Computer Vision

---*Camera Calibration III*

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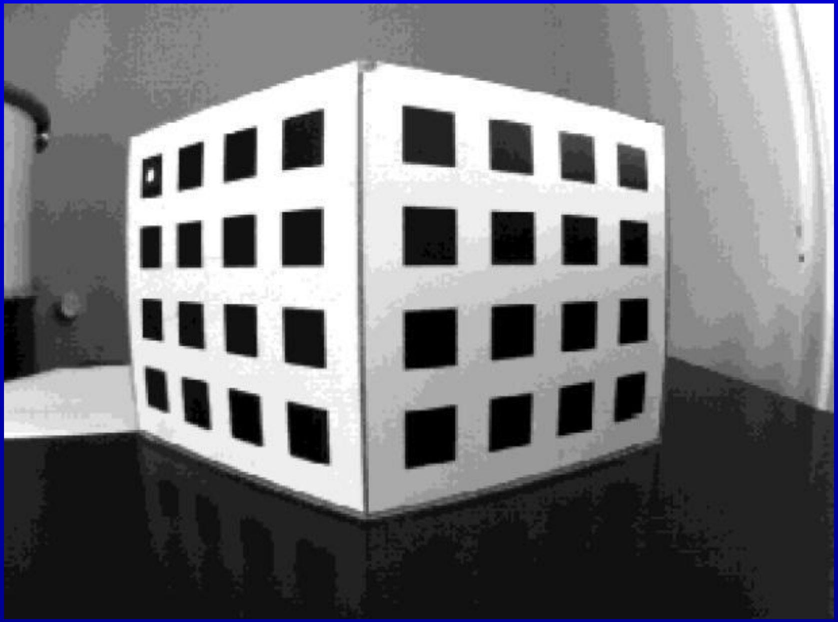
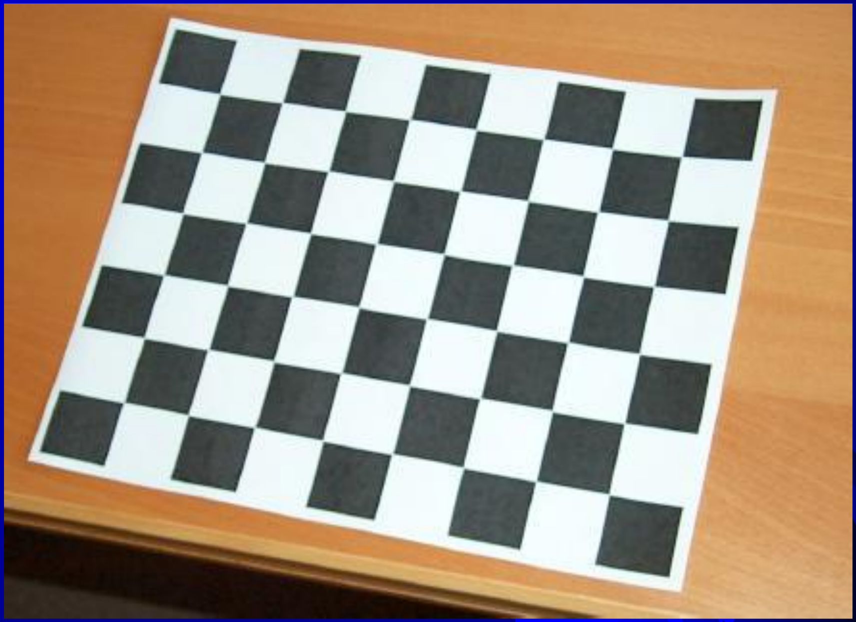
Camera Calibration

- The problem: compute the camera intrinsic and extrinsic parameters using only observed camera data.



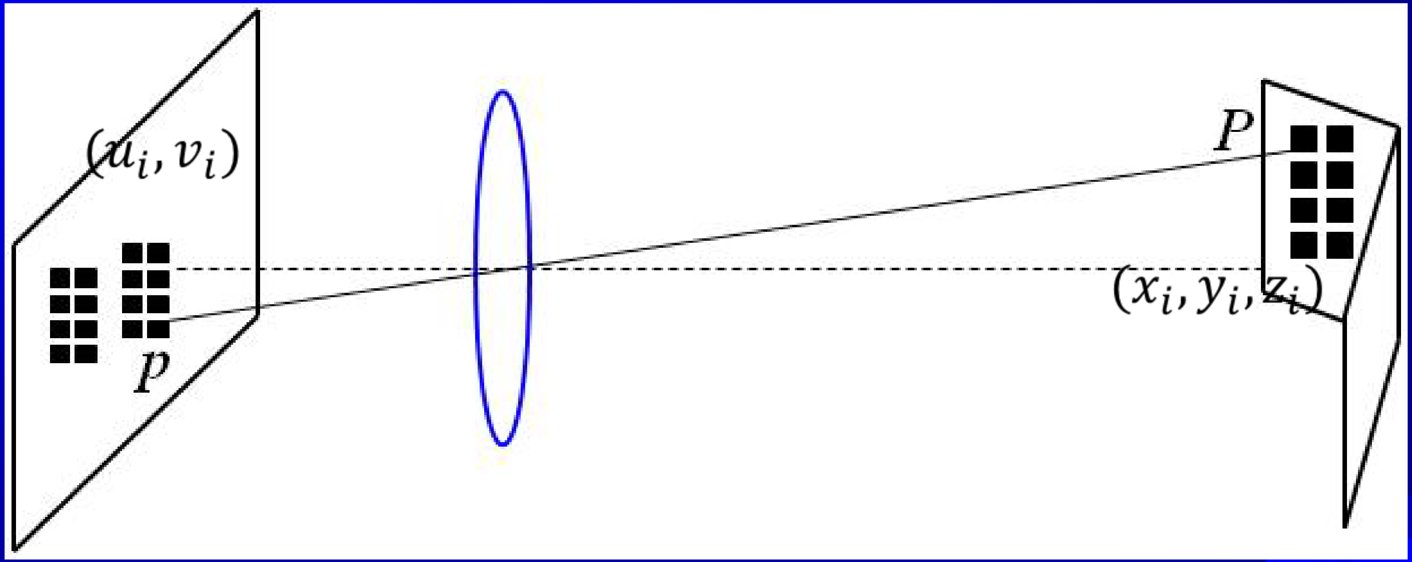


Calibration Classifications

- Calibration pattern based method
  - Feature: Utilize the structural information of the scene. The calibration target is often used.
  - Pros: Can be employed in any camera model with high calibration accuracy.
  - Cons: The calibration procedure is complex and the structural information should be highly accurate.
- Camera self-calibration method.
  - Feature: Using the correspondences between multi-images to calibrate.
  - Pro: Only setup the correspondences between multi-images with high flexibility and potential use in wide range of applications.
  - Con: Nonlinear, low robustness.

Linear camera calibration

- Assume we have known the image positions (u_i, v_i) of n points P_i (automatically or by hand)



$$\boldsymbol{Ax} = 0 \quad \text{or} \quad \boldsymbol{Ax} = \boldsymbol{b}$$

Linear camera calibration

1. Two procedures of calibration process: (1) compute the perspective projection matrix $\mathcal{M}$, (2) estimate intrinsic and extrinsic parameters from $\mathcal{M}$.
2. From before, we had these equations relating image positions, u, v, to points at 3d positions P in homogeneous coordinates.

$$u = \frac{\boldsymbol{m}_1 \cdot \vec{P}}{\boldsymbol{m}_3 \cdot \vec{P}} \qquad v = \frac{\boldsymbol{m}_2 \cdot \vec{P}}{\boldsymbol{m}_3 \cdot \vec{P}} \tag{1}$$

3. for each feature point i we have

$$(\boldsymbol{m}_1 - u_i \boldsymbol{m}_3) \cdot \vec{P}_i = 0$$
$$(\boldsymbol{m}_2 - v_i \boldsymbol{m}_3) \cdot \vec{P}_i = 0$$

Linear camera calibration

1. stack all these measurements of $i = 1, \cdots, n$ points

$$(\boldsymbol{m}_1 - u_i \boldsymbol{m}_3) \cdot \vec{P}_i = 0$$
$$(\boldsymbol{m}_2 - v_i \boldsymbol{m}_3) \cdot \vec{P}_i = 0$$

into a big matrix:

$$\begin{pmatrix} \boldsymbol{P}_1^T & 0^T & -u_1 \boldsymbol{P}_1^T \\ 0^T & \boldsymbol{P}_1^T & -v_1 \boldsymbol{P}_1^T \\ \cdots & \cdots & \cdots \\ \boldsymbol{P}_n^T & 0^T & -u_n \boldsymbol{P}_n^T \\ 0^T & 0^T & -v_n \boldsymbol{P}_n^T \end{pmatrix} \begin{pmatrix} \boldsymbol{m}_1 \\ \boldsymbol{m}_2 \\ \boldsymbol{m}_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} \quad (2)$$

2. That is $\mathcal{P}\boldsymbol{m} = 0$, when $n \geqslant 6$, homogeneous linear least-squares can be used to solve the unit vector $\boldsymbol{m}$, hence project matrix $\mathcal{M}$

Linear camera calibration

- Showing all the elements

$$\begin{pmatrix} P_{1x} & P_{1y} & P_{1z} & 1 & 0 & 0 & 0 & 0 & -u_1 P_{1x} & -u_1 P_{1y} & -u_1 P_{1z} & -u_1 \\ 0 & 0 & 0 & 0 & P_{1x} & P_{1y} & P_{1z} & 1 & -v_1 P_{1x} & -v_1 P_{1y} & -v_1 P_{1z} & -v_1 \\ & & & & & & \cdots & \cdots & \cdots & & & \\ P_{nx} & P_{ny} & P_{nz} & 1 & 0 & 0 & 0 & 0 & -u_n P_{nx} & -u_n P_{ny} & -u_n P_{nz} & -u_n \\ 0 & 0 & 0 & 0 & P_{nx} & P_{ny} & P_{nz} & 1 & -v_n P_{nx} & -v_n P_{ny} & -v_n P_{nz} & -v_n \end{pmatrix} \begin{pmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}$$

- We have $\mathcal{P}\boldsymbol{m} = 0$, we want to solve for the unit vector $\boldsymbol{m}$ that minimizes $|\mathcal{P}\boldsymbol{m}|^2$ by linear least square.

Estimation of intr. and extr. parameters

- Write $\mathcal{M}$ as $\mathcal{M} = (\mathcal{A} \;\; \boldsymbol{b})$, with $\boldsymbol{a}_1^T, \boldsymbol{a}_2^T, \boldsymbol{a}_3^T$ denoting the rows of $\mathcal{A}$, and we have

$$\rho(\mathcal{A} \;\; \boldsymbol{b}) \;=\; \mathcal{K}(\mathcal{R} \;\; \boldsymbol{t}) \Leftrightarrow \qquad (3)$$

$$\rho \begin{pmatrix} \boldsymbol{a}_1^T \\ \boldsymbol{a}_2^T \\ \boldsymbol{a}_3^T \end{pmatrix} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T \\ \frac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T \\ \boldsymbol{r}_3^T \end{pmatrix} \qquad (4)$$

where ρ is an unknown scale factor to account for the fact that the recovered matrix $\mathcal{M}$ has unit Frobenius form since $|\mathcal{M}| = |\boldsymbol{m}| = 1$.

Estimation of intr. and extr. parameters

- Plus the fact rows of rotation matrix have unit length and are perpendicular to each other, get

$$\begin{cases} \rho = \epsilon / |\boldsymbol{a}_3| \\ \boldsymbol{r}_3 = \rho \boldsymbol{a}_3 \\ u_0 = \rho^2 (\boldsymbol{a}_1 \cdot \boldsymbol{a}_3) \\ v_0 = \rho^2 (\boldsymbol{a}_2 \cdot \boldsymbol{a}_3) \end{cases} \quad \text{where} \quad \epsilon = \mp 1. \tag{5}$$

Estimation of intr. and extr. parameters

- Since θ is always in the neighborhood of $\pi/2$ with positive sine, we have

$$\begin{cases} \rho^2(\boldsymbol{a}_1 \times \boldsymbol{a}_3) = -\alpha \boldsymbol{r}_2 - \alpha \cot\theta \boldsymbol{r}_1 \\ \rho^2(\boldsymbol{a}_2 \times \boldsymbol{a}_3) = \dfrac{\beta}{\sin\theta} \boldsymbol{r}_1 \end{cases} \tag{6}$$

and

$$\begin{cases} \rho^2|\boldsymbol{a}_1 \times \boldsymbol{a}_3| = \dfrac{|\alpha|}{\sin\theta} \\ \rho^2|\boldsymbol{a}_2 \times \boldsymbol{a}_3| = \dfrac{|\beta|}{\sin\theta} \end{cases} \tag{7}$$

Estimation of intr. and extr. parameters

1. Thus

$$\begin{cases} \cos\theta = -\dfrac{(\boldsymbol{a}_1 \times \boldsymbol{a}_3)\cdot(\boldsymbol{a}_2 \times \boldsymbol{a}_3)}{|\boldsymbol{a}_1 \times \boldsymbol{a}_3||\boldsymbol{a}_2 \times \boldsymbol{a}_3|} \\ \alpha = \rho^2|\boldsymbol{a}_1 \times \boldsymbol{a}_3|\sin\theta, \\ \beta = \rho^2|\boldsymbol{a}_2 \times \boldsymbol{a}_3|\sin\theta. \end{cases} \quad (8)$$

2. Compute $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ from the second part of equation (6) and (7) .

$$\begin{cases} \boldsymbol{r}_1 = \dfrac{\rho^2\sin\theta}{\beta}(\boldsymbol{a}_2 \times \boldsymbol{a}_3) = \dfrac{1}{|\boldsymbol{a}_2 \times \boldsymbol{a}_3|}(\boldsymbol{a}_2 \times \boldsymbol{a}_3) \\ \boldsymbol{r}_2 = \boldsymbol{r}_3 \times \boldsymbol{r}_1. \end{cases} \quad (9)$$

Estimation of intr. and extr. parameters

1. The translation parameters can be recovered by writing $\mathcal{K}\boldsymbol{t} = \rho\boldsymbol{b}$, and hence $\boldsymbol{t} = \rho\mathcal{K}^{-1}\boldsymbol{b}$.
2. In practice, we know the sign of t_z (determined by the origin of world coordinate system whether it locates before or after the camera), then the camera parameters can be uniquely determined.

Calibration Example

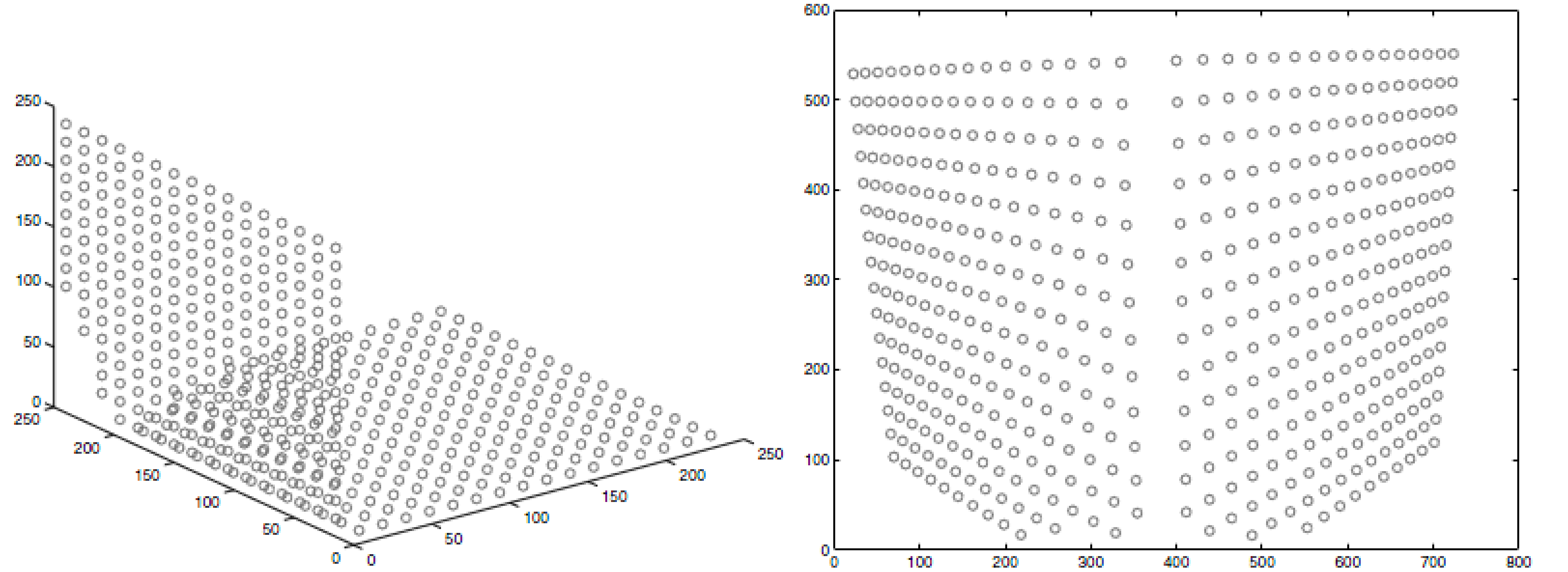
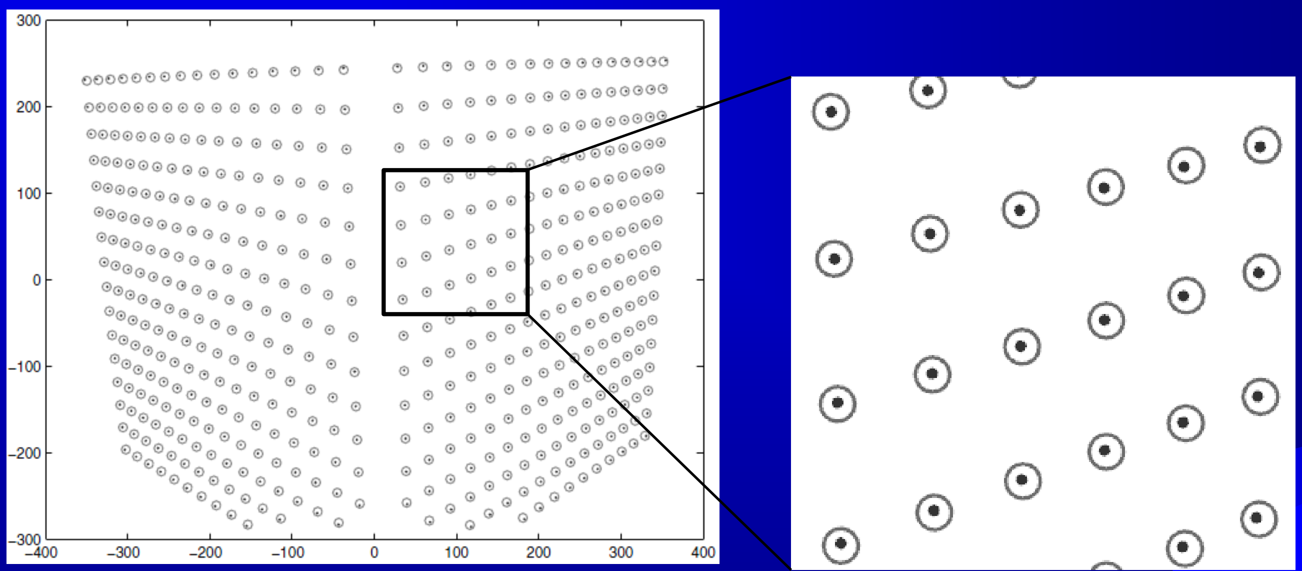


FIGURE 1.17: Camera calibration data. Left: A rendering of 491 3D fiducial points measured on a calibration rig. Right: The corresponding image points. Data courtesy of Janne Heikkilä; data copyright ©2000 University of Oulu.

Calibration Example

$$K = \begin{pmatrix} 970.2841 & 0.0986 & 372.0050 \\ 0 & 963.3466 & 299.2921 \\ 0 & 0 & 1 \end{pmatrix}$$



The original data points (circles) are overlaid with the reprojected 3D points (dots). The root-mean-squared error is 0.96 pixel for this 768 $\times$ 576 image.

Linear camera calibration II

1. Compute projection matrix $\mathcal{M}$ from reference image. The projection equation

$$z\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} P_{ix} \\ P_{iy} \\ P_{iz} \\ 1 \end{bmatrix} \quad (10)$$

2. There are three functions

$$\begin{cases} zu_i = m_{11}P_{ix} + m_{12}P_{iy} + m_{13}P_{iz} + m_{14} \\ zv_i = m_{21}P_{ix} + m_{22}P_{iy} + m_{23}P_{iz} + m_{24} \\ z = m_{31}P_{ix} + m_{32}P_{iy} + m_{33}P_{iz} + m_{34} \end{cases} \quad (11)$$

3. The first equation divides the third, and the second divides the third, we can get.

Linear camera calibration II

$$P_{ix}m_{11} + P_{iy}m_{12} + P_{iz}m_{13} + m_{14} - u_iP_{ix}m_{31} - u_iP_{iy}m_{32} - u_iP_{iz}m_{33} = u_im_{34}$$

$$P_{ix}m_{21} + P_{iy}m_{22} + P_{iz}m_{23} + m_{24} - v_iP_{ix}m_{31} - v_iP_{iy}m_{32} - v_iP_{iz}m_{33} = v_im_{34}$$

- Have n known points in calibration target (P_{ix}, P_{iy}, P_{iz}) and the corresponding image coordinates (u_i, v_i), get

$$\begin{bmatrix} P_{x1} & P_{y1} & P_{z1} & 1 & 0 & 0 & 0 & 0 & -u_1P_{x1} & -u_1P_{y1} & -u_1P_{z1} \\ 0 & 0 & 0 & 0 & P_{x1} & P_{y1} & P_{z1} & 1 & -v_1P_{x1} & -v_1P_{y1} & -v_1P_{z1} \\ & & \cdots & & \cdots & & \cdots & & & & \\ P_{xn} & P_{yn} & P_{zn} & 1 & 0 & 0 & 0 & 0 & -u_nP_{xn} & -u_nP_{yn} & -u_nP_{zn} \\ 0 & 0 & 0 & 0 & P_{xn} & P_{yn} & P_{zn} & 1 & -v_nP_{xn} & -v_nP_{yn} & -v_nP_{zn} \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \end{bmatrix} = \begin{bmatrix} u_1m_{34} \\ v_1m_{34} \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ u_nm_{34} \\ v_nm_{34} \end{bmatrix}$$

Linear camera calibration II

1. From $\boldsymbol{p} = \frac{1}{z}\mathcal{M}\boldsymbol{P}$, multiplying a constant to $\mathcal{M}$ does not affect the relation between $\boldsymbol{P}$ and $\boldsymbol{p}$. We assume $m_{34} = 1$.
2. There are 11 unknowns, denoted as $\boldsymbol{m}$. The matrix form is rewritten as $\mathcal{K}\boldsymbol{m} = \mathcal{U}$ where $\mathcal{K}$ is $2n \times 11$, $\mathcal{U}$ is $2n$. $\mathcal{K}$ and $\mathcal{U}$ are known. When $2n > 11$, $\boldsymbol{m}$ can be solved using LS.

$$\boldsymbol{m} = (\mathcal{K}^T\mathcal{K})^{-1}\mathcal{K}^T\mathcal{U} \tag{12}$$

for assumed $m_{34} = 1$, the matrix $\mathcal{M}$ can be constructed.

Linear camera calibration II

- The matrix $\mathcal{M}$ is decomposed as

$$m_{34}\begin{bmatrix} \boldsymbol{m}_1^T & m_{14} \\ \boldsymbol{m}_2^T & m_{24} \\ \boldsymbol{m}_3^T & 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \boldsymbol{r}_1^T & t_x \\ \boldsymbol{r}_2^T & t_y \\ \boldsymbol{r}_3^T & t_z \\ 0 & 1 \end{bmatrix} \quad (13)$$

where $\boldsymbol{m}_i^T (i = 1, 2, 3)$ are columns made up with the first three elements in the ith row. $m_{i4}(i = 1, 2, 3)$ is the elements of the 4th column in the ith row.

Linear camera calibration II

1. From equation (13) we get

$$m_{34}\begin{bmatrix} \boldsymbol{m}_1^T & m_{14} \\ \boldsymbol{m}_2^T & m_{24} \\ \boldsymbol{m}_3^T & 1 \end{bmatrix} = \begin{bmatrix} \alpha\boldsymbol{r}_1^T + u_0\boldsymbol{r}_3^T & \alpha t_x + u_0 t_z \\ \beta\boldsymbol{r}_2^T + v_0\boldsymbol{r}_3^T & \beta t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{bmatrix} \quad (14)$$

from (14), we get $m_{34}\boldsymbol{m}_3 = \boldsymbol{r}_3$, for normalized orthogonality of $\mathcal{R}$, $|\boldsymbol{r}_3| = 1$, so $m_{34}|\boldsymbol{m}_3| = 1$ and $m_{34} = \frac{1}{|\boldsymbol{m}_3|}$.

Linear camera calibration II

- $\boldsymbol{r}_3, u_0, v_0, \alpha, \beta$ can be solved

$$\begin{cases} \boldsymbol{r}_3 = m_{34}\boldsymbol{m}_3 \\ \alpha = m_{34}^2 |\boldsymbol{m}_1 \times \boldsymbol{m}_3| \\ \beta = m_{34}^2 |\boldsymbol{m}_2 \times \boldsymbol{m}_3| \\ u_0 = (\alpha \boldsymbol{r}_1^T + u_0 \boldsymbol{r}_3^T) \cdot \boldsymbol{r}_3 = m_{34}^2 \boldsymbol{m}_1^T \boldsymbol{m}_3 \\ v_0 = (\beta \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T) \cdot \boldsymbol{r}_3 = m_{34}^2 \boldsymbol{m}_2^T \boldsymbol{m}_3 \end{cases} \quad (15)$$

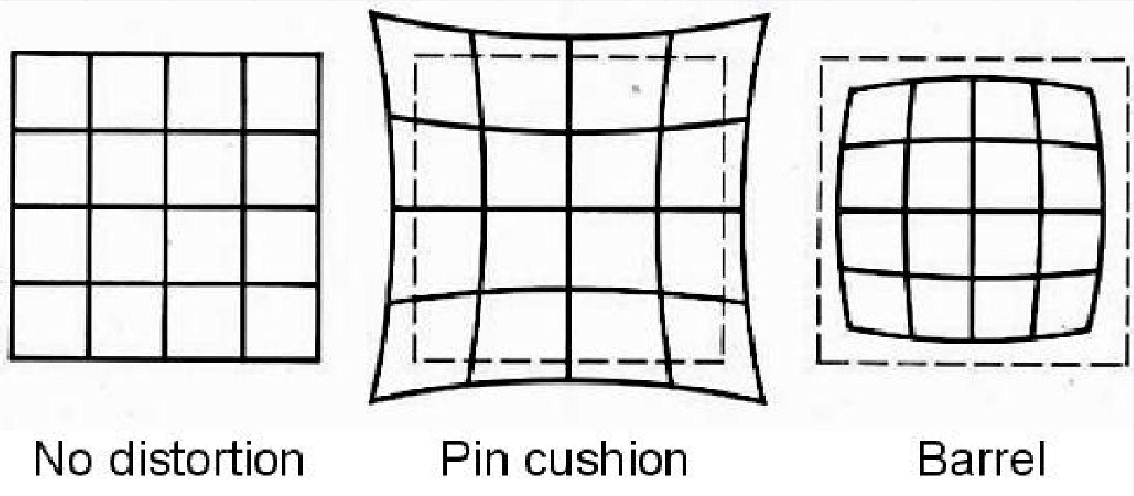
Linear camera calibration II

- Other parameters can also be solved

$$\begin{cases} \boldsymbol{r}_1 = \dfrac{m_{34}}{\alpha}(\boldsymbol{m}_1 - u_0\boldsymbol{m}_3) \\ \boldsymbol{r}_2 = \dfrac{m_{34}}{\beta}(\boldsymbol{m}_2 - v_0\boldsymbol{m}_3) \\ t_z = m_{34} \\ t_x = \dfrac{m_{34}}{\alpha}(m_{14} - u_0) \\ t_y = \dfrac{m_{34}}{\beta}(m_{24} - v_0) \end{cases} \tag{16}$$

Take into account radial aberration

1. Radial distortion (径向畸变)



2. Caused by imperfect lenses
3. Deviations are most noticeable for rays that pass through the edge of the lens

Take into account radial aberration

1. Assume image center is known and let $u_0 = v_0 = 0$, and the model projection is

$$\boldsymbol{p} = \frac{1}{z}\begin{pmatrix} 1/\lambda & 0 & 0 \\ 0 & 1/\lambda & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathcal{M}\boldsymbol{P} \tag{17}$$

where λ is a polynomial function of the squared distance d^2 between the image center and the image point p.

2. A low degree polynomial is often used, $(\lambda = 1 + \sum_{p=1}^{q} \kappa_p d^{2p},$ with $q \leqslant 3)$ and the *distortion coefficients* $\kappa_p (p = 1, \cdots, q)$ are small.

3. d^2 is naturally expressed in terms of the *normalized* image coordinates of the point p, i.e., $d^2 = \hat{u}^2 + \hat{v}^2$.

Take into account radial aberration

1. According to the coordinate change between the physical image frame and the normalized one, from Equation $p = \mathcal{K}\hat{p}$ we can get

$$\hat{u} = \frac{u}{\alpha} + \frac{v\cos\theta}{\beta}; \;\; \hat{v} = \frac{v\sin\theta}{\beta}$$

2. From $d^2 = \hat{u}^2 + \hat{v}^2$, so

$$d^2 = \frac{u^2}{\alpha^2} + 2\frac{uv\cos\theta}{\alpha\beta} + \frac{\cos^2\theta v^2}{\beta^2} + \frac{v^2\sin^2\theta}{\beta^2}$$

it can be easily got

$$d^2 = \frac{u^2}{\alpha^2} + \frac{v^2}{\beta^2} + 2\frac{uv}{\alpha\beta}\cos\theta \qquad (18)$$

Take into account radial aberration

1. λ is an explicit function of u and v, which is highly nonlinear constraints on the $q + 11$ camera parameters.
2. One is using *nonlinear least-square* to solve parameters.
3. More preferable to a two stage tailored approach: 1) eliminating λ from (17) to use LLS to estimate nine of camera parameters. 2) using simple nonlinear process to solve the remains.

Take into account radial aberration

1. *radial alignment constraint*

$$\lambda \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \frac{\boldsymbol{m}_1 \cdot \boldsymbol{P}}{\boldsymbol{m}_3 \cdot \boldsymbol{P}} \\ \frac{\boldsymbol{m}_2 \cdot \boldsymbol{P}}{\boldsymbol{m}_3 \cdot \boldsymbol{P}} \end{pmatrix} \Rightarrow v(\boldsymbol{m}_1 \cdot \boldsymbol{P}) - u(\boldsymbol{m}_2 \cdot \boldsymbol{P}) = 0 \quad (19)$$

2. Given n fiducial points, we have n linear equations in the eight coefficients of the vector $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$, so $\mathcal{Q}\boldsymbol{n} = 0$

$$\text{where } \mathcal{Q} \stackrel{\text{def}}{=} \begin{pmatrix} v_1 \boldsymbol{P}_1^T & -u_1 \boldsymbol{P}_1^T \\ \dots & \dots \\ v_n \boldsymbol{P}_n^T & -u_n \boldsymbol{P}_n^T \end{pmatrix} \text{ and } \boldsymbol{n} = \begin{pmatrix} \boldsymbol{m}_1 \\ \boldsymbol{m}_2 \end{pmatrix} \quad (20)$$

3. when $n \geqslant 8$, equation (20) is overconstrained, using *LLS* to get the solution.

Take into account radial aberration

1. Once having $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$, defining $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$. Like Equation (3), we have

$$\rho \begin{pmatrix} \boldsymbol{a}_1^T \\ \boldsymbol{a}_2^T \end{pmatrix} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T \\ \dfrac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T \end{pmatrix} \tag{21}$$

2. Calculating the norm and dot product of $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ yields the aspect ratio and skew of the camera.

$$\frac{\beta}{\alpha} = \frac{|\boldsymbol{a}_2|}{|\boldsymbol{a}_1|} \text{ and } \cos\theta = -\frac{\boldsymbol{a}_1 \cdot \boldsymbol{a}_2}{|\boldsymbol{a}_1| \cdot |\boldsymbol{a}_2|} \tag{22}$$

3. As $|\boldsymbol{r}_2^T| = 1$, so

$$\begin{aligned} \alpha &= \epsilon\rho|\boldsymbol{a}_1|\sin\theta \\ \beta &= \epsilon\rho|\boldsymbol{a}_2|\sin\theta \end{aligned} \tag{23}$$

where $\epsilon = \mp 1$.

Take into account radial aberration

1. we can get

$$\begin{cases} \boldsymbol{r}_1 = \dfrac{\epsilon}{\sin\theta}\left(\dfrac{1}{|\boldsymbol{a}_1|}\boldsymbol{a}_1 + \dfrac{\cos\theta}{|\boldsymbol{a}_2|}\boldsymbol{a}_2\right) \\ \boldsymbol{r}_2 = \dfrac{\epsilon}{|\boldsymbol{a}_2|}\boldsymbol{a}_2 \end{cases} \tag{24}$$

2. Using above equation and $\boldsymbol{r}_3 = \boldsymbol{r}_1 \times \boldsymbol{r}_2$, we can recover the rotation matrix $\mathcal{R}$, but twofold ambiguity. Two of the translation parameters can also be recovered by writing

$$\begin{pmatrix} \alpha t_x - \alpha \cot\theta t_y \\ \dfrac{\beta}{\sin\theta} t_y \end{pmatrix} = \rho \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \tag{25}$$

Take into account radial aberration

1. Let $\boldsymbol{b} = (b_1, b_2)$, compute t_x and t_y.

$$\begin{cases} t_x = \dfrac{\epsilon}{\sin\theta}\left(\dfrac{b_1}{|\boldsymbol{a}_1|} + \dfrac{b_2\cos\theta}{|\boldsymbol{a}_2|}\right) \\ t_y = \dfrac{\epsilon b_2}{|\boldsymbol{a}_2|} \end{cases} \quad (26)$$

Take into account radial aberration

1. Impossible to recover t_z and ρ only from $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$.
2. Rewrite (19) as

$$\begin{cases} (\boldsymbol{m}_1 - \lambda u \boldsymbol{m}_3) \cdot \boldsymbol{P} = 0 \\ (\boldsymbol{m}_2 - \lambda v \boldsymbol{m}_3) \cdot \boldsymbol{P} = 0 \end{cases} \tag{27}$$

3. $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$ are known, and $\rho \boldsymbol{m}_3^T = (\boldsymbol{r}_3^T \quad t_z)$ according to projection matrix $\mathcal{M}$, and $\boldsymbol{r}_3$ is also known.
4. Combining (18), (22) and (13), we get

$$d^2 = \frac{1}{\rho^2} \frac{|u\boldsymbol{a}_2 - v\boldsymbol{a}_1|^2}{|\boldsymbol{a}_1 \times \boldsymbol{a}_2|^2} \tag{28}$$

5. substituting (28) into λ yields a nonlinear equation in ρ, t_z and $\kappa_p (p = 1, \cdots, q)$. Having enough data, we can solve these parameters by using nonlinear least square.

Homework

- From the following equation to derive the intrinsic and extrinsic parameters and m_{34}. Assume $\boldsymbol{m}_1^T, \boldsymbol{m}_2^T, \boldsymbol{m}_3^T$, and m_{14}, m_{24} are solved.

$$m_{34}\begin{bmatrix} \boldsymbol{m}_1^T & m_{14} \\ \boldsymbol{m}_2^T & m_{24} \\ \boldsymbol{m}_3^T & 1 \end{bmatrix} = \begin{bmatrix} \alpha\boldsymbol{r}_1^T + u_0\boldsymbol{r}_3^T & \alpha t_x + u_0 t_z \\ \beta\boldsymbol{r}_2^T + v_0\boldsymbol{r}_3^T & \beta t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{bmatrix}$$

See You