

## Description of the cross-correlaiton functions in Fig. 5C

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(Dated: July 1, 2021)

The 3 cross-correlation functions in Fig. 5C in the main text are described below.

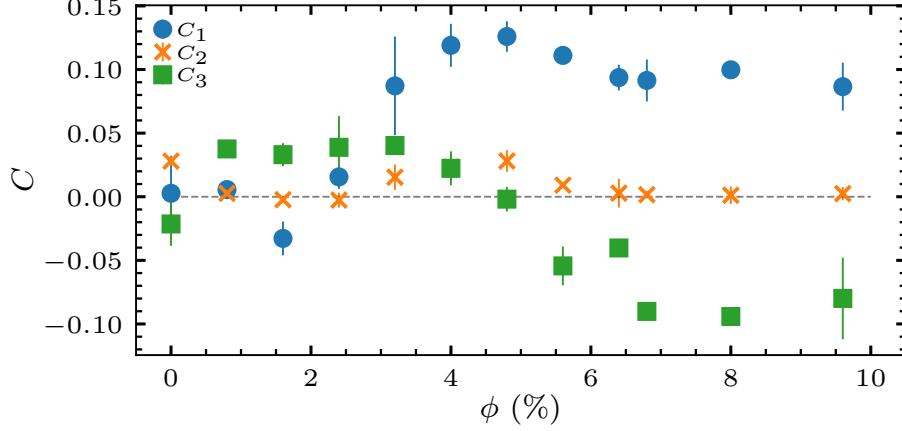


FIG. 1. Figure 5C in main text.

$C_1$  is the cross-correlation function between local density fluctuation  $\delta N$  and kinetic energy  $E$ .  $C_1$  shown in Fig. 5C is an average over 1000 single frame correlation  $C'_1$ , and  $C'_1$  is defined as

$$C'_1 = \frac{\langle (\delta N - \overline{\delta N})(E - \overline{E}) \rangle}{\sigma_{\delta N} \sigma_E} \quad (1)$$

$C_2$  is the cross-correlation function between local density fluctuation  $\delta N$  and divergence of bacterial flux  $\nabla \cdot (n\mathbf{v})$ .  $C_3$  is the cross-correlation function between local density  $N$  and local velocity (magnitude)  $v$ .  $C'_2$  and  $C'_3$  are single frame correlations similar to  $C'_1$ , and are defined as

$$C'_2 = \frac{\langle (\delta N - \overline{\delta N})(\nabla \cdot (n\mathbf{v}) - \overline{\nabla \cdot (n\mathbf{v})}) \rangle}{\sigma_{\delta N} \sigma_{\nabla \cdot (n\mathbf{v})}} \quad (2)$$

$$C'_3 = \frac{\langle (N - \overline{N})(v - \overline{v}) \rangle}{\sigma_N \sigma_v} \quad (3)$$

$\overline{A}$  indicates the mean of variable  $A$ ,  $\sigma_A$  indicates the standard deviation of  $A$ , and  $\langle A \rangle$  denotes the average of  $A$  over all the positions.