Bifurcation energy model

We seek a pseudo-equilibrium model to describe the flow behavior of active nematics in bifurcation channels. The model considers 3 components: straight channel, continuity and diode channel. Each component contributes to the final flow configuration in the form of energy penalty. Their coupling (relative importance) is specified by a set of coupling coefficients.

The components

Straight channel: double well potential

$$H_s = \sum_{e \in E} V(\phi_e)$$

Either order 6 or order 4.

$$V_6(\phi_e) = -\frac{1}{4}\phi_e^4 + \frac{1}{6}\phi_e^6$$

$$V_4(\phi_e) = -\frac{1}{2}\phi_e^2 + \frac{1}{4}\phi_e^4$$

Continuity

N denotes the collection of all nodes and n represents single element in N. In the bifurcation network specifically, only one node exists.

$$H_c = \sum_{n \in N} (\mathbf{D} \cdot \mathbf{\Phi})_n^2$$

where $\mathbf{D}$ is the sign vector of each node and $\mathbf{\Phi}$ is the flow rates in all the channels connected to the node.

Diode channel: hard or soft

Hard diode:

$$H_+ = \begin{cases} +\infty, & \text{if } \phi_e \cdot \phi_d < 0 \\ 0, & \text{if } \phi_e \cdot \phi_d \geq 0 \end{cases}$$

where ϕ_d is the favored flow direction (the diode + direction).

Soft diode:

$$H_+ = (\phi_e - \phi_d)^2$$

where ϕ_d is the favored flow rate. If ϕ_e deviate from ϕ_d, energy penalty shoots up.

Coupling

We use coupling coefficients a, b and c to couple the effects from straight channel, continuity and diode channel:

$$H = aH_s + bH_c + cH_+$$

Model versions

We vary the coupling coefficients, straight channel models as well as the diode channel models. There are infinite possibilities, but we try to keep our trials clean.

- v0: $a = 10, b = 1000, c = 100$, order 6, hard diode
- v1: $a = 10, b = 1000, c = 100$, order 6, soft diode
- v2: $a = 10, b = 1000, c = 100$, order 4, soft diode

Known issues

- Ratchet length is not considered
- angle is not considered
- in ratchet-rich models (e.g. model_13), all the three flow rates are not 1