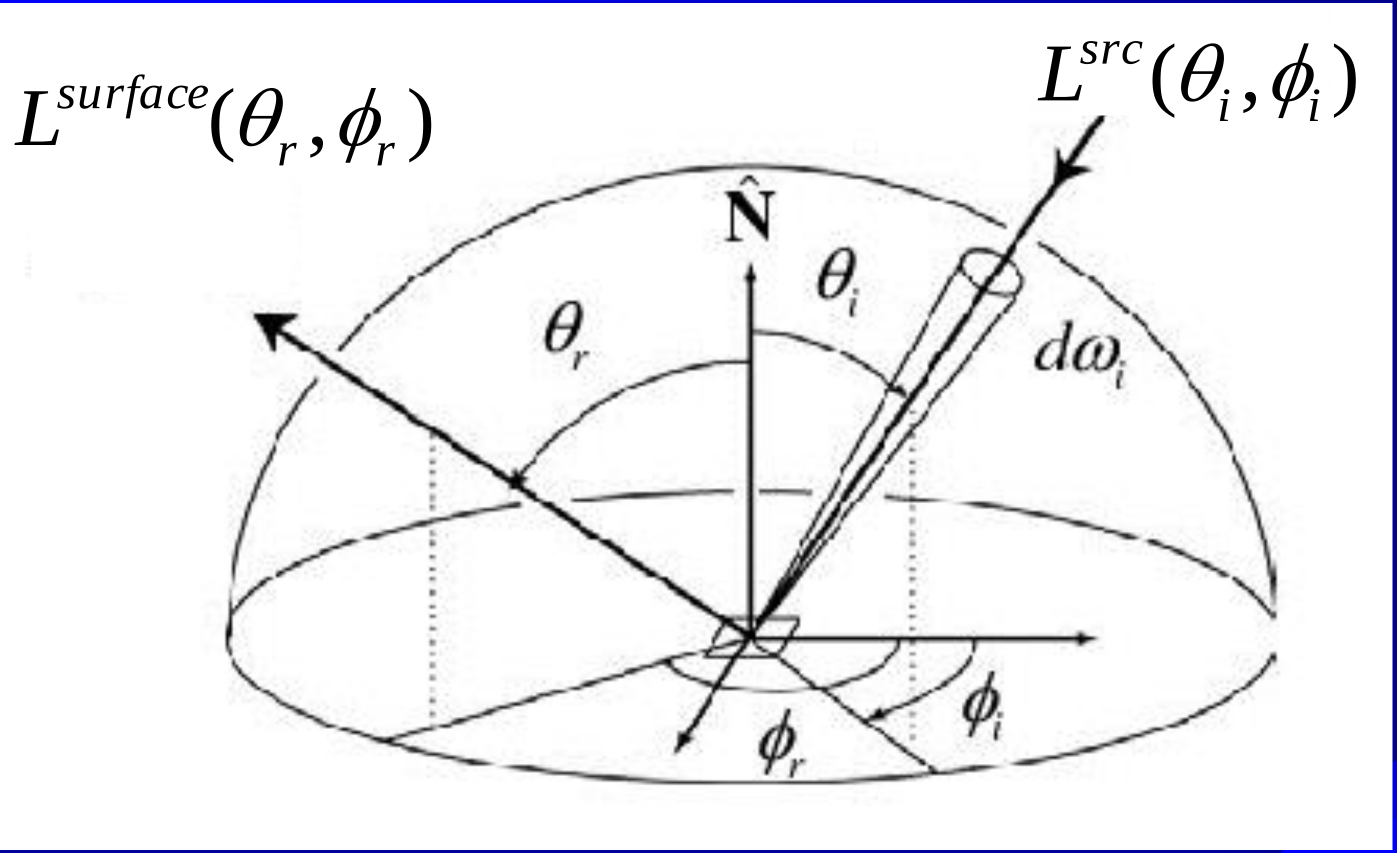


$E^{surface}(\theta_i, \phi_i)$ Irradiance at Surface in direction (θ_i, ϕ_i)

$L^{surface}(\theta_r, \phi_r)$ Radiance of Surface in direction (θ_r, ϕ_r)

BRDF : $$f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{L^{surface}(\theta_r, \phi_r)}{E^{surface}(\theta_i, \phi_i)}$$

Derivation of the Scene Radiance Equation



From the definition of BRDF:

$$L^{surface}(\theta_r,\phi_r) \quad = \quad E^{surface}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)$$

Derivation of the Scene Radiance Equation

From the definition of BRDF:

$$L^{surface}(\theta_r,\phi_r) \quad = \quad \underline{E^{surface}(\theta_i,\phi_i)}\, f(\theta_i,\phi_i;\theta_r,\phi_r)$$

Write Surface Irradiance in terms of Source Radiance:

$$L^{surface}(\theta_r,\phi_r) \quad = \quad L^{src}\underline{(\theta_i,\phi_i)}\, f(\theta_i,\phi_i;\theta_r,\phi_r)\,\underline{\cos\theta_i d\omega_i}$$

Integrate over entire hemisphere of possible source directions:

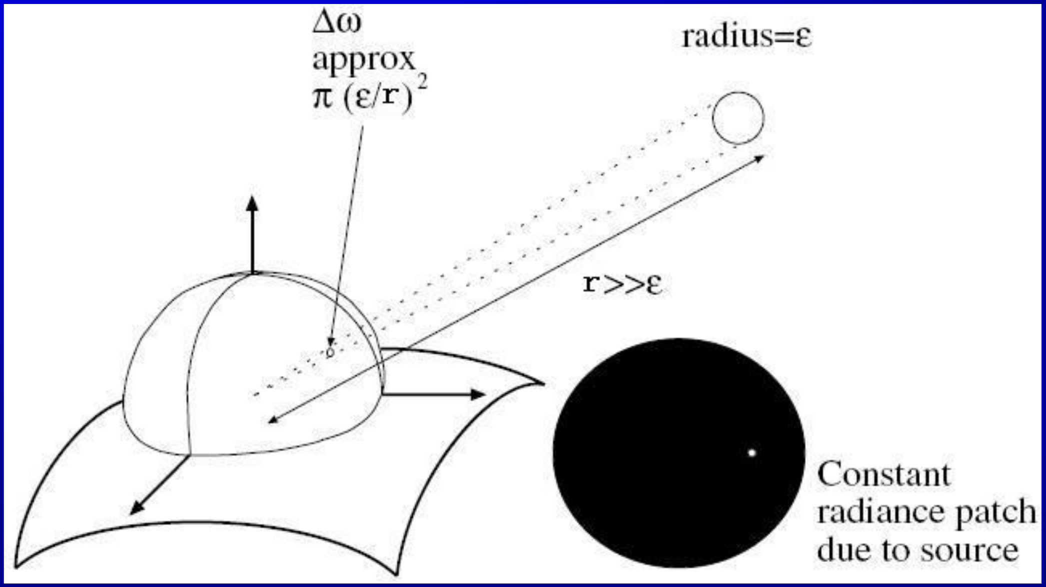
$$L^{surface}(\theta_r,\phi_r) \quad = \quad \int_{2\pi} L^{src}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)\cos\theta_i\ \underline{d\omega_i}$$

Convert from solid angle to theta-phi representation:

$$L^{surface}(\theta_r,\phi_r) \quad = \quad \int_{-\pi}^{\pi}\int_{0}^{\pi/2} L^{src}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)\cos\theta_i\ \underline{\sin\theta_i d\theta_i d\phi_i}$$

Source and their effects

- A light source is defined as anything that emits light that is internally generated (not just reflected).
- Point source (点光源)
- Assume that a surface patch is viewing a sphere of radius ε, at a distance r away, and that $\varepsilon \ll r$. Illustrated as follows.

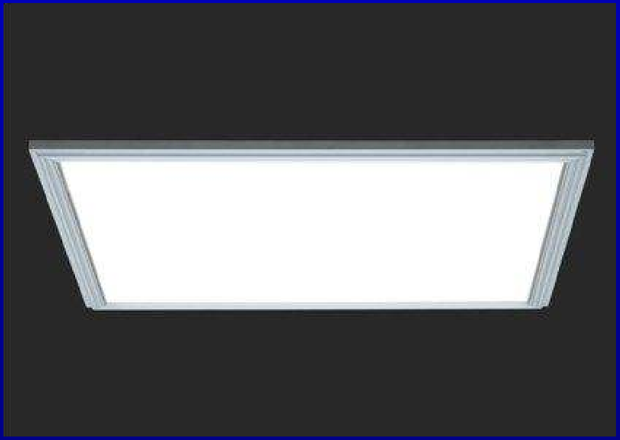
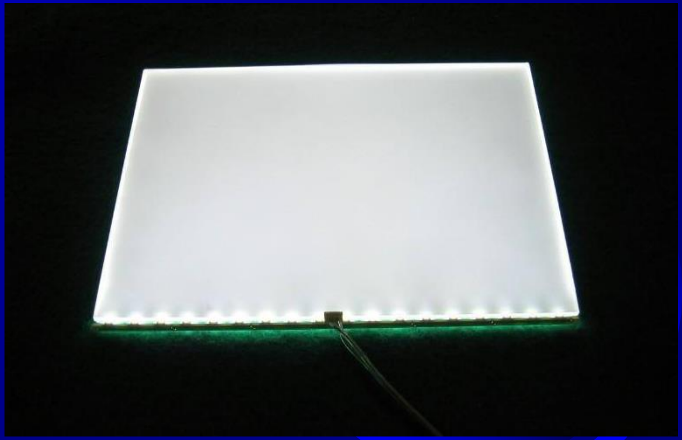


Source and their effects

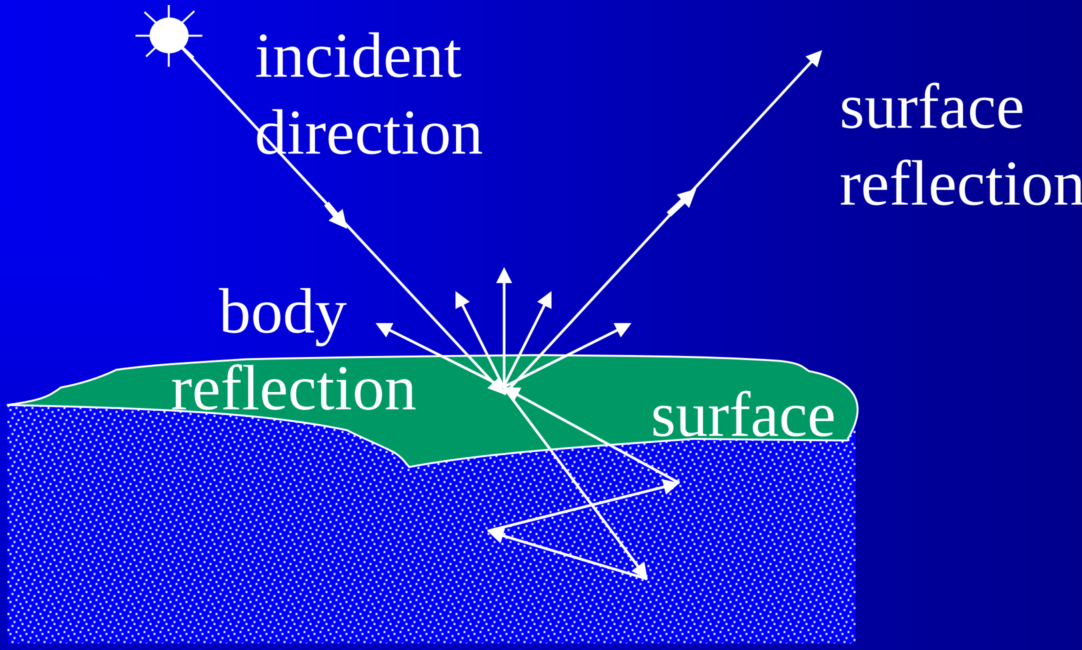
- Line sources
- A line source has the geometry of a line ---- single fluorescent light bulb.
- Line sources are not terribly common in natural scenes or in synthetic environments.
- A line source is modeled as a thin cylinder with diameter ϵ.
- The line source is infinitely long and a patch viewing the source frontally.

Source and their effects

- Area sources
- An area source is an area radiating light.
- They occur quite commonly in natural scenes ----- an overcast sky, or in synthetic environment-----fluorescent light boxes.
- Allow us to explain various shadowing and interreflection effect.
- Area sources emits radiance independent of position and of direction------they can be described by their exitance（出射度）.

Mechanisms of Reflection



- Body Reflection:

  Diffuse Reflection (漫反射)
  Matte Appearance (粗糙表面)
  Non-Homogeneous Medium
  Clay, paper, etc

- Surface Reflection:

  Specular Reflection (镜面反射)
  Glossy Appearance (光滑表面)
  Highlights （高光）
  Dominant for Metals

Image Intensity = Body Reflection + Surface Reflection

Mechanisms of Reflection

- Example Surfaces

Body Reflection:

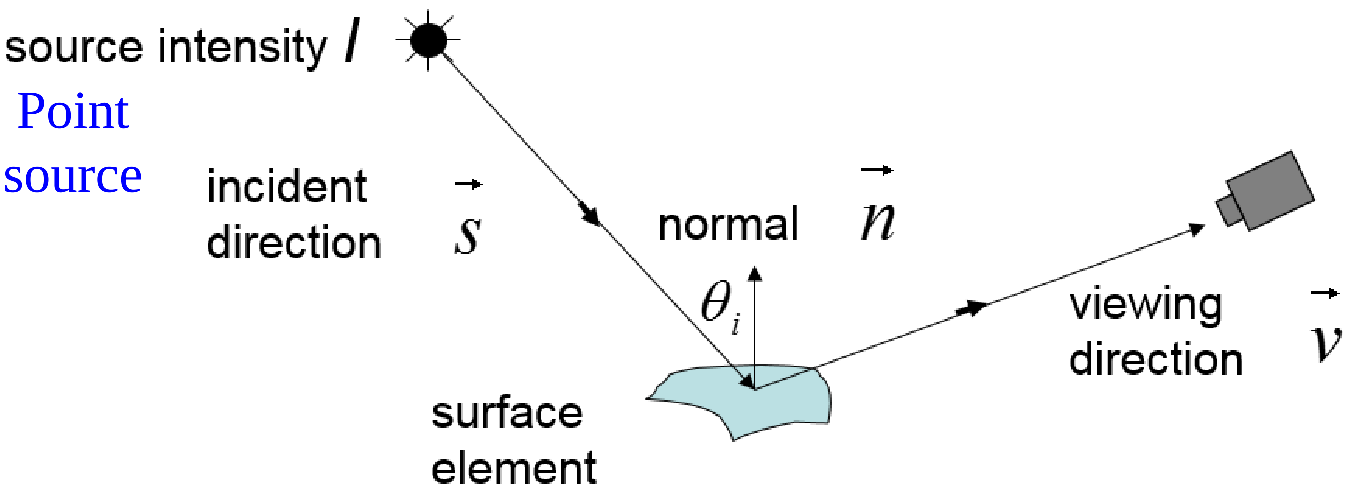
Diffuse Reflection
Matte Appearance
Non-Homogeneous Medium
Clay, paper, etc

Surface Reflection:

Specular Reflection
Glossy Appearance
Highlights
Dominant for Metals

Many materials exhibit both Reflections:

Diffuse Reflection and Lambertian BRDF



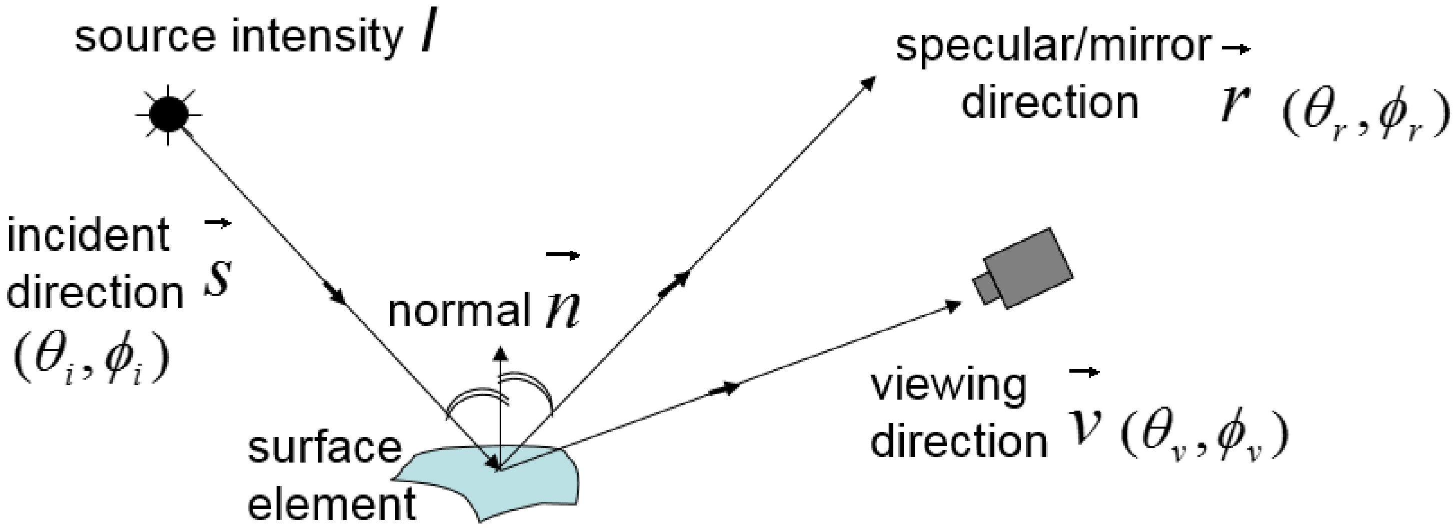
- Surface appears equally bright from ALL directions! (independent of $\vec{v}$)

- Lambertian BRDF is simply a constant : $f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{\rho_d}{\pi}$ albedo 反射率

- Surface Radiance : $L = \frac{\rho_d}{\pi} I \cos\theta_i = \frac{\rho_d}{\pi} I \; \vec{n} \cdot \vec{s}$ source intensity

- Commonly used in Vision and Graphics!

Specular Reflection and Mirror BRDF



- Valid for very smooth surfaces.

- All incident light energy reflected in a SINGLE direction (only when $\vec{v} = \vec{r}$).

- Mirror BRDF is simply a double-delta function :

specular albedo

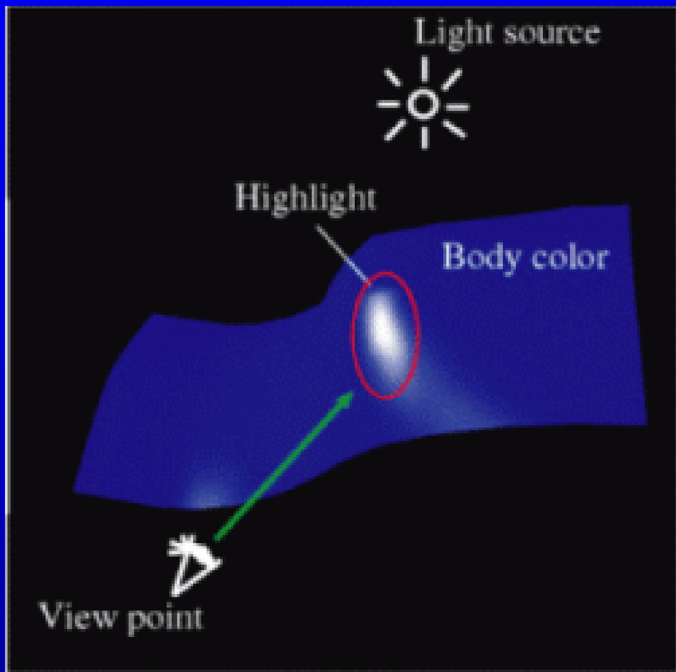
$$f(\theta_i, \phi_i; \theta_v, \phi_v) = \rho_s \, \delta(\theta_i - \theta_v) \, \delta(\phi_i + \pi - \phi_v)$$

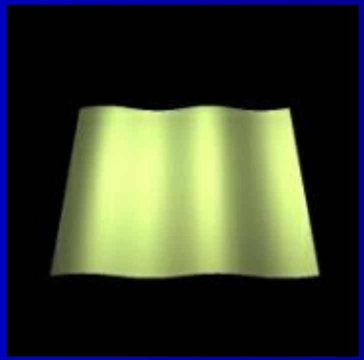
- Surface Radiance : $L = I \, \rho_s \, \delta(\theta_i - \theta_v) \, \delta(\phi_i + \pi - \phi_v)$

Combing Specular and Diffuse: Dichromatic Reflection

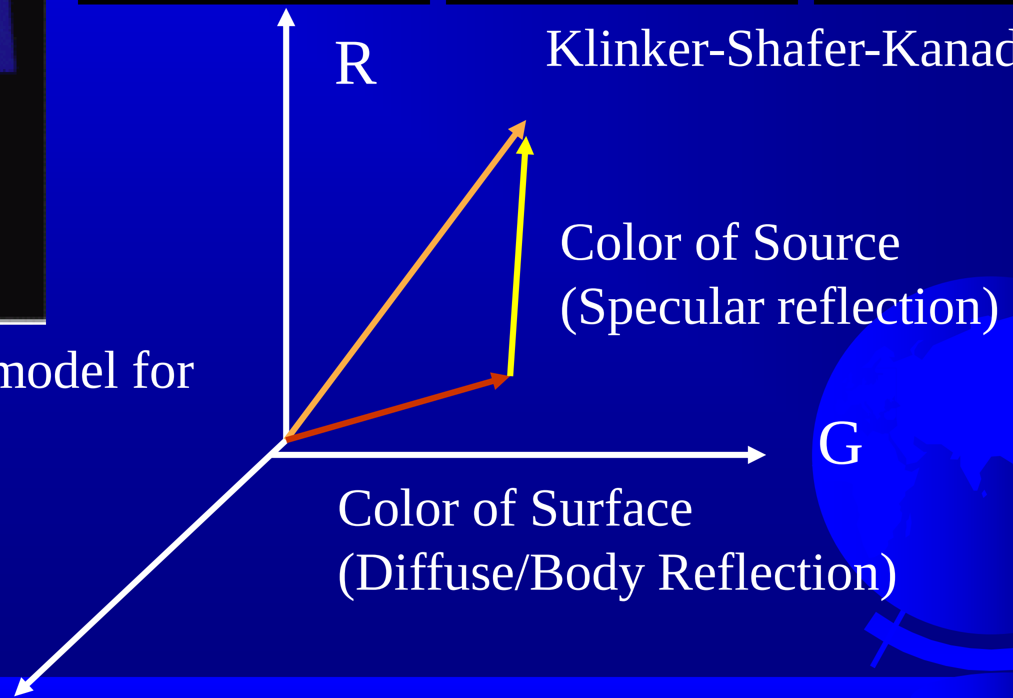
Dichromatic Reflection（双色反射模型）

Observed Image Color = a* Body Color + b*Specular Reflection Color





Klinker-Shafer-Kanade 1988

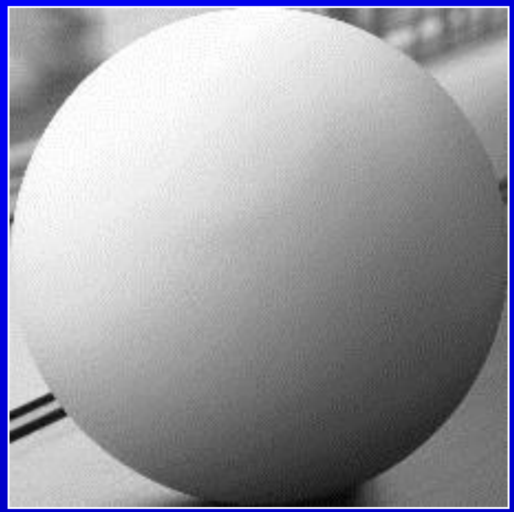


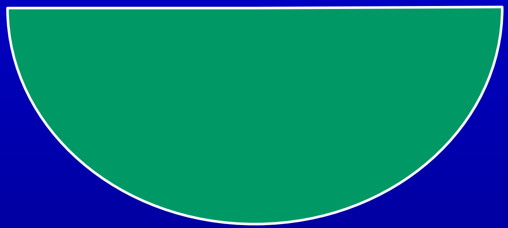
Does not specify any specific model for Diffuse/specular reflection

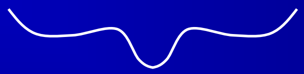
Diffuse and Specular Reflection

diffuse   specular   diffuse+specular

Image Intensity and 3D Geometry





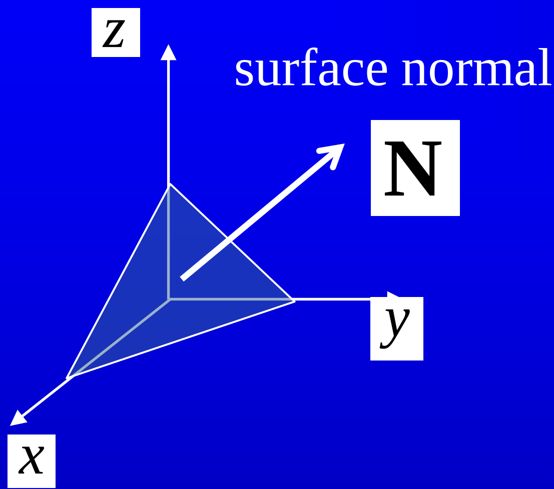


Shading as a cue for shape reconstruction

What is the relation between intensity and shape?

$\boxed{\text{Reflectance Map}}$

Surface Normal



Equation of plane $$Ax + By + Cz + D = 0$$

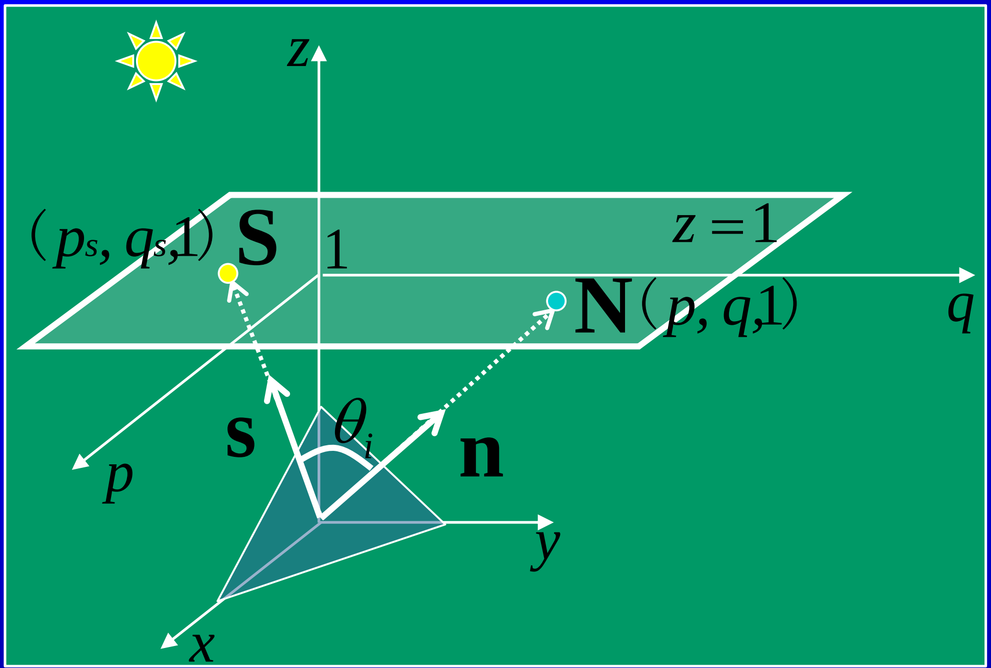
or $$\frac{A}{C}x + \frac{B}{C}y + z + \frac{D}{C} = 0$$

$$z = -\frac{A}{C}x - \frac{B}{C}y - D$$

Let $$-\frac{\partial z}{\partial x} = \frac{A}{C} = p \qquad -\frac{\partial z}{\partial y} = \frac{B}{C} = q$$

Surface normal $$\mathbf{N} = \left(\frac{A}{C}, \frac{B}{C}, 1\right) = (p, q, 1)$$

Surface Normal



Normal vector

$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = \frac{(p, q, 1)}{\sqrt{p^2 + q^2 + 1}}$$

Source vector

$$\mathbf{s} = \frac{\mathbf{S}}{|\mathbf{S}|} = \frac{(p_S, q_S, 1)}{\sqrt{p_S^2 + q_S^2 + 1}}$$

$$z = -\frac{A}{C}x - \frac{B}{C}y - D$$

$$\cos\theta_i = \mathbf{n} \cdot \mathbf{s} = \frac{(pp_S + qq_S + 1)}{\sqrt{p^2 + q^2 + 1}\sqrt{p_S^2 + q_S^2 + 1}}$$

$z = 1$ plane is called the Gradient Space (*pq* plane)

- Every point on it corresponds to a particular surface orientation

Reflectance Map

- Relates image irradiance *I*(*x*,*y*) to surface orientation (*p*,*q*) for given source direction **s** and surface reflectance **n**.

Assume: Source at infinity

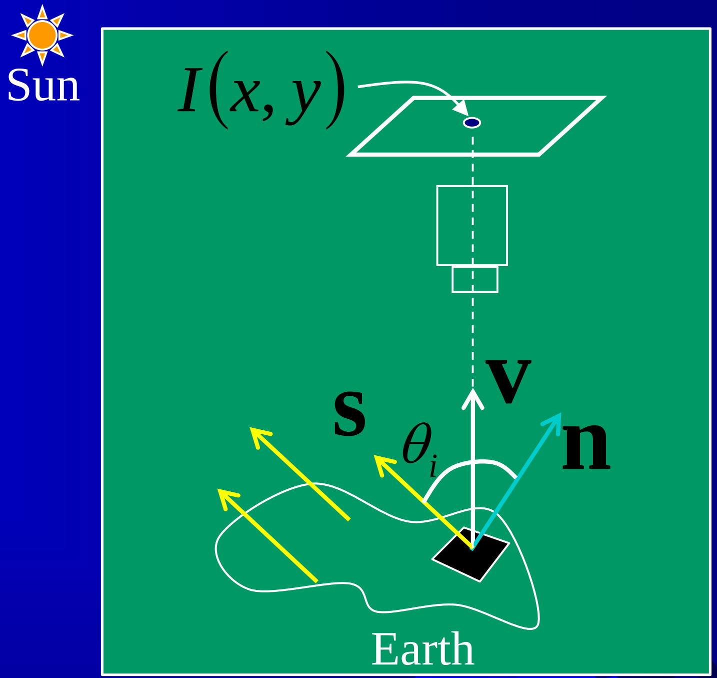


Image irradiance:

$$I = \frac{\rho}{\pi} kc \cos\theta_i = \frac{\rho}{\pi} kc\,\mathbf{n}\cdot\mathbf{s}$$

- Lambertian case:

k : source brightness

ρ : surface albedo (reflectance)

c : constant (optical system)

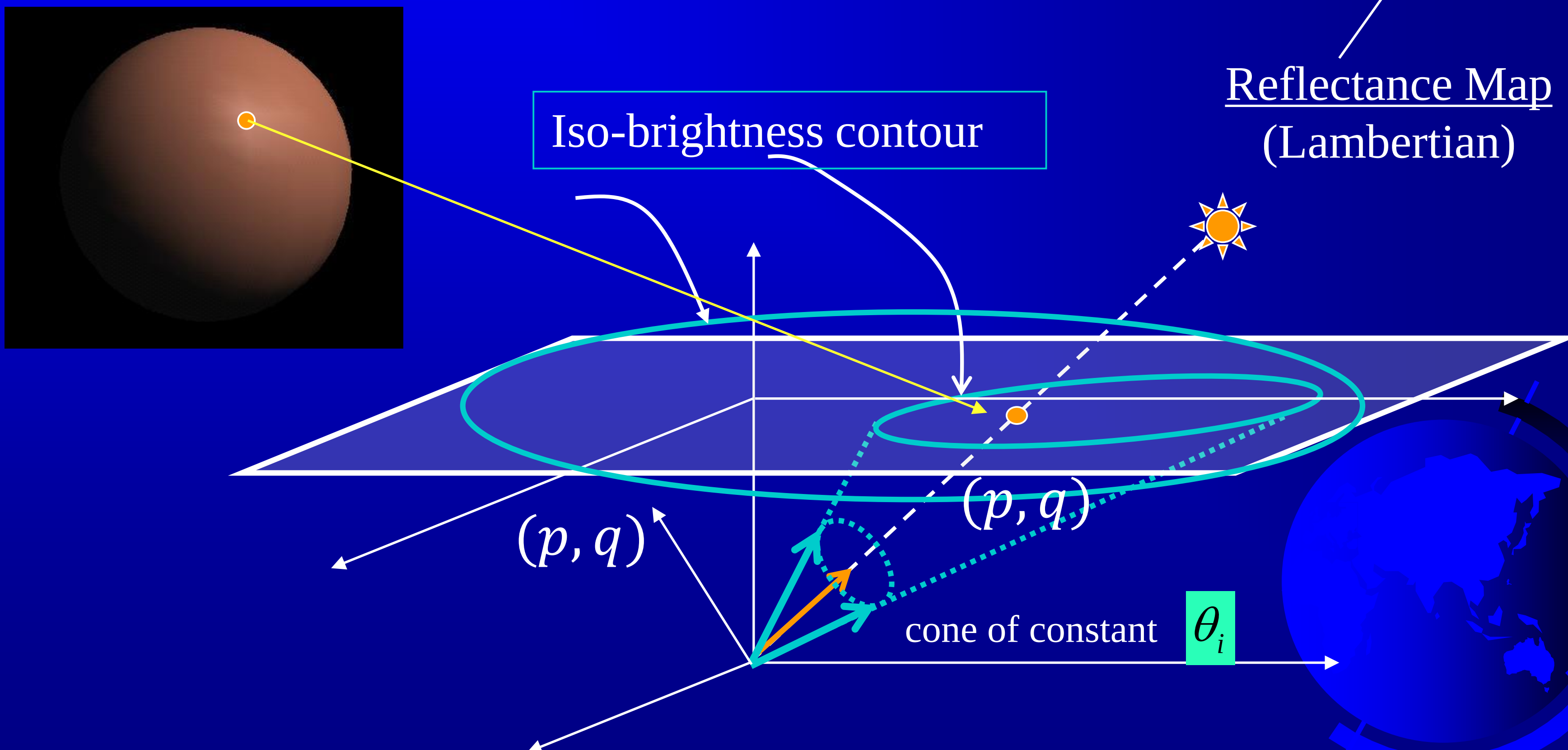
Let $\frac{kc}{\pi} = 1$ then $I = \rho\,\mathbf{n}\cdot\mathbf{s} = \rho cos\theta_i$

Reflectance Map

- Lambertian case

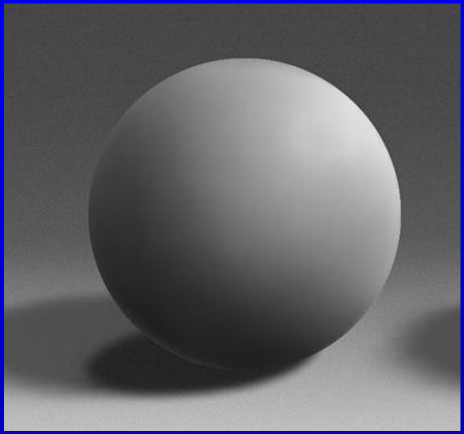
$$I = \rho cos\theta_i = \rho \mathbf{n} \cdot \mathbf{s} = \frac{(pp_s + qq_s + 1)}{\sqrt{p^2 + q^2 + 1}\sqrt{p_s^2 + q_s^2 + 1}} = \boxed{R(p, q)}$$

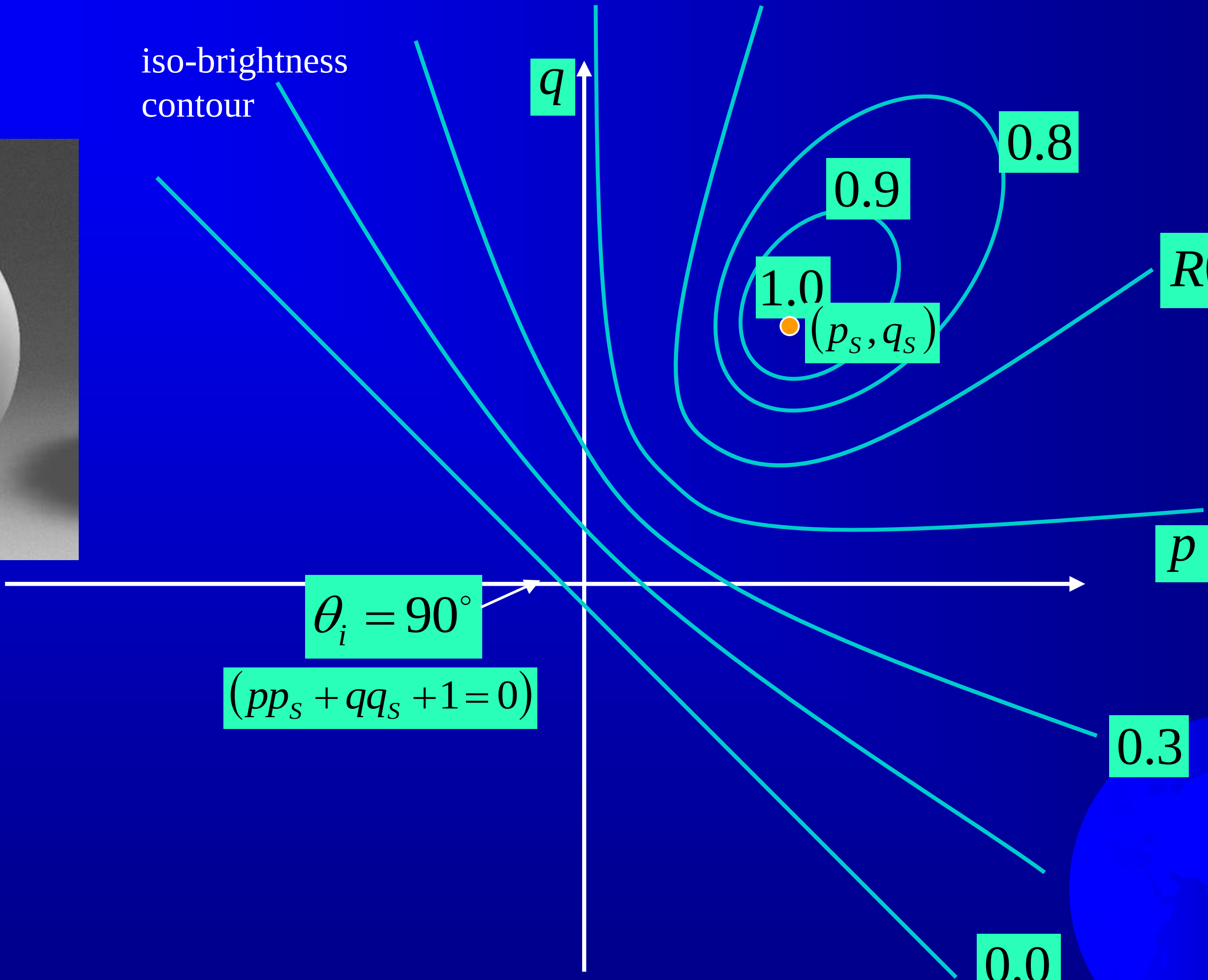
Reflectance Map
(Lambertian)



Reflectance Map

- Lambertian case

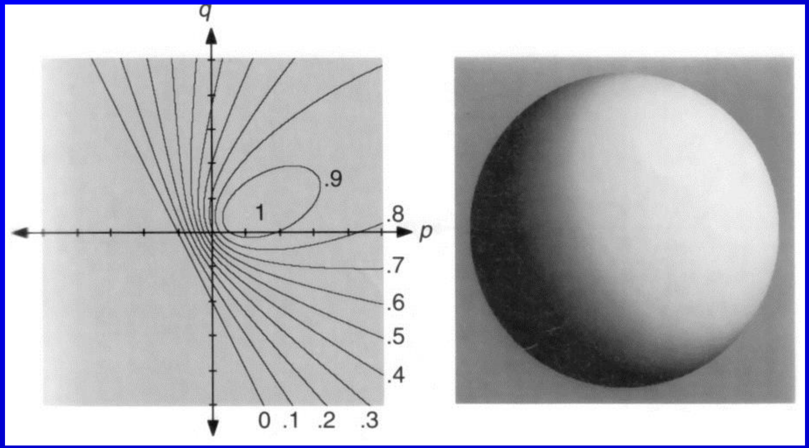




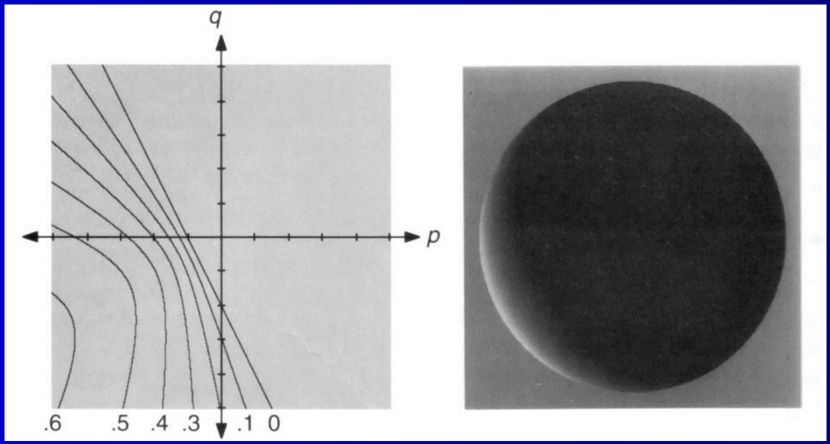
Note: $R(p,q)$ is maximum when $(p,q) = (p_S, q_S)$

Reflectance Map

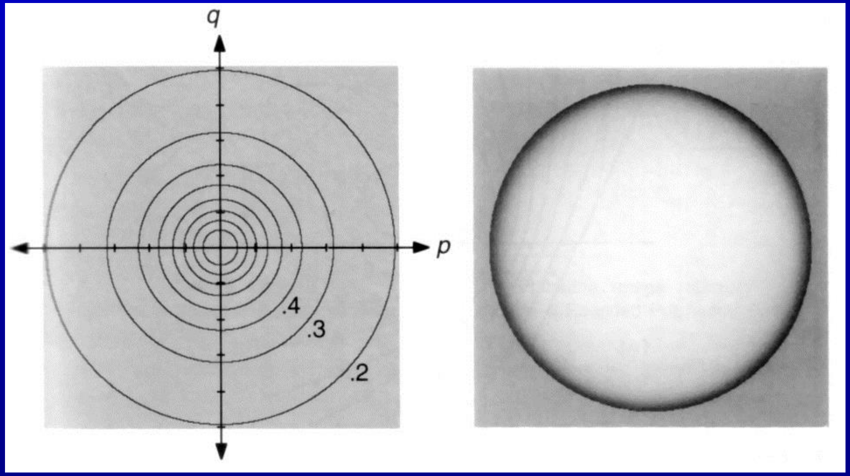
- Lambertian case



Illuminant direction: [1 0.5 -1]



Illuminated in the direction [1 0.5 -1] (from behind).



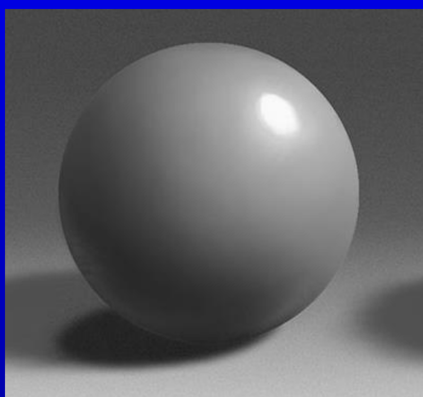
Scene lit from [0 0 -1].

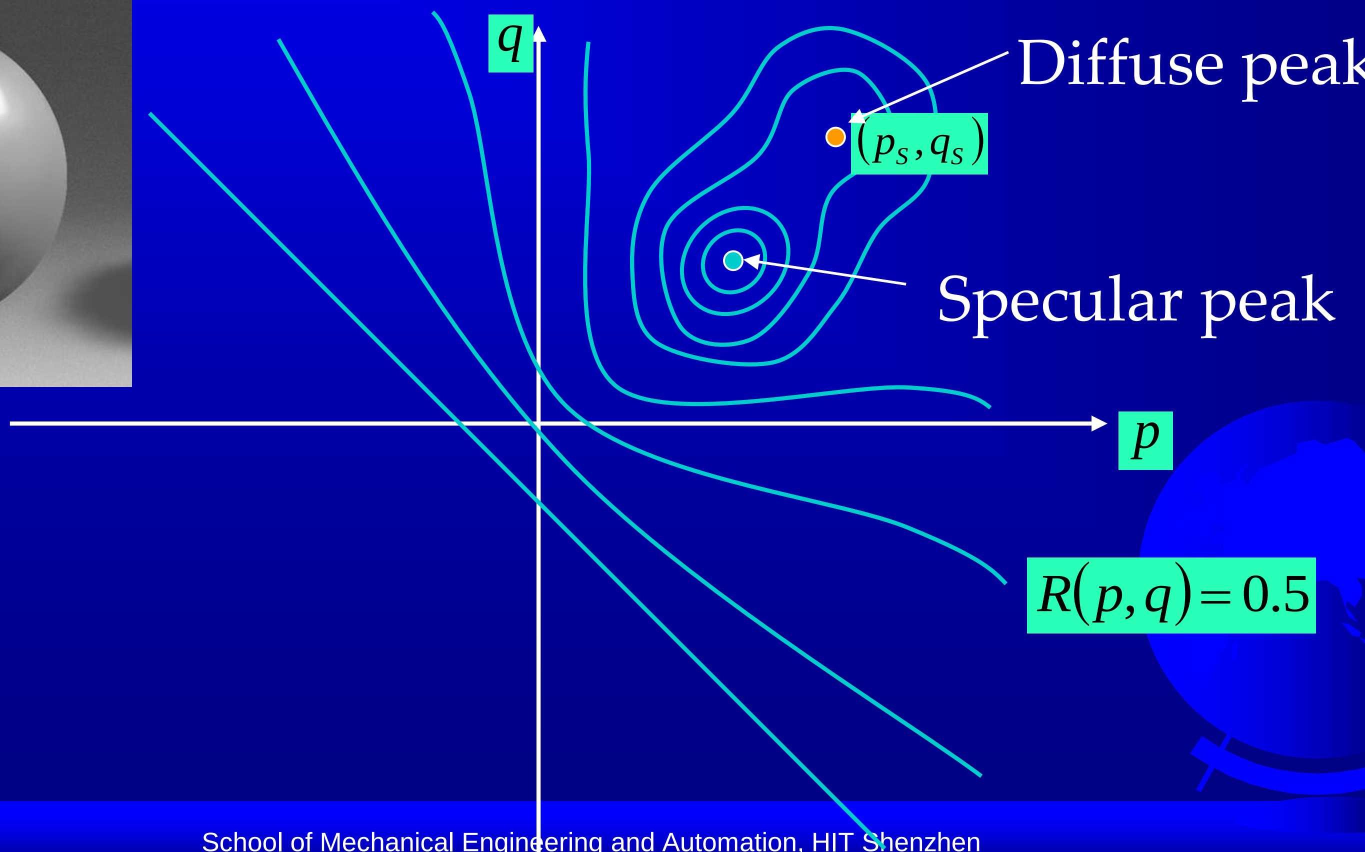
Reflectance Map

- Glossy surfaces (Torrance-Sparrow reflectance model)

$$I = \boxed{\frac{\rho_d}{\pi} kc \cos\theta_i} + \boxed{\frac{\rho_s kc}{\cos\theta_r} p(\beta) G} = R(p,q)$$

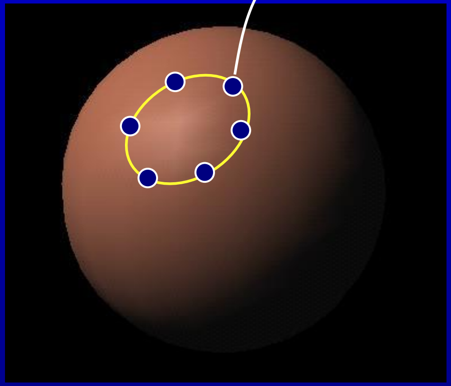
diffuse term　　specular term

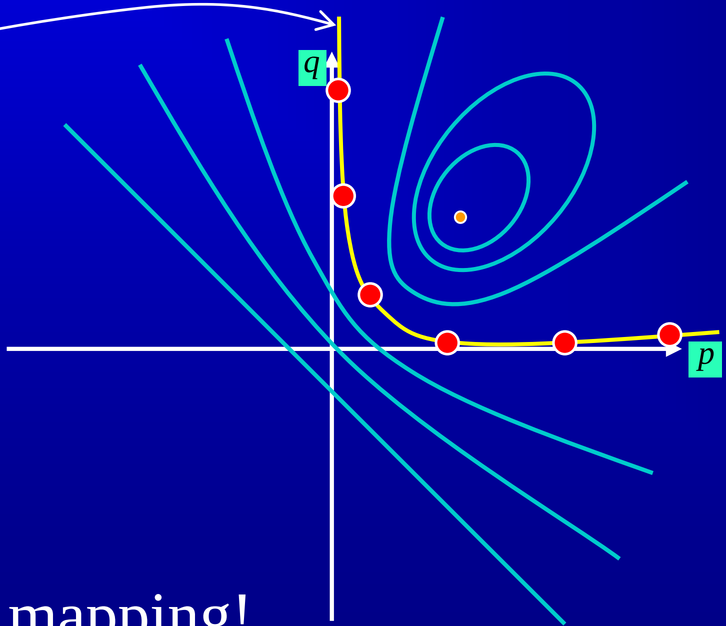




Shape from a Single Image?

- Given a single image of an object with known surface reflectance taken under a known light source, can we recover the shape of the object?
- Given $R(p,q)$ ((p_s,q_s) and surface reflectance) can we determine (p,q) uniquely for each image point?



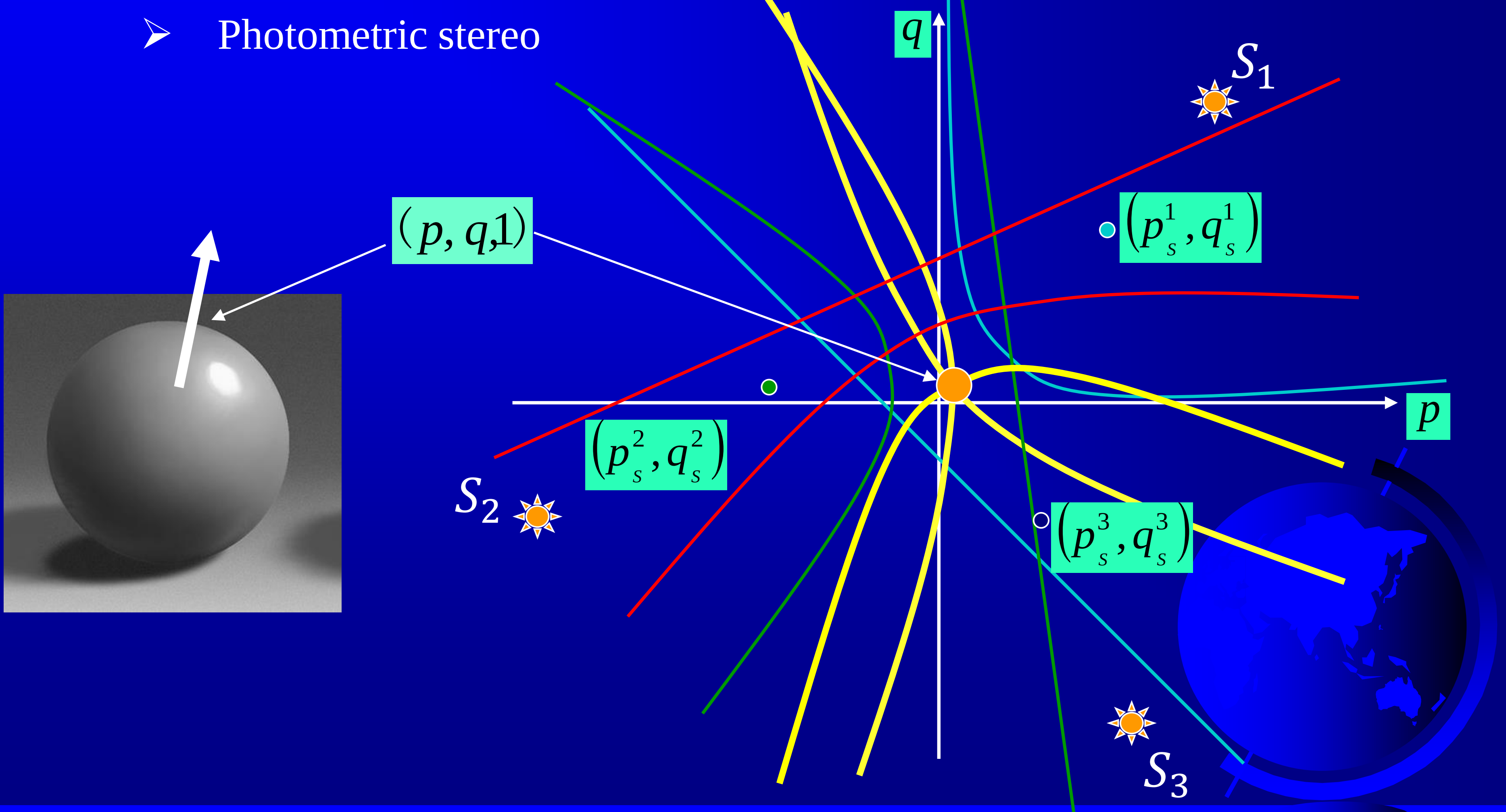


Not a one to one mapping!

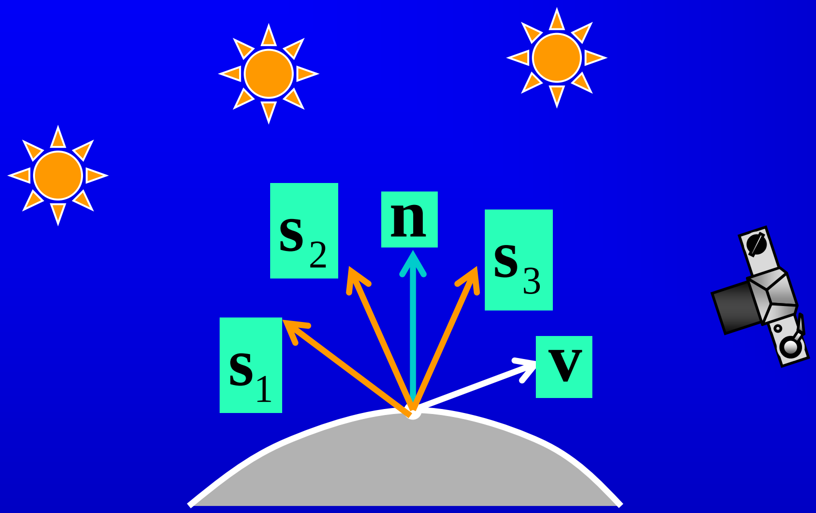
NO

Shape from a Single Image?

- Solution
- Take more images in different orientations.
  - Photometric stereo



Photometric stereo



Lambertian case:

$$I = \frac{\rho}{\pi} kc \cos\theta_i = \rho \mathbf{n} \cdot \mathbf{s} \qquad \left(\frac{kc}{\pi} = 1 \right)$$

Image irradiance:

$$I_1 = \rho \mathbf{n} \cdot \mathbf{s}_1$$

$$I_2 = \rho \mathbf{n} \cdot \mathbf{s}_2$$

$$I_3 = \rho \mathbf{n} \cdot \mathbf{s}_3$$

- We can write this in matrix form:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \rho \begin{bmatrix} \mathbf{s}_1^T \\ \mathbf{s}_2^T \\ \mathbf{s}_3^T \end{bmatrix} \mathbf{n}$$

Photometric stereo

- Solving the equation

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{\substack{\mathbf{I} \\ 3\times 1}} = \underbrace{\begin{bmatrix} s_1^T \\ s_2^T \\ s_3^T \end{bmatrix}}_{\substack{\mathbf{S} \\ 3\times 3}} \underbrace{\rho\mathbf{n}}_{\substack{\tilde{\mathbf{n}} \\ 3\times 1}}$$

$$\tilde{\mathbf{n}} = \rho\mathbf{n}$$

$$\tilde{\mathbf{n}} = \mathbf{S}^{-1}\mathbf{I}$$

inverse

$$\rho = |\tilde{\mathbf{n}}| \qquad (1)$$

$$\mathbf{n} = \frac{\tilde{\mathbf{n}}}{|\tilde{\mathbf{n}}|} = \frac{\tilde{\mathbf{n}}}{\rho} \qquad (2)$$

Photometric stereo

- More than Three Light Sources
- Get better results by using more lights

$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} \mathbf{s}_1^T \\ \vdots \\ \mathbf{s}_N^T \end{bmatrix} \rho \mathbf{n}$$

- Least squares solution:

$$\mathbf{I} = \mathbf{S}\tilde{\mathbf{n}} \quad \longleftarrow \quad N \times 1 = \underline{(N \times 3)}(3 \times 1)$$

$$\mathbf{S}^T \mathbf{I} = \mathbf{S}^T \mathbf{S} \tilde{\mathbf{n}}$$

$$\tilde{\mathbf{n}} = \boxed{\left(\mathbf{S}^T \mathbf{S}\right)^{-1} \mathbf{S}^T} \mathbf{I}$$

Moore-Penrose pseudo inverse

- Solve for $\rho, \mathbf{n}$ as Eq. (1)(2)

Photometric stereo

- Color Images
- The case of RGB images
  - Get three sets of equations, one per color channel:

$$\mathbf{I}_R = \rho_R \mathbf{S}\,\mathbf{n} \qquad \mathbf{I}_G = \rho_G \mathbf{S}\,\mathbf{n} \qquad \mathbf{I}_B = \rho_B \mathbf{S}\,\mathbf{n}$$

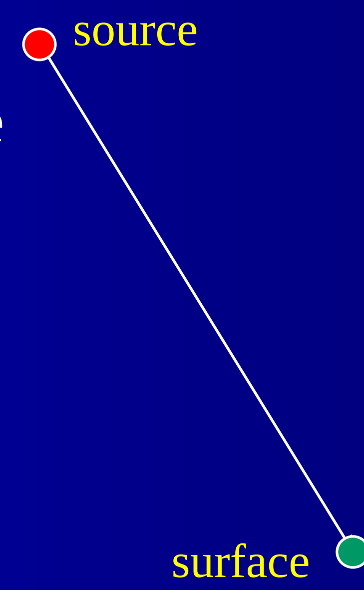
  - Simple solution: first solve for $\mathbf{n}$ using one channel
  - Then substitute known $\mathbf{n}$ into above equations to get

$$(r_R, r_G, r_B)$$

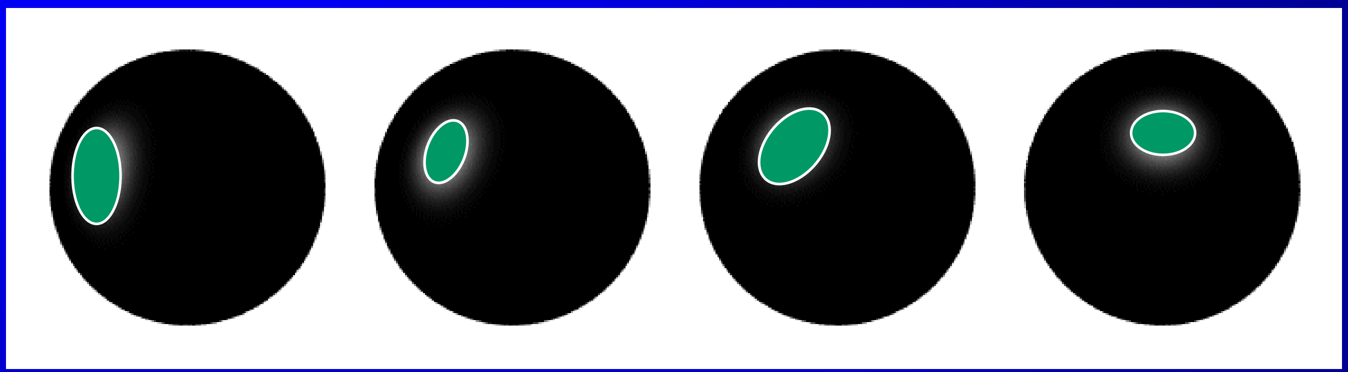
  - Or combine three channels and solve for $\mathbf{n}$

$$\mathbf{I} = \sqrt{\mathbf{I}_R^2 + \mathbf{I}_G^2 + \mathbf{I}_B^2} = r\mathbf{S}\mathbf{n}$$

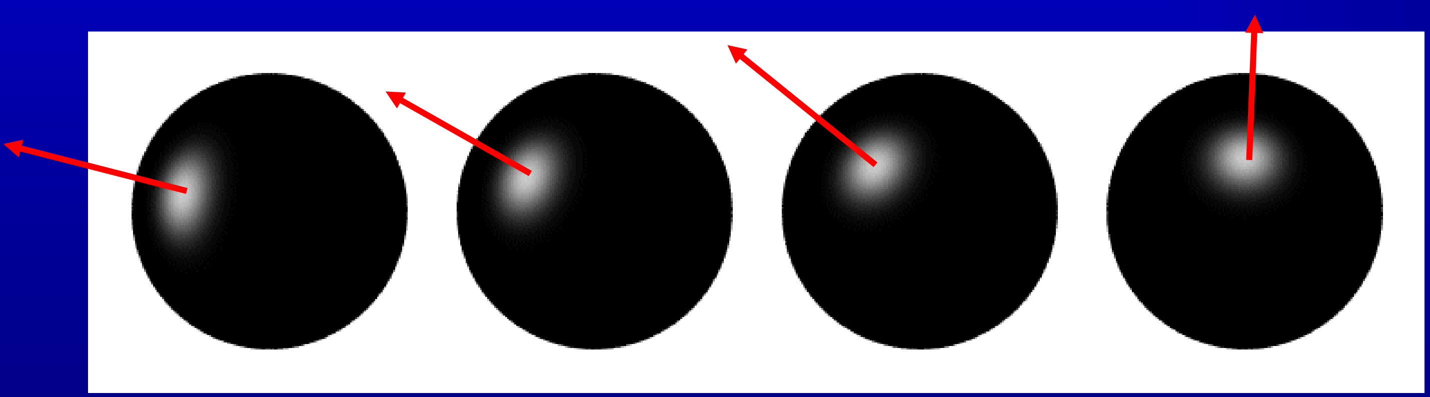
Computing light source directions



- Trick: place a chrome sphere（镀铬球）in the scene

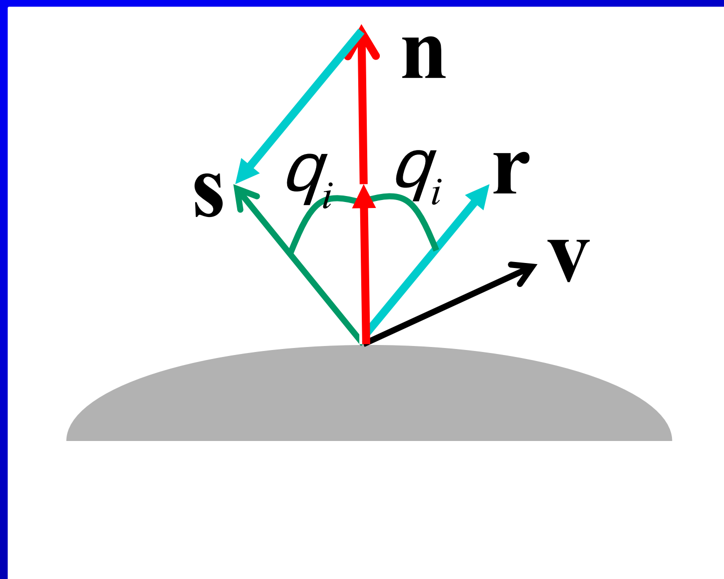


- The location of the highlight tells you the source direction



Specular Reflection - Recap

- For a perfect mirror, light is reflected about **n**



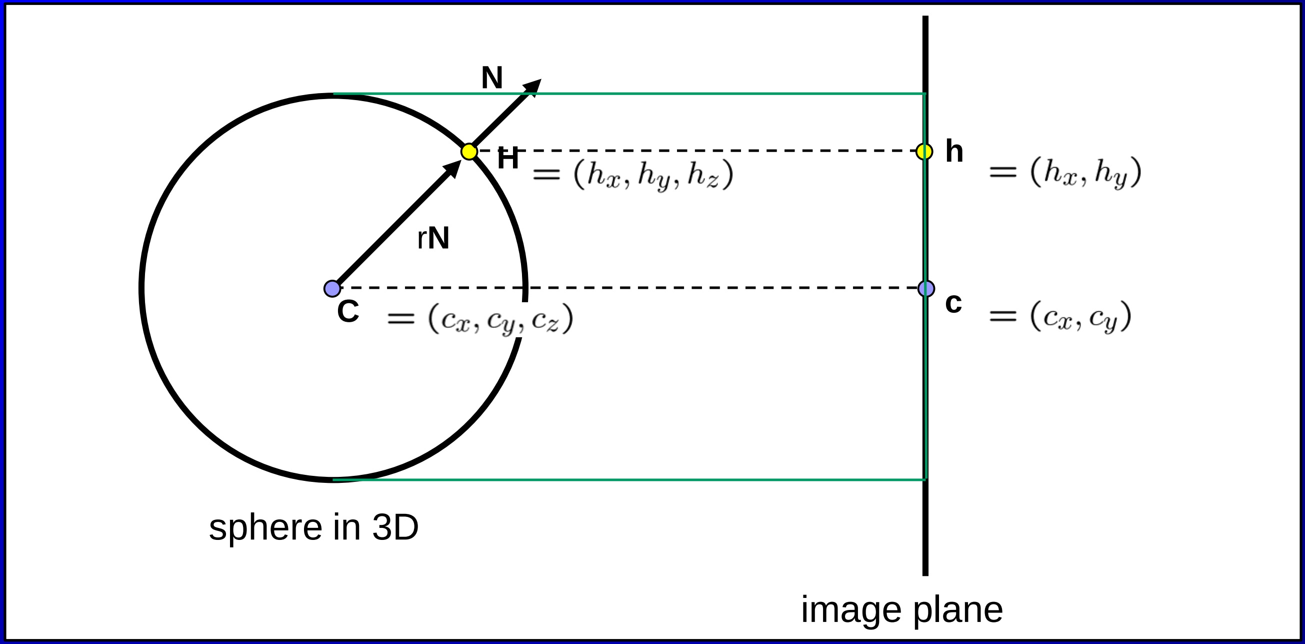
$$R_e = \begin{cases} R_i & \text{if } \mathbf{v} = \mathbf{r} \\ 0 & \text{otherwise} \end{cases}$$

- We see a highlight when $\mathbf{v} = \mathbf{r}$
- Then $\mathbf{s}$ is given as follows:

$$\mathbf{s} = 2(\mathbf{n} \cdot \mathbf{r})\mathbf{n} - \mathbf{r}$$

Computing the Light Source Direction

- Chrome sphere that has a highlight at position **h** in the image



- Can compute **N** by studying this figure

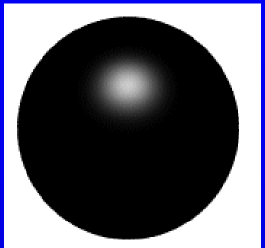
  Hints:

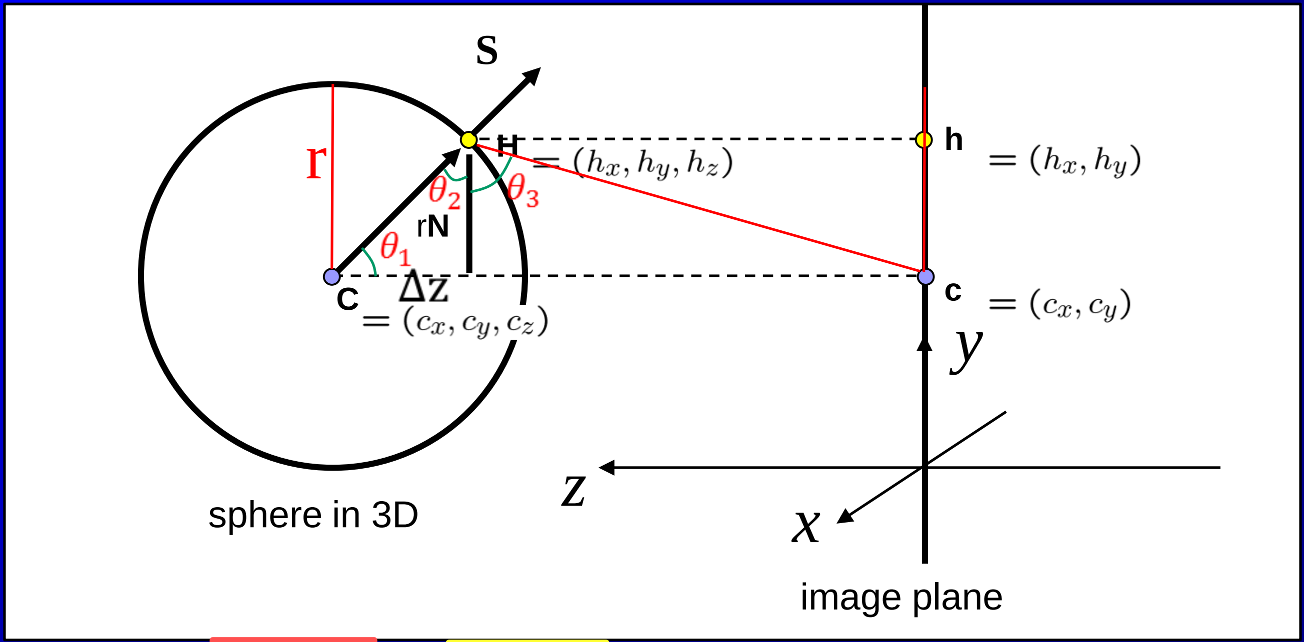
  use this equation: $\|H - C\| = r$

  can measure **c**, **h**, and r in the image

Computing the Light Source Direction

- Chrome sphere that has a highlight at position **h** in the image



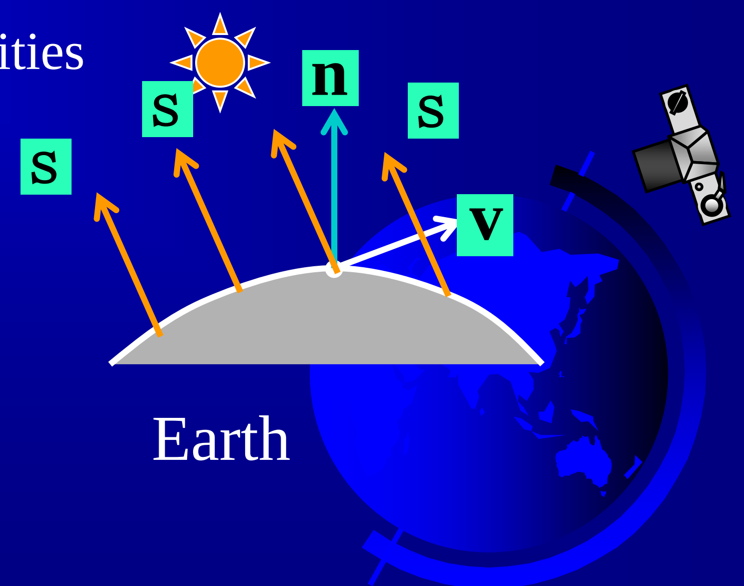


- can measure c(c_x, c_y), h(h_x, h_y) and r in the image
- C(c_x, c_y, c_z),H(h_x, h_y, h_z). $c_z = h_z + \Delta z$

$$\begin{cases} r \\ (h_y - c_y) \end{cases} \Rightarrow \Delta z \Rightarrow h_z - c_z = -\Delta z \Rightarrow \begin{matrix} (h_x - c_x) \\ (h_y - c_y) \\ (h_z - c_z) \end{matrix} \Rightarrow \mathbf{S}$$

Limitations

- Big problems
  - Doesn't work for shiny things, semi-translucent things
  - Shadows, inter-reflections
- Smaller problems
  - Camera and lights have to be distant
  - Calibration requirements
    - measure light source directions, intensities
    - camera response function



Trick for Handling Shadows

- Weight each equation by the pixel brightness:

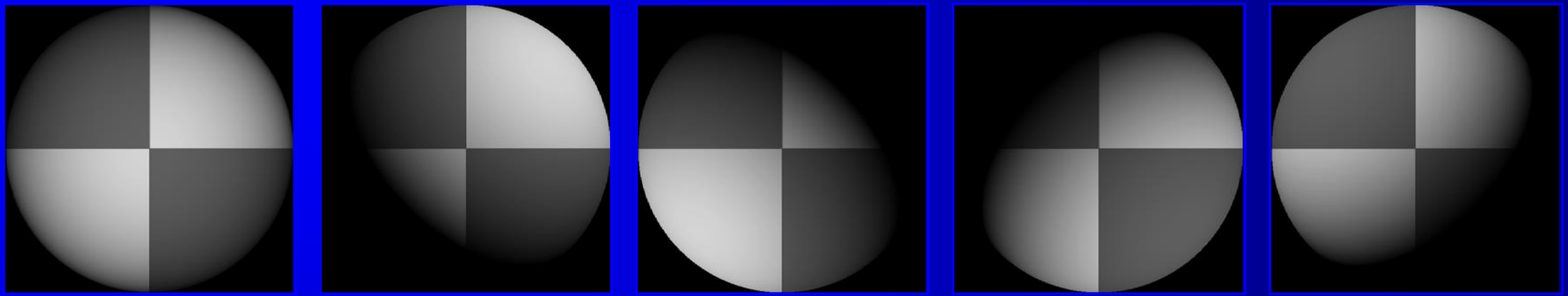
$$I_i\left(I_i\right) = I_i\left(\rho\,\mathbf{n}\cdot\mathbf{s}_i\right)$$

- Gives weighted least-squares matrix equation:

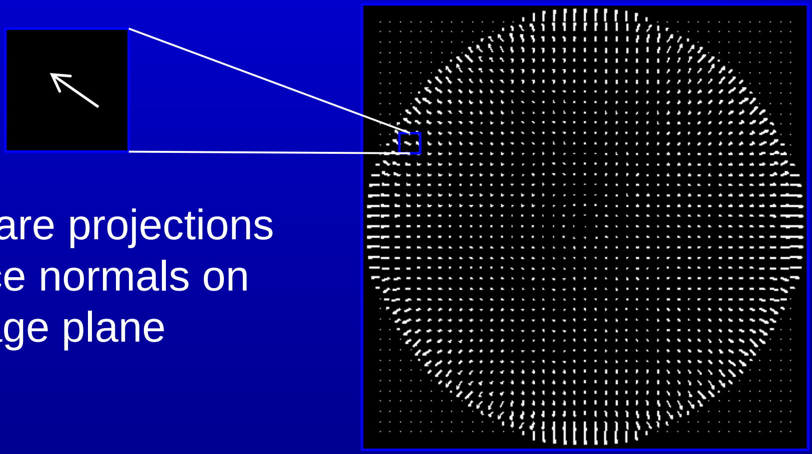
$$\begin{bmatrix} I_1^2 \\ \vdots \\ I_N^2 \end{bmatrix} = \begin{bmatrix} I_1\mathbf{s}_1^T \\ \vdots \\ I_N\mathbf{s}_N^T \end{bmatrix} \rho\,\mathbf{n}$$

- Solve for $\rho, \mathbf{n}$ as Eq. (1)(2).

Results: Lambertian Sphere

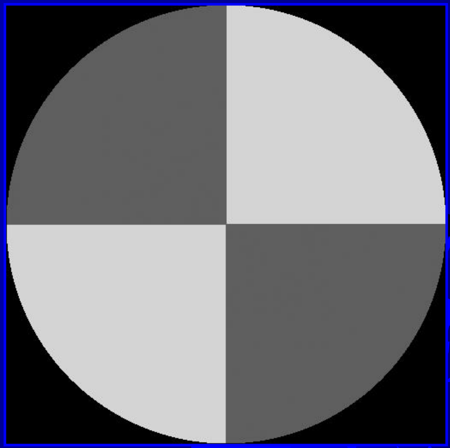


Input Images



Needles are projections of surface normals on image plane

Estimated Surface Normals $\mathbf{n}$



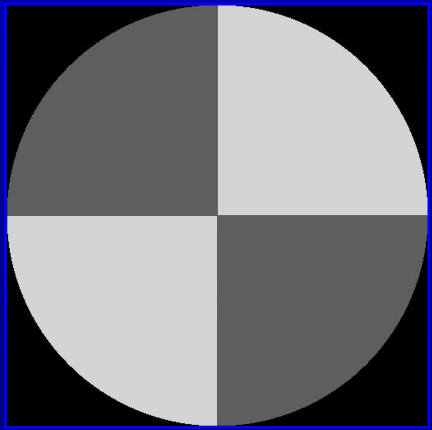
Estimated Albedo ρ

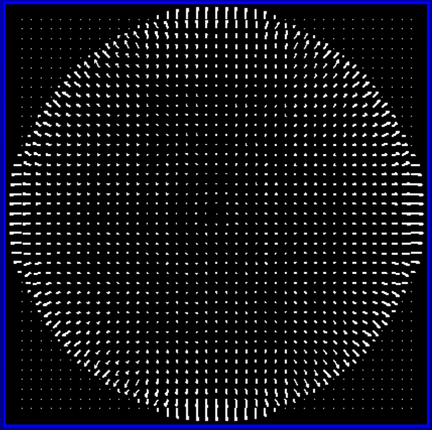
Results: Lambertian Sphere

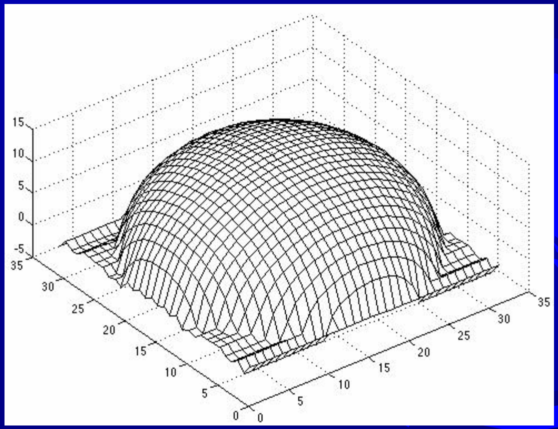
- We can now recover the surface height at any point by integration along some path. For example, we can reconstruct the surface at (u, v) by starting at $(0, 0)$, summing the y derivative along the line $x = 0$ to the point $(0, v)$, and then summing the x derivative along the line $y = v$ to the point (u, v).

$$f(u,v) = \int_0^v f_y(0,y)dy + \int_0^u f_x(x,v)dx + c$$

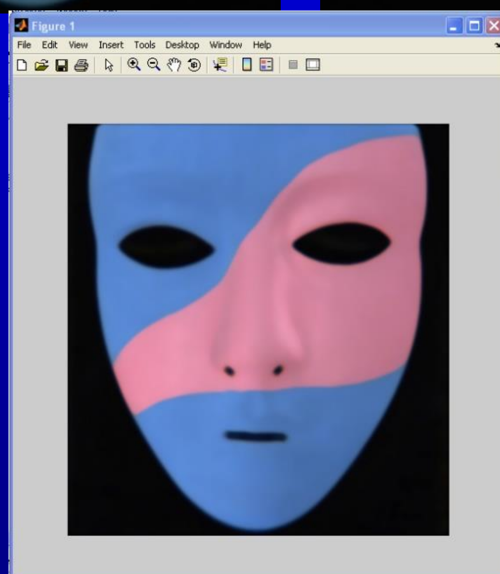
where c is a constant of integration.

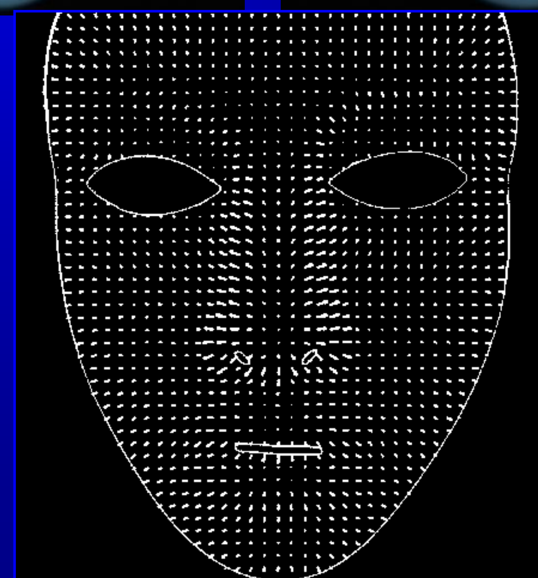






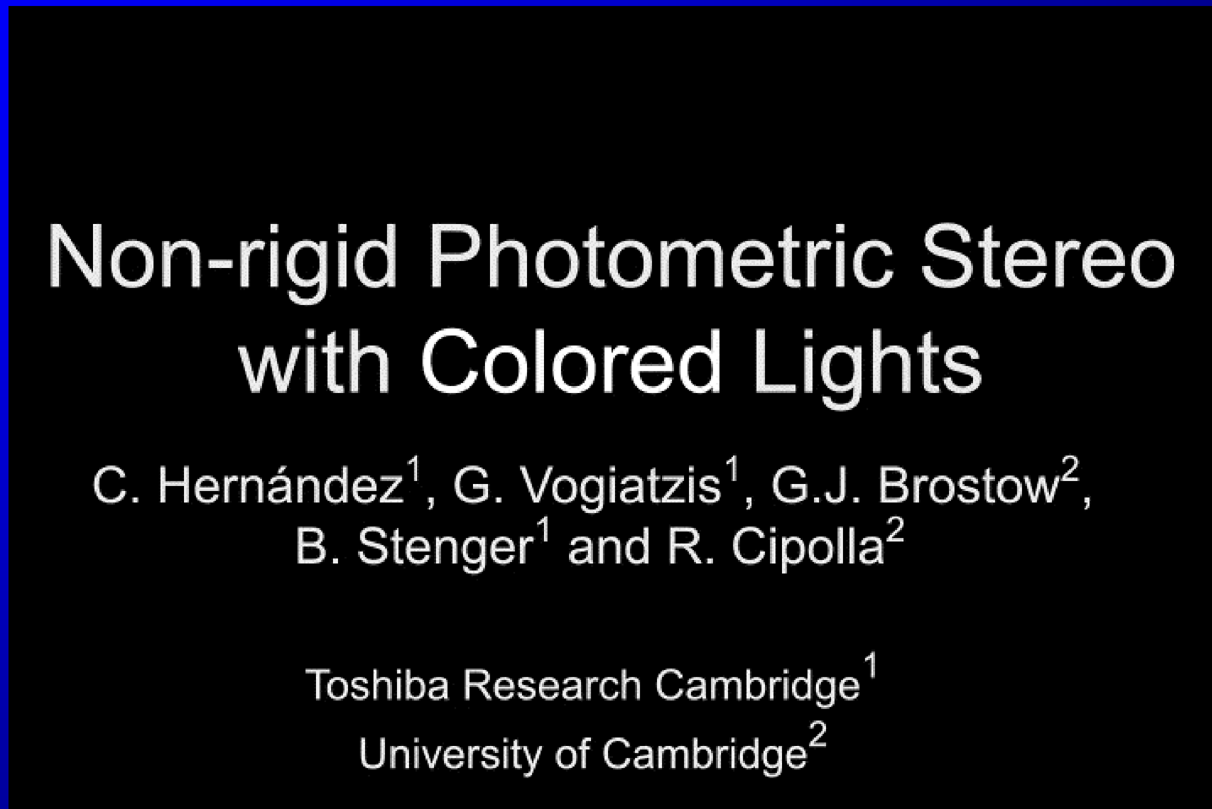
Results: Lambertain Mask





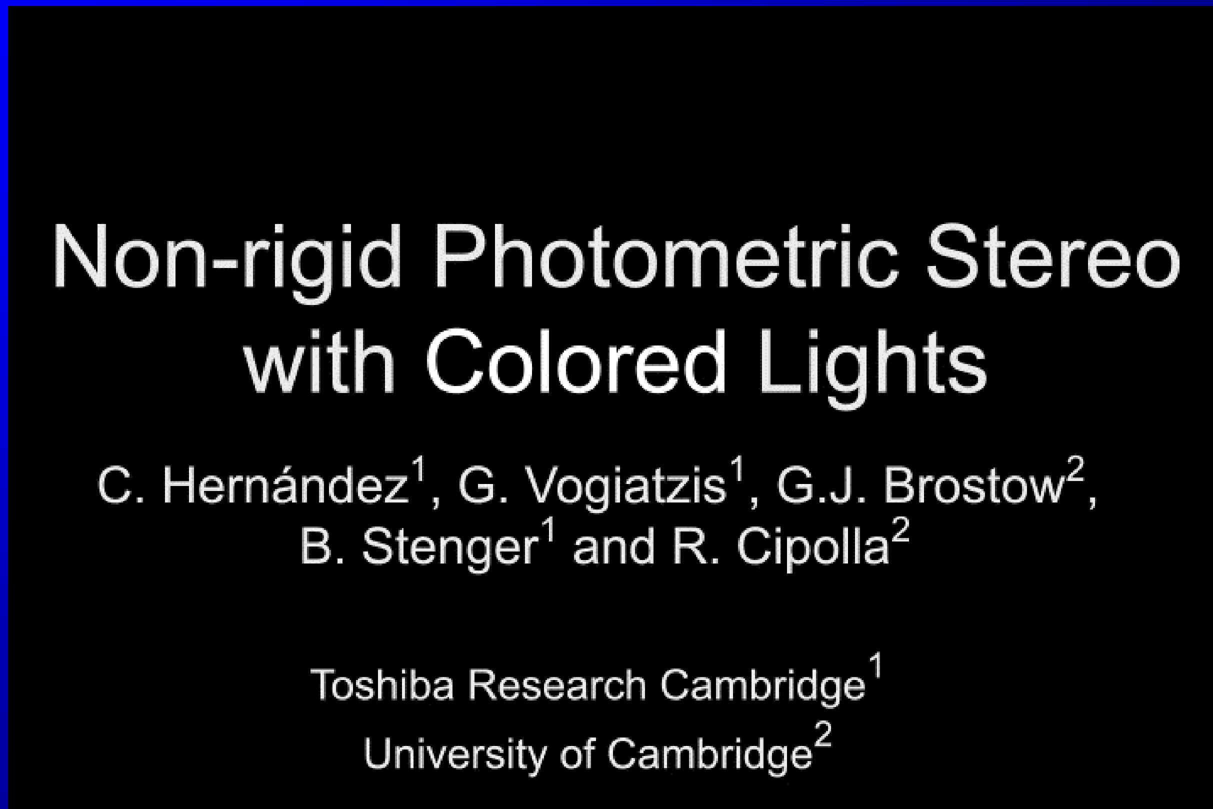
Albedo and Surface Normal

Results: Lambertain Mask



Carlos Hernandez, George Vogiatzis, Gabriel J. Brostow, Bjorn Stenger, Roberto Cipolla. Non-rigid Photometric Stereo with Colored Lights. 2007 IEEE 11th International Conference on Computer Vision.

Results: Lambertain Mask



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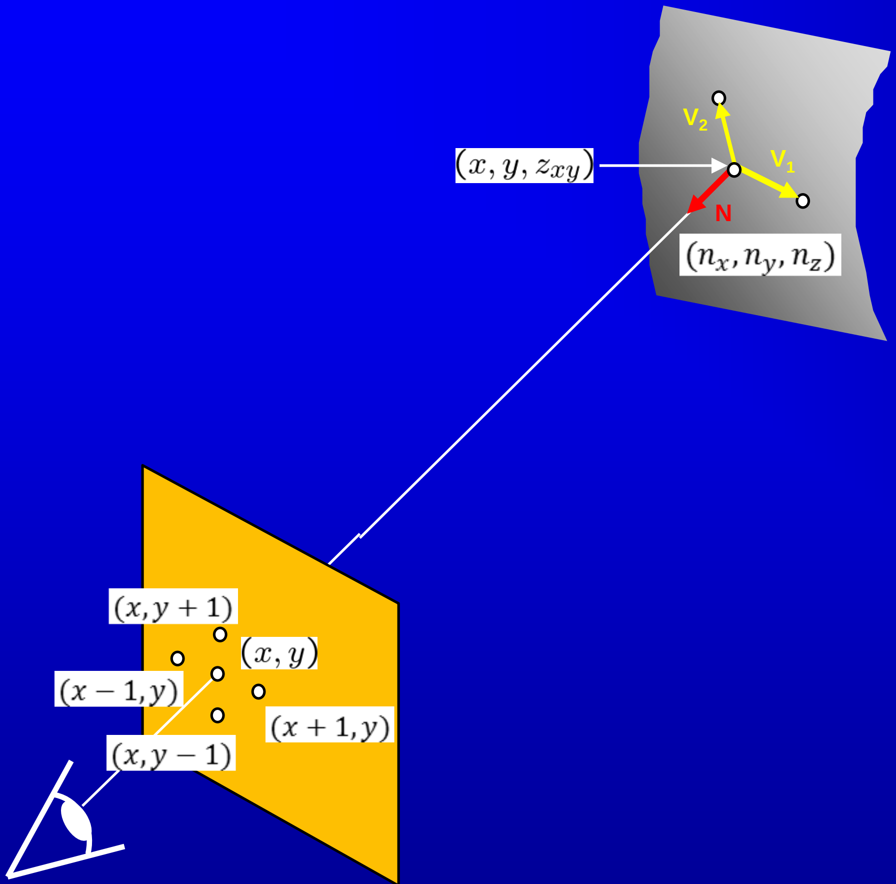
Depth from Normals



$$\begin{aligned} V_1 &= (x+1, y, z_{x+1,y}) - (x, y, z_{xy}) \\ &= (1, 0, z_{x+1,y} - z_{xy}) \end{aligned}$$

$$\begin{aligned} 0 &= N \cdot V_1 \\ &= (n_x, n_y, n_z) \cdot (1, 0, z_{x+1,y} - z_{xy}) \\ &= n_x + n_z(z_{x+1,y} - z_{xy}) \end{aligned}$$

$$\begin{aligned} V_2 &= (x, y+1, z_{x,y+1} - (x, y, z_{x,y}) \\ &= (0, 1, z_{x,y+1} - z_{x,y}) \end{aligned}$$

$$\begin{aligned} 0 &= N.V_2 \\ &= (n_x, n_y, n_z).(0, 1, z_{x,y+1} - z_{x,y}) \\ &= n_y + n_z(z_{x,y+1} - z_{x,y}) \end{aligned}$$

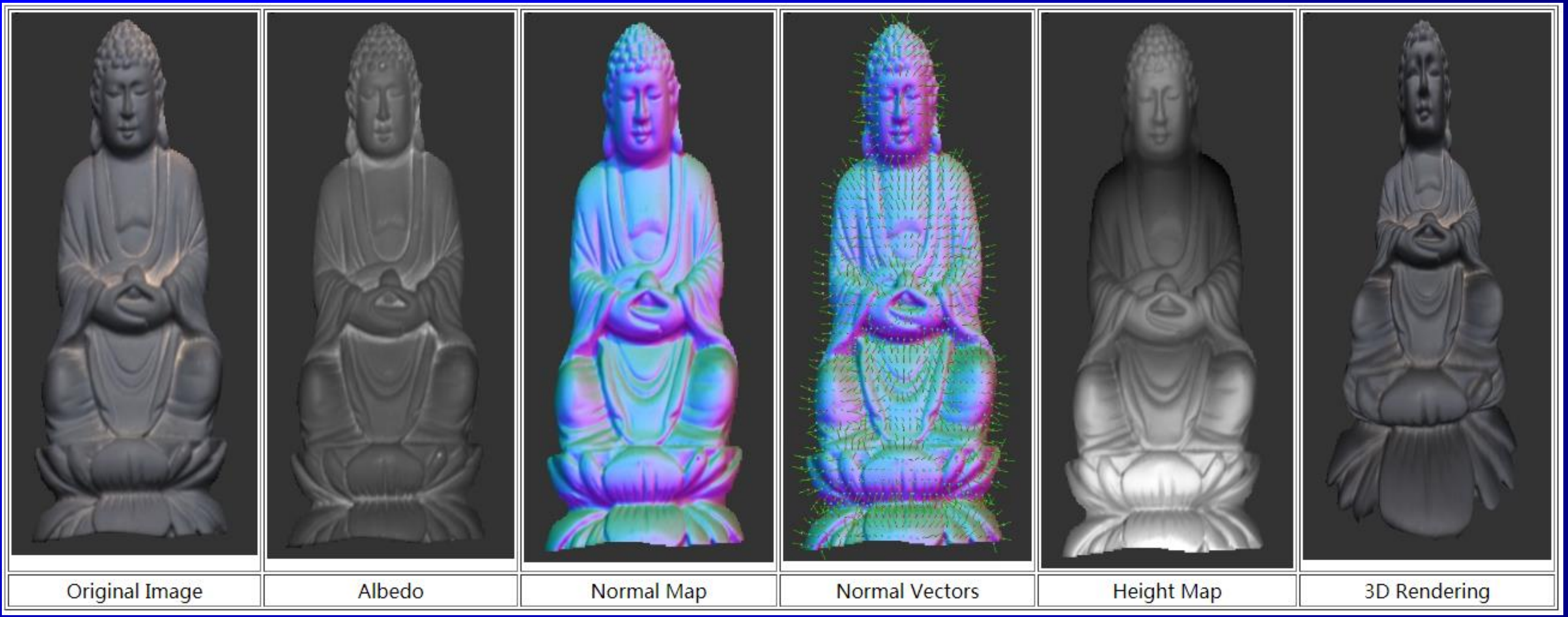
Trick for Handling Shadows

- Boundary conditions or pixels where either (n_x, n_y, n_z) values are not available must be taken into consideration. At those pixels, instead of taking forward step for depth calculation we take backward direction that modifies the above equations as

$$\begin{aligned} -n_x + n_z(z_{x-1,y} - z_{x,y}) &= 0 \\ -n_y + n_z(z_{x,y-1} - z_{x,y}) &= 0 \end{aligned}$$

- Each normal gives us two linear constraints on *z*
- Compute *z* values by solving a matrix equation

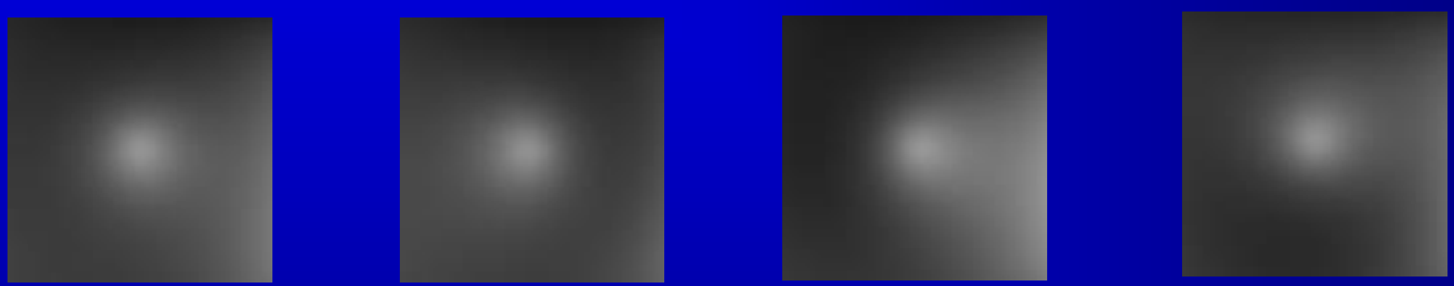
Photometric Stereo Example



1. Estimate light source directions
2. Compute surface normals
3. Compute albedo values
4. Estimate depth from surface normals
5. Relight the object (with original texture and uniform albedo)

Homework

- Write a program to reconstruct surface patch from the following images by using the photometric stereo method.
- Requirements: 1)the program can read images and show the reconstructed normals and surface. 2)turn in the code (matlab or c++), if c++, turn in a executable file.



(a) S1(0 0 -1)　(b) S2(0 0.2 -1)　(c) S3(0 -0.2 -1)　(d) S4(0.2 0 -1)

See You