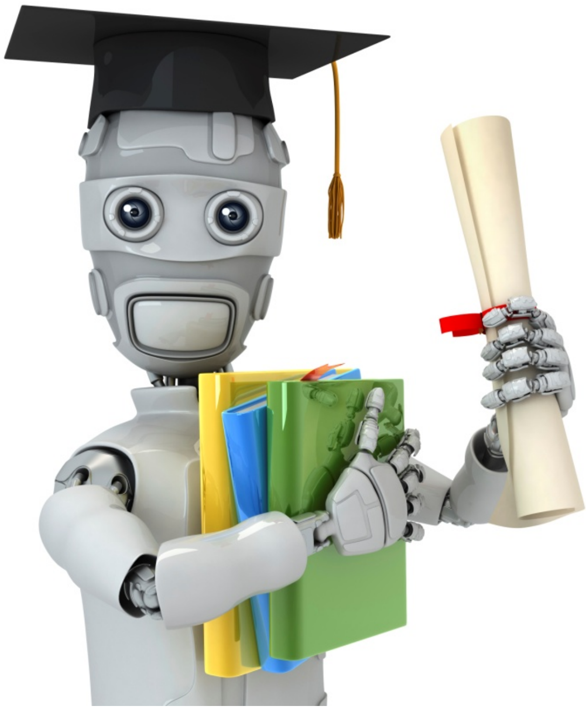


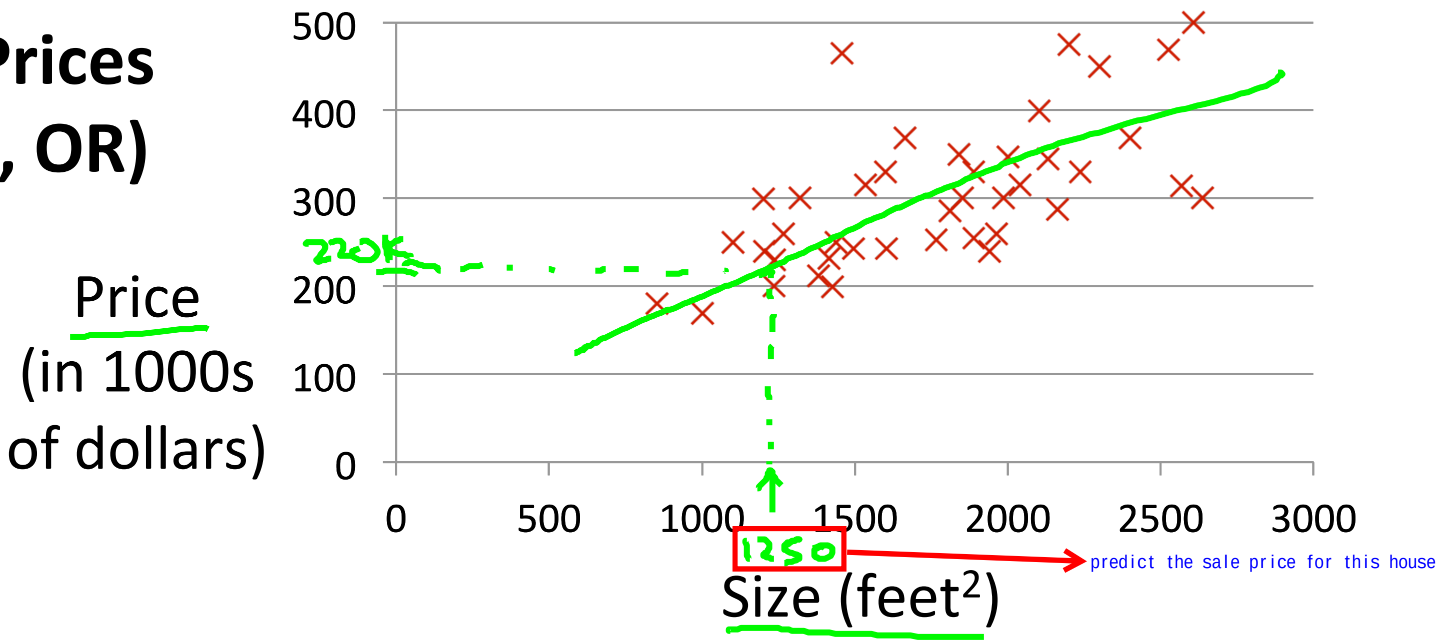
Machine Learning

supervised learning

Linear regression with one variable

Model representation

Housing Prices (Portland, OR)



Supervised Learning

Given the "right answer" for each example in the data.

Regression Problem

Predict real-valued output → continuous

Classification: Discrete-valued output

Training set of housing prices (Portland, OR)

Size in feet^2 (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178
…	…

$m = 47$

Notation:

- **m** = Number of training examples
- **x**'s = "input" variable / features
- **y**'s = "output" variable / "target" variable

(x, y) - one training example

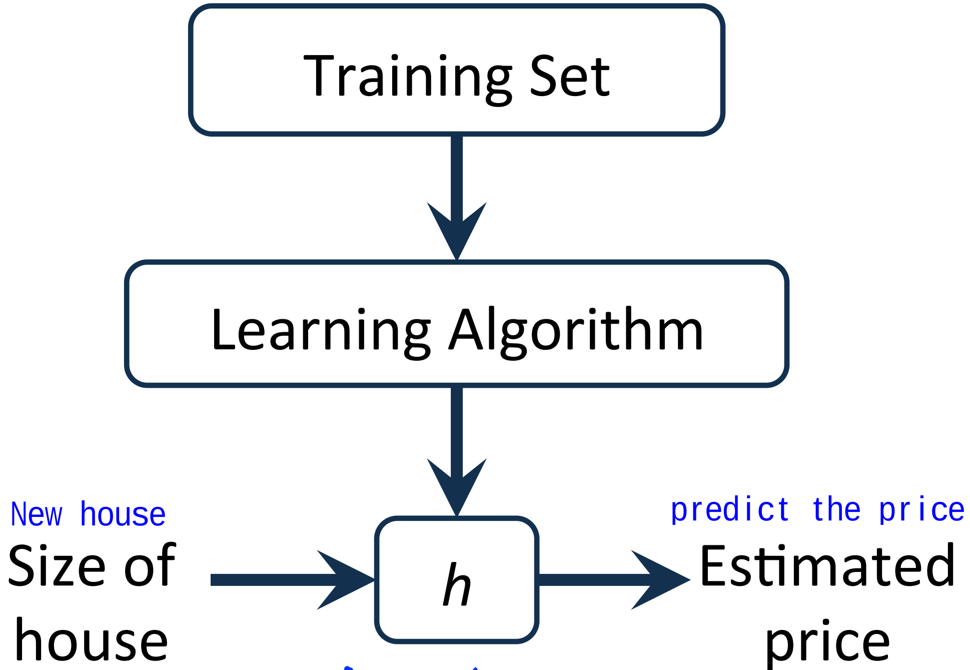
$(x^{(i)}, y^{(i)})$ - i^{th} training example

row

$x^{(1)} = 2104$

$x^{(2)} = 1416$

$y^{(1)} = 460$



$\underline{x}$ | $\underline{\text{hypothesis}}$ | $\underline{\text{(estimated value of } y)}$

h maps from x's to y's.

maps

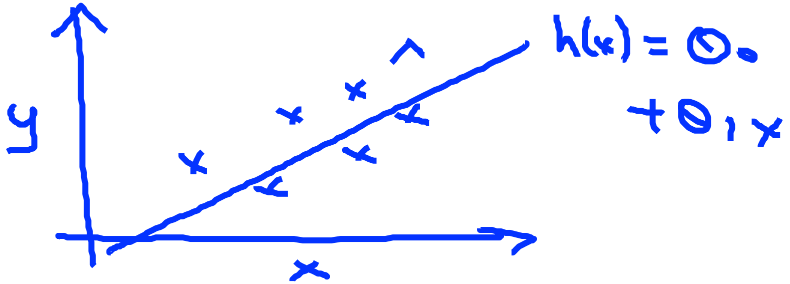
is a function

How do we represent *h* ?

assume a linear function: Simple

$$h_\theta(x) = \underline{\theta_0 + \theta_1 x}$$

Shorthand: $h(x)$

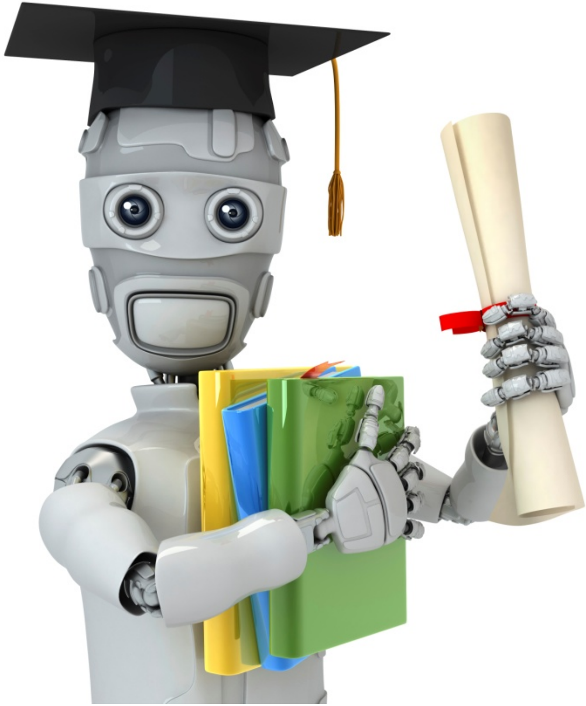


Equivalent names

Linear regression with one variable. (x)

Univariate linear regression.

└ one variable



Machine Learning

Linear regression with one variable

Cost function

Training Set

Size in feet^2 (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178
…	…

$m = 47$

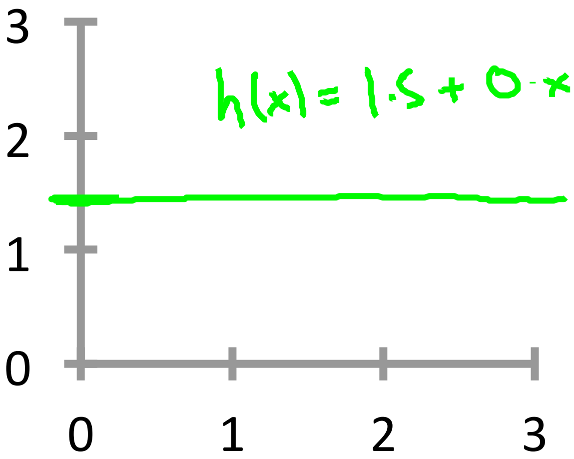
Hypothesis: $\boxed{h_\theta(x) = \theta_0 + \theta_1 x}$

θ_i's: Parameters

Model parameters

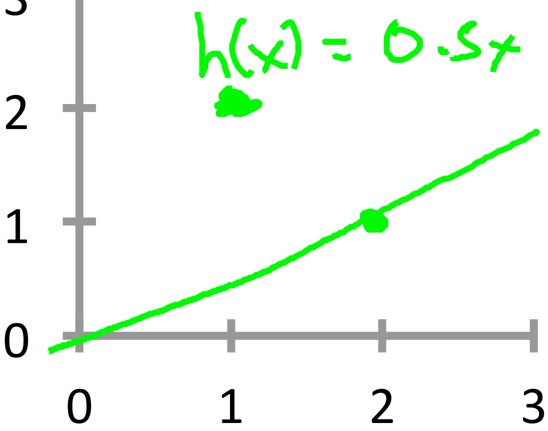
How to choose θ_i's ?

$$h_\theta(x) = \theta_0 + \theta_1 x$$



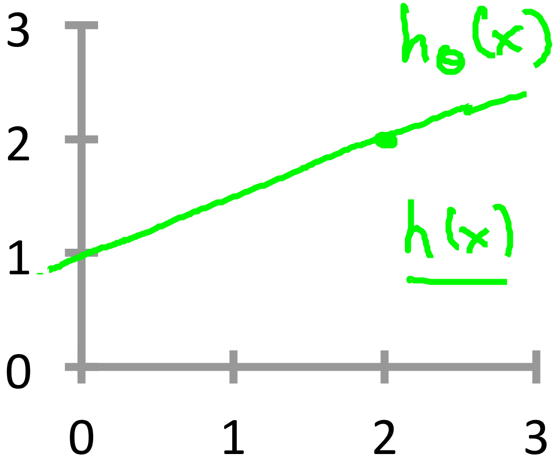
$\theta_0 = 1.5$

$\theta_1 = 0$



$\theta_0 = 0$

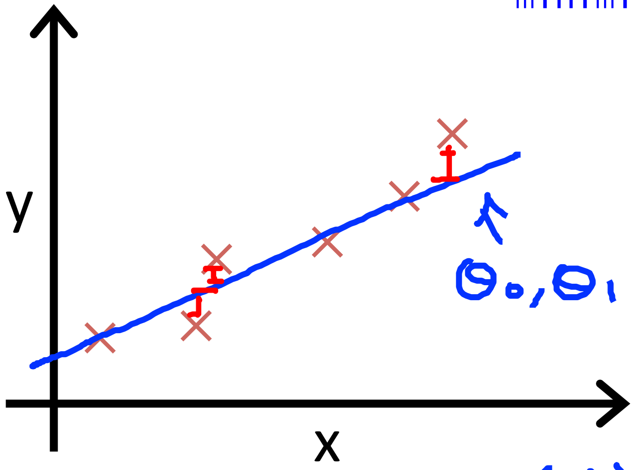
$\theta_1 = 0.5$



$\theta_0 = 1$

$\theta_1 = 0.5$

Linear regression



Optimization
minimization

$$\underset{\theta_0,\theta_1}{\text{minimize}} \ \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

\# training examples

makes math easier

$$h_\theta(x^{(i)}) = \theta_0 + \theta_1 x^{(i)}$$

$(x^{(i)}, y^{(i)})$

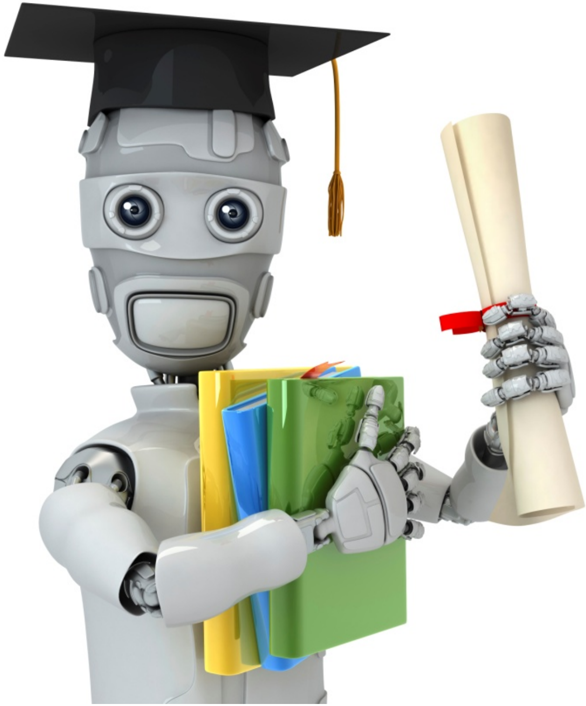
$$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ is close to y for our training examples (x, y)

x, y

$$\underset{\theta_0,\theta_1}{\text{minimize}} \ \underbrace{J(\theta_0,\theta_1)}_{\text{Cost function}}$$

Squared error function



Machine Learning

Linear regression with one variable

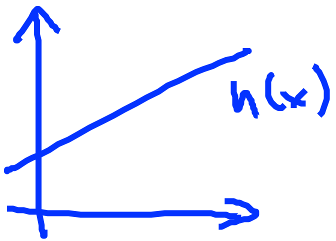
Cost function intuition I

Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Parameters:

$$\theta_0, \theta_1$$



Cost Function:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

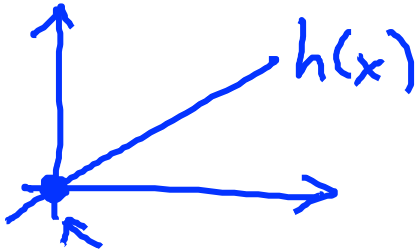
Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

Simplified

$$h_\theta(x) = \theta_1 x$$

$\theta_0 = 0$

$$\theta_1$$



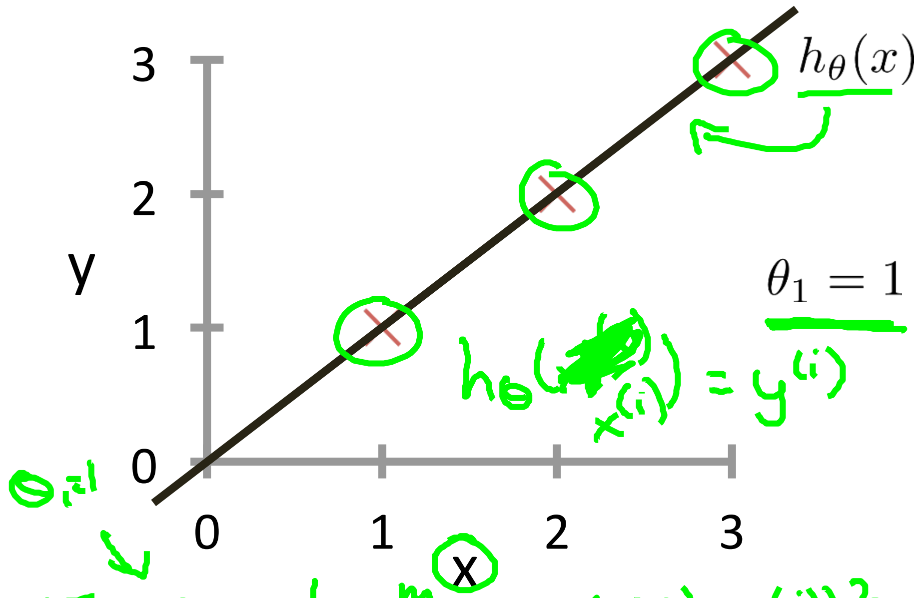
$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$\theta, x^{(i)}$

$$\underset{\theta_1}{\text{minimize}}\ J(\theta_1)$$

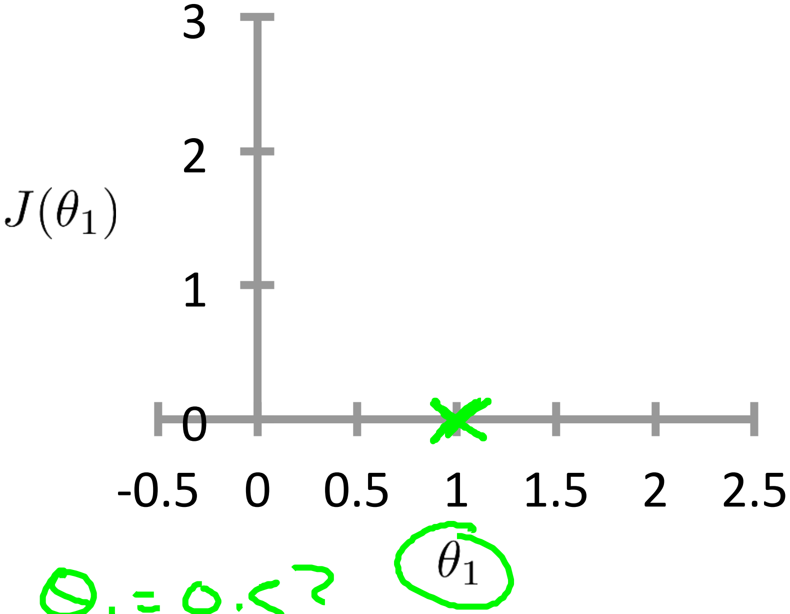
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



$J(\theta_1)$

(function of the parameter θ_1)



$$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

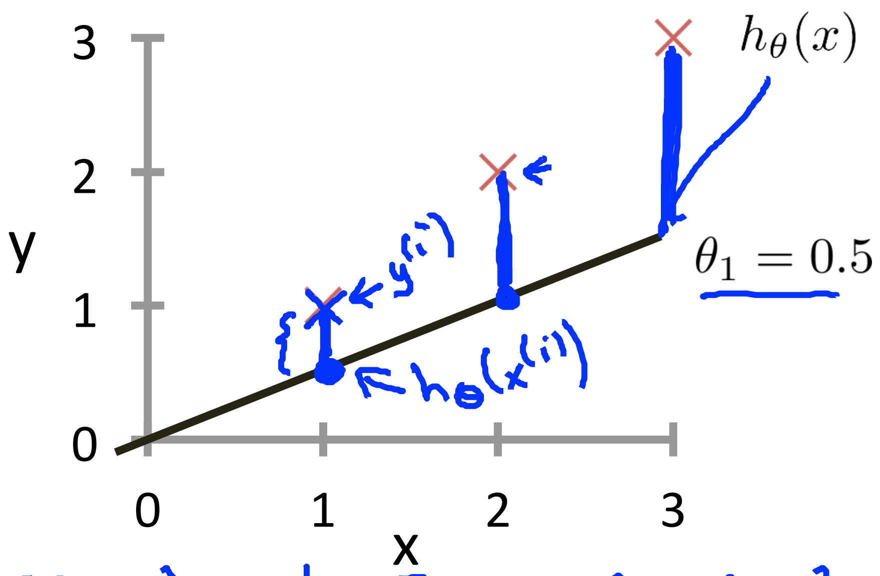
$$= \frac{1}{2m}\sum_{i=1}^{m}\left(\theta_1 x^{(i)} - y^{(i)}\right)^2 = \frac{1}{2m}(0^2+0^2+0^2) = 0$$

$\theta_1 = 0.5$?

$J(1) = 0$

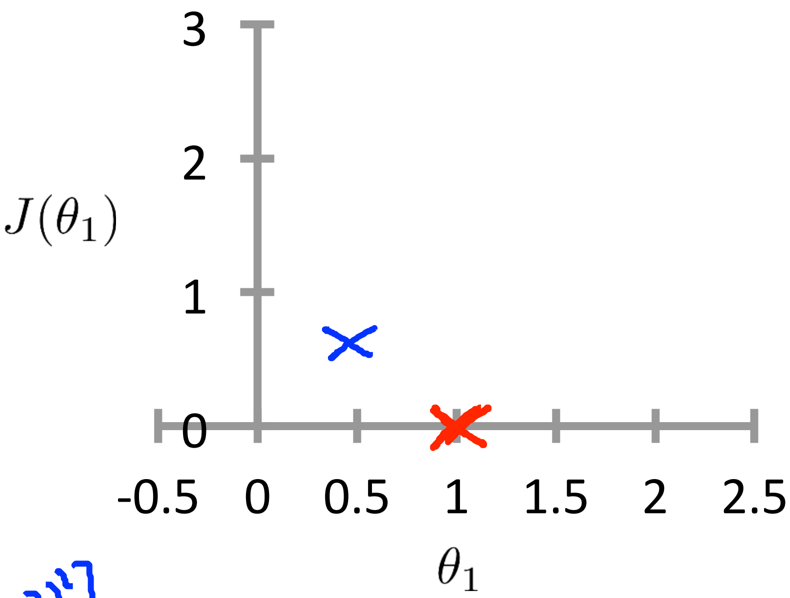
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



$J(\theta_1)$

(function of the parameter θ_1)



$$J(0.5) = \frac{1}{2m}\left[(0.5-1)^2 + (1-2)^2 + (1.5-3)^2\right]$$

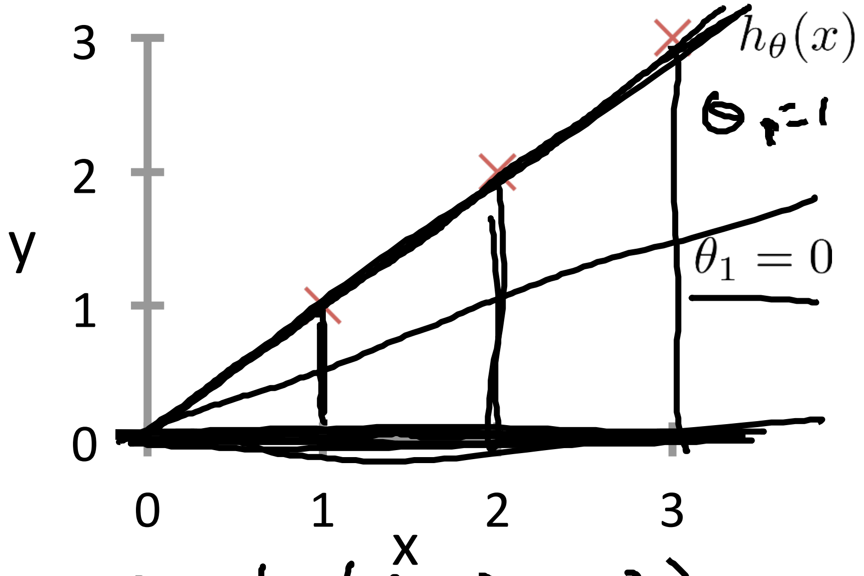
$$= \frac{1}{2\times 3}(3.5) = \frac{3.5}{6} \approx 0.58$$

$\theta_1 = 0$?

$J(0) = $?

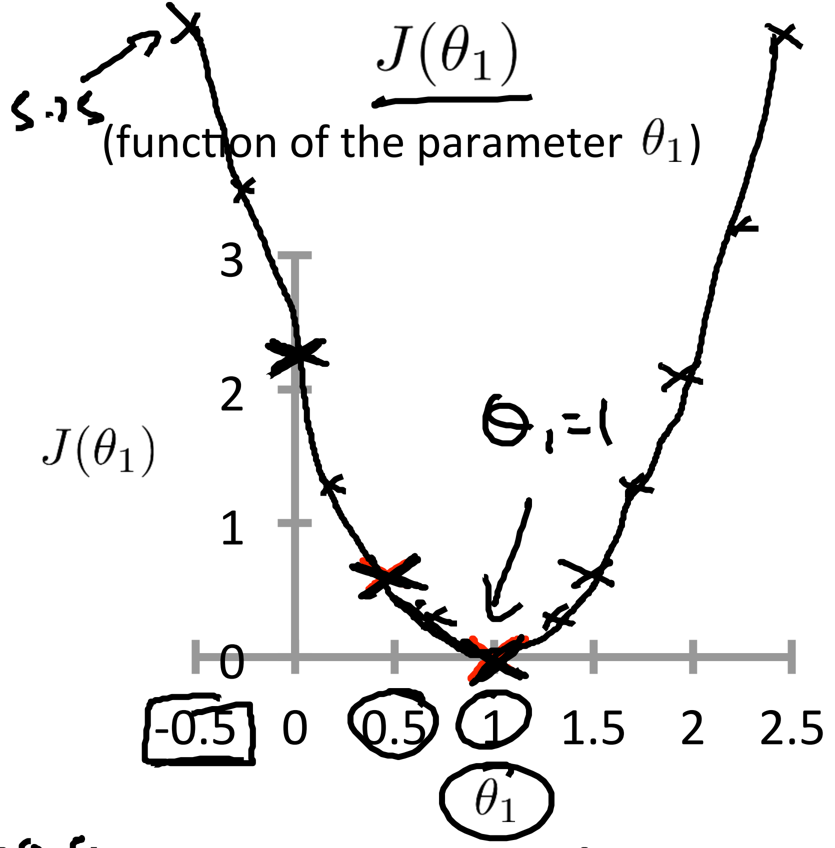
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



$J(\theta_1)$

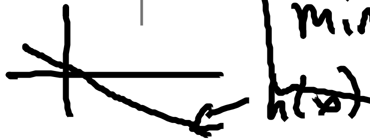
(function of the parameter θ_1)



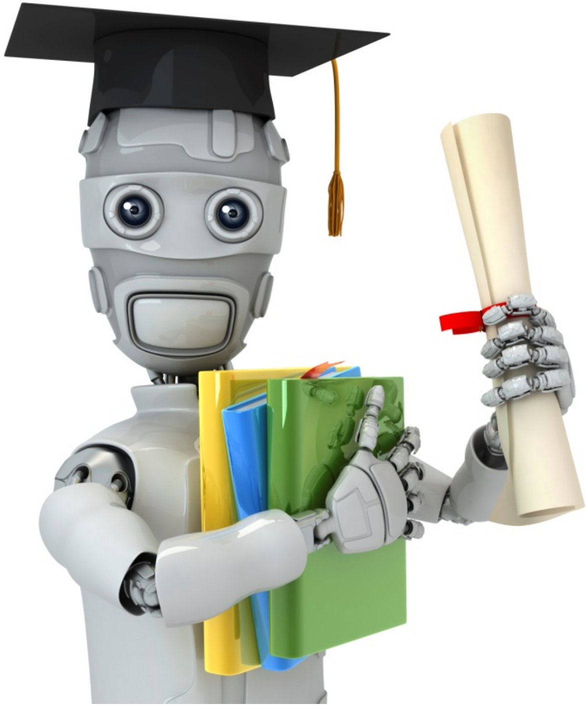
$J(0) = \frac{1}{2m}(1^2 + 2^2 + 3^2)$

$= \frac{1}{6} \cdot 14 \approx 2.3$

$h(x) = -0.5x$



$\underset{\theta_1}{\text{minimize}}\ J(\theta_1)$



Machine Learning

Linear regression with one variable

Cost function intuition II

Problem formulation

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

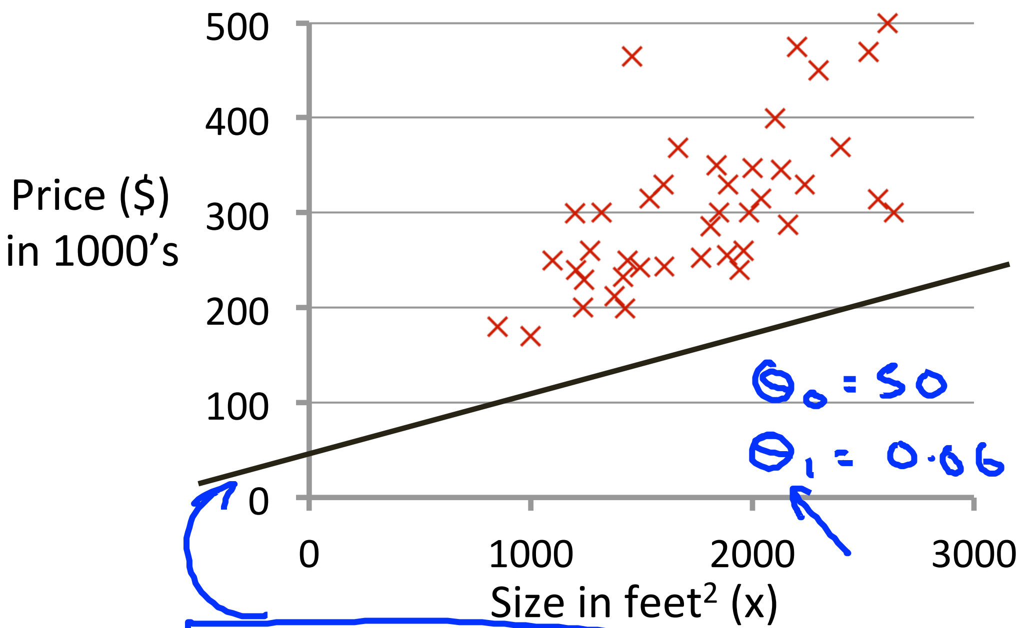
Parameters: $\underline{\theta_0, \theta_1}$

Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

$h_\theta(x)$

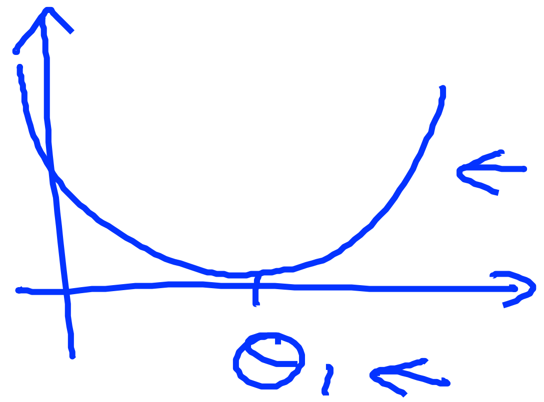
(for fixed θ_0, θ_1, this is a function of x)



$$\boxed{h_\theta(x) = 50 + 0.06x}$$

$J(\theta_0, \theta_1)$

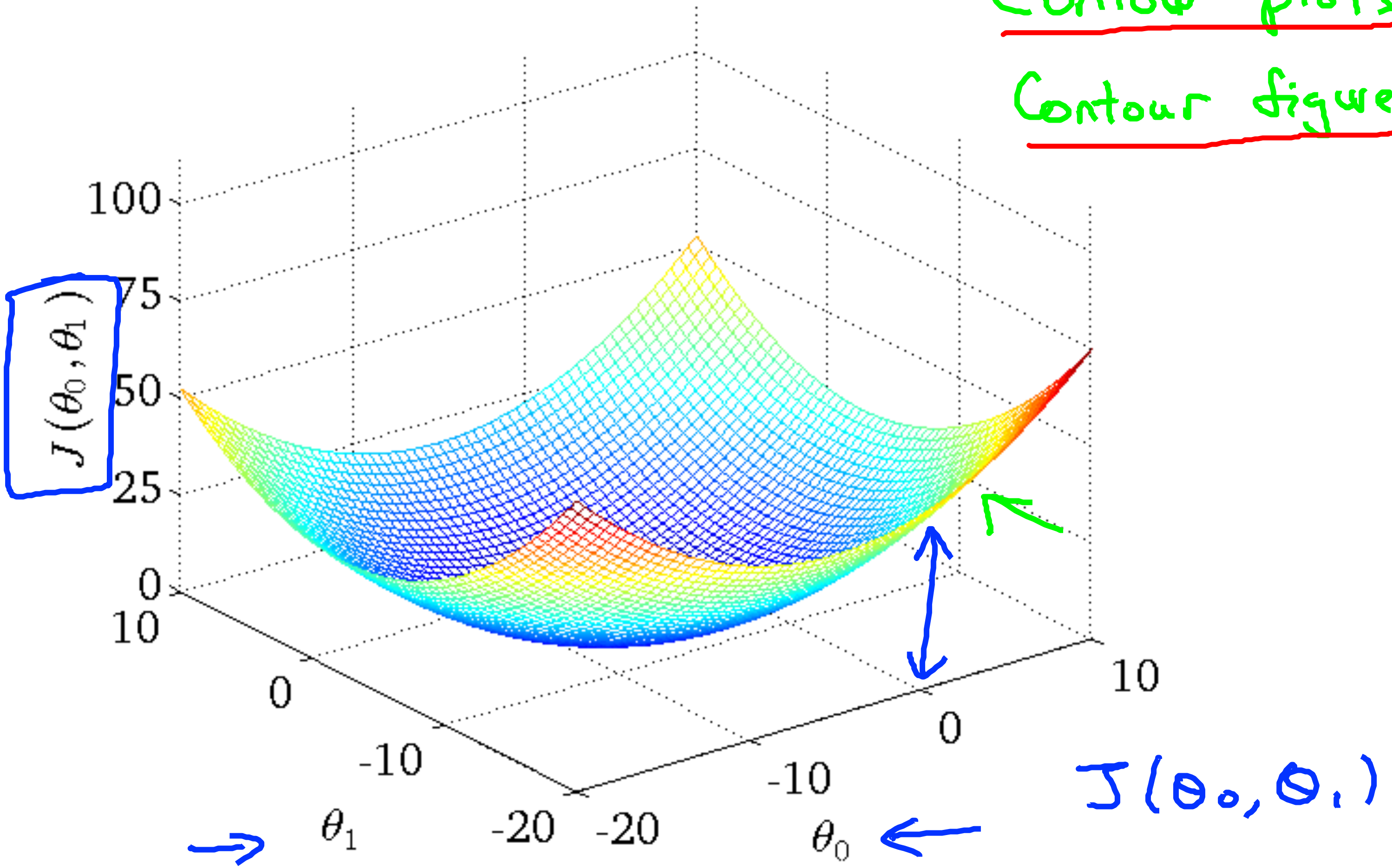
(function of the parameters θ_0, θ_1)



$\boxed{\theta_0, \theta_1}$

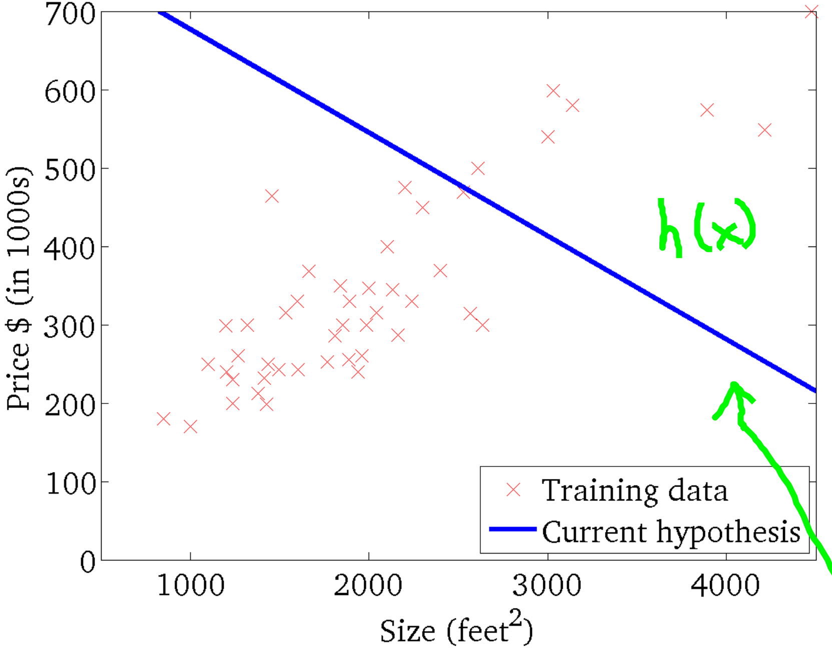
Contour plots

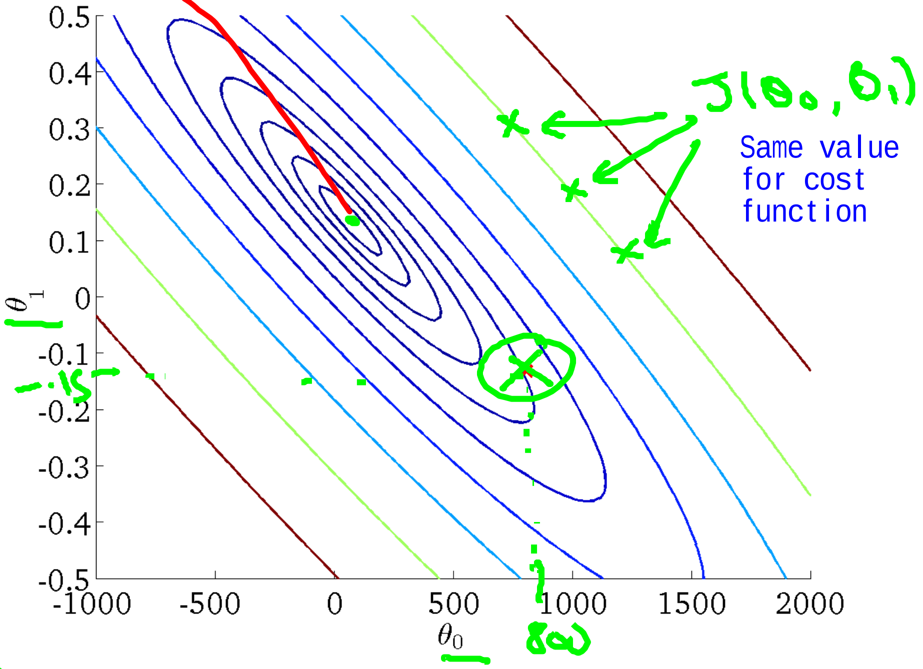
Contour figures -



$h_\theta(x)$ minimal cost function $J(\theta_0, \theta_1)$

(for fixed θ_0, θ_1, this is a function of x) (function of the parameters θ_0, θ_1)



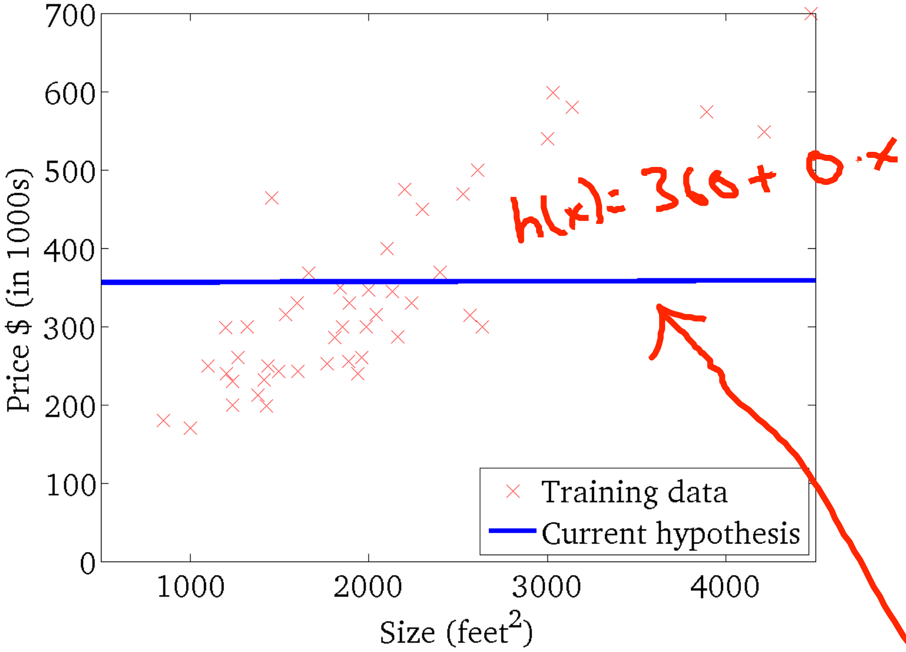


θ_0, θ_1

Andrew Ng

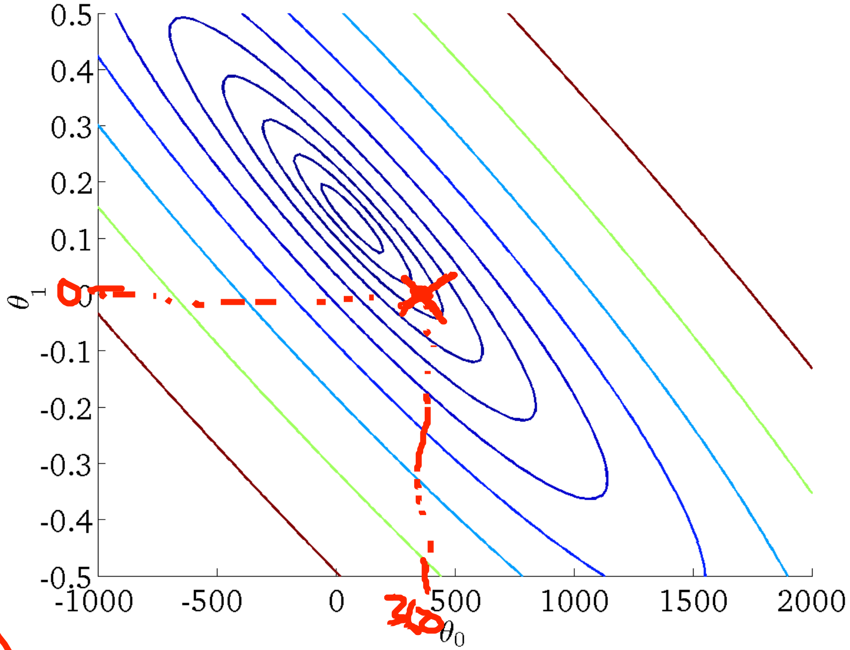
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



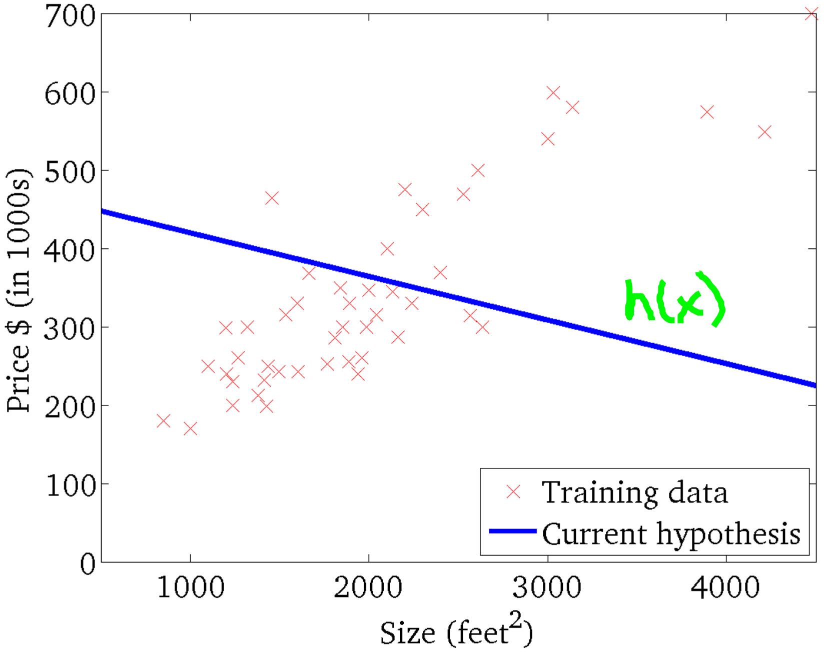
$\theta_0 = 360$

$\theta_1 = 0$

Andrew Ng

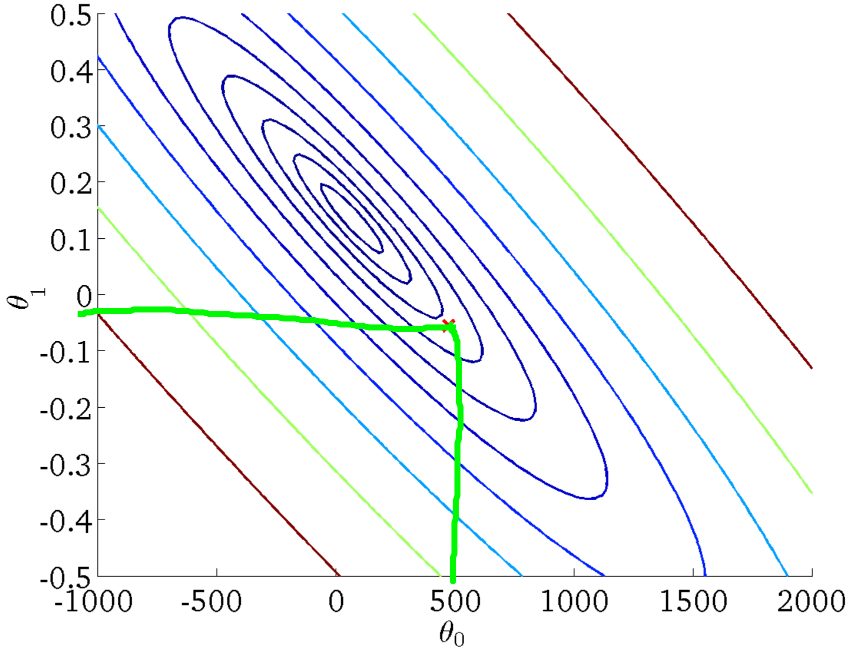
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



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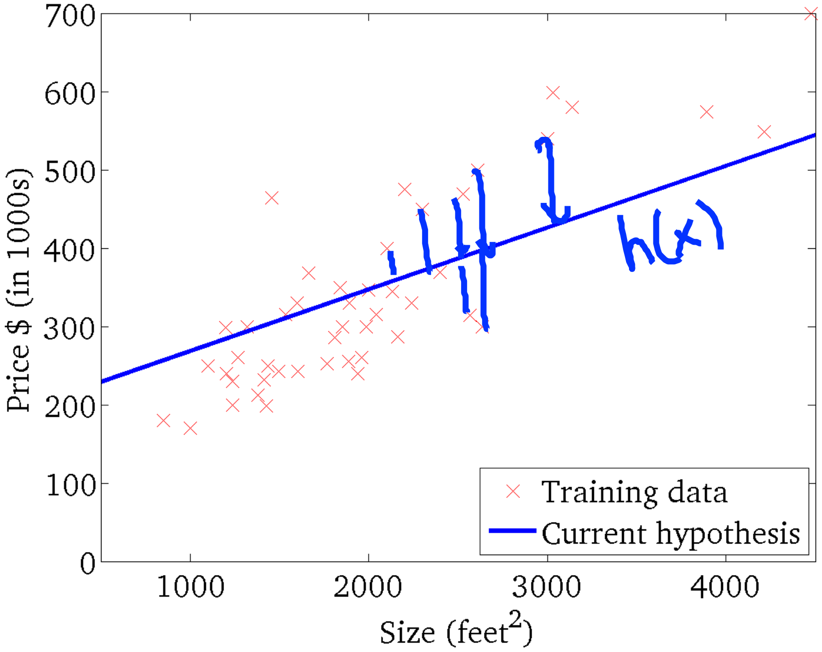
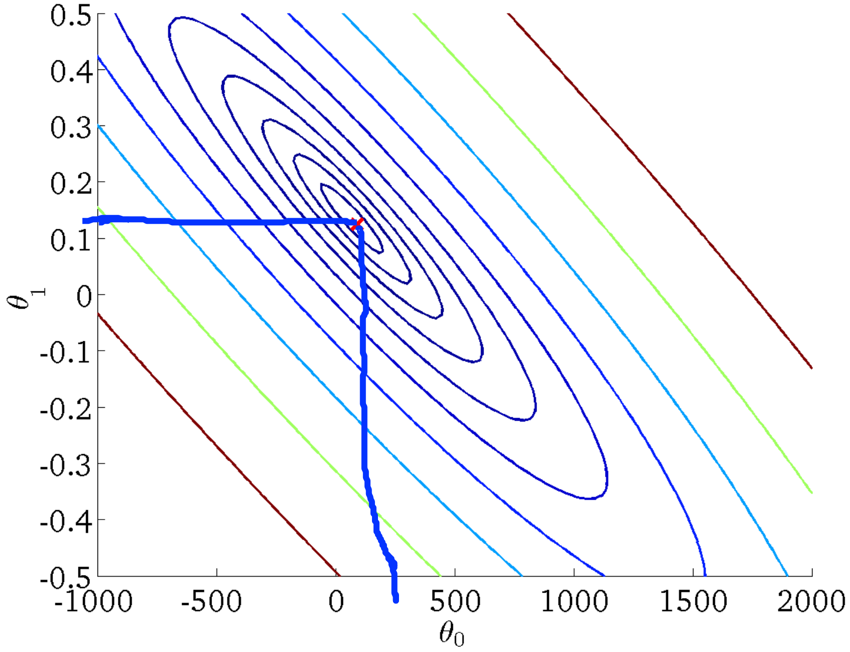
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



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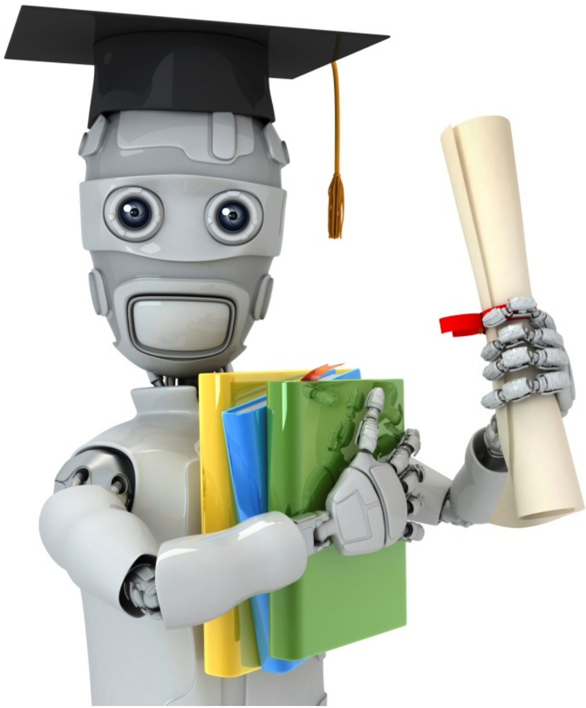


Machine Learning

Linear regression with one variable

Gradient descent

Problem setup

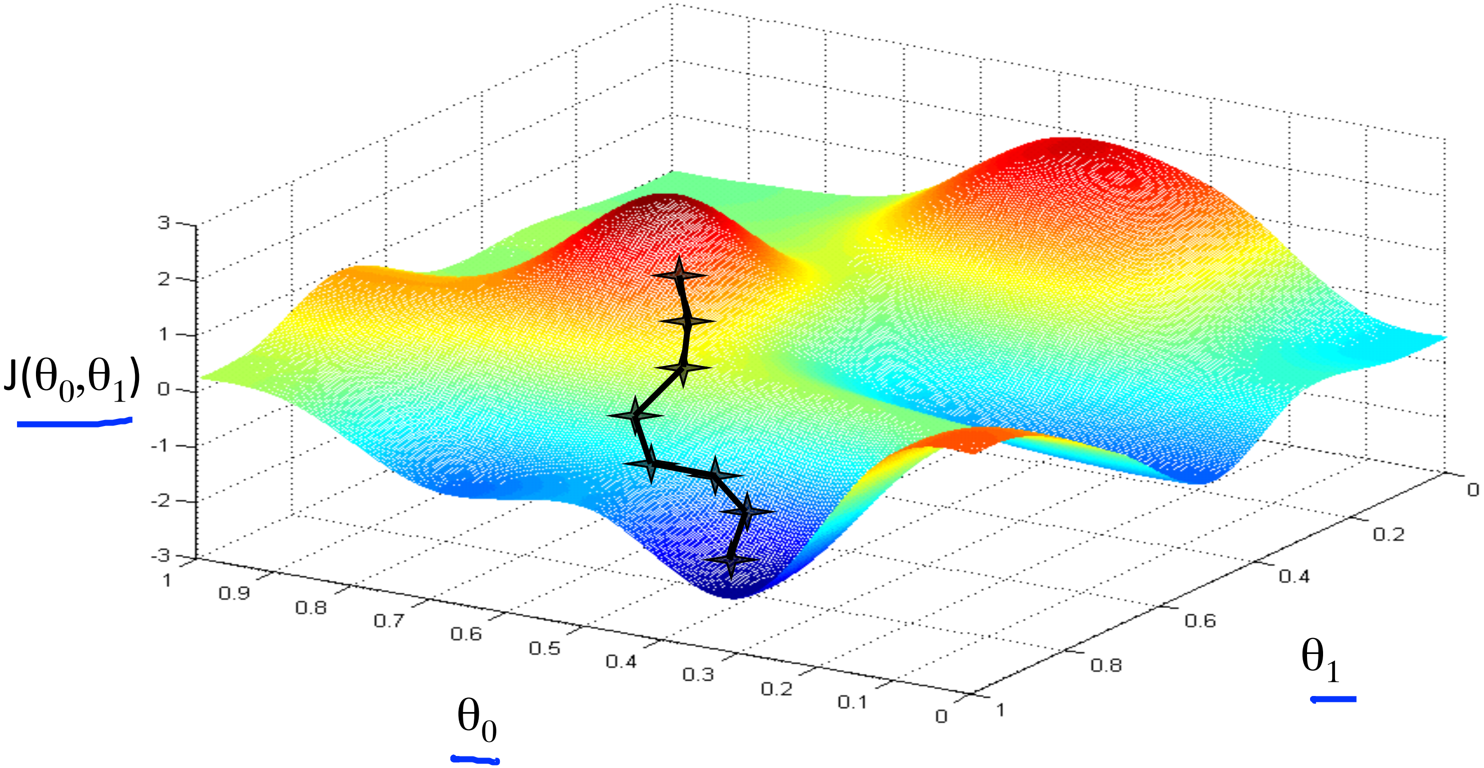
Have some function $J(\theta_0, \theta_1)$ $J(\theta_0, \theta_1, \theta_2, \ldots, \theta_n)$

Want $\min_{\theta_0, \theta_1} J(\theta_0, \theta_1)$ $\min_{\theta_0 \ldots \theta_n} J(\theta_0, \ldots, \theta_n)$

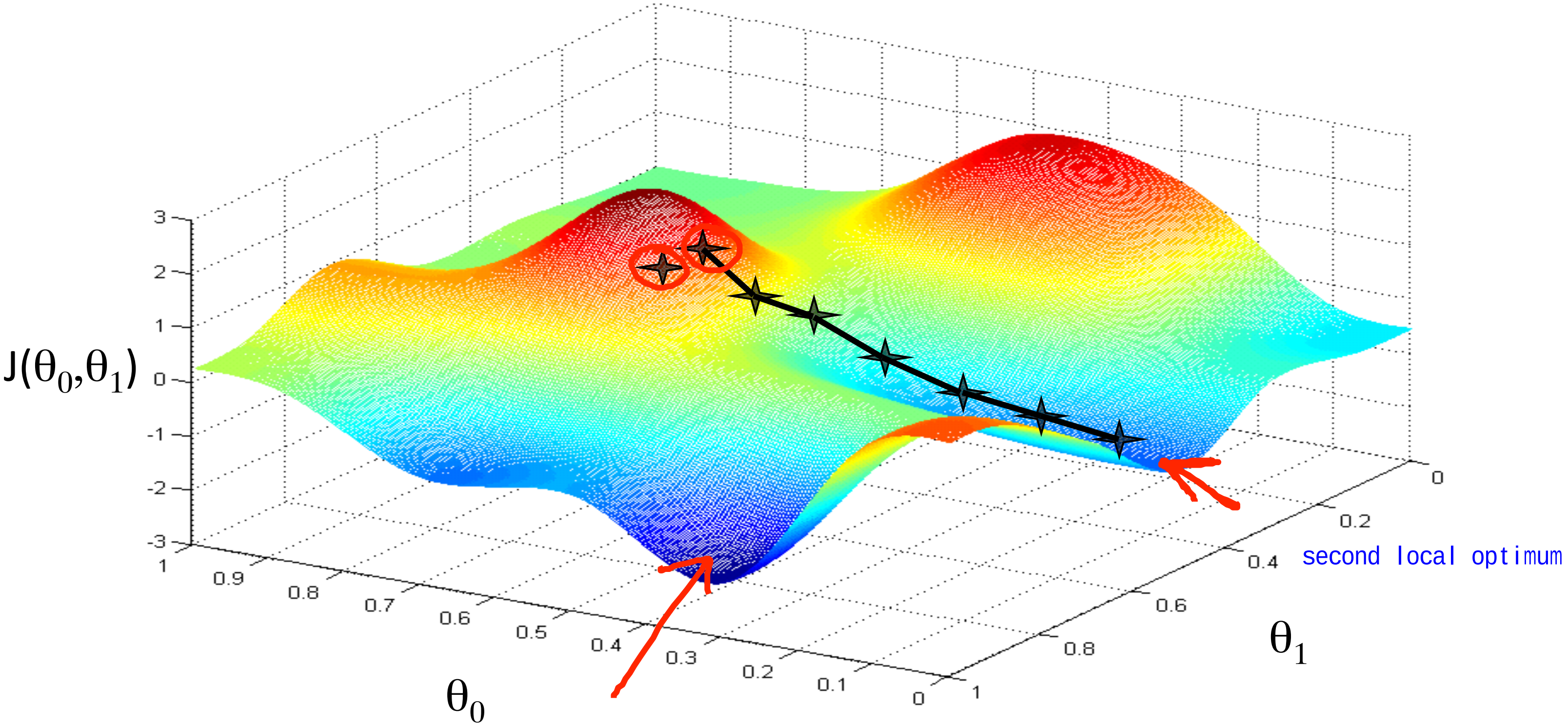
Outline:

give some initial values

- Start with some θ_0, θ_1 (say $\theta_0 = 0, \theta_1 = 0$)
- Keep changing θ_0, θ_1 to reduce $J(\theta_0, \theta_1)$ until we hopefully end up at a minimum



3
2
1
0
-1
-2
-3
J(θ0,θ1)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ0
0
0.2
0.4
0.6
0.8
1
θ1



J(θ0,θ1)
3
2
1
0
-1
-2
-3
θ0
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ1
0
0.2
0.4
0.6
0.8
1
second local optimum

Gradient descent algorithm

Assignment | Truth assertion
$a := b$ | $a = b$
$a := a+1$ | $a = a+1$ X

θ_0, θ_1

assignment operator

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \quad (\text{for } j = 0 \text{ and } j = 1)$$

}

learning rate → determine speed

Simultaneously update θ_0 and θ_1

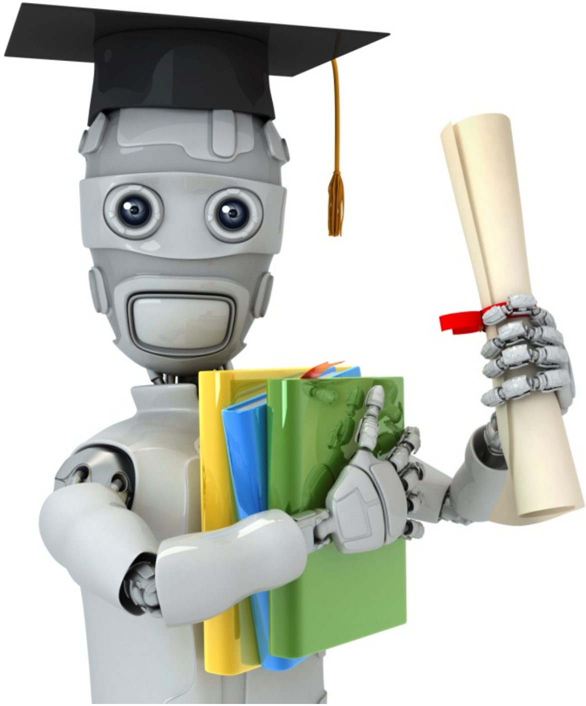
Correct: Simultaneous update

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$\theta_0 := \text{temp0}$$
$$\theta_1 := \text{temp1}$$

Incorrect:

X

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\theta_0 := \text{temp0}$$
$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$\theta_1 := \text{temp1}$$



Machine Learning

Linear regression with one variable

Gradient descent intuition

Gradient descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \boxed{\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)} \quad \text{(simultaneously update } j = 0 \text{ and } j = 1\text{)}$$

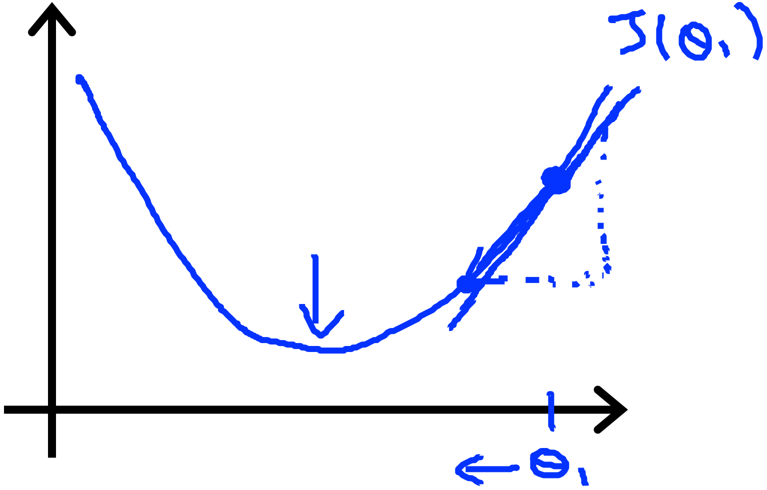
}

learning rate

derivative

$\min_{\theta_1} J(\theta_1)$

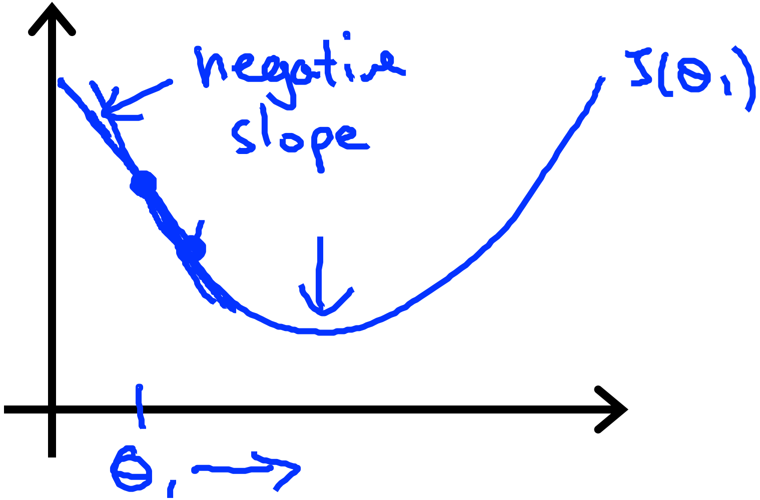
$\theta_1 \in \mathbb{R}$



$(\theta_1 \in \mathbb{R})$

$$\theta_1 := \theta_1 - \alpha \underbrace{\boxed{\frac{d}{d\theta_1} J(\theta_1)}}_{\geq 0} \qquad \frac{\partial}{\partial\theta_1}$$

$$\theta_1 := \theta_1 - \alpha \cdot (\text{positive number})$$

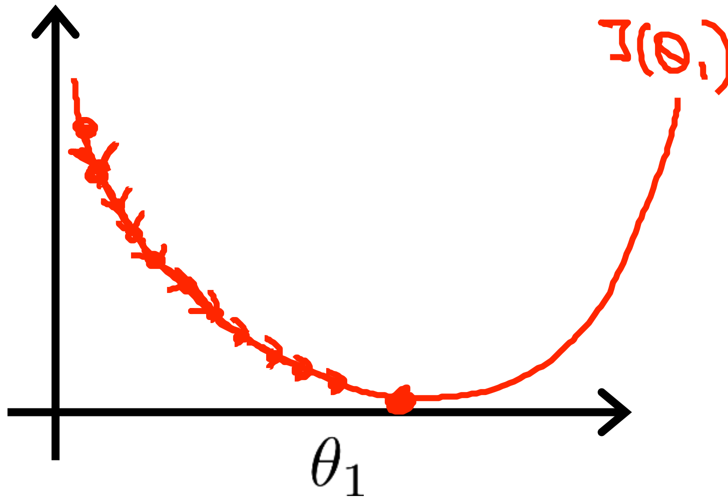


$$\underbrace{\frac{d}{d\theta_1} J(\theta_1)}_{\leq 0}$$

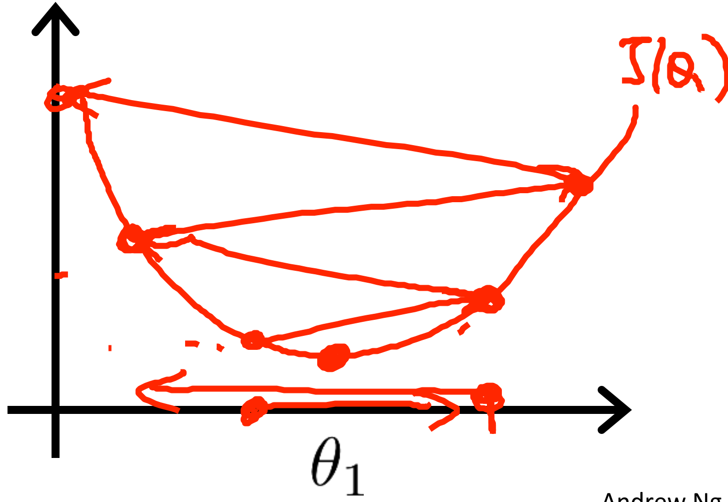
$$\theta_1 := \theta_1 - \alpha \,(\text{negative number})$$

$$\boxed{\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)}$$

If α is too small, gradient descent can be slow.



If α is too large, gradient descent can overshoot the minimum. It may fail to converge, or even diverge.



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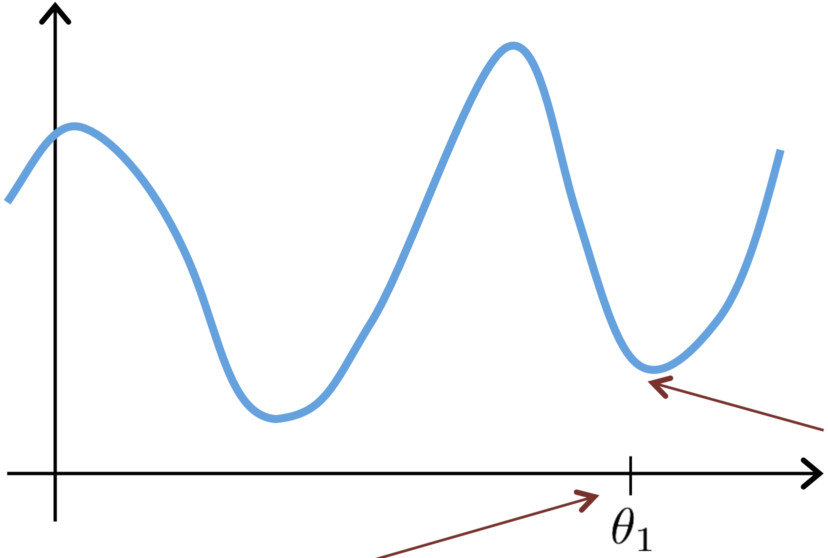


θ_1 at local optima

Current value of θ_1

$$\theta_1 := \theta_1 - \alpha \underbrace{\frac{d}{d\theta_1} J(\theta_1)}_{=0}$$

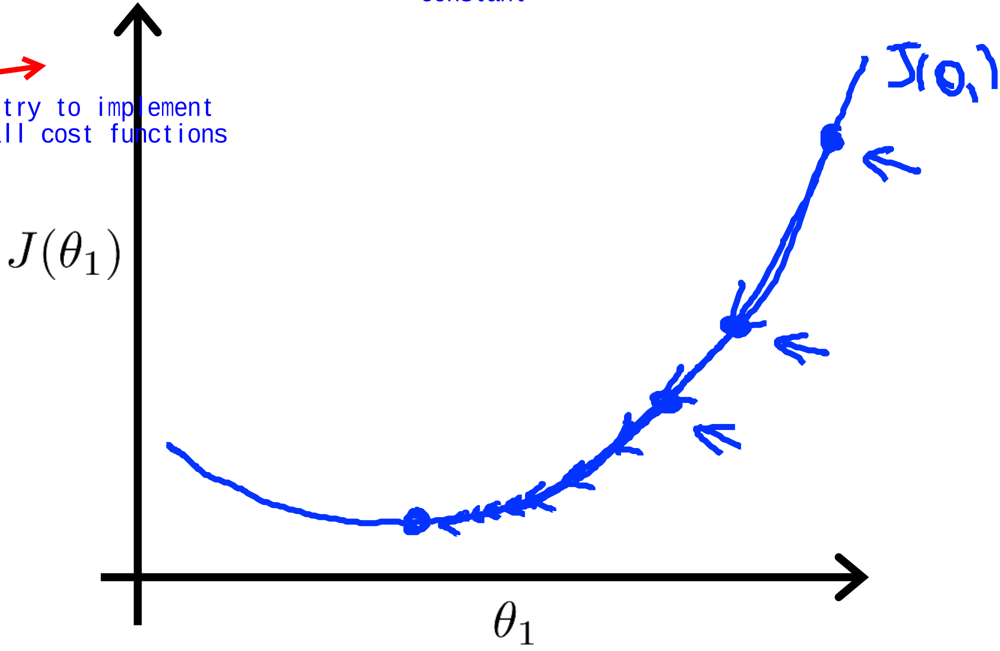
Gradient descent can converge to a local minimum, even with the learning rate α fixed.

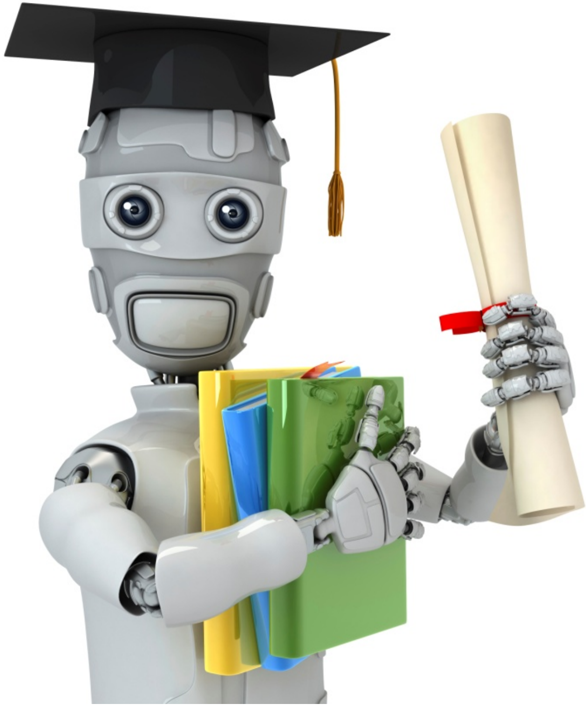
constant

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

can try to implement
to all cost functions

As we approach a local minimum, gradient descent will automatically take smaller steps. So, no need to decrease α over time.





Machine Learning

Linear regression with one variable

Gradient descent for linear regression

Gradient descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \boxed{\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)}$$

$$(\text{for } j = 1 \text{ and } j = 0)$$

}

Linear Regression Model

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$\frac{\partial}{\partial\theta_j} \underline{J(\theta_0,\theta_1)} = \frac{\partial}{\partial\theta_j} \underline{\frac{1}{2m} \sum_{i=1}^{m} \left(\underline{h_\theta(x^{(i)}}) - y^{(i)}\right)^2}$$

$$= \frac{\partial}{\partial\theta_j} \frac{1}{2m} \sum_{i=1}^{m} \left(\underline{\theta_0 + \theta_1 x^{(i)}} - y^{(i)}\right)^2$$

$$\boxed{\begin{aligned} j = 0 : \underline{\frac{\partial}{\partial\theta_0} J(\theta_0,\theta_1)} &= \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right) \\ j = 1 : \underline{\frac{\partial}{\partial\theta_1} J(\theta_0,\theta_1)} &= \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right) \cdot x^{(i)} \end{aligned}}$$

Gradient descent algorithm

$$\frac{\partial}{\partial\theta_0} J(\theta_0, \theta_1)$$

repeat until convergence {

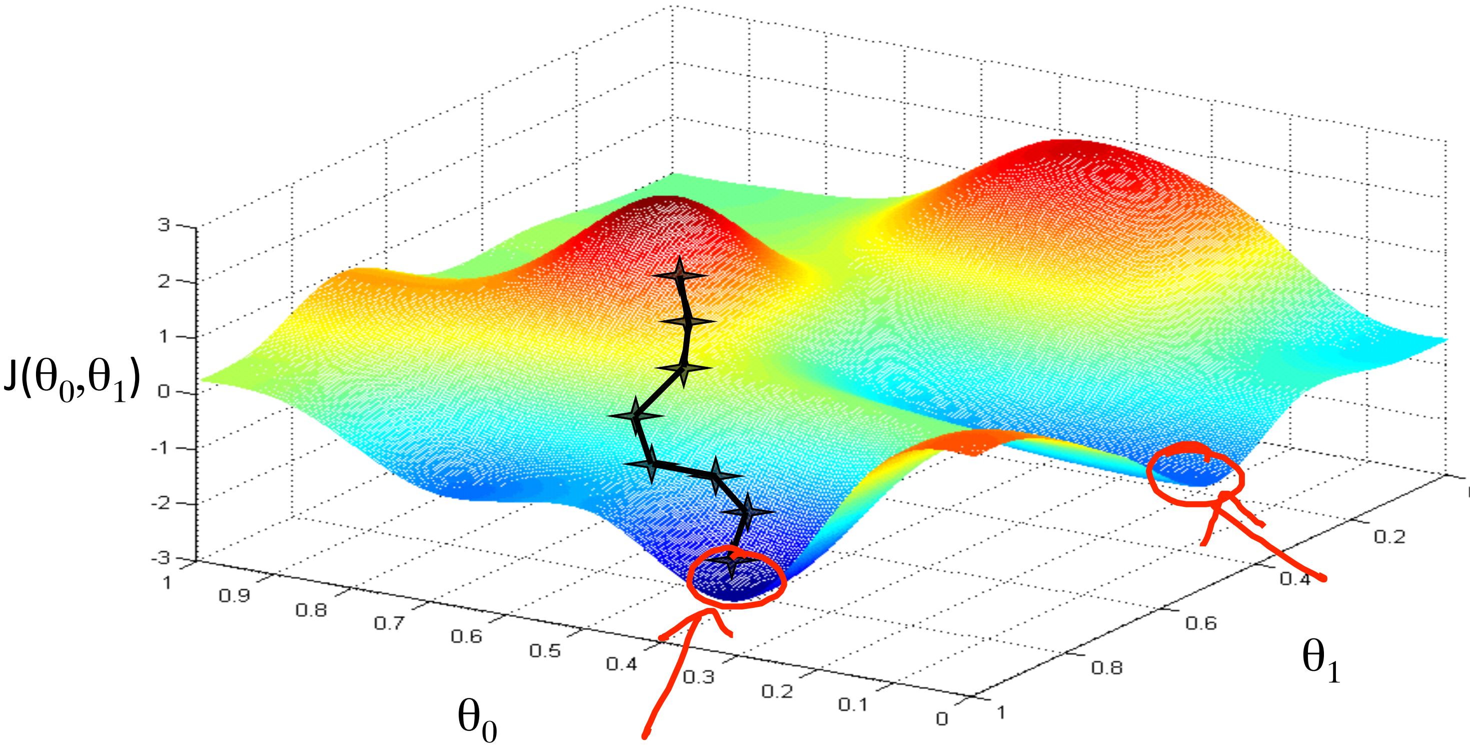
$$\theta_0 := \theta_0 - \alpha \boxed{\frac{1}{m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)}$$

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{1}{m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)\cdot x^{(i)}}$$

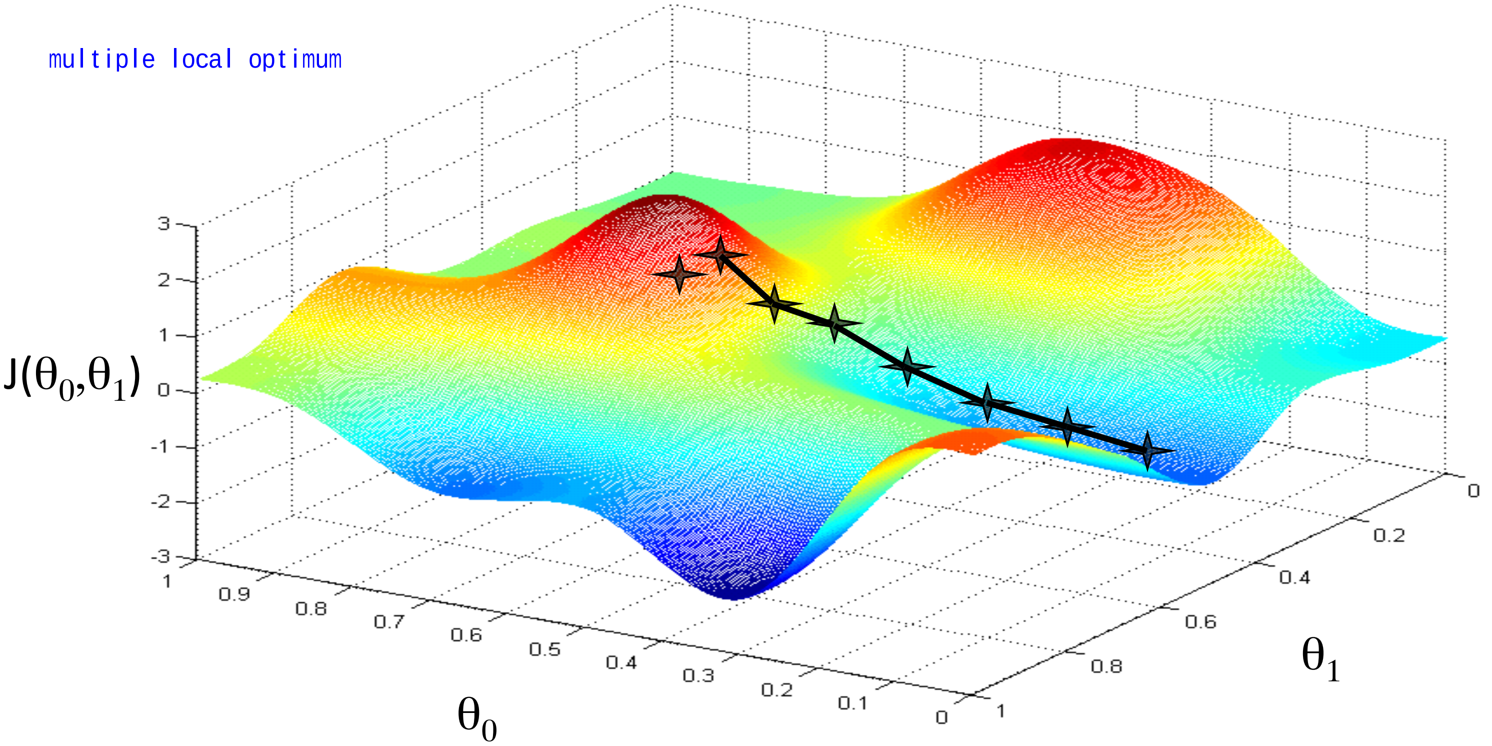
}

$$\frac{\partial}{\partial\theta_1} J(\theta_0, \theta_1)$$

update θ_0 and θ_1 simultaneously



3
2
1
0
-1
-2
-3
$J(\theta_0,\theta_1)$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1


multiple local optimum
$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1

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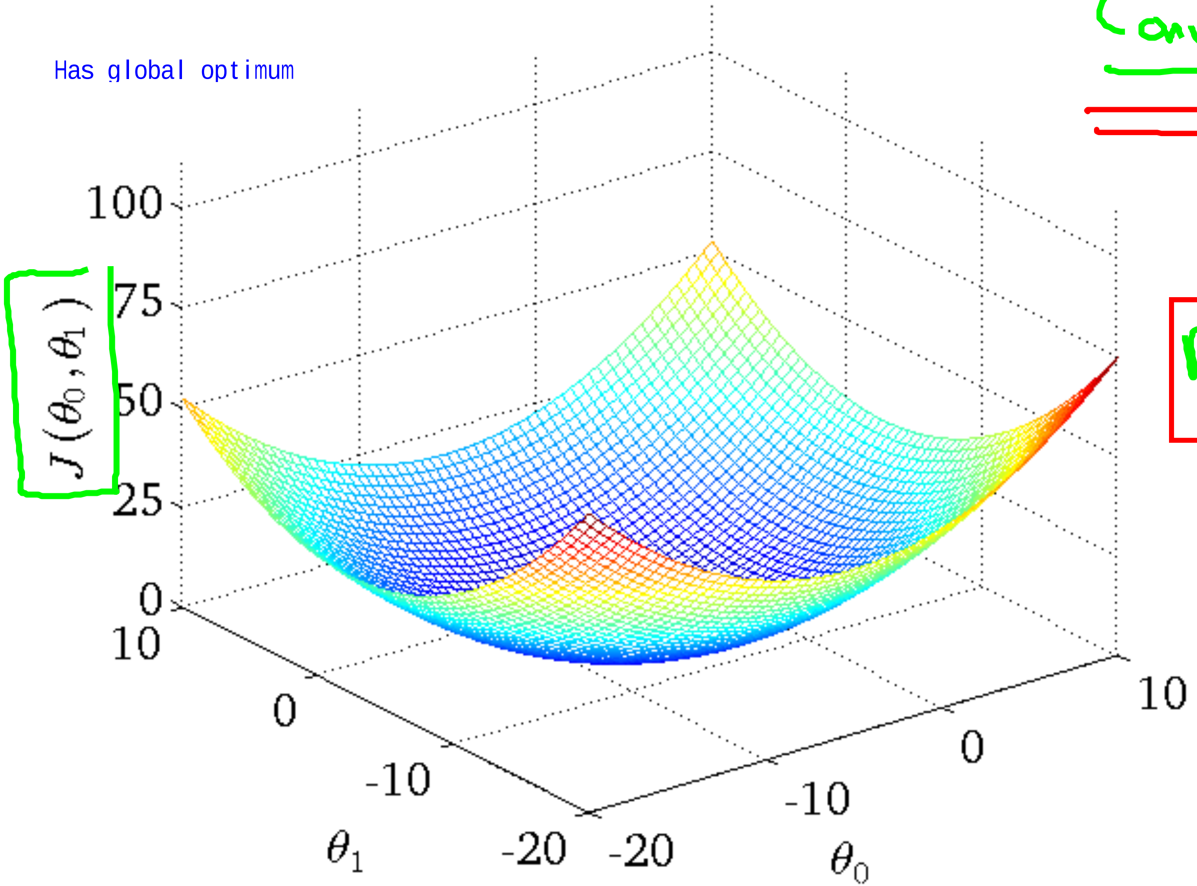
For linear regression

"Convex function"

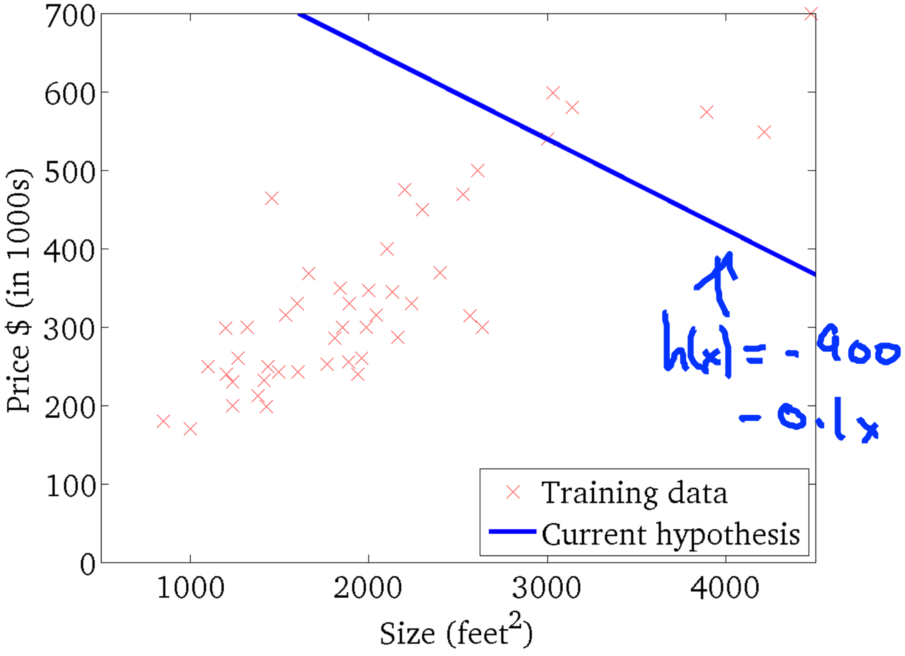
Bowl-shaped

Has global optimum



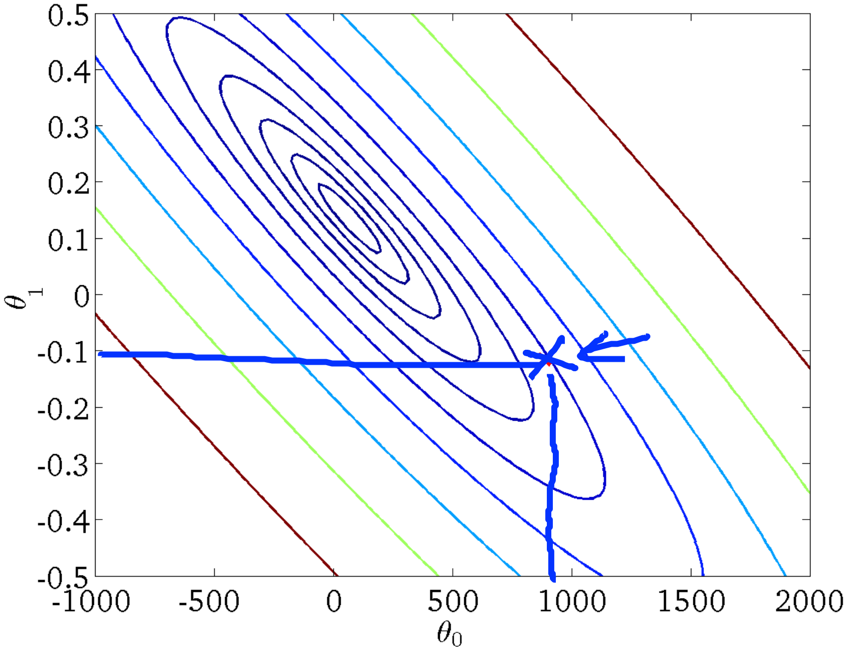
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



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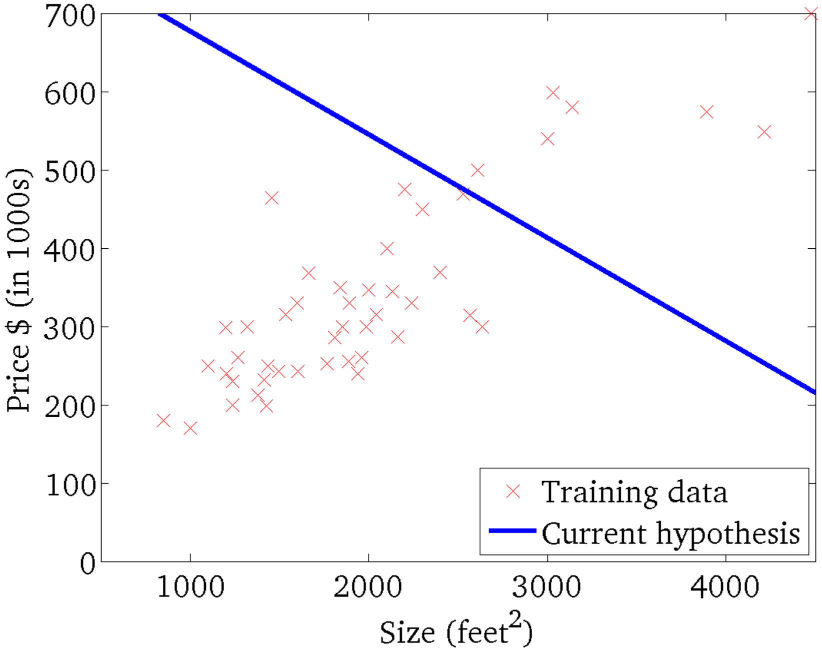
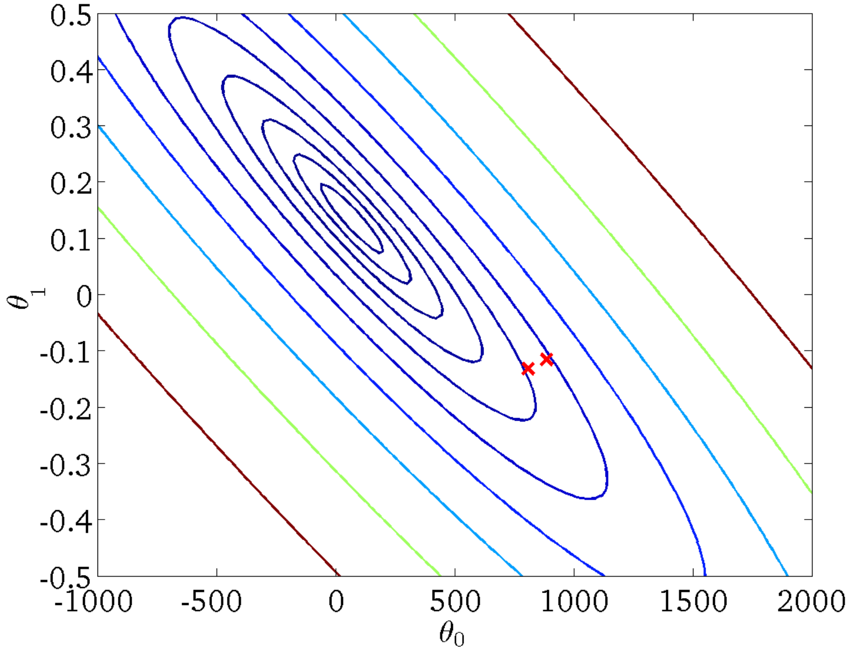
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



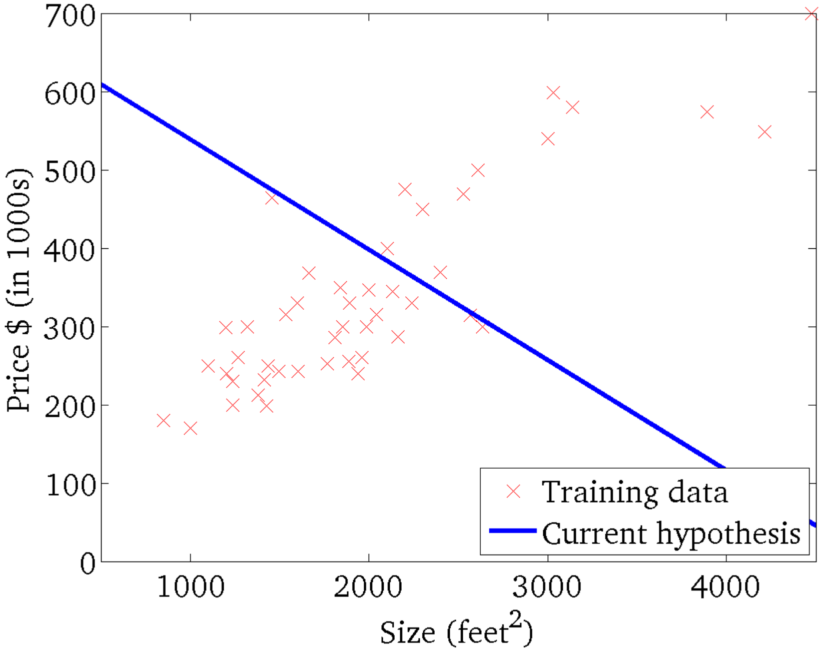
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



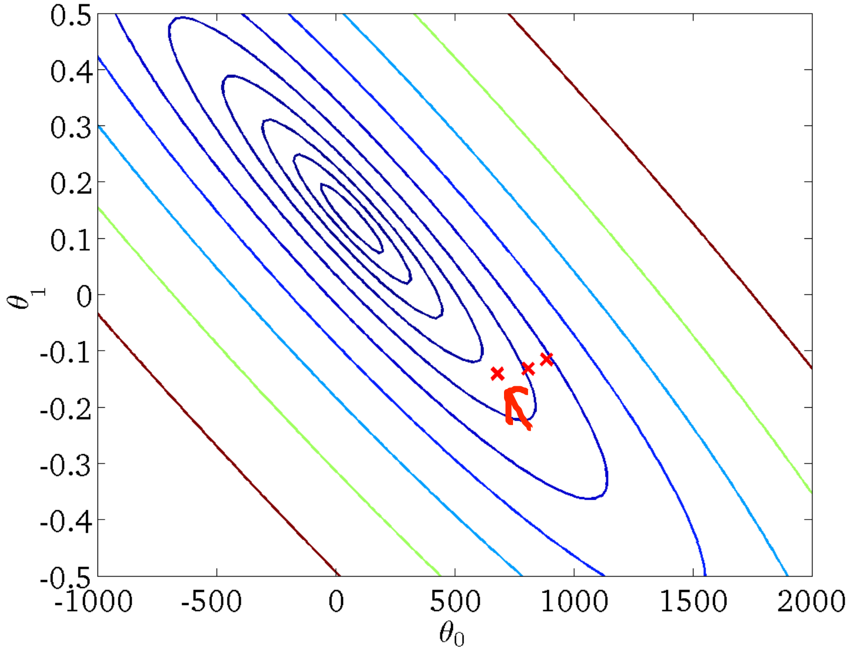
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



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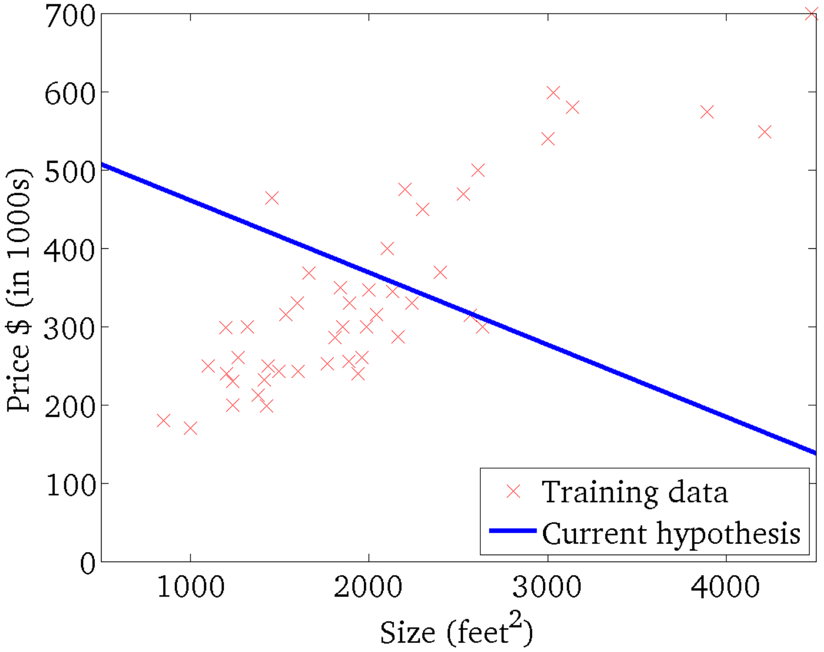
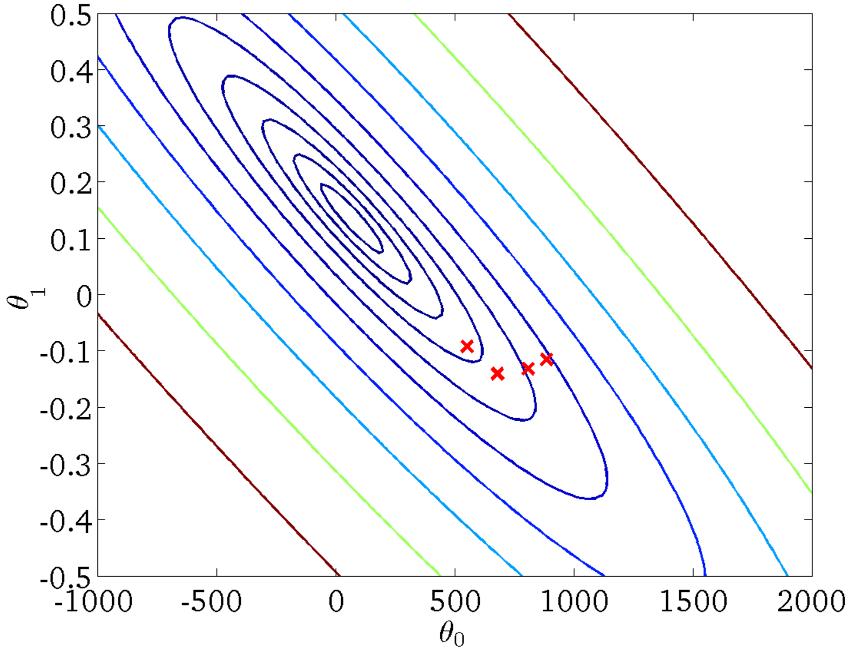
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



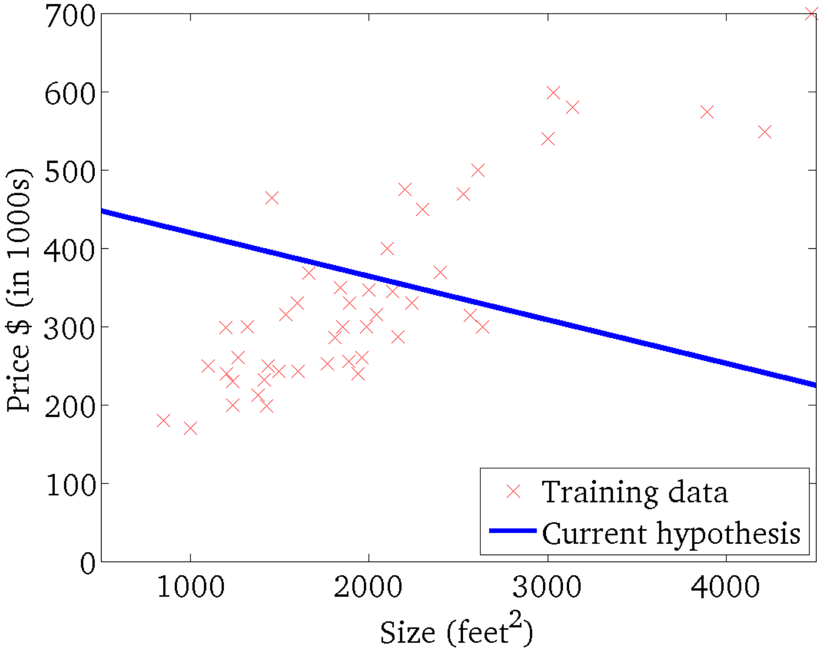
$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



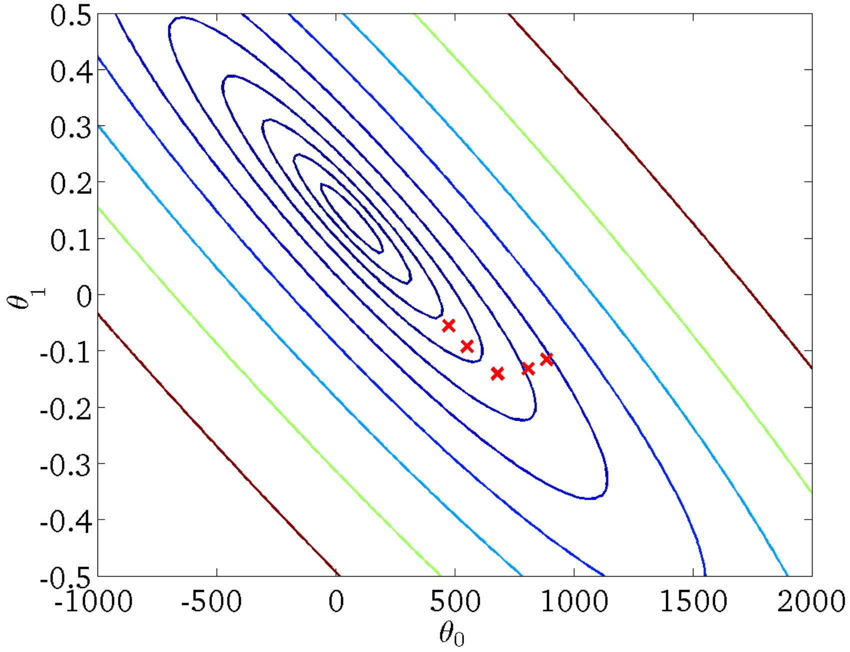
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



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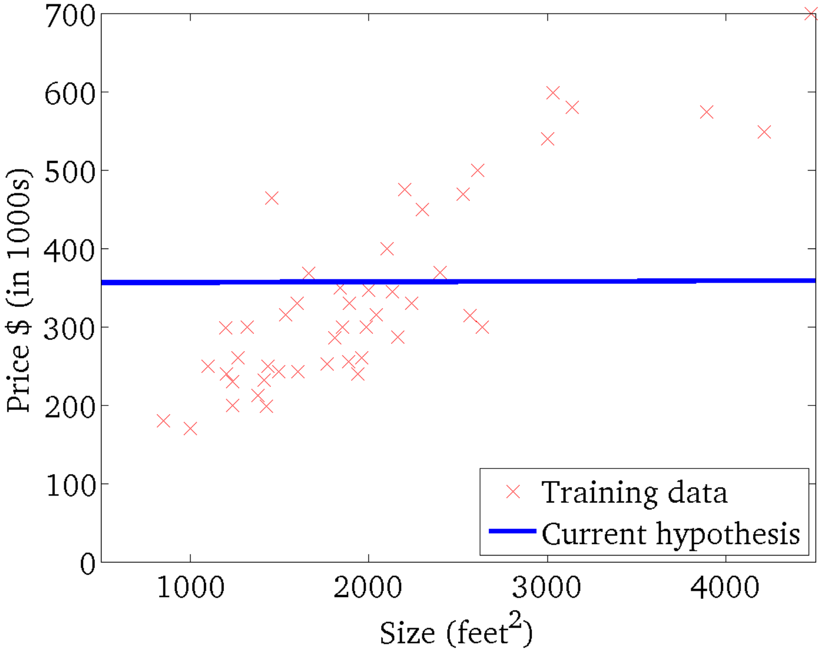
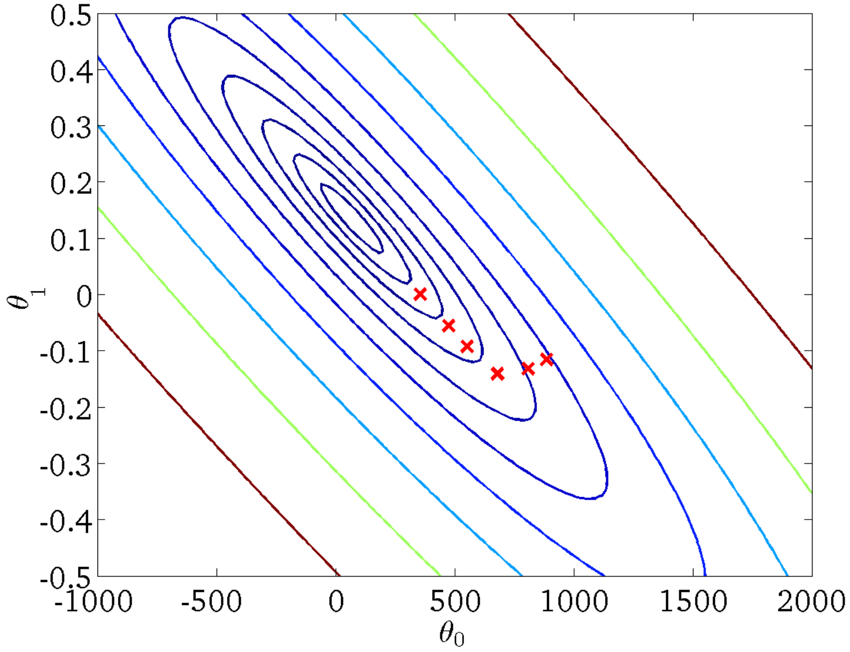
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



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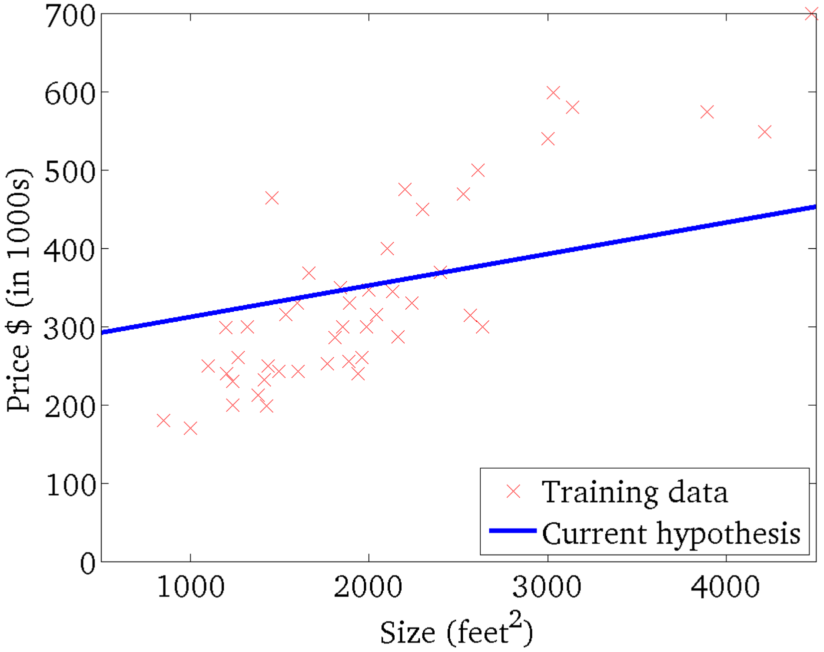
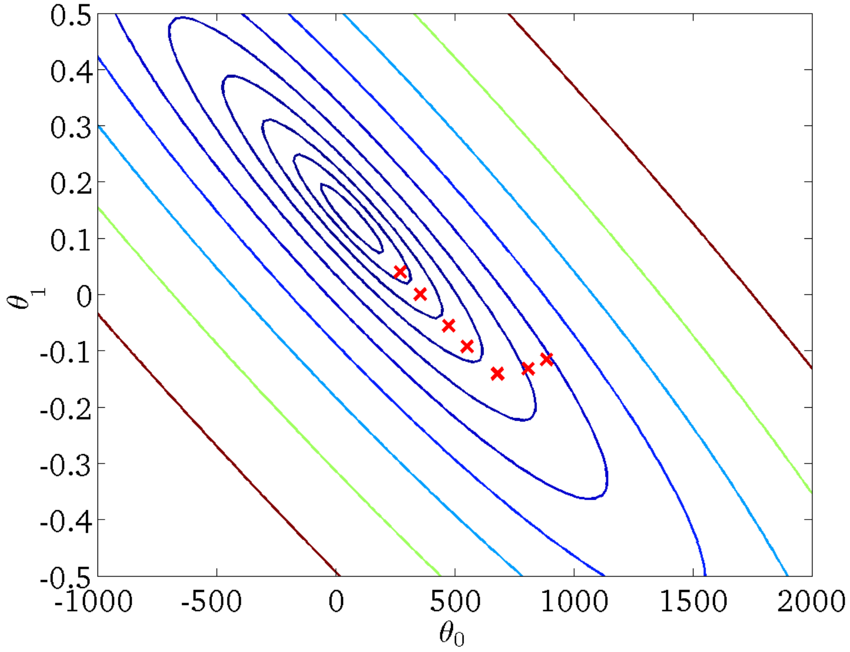
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



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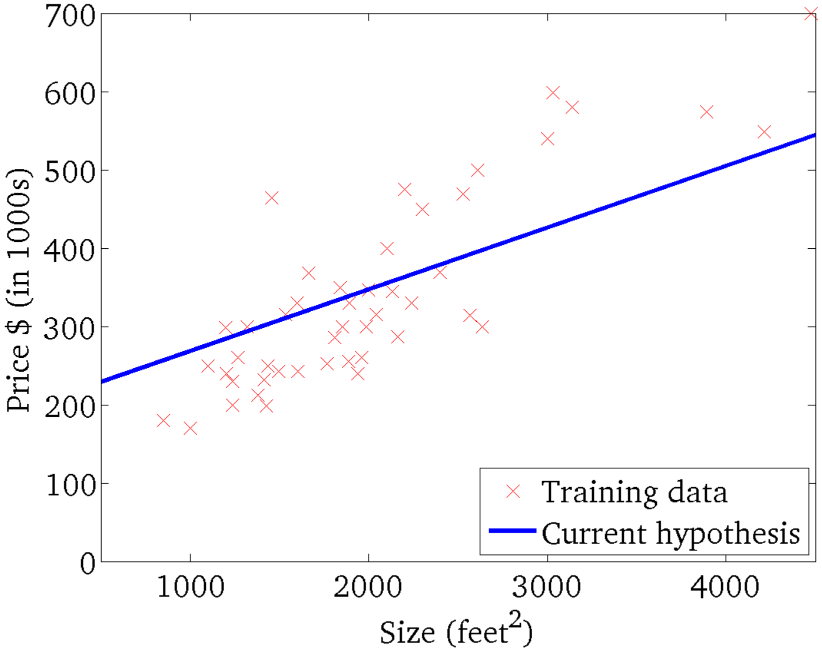
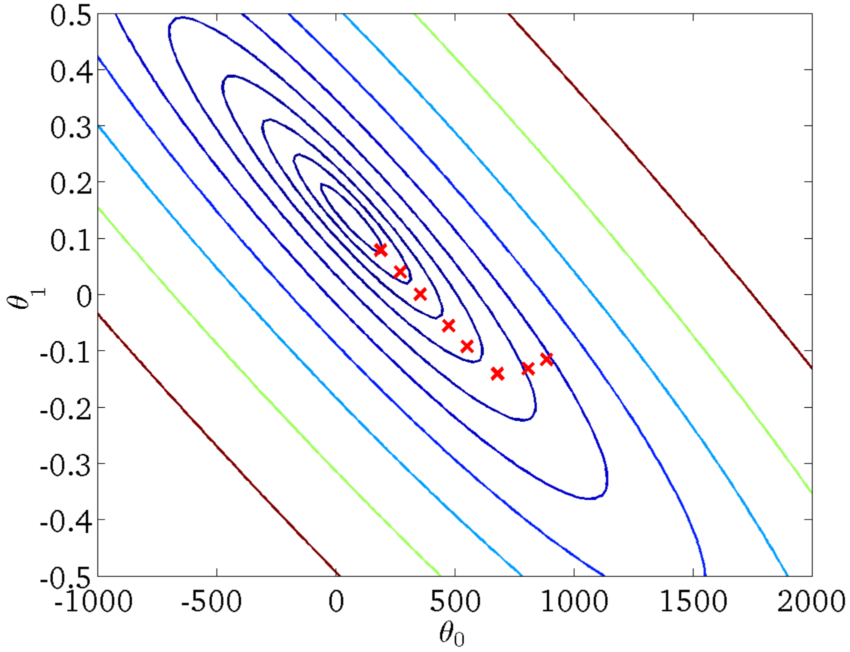
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

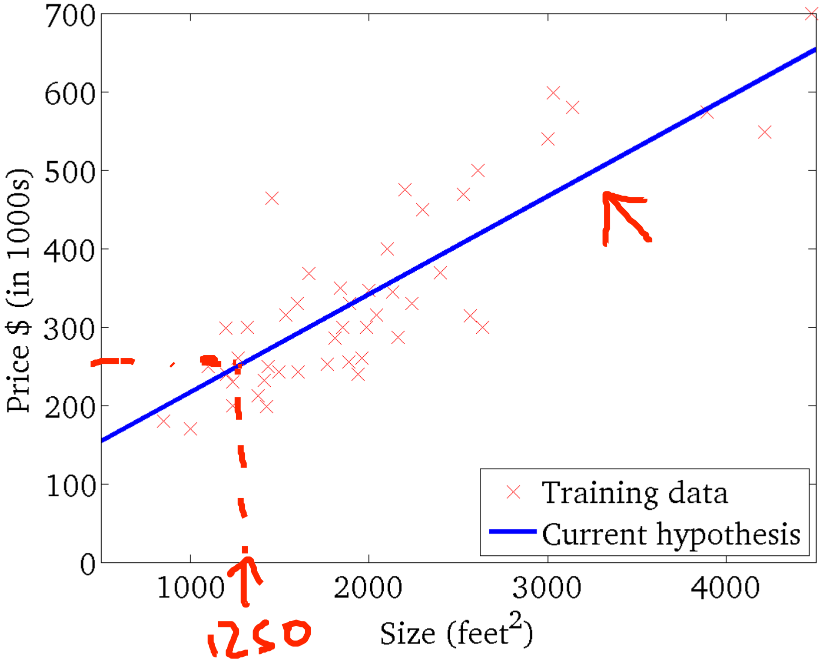
(function of the parameters θ_0, θ_1)



global optimum has reached

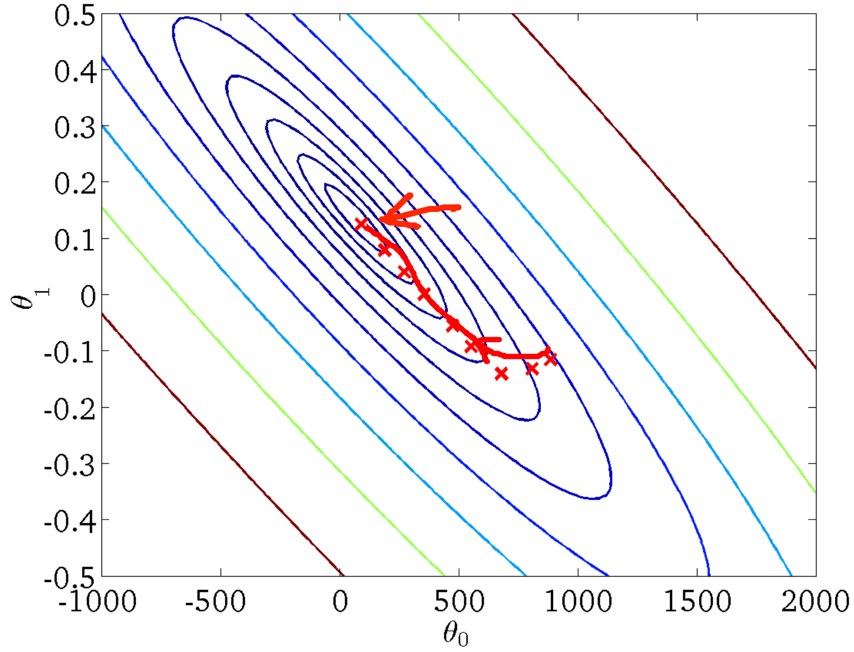
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



refers to we are looking at our entire batch of training examples

"Batch" Gradient Descent

Alternative name

"Batch": Each step of gradient descent uses all the training examples.

$$\sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})$$