Stochastic model discussion summary

(Feb 25, 2022)

Formulation and solution

We will continue the discussion with the equations and solutions given by Maggi 2014:

$$\dot{y} = -\mu k y + \eta^T + \eta^A,$$

where η^A is the active noise, satisfying $\left\langle \eta^A(t)\eta^A(t') \right\rangle = (D_A/\tau)e^{-|t-t'|/\tau}$. The active part of the solution is

$$\left\langle \Delta y^2(t) \right\rangle = \frac{2D_A}{\mu k}\frac{1 - e^{-\mu k t} - \mu k \tau (1 - e^{-t/\tau})}{1 - (\mu k \tau)^2}.$$

We let $\mu k = 1/\tau^*$, and it can be shown that the τ^* here is the saturation time scale of inner droplet motion.

Limit cases

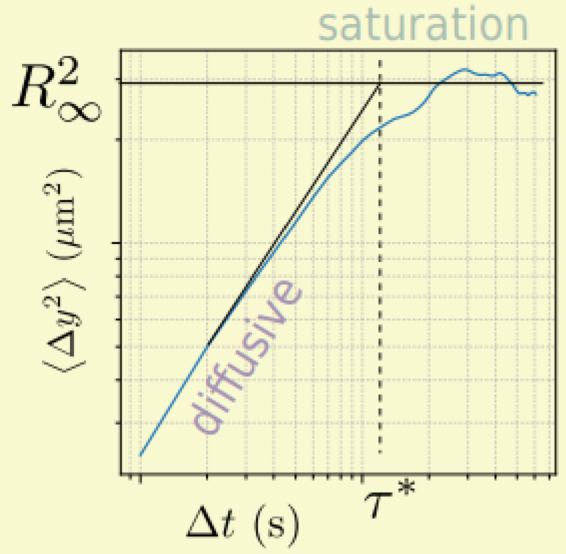
$$t \to 0: \left\langle \Delta y^2(t) \right\rangle = \frac{D_A}{\tau(1+\tau/\tau^*)}t^2$$

$$\tau^* \gg t \gg \tau: \left\langle \Delta y^2(t) \right\rangle = \frac{2D_A}{1-(\tau/\tau^*)^2}(t-\tau)$$

$$t \to \infty: \left\langle \Delta y^2(t) \right\rangle = \frac{2D_A \tau^*}{1+\tau/\tau^*}$$

Evaluate R_∞ and τ^*

A typical MSD curve can be described by the saturation value R_∞^2 and the transition time τ^*. The ballistic regime, where $\left\langle \Delta y^2(t) \right\rangle \propto t^2$, is usually within 1 second and is difficult to measure accurately in experiment.



The transition time τ^* comes out directly from the model as

$$\tau^* = \frac{1}{\mu k},$$

where $\mu = (6\pi\eta r_i)^{-1}$ is the particle mobility and $k = m^* g/(r_o - r_i)$ is the effective spring constant. Notice that m^* is the buoyant mass of the oil droplet in water, and can be computed as $m^* = \Delta\rho \frac{4}{3}\pi r_i^3$. Taken together, τ^* can be expressed as

$$\tau^* = \frac{9\eta}{2\Delta\rho g}\frac{r_o - r_i}{r_i^2} = \frac{9\eta}{\Delta\rho g}\frac{D-d}{d^2}$$

Use water viscosity $\eta = 0.001$ Pa s, we can compute the theoretical coefficient $K = 9\eta/\rho^* g = 3.9$ μm s.

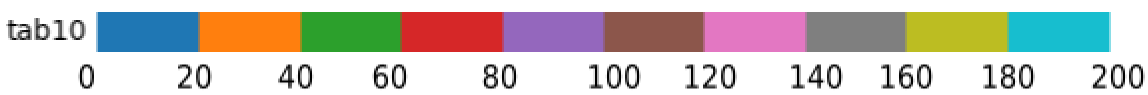
In the limit $\tau^* \gg \tau$, the saturation value $R_\infty^2 \approx 2D_A\tau^* \propto \tau^* \propto (D-d)/d^2$,

$$R_\infty \propto \frac{\sqrt{D-d}}{d}.$$

When $\tau^* \sim \tau$, the experimental determination of τ^* becomes problematic.

Experimental data

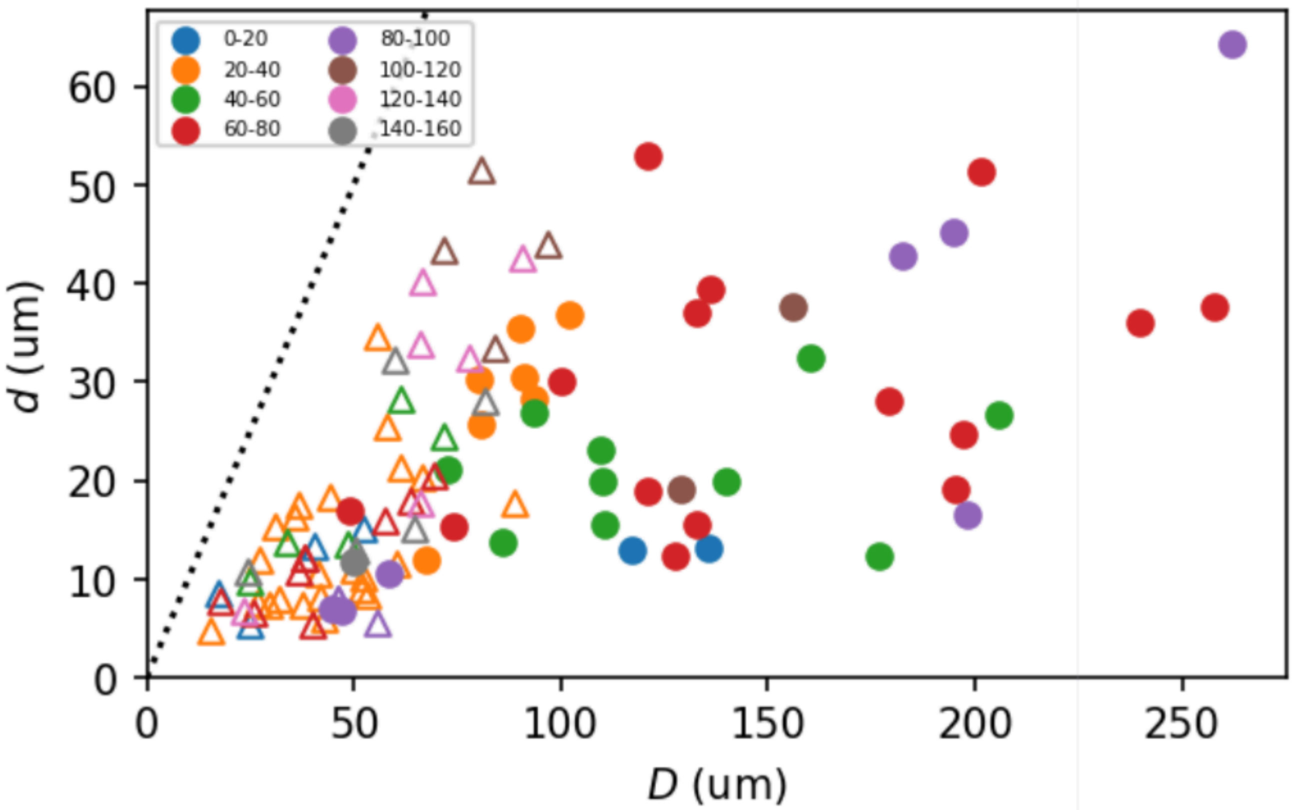
The current data set presents 3 parameters: D, d, n and two observables: R_∞ and τ^*. D and d are always combined as $(D-d)/d^2$ as x-axis. Bacterial concentration n is encoded using `tab10` colormap, with the following mapping.



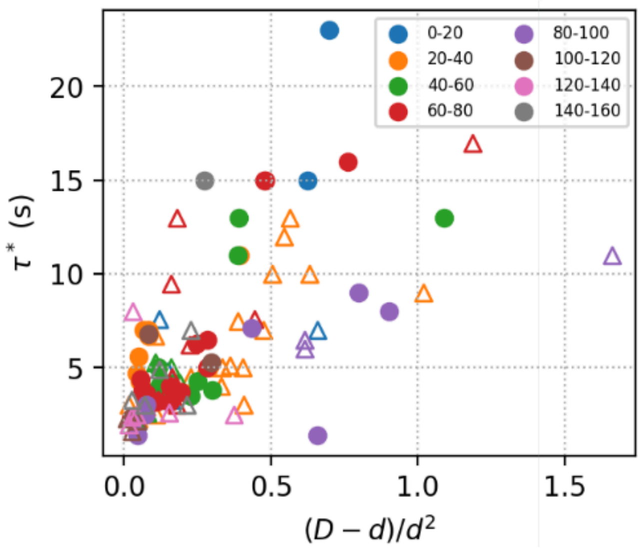
The data here combines Chile and Paris data. To discern them, I use solid circle to plot

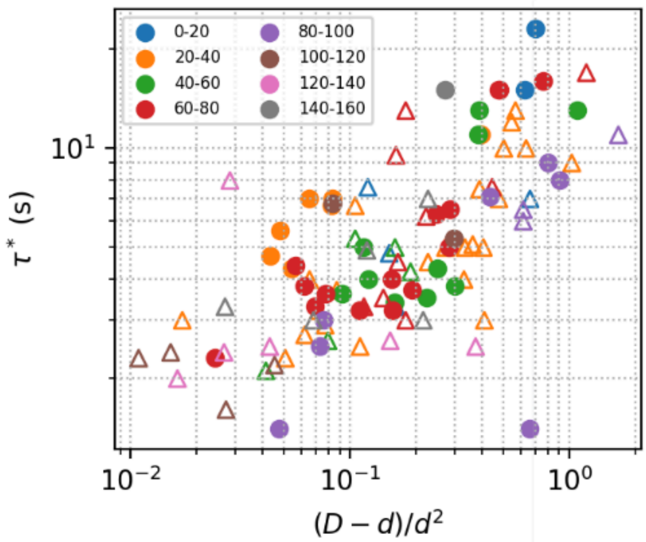
Paris data, and empty triangle to plot Chile data.

Parameter distribution:

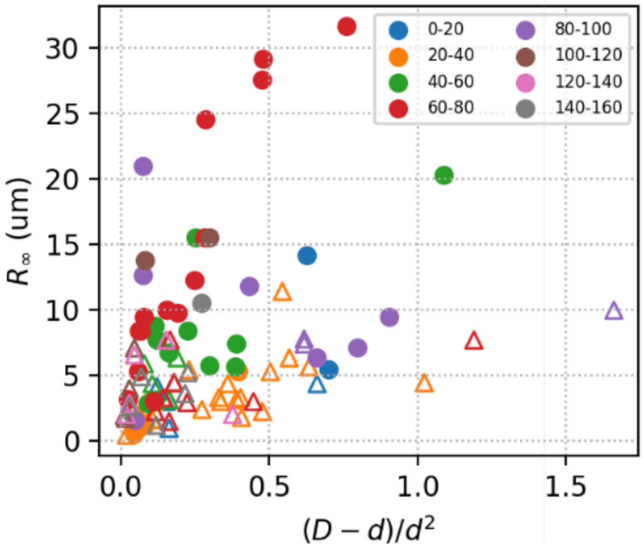


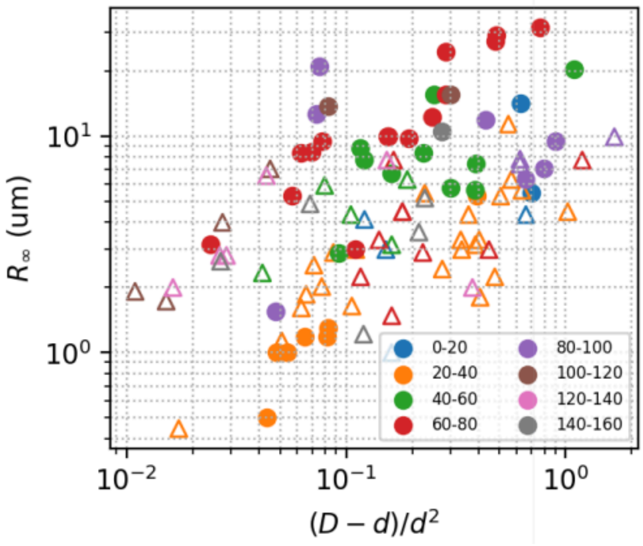
τ^* vs. $(D-d)/d^2$



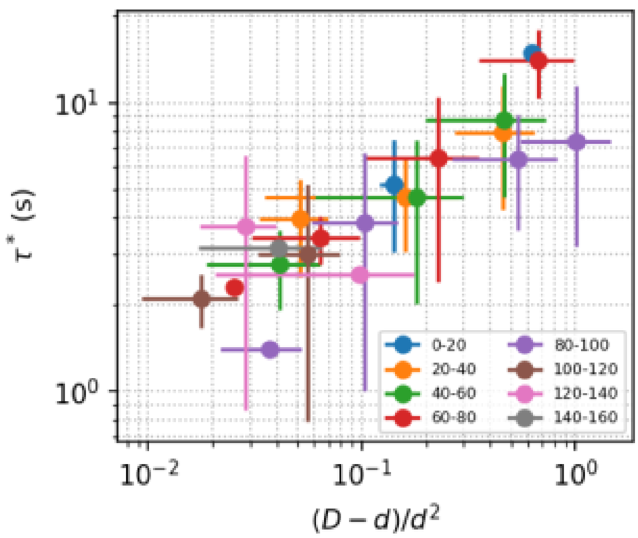


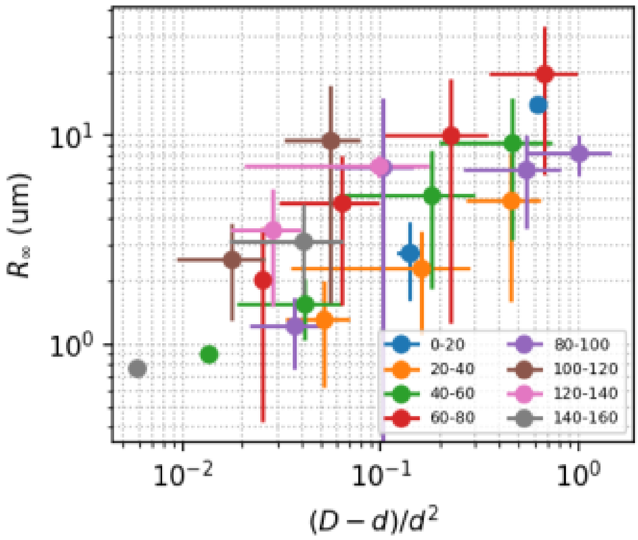
R_∞ vs. $(D-d)/d^2$





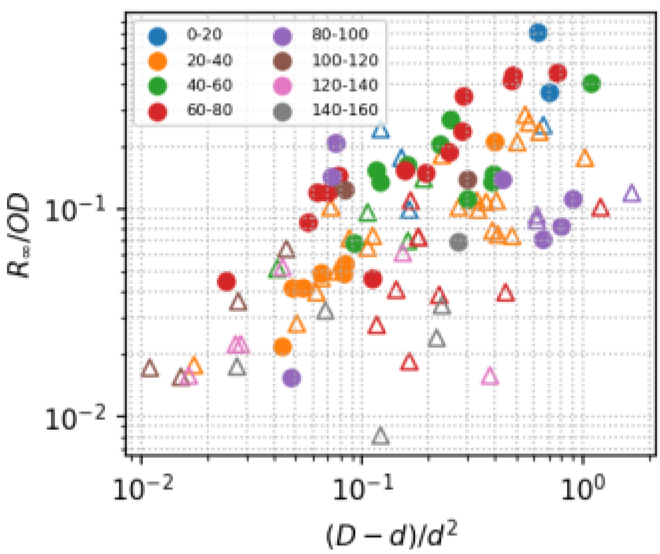
Plot the scattered data in bins



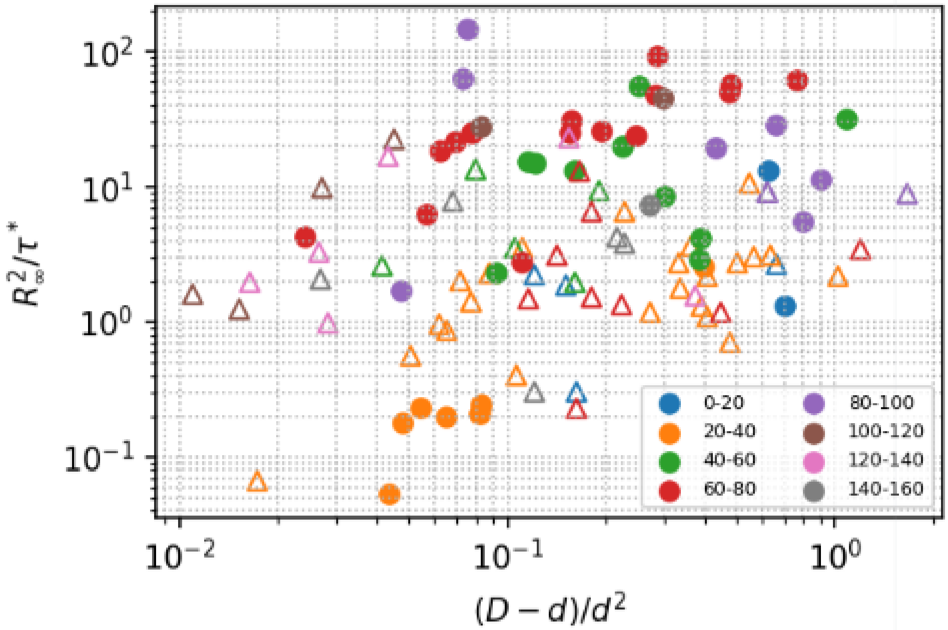


Some attempts

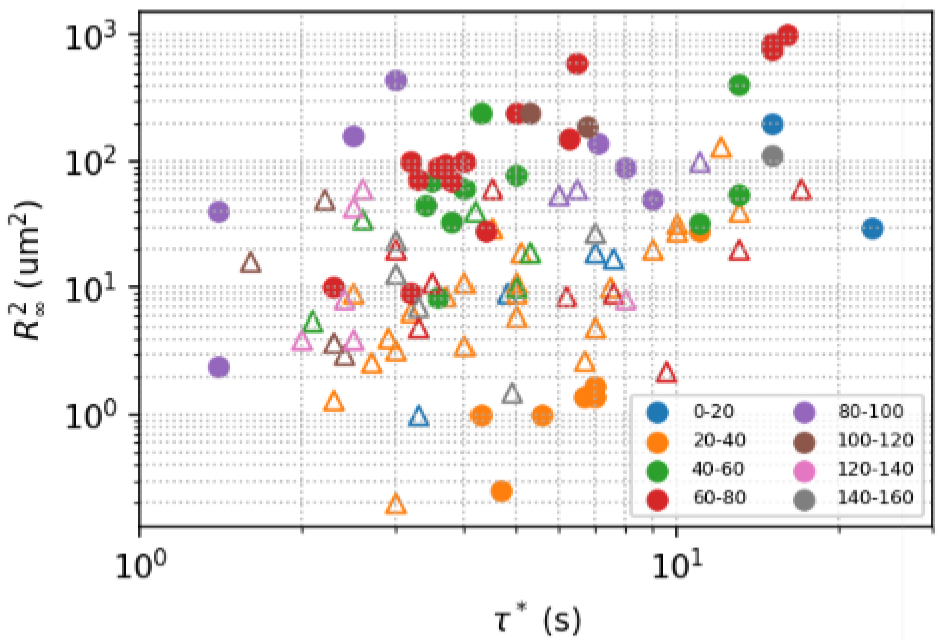
Rescale R_∞ with n (OD)



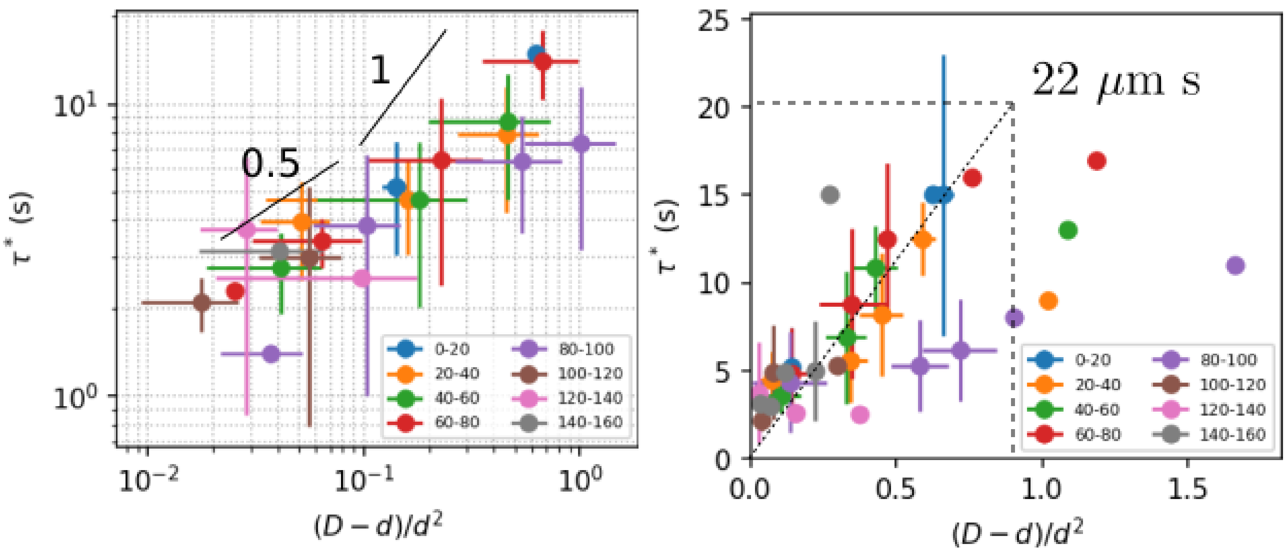
Plot R_∞^2/τ^* vs. $(D-d)/d^2$ ($=\frac{2D_A}{1+\tau/\tau^*}$)



Plot R_∞^2 vs. τ^* (similar to above, expect a linear regime)



Separate $(D-d)/d^2$ regimes (already evidenced in R_∞^2/τ^* vs. $(D-d)/d^2$ plot, and in τ^* vs. $(D-d)/d^2$ plot). A linear regime is seen for τ^* at intermediate $(D-d)/d^2$. Fit the linear regime, we obtain a slope ~ 22 μm s. The stochastic model predicts this slope to be $9\eta/\Delta\rho g \approx 3.9$ μm s. This discrepancy is interesting to look into.



τ^* (s)
$(D-d)/d^2$
1
0.5
0-20
20-40
40-60
60-80
80-100
100-120
120-140
140-160
$22\ \mu$m s