

Stochastic model discussion summary

Formulation and solution

We will continue the discussion with the equations and solutions given by Maggi 2014:

$$\dot{y} = -\mu k y + \eta^T + \eta^A,$$

where η^A is the active noise, satisfying $\langle \eta^A(t) \eta^A(t') \rangle = (D_A/\tau) e^{-|t-t'|/\tau}$. The active part of the solution is

$$\langle \Delta y^2(t) \rangle = \frac{2D_A}{\mu k} \frac{1 - e^{-\mu k t} - \mu k \tau (1 - e^{-t/\tau})}{1 - (\mu k \tau)^2}.$$

We let $\mu k = 1/\tau^*$, and it can be shown that the τ^* here is the saturation time scale of inner droplet motion.

Limit cases

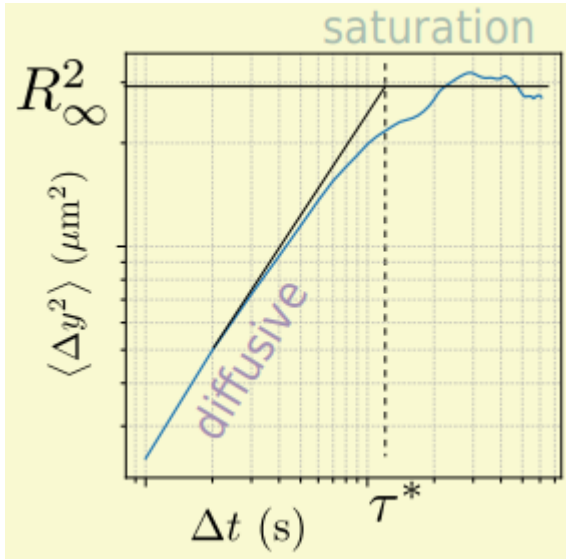
$$t \rightarrow 0 : \langle \Delta y^2(t) \rangle = \frac{D_A}{\tau(1 + \tau/\tau^*)} t^2$$

$$\tau^* \gg t \gg \tau : \langle \Delta y^2(t) \rangle = \frac{2D_A}{1 - (\tau/\tau^*)^2} (t - \tau)$$

$$t \rightarrow \infty : \langle \Delta y^2(t) \rangle = \frac{2D_A \tau^*}{1 + \tau/\tau^*}$$

Evaluate R_∞ and τ^*

A typical MSD curve can be described by the saturation value R_∞^2 and the transition time τ^* . The ballistic regime, where $\langle \Delta y^2(t) \rangle \propto t^2$, is usually within 1 second and is difficult to measure accurately in experiment.



The transition time τ^* comes out directly from the model as

$$\tau^* = \frac{1}{\mu k},$$

where $\mu = (6\pi\eta r_i)^{-1}$ is the particle mobility and $k = m^*g/(r_o - r_i)$ is the effective spring constant. Notice that m^* is the buoyant mass of the oil droplet in water, and can be computed as $m^* = \Delta\rho \frac{4}{3}\pi r_i^3$. Taken together, τ^* can be expressed as

$$\tau^* = \frac{9\eta}{2\Delta\rho g} \frac{r_o - r_i}{r_i^2} = \frac{9\eta}{\Delta\rho g} \frac{D - d}{d^2}$$

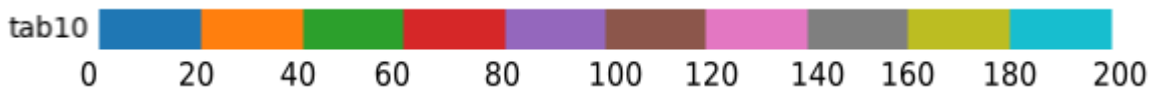
In the limit $\tau^* \gg \tau$, the saturation value $R_\infty^2 \approx 2D_A\tau^* \propto \tau^* \propto (D - d)/d^2$,

$$R_\infty \propto \frac{\sqrt{D - d}}{d}.$$

When $\tau^* \sim \tau$, the experimental determination of τ^* becomes problematic.

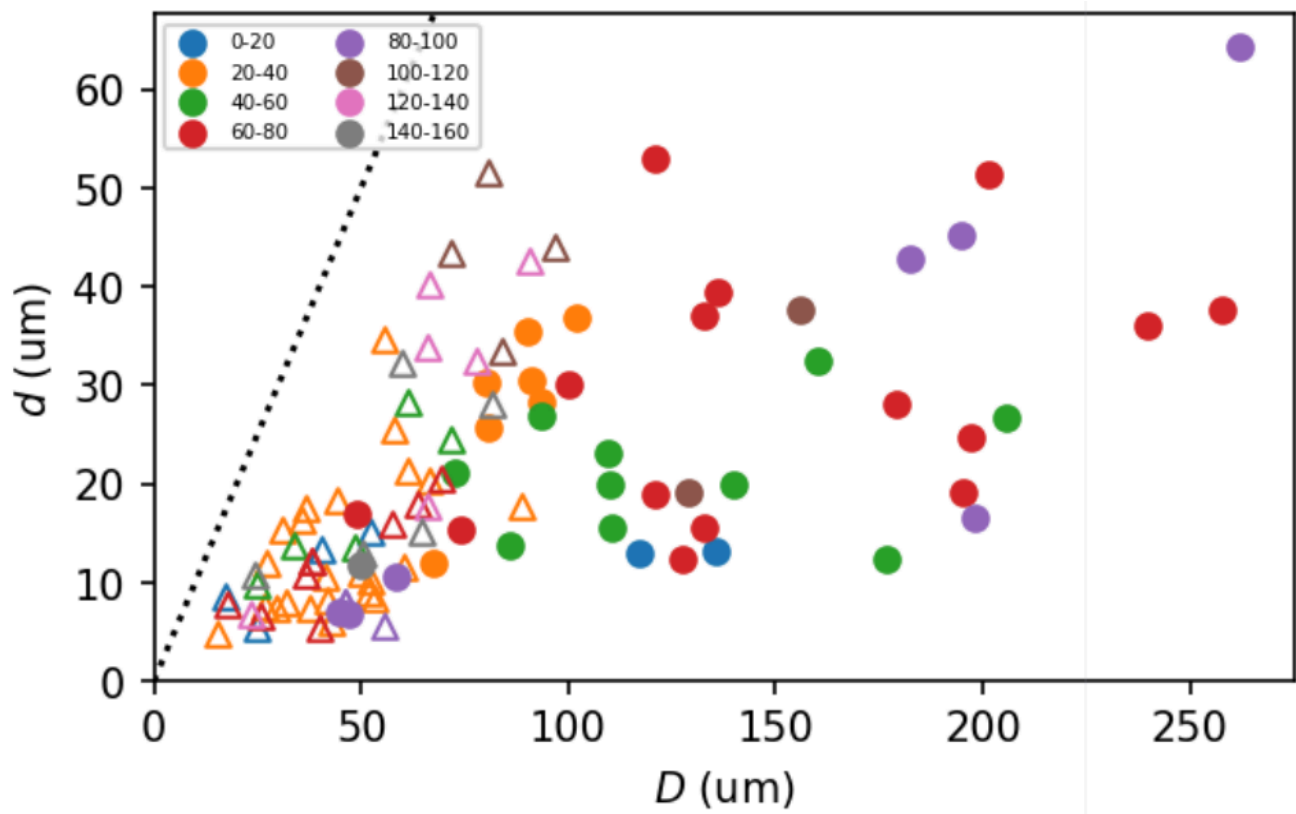
Experimental data

The current data set presents 3 parameters: D, d, n and two observables: R_∞ and τ^* . D and d are always combined as $(D - d)/d^2$ as x -axis. Bacterial concentration n is encoded using `tab10` colormap, with the following mapping.

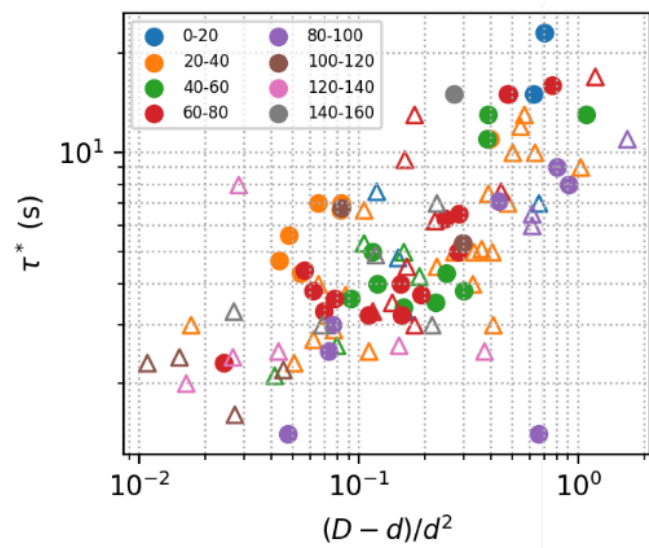
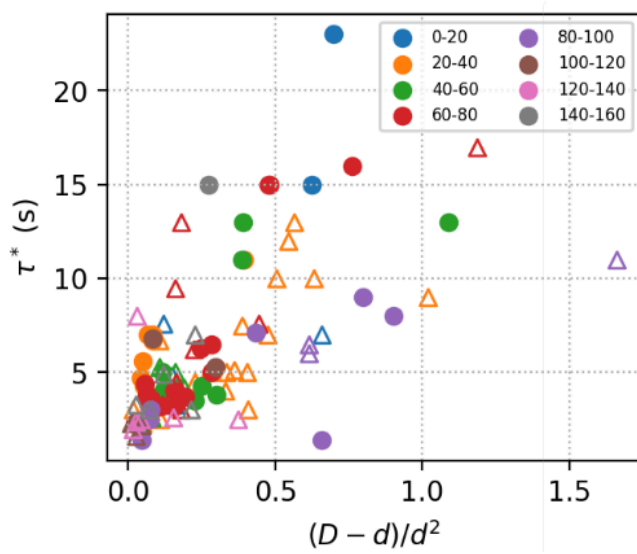


The data here combines Chile and Paris data. To discern them, I use solid circle to plot Paris data, and empty triangle to plot Chile data.

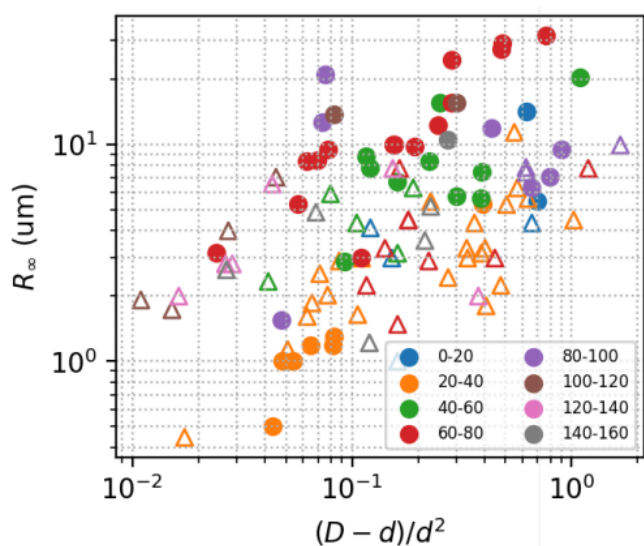
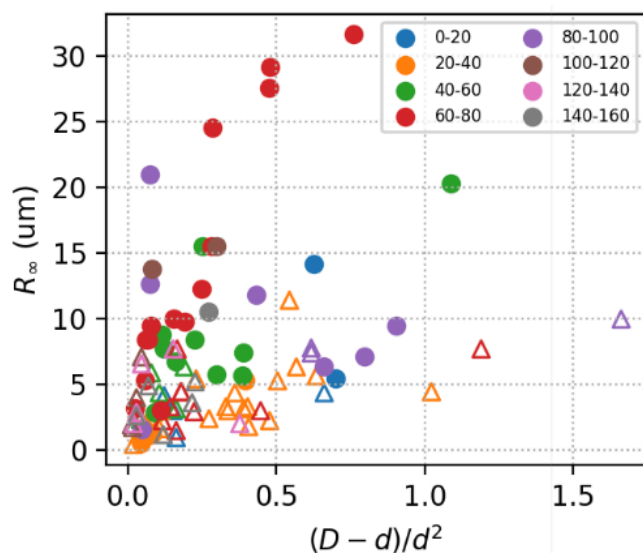
Parameter distribution:



τ^* vs. $(D - d)/d^2$

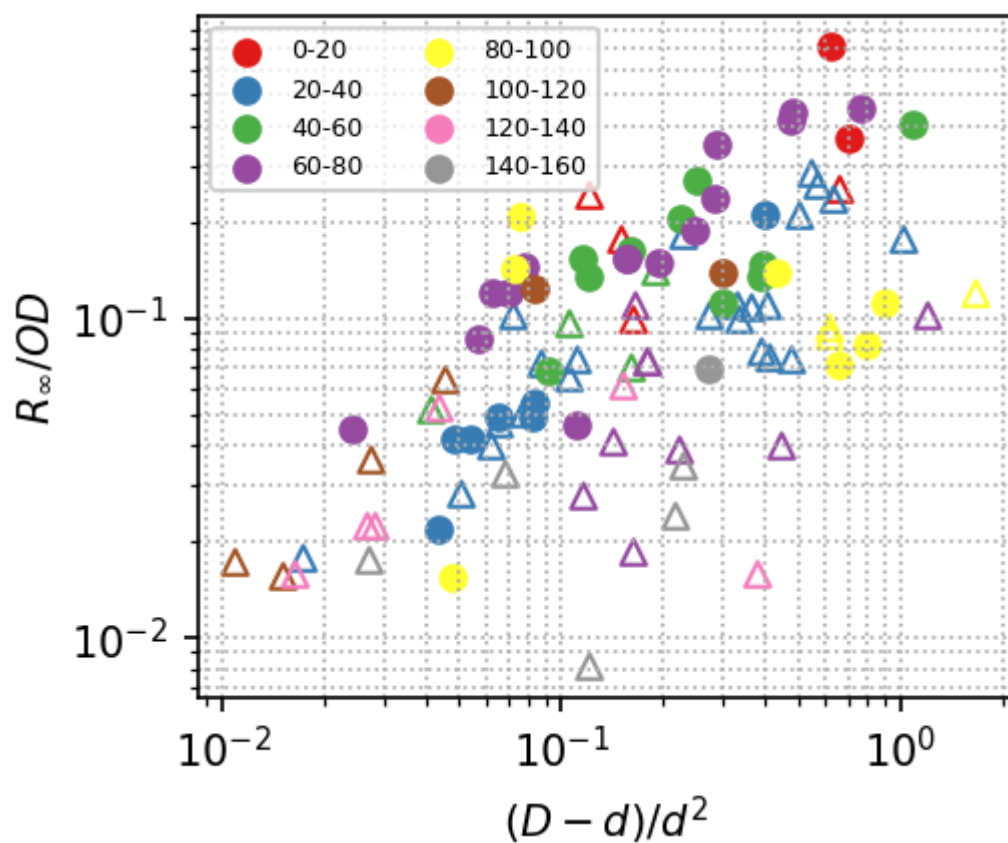


R_∞ vs. $(D - d)/d^2$

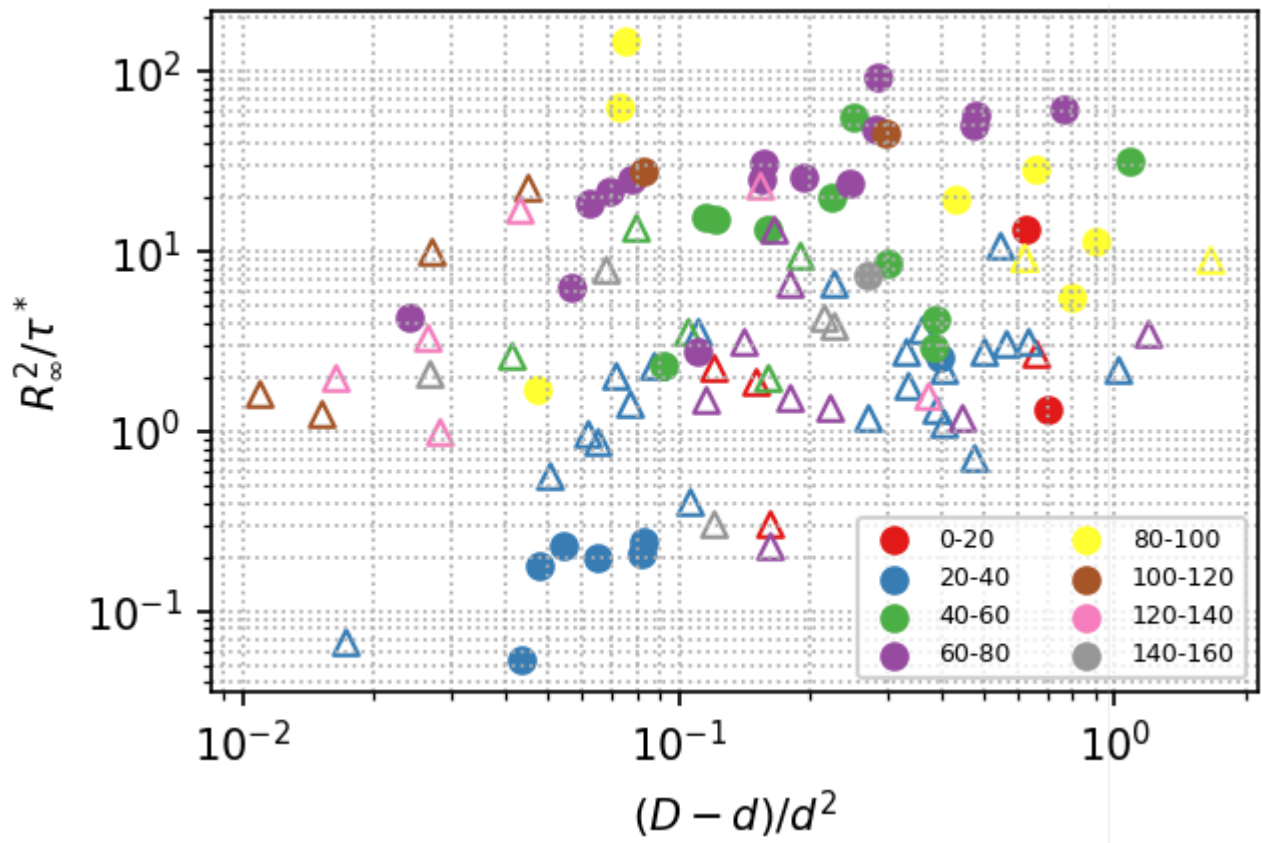


Some attempts

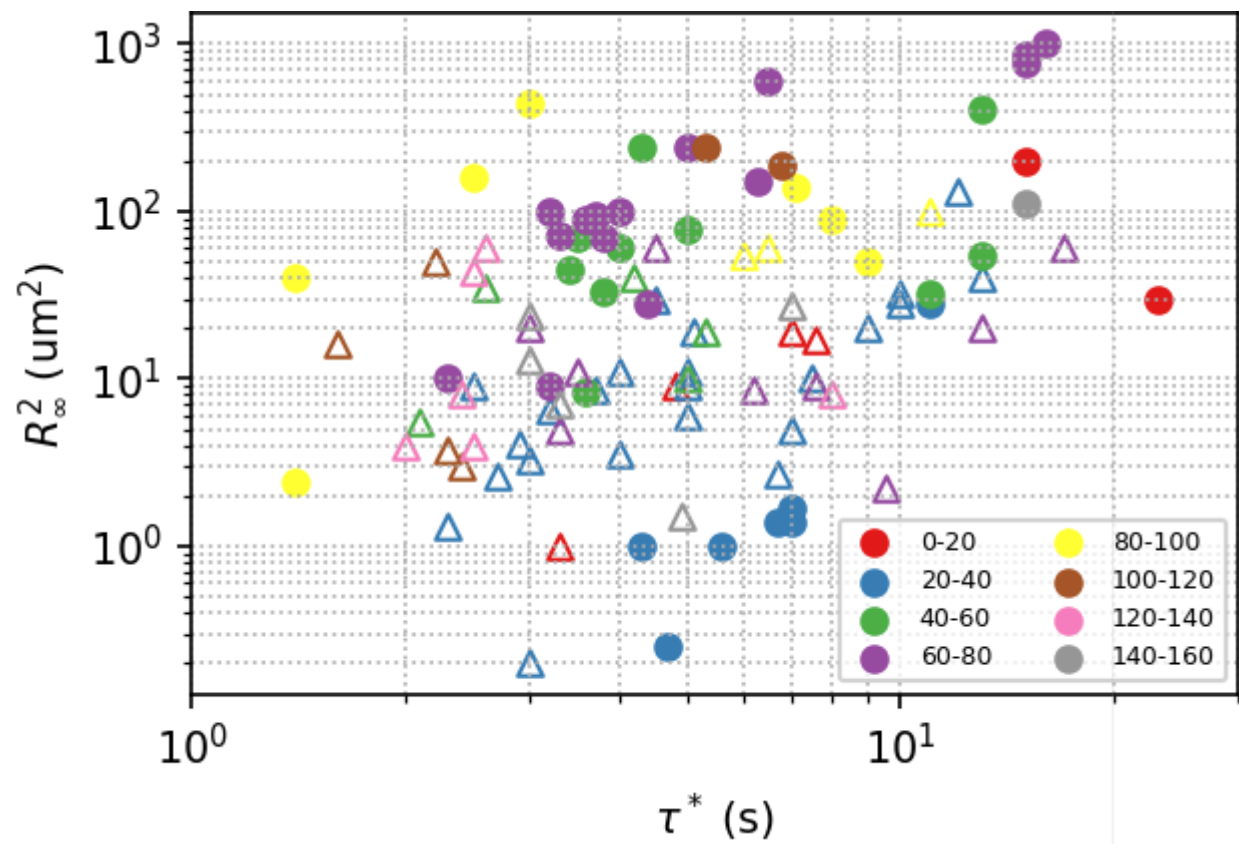
Rescale R_∞ with n (OD)



Plot R_∞^2/τ^* vs. $(D-d)/d^2$ ($= \frac{2D_A}{1+\tau/\tau^*}$)



Plot R_{∞}^2 vs. τ^* (similar to above, expect a linear regime)



Separate $(D - d)/d^2$ regimes (already evidenced in R_{∞}^2 / τ^* vs. $(D - d)/d^2$ plot)