Iterative Learning Distributed Model Predictive Control for Autonomous Vehicle Platoons with Applications to Repetitive Tasks

A fundamental analysis towards systems with disturbances based on this work.

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1. A primary analysis about the recursive feasibility considering the disturbance brought by the stochasticity and randomness from the vehicle model and traffic.

A primary analysis about the recursive feasibility considering the disturbance brought by the stochasticity and randomness from the vehicle model and traffic is conducted as follow. For system (4), a bounded disturbance $z_i^{[j]}(k)$ is introduced to describe the affects brought by stochasticity and randomness from the vehicle model and traffic, which satisfies $|z_i^{[j]}(k)| < \widetilde{z}$ and results in $X(k+1) = f(X(k), u(k), r(q(k))) + Ez(k)$ based on (2). The matrix E represents a disturbance matrix, which can be set as an identity matrix. The recorded sequences in $\mathcal{SS}_i^{[j-1]}$ is extended as follow

$$
\begin{aligned}
\widetilde{\mathcal{X}}_i^{[j-1]} &= \{\widetilde{X}_i^{[j-1]}(1), \widetilde{X}_i^{[j-1]}(2), \cdots, \widetilde{X}_i^{[j-1]}(k'), \cdots\} \\
\mathcal{X}_i^{[j-1]} &= \{X_i^{[j-1]}(1), X_i^{[j-1]}(2), \cdots, X_i^{[j-1]}(k'), \cdots\} \\
\mathcal{U}_i^{[j-1]} &= \{u_i^{[j-1]}(1), u_i^{[j-1]}(2), \cdots, u_i^{[j-1]}(k'), \cdots\} \\
\mathcal{Z}_i^{[j-1]} &= \{z_i^{[j-1]}(1), z_i^{[j-1]}(2), \cdots, z_i^{[j-1]}(k'), \cdots\}
\end{aligned}
$$

where $\widetilde{X}$ represents the nominal states calculated by (2), and satisfies $X = \widetilde{X} + Ez$ for arbitrary i, j, k. Suppose that $\mathbf{U}_i^{[j],*}(k)$ is the optimal solution of the ILDMPC problem (36) at instant k in the jth iteration. The corresponding predicted state sequence is generated as follow.

$$
\widetilde{\mathbf{X}}_i^{[j],*}(k) = \{\widetilde{X}_i^{[j],*}(1|k), \widetilde{X}_i^{[j],*}(2|k), \cdots, \widetilde{X}_i^{[j],*}(N_P+1|k)\}
$$

According to the terminal cost (34), $\widetilde{X}_i^{[j],*}(N_P+1|k) = \widetilde{X}_i^{[j-1],\dagger}(\iota) + E_i z_i^{[j-1],\dagger}(\iota)$ holds. Supposing the bounded disturbance can always be compensated by the control input within the control space, it is noted that $\exists u_i^{[j-1],\dagger}(\iota), \widetilde{u}_i^{[j-1],\dagger}(\iota), u_i^{[j],*}(N_p|k), \widetilde{u}_i^{[j],*}(N_p|k)$ satisfy

$$
\begin{aligned}
X_i^{[j-1],\dagger}(\iota+1) &= f_i(X_i^{[j-1],\dagger}(\iota), u_i^{[j-1],\dagger}(\iota), r(q_i^{[j-1],\dagger}(\iota))) + E_i z_i^{[j-1],\dagger}(\iota) \\
\widetilde{X}_i^{[j-1],\dagger}(\iota+1) &= f_i(\widetilde{X}_i^{[j-1],\dagger}(\iota), \widetilde{u}_i^{[j-1],\dagger}(\iota), r(q_i^{[j-1],\dagger}(\iota))) \\
X_i^{[j],*}(N_P+2|k) &= f_i(X_i^{[j],*}(N_P+1|k), u_i^{[j],*}(N_p|k), r(q_i^{[j]}(k))) + E_i z_i^{[j]}(k) \\
\widetilde{X}_i^{[j],*}(N_P+2|k) &= f_i(\widetilde{X}_i^{[j],*}(N_P+1|k), \widetilde{u}_i^{[j],*}(N_p|k), r(q_i^{[j]}(k))) \\
\widetilde{u}_i^{[j-1],\dagger}(\iota) &= \widetilde{u}_i^{[j],*}(N_p|k)
\end{aligned}
$$

where the nominal control input $\widetilde{u}$ and actual control input u can be extended as follow with a generalized inverse function $f_i^-(\cdot)$

$$
\begin{aligned}
u_i^{[j-1],\dagger}(\iota) &= \widetilde{u}_i^{[j-1],\dagger}(\iota) + f_i^-(\widetilde{X}_i^{[j-1],\dagger}(\iota), z_i^{[j-1],\dagger}(\iota), r(q_i^{[j-1],\dagger}(\iota))) \\
u_i^{[j],*}(N_p|k) &= \widetilde{u}_i^{[j],*}(N_p|k) + f_i^-(\widetilde{X}_i^{[j],*}(N_P+1|k), z_i^{[j]}(k), r(q_i^{[j]}(k)))
\end{aligned}
$$

At instant $k+1$ in the jth iteration, a feasible solution $\mathbf{U}_i^{[j],\dagger}(k+1)$ and the corresponding state $\mathbf{X}_i^{[j],\dagger}(k+1)$ can be generated as follows

$$
\begin{aligned}
\mathbf{U}_i^{[j],\dagger}(k+1) =& \{u_i^{[j],*}(2|k), u_i^{[j],*}(3|k), \cdots, u_i^{[j],*}(N_P|k), u_i^{[j],\dagger}(N_P|k+1)\} \\
\mathbf{X}_i^{[j],\dagger}(k+1) =& \{X_i^{[j],*}(2|k), X_i^{[j],*}(3|k), \cdots, X_i^{[j],*}(N_P+1|k), X_i^{[j-1],\dagger}(\iota+1)\}
\end{aligned}
$$

where

$$u_i^{[j],\dagger}(N_P|k+1) = \widetilde{u}_i^{[j-1],\dagger}(\iota) + f_i^{-}(\widetilde{X}_i^{[j],*}(N_P+1|k), z_i^{[j]}(k+1), r(q_i^{[j]}(k+1)))$$

Therefore, as an extension, if the ILDMPC problem (36) is successfully optimized with the solution $\mathbf{U}_i^{[j],*}(k)$ and the corresponding state $\mathbf{X}_i^{[j],*}(k)$, there will be a feasible domain containing solutions $\mathbf{U}_i^{[j],\dagger}(k+1)$ and satisfactory states $\mathbf{X}_i^{[j],\dagger}(k+1)$. This conclusion holds with the assumption that the affects brought by stochasticity and randomness from the vehicle model and traffic can be described by a bounded disturbance $z_i^{[j]}(k)$, and the bounded disturbance can always be compensated by the control input within the control space.

2. A primary analysis about the stability considering the disturbance brought by the stochasticity and randomness from the vehicle model and traffic.

   Similarly, considering the disturbance, a primary analysis is conducted as follow. With the definition of the loss function (35) and Lyapunov function (40), the difference of the Lyapunov function for the discrete system $X(k+1) = f(X(k), u(k), r(q(k)))$ can be scaled as (43). For the discrete system $X(k+1) = f(X(k), u(k), r(q(k))) + Ez(k)$, the difference is rewritten as follow

$$\begin{aligned}
&J_i^{[j]}|_{k:\infty} - J_i^{[j]}|_{k+1:\infty} \\
=&\lim_{N_p\to\infty} \sum_{n=k}^{N_p+k} l_{\text{loss}}(X_i^{[j]}(n), u_i^{[j]}(n), r(q_i^{[j]}(n))) - \sum_{n=k+1}^{N_p+k+1} l_{\text{loss}}(X_i^{[j]}(n), u_i^{[j]}(n), r(q_i^{[j]}(n))) \\
=&l_{\text{loss}}(X_i^{[j]}(k), u_i^{[j]}(k), r(q_i^{[j]}(k))) \\
&- \lim_{N_p\to\infty} l_{\text{loss}}(X_i^{[j]}(k+N_p+1), u_i^{[j]}(k+N_p+1), r(q_i^{[j]}(k+N_p+1))) \\
\geq&l_{\text{loss}}(X_i^{[j]}(k), u_i^{[j]}(k), r(q_i^{[j]}(k))) \\
&- \lim_{\iota\to\infty} l_{\text{loss}}(X_i^{[j-1],\dagger}(\iota+2), u_i^{[j-1],\dagger}(\iota+2), r(q_i^{[j-1],\dagger}(\iota+2))) - \Delta_i^{\delta} \\
=&l_{\text{loss}}(X_i^{[j]}(k), u_i^{[j]}(k), r(q_i^{[j]}(k))) - \Delta_i^{\delta}
\end{aligned}$$

   where Δ_i^{δ} is an additional term brought by the disturbance and can be extended as

$$\begin{aligned}
\Delta_i^{\delta} =& \lim_{\iota, N_P\to\infty} 2\left(C\widetilde{X}_i^{[j-1],\dagger}(\iota+2)\right)^{\mathrm{T}} \mathbf{P_1}\left(\Delta_i^{[j],\text{error}} - \Delta_i^{[j-1],\text{error}}\right) \\
&+ 2\left(\widetilde{u}_i^{[j-1],\dagger}(\iota+2)\right)^{\mathrm{T}} \mathbf{P_2}\left(\Delta_i^{[j],\text{input}} - \Delta_i^{[j-1],\text{input}}\right) \\
&+ \|\Delta_i^{[j],\text{error}}\|_{\mathbf{P_1}} - \|\Delta_i^{[j-1],\text{error}}\|_{\mathbf{P_1}} + \|\Delta_i^{[j],\text{input}}\|_{\mathbf{P_2}} - \|\Delta_i^{[j-1],\text{input}}\|_{\mathbf{P_2}} \\
\Delta_i^{[j],\text{error}} =& z_i^{[j]}(k+N_P+1) - \widetilde{Y}_i^{[j]}(k+N_P+1) \\
\Delta_i^{[j-1],\text{error}} =& z_i^{[j-1],\dagger}(\iota+2) - \widetilde{Y}_i^{[j-1],\dagger}(\iota+2) \\
\Delta_i^{[j],\text{input}} =& f_i^{-}(\widetilde{X}_i^{[j-1],\dagger}(\iota+2), z_i^{[j]}(k+N_P+1), r(q_i^{[j]}(k+N_P+1))) \\
\Delta_i^{[j-1],\text{input}} =& f_i^{-}(\widetilde{X}_i^{[j-1],\dagger}, z_i^{[j-1],\dagger}(\iota+2), r(q_i^{[j-1],\dagger}(\iota+2))) \\
C =& \text{diag}(1,1,1,1,1,1,1,1,0,0)
\end{aligned}$$

Therefore, considering the unknown disturbance $z_i^{[j]}(k)$, the controlled system can be theoretically stable only if $l_{\text{loss}}(X_i^{[j]}(k), u_i^{[j]}(k), r(q_i^{[j]}(k))) - \Delta_i^{\delta} > 0$ holds. In fact, the additional term Δ_i^{δ} is quite small because it mainly calculated from the differences of differences, which are relatively negligible in the control process. However, for a rigorous theoretical proof, there are more efforts required for terminal cost design to guarantee $l_{\text{loss}}(X_i^{[j]}(k), u_i^{[j]}(k), r(q_i^{[j]}(k))) - \Delta_i^{\delta} > 0$, which is planned in the future work.

3. An exploratory simulation about the potential considering the disturbance brought by the stochasticity and randomness from the vehicle model and traffic.

   To support our analysis about the systems with disturbance, simulations are carried out with random disturbance $z_i^{[j]}(k)$ obeying the average distribution, whose boundaries are chosen as $\widetilde{z}$ satisfying $|z_i^{[j]}(k)| < \widetilde{z}$. $\widetilde{z}$ is extended as follow

$$\widetilde{z} = [0.05, 0.05, 0.01, 0.05, 0.05, 0.005, 0.025, 0.025]^{\mathrm{T}}$$

   where the detail values represent the maximum disturbance for position is less than 0.1m, the maximum disturbance for velocity is less than 0.1m/s, the maximum disturbance for acceleration is less than 0.05m/s^2, the maximum disturbance for heading location is less than 0.02rad, the maximum disturbance for yaw rotation is less than 0.01rad/s.

   The results are shown as Fig .1, Fig .2, Fig .3 and Fig .4. As shown in Fig .1 with the disturbance, controllers are successful to guarantee path tracking within the given lane, and maintain a desired inter-vehicle distance and identical velocity. For the existing of disturbance, ILDMPC can not perform obvious improvement because the random disturbance play an important part in lane-keeping process. As shown in Fig .2, ILDMPC and DMPC still have better performances compared with the feedback controller on path tracking, which results in smaller tracking error and shorter traveling distance.

   As shown in Fig .3 and Fig .4, ILDMPC have the best performance compared with the DMPC and feedback controller on platoon control, which results in smaller platooning error and lower air resistance. In detail, for the desired inter-vehicle distance, ILDMPC performs closer to the reference value, which is similar for the 3 followers. For the identical velocity, ILDMPC performs closer to the reference value and has more stable profile, which shows improvement for both the leader and followers.

   Quantifying by the integrated indicator, the comparison is shown as Fig .5, where the ILDMPC has the best performance comparing with the feedback controller and DMPC. More detail results are uploaded and accessible at https://github.com/ZNianHua/ILDMPC.

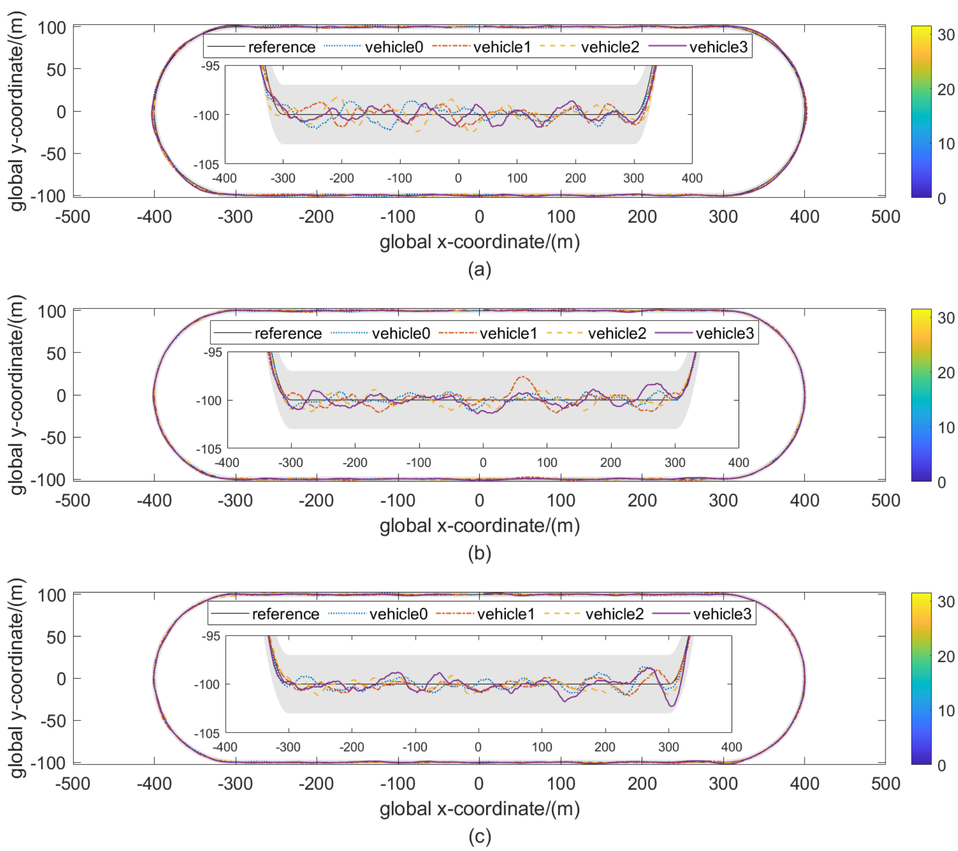

Figure 1: Traveling trajectories of the autonomous vehicle platoon with (a) the feedback controller (b) the traditional DMPC (c) the ILDMPC.

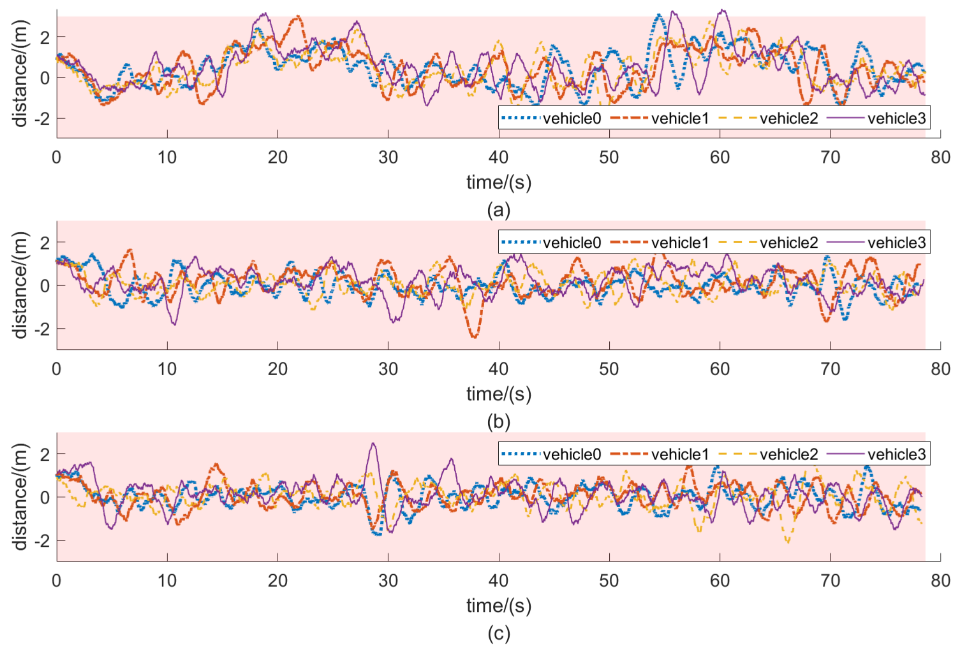

Figure 2: Lateral distances of the autonomous vehicle platoon with (a) the feedback controller (b) the traditional DMPC (c) the ILDMPC.

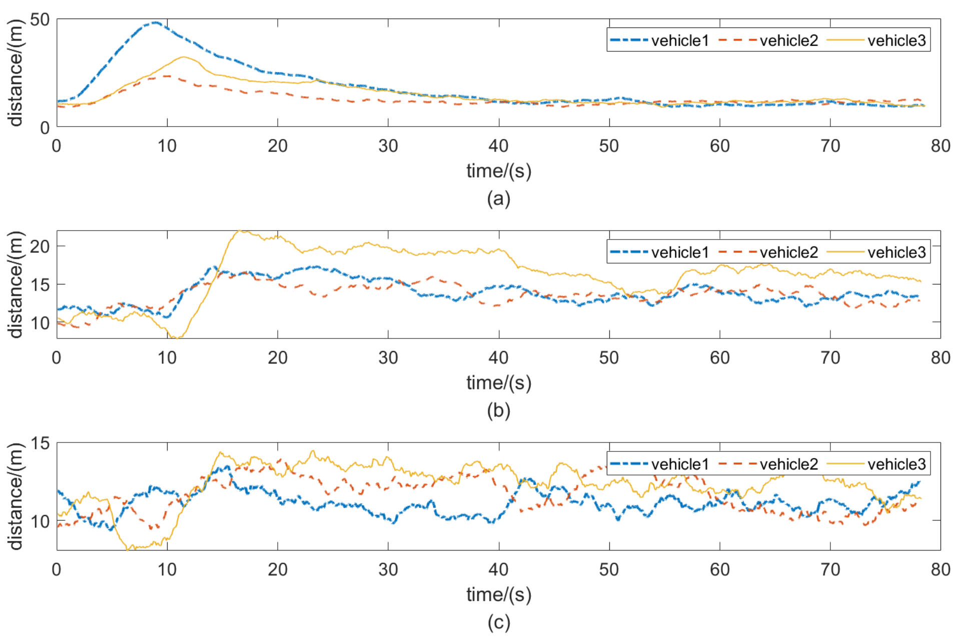

Figure 3: Inter-vehicle distances of the autonomous vehicle platoon with (a) the feedback controller (b) the traditional DMPC (c) the ILDMPC.

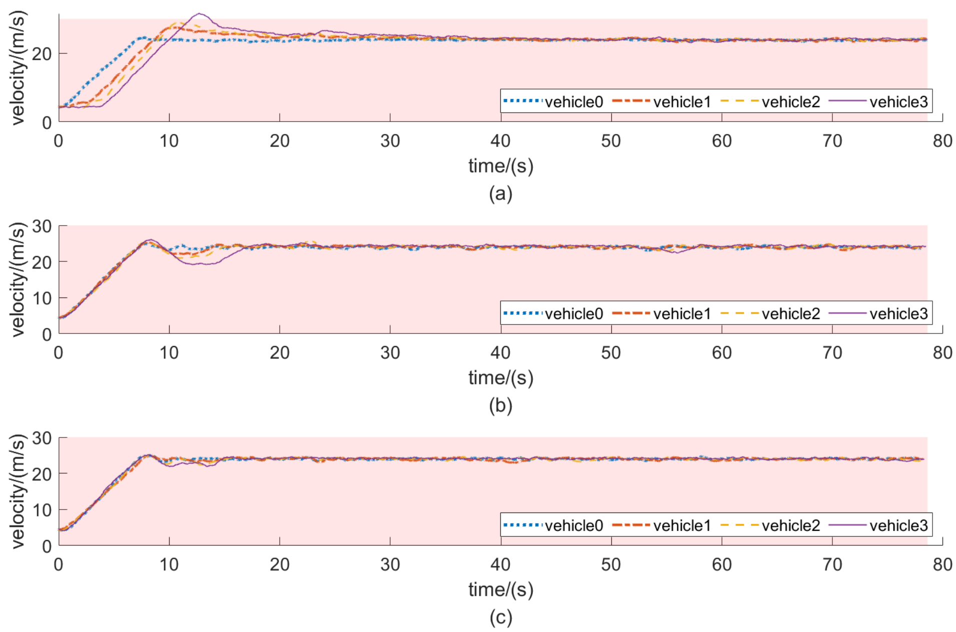

Figure 4: Traveling velocities of the autonomous vehicle platoon with (a) the feedback controller (b) the traditional DMPC (c) the ILDMPC.

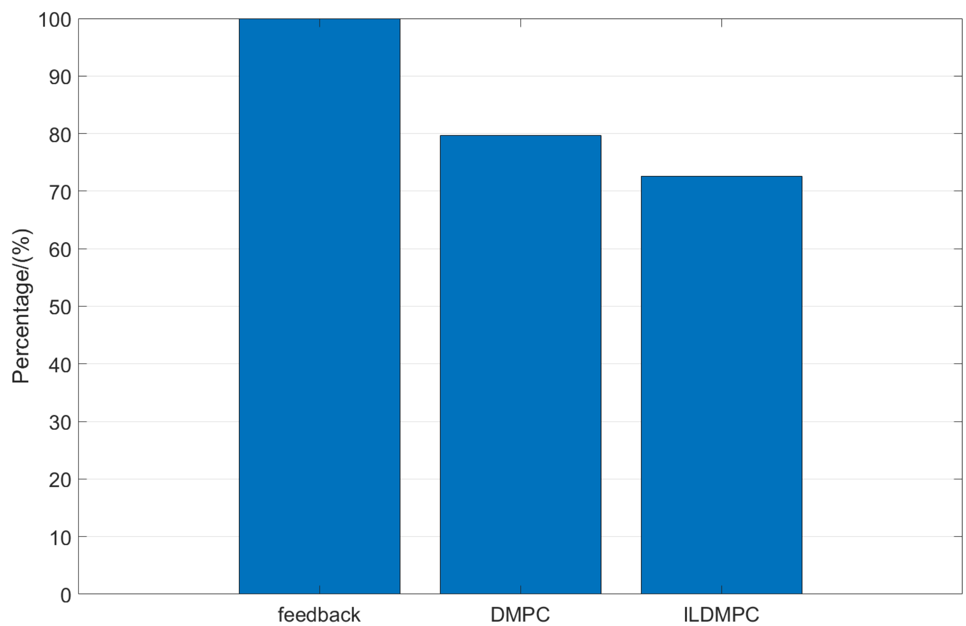

Figure 5: Performance comparison of controllers.