Algorithm MCL(X_{t-1}, u_t, z_t):

$\overline{X}_t = X_t = \emptyset$

for $m=1$ to M:

$x_t^{[m]}$ = motion_update($u_t, x_{t-1}^{[m]}$) $\Leftrightarrow$ sample $x_t^{[m]} \sim p(x_t | x_{t-1}^{[m]}, u_t)$

$w_t^{[m]}$ = sensor_update($z_t, x_t^{[m]}$) $\Leftrightarrow$ $w_t^{[m]} = p(z_t | x_t^{[m]})$

$\overline{X}_t = \overline{X}_t \oplus \langle x_t^{[m]}, w_t^{[m]} \rangle$

endfor

for $m=1$ to M:

draw $x_t^{[i]}$ from $\overline{X}_t$ with probability $\propto w_t^{[i]}$

$X_t = X_t \oplus x_t^{[i]}$

end for

return X_t

end Algorithm

$X_t = \{x_t^{[1]}, x_t^{[2]}, \dots, x_t^{[M]}\}$

u_t = actuation command

z_t = sensor data

$x_t^{[i]} = [x, y, \theta]^T$

kidnap variant

- randomly add extra particles every iteration

$\boxed{k = t}$

1) Prediction Phase

use motion model to get $p(x_k | Z^{k-1}) = \sum_{i=1}^{M} p(x_k | x_{k-1}, u_{k-1}) P(x_{k-1})$

2) Update Phase

use measurement model to get $P(x_k | Z^k) \propto \cancel{p(z_k|x_k) p(x_k|Z^{k-1})}$ $p(z_k | x_k) P(x_k)$

x_k is current state

u_{k-1} is control input

z_k is a single measurement

Z_k is set of measurements

have $P(x_0)$

$$w = \frac{p(x_k | Z^k)}{p(x_k | Z^{k-1})}$$

- Move then add random noise
- Robot has to move or problems!

Predict $\hat{r}$ measure $\tilde{r}$ $\frac{1}{\sigma}(\tilde{r} - \hat{r})$

Particle Problems

- Weights go to 0
  - → re sample
- No diversity
  - → only resample with large $\frac{\max w}{\min w}$
  - → resample with variance
- Particle Depravation
  - → Increase number of particles
- Sensor are to percise
  - → artificially increase sensor uncertainty

$$w^{[m]} = \frac{f(x^{[m]})}{g(x^{[m]})}$$

f is target distribution
g is proposal distribution

$$w_t^{[m]} = \eta \, p(z_t | x_t)$$

resampling:
→ Never when stopped (suspend measurements too)

```
Low_var_sampling(X_t, W_t)
    X̄_t = ∅
    r = rand(0, M^-1)
    c = w_t^[1]
    i = 1
    for m = 1:M
        u = r + (m-1) M^-1
        while u > c
            i++
            c += w_t^[i]
        end while
        add x_t^[i] to X̄_t
    end for
    return X̄_t
```

motion_model_odometry(x_t, u_t, x_{t-1})

$$\begin{cases} \delta_{rot1} = \operatorname{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta} \\ \delta_{trans} = \sqrt{(\bar{x}-\bar{x}')^2 + (\bar{y}-\bar{y}')^2} \\ \delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1} \end{cases}$$

$$\hat{\delta}_{rot1} = \operatorname{atan2}(y' - y, x' - x) - \theta$$
$$\hat{\delta}_{trans} = \sqrt{(x-x')^2 + (y-y')^2}$$
$$\hat{\delta}_{rot2} = \theta' - \theta - \hat{\delta}_{rot1}$$

$$p_1 = \text{prob}(\delta_{rot1} - \hat{\delta}_{rot1}, \alpha_1 \hat{\delta}_{rot1} + \alpha_2 \hat{\delta}_{trans})$$
$$p_2 = \text{prob}(\delta_{trans} - \hat{\delta}_{trans}, \alpha_3 \hat{\delta}_{trans} + \alpha_4 (\hat{\delta}_{rot1} + \hat{\delta}_{rot2}))$$
$$p_3 = \text{prob}(\delta_{rot2} - \hat{\delta}_{rot2}, \alpha_1 \hat{\delta}_{rot2} + \alpha_2 \hat{\delta}_{trans})$$

return $p_1 \cdot p_2 \cdot p_3$

sample_motion_model_odometry(u_t, x_{t-1})

$$\hat{\delta}_{rot1} = \delta_{rot1} - \text{sample}(\alpha_1 \delta_{rot1} + \alpha_2 \delta_{trans})$$
$$\hat{\delta}_{trans} = \delta_{trans} - \text{sample}(\alpha_3 \delta_{trans} + \alpha_4 (\delta_{rot1} + \delta_{rot2}))$$
$$\hat{\delta}_{rot2} = \delta_{rot2} - \text{sample}(\alpha_1 \delta_{rot2} + \alpha_2 \delta_{trans2})$$

$$x' = x + \hat{\delta}_{trans} \cos(\theta + \hat{\delta}_{rot1})$$
$$y' = y + \hat{\delta}_{trans} \sin(\theta + \hat{\delta}_{rot1})$$
$$\theta' = \theta + \hat{\delta}_{rot1} + \hat{\delta}_{rot2}$$

return $x_t = (x', y', \theta')^T$

Measurement Errors

1) Small measurement noise - gaussian
2) errors from unexpected objects - exponential
3) errors due to failures to detect objects - uniform (at max)
4) random noise - uniform over entire range

1) $p_{hit} = \begin{cases} \eta \, \mathcal{N}(\underset{\text{reading}}{z_t^k}; \underset{\text{exact (ray casting)}}{z_t^{k*}}, \sigma_{hit}^2) & \text{if } 0 \le z_t^k \le z_{max} \\ 0 & \text{else} \end{cases}$

$$\mathcal{N}(z_t^k; z_t^{k*}, \sigma_{hit}^2) = \frac{1}{\sqrt{2\pi\sigma_{hit}^2}} \cdot \exp\left(\frac{-1}{2} \cdot \frac{(z_t^k - z_t^{k*})^2}{\sigma_{hit}^2}\right)$$

$$\eta = \left(\int_0^{z_{max}} \mathcal{N} \, dz_t^k\right)^{-1}$$

2) $p_{short} = \begin{cases} \eta \lambda_{short} \exp(-\lambda_{short} z_t^k) & \text{if } 0 \le z_t^k \le z_t^{k*} \\ 0 & \text{else} \end{cases}$

$$\eta = \left(1 - \exp(-\lambda_{short} z_t^{k*})\right)^{-1}$$

3) $p_{max} = \begin{cases} 1 & \text{if } z_t^k >= z_{max} \\ 0 & \text{else} \end{cases}$

4) $p_{rand} = \begin{cases} \frac{1}{z_{max}} & \text{if } 0 \le z_t^k < z_{max} \\ 0 & \text{else} \end{cases}$

Combined

$$p(z_t^k | x_t, m) = \begin{pmatrix} \alpha_{hit} \\ \alpha_{short} \\ \alpha_{max} \\ \alpha_{rand} \end{pmatrix}^T \cdot \begin{pmatrix} p_{hit} \\ p_{short} \\ p_{max} \\ p_{rand} \end{pmatrix} \qquad \alpha_{hit} + \alpha_{short} + \alpha_{max} + \alpha_{rand} = 1$$

beam_range_finder_model (z_t, x_t, m):

q=1
for k=1 to K
    get z_t^{k*} using ray casting
    get $p(z_t^k | x_t, m)$; q•=p
return q

learn_intrinsic_parameters (Z, X, m)

repeat until some convergence criterion

for all z_i in Z do

$\eta = (p_{hit} + p_{short} + p_{max} + p_{rand})^{-1}$

calc z_i^*

$e_{i,hit} = \eta\, p_{hit}$

$\vdots$

$e_{i,rand} = \eta\, p_{rand}$

$z_{hit} = |Z|^{-1} \sum_i e_{i\,hit}$

$\vdots$

$z_{rand} = |Z|^{-1} \sum_i e_{i\,rand}$

$$\sigma_{hit} = \sqrt{\frac{\sum_i e_{i\,hit} (z_i - z_i^*)^2}{\sum e_{i\,hit}}}$$

$$\lambda_{short} = \frac{\sum e_{i\,short}}{\sum e_{i\,short}\, z_i}$$

return $\Theta = \{z_{hit}, z_{short}, z_{max}, z_{rand}, \sigma_{hit}, \lambda_{short}\}$

---

monitor p(sensor measurments) $\approx \frac{1}{M} \sum_i^M w_t^{[m]}$ (average over a few time steps)

Augmented_MCL (X_{t-1}, u_t, z_t, m)

Static w_{slow}, w_{fast}; $\bar{X}_t = X_t = \emptyset$

for $m = 1 : M$

$x_t^{[m]}$ = sample_motion_model $(u_t, x_{t-1}^{[m]})$

$w_t^{[m]}$ = measurement_model $(z_t, x_t^{[m]}, M)$

$\bar{X}_t = \bar{X}_t \oplus \langle x_t^{[m]}, w_t^{[m]} \rangle$

$w_{avg} = w_{avg} + \frac{1}{M} w_t^{[m]}$

w_{slow} += $\alpha_{slow}(w_{avg} - w_{slow})$

w_{fast} += $\alpha_{fast}(w_{avg} - w_{fast})$

for $m = 1 : M$

with propability = $\max\left(0, 1 - \frac{w_{fast}}{w_{slow}}\right)$

add random pose to X_t

else

draw $i \in \{1, \dots, N\}$ w/ prob $\propto w_t^{[i]}$

add $x_t^{[i]}$ to X_t

return X_t