# 第四章 数据结构目录

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#### 1、划分树,查询区间第 K 大

```
/*划分树(查询区间第 k 大) */
const int MAXN = 100010;
                         // 表示每层每个位置的值
int tree[20][MAXN];
int sorted[MAXN];
                         // 已经排序好的数
int toleft[20][MAXN];
                        // toleft[p][i]表示第 i 层从 1 到 i 有数分入左边
void build(int I, int r, int dep)
{
     if (I == r)
                 return;
     int mid = (l + r) \gg 1;
                                    // 表示等于中间值而且被分入左边的个数
     int same = mid - l + 1;
     for (int i = l; i <= r; i++){
                               // 注意是 I,不是 one
         if (tree[dep][i] < sorted[mid])</pre>
                                          same--;
    }
     int lpos = 1;
    int rpos = mid + 1;
     for (int i = 1; i <= r; i++){
         if (tree[dep][i] < sorted[mid])</pre>
                                         tree[dep + 1][lpos++] = tree[dep][i];
         else if (tree[dep][i] == sorted[mid] && same > 0){
              tree[dep + 1][lpos++] = tree[dep][i];
              same--;
         }
                 tree[dep + 1][rpos++] = tree[dep][i];
         else
         toleft[dep][i] = toleft[dep][I - 1] + lpos - I;
    }
     build(l, mid, dep + 1);
     build(mid + 1, r, dep + 1);
     return;
}
// 查询区间第 k 大的数,[L,R]是大区间,[I,r]是要查询的小区间
int query(int L, int R, int I, int r, int dep, int k){
     if(l == r)
                return tree[dep][l];
     int mid = (L + R) >> 1;
     int cnt = toleft[dep][r] - toleft[dep][l - 1];
     if (cnt >= k){
         int newl = L + toleft[dep][I - 1] - toleft[dep][L - 1];
         int newr = newl + cnt - 1;
         return query(L, mid, newl, newr, dep + 1, k);
    }
     else{
         int newr = r + toleft[dep][R] - toleft[dep][r];
         int newl = newr - (r - I - cnt);
         return query(mid + 1, R, newl, newr, dep + 1, k - cnt);
```

```
}
}
                         // 表示每层每个位置的值
/*int tree[20][MAXN];
  int sorted[MAXN];
                          // 已经排序好的数
                         // toleft[p][i]表示第 i 层从 1 到 i 有数分入左边*/
  int toleft[20][MAXN];
int main(){
    int n, m; //n 个数 , m 次查询
    while (scanf("%d%d", &n, &m) == 2){
         memset(tree, 0, sizeof(tree));
         for (int i = 1; i <= n; i++){
              scanf("%d", &tree[0][i]);
              sorted[i] = tree[0][i];
         }
         sort(sorted + 1, sorted + n + 1);
         build(1, n, 0);
         int s, t, k;
         while(m--){
              scanf("%d%d%d", &s, &t, &k);
              printf("%d\n", query(1, n, s, t, 0, k));
         }
    }
    return 0;
}
```

### 2、伸展树,区间子序列最小值

```
/* 伸展树(Splay Tree)
 * 题目:维修数列。
 * 经典题,插入、删除、修改、翻转、求和、求和最大的子序列*/
#define Key value ch[ch[root][1]][0]
const int MAXN = 500010;
const int INF = 0x3f3f3f3f;
int pre[MAXN], ch[MAXN][2], key[MAXN], size[MAXN];
int root, tot1;
int sum[MAXN], rev[MAXN], same[MAXN];
int lx[MAXN], rx[MAXN], mx[MAXN];
int s[MAXN], tot2;
                   // 内存池和容量
int a[MAXN];
int n, q;
// debug Start**********************
void Treavel(int x){
    if (x){
        Treavel(ch[x][0]);
        printf("结点:%2d: 左儿子 %2d 右儿子 %2d 父结点 %2d size = %2d\n", x, ch[x][0],
```

```
ch[x][1], pre[x], size[x]);
          Treavel(ch[x][1]);
    }
     return;
}
void debug(){
     printf("root:%d\n", root);
    Treavel(root);
    return;
}
// debug End**********************
void NewNode(int &r, int father, int k){
     if (tot2) r = s[tot2--]; // 取的时候是 tot2--,存的时候就是++tot2
          r = ++tot1;
     pre[r] = father;
     ch[r][0] = ch[r][1] = 0;
     key[r] = k;
    sum[r] = k;
     rev[r] = same[r] = 0;
    lx[r] = rx[r] = mx[r] = k;
     size[r] = 1;
     return;
void Update_Rev(int r){
     if (!r)
            return ;
    swap(ch[r][0], ch[r][1]);
    swap(lx[r], rx[r]);
     rev[r] ^= 1;
     return;
void Update_Same(int r, int v)
{
     if (!r) return;
     key[r] = v;
     sum[r] = v * size[r];
    lx[r] = rx[r] = mx[r] = max(v, v * size[r]);
    same[r] = 1;
     return;
void push_up(int r){
    int lson = ch[r][0], rson = ch[r][1];
    size[r] = size[lson] + size[rson] + 1;
     sum[r] = sum[lson] + sum[rson] + key[r];
     lx[r] = max(lx[lson], sum[lson] + key[r] + max(0, lx[rson]));
```

```
rx[r] = max(rx[rson], sum[rson] + key[r] + max(0, rx[lson]));
     mx[r] = max(0, rx[lson]) + key[r] + max(0, lx[rson]);
     mx[r] = max(mx[r], max(mx[lson], mx[rson]));
     return;
}
void push_down(int r){
     if (same[r]){
          Update_Same(ch[r][0], key[r]);
          Update_Same(ch[r][1], key[r]);
          same[r] = 0;
     }
     if(rev[r]){
          Update_Rev(ch[r][0]);
          Update_Rev(ch[r][1]);
          rev[r] = 0;
     }
     return;
}
void Build(int &x, int I, int r, int father){
     if (l > r)
                 return;
     int mid = (I + r) / 2;
     NewNode(x, father, a[mid]);
     Build(ch[x][0], I, mid - 1, x);
     Build(ch[x][1], mid + 1, r, x);
     push_up(x);
     return;
}
void Init(){
     root = tot1 = tot2 = 0;
     ch[root][0] = ch[root][1] = size[root] = pre[root] = 0;
     same[root] = rev[root] = sum[root] = key[root] = 0;
     lx[root] = rx[root] = mx[root] = -INF;
     NewNode(root, 0, -1);
     NewNode(ch[root][1], root, -1);
     for (int i = 0; i < n; i++)
                                scanf("%d", &a[i]);
     Build(Key_value, 0, n - 1, ch[root][1]);
     push up(ch[root][1]);
     push_up(root);
}
// 旋转,0 为左旋,1 为右旋
void Rotate(int x,int kind){
     int y = pre[x];
     push_down(y);
     push_down(x);
```

```
ch[y][!kind] = ch[x][kind];
     pre[ch[x][kind]] = y;
     if (pre[y])
          ch[pre[y]][ch[pre[y]][1]==y] = x;
     pre[x] = pre[y];
     ch[x][kind] = y;
     pre[y] = x;
     push_up(y);
}
// Splay 调整,将 r 结点调整到 goal 下面
void Splay(int r, int goal){
     push_down(r);
     while (pre[r] != goal){
          if (pre[pre[r]] == goal){
               push_down(pre[r]);
               push_down(r);
               Rotate(r, ch[pre[r]][0] == r);
          }
          else{
               push_down(pre[pre[r]]);
               push_down(pre[r]);
               push_down(r);
               int y = pre[r];
               int kind = ch[pre[y]][0] == y;
               if (ch[y][kind] == r){
                    Rotate(r, !kind);
                    Rotate(r, kind);
               }
               else{
                    Rotate(y, kind);
                    Rotate(r, kind);
               }
          }
     }
     push_up(r);
     if (goal == 0)
                       root = r;
     return;
}
int Get_kth(int r, int k){
     push_down(r);
     int t = size[ch[r][0]] + 1;
     if (t == k)
                  return r;
     if (t > k)
                  return Get_kth(ch[r][0], k);
     else
              return Get_kth(ch[r][1], k - t);
```

```
}
// 在第 pos 个数后面插入 tot 个数
void Insert(int pos, int tot){
    for (int i = 0; i < tot; i++)
                               scanf("%d",&a[i]);
    Splay(Get_kth(root, pos + 1), 0);
    Splay(Get_kth(root, pos + 2), root);
    Build(Key_value, 0, tot - 1, ch[root][1]);
    push_up(ch[root][1]);
    push_up(root);
    return;
}
// 删除子树
void erase(int r){
              return;
    if (!r)
    s[++tot2] = r;
    erase(ch[r][0]);
    erase(ch[r][1]);
    return;
}
// 从第 pos 个数开始连续删除 tot 个数
void Delete(int pos, int tot){
    Splay(Get_kth(root, pos), 0);
    Splay(Get_kth(root, pos + tot + 1), root);
    erase(Key_value);
    pre[Key_value] = 0;
    Key_value = 0;
    push_up(ch[root][1]);
    push_up(root);
    return;
}
// 将从第 pos 个数开始的连续的 tot 个数修改为 c
void Make Same(int pos, int tot, int c){
    Splay(Get_kth(root, pos), 0);
    Splay(Get_kth(root, pos + tot + 1), root);
    Update_Same(Key_value, c);
    push_up(ch[root][1]);
    push up(root);
    return;
}
// 将第 pos 个数开始的连续 tot 个数进行反转
void Reverse(int pos, int tot){
    Splay(Get_kth(root, pos), 0);
    Splay(Get_kth(root,pos+tot + 1), root);
    Update_Rev(Key_value);
```

```
push_up(ch[root][1]);
    push_up(root);
    return;
}
// 得到第 pos 个数开始的 tot 个数的和
int Get_Sum(int pos, int tot){
    Splay(Get_kth(root, pos), 0);
    Splay(Get_kth(root, pos + tot + 1), root);
    return sum[Key_value];
}
// 得到第 pos 个数开始的 tot 个数中最大的子段和
int Get_MaxSum(int pos, int tot){
    Splay(Get_kth(root, pos), 0);
    Splay(Get_kth(root, pos + tot + 1), root);
    return mx[Key_value];
}
void InOrder(int r){
    if (!r)
              return;
    push_down(r);
    InOrder(ch[r][0]);
    printf("%d ",key[r]);
    InOrder(ch[r][1]);
    return;
}
int main(){
    // freopen("in.txt", "r", stdin);
    // freopen("out.txt", "w", stdout);
    while (scanf("%d%d", &n, &q) == 2){
         Init();
         char op[20];
         int x, y, z;
         while (q--){
              scanf("%s", op);
              if (strcmp(op, "INSERT") == 0){
                   scanf("%d%d", &x, &y);
                   Insert(x, y);
              }
              else if (strcmp(op, "DELETE") == 0){
                   scanf("%d%d", &x, &y);
                   Delete(x,y);
              }
              else if (strcmp(op, "MAKE-SAME") == 0){
                   scanf("%d%d%d", &x, &y, &z);
                   Make_Same(x, y, z);
```

```
}
              else if (strcmp(op, "REVERSE") == 0){
                   scanf("%d%d", &x, &y);
                   Reverse(x, y);
              }
              else if (strcmp(op, "GET-SUM") == 0){
                   scanf("%d%d", &x, &y);
                   printf("%d\n", Get_Sum(x, y));
              }
              else if (strcmp(op, "MAX-SUM") == 0){
                   printf("%d\n", Get_MaxSum(1, size[root] - 2));
              }
         }
    }
     return 0;
}
```

#### 3、树状数组(全功能)

```
#define Max 500010
int N;
int data[Max] = \{0\};
int C[Max] = {0}; //差分数组
int C2[Max] = \{0\}; // C2[i] = (i-1)*C[i]
int BIT[Max] = \{0\};
int BIT1[Max] = \{0\};
int BIT2[Max] = \{0\};
int lowbit(int x){ return (x)&(-x);}
int getsum(int x){ // 数据数组求和
     int sum = 0;
    for(;x > 0;x-=lowbit(x)){
                               sum+=BIT[x]; }
     return sum;
int getsum1(int x){ // 差分数组求和
     int sum = 0;
    for(;x > 0;x=lowbit(x))
                                sum+=BIT1[x]; }
     return sum;
}
int getsum2(int x){
     int sum = 0;
    for(;x > 0;x=lowbit(x))
                                sum+=BIT2[x]; }
     return sum;
```

void add(int i,int add){ // 数据 BIT 更新

```
for (;i <= N;i+=lowbit(i)){
                                BIT[i]+=add; }
}
void add1(int i,int add){// 差分 BIT 更新
     for(;i <= N;i+=lowbit(i)){</pre>
                                 BIT1[i]+=add;}
}
void add2(int i,int add){
     for(;i <= N;i+=lowbit(i)){
                               BIT2[i]+=add;}
}
int main(){
    //输入数据到 data, 更新 BIT 数组
     /* 区间修改, 单点查询
      * C[i] = data[i]-data[i-1]
      * add1(i,C[i])
      * 第 x 个数为 getsum1(x)*/
    /*区间修改,区间查询
      * C2[i] = (i-1)*C[i]
      * add2(i,C2[i])
      * 前 n 项和为 n*getsum1(n)-getsum2(n)*/
     /*cin >> N;
       for(int i = 1; i \le N; i++){
                                 输入数据
         cin >> data[i];
         C[i] = data[i]-data[i-1];
         C2[i] = (i-1)*C[i];
         add1(i,C[i]);
         add2(i,C2[i]);
    }
    int l,r;
     cin >> I >> r;修改区间
     int v;
     cin >> v;
     C[I] +=v;
     C[r+1] -=v;
     add1(l,v);
     add1(r+1,-v);
    int x;
    cin >> x; 查询单点
     cout << getsum1(x) << endl;
     C2[I] += v*(I-1);
     C2[r+1] += (-v)*(r);
     add2(l,v*(l-1));
     add2(r+1,(-v)*r);
    cin >> I >> r; 查询区间
    cout << r^*getsum1(r)-getsum2(r)-((l-1)^*getsum1(l-1)-getsum2(l-1)) << endl; */
}
```

#### 4、线段树

```
//tree[]表示每个区间的最小值,sum[]区间和,col[]打标记,表明该区间每个元素增加了多少
//懒标记用在 update 上,意思是可以不用都更新,用到的时候再更新
void build(int node,int begin,int end){
    if(begin==end){
        tree[node]=array[begin];
        sum[node]=array[begin];
        return:
    }
    int mid=begin+(end-begin)
    build(2*node,begin,mid);
    build(2*node+1,mid+1,end);
    //用 tree 记录每个区间的最值
    if(tree[node*2]<=tree[node*2+1]) tree[node]=tree[2*node];
    else tree[node]=tree[2*node+1];
}
//区间查询
int query1(int node,int begin,int end,int left,int right){//查询 3-5 之间的最小值
    int p1,p2;
    if(left>end | | right<begin) return -1;
    if(begin>=right && end<=right)</pre>
                                   return tree[node];
    int mid=begin+(end-begin)/2;
    p1=query(2*node,begin,mid);
    p2=query(2*node+1,mid+1,end);
    if(p1==-1)return p2;
    if(p2==-1)return p1;
    if(p1<=p2)return p1;
    else return p2;
//单点更新
void updata(int node ,int begin,int end,int ind,int add){//将哪个点 ind 更新了 add
    if(begin==end) tree[node]+=add;
    int mid=begin+(end-begin)/2;
    if(ind<=m) updata(node*2,left,mid,ind,add);</pre>
           updata(node*2+1,mid+1,right,ind,add);
    tree[node]=min(tree[node*2,node*2+1]);
}
//下放标记
void pushdown(int rt,int m){
    if(col[rt]){
        col[rt<<1]=col[rt];
        col[rt<<1|1]=col[rt];
        sum[rt<<1]=col[rt]*(m-m/2);//这个区间要稍微大一点
```

```
sum[rt<<1|1]=col[rt]*(m/2);
        col[rt]=0;
    }
}
//区间修改
//以修改区间的值并求整个数组的和为例
void change(int node,int begin,int end,int left,int right,int c){
    if(left<=begin && end >=right){
        col[node]=c;//在该节点上打一个标记标记打给谁了,就一个么?
        sum[node]=c*(end-begin+1); //做和的时候用了,但是查询的时候不是这个区间了,
这时就用到标记下放
        return;
    }
    pushdown(node,end-begin+1);
    int mid=begin+(end-begin)/2;
    if(left<=mid) change(node*2,begin,end,left,mid,c);</pre>
    if(right>mid) change(node*2+1,begin,end,mid+1,right,c);
    sum[node]=sum[node*2]+sum[node*2+1];
}
//查询区间之和
int query2(int node,int begin,int end,int left,int right){
    if(left<=begin && end>=right) return sum[node]; //不需要就不下放了
    Pushdown(node,end-begin+1);//需要的时候再下放
    int ret=0;
    int mid=begin+(end-begin)/2;
    if(left<mid) ret+=query2(node*2,begin,end,left,mid,c);</pre>
    if(right>mid) ret+=query2(node*2+1,begin,end,mid+1,right,c);
    return ret;
}
```

## 5、主席树,静态区间第 k 小

```
if (1 != r){
          int mid = (l + r) >> 1;
          lson[root] = build(I, mid);
          rson[root] = build(mid + 1, r);
     }
     return root;
int hash_(int x){
     return (int)(lower_bound(t + 1, t + 1 + m, x) - t);
}
int update(int root, int pos, int val){
     int newroot = tot++, tmp = newroot;
     c[newroot] = c[root] + val;
     int I = 1, r = m;
     while (I < r){
          int mid = (l + r) >> 1;
          if (pos \le mid)
               Ison[newroot] = tot++;
               rson[newroot] = rson[root];
               newroot = Ison[newroot];
               root = Ison[root];
               r = mid;
          }
          else{
               rson[newroot] = tot++;
               lson[newroot] = lson[root];
               newroot = rson[newroot];
               root = rson[root];
               I = mid + 1;
          c[newroot] = c[root] + val;
     }
     return tmp;
}
int query(int left_root, int right_root, int k){
     int I = 1, r = m;
     while (1 < r)
          int mid = (l + r) >> 1;
          if (c[lson[left_root]] - c[lson[right_root]] >= k ){
               r = mid;
               left_root = lson[left_root];
               right_root = lson[right_root];
          }
          else{
```

```
I = mid + 1;
                k -= c[lson[left_root]] - c[lson[right_root]];
                left_root = rson[left_root];
                right_root = rson[right_root];
          }
     }
     return I;
}
int main(){
     // freopen("in.txt","r",stdin);
     // freopen("out.txt","w",stdout);
     while (scanf("%d%d", &n, &q) == 2){
          tot = 0;
          for (int i = 1; i <= n; i++)
                                        scanf("%d", &a[i]);
          Init_hash();
          T[n + 1] = build(1, m);
          for (int i = n; i; i--){
               int pos = hash_(a[i]);
               T[i] = update(T[i + 1], pos, 1);
          }
          while (q--){
                int l, r, k;
                scanf("%d%d%d", &I, &r, &k);
                printf("%d\n", t[query(T[I], T[r + 1], k)]);
          }
     }
     return 0;
}
```

## 6、主席树,区间有多少不重复的数

```
/*给出一个序列,查询区间内有多少个不相同的数 */
const int MAXN = 30010;
const int M = MAXN * 100;
int n, q, tot;
int a[MAXN];
int T[MAXN], lson[M], rson[M], c[M];
int build(int l, int r){
    int root = tot++;
    c[root] = 0;
    if (I != r){
        int mid = (I + r) >> 1;
        lson[root] = build(I, mid);
        rson[root] = build(mid + 1, r);
```

```
}
     return root;
}
int update(int root, int pos, int val){
     int newroot = tot++, tmp = newroot;
     c[newroot] = c[root] + val;
     int I = 1, r = n;
     while (l < r){
          int mid = (l + r) >> 1;
          if (pos \le mid)
               Ison[newroot] = tot++;
               rson[newroot] = rson[root];
               newroot = Ison[newroot];
               root = Ison[root];
               r = mid;
          }
          else{
               rson[newroot] = tot++;
               lson[newroot] = lson[root];
               newroot = rson[newroot];
               root = rson[root];
               I = mid + 1;
          c[newroot] = c[root] + val;
     }
     return tmp;
}
int query(int root, int pos){
     int ret = 0;
     int I = 1, r = n;
     while (pos < r){
          int mid = (l + r) >> 1;
          if (pos \le mid){
               r = mid;
               root = Ison[root];
          }
          else{
               ret += c[lson[root]];
               root = rson[root];
               I = mid + 1;
          }
     }
     return ret + c[root];
}
```

```
int main(){
     // freopen("in.txt", "r", stdin);
     // freopen("out.txt", "w", stdout);
     while (scanf("%d", &n) == 1){
          tot = 0;
          for (int i = 1; i <= n; i++)
                                       scanf("%d", &a[i]);
          T[n + 1] = build(1, n);//T 为树
          map<int,int> mp;
          for (int i = n; i >= 1; i--){
                if (mp.find(a[i]) == mp.end()) T[i] = update(T[i + 1], i, 1);
                     int tmp = update(T[i + 1], mp[a[i]], -1);
                     T[i] = update(tmp, i, 1);
               }
                mp[a[i]] = i;
          }
          scanf("%d", &q);
          while (q--){
               int l, r;
                scanf("%d%d", &I, &r);
                printf("%d\n", query(T[I], r));
          }
     }
     return 0;
}
```

## 7、主席树+树状数组,动态区间第 k 大

```
/*树状数组套主席树*/
const int MAXN = 60010;
const int M = 2500010;
int n, q, m, tot;
int a[MAXN], t[MAXN];
int T[MAXN], Ison[M], rson[M],c[M];
int S[MAXN];
struct Query{
    int kind;
     int l, r, k;
} query[10010];
void Init_hash(int k){
    sort(t, t + k);
     m = (int)(unique(t, t + k) - t);
     return;
}
```

```
int hash_(int x){
     return (int)(lower_bound(t, t + m, x) - t);
int build(int I, int r){
     int root = tot++;
     c[root] = 0;
     if (1 != r){
          int mid = (I + r) / 2;
          lson[root] = build(I, mid);
          rson[root] = build(mid + 1, r);
     }
     return root;
}
int Insert(int root, int pos, int val){
     int newroot = tot++, tmp = newroot;
     int l = 0, r = m - 1;
     c[newroot] = c[root] + val;
     while (I < r)
          int mid = (l + r) >> 1;
          if (pos \le mid)
               Ison[newroot] = tot++;
               rson[newroot] = rson[root];
               newroot = Ison[newroot];
               root = Ison[root];
               r = mid;
          }
          else{
               rson[newroot] = tot++;
               lson[newroot] = lson[root];
               newroot = rson[newroot];
               root = rson[root];
               I = mid + 1;
          c[newroot] = c[root] + val;
     }
     return tmp;
}
int lowbit(int x){ return x & (-x);}
int use[MAXN];
void add(int x, int pos, int val){
     while (x \le n){
          S[x] = Insert(S[x], pos, val);
          x += lowbit(x);
     }
```

```
return;
}
int sum(int x){
     int ret = 0;
     while (x > 0)
           ret += c[lson[use[x]]];
           x = lowbit(x);
     }
     return ret;
}
int Query(int left, int right, int k){
     int left_root = T[left - 1];
     int right_root = T[right];
     int l = 0, r = m - 1;
     for (int i = left - 1; i; i -= lowbit(i))
                                              use[i] = S[i];
     for (int i = right; i; i -= lowbit(i))
                                             use[i] = S[i];
     while (I < r){
           int mid = (I + r) / 2;
           int tmp = sum(right) - sum(left - 1) + c[lson[right_root]] - c[lson[left_root]];
           if (tmp >= k){
                r = mid;
                for (int i = left - 1; i; i -= lowbit(i))
                                                         use[i] = lson[use[i]];
                for (int i = right; i; i -= lowbit(i))
                                                       use[i] = lson[use[i]];
                left_root = lson[left_root];
                right root = lson[right root];
           }
           else{
                I = mid + 1;
                k = tmp;
                for (int i = left - 1; i; i -= lowbit(i))
                                                         use[i] = rson[use[i]];
                for (int i = right; i; i -= lowbit(i))
                                                      use[i] = rson[use[i]];
                left root = rson[left root];
                right_root = rson[right_root];
           }
     }
     return I;
}
void Modify(int x, int p, int d){
     while (x \le n)
           S[x] = Insert(S[x], p, d);
           x += lowbit(x);
     }
     return;
}
```

```
int main(){
     // freopen("in.txt", "r", stdin);
     // freopen("out.txt", "w", stdout);
     int Tcase;
     scanf("%d", &Tcase);
     while (Tcase--){
          scanf("%d%d", &n, &q);
          tot = 0;
          m = 0;
          for (int i = 1; i \le n; i++){
                scanf("%d", &a[i]);
                t[m++] = a[i];
          }
          char op[10];
          for (int i = 0; i < q; i++){
                scanf("%s", op);
                if (op[0] == 'Q'){}
                     query[i].kind = 0;
                     scanf("%d%d%d", &query[i].I, &query[i].r, &query[i].k);
                }
                else{
                     query[i].kind = 1;
                     scanf("%d%d", &query[i].I, &query[i].r);
                     t[m++] = query[i].r;
                }
          }
          Init_hash(m);
          T[0] = build(0, m - 1);
          for (int i = 1; i \le n; i++) T[i] = Insert(T[i-1], hash_(a[i]), 1);
          for (int i = 1; i \le n; i++)
                                      S[i] = T[0];
          for (int i = 0; i < q; i++){
                if (query[i].kind == 0){
                     printf("%d\n", t[Query(query[i].l, query[i].r, query[i].k)]);\\
                }
                else{
                     Modify(query[i].l, hash_(a[query[i].l]), -1);
                     Modify(query[i].l, hash (query[i].r), 1);
                     a[query[i].l] = query[i].r;
                }
          }
     }
     return 0;
}
```

## 8、左偏树 ,小堆的合并,有序序列合并

```
/* 合并复杂度 O(log N)
 * INIT: init()读入数据并进行初始化;
    CALL: merge() 合并两棵左偏树;
           ins() 插入一个新节点;
           top() 取得最小结点;
           pop() 取得并删除最小结点;
           del() 删除某结点;
           add() 增/减一个结点的键值;
           iroot() 获取结点 i 的根;*/
#define typec int
                      // type of key val
const int na = -1;
const int N = 1010;
struct node{
    typec key;
    int I, r, f, dist;
} tr[N];
int iroot(int i){ // find i's root
    if (i == na) return i;
    while (tr[i].f!= na){    i = tr[i].f;}
    return i;
int merge(int rx, int ry){
                        // two root:
                                           rx, ry
    if (rx == na)
                  return ry;
    if (ry == na)
                  return rx;
    if (tr[rx].key > tr[ry].key)
                              swap(rx, ry);
    int r = merge(tr[rx].r, ry);
    tr[rx].r = r;
    tr[r].f = rx;
    if (tr[r].dist > tr[tr[rx].l].dist)
                                   swap(tr[rx].l, tr[rx].r);
    if (tr[rx].r == na) tr[rx].dist = 0;
             tr[rx].dist = tr[tr[rx].r].dist + 1;
    return rx; // return new root
}
int ins(int i, typec key, int root)
{ // add a new node(i, key)
    tr[i].key = key;
    tr[i].I = tr[i].r = tr[i].f = na;
    tr[i].dist = 0;
    return root = merge(root, i); // return new root
}
int del(int i)
{ // delete node i
```

```
if (i == na)
                      return i;
     int x, y, l, r;
     I = tr[i].I;
     r = tr[i].r;
     y = tr[i].f;
     tr[i].I = tr[i].r = tr[i].f = na;
     tr[x = merge(I, r)].f = y;
     if (y != na \&\& tr[y].l == i) tr[y].l = x;
     if (y != na && tr[y].r == i)
                                      tr[y].r = x;
     for (; y != na; x = y, y = tr[y].f){
           if (tr[tr[y].l].dist < tr[tr[y].r].dist) swap(tr[y].l, tr[y].r);
           if (tr[tr[y].r].dist + 1 == tr[y].dist)
                                                   break;
           tr[y].dist = tr[tr[y].r].dist + 1;
     }
                      return iroot(x); // return new root
     if (x != na)
     else return iroot(y);
}
node top(int root){     return tr[root];}
node pop(int &root){
     node out = tr[root];
     int I = tr[root].I, r = tr[root].r;
     tr[root].l = tr[root].r = tr[root].f = na;
     tr[I].f = tr[r].f = na;
     root = merge(I, r);
     return out;
}
int add(int i, typec val){  // tr[i].key += val
     if (i == na) return i;
     if (tr[i].l == na && tr[i].r == na && tr[i].f == na){
           tr[i].key += val;
           return i;
     }
     typec key = tr[i].key + val;
     int rt = del(i);
     return ins(i, key, rt);
}
void init(int n){
     for (int i = 1; i \le n; i++){
           scanf("%d", &tr[i].key);
                                       // %d: type of key
           tr[i].l = tr[i].r = tr[i].f = na;
           tr[i].dist = 0;
     }
     return;
}
```

### 9、背包相关

```
int dp[SIZE];
int volume[MAXN], value[MAXN], c[MAXN];
                    // 总物品数,背包容量
int n, v;
// 01背包
void ZeroOnepark(int val, int vol){
    for (int j = v ; j >= vol; j--){
            dp[j] = max(dp[j], dp[j - vol] + val);
    }
}
// 完全背包
void Completepark(int val, int vol){
     for (int j = vol; j \le v; j++){
            dp[j] = max(dp[j], dp[j - vol] + val);
    }
}
// 多重背包
void Multiplepark(int val, int vol, int amount){
     if (vol * amount >= v) Completepark(val, vol);
     else{
         int k = 1;
         while (k < amount){
              ZeroOnepark(k * val, k * vol);
              amount -= k;
              k <<= 1;
         }
         if (amount > 0){
              ZeroOnepark(amount * val, amount * vol);
         }
    }
}
int main(){
     while (cin >> n >> v){
         for (int i = 1; i \le n; i++){
              cin >> volume[i] >> value[i] >> c[i]; // 费用,价值,数量
         }
         memset(dp, 0, sizeof(dp));
         for (int i = 1; i <= n; i++){
              Multiplepark(value[i], volume[i], c[i]);
         }
         cout << dp[v] << endl;
     }
     return 0;}
```

## 10、并查集

```
/* INIT: makeset(n);
 * CALL: findset(x); unin(x, y);*/
const int N = 1010;
struct lset{
     int p[N], rank[N], sz;
     void link(int x, int y){
          if (x == y) return;
          if (rank[x] > rank[y])
                                   p[y] = x;
          else
                   p[x] = y;
          if (rank[x] == rank[y])
                                    rank[y]++;
          return;
     }
     void makeset(int n){
          sz = n;
          for (int i = 0; i < sz; i++){
               p[i] = i;
               rank[i] = 0;
          }
          return;
     }
     int findset(int x){
          if (x != p[x])
                            p[x] = findset(p[x]);
          return p[x];
     }
     void unin(int x, int y){
          link(findset(x), findset(y));
          return;
     }
     void compress(){
          for (int i = 0; i < sz; i++)
                                      findset(i);
          return;
     }
};
```

#### 11、使序列有序的最小交换次数

```
int IT_MAX = 1 << 19;
int MOD = 1000000007;
const int INF = 0x3f3f3f3f;
const db PI = acos(-1);
const db ERR = 1e-10;
const int MAX N = 100005;
bool cmp (int a , int b){
/*交换任意两数的本质是改变了元素位置,
 * 故建立元素与其目标状态应放置位置的映射关系*/
int getMinSwaps(vector<int> &A){ //排序
    vector<int> B(A);
    sort(B.begin(), B.end());
    map<int, int> m;
    int len = (int)A.size();
                            m[B[i]] = i; //建立每个元素与其应放位置的映射关系
    for (int i = 0; i < len; i++)
                     // 循环节个数
    int loops = 0;
    vector<bool> flag(len, false);
    // 找出循环节的个数
    for (int i = 0; i < len; i++){
        if (!flag[i]){
            int j = i;
            while (!flag[j]){
                flag[j] = true;
                j = m[A[j]];
                             // 原序列中 i 位置的元素在有序序列中的位置
            }
            loops++;
        }
    }
    return len - loops;
vector<int> nums;
int main(){
    nums.push_back(1);
    nums.push back(2);
    nums.push_back(4);
    nums.push_back(3);
    nums.push_back(5);
    int res = getMinSwaps(nums);
    cout << res << '\n';
    return 0;
}
```

#### 12、使序列有序的最小交换区间次数

- /\* 默认目标映射关系是 key 1 => val 1 ······ key n => val n
- \* 如果序列不是 1~n 可以通过 map 建立新的目标映射关系
- \* 交换任意区间的本质是改变了元素的后继,故建立元素与其初始状态后继的映射关系\*/

```
const int MAXN = 30;
int n;
int vis[MAXN];
int A[MAXN], B[MAXN];
int getMinSwaps(){
     memset(vis, 0, sizeof(vis));
     for (int i = 1; i <= n; i++)
                                  B[A[i]] = A[i \% n + 1];
     for (int i = 1; i <= n; i++) B[i] = (B[i] - 2 + n) \% n + 1;
     int cnt = n;
     for (int i = 1; i <= n; i++){
          if (vis[i])
                        continue;
          vis[i] = 1;
          cnt--;
          for (int j = B[i]; j != i; j = B[j]) vis[j] = 1;
     }
     return cnt;
}
int main(){
     cin >> n;
     for (int i = 1; i <= n; i++)
                                  cin >> A[i];
     int res = getMinSwaps();
     cout << res << '\n';
     return 0;
}
```