

Q1. VAR MODELLING

Download the adjusted closing prices for the following five stocks: AAPL, MSFT, IBM, Nvidia, Alphabet, Amazon for the period spanning January 1, 2014, to December 31, 2024. Then build an equally weighted portfolio and perform the following analysis discussing carefully the adopted procedure and your findings

1. Perform a statistical analysis of the portfolio returns, describing your main findings.
2. Using a rolling window of 6 months and starting on July 1st 2014, you have to estimate for each day the VaR at 90% and 99% confidence levels at one day horizon using at least four different methods.
3. Given your VaR forecasts, compute the number of VaR violations for each model and each confidence level.
4. Using the Kupiec, the conditional coverage test and the distributional tests described in the Christoffersen book, evaluate the backtesting performance of these models.
5. Discuss your results.

Q2. THE RISK PARITY PORTFOLIO

The **risk parity portfolio** is a portfolio in which each asset contributes equally to the Component Value at Risk (VaR). This portfolio is constructed by selecting weights that minimize the dispersion (or standard deviation) of the individual Conditional VaRs (CVaRs).

Using the dataset from **Question 1**, compute the Component VaR using both the parametric approach, which relies on the sample covariance matrix, and the non-parametric approach.

Another portfolio to consider is the **maximum diversification portfolio**, which aims to maximize the following ratio:

$$\max_{\mathbf{w}} \frac{\mathbf{w}'\boldsymbol{\sigma}}{\sqrt{\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}}}$$

subject to: $\mathbf{w}'\mathbf{1} = 1$ and $\mathbf{w} \geq \mathbf{0}$, where \mathbf{w} represents the portfolio weights, $\boldsymbol{\sigma}$ is the vector of asset volatilities, and $\boldsymbol{\Sigma}$ is the covariance matrix.

A third portfolio is the **equally-weighted portfolio**, where all assets are assigned equal weights.

PROCEDURE

1. Split the dataset into two parts:
 - Use the first half of the sample to determine the portfolio compositions.
 - Use the second half of the sample to compute the daily log-returns for each portfolio.
2. Evaluate the performance of the portfolios using the following metrics:
 - **Sharpe Ratio:** Measures risk-adjusted returns, assuming a zero risk-free rate.
 - **Maximum Drawdown:** Quantifies the largest peak-to-trough decline in portfolio value.
 - **Number of VaR Violations:** The count of instances where portfolio losses exceed the Value at Risk (VaR) at a 95% confidence level.
 - Other measures you believe to be relevant for your analysis

ANALYSIS

After calculating these metrics, analyze and interpret the results, comparing the performance of the different portfolio strategies. Discuss the trade-offs between risk parity, maximum diversification, and equally-weighted portfolios, focusing on their relative strengths and weaknesses in terms of risk-adjusted returns, drawdowns, and turnover.