
Market Finance

PROJECT : OPTIONS, DATA SET 2

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March 2025

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I. Equity + Put Strategy

1.1. Objective of Our Strategy and Qualitative Reaction

1.1.1. Objective of Our Strategy

Our Stocks + Put strategy aims to protect our investment against a decline in the index. To achieve this, we purchase one unit of the stock index, allowing us to benefit from market growth if the index rises. At the same time, we buy a Put option at 90% of the current price to safeguard against a potential drop in the index. This approach enables us to limit potential losses while maintaining exposure to possible market gains.

1.1.2. Reaction to Index Movements

The value of our portfolio depends on two key factors: the evolution of the index price and the price of the Put option.

- If the index increases, our stocks appreciate, generating a profit, although the Put loses value.
- If the index experiences a slight decrease, we incur a loss on the stocks, but the Put starts gaining value, mitigating the negative impact on our portfolio.
- In the event of a sharp decline in the index, the loss on our stocks is significant, but the Put appreciates considerably, effectively protecting us against excessive downturns.

1.1.3. Reaction to Implied Volatility Movements

When implied volatility rises, the Put price increases, strengthening our strategy's protection. However, this also implies a higher purchase cost. Conversely, when implied volatility decreases, the Put price declines, making our hedge less effective.

1.2. Estimation of P&L using sensitivities and the Black-Scholes formula

The tables below present the P&L calculation results obtained using two distinct approaches: the use of sensitivities or risk coefficients and the Black-Scholes model.

Table 1: Estimation of P&L using sensitivities

Estimation P&L		Implied Volatility Variation							
		-10%	-5%	-1%	0%	1%	5%	10%	
Index return	-39,62715095	-387,7683821	-326,9181958	-278,2380468	-266,0680095	-253,8979723	-205,2178232	-144,3676369	
	-10%	-260,2772083	-199,427022	-150,746873	-138,5768357	-126,4067985	-77,72664943	-16,87646313	
	-5%	-150,9334455	-90,08325921	-41,40311017	-29,23307291	-17,06303565	31,61711339	92,4672997	
	0%	-122,5765571	-61,72637076	-13,04622171	-0,876184453	11,29385281	59,97400185	120,8241881	
	1%	-93,8112895	-32,9611032	15,71904584	27,8890831	40,05912036	88,7392694	149,5894557	
	5%	25,33357171	86,18375801	134,8639071	147,0339443	159,2039816	207,8841306	268,7343169	
	10%	183,453178	244,3033643	292,9835133	305,1535506	317,3235878	366,0037369	426,8539232	

Table 2: Estimation of P&L using the Black-Scholes Formula

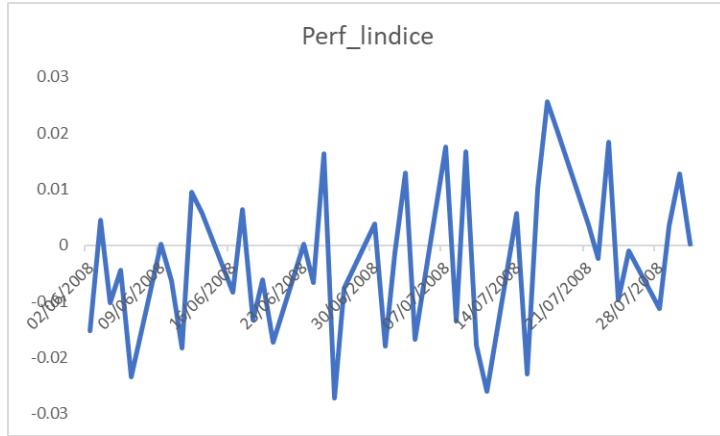
Estimation P&L		Implied Volatility Variation							
		-39,55	-10%	-5%	-1%	0%	1%	5%	10%
Index Return	-10%	-394,3195934	-329,2634385	-276,8067381	-263,6652389	-250,5175776	-197,900827	-132,1735305	
	-5%	-262,1609969	-201,5442986	-151,0679257	-138,2753237	-125,4294368	-73,63020046	-8,271014394	
	-1%	-144,0193654	-89,05282448	-41,46506935	-29,23432228	-16,89748173	33,30880351	97,43547721	
	0%	-113,0306822	-59,66589621	-12,93794337	-0,878959597	11,30057707	61,00035666	124,7069168	
	1%	-81,52470511	-29,81381066	16,00972074	27,88635484	39,89830457	89,05367728	152,2998063	
	5%	49,0672585	93,85314218	135,7121135	146,7721934	158,0278174	204,6795127	265,7249808	
	10%	220,5264908	256,6259694	293,0866211	302,995152	313,1710978	356,1479126	413,794668	

The P&L determined using the sensitivities method is -39.63, while the one calculated with the Black-Scholes formula is -39.55.

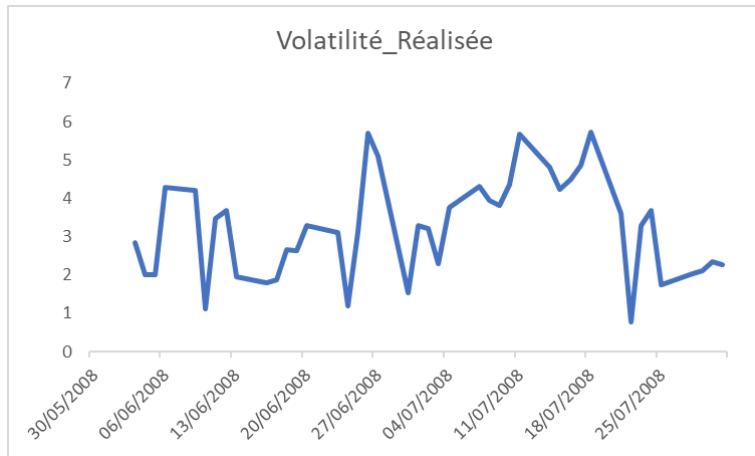
This difference arises from the use of two distinct approaches. On one hand, the Black-Scholes model relies on a mathematical framework that, while relevant, simplifies market dynamics. On the other hand, the sensitivities approach is based on historical data, making it more reflective of a dynamic and evolving market reality.

1.3. The index performance and its daily realized volatility over the period

The chart below illustrates the evolution of the daily returns of the Euro Stoxx 50 over the considered period.



The chart reveals high volatility in returns, with alternating periods of gains and losses. Indeed, multiple positive and negative spikes indicate an unstable market during this period, with instability often triggered by macroeconomic events, central bank announcements, or financial crises. The observed volatility reflects investor uncertainty, which leads to significant price fluctuations. Moreover, a return to calmer conditions could have a notable impact on investment strategies, particularly those involving options, as volatility increases option prices and the cost of hedging.



The following chart illustrates the dynamics of the index's realized volatility over the study period.

Realized volatility reflects the magnitude of the fluctuations experienced by the index. Periods of tension, characterized by sharper variations, alternate with phases of relative calm. Peaks highlight moments of uncertainty and concern, causing market turbulence and signaling heightened investor anxiety. A sudden rise in volatility is typically associated with economic announcements, financial crises, or political decisions that can impact the market. Conversely, low volatility indicates observed stability, supported by a climate of confidence among economic players. Analyzing volatility helps anticipate different types of risks and

adjust investment strategies based on variations, which can sometimes persist. Sustained realized volatility may signal a bearish trend or ongoing instability in financial markets.

Comparison with Strategy Parameters

The strategy, which involves purchasing a stock while simultaneously buying a put option with an average strike of 90%, aims to **limit losses** in the event of a decline in the underlying asset. The protection is most effective when **volatility is high**, as the put becomes particularly expensive and its value increases in a market where stocks experience significant drops. Conversely, in a **low-volatility environment**, the put's cost may negatively impact the overall profitability of the strategy.

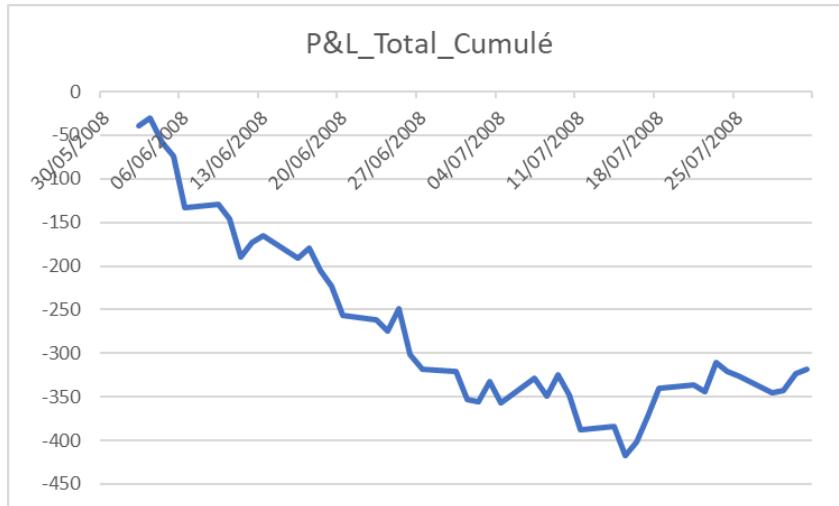
Expected Qualitative Results

- **In absolute terms:** The expected performance will depend on the **index dynamics and its volatility level**. During **high volatility**, losses will be better absorbed when the index declines. However, in **calm market conditions**, profitability may be significantly affected by the cost of the protective put.
- **Relative to the index:** The strategy is expected to **underperform** the index during bullish phases due to the cost of the put. However, it should **outperform** in bearish markets, benefiting from the protective effect of the option.

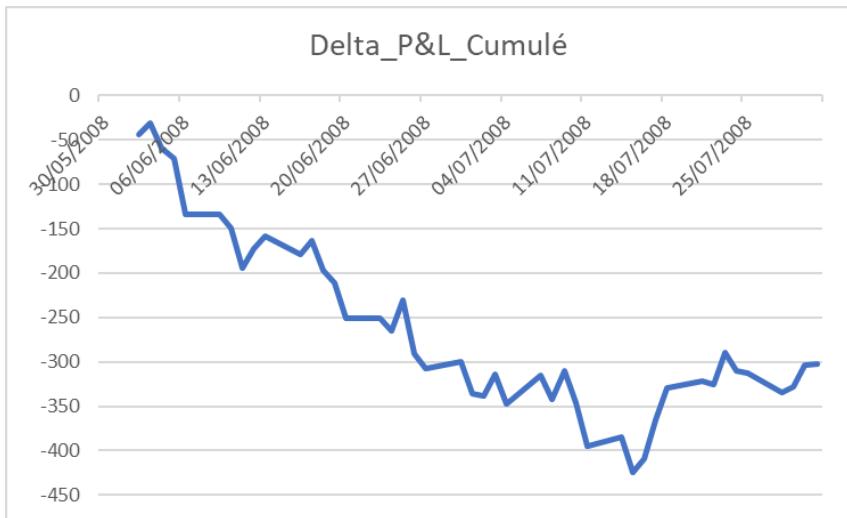
1.4. Backtesting of the strategy

Analysis of the Global P&L Evolution and Contribution of Risk Factors to Strategy Performance

- This **Cumulative Total P&L** graph represents the evolution of the overall performance of the strategy over the entire period. It shows whether our strategy has generated a gain or a loss over time.

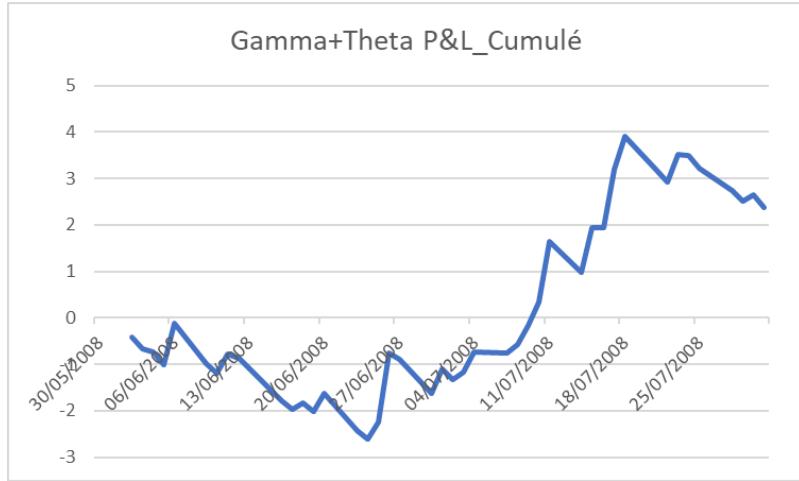


We observe that, from the very beginning, the **Cumulative Total P&L** drops sharply, indicating that our strategy quickly lost significant value. Subsequently, there are some attempts at stabilization and recovery, but they are insufficient to offset the accumulated losses. The overall trend remains negative, suggesting that the parameters of our strategy were not optimal for adapting to market conditions.



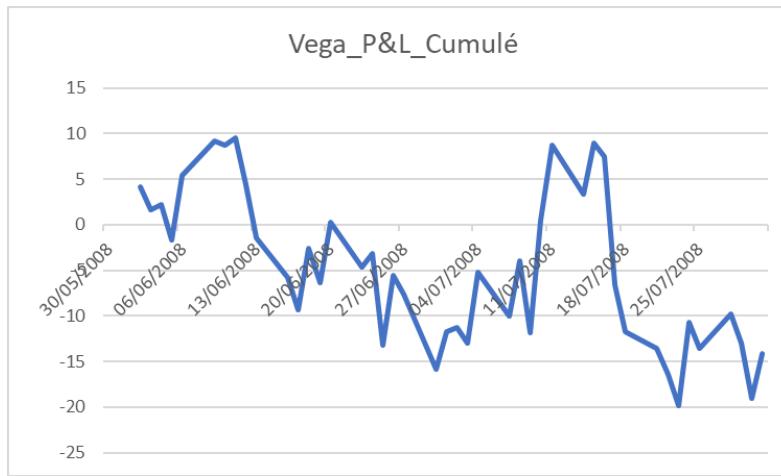
The following **Cumulative Delta P&L** graph measures the impact of index variations on our P&L. It indicates whether our directional market exposure has generated gains or losses over time.

We observe that the **Cumulative Delta P&L** follows almost exactly the same trend as the **Cumulative Total P&L**, indicating that the overall losses are largely driven by unfavorable index movements. From the very beginning, the graph drops sharply, proving that our Delta position was poorly calibrated to capture the market trend. There are a few recovery phases where the index moved in a more favorable direction, but they were not enough to reverse the negative trend.



The **Cumulative Gamma + Theta P&L** graph below illustrates the combined impact of **Gamma** and **Theta** on our P&L. **Theta** represents the loss of option value due to time decay, while **Gamma** measures the sensitivity of Delta to changes in the index.

Unlike the other curves, this one shows a slight **positive trend** towards the end of the period. This means that, although **Theta** caused a continuous loss by gradually reducing the value of our options each day, **Gamma** progressively offset this loss. Towards the end of the period, Gamma played a more significant role, helping to **limit the losses**. However, this effect came too late to reverse the overall negative trend of the strategy.

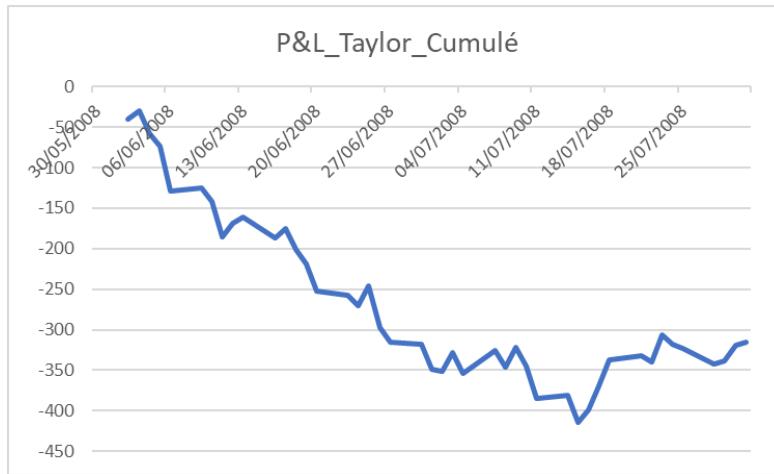


The **Cumulative Vega P&L** graph illustrates the impact of **implied volatility fluctuations** on our P&L. It helps us understand how our strategy's sensitivity to volatility has influenced our gains and losses.

The graph is highly volatile, indicating that our strategy was **strongly influenced by fluctuations in implied volatility**. We observe that when volatility increased, the **Vega P&L** was positive, meaning the strategy generated gains. Conversely, when volatility decreased,

the losses were significant. Towards the end of the period, the graph becomes more negative, suggesting that volatility declined, reducing the value of our options and negatively impacting the P&L.

The following **Cumulative Taylor P&L** graph is an approximation based on our strategy's sensitivities (**Delta, Gamma, Vega, and Theta**). It allows us to compare the **theoretical evolution** of the P&L with the **actual P&L**.



We observe that the **Cumulative Taylor P&L** generally follows the same trajectory as the **Cumulative Total P&L**, but with a discrepancy in the final value. This indicates that **estimations based on sensitivities do not perfectly capture real market conditions**. Certain market adjustments may have occurred that were not fully accounted for in this approximation. This demonstrates that relying solely on sensitivities is not always sufficient to accurately predict the **evolution of the P&L**.

In conclusion, the analysis of these graphs reveals that the **overall losses** of the strategy were primarily driven by the **negative impact of Delta**, while **Gamma provided a slight stabilizing effect**, and **Vega introduced significant fluctuations**, though they were not enough to offset the losses. Our strategy was penalized by a **misaligned Delta exposure**, a **constant erosion of the Put's value due to Theta**, and a **decline in volatility that reduced Vega-related gains**.

II. Equity + Put Spread Strategy

2.1. Objective & Qualitative Reactions

2.1.1. Objective

In this section, we analyzed the **Equity + Put Spread Strategy**, which consists of the following components:

- **Purchase of a stock index unit:** A long position in the **Euro Stoxx 50 index** is established.
- **Purchase of a Put Spread on the stock index:** A put option is **purchased at 90% of the index value** (strike bought), while another put option is **sold at 80% of the index value** (strike sold).

The main objective of this strategy is to provide a cost-effective hedge against moderate declines while maintaining exposure to the index. The strategy secures market gains by purchasing the index, while the put spread which consists of purchasing a put at 90% and selling a put at 80% of the index value offers partial downside protection.

In comparison to a single put, this structure reduces hedging costs and limits losses within the 90%-80% range. It is appropriate for investors who are moderately optimistic and are interested in risk management without completely foregoing upside potential.

2.1.2. Qualitative Reactions to Index Moves

Absolute Reaction to Index Movements

- **Index Rises Above 90%:** The put spread expires worthless, and the strategy performs like a direct investment in the index. The investor benefits fully from price appreciation.
- **Index Falls Between 90% and 80%:** The put spread starts gaining value, partially offsetting losses from the index decline. The strategy outperforms a direct index position due to the hedge.
- **Index Falls Below 80%:** The put spread reaches its maximum payout. Further declines result in additional losses, as downside protection is no longer effective beyond the lower strike.

Relative Reaction to Index Movements

- **Above 90%:** The strategy performs **identically** to the index.
- **Between 90% and 80%:** The strategy **outperforms** a pure index position, as the put spread cushions losses.
- **Below 80%:** The strategy **underperforms a fully hedged position**, as losses from

the index continue beyond the put spread's protective range.

2.1.3. Qualitative Reactions to Movements in Implied Volatility

Absolute Reaction to Implied Volatility Movements

- **Increase in Implied Volatility:**

- The value of both the long put (90%) and the short put (80%) increases.
- Since the long put is at a higher strike, it gains more value than the short put.
- As a result, the put spread appreciates, enhancing downside protection.
- The cost of entering the strategy (if established after a volatility rise) would be higher.

Decrease in Implied Volatility:

- The value of both put options declines, reducing the hedge's effectiveness.
- The long put loses more value than the short put, decreasing the net value of the put spread.
- If volatility drops after the strategy is implemented, downside protection becomes weaker.

Relative Reaction to Implied Volatility Movements

- **In Stable or Rising Markets:**

- Implied volatility tends to be lower. The put spread has minimal value, and the strategy behaves similarly to a direct index investment.
- The hedge remains inexpensive but less effective if a sudden downturn occurs.

In Declining Markets:

- Implied volatility usually rises, increasing the value of the put spread.
- The hedge becomes more effective, mitigating index losses.
- However, because the short put (80%) also gains value, the benefit of increased volatility is capped compared to a standalone put.

2.2. Sensitivities & Daily P&L Matrix

2.2.1. Calculating the Greeks at Inception

On the first day of the strategy, we compute the **Delta**, **Gamma**, **Theta**, and **Vega** of each option using the Black–Scholes formula. Since our strategy is:

- +1 Euro Stoxx 50 index
- +1 Put at 90% money
- -1 Put at 80% money

The **net Greeks** are:

$$\text{Delta_Spread} = 1 + \text{Delta(Put_90\%)} - \text{Delta(Put_80\%)} = 0.898376$$

This indicates that the overall position has a **directional exposure** slightly below 1. Therefore, for small changes in the underlying index, the portfolio's value will not shift as drastically as holding a full share of the index. The long 90% put dampens some downside exposure, while the short 80% put reintroduces a portion of that Delta, resulting in a net Delta under 1.

Gamma_Spread = Gamma (Put_90%) – Gamma (Put_80%) = **0.000095** ⇒ The net Gamma is **very low**, reflecting **limited convexity**. If the market moves significantly, the strategy's Delta will not adjust as sharply as a single long put would. This is because the short 80% put offsets part of the 90% put's Gamma, yielding a position that is overall less "convex."

Theta_Spread = Theta (Put_90%) – Theta (Put_80%) = **-13.695549** ⇒ The net Theta is **negative**, signifying a **daily time decay** cost. While selling the 80% put helps collect premium and thereby reduces Theta relative to a standalone long put, the overall position still loses value over time, since there is a long put in the mix.

Vega_Spread = Vega (Put_90%) – Vega (Put_80%) = **343.446988** ⇒ With a **positive and relatively large Vega**, the strategy is effectively **long volatility**. If implied volatility rises, the 90% put gains more than the 80% put loses, so the net effect is beneficial. Still, the short put constrains the overall volatility upside compared to holding just the long put.

Here, "+1" in Delta_Spread accounts for the action (the index). We store these initial sensitivities to see how exposed the position is to changes in price (Delta, Gamma), time decay (Theta), and volatility (Vega). Putting all these together, we see that the (Equity + Long Put 90% – Short Put 80%) configuration is **less directional** (Delta < 1) than the pure index, has **reduced convexity** (low Gamma), incurs **negative time decay** (Theta), but maintains a **positive Vega** bias, benefiting to a degree from volatility increases.

2.2.2. Building a Daily P&L Matrix with Spot/Vol Variations

We then create a **scenario table** where:

Rows represent hypothetical changes in implied volatility (-10%, -5%, -1%, 0%, +1%, +5%, +10%) and **Columns** represent hypothetical changes in the performance of the index (-10%, -5%, -1%, 0%, +1%, +5%, +10%).

For each combination (ΔVol , $\Delta Spot$),

Greeks Approximation :

$$\Delta P\&L \approx \text{Delta_Spread} * \Delta S + \frac{1}{2} \text{Gamma_Spread} * (\Delta S)^2 + \text{Theta_Spread} * \Delta t + \text{Vega_Spread} * \Delta \sigma.$$

This approach linearly (or quadratically, via Gamma) estimates how the strategy's value changes given small variations in the index price (ΔS) and implied volatility ($\Delta \sigma$).

Estimation P&L	-50,0471476	Implied Volatility Variation						
		-10%	-5%	-1%	0%	1%	5%	10%
Index Return	-10%	-367,115652	-349,943554	-336,2058755	-332,7714559	-329,3370363	-315,5993579	-298,4272598
	-5%	-202,4722667	-185,3001687	-171,5624903	-168,1280706	-164,693651	-150,9559726	-133,7838746
	-1%	-68,32948104	-51,15738299	-37,41970455	-33,98528495	-30,55086534	-16,8131869	0,358911146
	0%	-34,45655163	-17,28445358	-3,546775146	-0,112355537	3,322064072	17,05974251	34,23184055
	1%	-0,448729027	16,72336902	30,46104746	33,89546706	37,32988667	51,06756511	68,23966316
	5%	136,9314933	154,1035914	167,8412698	171,2756894	174,710109	188,4477874	205,6198855
	10%	311,6918681	328,8639661	342,6016446	346,0360642	349,4704838	363,2081622	380,3802603

2.2.3. Black–Scholes Revaluation:

We recalculate the **price of each put** under the new spot ($S+\Delta S$) and new volatility ($\sigma+\Delta\sigma$) still adjusting T by one day if needed. The **new strategy value** = $(S+\Delta S) + \text{new Put (90\%)} \text{ price} - \text{new Put (80\%)} \text{ price}$. The **P&L** is the difference between this “exact revaluation” and the original strategy value.

Estimation P&L	-50,43	Implied Volatility Variation						
		-10%	-5%	-1%	0%	1%	5%	10%
Index Return	-10%	-364,736706	-346,5471567	-335,0237707	-332,448106	-329,9744424	-320,9466907	-311,1752908
	-5%	-206,6423333	-185,2212118	-171,1901275	-168,0277965	-164,9855237	-153,8649283	-141,8589528
	-1%	-74,44954012	-52,48119872	-37,41952412	-33,9735201	-30,6440942	-18,37878505	-5,028322044
	0%	-40,67034965	-18,80146413	-3,606152257	-0,113560985	3,26547835	15,74579366	29,37153754
	1%	-6,619830889	15,06885287	30,35221469	33,88223705	37,30248879	49,97050076	63,84779002
	5%	132,0585052	152,3620654	167,5883839	171,1836605	174,6906475	187,8517041	202,5067629
	10%	310,0662606	327,6529915	342,0720001	345,5882962	349,0528884	362,3176972	377,4731072

2.2.4. Why the difference in results?

The differences between Table 1 (Sensitivity-Based P&L Estimation) and Table 2 (Black–Scholes P&L Estimation) arise from the non-linearity of options pricing and the interaction between the long and short put positions in the put spread. The sensitivity-based method uses Delta and Vega to approximate P&L changes, assuming linear adjustments, while the Black–Scholes model fully revalues the put spread, incorporating Gamma, Theta, and dynamic Vega adjustments.

For the put spread strategy, the key source of difference is the offsetting effect between the long put (90%) and the short put (80%), which creates non-linear pricing behavior that the sensitivity-based method does not fully capture.

Gamma Effect on the Put Spread: In large index declines (-10%, -5%), Table 1 overestimates losses (e.g., at -10% index return and 0% volatility, Table 1 estimates -332.771 vs. -332.448 in Table 2). This happens because the sensitivity method assumes a constant Delta, while in reality, Delta changes non-linearly as both the long put (90%) and short put (80%) move deeper in the money. Black–Scholes dynamically adjusts for this, making it more accurate.

Vega Sensitivity on the Put Spread: When implied volatility drops, Table 1 underestimates losses (e.g., at -10% index return and -10% volatility, Table 1 estimates -367.116 vs. -364.737 in Table 2). The sensitivity method assumes a fixed Vega impact, but Black-Scholes correctly reflects how the long put (90%) loses more value than the short put (80%) gains, because the short put, being further out of the money, is less sensitive to volatility changes.

Theta Differences in the Put Spread: Time decay affects the two options at different rates, but the sensitivity-based approach applies a uniform Theta adjustment. Black-Scholes, on the other hand, accounts for the fact that the long put (90%) has a higher Theta decay than the short put (80%), which explains small variations in results (e.g., at 0% index return, Table 1 estimates -0.112 vs. -0.114 in Table 2).

Interaction Between the Two Puts in the Spread: The sensitivity-based approach treats each put option independently, while Black-Scholes revalues the spread as a whole, factoring in how the short put starts to reduce the effectiveness of the hedge as the index moves below 80%. This results in minor differences for moderate index moves (e.g., at -1% index return and 0% volatility, Table 1 estimates -33.985 vs. -33.974 in Table 2).

2.3. Index Performance & Realized Volatility

Over our sample period, the Euro Stoxx 50 declined by **-10.85%**, while the **average realized volatility** (calculated from the daily returns, annualized) was around **22.60%**, slightly lower than the implied volatilities used initially in our option pricing. Meanwhile, our Put Spread strategy (one share + long put at 90% – short put at 80%) achieved a **performance of about -9.29%**, thus slightly outperforming the index.

Relatively speaking, this outcome shows that selling the 80% put reduces the total cost of protection (less negative Theta than a single long put) but limits coverage if the market were to fall below 80% of the initial spot. Because the index dropped by -10.85% (remaining above or near the sold strike), the Put Spread managed to soften the decline (to about -9.29%), indicating a fairly cost-efficient hedge for a moderate down move. However, had volatility risen more sharply or had the market declined much further (below 80% of the initial spot), the protection might have been weaker than a single put. Thus, **in absolute terms**, the strategy reduced losses while containing costs; and **in relative terms**, it slightly outperformed the index, thanks to premium collection from the short 80% put.

2.4. Backtest & Greeks Decomposition

Over our sample, the **Equity + Put Spread** strategy ended with a **cumulative P&L** of about -

50, compared to **-57** for the index alone, indicating a slightly softer decline. To understand how this daily performance evolved, we **decompose** the P&L into four main components **Delta**, **Gamma+Theta**, and **Vega** now each directly visible in our table:

a. **Delta**

- For example, on **June 4, 2008**, the index dropped about **1.2%** (from 3737.34 to 3699.05).
- The **Delta P&L** for that day is approximately **+4.05**, because the long 90% put partially compensated for the action's loss, though the short 80% put slightly reduced that offset.

b. **Gamma+Theta**

- On the same **June 4**, the **Gamma+Theta** entry was about **-0.03**, indicating modest convexity/time effects.
- By selling the 80% put, the net position's convexity (Gamma) is reduced, and the daily time decay (Theta) is partially offset.

c. **Theta** (included in Gamma+Theta)

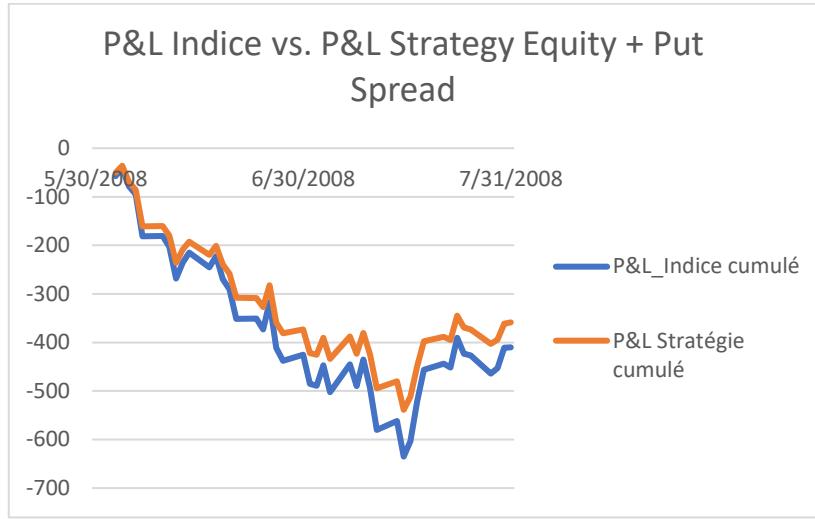
- We now record the daily **Gamma+Theta** sum directly, showing how the put sold reduces the net cost of time.
- For instance, on **June 10**, (Gamma+Theta) was around **-0.013**, whereas a single long put might have had a more negative daily decay.

d. **Vega**

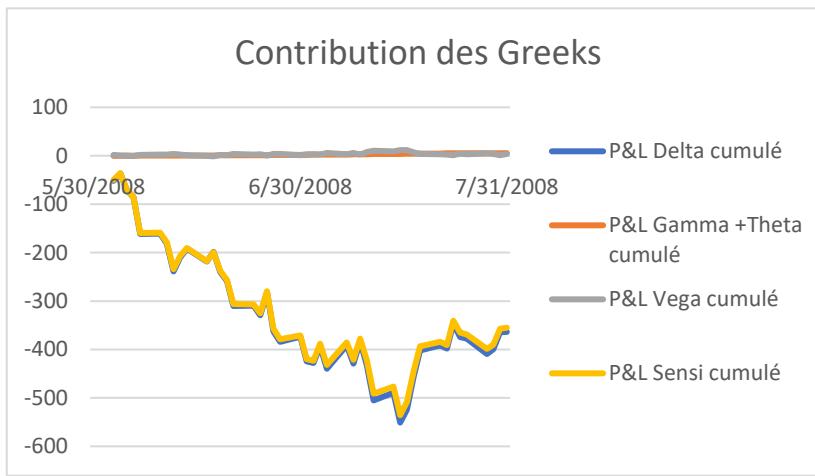
- When implied volatility changes (e.g., on **June 6**, rising from 28.4% to 28.7%), the **P&L Vega** column displays approximately **+2.25**, reflecting gains from the 90% put's volatility exposure.
- However, the short 80% put cuts into that positive Vega, leading to a net figure less than it would be with a standalone long put.

Hence, each day's **Strategy P&L** is the **sum of Delta P&L, Gamma+Theta P&L, and Vega P&L (P&L Sensi)**. Observing these fields, we confirm that although the put spread **reduces negative Theta** (cheaper daily cost), it likewise has **lower Gamma** and **lower Vega**, offering weaker gains if the index plunges or volatility spikes. The resulting **cumulative P&L** of about **-50** illustrates this "**less expensive but partially less protective**" characteristic of the Put Spread.

Visualization:



The first chart illustrates the cumulative P&L evolution for both the Euro Stoxx 50 index and the Equity + Put Spread strategy. The strategy exhibits a slightly softer decline compared to the index, as reflected by its final cumulative P&L of approximately -50 versus -57 for the index. This confirms that the Put Spread strategy successfully mitigated losses, albeit with a limited downside buffer due to the short 80% put. The protection remains effective as long as the index does not fall significantly below the sold strike level.



The second chart presents the cumulative contributions of the Greeks (Delta, Gamma+Theta, and Vega) to the strategy's overall P&L. The Delta P&L (blue line) closely tracks the overall P&L movement, reinforcing that market directionality remains the dominant factor. Gamma+Theta (orange line) stays close to zero, confirming that the short put reduces time decay but simultaneously diminishes the strategy's convexity. Vega (gray line) shows a modest contribution, reflecting the limited sensitivity of the strategy to volatility fluctuations. Lastly, the total P&L with sensitivities (yellow line) follows a trajectory almost identical to Delta, indicating that Gamma, Theta, and Vega have only marginal influences on the final outcome.

Overall, these visualizations validate the cost-efficient nature of the Put Spread, which reduces negative Theta effects while maintaining partial downside protection. However, in cases of sharp market declines or volatility surges, the hedge would have been less effective than a simple long put strategy.

III. Conclusion

The backtesting data show the trade-offs in controlling downside risk between the Equity + Put and Equity + Put Spread approaches. Benefiting from option appreciation amid market dips, the Equity + option approach offered the most protection. Its cost is therefore important, particularly in low-volatility times as it suffered more Theta decay and was more vulnerable to Vega changes.

By selling a put at 80%, the Put Spread approach lessened the hedging expenses and hence the Theta losses. This limited downside protection, therefore, reduces its efficacy in severe downturns. Although it beat the index, in volatile markets it underperformed the whole put strategy.

Delta drove performance mostly for both approaches; Theta degraded returns, especially for the full put method. As hedges, the realized volatility (22.60%) confirmed the efficiency of both strategies.

All things considered, the Equity + Put approach is better suited for complete downside protection and the Put Spread strategy provides a reasonably priced hedge for modest falls. The decision relies on market view, cost concerns, and risk tolerance.