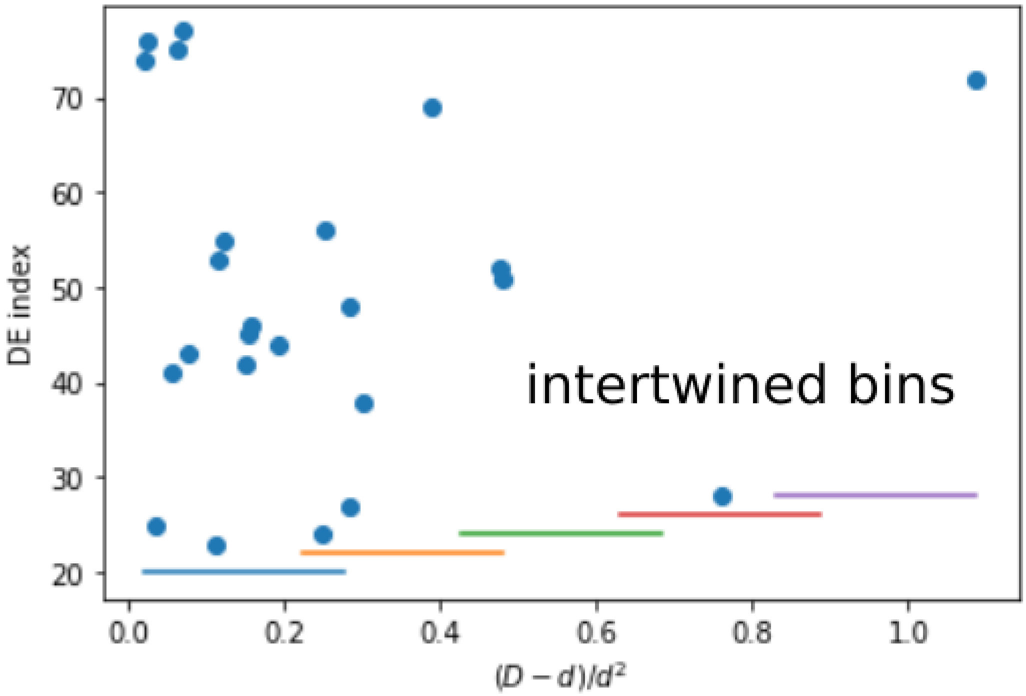
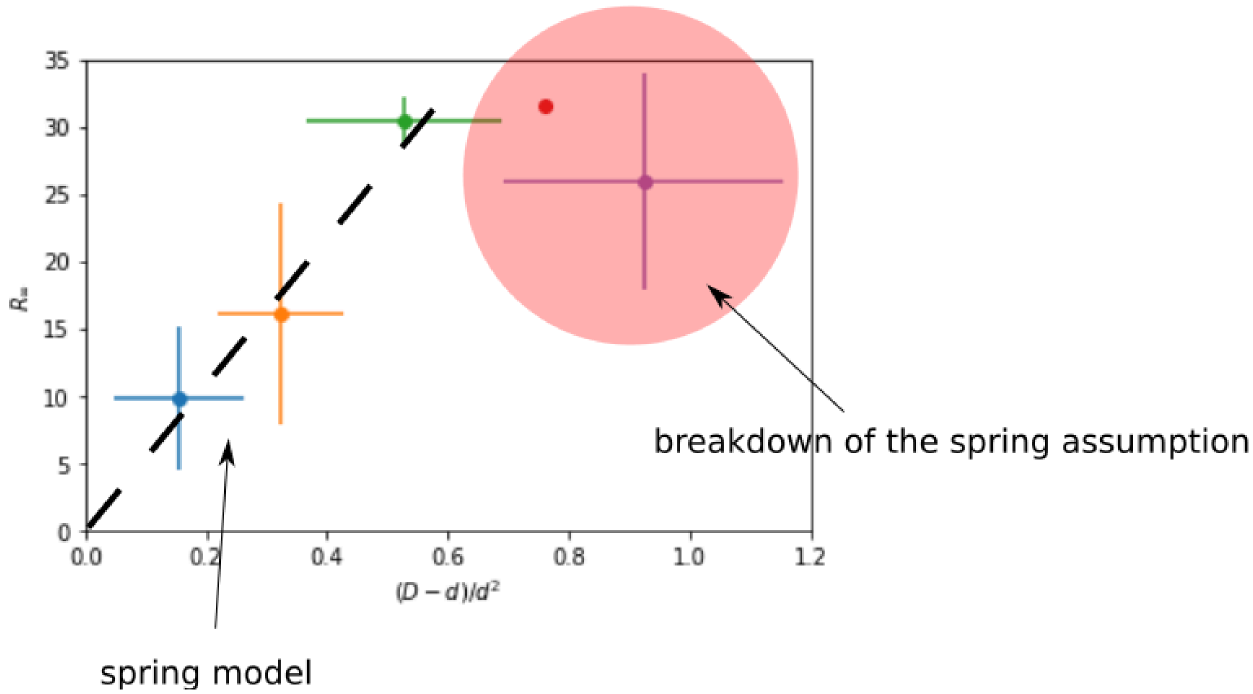
Model the confinement effect

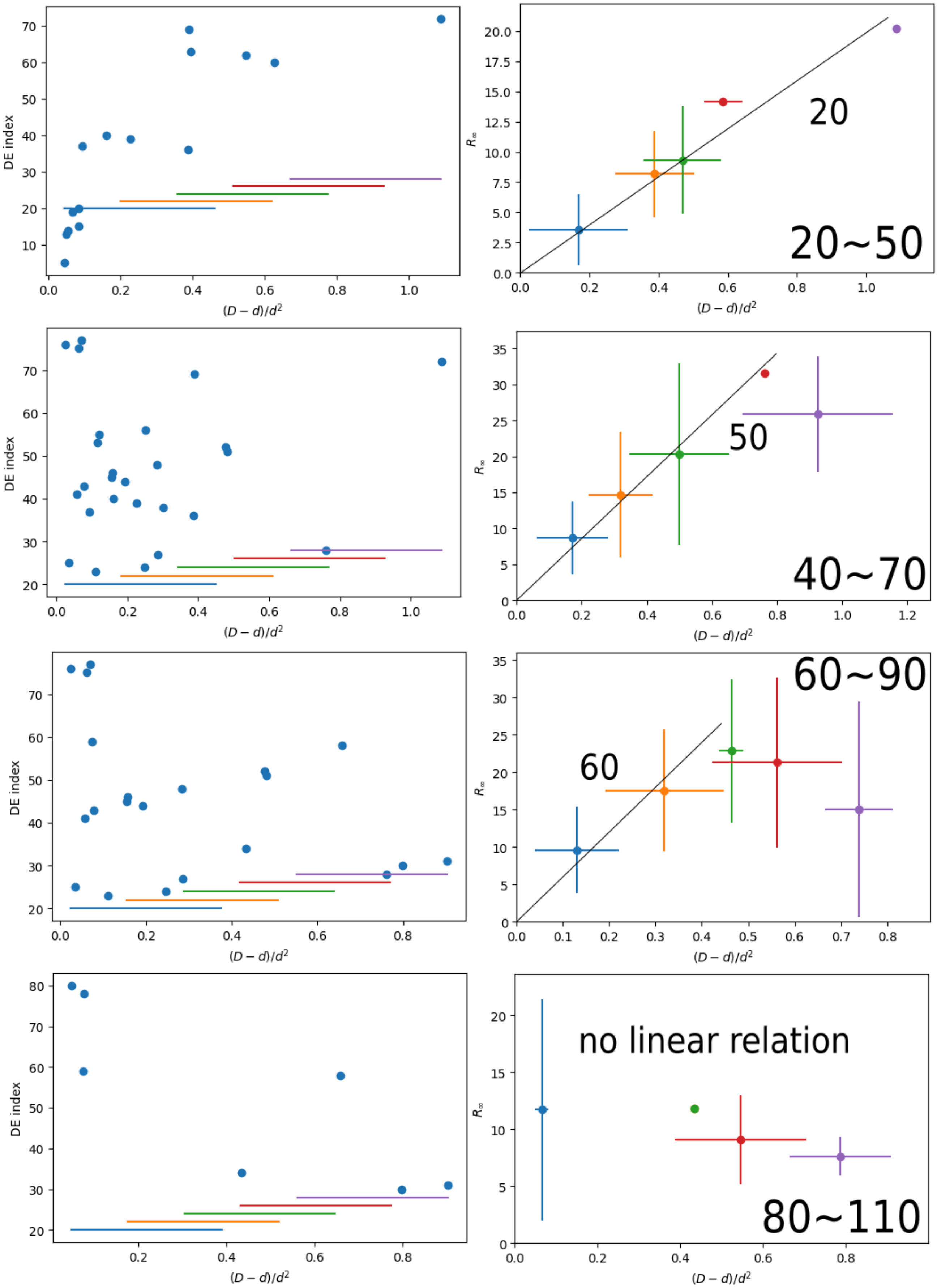
Intertwined bins for $(D-d)/d^2$ (OD=50~70)



Saturation length R_∞ vs. $(D-d)/d^2$



We can also look at data from other concentrations.



As expected, at low concentrations, the displacements of inner droplets are so small, so that the Langevin equation takes the following form

$$\dot{x} = \eta(t) + \gamma x,$$

where $\eta(t)$ is assumed to be an exponentially correlated noise, satisfying $\langle \eta(t)\eta(t') \rangle = Ae^{-\nu|t-t'|}$. It can be shown that this equation predicts the following MSD saturation value R_∞ and transition time τ^*:

$$R_\infty = \frac{A}{\gamma(\nu + \gamma)},$$

$$\tau^* = \frac{1}{\gamma} = \frac{\Gamma}{k},$$

where $\Gamma = 6\pi\eta r_i$ is the drag coefficient and $k = m^* g/(r_o - r_i)$ is the effective spring constant. Notice that m^* is the buoyant mass of the oil droplet in water, and can be computed as $m^* = \rho^* \frac{4}{3}\pi r_i^3$. Taken together, τ^* can be expressed as

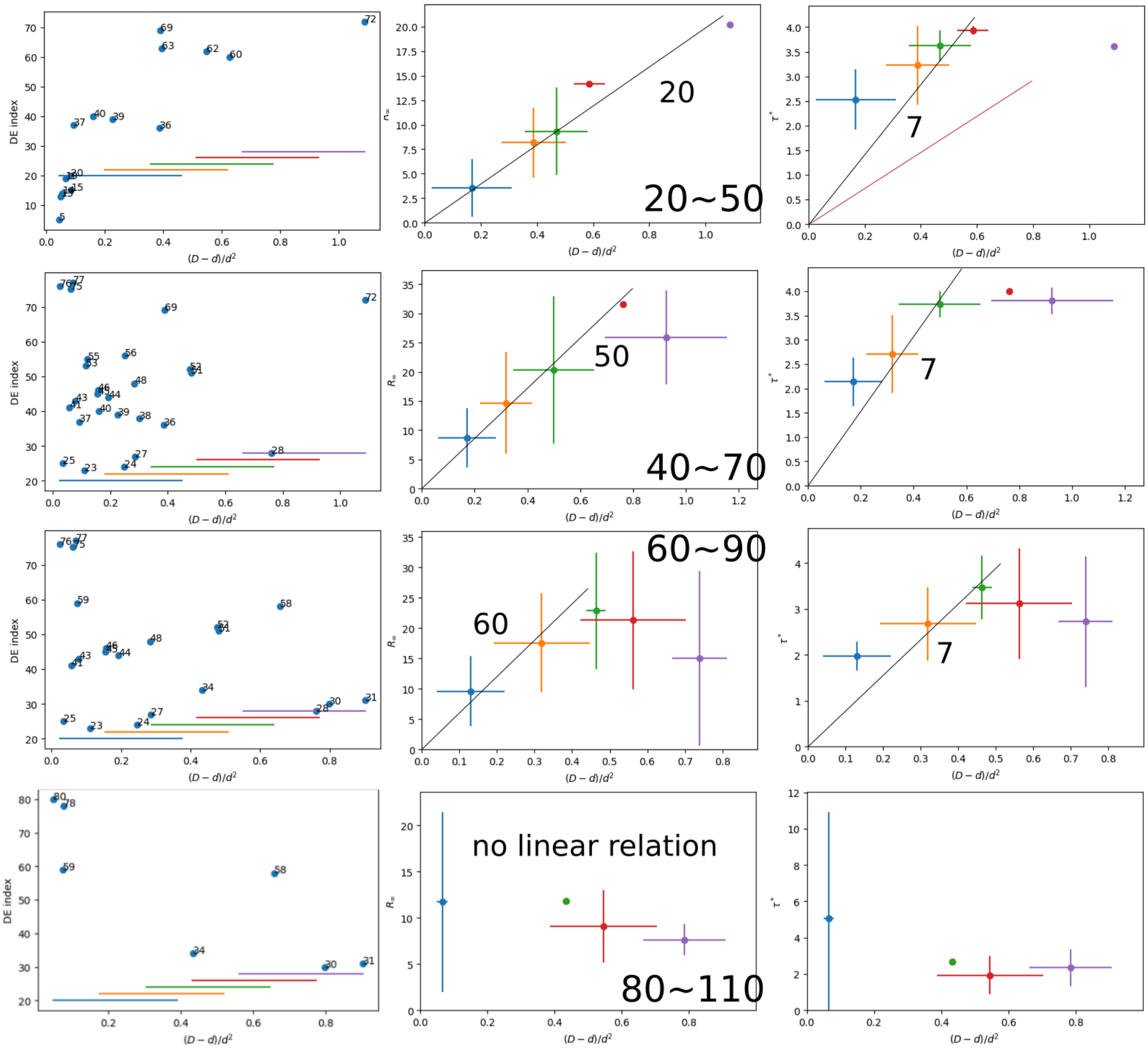
$$\tau^* = \frac{9\eta}{2\rho^* g} \frac{r_o - r_i}{r_i^2}.$$

To be consistent with my diameter representation,

$$\tau^* = \frac{9\eta}{\rho^* g} \frac{D - d}{d^2}.$$

Use water viscosity $\eta = 0.001$ Pa s, we can compute the theoretical coefficient $K = 9\eta/\rho^* g = 3.9$ μm s. τ^* is plotted against $(D - d)/d^2$ as below:

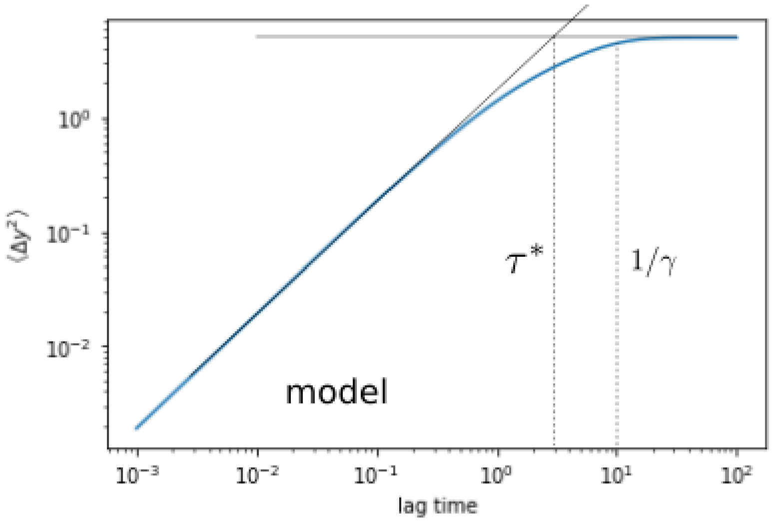
R_∞ and τ^* scaling



The nice linear relation observed at low OD's in the R_∞ vs. $(D-d)/d^2$ plots is not observed in the τ^* plots. If we force a linear fitting to the data points, we obtain a constant slope ≈ 7 μm s for OD up to 90. Although the linear relation is not very pronounced, the value of the prefactor ≈ 7 μm s is quite close to the theoretical prediction 3.9 μm s.

Difference between τ^* and $1/\gamma$

Note that although we stated $\tau^* = 1/\gamma$, it is actually not exact. The figure below plots a typical MSD predicted by the Langevin model, with the definitions of both τ^* and $1/\gamma$ illustrated.



If we use the $1/\gamma$ definition to measure the time scale τ^*, the τ^* curves will shift up and deviate more from the theoretical prediction.

Discuss the theoretical saturation value R_∞

In the previous section, I show that the time scale τ^* (or $1/\gamma$) scales linearly with $(D-d)/d^2$. The Langevin model predicts that

$$R_\infty = \frac{A}{\gamma(\nu+\gamma)}$$

where $\nu = 1/\tau > 1$ is roughly a constant according to current data. Since $\gamma \ll 1$ holds true in most scenarios, we can assume $\nu \gg \gamma$. The saturation value R_∞ can be approximated as $A\tau\tau^*$. It is also assumed that A, the activity of the active bath, is only a function of the bacterial concentration OD, and does not depend on the confinement. Therefore, at a fixed OD, we expect

$$R_\infty \approx A\tau\tau^* \propto \tau^* \propto \frac{D-d}{d^2}$$

Look at the R_∞ vs. $(D-d)/d^2$ plots, this is indeed a good prediction for the low OD and large inner size regime. More interestingly, although we derive how R_∞ depends on the confinement through τ^*, R_∞ actually shows better linear relations than τ^*. This has brought new questions:

Why does R_∞ show better linear relation with $(D-d)/d^2$ than τ^*? Is it because our τ^* is not exactly the $1/\gamma$ in the model? Or $A\tau$ is not a constant, but also depends on the confinement? Or more simply, the model is wrong?

How to understand the limit where the spring assumption breaks down?