How to understand confinement dependent D_A

In the stochastic model

$$\dot{y} = -\mu k y + \eta^T + \eta^A$$

D_A is the prefactor of the exponentially correlated active noise velocity η^A:
$\langle \eta^A(t)\eta^A(t')\rangle = (D_A/\tau)e^{-|t-t'|/\tau}$.

When applying this model to **active particles** (e.g. the self-propelled Janus particles in Palacci et al. 2010), D_A and τ are intrinsic properties of an active particle and can be measured by tracking individual particles.

People have been borrowing this model to understand the motion of **passive particles in an active bath** (Wu 2000, Maggi 2014, Ye 2020). Qualitatively, the superdiffusive-diffusive type of MSDs are well captured by the model. In the experiment of Wu 2000, they study the diffusivity of particles of two different sizes: 4.5 μm and 10 μm in diameter. Their results suggest that the effective diffusivity $D_{\text{eff}} \propto 1/R$, where R is the particle radius. In their paper, they commented that

> ...which has the same R dependence as the Stokes-Einstein relation, $D_T = k_B T/6\pi\eta R$. Such a result is expected for a system at thermal equilibrium, but not for a nonequilibrium system such as ours.

This $1/R$ dependence coincide with our data. If we look at the D_A vs. $D - d$ plot in the left below,
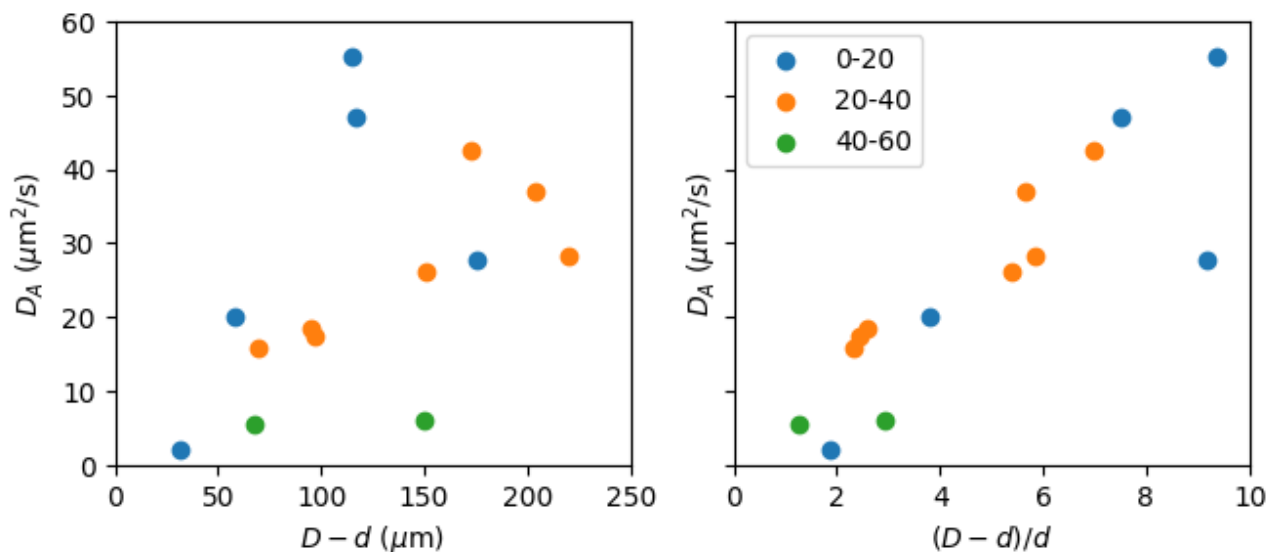
the D_A data are very scattered until we rescale the $D - d$ with d, the inner droplet diameter. If we fix $D - d$ in the left panel, and plot D_A as a function of d, what we get is

essentially $D_A \propto 1/R$. Below is a direct verification of such dependence, where we fix OD and D in a narrow range.
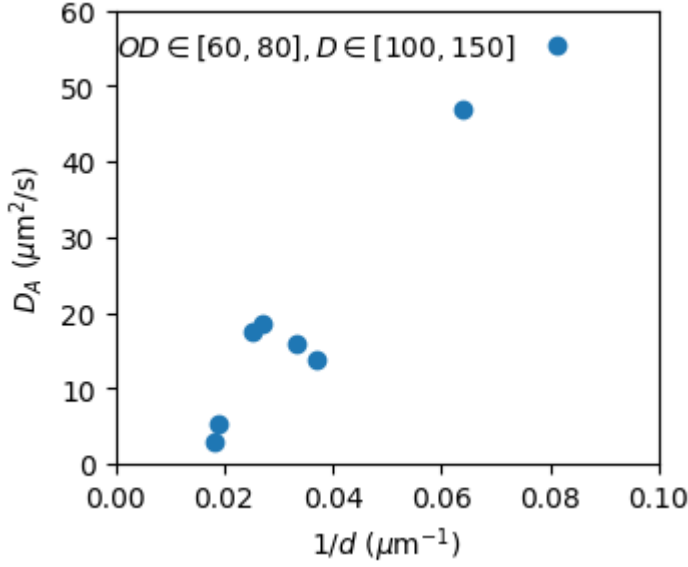
It is encouraging to see that a previously identified relation $D \propto 1/R$ is found again in our experiment. Being very similar to the form of Stokes-Einstein relation, this observation may suggest that the active bath results in a fixed amount of energy flux to the particle, which then dissipates through viscous drag.

Energy equipartition

In equilibrium system, energy is equally distributed to all degrees of freedom. Maggi et al. 2014 investigated the energy distribution of a silica particle in a glass capillary parallel and perpendicular to the capillary axis. When moving parallel to capillary axis, the motion is unconstrained, and the energy associated with this degree of freedom is $E_\parallel = (D_T + D_A)/\mu$. When moving perpendicular to capillary axis, the motion is effectively constrained by a harmonic potential, and the energy is $E_\perp = D_A/[\mu(1 + \mu k \tau)] + D_T/\mu$. This energy partition comes directly from the solution of the stochastic equation, and provide a picture of how external potential can modify the energy equipartition in an active system.

In their system, they have access to 2 different curvatures - zero and nonzero, along x and y. They did state in the paper that both x and y MSD's fit well to the theoretical prediction. The two fittings should independently give two sets of fitting parameters (D_A, τ, τ^*). However, they only present one D_A and do not comment on the other, implying that the results from both fittings are identical.

This is on the contrary to our result, where D_A increases linearly with $(D - d)/d$. The bacterial concentration in their experiment is $\sim 10^{10}$ cells/ml, roughly corresponding to $OD \sim 15$. This is the dilute regime before the onset of collective motion. Below I plot our

D_A vs. $D - d$ data in other concentration bracket:
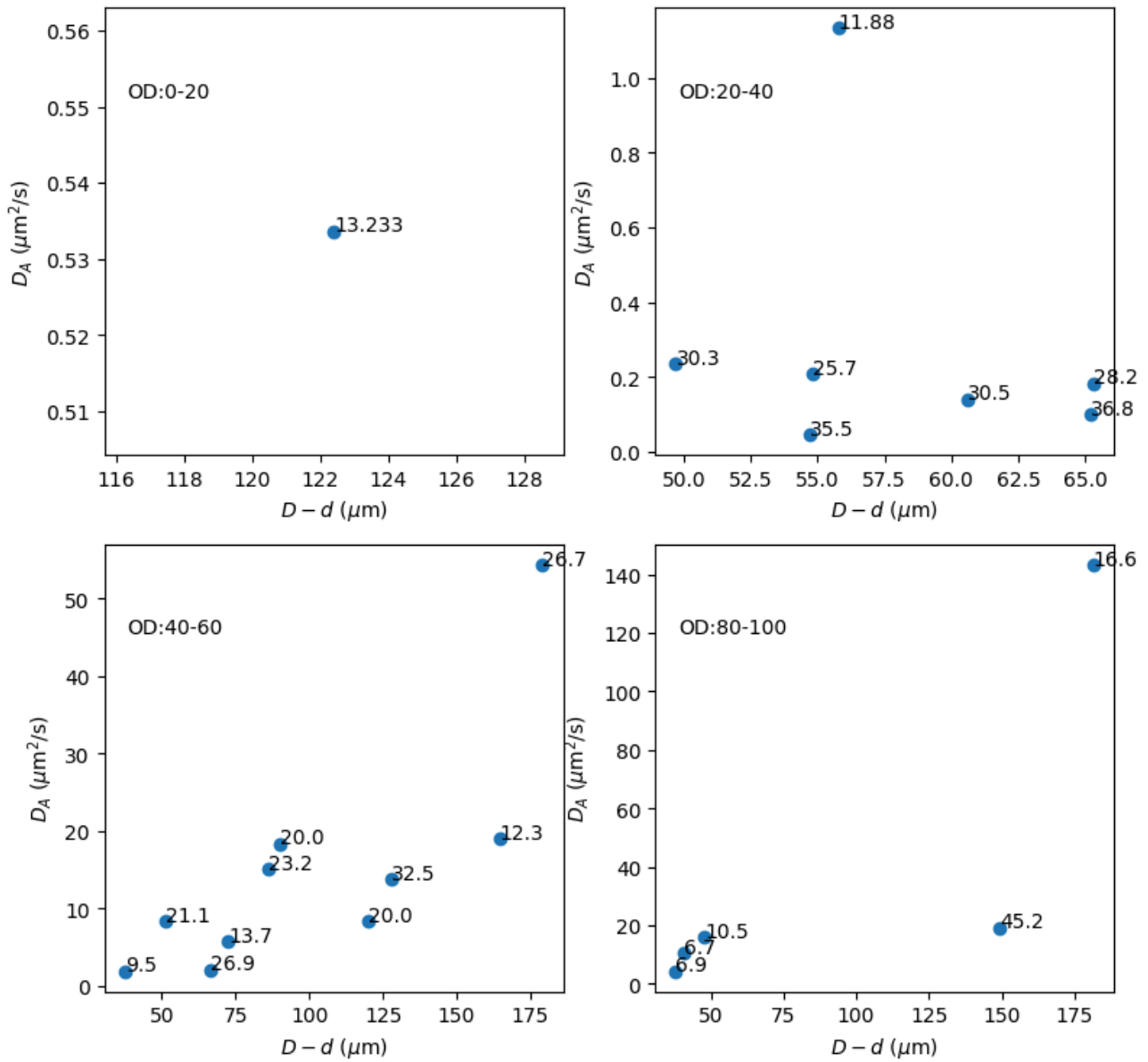
As can be seen, the dependence on $D - d$ only start to be observed when $OD > 40$. When $OD \in [20, 40]$, D_A shows no pronounced dependence on confinement.

Maybe collective motion is where confinement really change the bacterial behavior.