Iterative Learning Distributed Model Predictive Control for Autonomous Vehicle Platoons with Applications to Repetitive Tasks

A fundamental analysis towards systems with disturbances based on this work.

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1. A primary analysis about the recursive feasibility considering the disturbance brought by the stochasticity and randomness from the vehicle model and traffic.

   A primary analysis about the recursive feasibility considering the disturbance brought by the stochasticity and randomness from the vehicle model and traffic is conducted as follow. For system (4), a bounded disturbance $z_i^{[j]}(k)$ is introduced to describe the affects brought by stochasticity and randomness from the vehicle model and traffic, which satisfies $|z_i^{[j]}(k)| < \widetilde{z}$ and results in $X(k+1) = f(X(k), u(k), r(q(k))) + Ez(k)$ based on (2). The matrix E represents a disturbance matrix, which can be set as an identity matrix. The recorded sequences in $\mathcal{SS}_i^{[j-1]}$ is extended as follow

$$
\begin{aligned}
\widetilde{\mathcal{X}}_i^{[j-1]} &= \{\widetilde{X}_i^{[j-1]}(1), \widetilde{X}_i^{[j-1]}(2), \cdots, \widetilde{X}_i^{[j-1]}(k'), \cdots\} \\
\mathcal{X}_i^{[j-1]} &= \{X_i^{[j-1]}(1), X_i^{[j-1]}(2), \cdots, X_i^{[j-1]}(k'), \cdots\} \\
\mathcal{U}_i^{[j-1]} &= \{u_i^{[j-1]}(1), u_i^{[j-1]}(2), \cdots, u_i^{[j-1]}(k'), \cdots\} \\
\mathcal{Z}_i^{[j-1]} &= \{z_i^{[j-1]}(1), z_i^{[j-1]}(2), \cdots, z_i^{[j-1]}(k'), \cdots\}
\end{aligned}
$$

   where $\widetilde{X}$ represents the nominal states calculated by (2), and satisfies $X = \widetilde{X} + Ez$ for arbitrary i, j, k. Suppose that $\mathbf{U}_i^{[j],*}(k)$ is the optimal solution of the ILDMPC problem (36) at instant k in the jth iteration. The corresponding predicted state sequence is generated as follow.

$$
\widetilde{\mathbf{X}}_i^{[j],*}(k) = \{\widetilde{X}_i^{[j],*}(1|k), \widetilde{X}_i^{[j],*}(2|k), \cdots, \widetilde{X}_i^{[j],*}(N_P+1|k)\}
$$

   According to the terminal cost (34), $\widetilde{X}_i^{[j],*}(N_P+1|k) = \widetilde{X}_i^{[j-1],\dagger}(\iota) + E_i z_i^{[j-1],\dagger}(\iota)$ holds. Supposing the bounded disturbance can always be compensated by the control input within the control space, it is noted that $\exists u_i^{[j-1],\dagger}(\iota), \widetilde{u}_i^{[j-1],\dagger}(\iota), u_i^{[j],*}(N_p|k), \widetilde{u}_i^{[j],*}(N_p|k)$ satisfy

$$
\begin{aligned}
X_i^{[j-1],\dagger}(\iota+1) &= f_i(X_i^{[j-1],\dagger}(\iota), u_i^{[j-1],\dagger}(\iota), r(q_i^{[j-1],\dagger}(\iota))) + E_i z_i^{[j-1],\dagger}(\iota) \\
\widetilde{X}_i^{[j-1],\dagger}(\iota+1) &= f_i(\widetilde{X}_i^{[j-1],\dagger}(\iota), \widetilde{u}_i^{[j-1],\dagger}(\iota), r(q_i^{[j-1],\dagger}(\iota))) \\
X_i^{[j],*}(N_P+2|k) &= f_i(X_i^{[j],*}(N_P+1|k), u_i^{[j],*}(N_p|k), r(q_i^{[j]}(k))) + E_i z_i^{[j]}(k) \\
\widetilde{X}_i^{[j],*}(N_P+2|k) &= f_i(\widetilde{X}_i^{[j],*}(N_P+1|k), \widetilde{u}_i^{[j],*}(N_p|k), r(q_i^{[j]}(k))) \\
\widetilde{u}_i^{[j-1],\dagger}(\iota) &= \widetilde{u}_i^{[j],*}(N_p|k)
\end{aligned}
$$

   where the nominal control input $\widetilde{u}$ and actual control input u can be extended as follow with a generalized inverse function $f_i^-(\cdot)$

$$
\begin{aligned}
u_i^{[j-1],\dagger}(\iota) &= \widetilde{u}_i^{[j-1],\dagger}(\iota) + f_i^-(\widetilde{X}_i^{[j-1],\dagger}(\iota), z_i^{[j-1],\dagger}(\iota), r(q_i^{[j-1],\dagger}(\iota))) \\
u_i^{[j],*}(N_p|k) &= \widetilde{u}_i^{[j],*}(N_p|k) + f_i^-(\widetilde{X}_i^{[j],*}(N_P+1|k), z_i^{[j]}(k), r(q_i^{[j]}(k)))
\end{aligned}
$$

   At instant $k+1$ in the jth iteration, a feasible solution $\mathbf{U}_i^{[j],\dagger}(k+1)$ and the corresponding state $\mathbf{X}_i^{[j],\dagger}(k+1)$ can be generated as follows

$$
\begin{aligned}
\mathbf{U}_i^{[j],\dagger}(k+1) =& \{u_i^{[j],*}(2|k), u_i^{[j],*}(3|k), \cdots, u_i^{[j],*}(N_P|k), u_i^{[j],\dagger}(N_P|k+1)\} \\
\mathbf{X}_i^{[j],\dagger}(k+1) =& \{X_i^{[j],*}(2|k), X_i^{[j],*}(3|k), \cdots, X_i^{[j],*}(N_P+1|k), X_i^{[j-1],\dagger}(\iota+1)\}
\end{aligned}
$$

where

$$u_i^{[j],\dagger}(N_P|k+1) = \widetilde{u}_i^{[j-1],\dagger}(\iota) + f_i^-(\widetilde{X}_i^{[j],*}(N_P+1|k), z_i^{[j]}(k+1), r(q_i^{[j]}(k+1)))$$

Therefore, as an extension, if the ILDMPC problem (36) is successfully optimized with the solution $\mathbf{U}_i^{[j],*}(k)$ and the corresponding state $\mathbf{X}_i^{[j],*}(k)$, there will be a feasible domain containing solutions $\mathbf{U}_i^{[j],\dagger}(k+1)$ and satisfactory states $\mathbf{X}_i^{[j],\dagger}(k+1)$. This conclusion holds with the assumption that the affects brought by stochasticity and randomness from the vehicle model and traffic can be described by a bounded disturbance $z_i^{[j]}(k)$, and the bounded disturbance can always be compensated by the control input within the control space.

2. A primary analysis about the stability considering the disturbance brought by the stochasticity and randomness from the vehicle model and traffic.

   Similarly, considering the disturbance, a primary analysis is conducted as follow. With the definition of the loss function (35) and Lyapunov function (40), the difference of the Lyapunov function for the discrete system $X(k+1) = f(X(k), u(k), r(q(k)))$ can be scaled as (43). For the discrete system $X(k+1) = f(X(k), u(k), r(q(k))) + Ez(k)$, the difference is rewritten as follow

$$\begin{aligned}
&J_i^{[j]}|_{k:\infty} - J_i^{[j]}|_{k+1:\infty} \\
=&\lim_{N_p\to\infty} \sum_{n=k}^{N_p+k} l_{\text{loss}}(X_i^{[j]}(n), u_i^{[j]}(n), r(q_i^{[j]}(n))) - \sum_{n=k+1}^{N_p+k+1} l_{\text{loss}}(X_i^{[j]}(n), u_i^{[j]}(n), r(q_i^{[j]}(n))) \\
=&l_{\text{loss}}(X_i^{[j]}(k), u_i^{[j]}(k), r(q_i^{[j]}(k))) \\
&- \lim_{N_p\to\infty} l_{\text{loss}}(X_i^{[j]}(k+N_p+1), u_i^{[j]}(k+N_p+1), r(q_i^{[j]}(k+N_p+1))) \\
\geq&l_{\text{loss}}(X_i^{[j]}(k), u_i^{[j]}(k), r(q_i^{[j]}(k))) \\
&- \lim_{\iota\to\infty} l_{\text{loss}}(X_i^{[j-1],\dagger}(\iota+2), u_i^{[j-1],\dagger}(\iota+2), r(q_i^{[j-1],\dagger}(\iota+2))) - \Delta_i^\delta \\
=&l_{\text{loss}}(X_i^{[j]}(k), u_i^{[j]}(k), r(q_i^{[j]}(k))) - \Delta_i^\delta
\end{aligned}$$

where Δ_i^δ is an additional term brought by the disturbance and can be extended as

$$\begin{aligned}
\Delta_i^\delta =& \lim_{\iota, N_P\to\infty} 2\left(C\widetilde{X}_i^{[j-1],\dagger}(\iota+2)\right)^{\mathrm{T}} \mathbf{P_1}\left(\Delta_i^{[j],\text{error}} - \Delta_i^{[j-1],\text{error}}\right) \\
&+ 2\left(\widetilde{u}_i^{[j-1],\dagger}(\iota+2)\right)^{\mathrm{T}} \mathbf{P_2}\left(\Delta_i^{[j],\text{input}} - \Delta_i^{[j-1],\text{input}}\right) \\
&+ \|\Delta_i^{[j],\text{error}}\|_{\mathbf{P_1}} - \|\Delta_i^{[j-1],\text{error}}\|_{\mathbf{P_1}} + \|\Delta_i^{[j],\text{input}}\|_{\mathbf{P_2}} - \|\Delta_i^{[j-1],\text{input}}\|_{\mathbf{P_2}} \\
\Delta_i^{[j],\text{error}} =& z_i^{[j]}(k+N_P+1) - \widetilde{Y}_i^{[j]}(k+N_P+1) \\
\Delta_i^{[j-1],\text{error}} =& z_i^{[j-1],\dagger}(\iota+2) - \widetilde{Y}_i^{[j-1],\dagger}(\iota+2) \\
\Delta_i^{[j],\text{input}} =& f_i^-(\widetilde{X}_i^{[j-1],\dagger}(\iota+2), z_i^{[j]}(k+N_P+1), r(q_i^{[j]}(k+N_P+1))) \\
\Delta_i^{[j-1],\text{input}} =& f_i^-(\widetilde{X}_i^{[j-1],\dagger}, z_i^{[j-1],\dagger}(\iota+2), r(q_i^{[j-1],\dagger}(\iota+2))) \\
C =& \text{diag}(1,1,1,1,1,1,1,1,0,0)
\end{aligned}$$

Therefore, considering the unknown disturbance $z_i^{[j]}(k)$, the controlled system can be theoretically stable only if $l_{\text{loss}}(X_i^{[j]}(k), u_i^{[j]}(k), r(q_i^{[j]}(k))) - \Delta_i^{\delta} > 0$ holds. In fact, the additional term Δ_i^{δ} is quite small because it mainly calculated from the differences of differences, which are relatively negligible in the control process. However, for a rigorous theoretical proof, there are more efforts required for terminal cost design to guarantee $l_{\text{loss}}(X_i^{[j]}(k), u_i^{[j]}(k), r(q_i^{[j]}(k))) - \Delta_i^{\delta} > 0$, which is planned in the future work.

References