doc_draft

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The idea is to derive the energy spectrum definition in Yi's formula to the velocity correlation definition.

$$E(k_x, k_y) = u_k(k_x, k_y)u_k^*(k_x, k_y) = \iint u(x, y)e^{-ik_x x}e^{-k_y y}dxdy \left[\iint u(x', y')e^{-ik_x x'}e^{-ik_y y'}dx'dy'\right]^* = \iint u(x, y)e^{-}$$

here, we change variable and let $x'' = x - x'$ and $y'' = y - y'$ the original expression can be rearranged into

$$\iiiint u(x'+x'', y'+y'')u(x', y')e^{-ik_x x''}e^{-k_y y''}d(x'+x'')d(y'+y'')dx'dy' = \iint \left[\iint u(x' + x'', y' + y'')u(x', y')dx'dy'\right.$$

using the definition of velocity correlation function (average all possible pairs over available space):

$$\langle u(x, y)u(x + x'', y + y'')\rangle = \frac{\iint u(x' + x'', y' + y'')u(x', y')dx'dy'}{\iint dx'dy'}$$

we obtain

$$\iint dx'dy' \iint \langle u(x, y)u(x + x'', y + y'')\rangle e^{-ik_x x''}e^{-k_y y''}dx''dy''$$

the first integration is the available space size of velocity field, in this case the size of field of view A. In the code, A should be step size s times the row number r and column number c of velocity matrix size:

$$A = rcss$$

Note that r and s should have no unit and s should have unit um.

Let's draw a comparison between the two methods. Method I:

$$E_1(k_x, k_y) = \iint \langle u(x, y)u(x + x'', y + y'')\rangle e^{-ik_x x''}e^{-k_y y''}dx''dy''$$

Method II:

$$E_2(k_x, k_y) = \iint dx'dy' \iint \langle u(x, y)u(x+x'', y+y'')\rangle e^{-ik_x x''}e^{-k_y y''}dx''dy'' = A \iint \langle u(x, y)u(x+x'', y+y'')\rangle e^{-ik_x x''}e^{-}$$

Thus

$$E_1(k_x, k_y) = \frac{E_2(k_x, k_y)}{A}$$