Q3 VaR of a bond

Consider a bond with a 10-year maturity and annual coupon payments. This bond has a face value of 100 and a coupon rate of 5%. Currently, the gross price of the bond is 99. Assume a year consists of 360 days.

Compute the yield to maturity (YTM) for this bond, under the assumption that the daily fluctuations in YTM follow independent and identically distributed (i.i.d.) Gaussian random variables with a mean of 0 and a standard deviation of 0.006.

- Estimate the probability of a 10% decline in the bond price within a 30-day period.
- Compute the Value at Risk (VaR) for your bond at a 99% confidence level across various horizons (1, 10, 20, 30,...,90 days) using the following methods:
  1. Exact formula (provide a detailed explanation of the procedure).
  2. Exact formula via delta approximation (i.e., exploiting Taylor's formula truncated to the first order).
  3. Exact formula via delta-gamma approximation (i.e., exploiting Taylor's formula truncated to the second order).
  4. Monte Carlo simulation with at least 10,000 simulations and delta approximation.
  5. Monte Carlo simulation with at least 10,000 simulations and delta-gamma approximation.
  6. Monte Carlo simulation with at least 10,000 simulations and full revaluation.
  7. Discuss and compare your results.
- Compute the Expected Shortfall of your bond at a 99% confidence level for different horizons (1, 10, 20, 30,...,90 days) using your preferred method. Provide a detailed explanation of the adopted procedure.

Monte Carlo simulations must be performed using at least 10,000 simulations. Greeks (i.e. Delta, Gamma and Theta) can be computed analytically or via finite differences.

Q4. VaR of a Portfolio with Options

Consider the following tickers: INTC, JPM, AA, and PG. Using two years of historical data, estimate the covariance matrix based on your preferred approach.

On February 21st, 2024, you hold a portfolio consisting of the following derivatives:

- INTC: Short 3 call options with a strike price at 90% of the stock price and a time to maturity of 9 months.
- JPM: Long 6 at-the-money put options with a time to maturity of 6 months.
- AA: Long 6 call options with a strike price at 105% of the underlying stock price and a time to maturity of 12 months.
- PG: Short 2 put options with a strike price at 110% of the underlying stock price and a time to maturity of 9 months.

The options are priced using the Black-Scholes model, with annualized historical volatility as an input. The annualized risk-free rate is set at 4.0%, and we assume that the stocks do not pay dividends.

Simulate stock returns using at least 10,000 simulations over a 10-day horizon. Compute the Value at Risk (VaR) and Expected Shortfall (ES) for the options portfolio at a 99% confidence level.

Next, calculate the Marginal and Component VaR/ES using the appropriate formula for the non-Gaussian case.

Provide a step-by-step explanation of the procedure, detailing your calculations and offering insights into the results.