Giant Number Fluctuations and Energy Spectra in 3-D Bacterial Turbulence

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Giant number fluctuations, a landmark of collectively moving active particles, is universal in active systems across multiple length scales. Here, we present the first experimental study on the giant number fluctuation in 3-dimensional bacterial suspensions. Our measurements show that the number fluctuation scaled by the square root of mean number $\Delta N/\sqrt{N}$ scales with $N^{0.32}$ at high concentrations, confirming the theoretical predictions. Near the phase boundary, we observe a simultaneous increase of the scaling exponent and the flow induced by bacterial motions, suggesting a strong interplay between giant number fluctuations and flow. We show that this interplay spans all length scales in an active turbulence, by analyzing the kinetic energy spectra.

Giant number fluctuations (GNF), formally defined as the anomalously strong dependence of the variance of the number of particles on the mean number, is a universal phenomenon in active systems across multiple length scales, ranging from birds, fish and driven granules to bacteria, biological macromolecules and active synthetic particles [1–12]. Predicted by the seminal works from Toner and Tu [13–15], this phenomenon has stimulated extensive research interest and has become a landmark of ordered collectively moving particles [16–20].

Over the last 20 years, the understanding of GNF has been deepened significantly. Despite the progress, two important questions are still awaiting definite answers. First, the exact value of the scaling exponent of the variance on the mean has not reached an agreement. While the GNF is a seemingly universal in many systems, the scaling exponents measured or calculated in different systems show remarkable discrepancy [1, 6, 7, 9–12, 21–27]. In particular, the experimental works so far have reported scaling exponent ranging from 0.13 to 0.5 (the scaling exponent α is defined as $\Delta N/\sqrt{N} \propto N^{\alpha}$, ΔN is the standard deviation of particle number and N is the mean number) [1, 6, 7, 9–12], making it hard to compare experimental results with theory and simulations. We understand this situation by realizing that all the experiments have been done in 2-dimensional systems, with one or more frictional walls in direct contact to the active particles. We also notice that, although several experimental studies on bacterial systems have been reported, the GNF in bacterial turbulence - arguably the most fascinating and striking manifestation of microswimmer collective motions - has not been investigated (Supplementary movie 1 shows a vigorous bacterial turbulence in motion). Therefore in this letter, we present an experimental measurement of GNF in 3-dimensional bacterial turbulence. Due to the absence of frictional walls, our results can provide a solid benchmark for other works to compare to. Meanwhile, our 3-D GNF measurement enables deeper studies on this universal phenomenon, such as the dimensionality effect.

Second, the driving force of GNF remains largely unclear. Although mechanisms based on nematic instability [1, 21, 22] and topological defects [9, 28] have been proposed or observed in specific systems, the origin of GNF in other active systems, especially bacterial turbulence, remains unknown. Here, we present a detailed analysis on the velocity fields in active turbulence and show that the flow strength correlates with GNF at multiple length scales. We hope this result deepens the current understanding of GNF in bacterial active turbulence, and will eventually lead to a theory that quantitatively captures experimental results.

Counting particle numbers poses a major challenge in measuring the GNF of 3-dimensional bacterial turbulence. Watching (Supplementary movie 1), one immediately realizes that it is not possible to direct count the number of bacteria. Fortunately, the spatial distribution of bacterial concentrations can be inferred from optical microscopy images, where dark region indicates high local concentration and bright region indicates low local concentration. This idea is an extension of Beer-Lambert law which correlates solution concentration with its light attenuation. Similar principle has been used in some inspiring experimental works using image intensity as local concentration indicators [9, 29]. To be more quantitative on the relation between concentration and image pixel intensity, we did a calibration experiment by preparing bacterial suspensions of volume fractions ranging from 1.6% to 7.2% and take images of them under the same illumination. The images corresponding to different volume fractions are shown in Fig. 1a. As expected, when volume fraction gets higher, the image becomes dimmer. We plot the volume fractions as a function of the mean pixel intensities in the corresponding images, and find that the relation is almost linear, as shown in Fig. 1b.

[1] V. Narayan, S. Ramaswamy, and N. Menon, Science **317**, 105 (2007), arXiv:0612020 [cond-mat].
[2] I. Aranson, A. Snezhko, J. S. Olafsen, and J. S. Urbach3, **320**, 1 (2008).

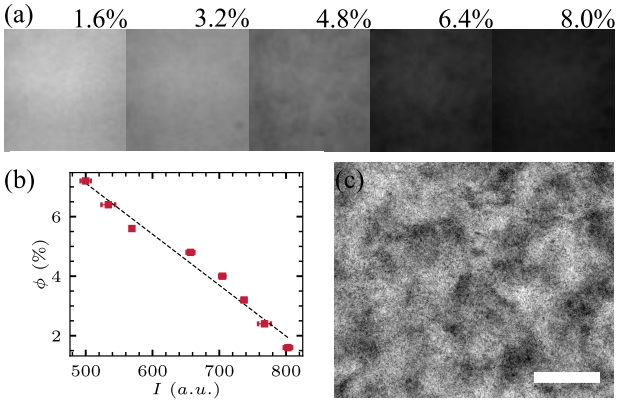

FIG. 1. (a) Bacterial suspensions of different volume fractions under the same illumination conditions. (b) Volume fraction as a function of averaged pixel intensities. (c) Bacterial active turbulence displaying constantly varying concentration inhomogeneity (6.4%). Scale bar is 100 µm. (d) Velocity field of a dilute bacterial suspension (1.6%). (e) Velocity field of a dense bacterial suspension (6.4%).

[3] A. J. Ward, D. J. Sumpter, I. D. Couzin, P. J. Hart, and J. Krause, Proceedings of the National Academy of Sciences of the United States of America **105**, 6948 (2008).

[4] M. Ballerini, N. Cabibbo, R. Candelier, A. Cavagna, E. Cisbani, I. Giardina, V. Lecomte, A. Orlandi, G. Parisi, A. Procaccini, M. Viale, and V. Zdravkovic, Proceedings of the National Academy of Sciences of the United States of America **105**, 1232 (2008).

[5] A. Sokolov, R. E. Goldstein, F. I. Feldchtein, and I. S. Aranson, Physical Review E - Statistical, Nonlinear, and Soft Matter Physics **80**, 1 (2009).

[6] J. Deseigne, O. Dauchot, and H. Chaté, Physical Review Letters **105**, 1 (2010), arXiv:1004.1499.

[7] H. P. Zhang, A. Be'er, E. L. Florin, and H. L. Swinney, Proceedings of the National Academy of Sciences of the United States of America **107**, 13626 (2010).

[8] J. Palacci, S. Sacanna, A. P. Steinberg, D. J. Pine, and P. M. Chaikin, Science **339**, 936 (2013).

[9] V. Schaller and A. R. Bausch, Proceedings of the National Academy of Sciences of the United States of America **110**, 4488 (2013).

[10] D. Nishiguchi, K. H. Nagai, H. Chaté, and M. Sano, Physical Review E **95**, 1 (2017), arXiv:1604.04247.

[11] K. Kawaguchi, R. Kageyama, and M. Sano, Nature **545**, 327 (2017).

[12] H. Karani, G. E. Pradillo, and P. M. Vlahovska, Physical Review Letters **123**, 208002 (2019).

[13] J. Toner and Y. Tu, Physical Review Letters **75**, 4326 (1995), arXiv:9506001 [adap-org].

[14] Y. Tu, J. Toner, and M. Ulm, Physical Review Letters **80**, 4819 (1998).

[15] J. Toner and Y. Tu, Physical Review E - Statistical Physics, Plasmas, Fluids, and Related Interdisciplinary Topics **58**, 4828 (1998), arXiv:9804180 [cond-mat].

[16] J. Toner, Y. Tu, and S. Ramaswamy, Annals of Physics **318**, 170 (2005).

[17] S. Ramaswamy, Annual Review of Condensed Matter Physics **1**, 323 (2010), arXiv:1004.1933.

[18] T. Vicsek and A. Zafeiris, Physics Reports **517**, 71 (2012), arXiv:1010.5017.

[19] D. Saintillan and M. J. Shelley, Comptes Rendus Physique **14**, 497 (2013).

[20] M. C. Marchetti, J. F. Joanny, S. Ramaswamy, T. B. Liverpool, J. Prost, M. Rao, and R. A. Simha, Reviews of Modern Physics **85**, 1143 (2013), arXiv:1207.2929.

[21] R. Aditi Simha and S. Ramaswamy, Physical review letters **89**, 058101 (2002), arXiv:0108301 [cond-mat].

[22] S. Ramaswamy, R. A. Simha, and J. Toner, Europhysics Letters **62**, 196 (2003), arXiv:0208573 [cond-mat].

[23] H. Chaté, F. Ginelli, G. Grégoire, and F. Raynaud, Physical Review E - Statistical, Nonlinear, and Soft Matter Physics **77**, 1 (2008), arXiv:0712.2062.

[24] S. Dey, D. Das, and R. Rajesh, Physical Review Letters **108**, 1 (2012).

[25] D. Saintillan and M. J. Shelley, Journal of the Royal Society Interface **9**, 571 (2012).

[26] S. Ngo, A. Peshkov, I. S. Aranson, E. Bertin, F. Ginelli, and H. Chaté, Physical Review Letters **113**, 1 (2014), arXiv:1312.1076.

[27] B. Mahault, F. Ginelli, and H. Chaté, Physical Review Letters **123**, 1 (2019), arXiv:1908.03794.

[28] D. Saintillan and M. J. Shelley, Physics of Fluids **20**, 10.1063/1.3041776 (2008).

[29] L. G. Wilson, V. A. Martinez, J. Schwarz-Linek, J. Tailleur, G. Bryant, P. N. Pusey, and W. C. Poon, Physical Review Letters **106**, 7 (2011), arXiv:1004.4764.