哈爾濱工業大學

运动控制

题　　目　运动控制作业三报告

专　　业　控制科学与工程

学　　号　20S053293

学　　生　张凌玮

指 导 教 师　李建刚

日　　期　2020 年 12 月 22 日

实验代码：

Main 函数：

```
clc;
clear;

syms theta1 theta2 theta3 theta4 theta5 l1 l2 l0;

%%
%通过ppt上的SCARA机器人来验证子函数是否写对
% q0 = [0 l1+l2 l0];
% q1 = [0 0 0];
% q2 = [0 l1 0];
% q3 = [0 l1+l2 0];
% g_st0 = [1 0 0 0;0 1 0 l1+l2;0 0 1 l0;0 0 0 1];
% matrix_1 = convert_exponential_rotation([0 0 1],theta1,q1);
% matrix_2 = convert_exponential_rotation([0 0 1],theta2,q2);
% matrix_3 = convert_exponential_rotation([0 0 1],theta3,q3);
% matrix_4 = convert_exponential_transfer(3,theta4);
% gst = matrix_1 * matrix_2 * matrix_3 * matrix_4 * g_st0;
%正确

%%
%读取数据
data = load('Answer of Question_1.txt');
num = size(data);

%初始化
point = zeros(num(1),3);
norm_vector = zeros(num(1),3);
theta_1_final = zeros(num(1),1);
theta_2_final = zeros(num(1),1);
theta_3_final = zeros(num(1),1);
theta_4_final = zeros(num(1),1);
theta_5_final = zeros(num(1),1);
solution = zeros(num(1),5);
o = zeros(1,num(1));

%初始化点
for i = 1 :1:num(1)
    point(i,:) = [data(i,1) data(i,2) data(i,3)];
    norm_vector(i,:) = [data(i,4) data(i,5) data(i,6)];
end
```

```
%计算平移矩阵
matrix_transfer_1 = convert_exponential_transfer(1,theta1);
matrix_transfer_2 = convert_exponential_transfer(2,theta2);
matrix_transfer_3 = convert_exponential_transfer(3,theta3);

%计算旋转矩阵
w_4 = [0 0 1];
w_5 = [0 sqrt(2)/2 sqrt(2)/2];
matrix_rotation_1 = convert_exponential_rotation(w_4,theta4,[0 0 0]);
matrix_rotation_2 = convert_exponential_rotation(w_5,theta5,[0 0 0]);

%计算g_st0与g_st
g_st0 = [1 0 0 0;...
      0 1 0 0;...
      0 0 1 0;...
      0 0 0 1];

g_st = matrix_transfer_1 * matrix_transfer_2 * matrix_transfer_3 * matrix_rotation_1 *
matrix_rotation_2 * g_st0;

%计算法向量矩阵
normal_vector_matrix= g_st * [0 0 1 0]';
transfer_matrix = g_st * [0 0 0 1]';


%使用解方程的方法来验证解析算法是否算对
% eq1 = normal_vector_matrix(2,1) - norm_vector(1,2);
% eq2 = normal_vector_matrix(3,1) - norm_vector(1,3);
% [theta4,theta5] = solve(eq1,eq2,theta4,theta5);
% theta4_final = theta4(1);
% double(theta4)
% double(theta5)

%%
%计算theta4与theta5
for i = 1 :num(1)
theta5(i,1) = acos(2 * norm_vector(i,3) - 1);
sol=solve((2^(1/2)*sin(theta4)*sin(theta5(i,1)))/2 - cos(theta4)*(cos(theta5(i,1))/2 - 1/2),theta4);
o(i) = double(sol(1));
end

%计算theta1、theta2和theta3
for i =1 :num(1)
```

```
        theta_1_final(i,1) = point(i,1);
        theta_2_final(i,1) = point(i,2);
        theta_3_final(i,1) = point(i,3);
        theta_4_final(i,1) = o(i);
        theta_5_final(i,1) = theta5(i,1);
        solution(i,:) = [theta_1_final(i,1) theta_2_final(i,1) theta_3_final(i,1) theta_4_final(i,1)
theta_5_final(i,1)];
end

disp(solution);

%%
%写入文件
fid = fopen('Rotation of Axis.txt','w');
for i=1:num(1)

fprintf(fid,'%f\t%f\t%f\t%f\t%f\n',solution(i,1),solution(i,2),solution(i,3),solution(i,4),solution(i,5)
);
end
fclose(fid);
```

计算平移矩阵函数

```
function matrix_transfer = convert_exponential_transfer(num,theta)
%判断字符串
if num == 1
        exponential_tranfer = [1 0 0 theta;0 1 0 0; 0 0 1 0;0 0 0 1];
end
if num == 2
        exponential_tranfer = [1 0 0 0;0 1 0 theta; 0 0 1 0;0 0 0 1];
end
if num == 3
        exponential_tranfer = [1 0 0 0;0 1 0 0; 0 0 1 theta;0 0 0 1];
end

matrix_transfer = exponential_tranfer;

end
```

计算旋转矩阵函数:

```
function matrix_rotation = convert_exponential_rotation(w,theta,q)

vos = 1- cos(theta);

%计算旋转矩阵R
```

```
exponential_rotation = [(w(1)^2)*vos + cos(theta) w(1)*w(2)*vos - w(3)*sin(theta)
w(1)*w(2)*vos + w(2)*sin(theta);...
    w(1)*w(2)*vos + w(3)*sin(theta) (w(2)^2)*vos + cos(theta) w(2)*w(3)*vos -
w(1)*sin(theta);...
    w(1)*w(3)*vos - w(2)*sin(theta) w(2)*w(3)*vos + w(1)*sin(theta) (w(3)^2)*vos +
cos(theta)];

%计算速度
velocity = -cross(w,q);

%e^(epsilon*theta)
exponential_epsilon = [exponential_rotation (eye(3) -
exponential_rotation)*cross(w,velocity)';0,0,0,1];
%exponential_epsilon  取消注释以显示输出

matrix_rotation = exponential_epsilon;
end
```

运行结果：（θ_4与θ_5只取其中一组解）

$$g_{st} = \begin{bmatrix} cos\theta_4 cos\theta_5 - \frac{\sqrt{2}sin\theta_4 sin\theta_5}{2} & -sin\theta_4\left(\frac{cos\theta_5+1}{2}\right) - \frac{\sqrt{2}cos\theta_4 sin\theta_5}{2} & sin\theta_4\left(\frac{cos\theta_5-1}{2}\right) + \frac{\sqrt{2}cos\theta_4 sin\theta_5}{2} & \theta_1 \\ cos\theta_5 sin\theta_4 + \frac{\sqrt{2}cos\theta_4 sin\theta_5}{2} & -cos\theta_4\left(\frac{cos\theta_5+1}{2}\right) - \frac{\sqrt{2}sin\theta_4 sin\theta_5}{2} & -cos\theta_4\left(\frac{cos\theta_5-1}{2}\right) + \frac{\sqrt{2}sin\theta_4 sin\theta_5}{2} & \theta_2 \\ -\frac{\sqrt{2}sin\theta_5}{2} & \frac{1-cos\theta_5}{2} & \frac{cos\theta_5+1}{2} & \theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$n = \begin{bmatrix} sin\theta_4\left(\frac{cos\theta_5 - 1}{2}\right) + \frac{\sqrt{2}cos\theta_4 sin\theta_5}{2} \\ -cos\theta_4\left(\frac{cos\theta_5 - 1}{2}\right) + \frac{\sqrt{2}sin\theta_4 sin\theta_5}{2} \\ \frac{cos\theta_5 + 1}{2} \\ 0 \end{bmatrix} \xrightarrow{n_x, n_y, n_z} \theta_4, \theta_5$$

$$\begin{cases} \theta_1 = X \\ \theta_2 = Y \\ \theta_3 = Z \end{cases}$$

运行结果数据（部分）：$[\theta_1, \theta_2, \theta_3, \theta_4, \theta_5]$ θ_4与θ_5单位为弧度

23.206603	19.921211	40.001934	-0.175022	0.490104
23.236618	19.853677	40.002435	-0.165667	0.464383
23.266633	19.808654	40.002924	-0.175336	0.490966
23.296648	19.763632	40.001231	-0.188620	0.527345
23.326663	19.741121	40.003864	-0.202568	0.565360
23.356678	19.718609	40.005945	-0.209371	0.583829
23.386693	19.696098	40.007473	-0.216133	0.602139
23.416708	19.651076	40.000285	-0.221270	0.616014
23.446723	19.628564	40.000151	-0.236045	0.655761
23.506753	19.606053	40.008025	-0.272886	0.753749
23.536768	19.583542	40.006766	-0.249289	0.691177
23.566783	19.561031	40.004946	-0.255785	0.708470
23.596798	19.538519	40.002563	-0.262235	0.725590
23.656828	19.516008	40.008154	-0.301337	0.828242
23.686843	19.493497	40.004631	-0.275134	0.759672
23.716858	19.470985	40.000542	-0.281445	0.776270
23.776888	19.448474	40.004396	-0.322065	0.881830
23.836918	19.425963	40.007661	-0.328962	0.899528
23.866933	19.403452	40.001839	-0.300401	0.825810
23.926963	19.380940	40.003927	-0.342409	0.933837
23.986993	19.358429	40.005419	-0.349129	0.950886
24.047024	19.335918	40.006310	-0.355791	0.967723
24.107054	19.313407	40.006599	-0.362395	0.984348
24.167084	19.290895	40.006283	-0.368940	1.000763
24.227114	19.268384	40.005358	-0.375427	1.016967
24.287144	19.245873	40.003822	-0.381854	1.032961
24.347174	19.223362	40.001672	-0.388225	1.048751
24.437219	19.200850	40.007300	-0.420618	1.128088
24.497249	19.178339	40.003900	-0.400992	1.080214
24.587294	19.155828	40.008245	-0.433680	1.159622
24.647324	19.133317	40.003582	-0.413536	1.110881
24.737369	19.110805	40.006629	-0.446484	1.190273
24.797399	19.088294	40.000688	-0.425858	1.140771
24.887444	19.065783	40.002422	-0.459031	1.220061
24.977489	19.043272	40.003487	-0.465322	1.234904
25.067534	19.020760	40.003876	-0.471551	1.249539
25.157579	18.998249	40.003588	-0.477720	1.263973
25.247624	18.975738	40.002616	-0.483828	1.278205
25.337669	18.953227	40.000957	-0.489876	1.292240
25.457729	18.930715	40.006821	-0.513764	1.347110
25.547774	18.908204	40.003757	-0.502027	1.320262
25.667834	18.885693	40.008171	-0.525916	1.374678

25.757879	18.863182	40.003681	-0.513947	1.347526
25.877939	18.840670	40.006624	-0.537821	1.401462
25.967984	18.818159	40.000690	-0.525640	1.374054
26.088044	18.795648	40.002140	-0.549487	1.427492
26.208104	18.773137	40.002825	-0.555350	1.440496
26.328164	18.750625	40.002741	-0.561156	1.453318
26.448224	18.728114	40.001880	-0.566904	1.465962
26.568284	18.705603	40.000238	-0.572594	1.478429
26.718359	18.683092	40.005810	-0.590702	1.517771
26.838419	18.660580	40.002563	-0.584049	1.503375
26.988494	18.638069	40.006476	-0.602062	1.542198
27.108554	18.615558	40.001599	-0.595283	1.527645
27.258629	18.593047	40.003824	-0.613196	1.565948
27.408704	18.570535	40.005184	-0.618800	1.577832
27.558779	18.548024	40.005669	-0.624351	1.589555
27.708854	18.525513	40.005274	-0.629846	1.601117
27.858929	18.503002	40.003990	-0.635286	1.612518
28.009005	18.480490	40.001810	-0.640673	1.623767
28.189095	18.457979	40.006499	-0.654993	1.653456
28.339170	18.435468	40.002477	-0.651531	1.646306
28.519260	18.412956	40.005258	-0.665728	1.675520
28.699350	18.390445	40.007058	-0.671136	1.686571
28.849425	18.367934	40.000198	-0.667544	1.679235
29.029515	18.345423	40.000037	-0.681554	1.707742
29.239620	18.322911	40.006485	-0.693533	1.731896
29.419710	18.300400	40.004271	-0.692243	1.729305
29.599800	18.277889	40.001030	-0.697392	1.739633
29.809905	18.255378	40.004285	-0.709179	1.763140
30.020010	18.232866	40.006431	-0.714417	1.773526
30.230115	18.210355	40.006773	-0.718659	1.781907
30.440220	18.187844	40.004836	-0.722847	1.790159
30.650325	18.165333	40.000626	-0.726987	1.798294
30.890445	18.142821	40.000856	-0.736115	1.816144
31.160580	18.120310	40.004898	-0.744242	1.831945
31.430715	18.097799	40.005850	-0.748365	1.839928
31.700850	18.075288	40.003731	-0.752425	1.847766
32.001001	18.052776	40.004316	-0.759675	1.861708
32.301151	18.030265	40.001394	-0.763660	1.869344
32.631316	18.007754	40.000233	-0.770261	1.881945
32.991496	17.985243	40.000064	-0.776404	1.893622
33.381691	17.962731	40.000075	-0.782163	1.904527
33.831916	17.940220	40.003713	-0.789039	1.917490
34.282141	17.917709	40.001124	-0.792722	1.924410
34.822411	17.895198	40.003159	-0.799651	1.937382