Data analysis

#msd #vacf #tesis #physics

With the current data we want to compare with the Numerical Model, to do that we will compare several quantities that we can compute from the experimental data.

We want to be sure that the quantities that we are computing are well computed after the data processing. For the double emulsion experiment, the main source of error came from the detection process. Filtering the data is an important step to get the physics behind the motion, but we have to be careful with this processing, and try to avoid to delete information from the data.

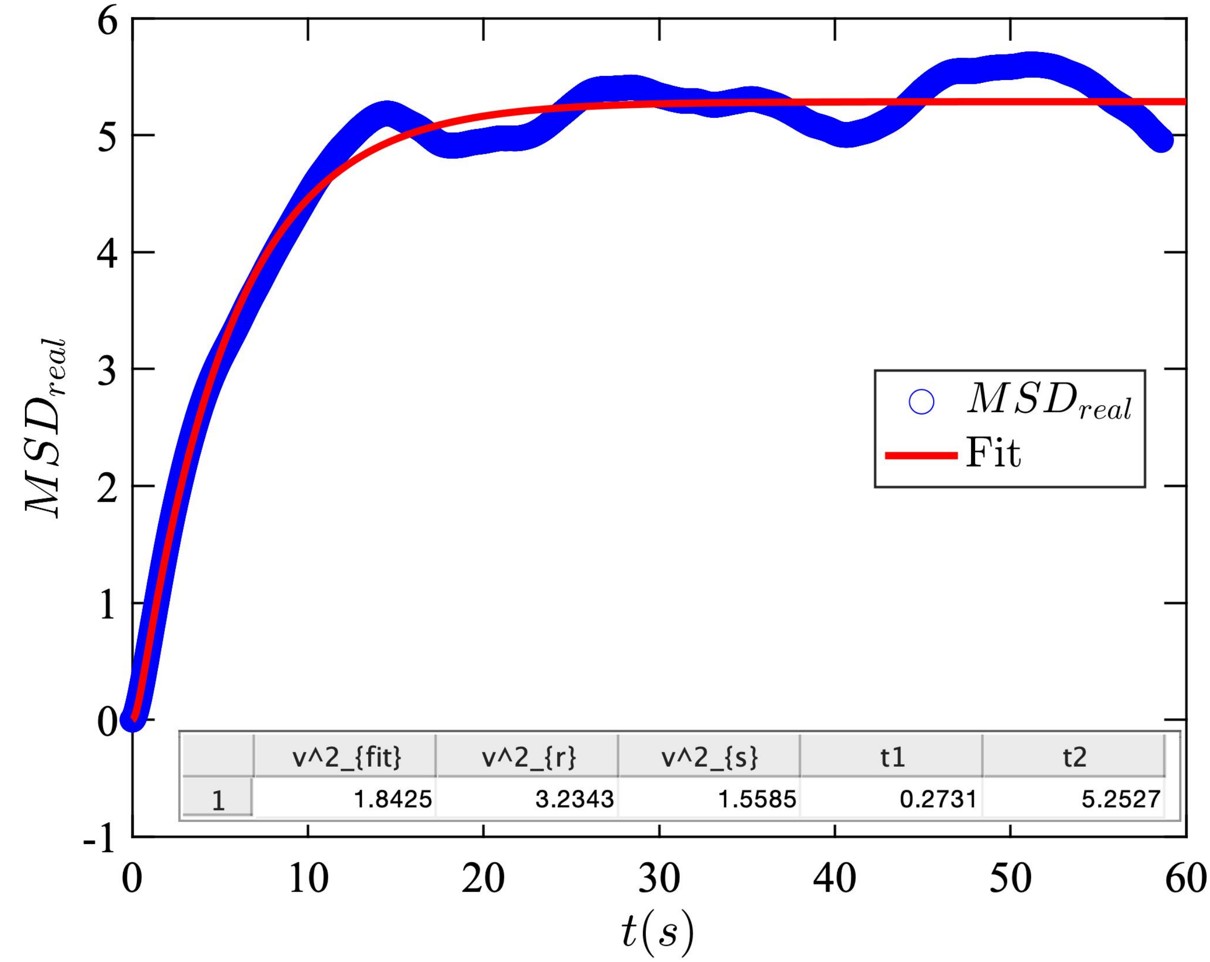
We decide to use the Mean square displacement instead of the Velocity autocorrelation function because it's less sensitive to noise, and we will use the data after doing the process of Detection Error.

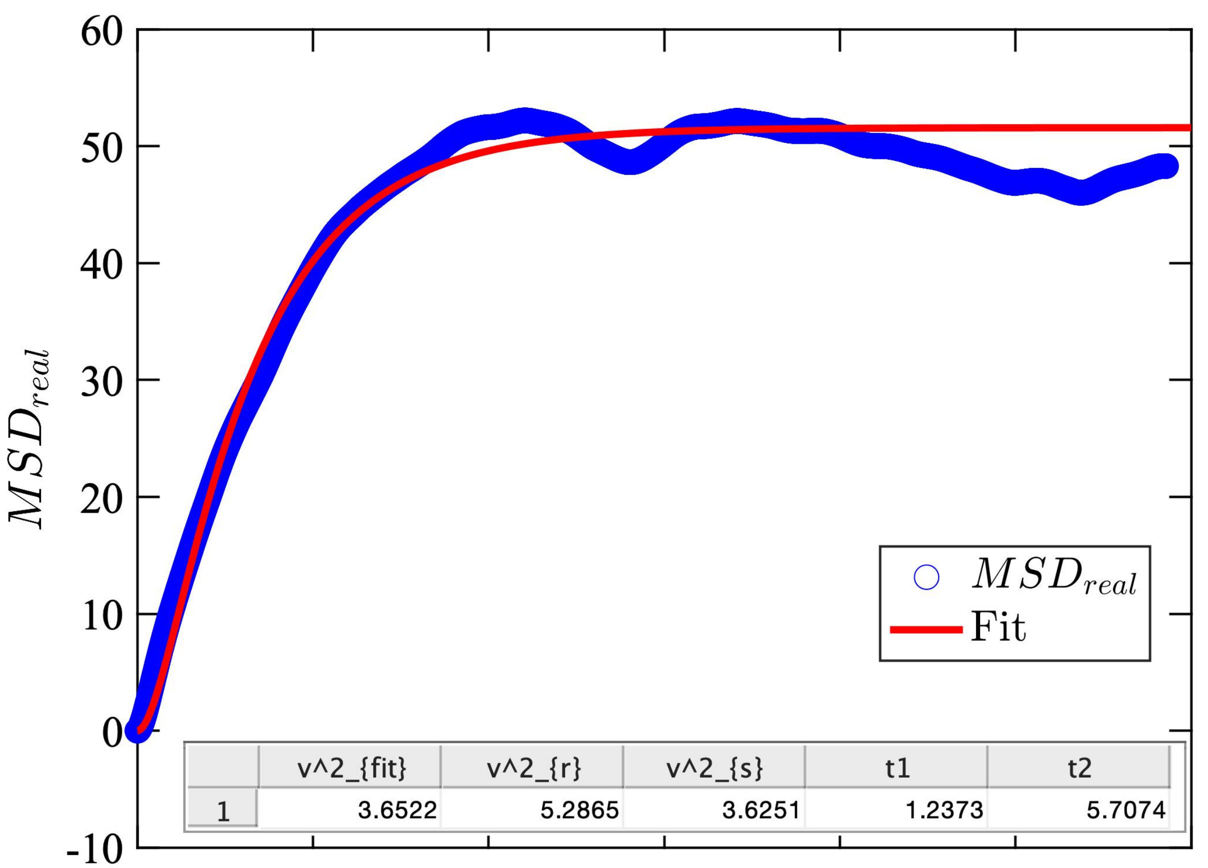
Fitting MSD

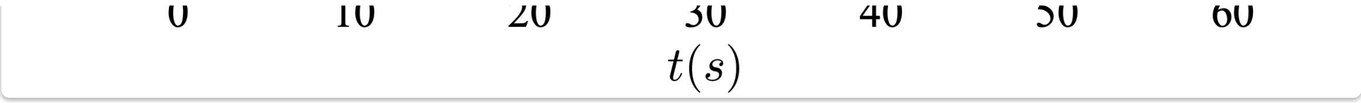
After to make the Detection Error process in the experimental results. I will fit the MSD_{real} with the function:

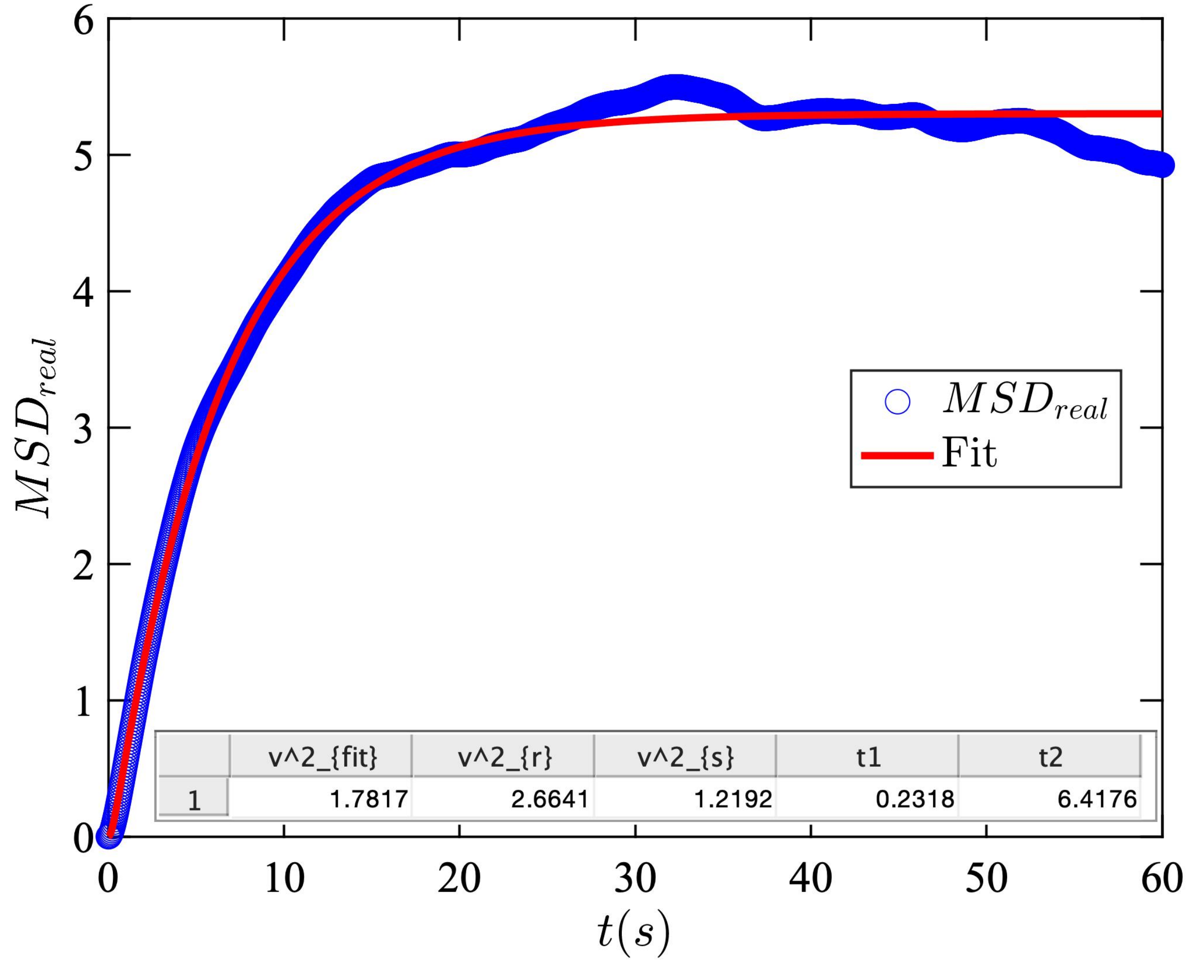
$$\langle \Delta x^2(t) \rangle = 2\langle v^2 \rangle \frac{t_2 t_1^2 e^{-t/t_1} - t_1 t_2^2 e^{-t/t_2} + t_1 t_2^2 - t_1^2 t_2}{t_2 - t_1}$$

We will assume that $t_1 < t_2$, and from the fitting of the MSD_{real} we can have an estimate of the value of the short time velocity, so we can use this value as starting point for the fit. In the images below, it is shown different experimental MSD and the respective fitted curve; Also is displayed the different values of the square velocity at short times, where v_{fit}^2 is from the fitting, v_r^2 is from the raw data, and v_s^2 is a smoothed data velocity.

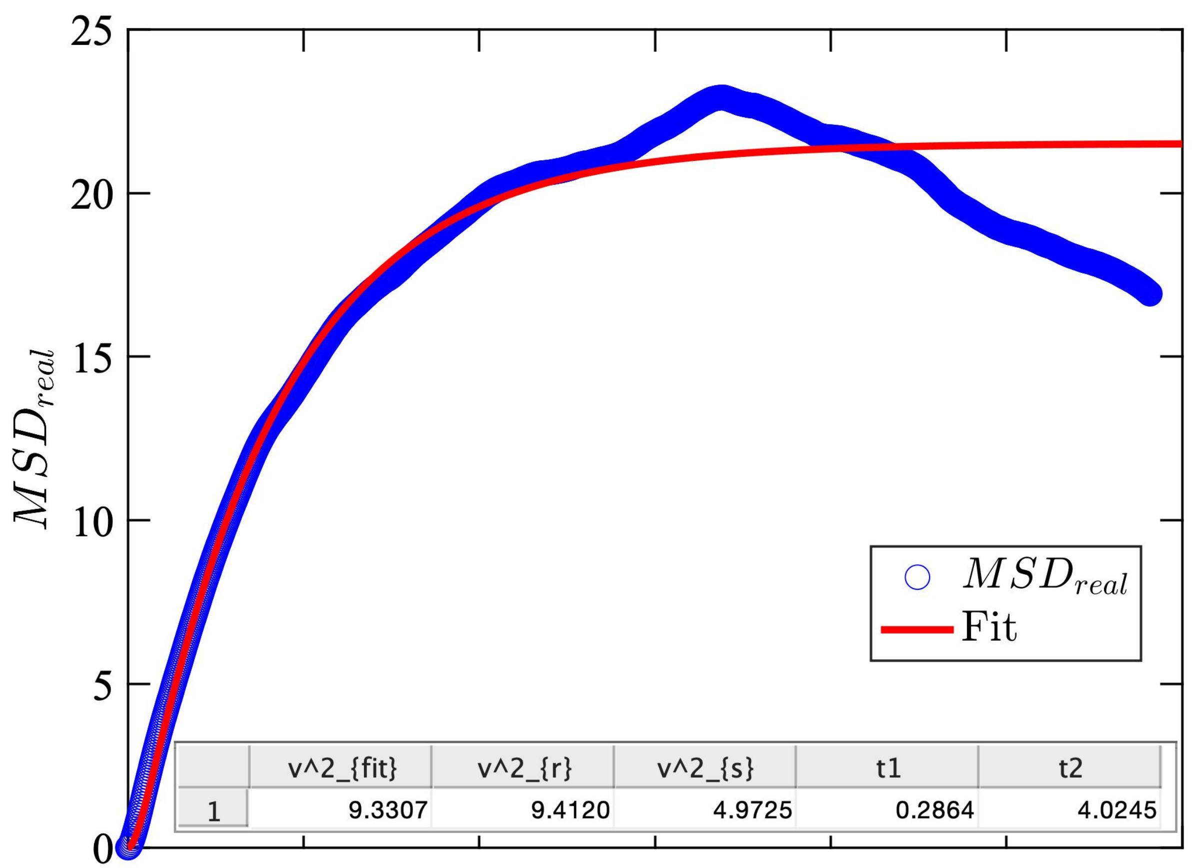

MSD_{real}
Fit
v^2_{fit} v^2_{r} v^2_{s} t1 t2
1 1.8425 3.2343 1.5585 0.2731 5.2527
MSD_{real}
$t(s)$


MSD_{real}
Fit
v^2_{fit} v^2_{r} v^2_{s} t1 t2
1 3.6522 5.2865 3.6251 1.2373 5.7074
MSD_{real}

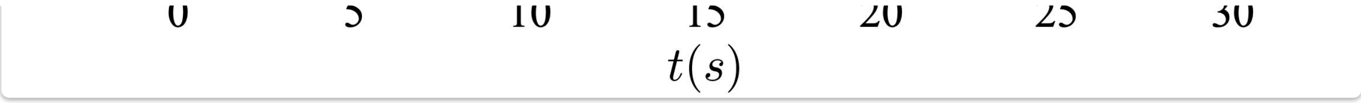

0 10 20 30 40 50 60
$t(s)$



MSD_{real}
Fit
MSD_{real}
$t(s)$
0
1
2
3
4
5
6
0
10
20
30
40
50
60
v^2_{fit} v^2_{r} v^2_{s} t1 t2
1 1.7817 2.6641 1.2192 0.2318 6.4176



MSD_{real}
Fit
MSD_{real}
0
5
10
15
20
25
v^2_{fit} v^2_{r} v^2_{s} t1 t2
1 9.3307 9.4120 4.9725 0.2864 4.0245



The fitting works well, but fitting the whole curve is not the best way to determine the constants. To improve the detection of the times, instead of fitting the whole curve, we will fit the MSD by part.
I'm trying to figure it out the best way to determine until when fit the MSD curve at short times, and then use that value of time as start point for t_1, to determine the amplitude of v_{bath}^2
.

To do that, I will compute the slope of the MSD over time. The slope will start in 2 and start to decrease until zero, when the MSD reach the plateau. I will determine the time to do the fit of the MSD at short times as the time when the slope reach the value $\Delta = 1.7$.

This method works very well with simulation where the MSD has a lot of point at short times, but for experiments we have to take care of the initial part since the we are computing a derivative of the MSD, can be a little noise if is made over the MSD_{real}.

In summary, we are using the following fact:

$$\lim_{t\to 0}\langle \Delta x^2(t)\rangle = \langle v^2\rangle t^2$$

Long time limits

$$\lim_{t\to\infty}\langle \Delta x^2(t)\rangle = 2\langle v^2\rangle t_1 t_2$$

We will use the expression for the MSD to compute the correlation times.

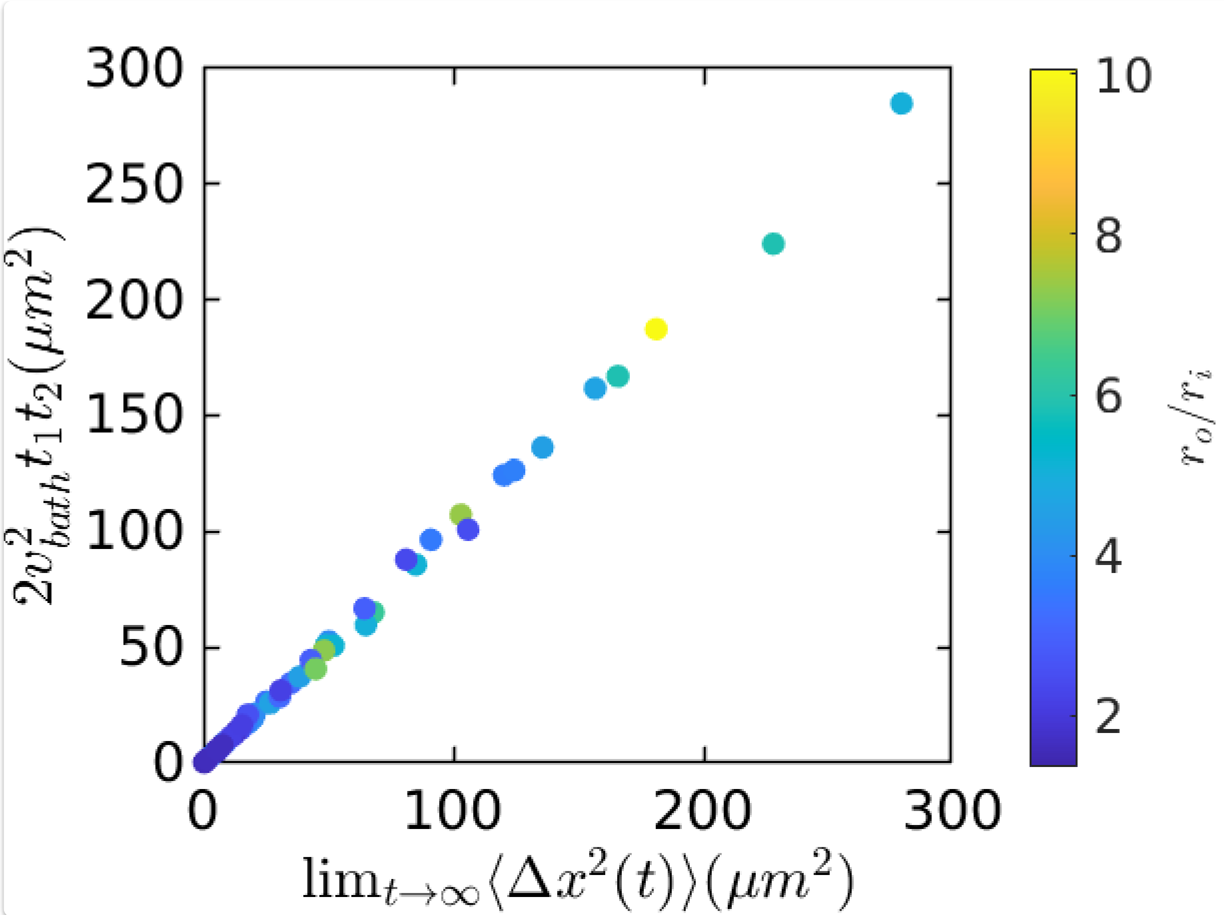
After the fit at short time, we will have t_1(assuming that t_1 is equal to the end of the ballistic part), and v_{bath}^2. Fixing these parameters we can fit the whole MSD only with t_2 as a fitting parameter using:

$$\langle \Delta x^2(t)\rangle = 2\langle v^2\rangle \frac{t_2 t_1^2 e^{-t/t_1} - t_1 t_2^2 e^{-t/t_2} + t_1 t_2^2 - t_1^2 t_2}{t_2 - t_1}$$

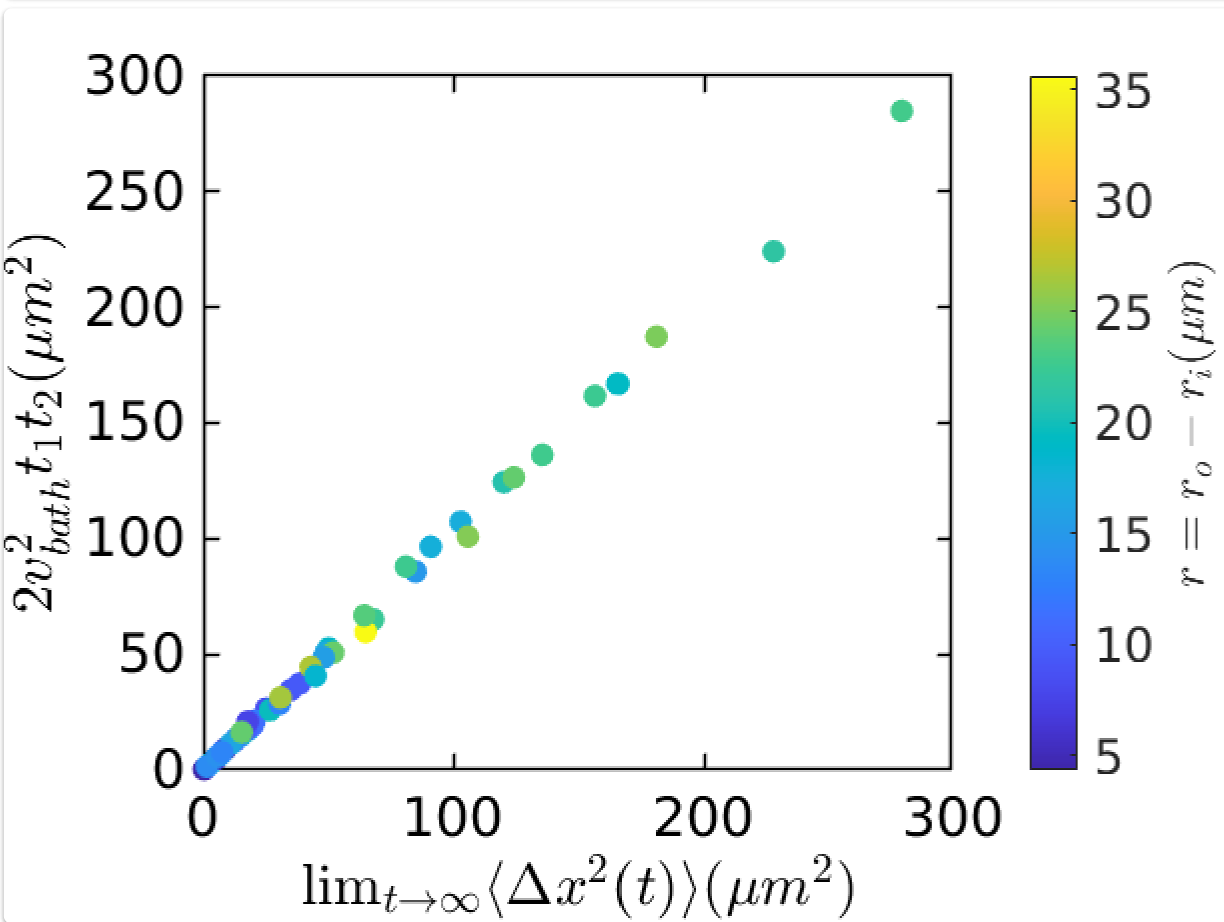
Using the long time limit, we can see if the value for t_2 obtained from the fitting is in agreement with the prediction. Computing the saturation value from the information extracted from the fitting, we will have:

$$lim_{t\to\infty}\langle \Delta x^2(t)\rangle = 2\langle v^2\rangle t_1 t_2$$

The right-hand part of the equation is obtained from the fitting, and the left part can be determined from the MSD curve.
Comparing these two values, we can determine if t_2 from the fit is representative.

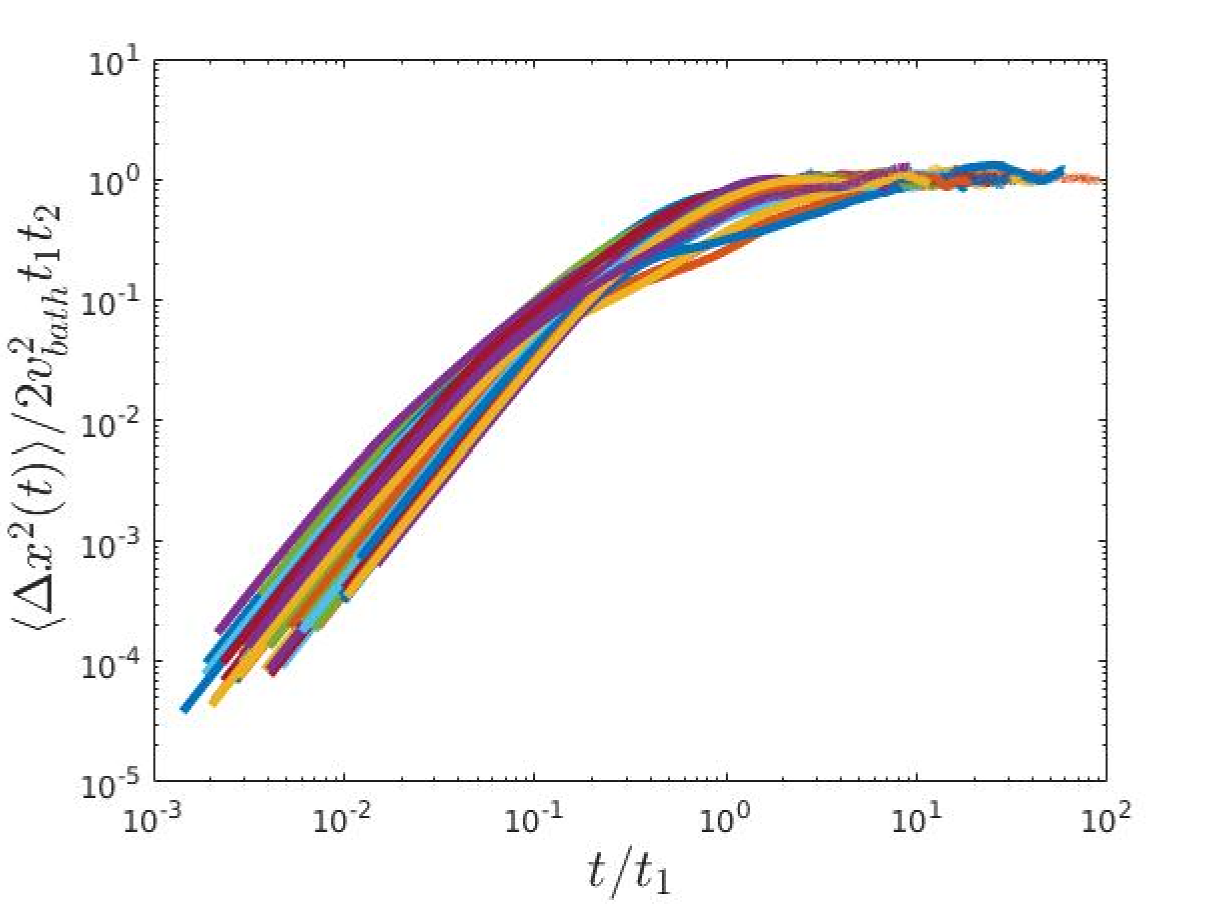


300
250
200
150
100
50
0
0
100
200
300
$2v_{bath}^2 t_1 t_2 (\mu m^2)$
$\lim_{t\to\infty}\langle \Delta x^2(t)\rangle (\mu m^2)$
10
8
6
4
2
r_o/r_i

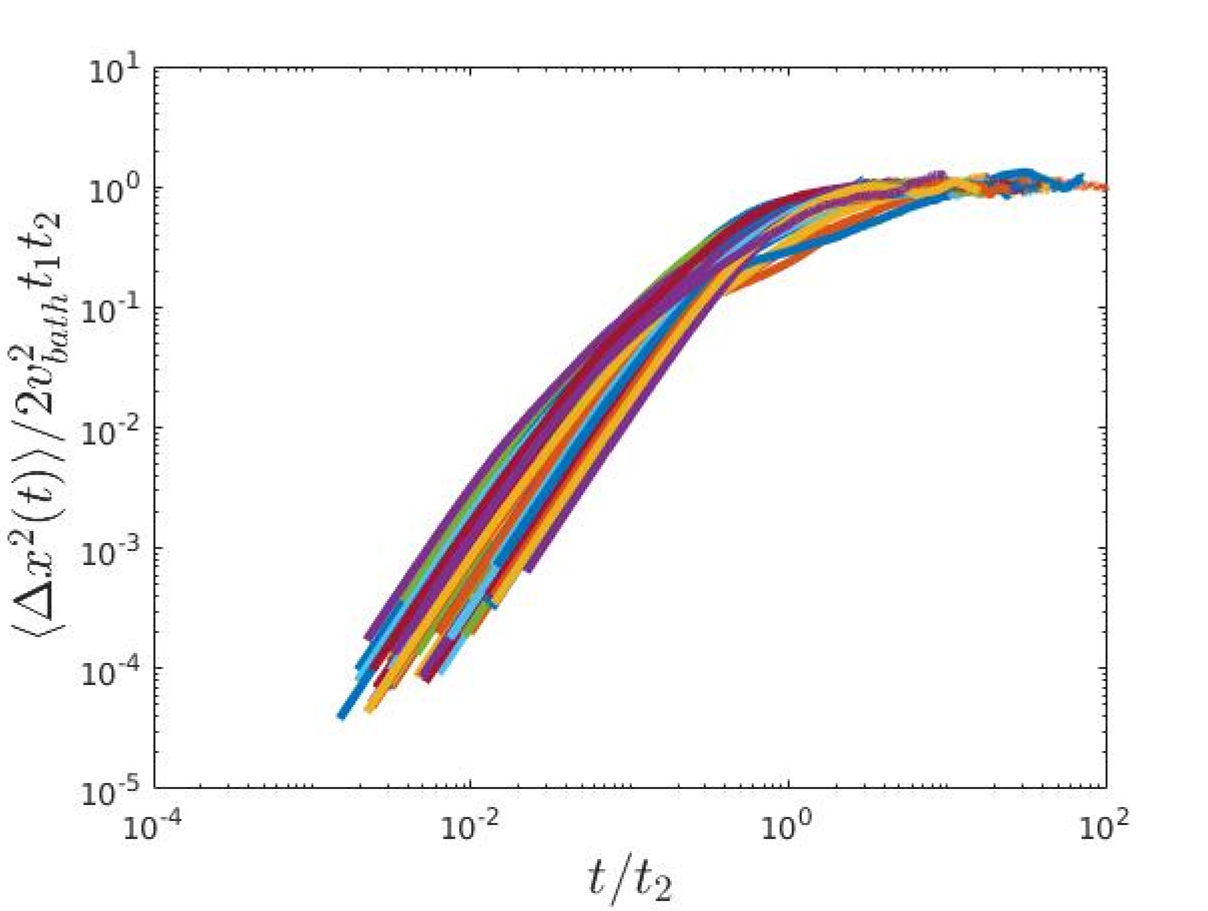


300
250
200
150
100
50
0
0
100
200
300
$2v_{bath}^2 t_1 t_2 (\mu m^2)$
$\lim_{t\to\infty}\langle \Delta x^2(t)\rangle (\mu m^2)$
35
30
25
20
15
10
5
$r = r_o - r_i (\mu m)$

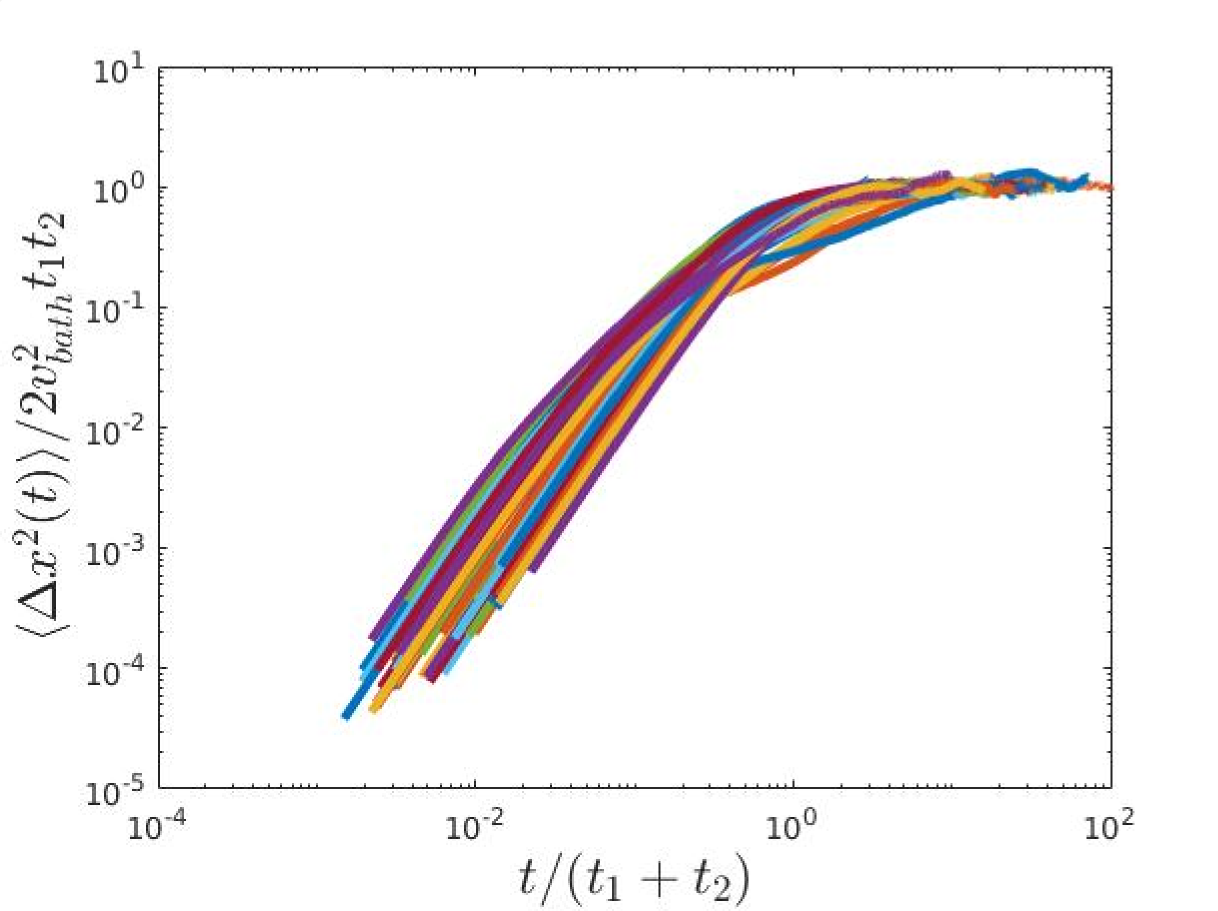
We can try to find a scaling for the MSD with the parameters, since the combination of $t_1, t_2, v_{\mathrm{bath}}$ are some kind of saturation value-persistence distance, we will have:



$\langle \Delta x^2(t) \rangle / 2v_{bath}^2 t_1 t_2$
t/t_1



$\langle \Delta x^2(t) \rangle / 2v_{bath}^2 t_1 t_2$
t/t_2



Let $t_2 = \Gamma/k$, where $\Gamma = 6\pi\eta r_i$, and $k = mg/r = \frac{4}{3}\pi r_i^3 \rho g$, we will have:

$$t_2 = \frac{6\pi\eta r_i}{\frac{4}{3}\pi r_i^3 \rho g}$$

$$t_2 = \frac{9\eta r}{2\rho g r_i^2}$$

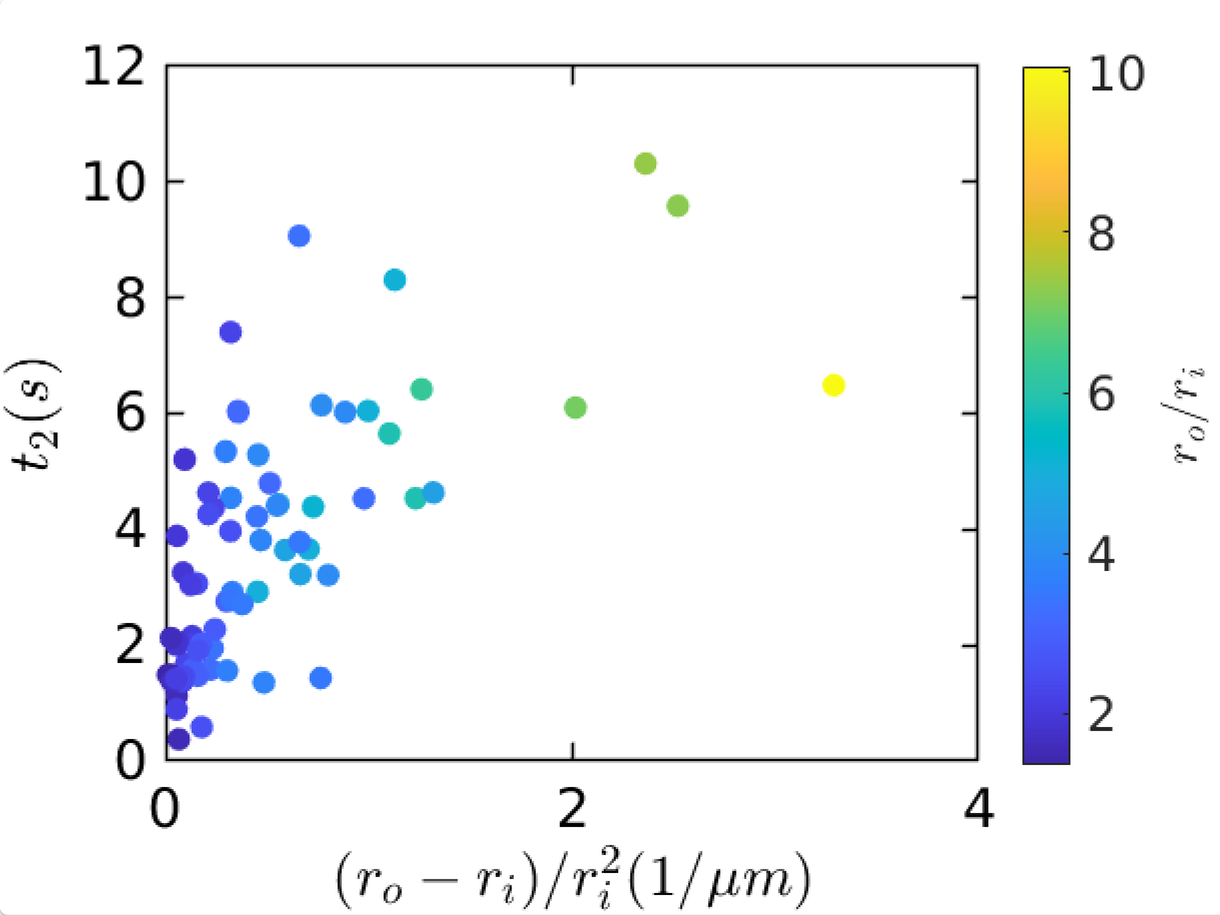
where $r = r_o - r_i$, so taking the limits we'll have:

$$\lim_{r_i \to 0} t_2 \propto \infty$$

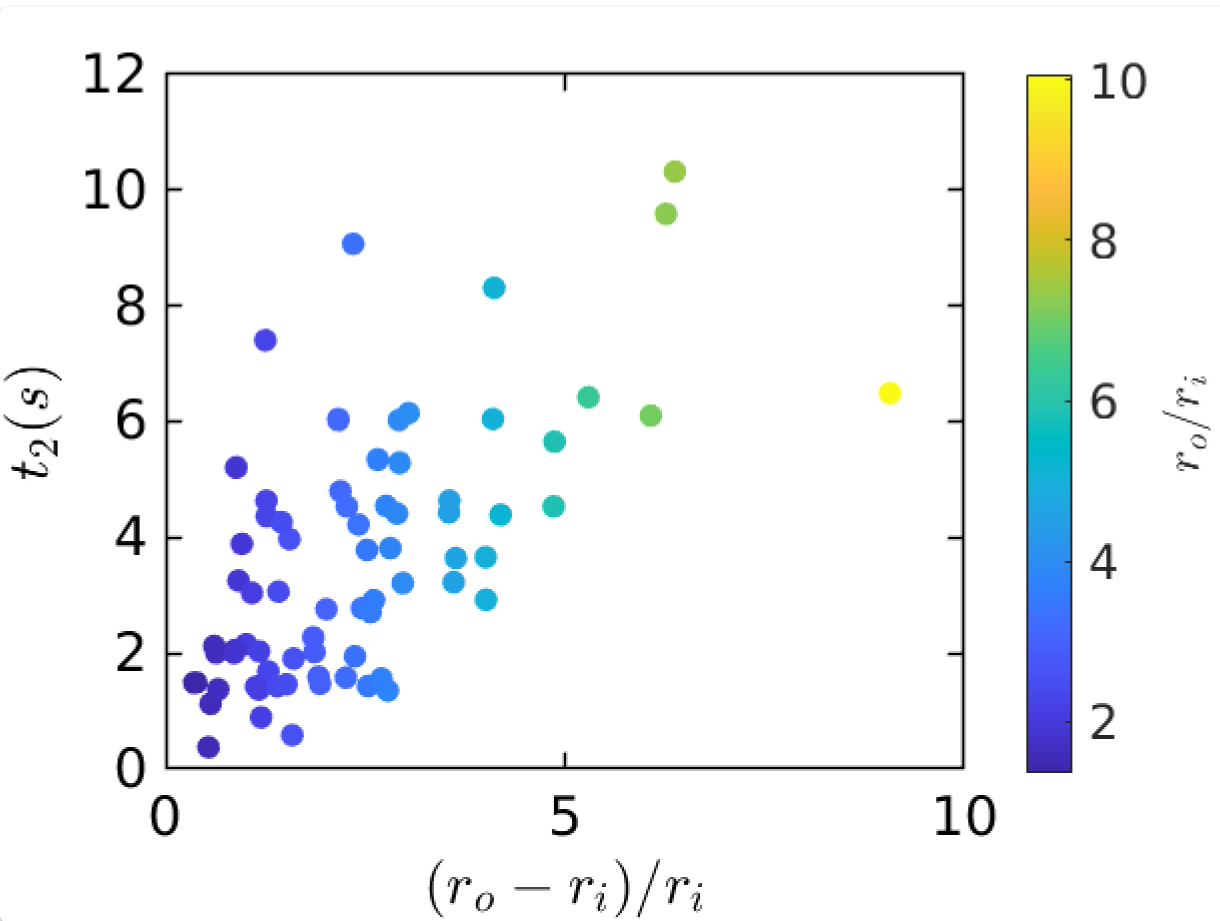
$$\lim_{r_i \to r_o} t_2 \propto 0$$

$$\lim_{r_o \gg r_i} t_2 \propto r_o$$

Which make sense, since r_i small, will take infinite time to reach the saturation or feel the confinement. If the inner droplet is too close to the size of the outer, will reach the plateau faster, so instantaneously saturate. Plotting t_2 we found:



$t_2(s)$
$(r_o - r_i)/r_i^2(1/\mu m)$
r_o/r_i



$t_2(s)$
$(r_o - r_i)/r_i$
r_o/r_i