Computer Vision
---*Lighting and photometric stereo I*

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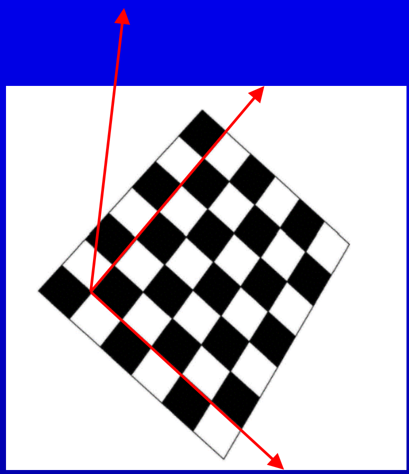
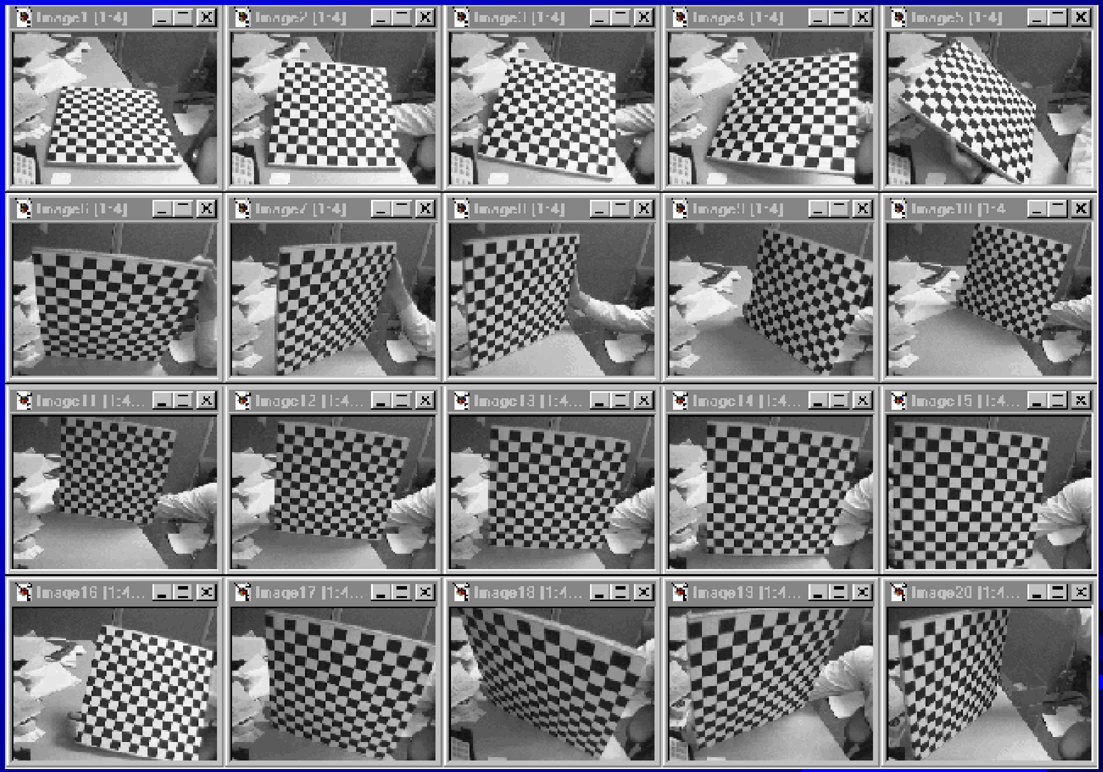
2019.9.30

Zhang's Calibration Method

- Planar pattern





Zhang's Calibration Method

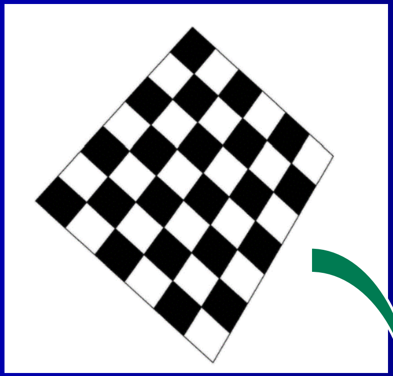
1. Notations: 2D point, $\boldsymbol{m} = [u, v]^T$.
2. 3D point, $\boldsymbol{M} = [X, Y, Z]^T$.
3. Augmented vector, $\widetilde{\boldsymbol{m}} = [u, v, 1]$, $\widetilde{\boldsymbol{M}} = [X, Y, Z, 1]$.
4. Relationship between 3D point $\boldsymbol{M}$ and image projection $\boldsymbol{m}$

$$s\widetilde{\boldsymbol{m}} = \boldsymbol{A}[\boldsymbol{R} \ \ \boldsymbol{t}]\widetilde{\boldsymbol{M}} \quad A = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

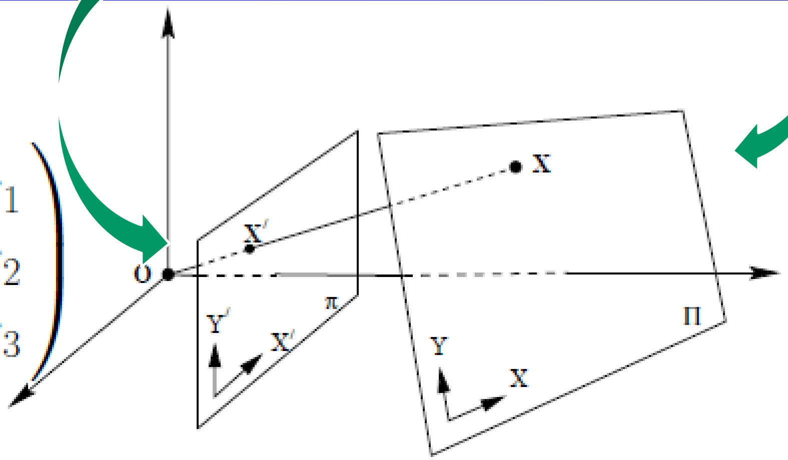
5. s is an arbitrary scale factor, $(\boldsymbol{R}, \boldsymbol{t})$ the extrinsic parameters. $\boldsymbol{A}$ is the intrinsic matrix, (u_0, v_0) the principal point, α and β teh scale factors and γ the skewness of the two image axes.

Zhang's Calibration Method

- Homography transformation



$$\begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$



or $\mathrm{x}' = \mathrm{H}\mathrm{x}$, where H is a 3×3 non-singular homogeneous matrix.

Zhang's Calibration Method

1. Constraints on intrinsic parameters.
2. Let H be $H = [\boldsymbol{h}_1\ \boldsymbol{h}_2\ \boldsymbol{h}_3]$, and $[\boldsymbol{h}_1\ \boldsymbol{h}_2\ \boldsymbol{h}_3] = \lambda A[\boldsymbol{r}_1\ \boldsymbol{r}_2\ \boldsymbol{t}]$.
3. Homography has 8 degree of freedom and 6 extrinsic parameters.
4. Two basic constraints on intrinsic parameter

$$\boldsymbol{h}_1^T A^{-T} A^{-1} \boldsymbol{h}_2 = 0 \tag{3}$$

$$\boldsymbol{h}_1^T A^{-T} A^{-1} \boldsymbol{h}_1 = \boldsymbol{h}_2^T A^{-T} A^{-1} \boldsymbol{h}_2 \tag{4}$$

$$\begin{bmatrix} v_{12}^T \\ (v_{11} - v_{22})^T \end{bmatrix} b = 0$$

$$B = A^{-T} A^{-1} \equiv \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{13} & B_{23} & B_{33} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{\alpha^2} & -\frac{\gamma}{\alpha^2\beta} & \frac{v_0\gamma - u_0\beta}{\alpha^2\beta} \\ -\frac{\gamma}{\alpha^2\beta} & \frac{\gamma^2}{\alpha^2\beta^2} + \frac{1}{\beta^2} & -\frac{\gamma(v_0\gamma - u_0\beta)}{\alpha^2\beta^2} - \frac{v_0^2}{\beta^2} \\ \frac{v_0\gamma - u_0\beta}{\alpha^2\beta} & -\frac{\gamma(v_0\gamma - u_0\beta)^2}{\alpha^2\beta^2} - \frac{v_0^2}{\beta^2} & \frac{(v_0\gamma - u_0\beta)^2}{\alpha^2\beta^2} + \frac{v_0^2}{\beta^2} + 1 \end{bmatrix}$$

Zhang's Calibration Method

- Complete Maximum Likelihood Estimation:
- The complete set of parameters by minimizing the following functional:

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\|m_{ij}-\breve{m}(A,k_1,k_2,R_i,t_i,M_j)\|^2 \tag{21}$$

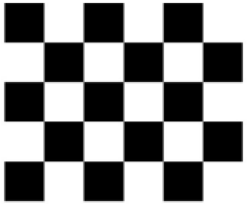
where $\breve{m}(A,k_1,k_2,R_i,t_i,M_j)$ is the projection of point M_j in image i according to equation $s\widetilde{m}=H\widetilde{M}$, followed by distortion according to equation (18). This is a nonlinear minimization problem, which is solved with the Leverberg-Marquardt algorithm.

Zhang's Calibration Method

1. Calibration procedure:
2. Print a pattern and attach to a planar surface.
3. Take few images of the model plane under different orientations.
4. Detect feature points in the images.
5. Estimate five intrinsic parameters and all the extrinsic parameters using the closed-form solution.
6. Refine all parameters by obtaining maximum-likelihood estimate.

Zhang's Calibration Method

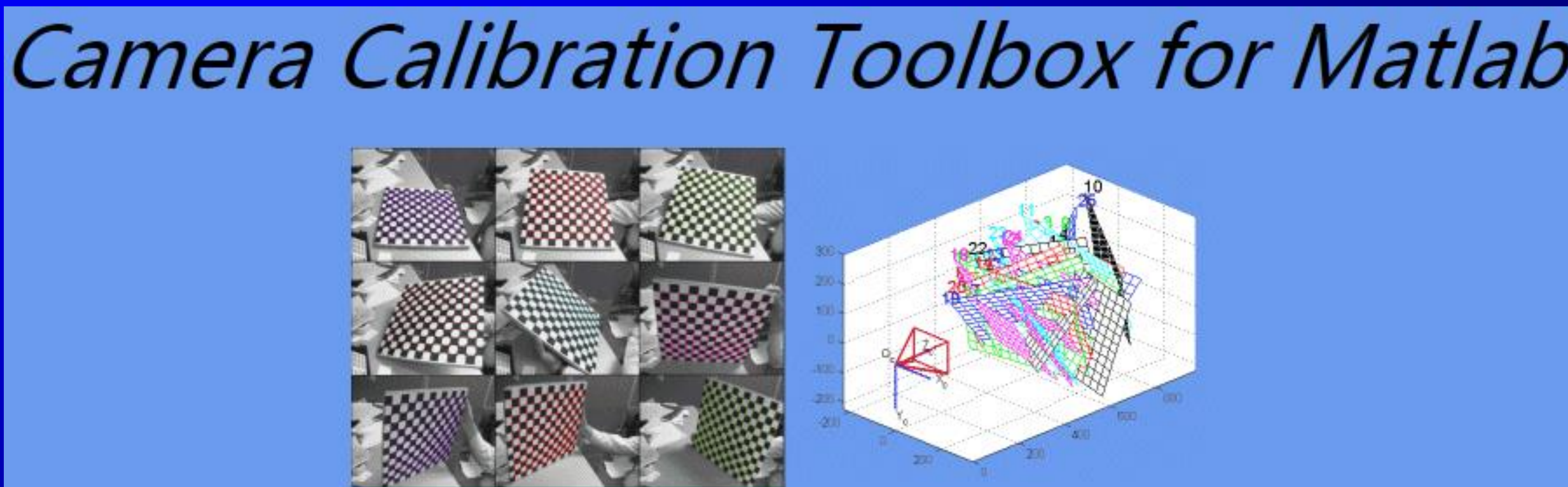
- OpenCV implementation



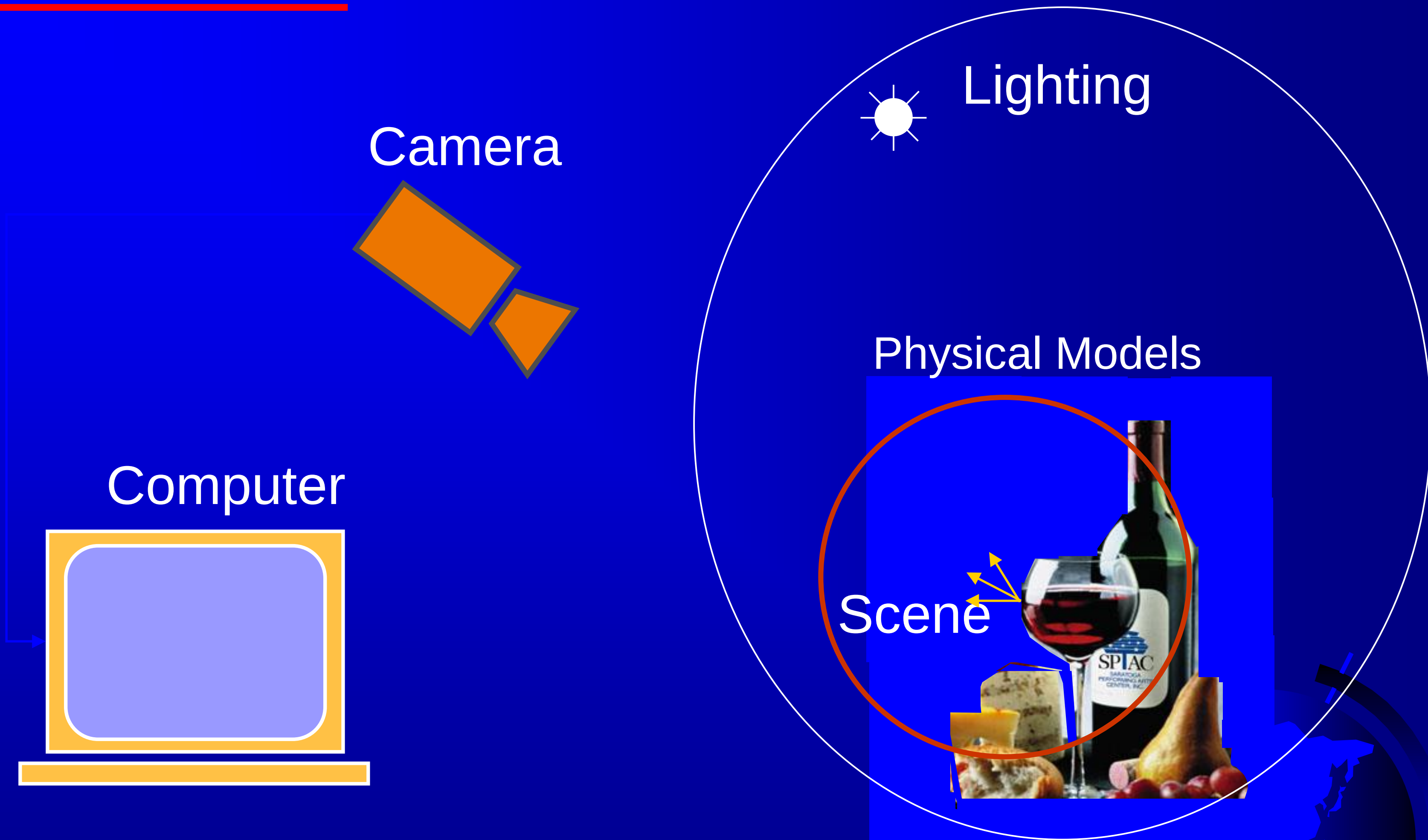
```
bool findChessboardCorners(image, patternSize, corners, flags);

double calibrateCamera(objectPoints, imgPoints, imgSize, camMatrix,
                       distCoeffs, rvecs, tvecs, flags, criteria);

Mat initCameraMatrix2D(objPoints, imgPoints, imgSize, aspectRatio);

void getOptimalNewCameraMatrix(camMatrix, distCoeffs, imgSize,
                               alpha, newImgSize, PixROI, cPP);

void undistort(src, dst, cameraMatrix, distCoeffs, newCameraMatrix);
```

Zhang's Calibration Method

- Camera Calibration Toolbox---matlab

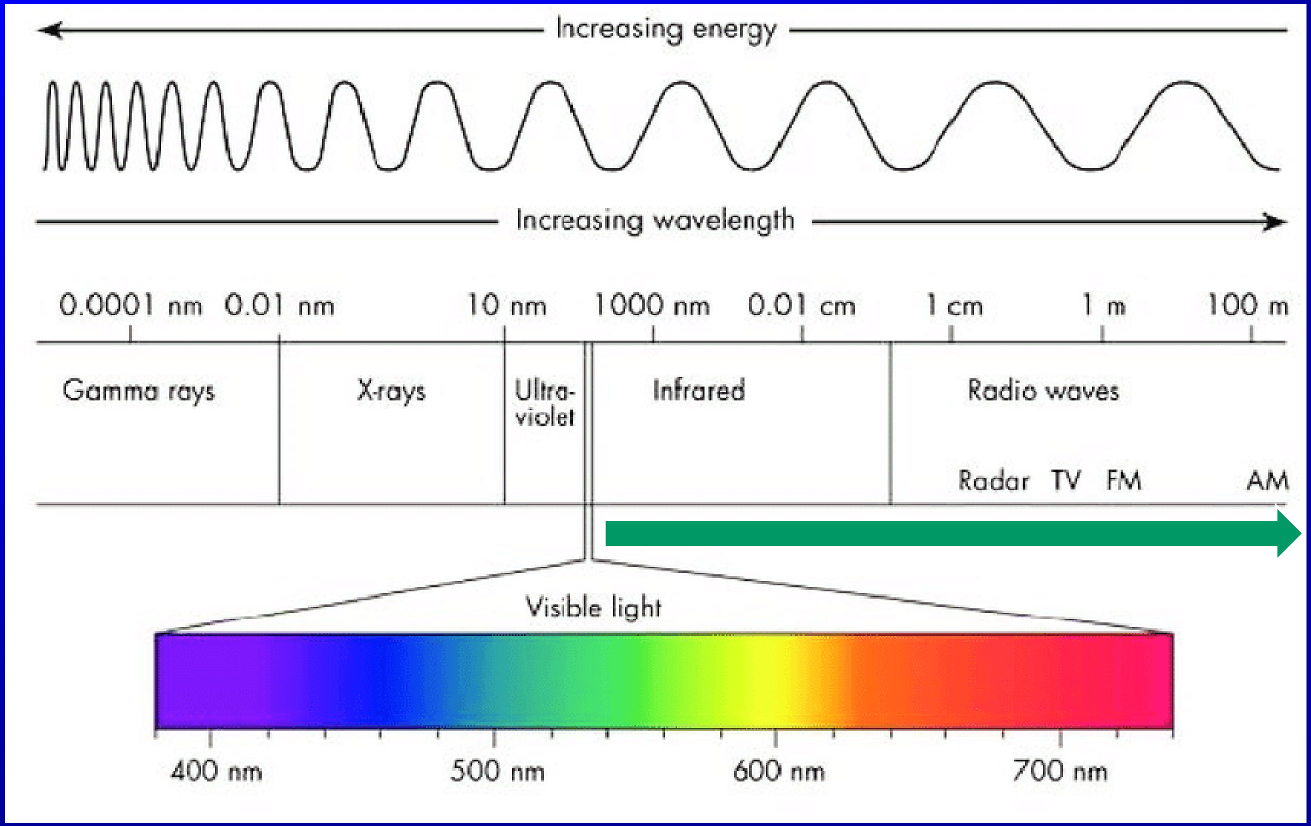


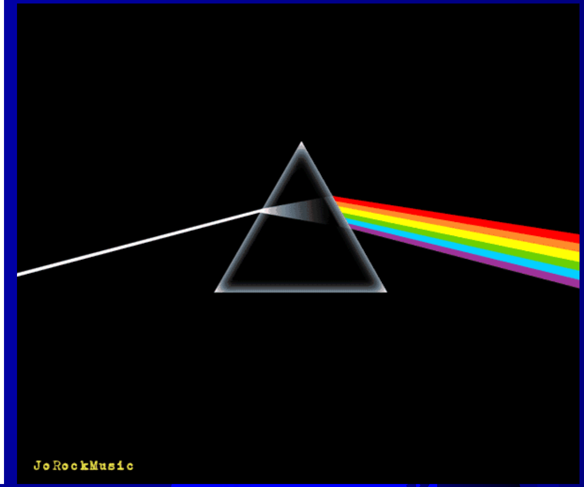
Lighting in Vision



We need to understand the relation between the lighting, surface reflectance, medium and the image of the scene.

Lighting in Vision





Why study the physics (optics) of the world?

Lets see some pictures!

Lighting in Vision

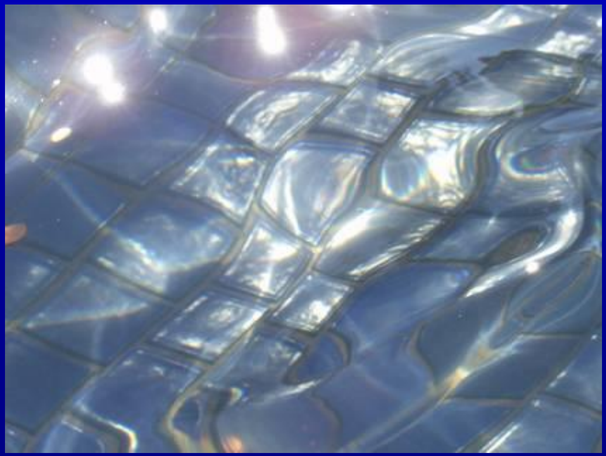
- Light and shadows

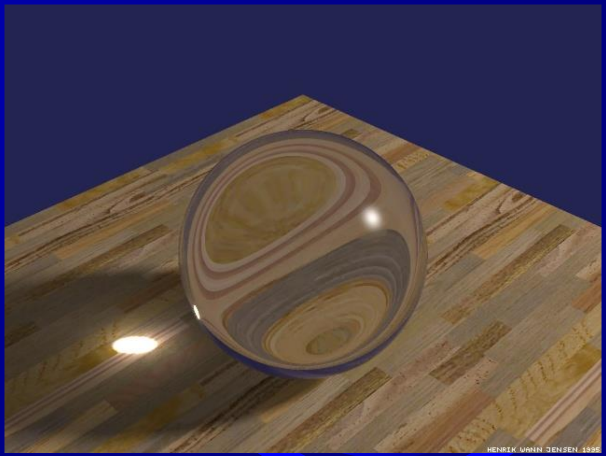


Lighting in Vision

- Reflections and Refractions



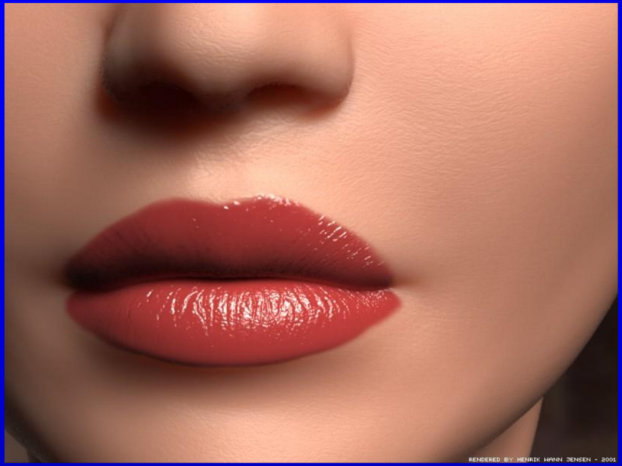


Lighting in Vision

- Interreflections and Scattering

Lighting in Vision

- More Complex Appearances

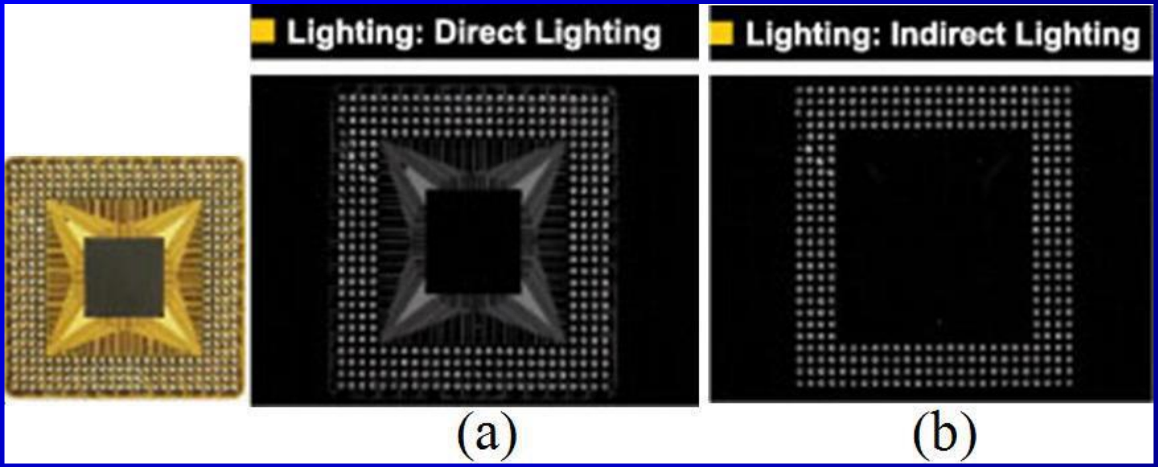


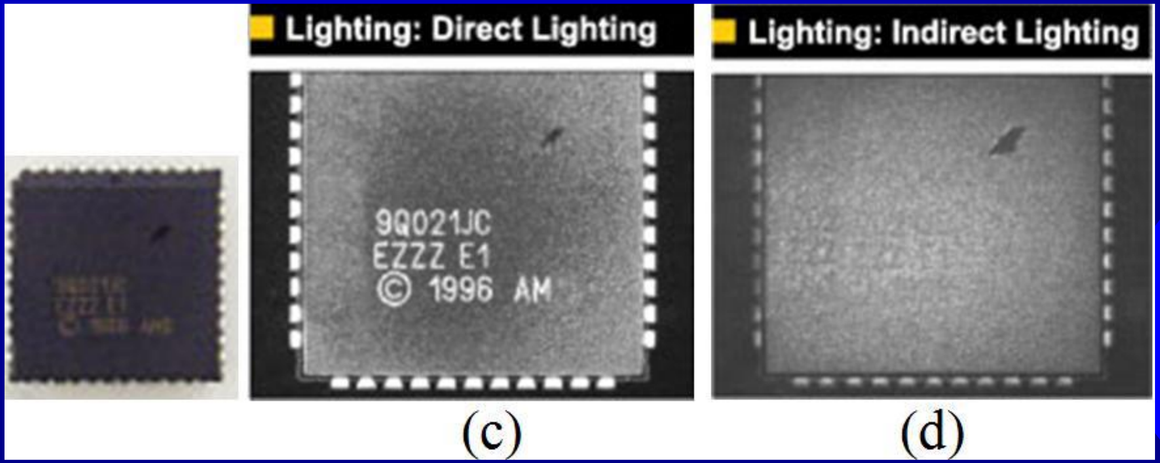




LED Lighting in Machine Vision

- The influence of lighting to image quality.





LED Lighting in Machine Vision

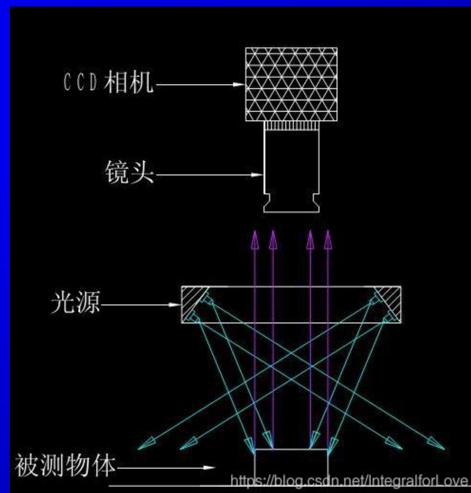
- The influence of lighting to image quality.

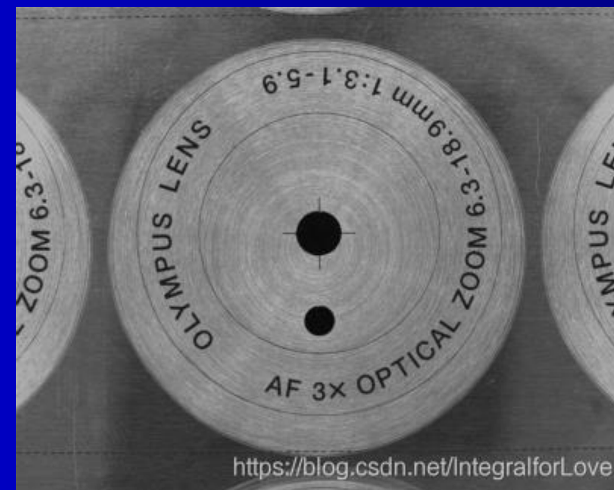
主要光源类型及其特性

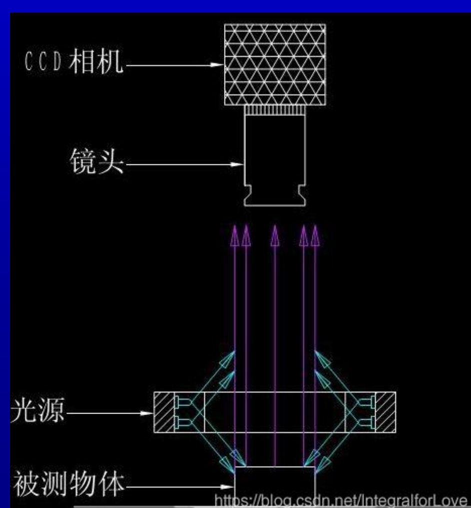
类型	光效(lm/W)	平均寿命/(h)	色温/K	特点
卤素灯	12~24	1000	2800~3000	发热量大，价格便宜，形体小
荧光灯	50~120	1500~3000	3000~6000	价格便宜，适用于大面积照射
LED灯	110~250	100000	全系列	功耗低，发热小，使用寿命长，价格便宜，使用范围广
氙灯	150~330	1000	5500~12000	光照强度高，可连续快速点亮
激光		50000	全系列	具有良好的方向性、单色性与相干性

LED Lighting in Machine Vision

- To control the light and improve the quality of images.







LED Lighting in Machine Vision

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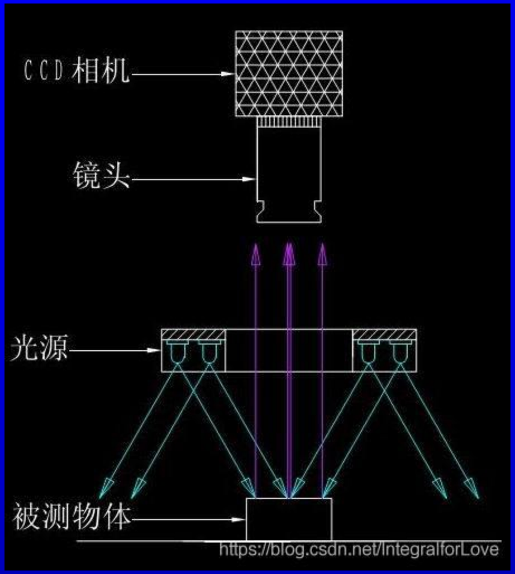
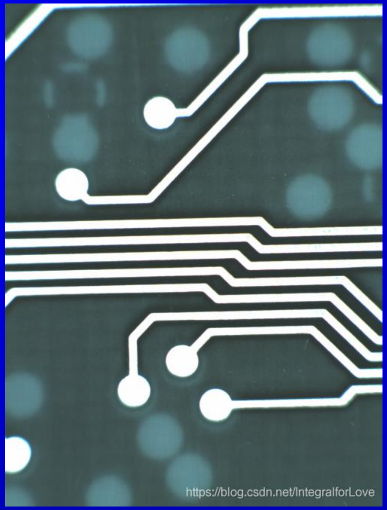
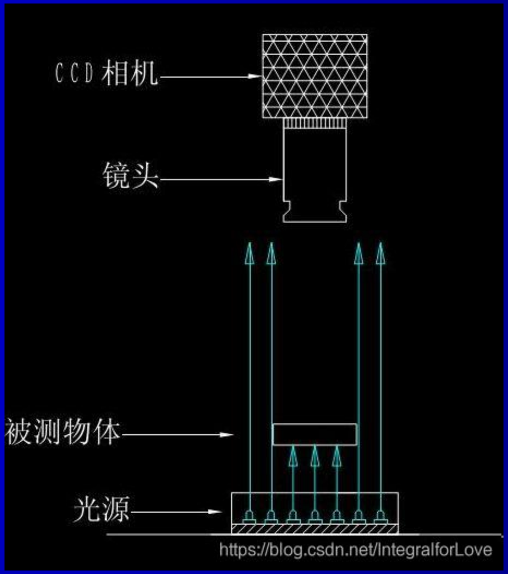
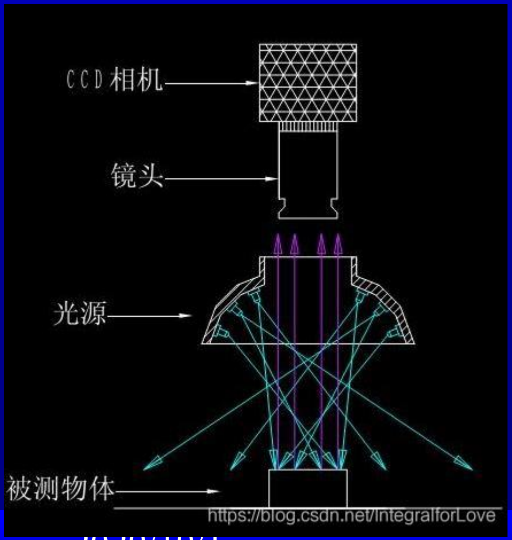
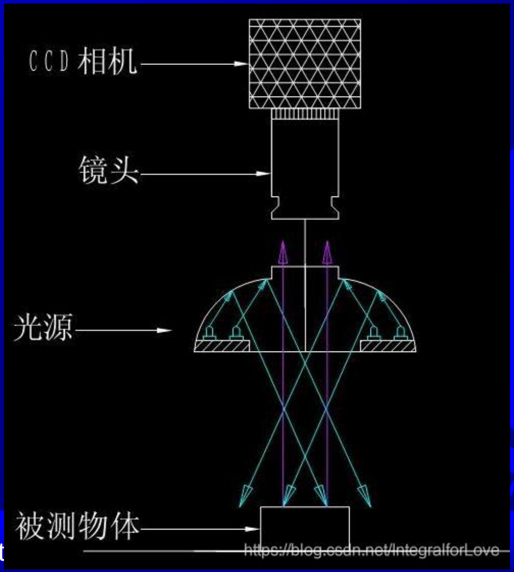




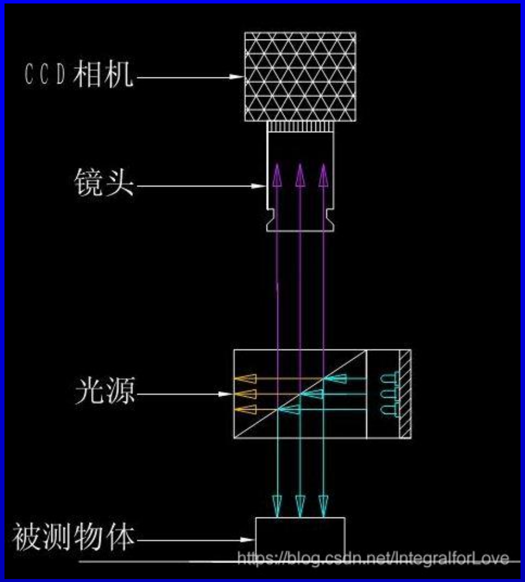


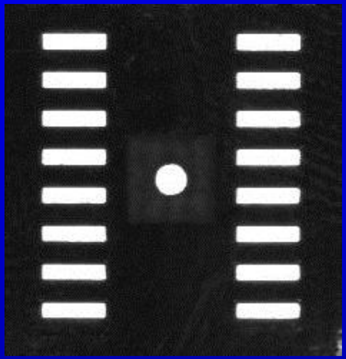


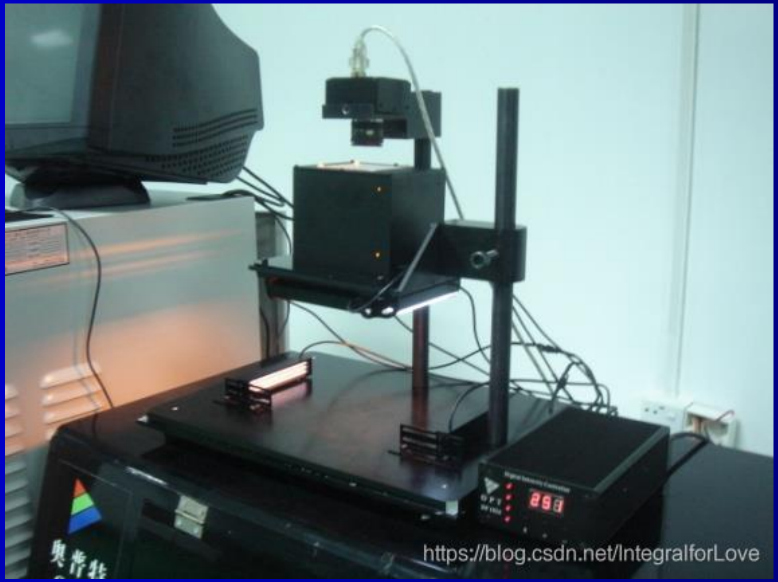


LED Lighting in Machine Vision

- To control the light and improve the quality of images.







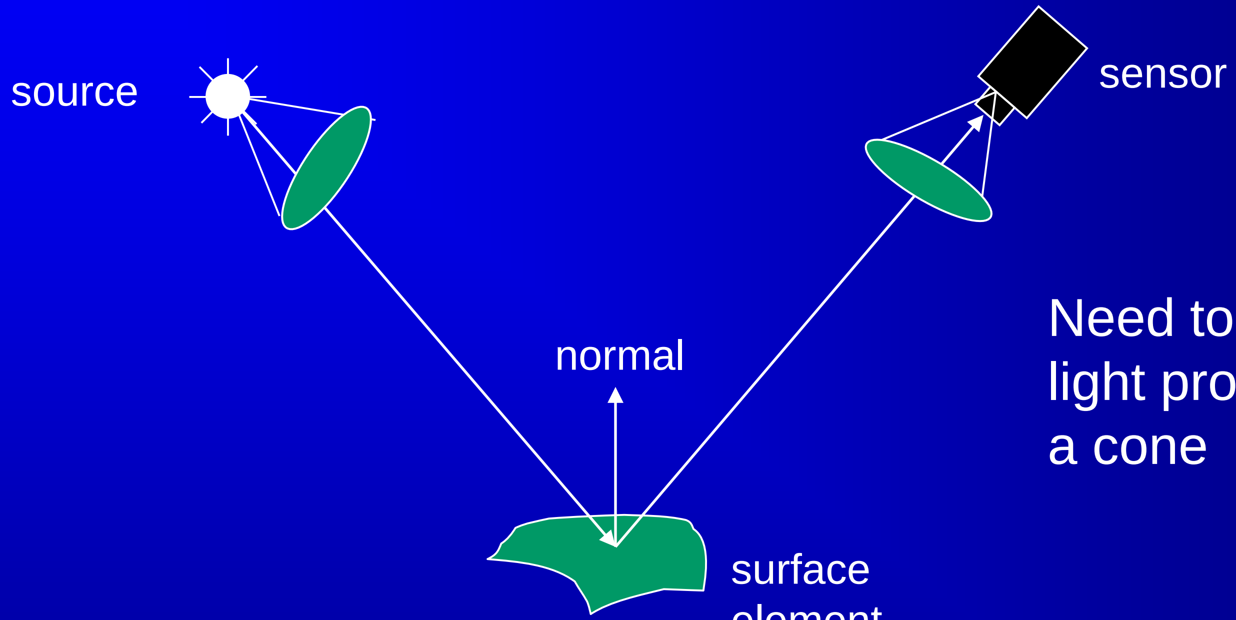
Radiometry(辐射度学) and Image Formation

- To interpret image intensities, we need to understand Radiometric Concepts and Reflectance Properties.

- Topics to be Covered:
- 1) Image Intensities: Overview
- 2) Radiometric Concepts:
- Radiant Intensity （辐射强度）
- Irradiance （辐照度）
- Radiance （辐射率）
- BRDF （双向反射分布函数）

- 3) Diffuse and Specular Reflectance (漫反射和镜面反射)

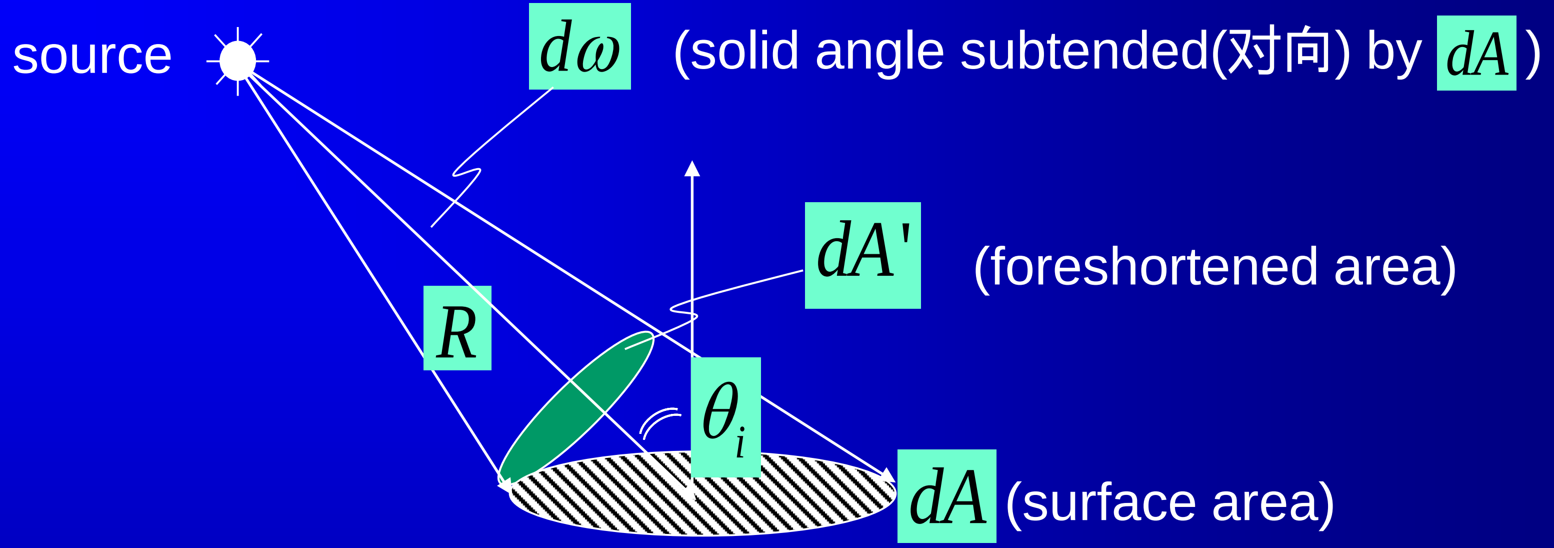
Image Intensities



Need to consider light propagation in a cone

Image intensities $= f$ (normal, surface reflectance, illumination)

Solid Angle（立体角）



Solid Angle :

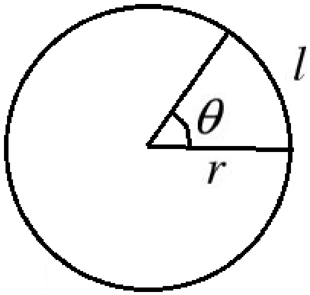
$$d\omega = \frac{dA'}{R^2} = \frac{dA\cos\theta_i}{R^2}$$

(steradian)

Angles and Solid Angles

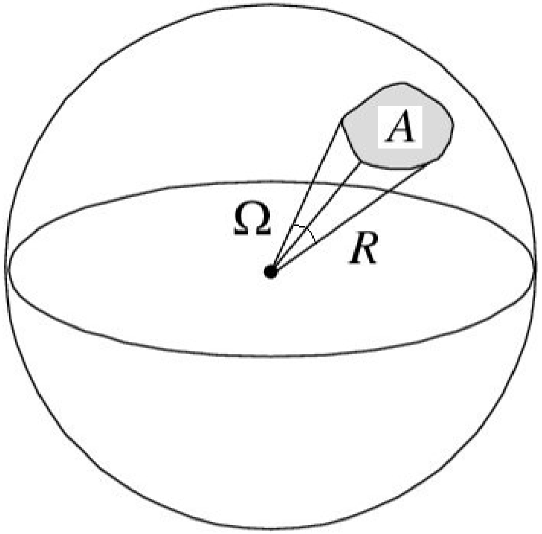
- **Angle** $\theta = \frac{l}{r}$

  $\Rightarrow$ **circle has 2π radians**



- **Solid angle** $\Omega = \frac{A}{R^2}$

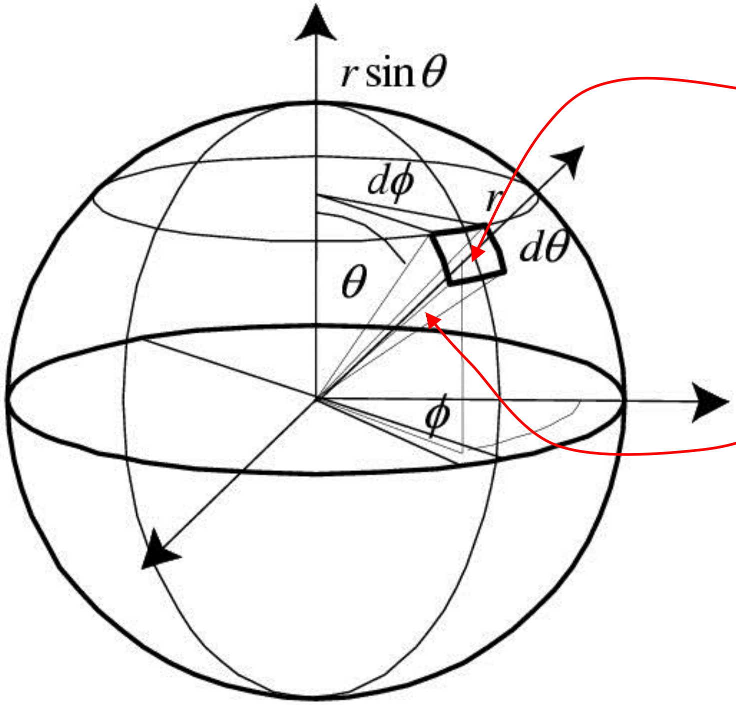
  $\Rightarrow$ **sphere has 4π steradians**



CS348B Lecture 4 Pat Hanrahan, Spring 2002

Differential Solid Angle and Spherical Polar Coordinates

Differential Solid Angles



$$dA = (r\,d\theta)(r\sin\theta\,d\phi) = r^2\sin\theta\,d\theta\,d\phi$$

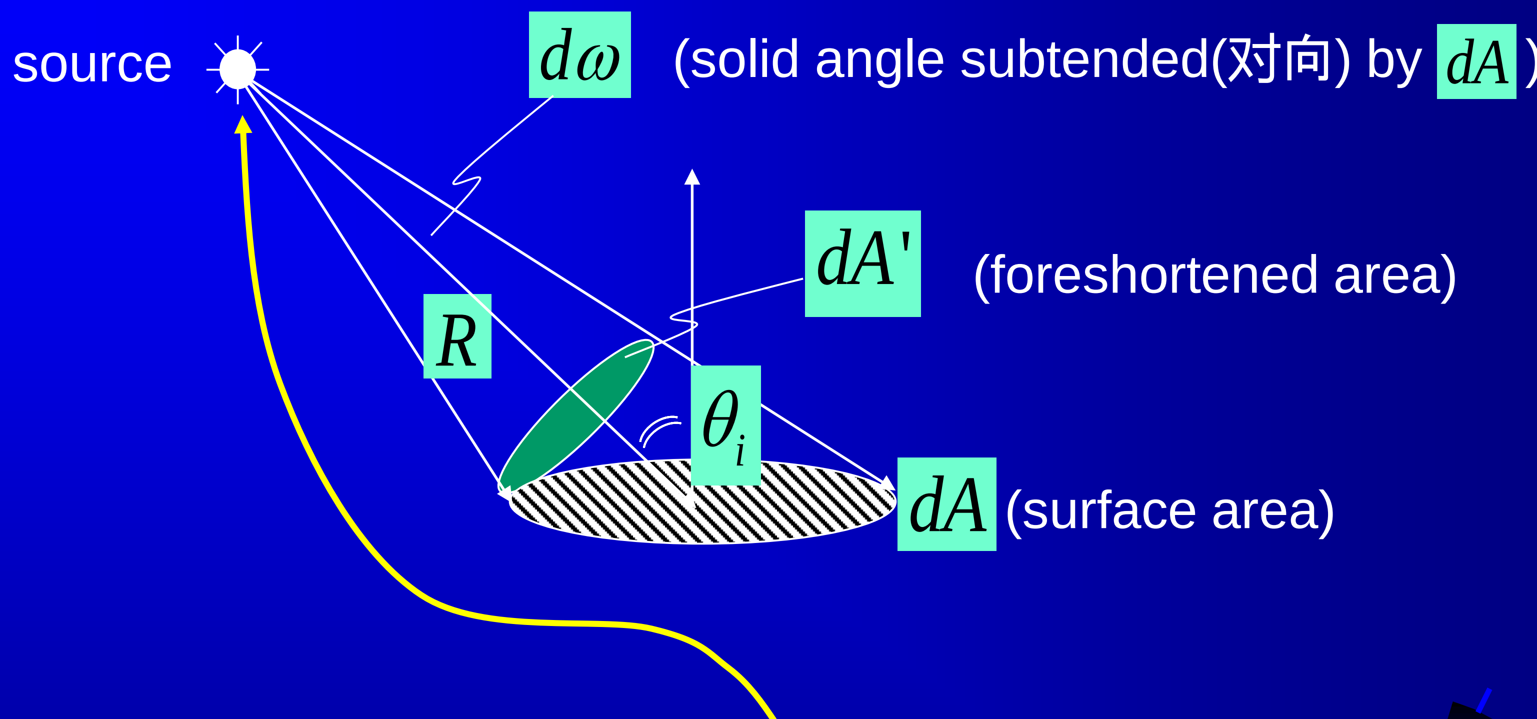
$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

$$S = \int_0^{\pi}\int_0^{2\pi}\sin\theta\,d\theta\,d\phi = 4\pi$$

CS348B Lecture 4

Pat Hanrahan, Spring 2002

Radiant Intensity of Source

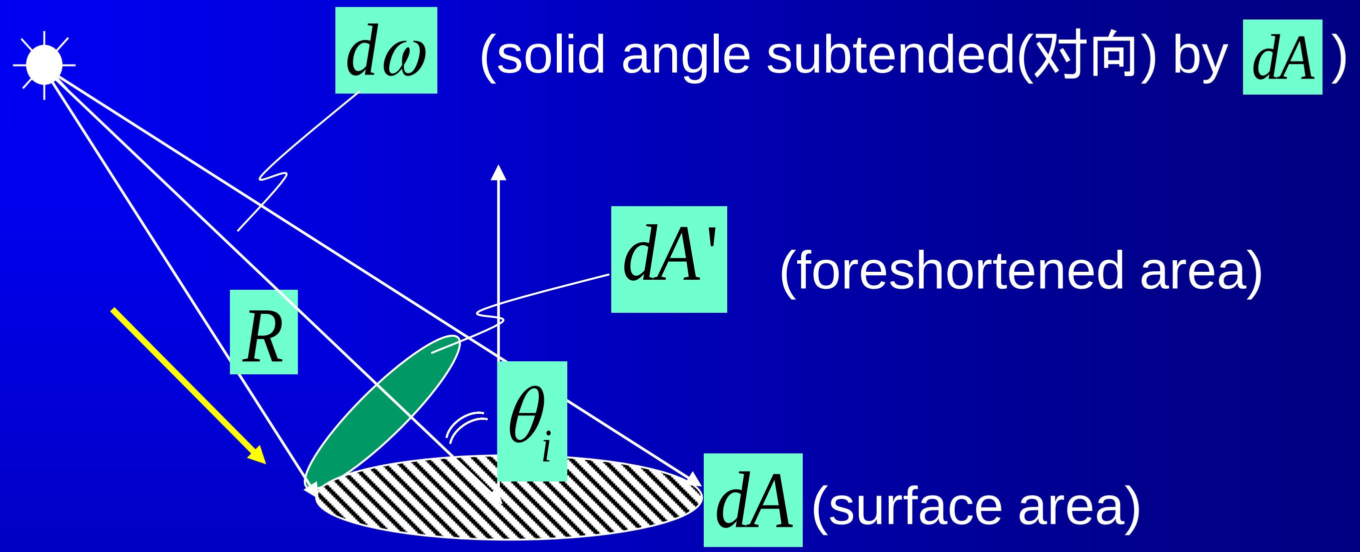


Radiant Intensity（辐射强度）of Source：(watts / steradian)

$$J = \frac{d\Phi}{d\omega}$$

Light Flux（光通量）(power) emitted per unit solid angle

Surface Irradiance(表面辐照度)



Light falling on a surface from all directions. How much?

Surface Irradiance（表面辐照度）: (watts / m^2)

$$E = \frac{d\Phi}{dA}$$

$d\Phi = \frac{dQ}{dt}$ ($d\Phi$光通量，dQ辐射能密度)

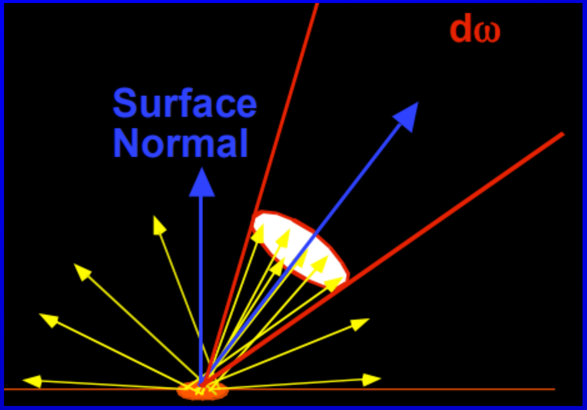
Light Flux (power) incident per unit surface area

Does not depend on where the light is coming from!

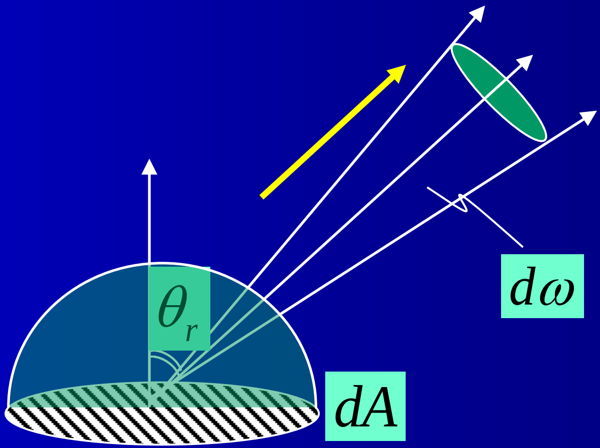
Surface Radiance (表面辐射率)

Surface acts as light source

Radiates over a hemisphere



(surface area) dA

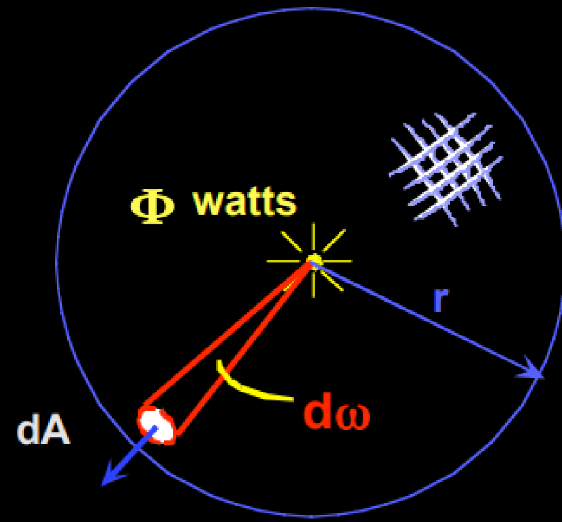


$$L = \frac{d^2\Phi}{(dA\cos\theta_r)\ \ d\omega}$$ (watts / m^2 steradian)

- Flux emitted per unit foreshortened area per unit solid angle.
- *L* depends on direction θ_r
- Surface can radiate into whole hemisphere.
- *L* depends on reflectance properties of surface.

Relationship Between Radiance and Irradiance

Relationship between radiance (radiant intensity) and irradiance



$$d\omega = \frac{dA}{r^2}$$

$$E = \frac{d\Phi}{dA}$$

J: Radiant Intensity

E: Irradiance

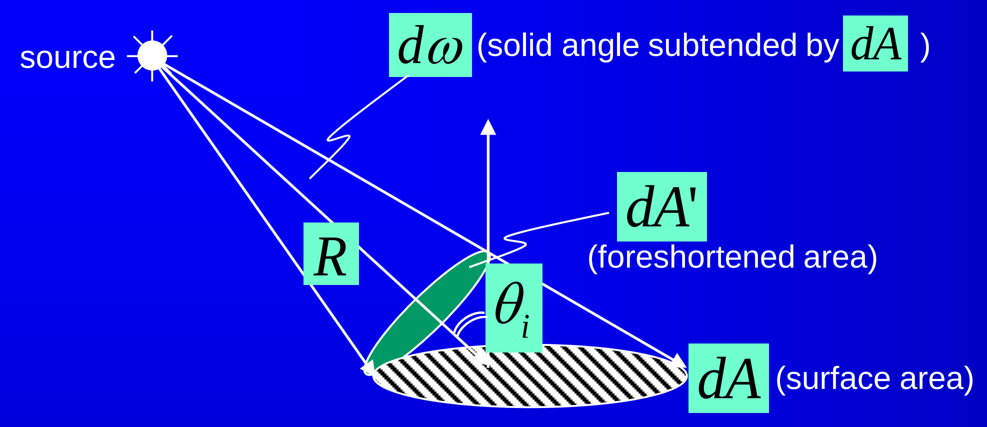
Φ: Watts

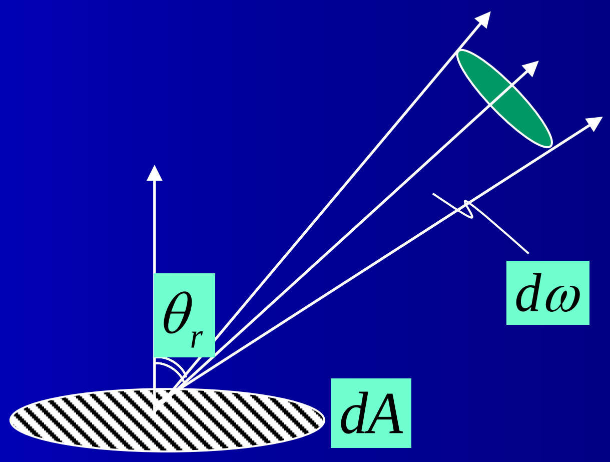
ω : Steradians

$$J = \frac{d\Phi}{d\omega} = \frac{r^2\, d\Phi}{dA} = r^2 E$$

$$\boxed{E = \frac{J}{r^2}}$$

Radiometric concepts – boring…but, important!





(1) Solid Angle : $d\omega = \frac{dA'}{R^2} = \frac{dA \cos\theta_i}{R^2}$ (steradian)

(2) Radiant Intensity of Source : $J = \frac{d\Phi}{d\omega}$ (watts / steradian)

Light Flux (power) emitted per unit solid angle

(3) Surface Irradiance : $E = \frac{d\Phi}{dA}$ (watts / m)

Light Flux (power) incident per unit surface area.

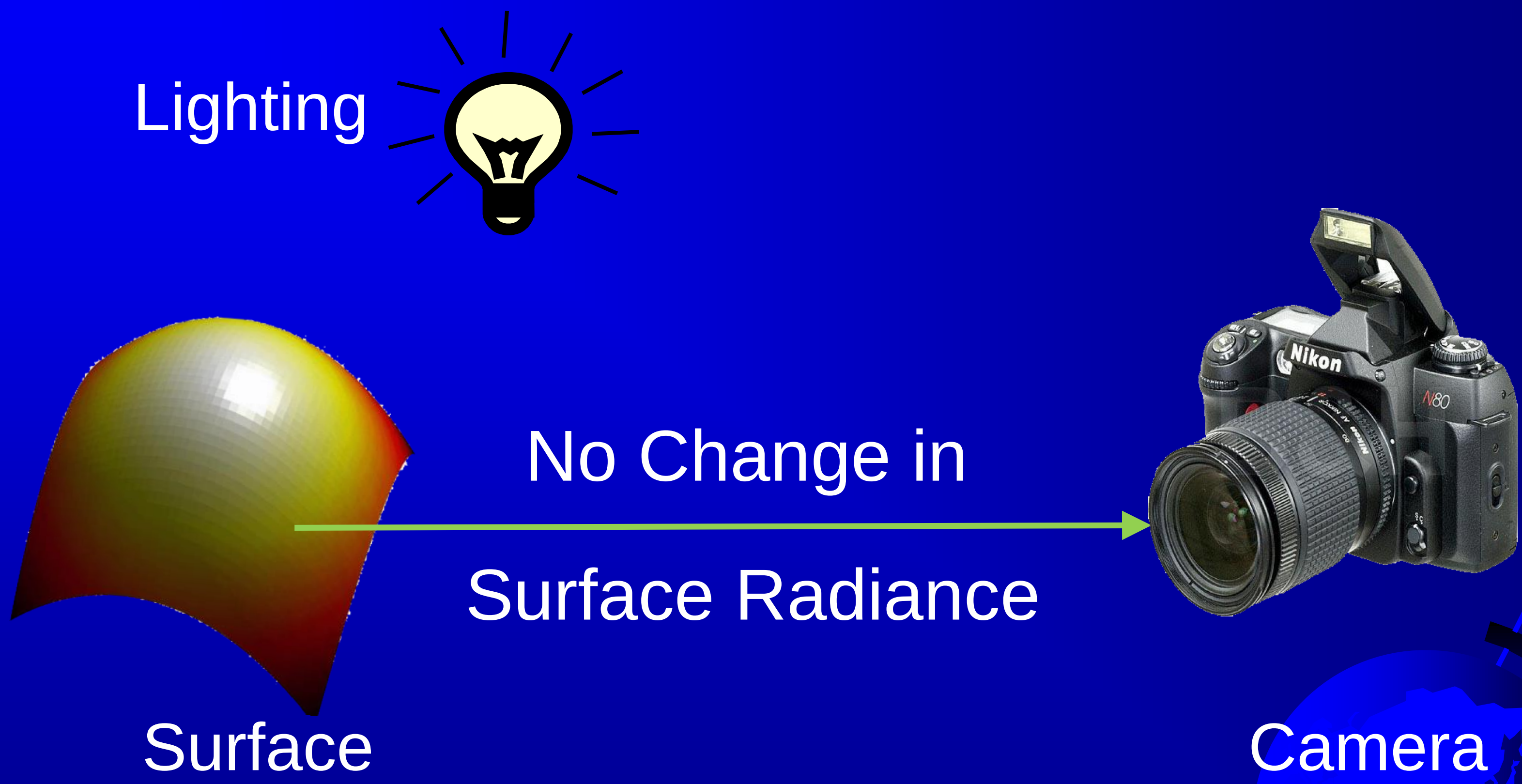
Does not depend on where the light is coming from!

(4) Surface Radiance (tricky) :

$$L = \frac{d^2\Phi}{(dA\cos\theta_r)\ d\omega}$$ (watts / m^2 steradian)

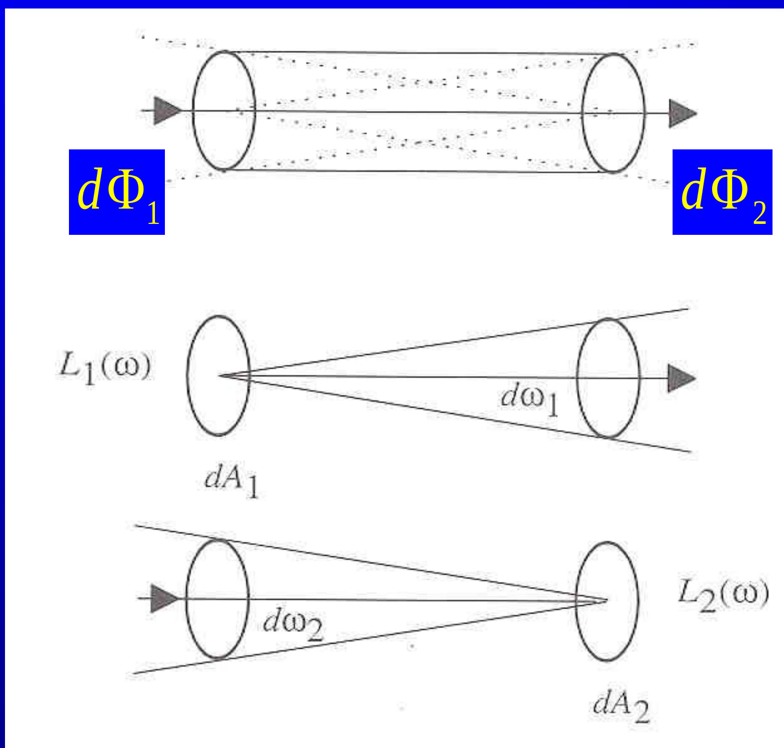
- Flux emitted per unit foreshortened area per unit solid angle.
- L depends on direction θ_r
- Surface can radiate into whole hemisphere.
- L depends on reflectance properties of surface.

The Fundamental Assumption in Vision



Radiance property

- Radiance is constant as it propagates along ray
  - Derived from conservation of flux
  - Fundamental in light transport



$$d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2$$

$$d\omega_1 = dA_2 / r^2 \qquad d\omega_2 = dA_1 / r^2$$

$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

$$\therefore L_1 = L_2$$

Relationship between Scene and Image Brightness

- Before light hits the image plane:

Scene → Scene Radiance L → Lens → Image Irradiance E

Linear Mapping!

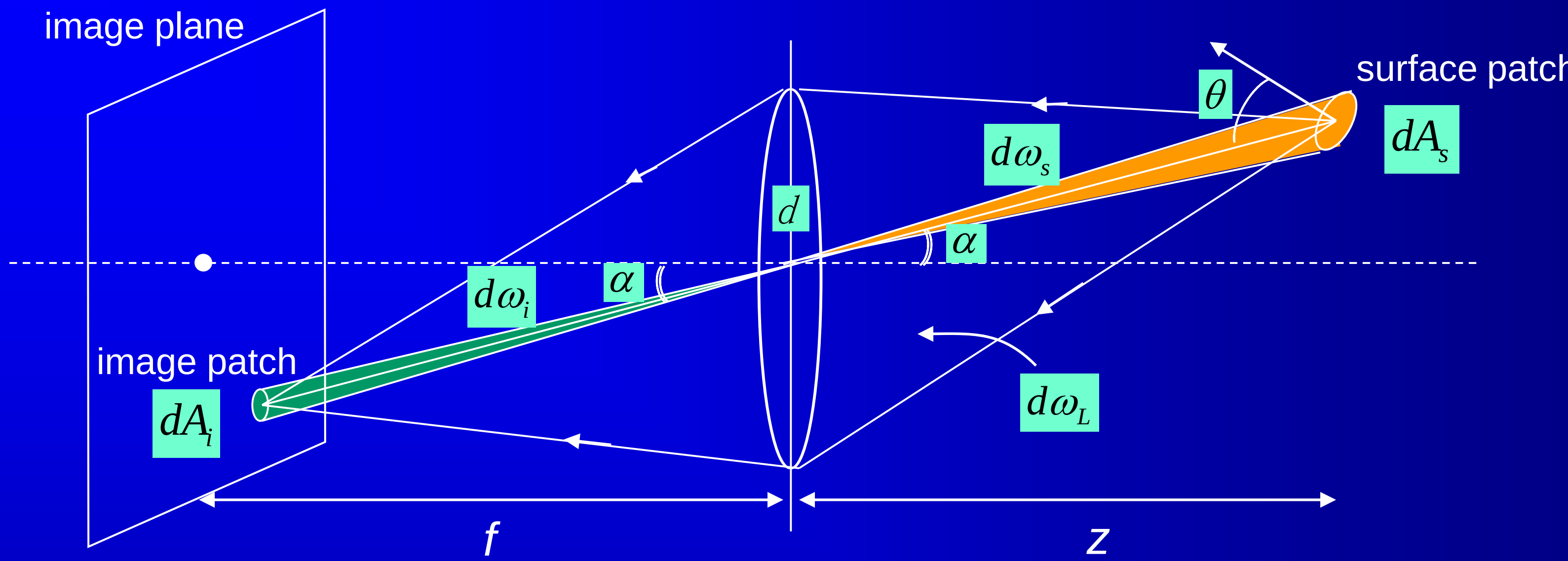
- After light hits the image plane:

Image Irradiance E → Camera Electronics → Measured Pixel Values, I

Non-linear Mapping!

Can we go from measured pixel value, I, to scene radiance, L?

Relationship between Scene and Image Brightness



- Solid angles of the double cone (orange and green):

$$d\omega_i = d\omega_s$$

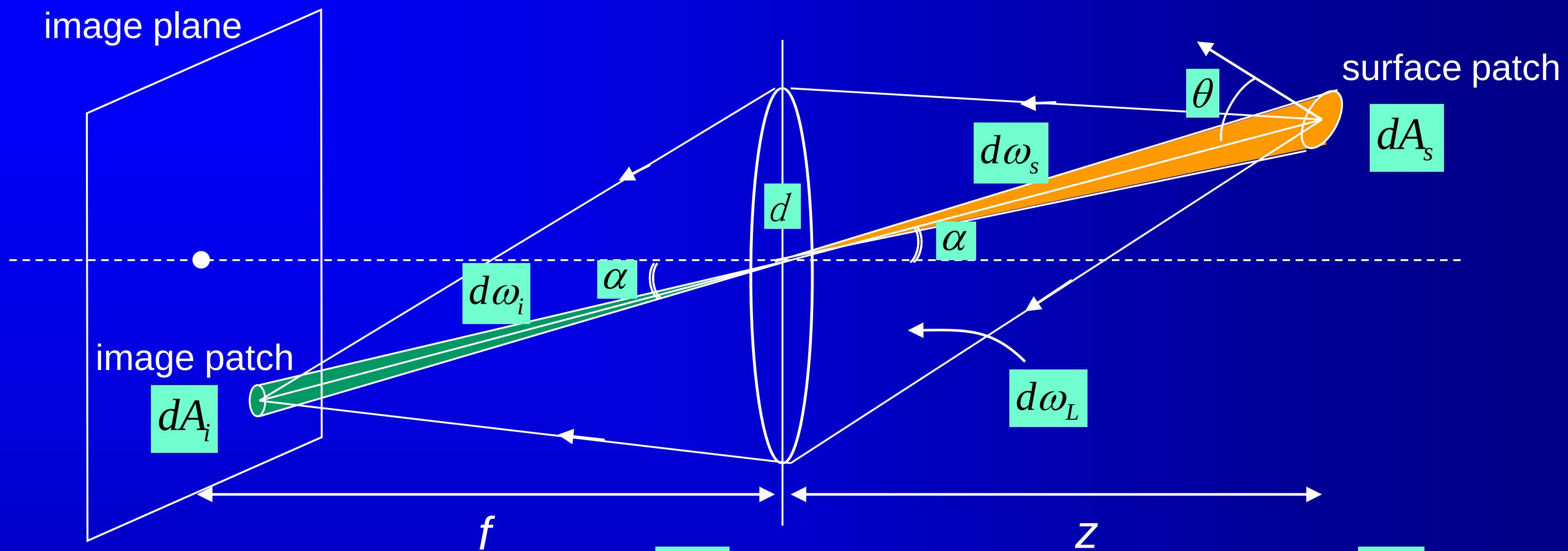
$$\frac{dA_i \cos\alpha}{(f/\cos\alpha)^2} = \frac{dA_s \cos\theta}{(z/\cos\alpha)^2}$$

$$\frac{dA_s}{dA_i} = \frac{\cos\alpha}{\cos\theta}\left(\frac{z}{f}\right)^2 \quad (1)$$

- Solid angle subtended by lens:

$$d\omega_L = \frac{\pi d^2}{4}\frac{\cos\alpha}{(z/\cos\alpha)^2} \quad (2)$$

Relationship between Scene and Image Brightness



- Flux received by lens from dA_s = Flux projected onto image dA_i

$$L\,(dA_s \cos\theta)\, d\omega_L = E\, dA_i \quad (3)$$

- From (1), (2), and (3):

$$\boxed{E = L\,\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos\alpha^4}$$

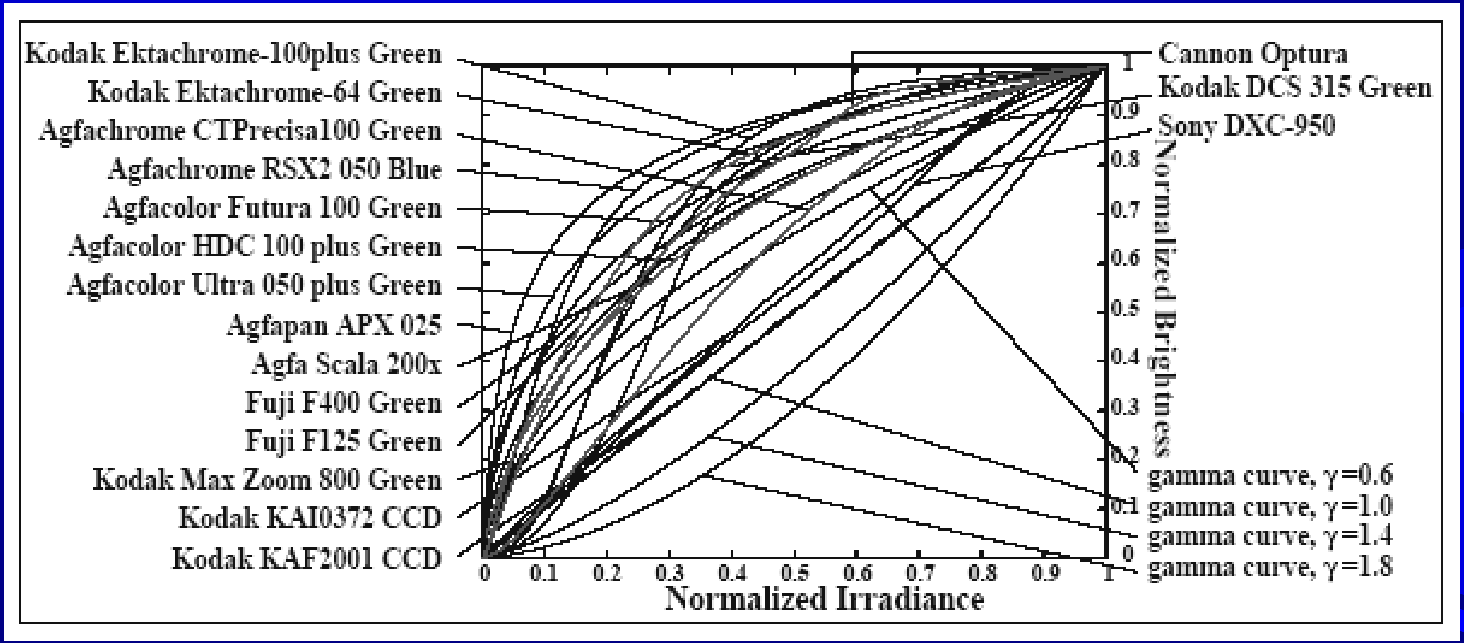
  - Image irradiance is proportional to Scene Radiance!
  - Small field of view → Effects of 4th power of cosine are small.

Relation between Pixel Values I and Image Irradiance E

Image Irradiance E → Camera Electronics → Measured Pixel Values, I

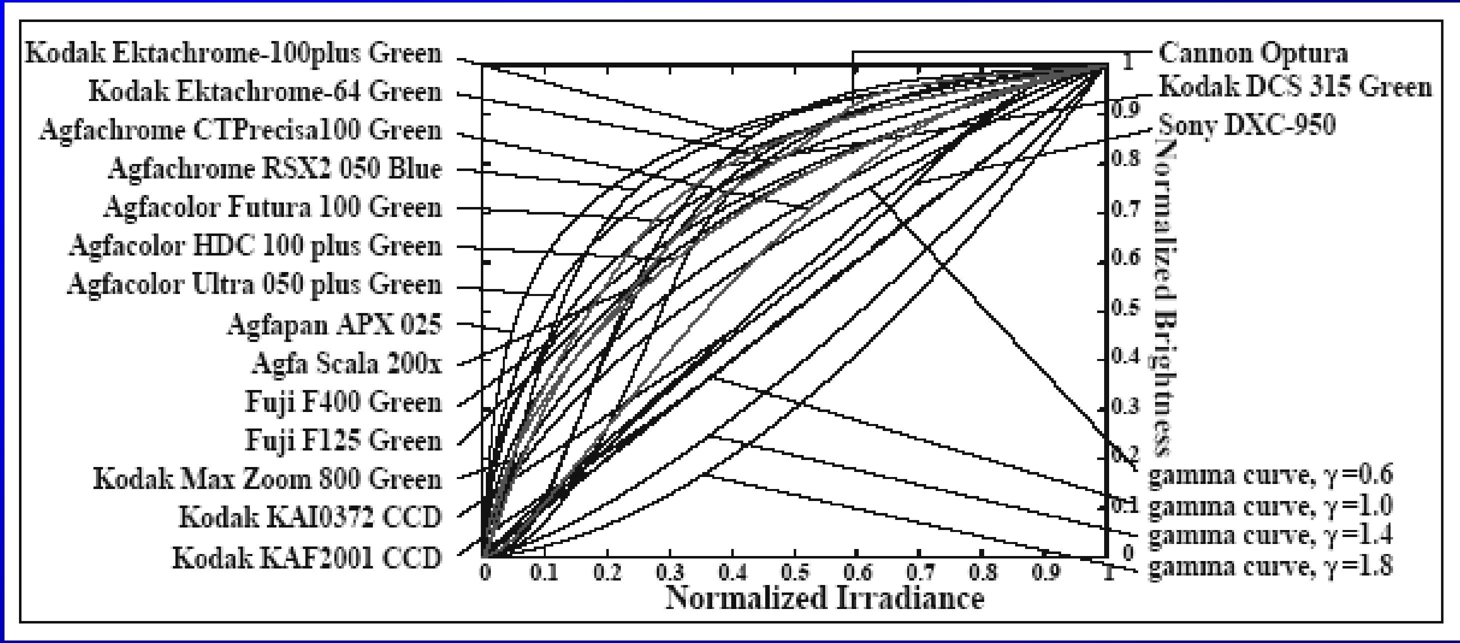
- The camera response function relates image irradiance at the image plane to the measured pixel intensity values.

$$g : E \rightarrow I$$



(Grossberg and Nayar)

Relation between Pixel Values I and Image Irradiance E



Real-world response functions (DoRF). The database includes photographic films, digital cameras, CCDs, and synthetic gamma curves. Note that even within a single brand of film, for example Agfa, there is considerable variation between response curves.

Radiometric Calibration

- Important preprocessing step for many vision and graphics algorithms such as photometric stereo, invariants, de-weathering, inverse rendering, image based rendering, etc.

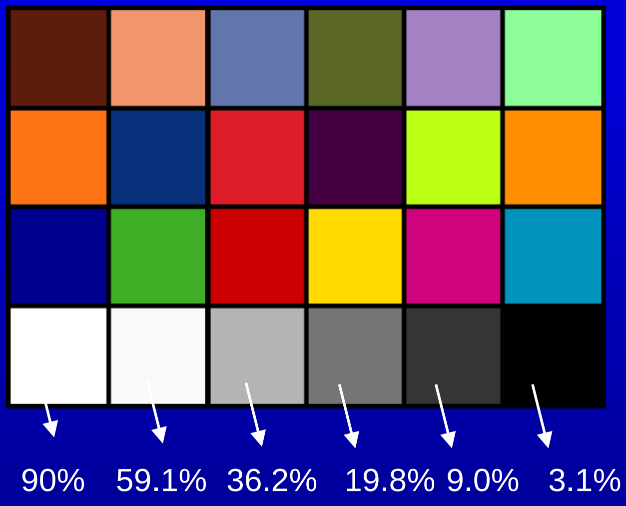
$$\boxed{g: E \rightarrow I} \Longrightarrow \boxed{g^{-1}: I \rightarrow E}$$

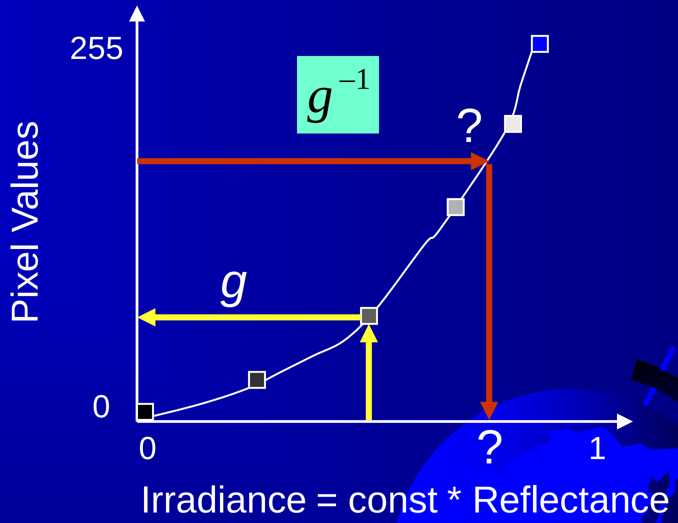
Radiometric Calibration

➢ Important preprocessing step for many vision and graphics algorithms such as photometric stereo, invariants, de-weathering, inverse rendering, image based rendering, etc.

$$g : E \rightarrow I \quad \Longrightarrow \quad g^{-1} : I \rightarrow E$$

•Use a color chart with precisely known reflectances.





- Use more camera exposures to fill up the curve.
- Method assumes constant lighting on all patches and works best when source is far away (example sunlight).

- Unique inverse exists because g is monotonic and smooth for all cameras.

Surface Appearance

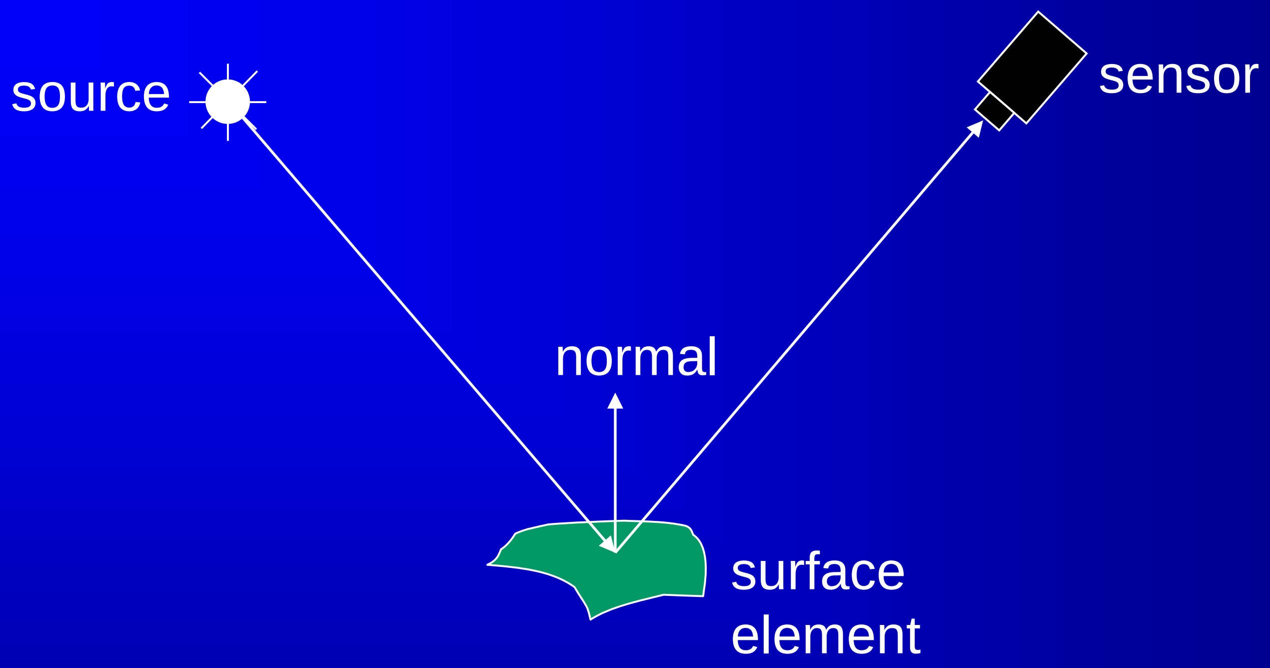
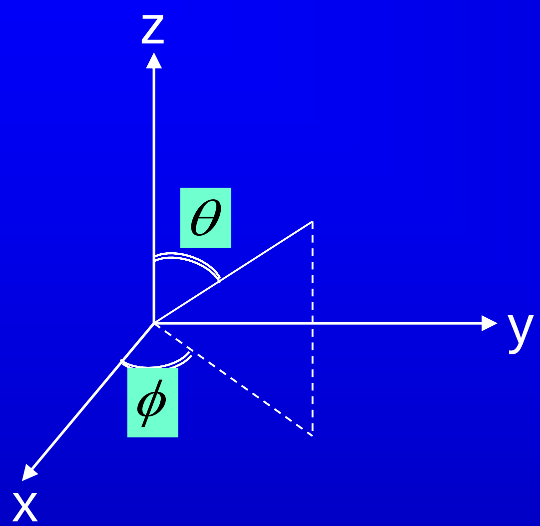
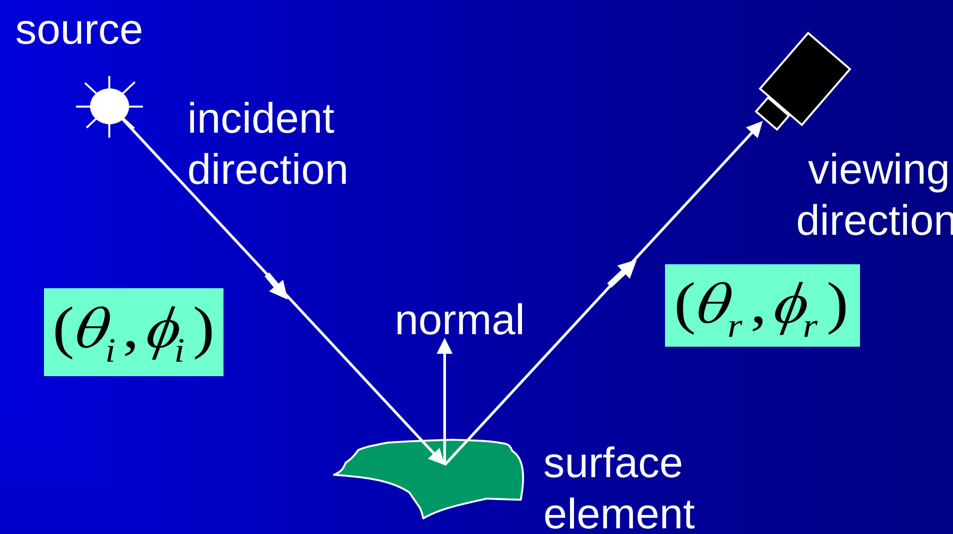


Image intensities = f (normal, surface reflectance, illumination)

Surface reflection depends on both the viewing and illumination directions.

BRDF: Bidirectional Reflectance Distribution Function





$E^{surface}(\theta_i, \phi_i)$ Irradiance at Surface in direction (θ_i, ϕ_i)

$L^{surface}(\theta_r, \phi_r)$ Radiance of Surface in direction (θ_r, ϕ_r)

$$\text{BRDF}: \; f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{L^{surface}(\theta_r, \phi_r)}{E^{surface}(\theta_i, \phi_i)}$$

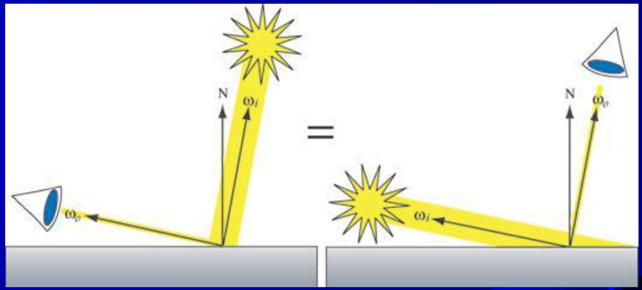
Important Properties of BRDFs

- Conservation of Energy:

$$\int_{hemisphere} f\left(\theta_i,\phi_i;\theta_r,\phi_r\right)d\omega_i \leq 1$$

- BRDF does not change when source and viewing directions are swapped.

$$f(\theta_i,\phi_i;\theta_r,\phi_r) \quad = \quad f(\theta_r,\phi_r;\theta_i,\phi_i)$$

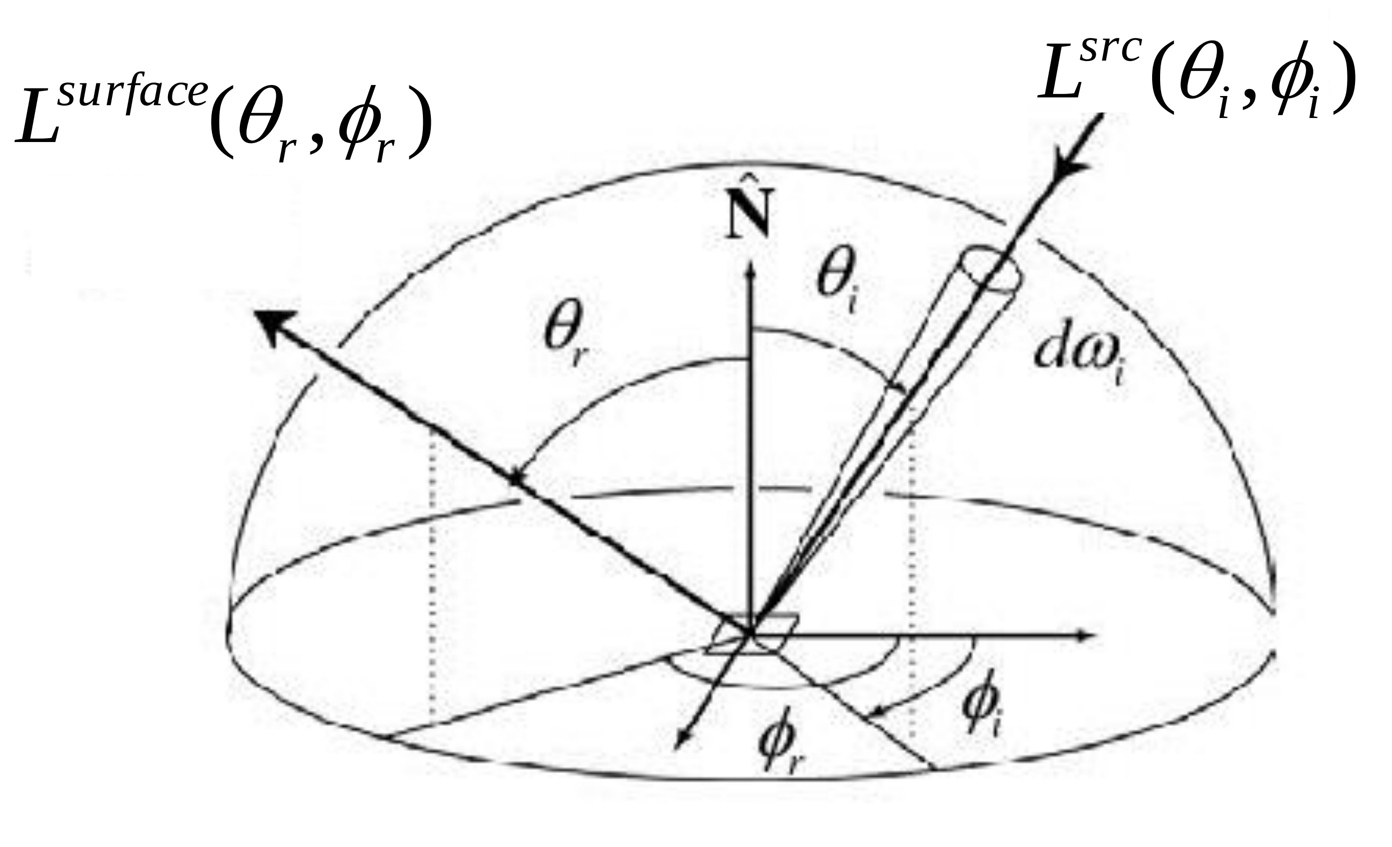


- Rotational Symmetry (Isotropy):

  BRDF does not change when surface is rotated about the normal.

  Can be written as a function of 3 variables : $f(\theta_i,\theta_r,\phi_i-\phi_r)$

Derivation of the Scene Radiance Equation



From the definition of BRDF:

$$L^{surface}(\theta_r,\phi_r) = E^{surface}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)$$

Derivation of the Scene Radiance Equation

From the definition of BRDF:

$$L^{surface}(\theta_r,\phi_r) \quad = \quad \underline{E^{surface}(\theta_i,\phi_i)}\, f(\theta_i,\phi_i;\theta_r,\phi_r)$$

Write Surface Irradiance in terms of Source Radiance:

$$L^{surface}(\theta_r,\phi_r) \quad = \quad \underline{L^{src}(\theta_i,\phi_i)}\, f(\theta_i,\phi_i;\theta_r,\phi_r)\,\underline{\cos\theta_i d\omega_i}$$

Integrate over entire hemisphere of possible source directions:

$$L^{surface}(\theta_r,\phi_r) \quad = \quad \int_{2\pi} L^{src}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)\cos\theta_i\;\underline{d\omega_i}$$

Convert from solid angle to theta-phi representation:

$$L^{surface}(\theta_r,\phi_r) \quad = \quad \int_{-\pi}^{\pi}\int_{0}^{\pi/2} L^{src}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)\cos\theta_i\;\underline{\sin\theta_i d\theta_i d\phi_i}$$

See You