Coverage Analysis of Joint Localization and Communication in THz Systems with 3D Arrays

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System Introduction

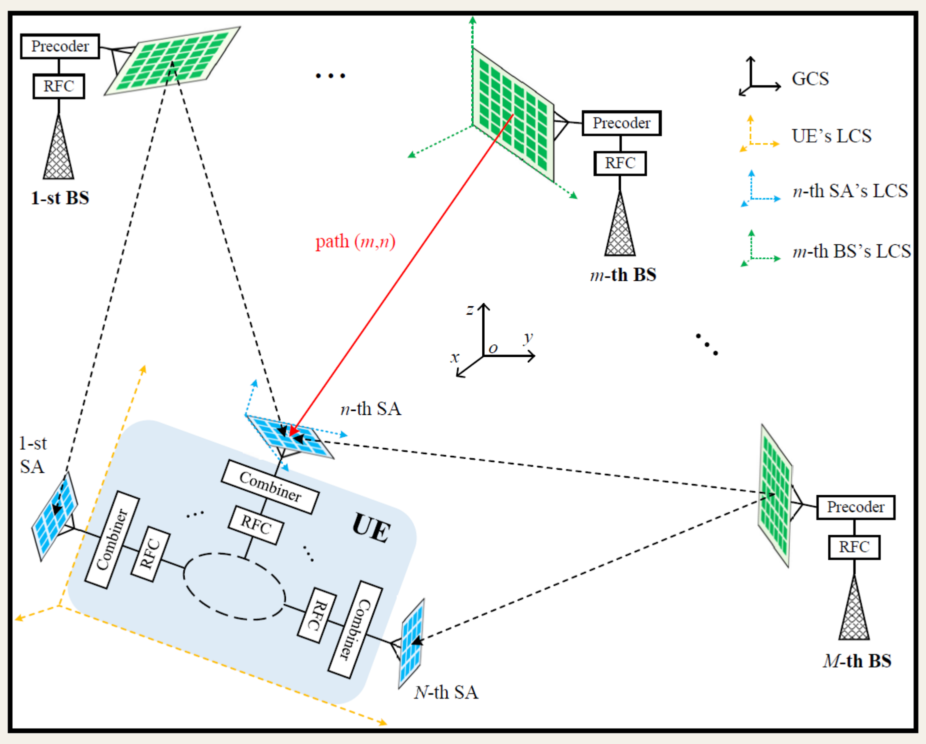

Fig. 1: Illustration of the considered geometric model.

- M base stations (BS) with known positions & orientations.
- N subarrays are arranged on the user equipment (UE).
- We consider a downlink scenario, where BSs are synchronized with each other.
- We use the Rician fading model to characterize the THz channel.
- **Localization** and **communication** performance are evaluated jointly.

THz MIMO Channel with 3D Array

Using analog beamforming, the received signal at the k-th subcarrier and the g-th transmission from the m-th BS is

$$\mathbf{y}_m^{(g)}[k] = \sqrt{P}\mathbf{W}^{(g)}\mathbf{H}_m[k]\mathbf{w}_{\mathrm{B},m}^{(g)}x_m^{(g)}[k] + \mathbf{W}^{(g)}\mathbf{n}^{(g)}[k],$$

$$\mathbf{H}_m[k] = \begin{bmatrix}\mathbf{H}_{m,1}[k]\\ \mathbf{H}_{m,2}[k]\\ \vdots \\ \mathbf{H}_{m,N}[k]\end{bmatrix}, \mathbf{W}^{(g)} = \begin{bmatrix}\mathbf{w}_{\mathrm{S},1}^{(g)\mathsf{T}} & 0 & \cdots & 0\\ 0 & \mathbf{w}_{\mathrm{S},2}^{(g)\mathsf{T}} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \mathbf{w}_{\mathrm{S},N}^{(g)\mathsf{T}}\end{bmatrix}.$$

Consider the Rician fading model,

$$\mathbf{H}_{m,n}[k] = \frac{G_{m,n}^k\sqrt{K_r}}{\sqrt{K_r+1}}\bar{\mathbf{H}}_{m,n}[k] + \frac{G_{m,n}^k}{\sqrt{K_r+1}}\tilde{\mathbf{H}}_{m,n}[k].$$

LOS component:

$$\bar{\mathbf{H}}_{m,n}[k] = e^{-j2\pi f_k\tau_{m,n}}\mathbf{a}_{\mathrm{S},n}\left(f_k,\mathbf{t}_m^{\mathrm{SA},n}\right)\mathbf{a}_{\mathrm{B},m}^{\mathsf{T}}\left(f_k,\mathbf{t}_n^{\mathrm{BS},m}\right),$$

NLOS component:

$$\tilde{\mathbf{H}}_{m,n}[k] = \mathbf{\Theta}_{\mathrm{R},n}^{1/2}\hat{\mathbf{H}}_{m,n}\mathbf{\Theta}_{\mathrm{T},m}^{1/2},$$

Path gain:

$$G_{m,n}^k = \begin{cases}\left(\frac{c}{4\pi f_k d_{m,n}}\right)^{\frac{\nu}{2}} e^{-\frac{1}{2}\mathcal{K}(f_k)d_{m,n}}\sqrt{G_{\mathrm{S},n}G_{\mathrm{B},m}}, & \text{if } (m,n)\in\mathcal{Q},\\ 0, & \text{if } (m,n)\notin\mathcal{Q},\end{cases}$$

Localization Performance Analysis

The localization problem: estimating the position and orientation of the UE based on the received signal.

Focusing on the LOS channels, we merge the NLOS component with the AWGN

$$\mathbf{y}_m^{(g)}[k] = \underbrace{\sqrt{P}\mathbf{W}^{(g)}\bar{\mathbf{H}}_m[k]\mathbf{w}_{\mathrm{B},m}^{(g)}x_m^{(g)}[k]}_{\boldsymbol{\varsigma}_m^{(g)}[k]} + \mathring{\mathbf{n}}_m^{(g)}[k],$$

$$\mathring{\mathbf{n}}_m^{(g)}[k] = \sqrt{P}\mathbf{W}^{(g)}\tilde{\mathbf{H}}_m[k]\mathbf{w}_{\mathrm{B},m}^{(g)}x_m^{(g)}[k] + \mathbf{W}^{(g)}\mathbf{n}^{(g)}[k]$$

Lemma 1.

$$\mathring{\mathbf{n}}^{(g)}[k] \sim \mathcal{CN}(\mathbf{0}, \underbrace{diag\{\Sigma_{m_1,n_1},\dots,\Sigma_{m_D,n_D}\}}_{\tilde{\mathbf{\Sigma}}}),$$

where

$$\Sigma_{m_i,n_i} = \sigma^2\left\|\mathbf{w}_{S,n_i}^{(g)}\right\|_2^2 + \frac{P(G_{m_i,n_i}^k)^2}{1+K_r} \cdot\left\|\mathbf{\Theta}_{T,m_i}^{1/2}\mathbf{w}_{B,m_i}^{(g)}x_{m_i}^{(g)}[k]\right\|_2^2\cdot\left\|\left(\mathbf{w}_{S,n_i}^{(g)}\right)^{\mathsf{T}}\mathbf{\Theta}_{R,n_i}^{1/2}\right\|_2^2, \quad i=1,\dots,D.$$

Before deriving the localization performance, Lemma 1 determines the statistics of the noise.

Derivation of the Cramer-Rao lower bound (CRLB)

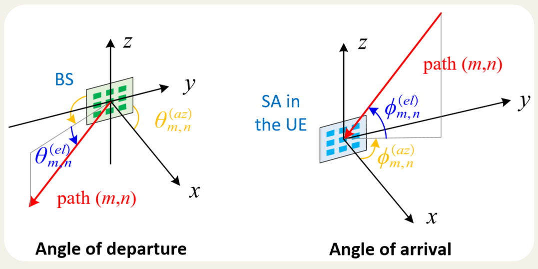

Fig. 2: The geometry of the azimuth and elevation components of the AOD and AOA.

Two-stage localization:

1. Estimate the channel parameters based on the received signal.
2. Estimate the UE position and orientation based on the estimated channel parameters.

Channel parameters:

$$\boldsymbol{\eta}_{\mathrm{ch}} \triangleq [\underbrace{\boldsymbol{\theta}_{\mathrm{az}}^{\mathsf{T}},\boldsymbol{\theta}_{\mathrm{el}}^{\mathsf{T}},\boldsymbol{\phi}_{\mathrm{az}}^{\mathsf{T}},\boldsymbol{\phi}_{\mathrm{el}}^{\mathsf{T}},\boldsymbol{\tau}^{\mathsf{T}}}_{\boldsymbol{\eta}\in\mathbb{R}^{5D\times1}},\mathbf{h}_a^{\mathsf{T}},\mathbf{h}_p^{\mathsf{T}}]^{\mathsf{T}} \in \mathbb{R}^{7D\times 1},$$

Localization parameters:

$$\mathbf{r} \triangleq \left[\mathbf{p}_{\mathrm{U}}^{\mathsf{T}},\rho,\mathrm{vec}(\mathbf{R}_{\mathrm{U}})^{\mathsf{T}}\right]^{\mathsf{T}}.$$

$$\theta_{m,n}^{(\mathrm{az})} = \mathrm{atan2}\Big(\mathbf{u}_2^{\mathsf{T}}\mathbf{R}_{\mathrm{B},m}^{\mathsf{T}}(\mathbf{p}_{\mathrm{U}}+\mathbf{R}_{\mathrm{U}}\mathbf{p}_{\mathrm{S},n}^{\mathrm{UE}}-\mathbf{p}_{\mathrm{B},m}), \mathbf{u}_1^{\mathsf{T}}\mathbf{R}_{\mathrm{B},m}^{\mathsf{T}}(\mathbf{p}_{\mathrm{U}}+\mathbf{R}_{\mathrm{U}}\mathbf{p}_{\mathrm{S},n}^{\mathrm{UE}}-\mathbf{p}_{\mathrm{B},m})\Big),$$

$$\theta_{m,n}^{(\mathrm{el})} = \mathrm{asin}\left(\frac{\mathbf{u}_3^{\mathsf{T}}\mathbf{R}_{\mathrm{B},m}^{\mathsf{T}}(\mathbf{p}_{\mathrm{U}}+\mathbf{R}_{\mathrm{U}}\mathbf{p}_{\mathrm{S},n}^{\mathrm{UE}}-\mathbf{p}_{\mathrm{B},m})}{\|\mathbf{p}_{\mathrm{U}}+\mathbf{R}_{\mathrm{U}}\mathbf{p}_{\mathrm{S},n}^{\mathrm{UE}}-\mathbf{p}_{\mathrm{B},m}\|_2}\right),$$

$$\phi_{m,n}^{(\mathrm{az})} = \mathrm{atan2}\Big(-\mathbf{u}_2^{\mathsf{T}}\left(\mathbf{R}_{\mathrm{S},n}^{\mathrm{UE}}\right)^{\mathsf{T}}\mathbf{R}_{\mathrm{U}}^{\mathsf{T}}(\mathbf{p}_{\mathrm{U}}+\mathbf{R}_{\mathrm{U}}\mathbf{p}_{\mathrm{S},n}^{\mathrm{UE}}-\mathbf{p}_{\mathrm{B},m}), -\mathbf{u}_1^{\mathsf{T}}\left(\mathbf{R}_{\mathrm{S},n}^{\mathrm{UE}}\right)^{\mathsf{T}}\mathbf{R}_{\mathrm{U}}^{\mathsf{T}}(\mathbf{p}_{\mathrm{U}}+\mathbf{R}_{\mathrm{U}}\mathbf{p}_{\mathrm{S},n}^{\mathrm{UE}}-\mathbf{p}_{\mathrm{B},m})\Big),$$

$$\phi_{m,n}^{(\mathrm{el})} = \mathrm{asin}\left(\frac{\mathbf{u}_3^{\mathsf{T}}\left(\mathbf{R}_{\mathrm{S},n}^{\mathrm{UE}}\right)^{\mathsf{T}}\mathbf{R}_{\mathrm{U}}^{\mathsf{T}}(\mathbf{p}_{\mathrm{U}}+\mathbf{R}_{\mathrm{U}}\mathbf{p}_{\mathrm{S},n}^{\mathrm{UE}}-\mathbf{p}_{\mathrm{B},m})}{-\|\mathbf{p}_{\mathrm{U}}+\mathbf{R}_{\mathrm{U}}\mathbf{p}_{\mathrm{S},n}^{\mathrm{UE}}-\mathbf{p}_{\mathrm{B},m}\|_2}\right),$$

Fisher information matrix (FIM) of the channel parameters:

$$\boldsymbol{\mathcal{I}}(\boldsymbol{\eta}_{ch}) = 2\sum_{g=1}^{G}\sum_{k=1}^{K} Re\left(\left(\frac{\partial\mathbf{m}^{(g)}[k]}{\partial\boldsymbol{\eta}_{ch}}\right)^{\mathsf{H}}\tilde{\mathbf{\Sigma}}^{-1}\frac{\partial\mathbf{m}^{(g)}[k]}{\partial\boldsymbol{\eta}_{ch}}\right).$$

$$\boldsymbol{\mathcal{I}}(\boldsymbol{\eta}) = \left(\left[\boldsymbol{\mathcal{I}}(\boldsymbol{\eta}_{\mathrm{ch}})^{-1}\right]_{1:5D,1:5D}\right)^{-1}.$$

FIM of the localization parameters:

$$\boldsymbol{\mathcal{I}}(\mathbf{r}) = \mathbf{T}^{\mathsf{T}}\boldsymbol{\mathcal{I}}(\boldsymbol{\eta})\mathbf{T}, \qquad [\mathbf{T}]_{i,j} = \partial\eta_i/\partial r_j$$

Constrained CRLB:

$$\boldsymbol{\mathcal{I}}_{\mathrm{const}}^{-1}(\mathbf{r}) = \mathbf{M}(\mathbf{M}^{\mathsf{T}}\boldsymbol{\mathcal{I}}(\mathbf{r})\mathbf{M})^{-1}\mathbf{M}^{\mathsf{T}}. \implies \mathrm{PEB} \triangleq \sqrt{\mathrm{tr}([\boldsymbol{\mathcal{I}}_{\mathrm{const}}^{-1}(\mathbf{r})]_{1:3,1:3})}, \quad \mathrm{OEB} \triangleq \sqrt{\mathrm{tr}([\boldsymbol{\mathcal{I}}_{\mathrm{const}}^{-1}(\mathbf{r})]_{5:13,5:13})}.$$

Localization coverage:

$$R^{\mathrm{p}}(\xi^{\mathrm{p}}) = \frac{\int_{\Omega_p}\int_{\Omega_o} H(\xi^{\mathrm{p}} - \mathrm{PEB}(\mathbf{p}_{\mathrm{U}},\mathbf{R}_{\mathrm{U}}))\mathrm{d}\mathbf{p}_{\mathrm{U}}\mathrm{d}\mathbf{R}_{\mathrm{U}}}{\int_{\Omega_p}\int_{\Omega_o}\mathrm{d}\mathbf{p}_{\mathrm{U}}\mathrm{d}\mathbf{R}_{\mathrm{U}}},$$

$$R^{\mathrm{o}}(\xi^{\mathrm{o}}) = \frac{\int_{\Omega_p}\int_{\Omega_o} H(\xi^{\mathrm{p}} - \mathrm{OEB}(\mathbf{p}_{\mathrm{U}},\mathbf{R}_{\mathrm{U}}))\mathrm{d}\mathbf{p}_{\mathrm{U}}\mathrm{d}\mathbf{R}_{\mathrm{U}}}{\int_{\Omega_p}\int_{\Omega_o}\mathrm{d}\mathbf{p}_{\mathrm{U}}\mathrm{d}\mathbf{R}_{\mathrm{U}}},$$

Table. 1: Default simulation parameters.

Parameter	Value
Propagation Speed c	2.9979×10^8 m/s
Carrier Frequency f_c	140 GHz
Bandwidth B	1000 MHz
# subcarriers K	128
# transmissions G	10
path loss component ν	2
Rician K-factor K_r	4
Transmit Power P	10 mW
Clock Offset ρ	100 ns
Noise PSD N_0	−173.855 dBm/Hz
UE Noise Figure	10 dB
Molecules of Medium	$\{N_2, O_2, H_2O, CO_2, CH_4\}$
Molecules Ratio	{76.6%, 21.0%, 1.6%, 0.03%, 0.77%}
Dimension of BS array	10×10
Dimension of UE's SA	4×4
Positions of BSs	$\mathbf{p}_{\mathrm{B},1} = [10.5, 10.5, 5]^{\mathsf{T}}$ $\mathbf{p}_{\mathrm{B},2} = [10.5, -10.5, 5]^{\mathsf{T}}$
Euler Angles of BSs	$\mathbf{R}_{\mathrm{B},1} = [0°, 135°, 45°]^{\mathsf{T}}$ $\mathbf{R}_{\mathrm{B},2} = [0°, 0°, 90°]^{\mathsf{T}}$

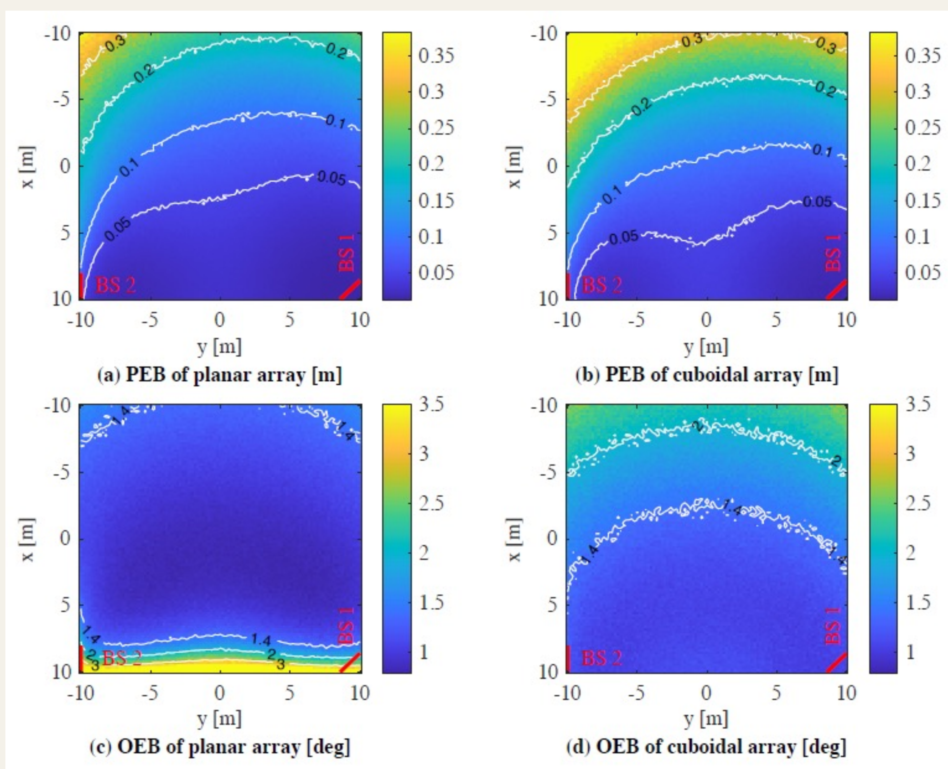

Fig. 3: The PEB and OEB distribution over different UE positions for the planar and cuboidal arrays.

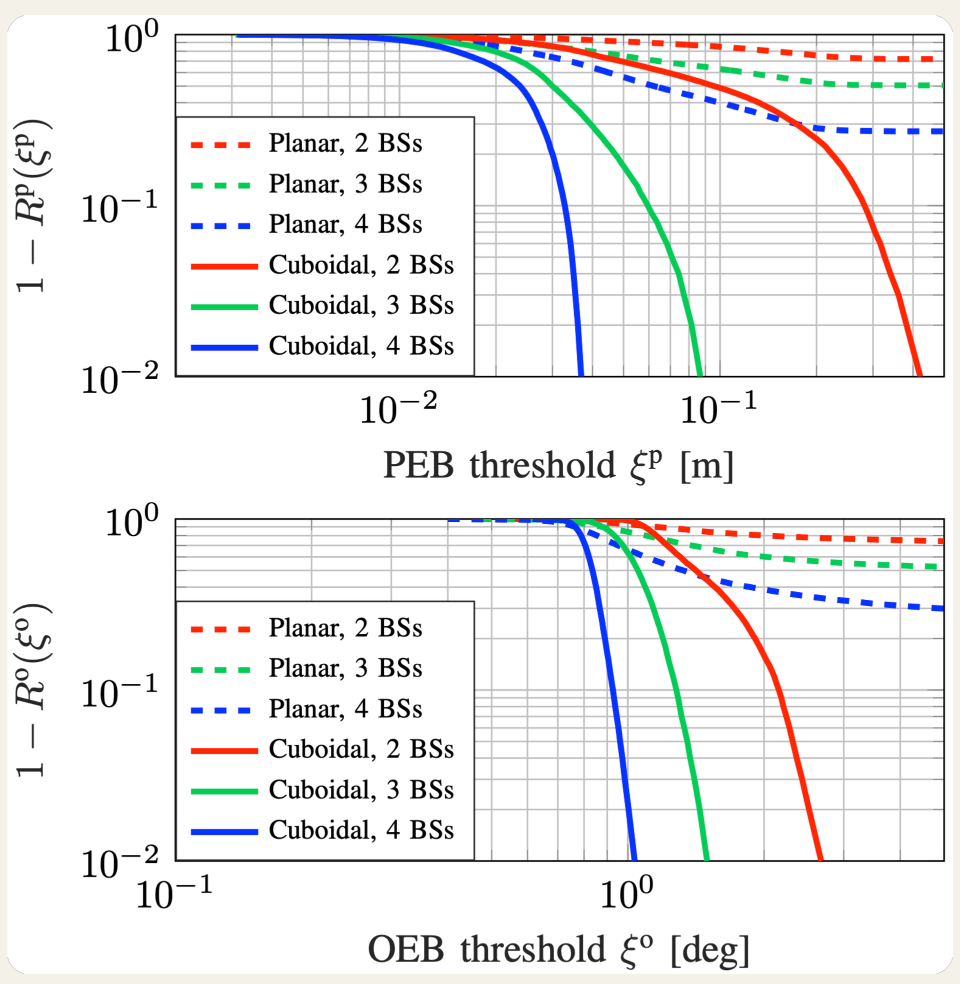

Fig. 4: PEB and OEB coverage of the two types of the array with {2, 3, 4} BSs.

Communication Performance Analysis

Instantaneous SNR:

$$\mathrm{SNR}_{m,n}^k \triangleq \frac{P\left|\mathbf{w}_{\mathrm{S},n}^{\mathsf{T}}\mathbf{H}_{m,n}[k]\mathbf{w}_{\mathrm{B},m}\right|^2}{\sigma^2},$$

Outage probability:

$$P_{m,n}^{\mathrm{out}}(\gamma_{\mathrm{th}},k) = 1, \ \text{if}(m,n)\notin\mathcal{Q}, \qquad P_{m,k,\gamma_{\mathrm{th}}}^{\mathrm{out}} \triangleq \prod_{n=1}^{N}P_{m,n}^{\mathrm{out}}(\gamma_{\mathrm{th}},k).$$

$$P_{m,n}^{\mathrm{out}}(\gamma_{\mathrm{th}},k) = \mathsf{P}(\mathrm{SNR}_{m,n}^k < \gamma_{\mathrm{th}})$$

Ergodic capacity:

$$C_m = \mathbb{E}\left[\frac{1}{K}\sum_{k=1}^{K}\log_2\det\left(\mathbf{I}_{N_S} + \frac{P}{\sigma^2 M}\mathbf{H}_m[k]\mathbf{H}_m^{\mathsf{H}}[k]\right)\right]. \qquad C = \sum_{m=1}^{M}C_m.$$

Non-Outage Coverage and Capacity Coverage:

$$R_{m,k,\gamma_{\mathrm{th}}}^{\mathrm{out}}(\xi_m^{\mathrm{out}}) = \frac{\int_{\Omega_p}\int_{\Omega_o} H(\xi_m^{\mathrm{out}} - P_{m,k,\gamma_{\mathrm{th}}}^{\mathrm{out}}(\mathbf{p}_{\mathrm{U}},\mathbf{R}_{\mathrm{U}}))\mathrm{d}\mathbf{p}_{\mathrm{U}}\mathrm{d}\mathbf{R}_{\mathrm{U}}}{\int_{\Omega_p}\int_{\Omega_o}\mathrm{d}\mathbf{p}_{\mathrm{U}}\mathrm{d}\mathbf{R}_{\mathrm{U}}},$$

$$R^{\mathrm{c}}(\xi^{\mathrm{c}}) = \frac{\int_{\Omega_p}\int_{\Omega_o} H(C(\mathbf{p}_{\mathrm{U}},\mathbf{R}_{\mathrm{U}}) - \xi^{\mathrm{c}})\mathrm{d}\mathbf{p}_{\mathrm{U}}\mathrm{d}\mathbf{R}_{\mathrm{U}}}{\int_{\Omega_p}\int_{\Omega_o}\mathrm{d}\mathbf{p}_{\mathrm{U}}\mathrm{d}\mathbf{R}_{\mathrm{U}}}.$$

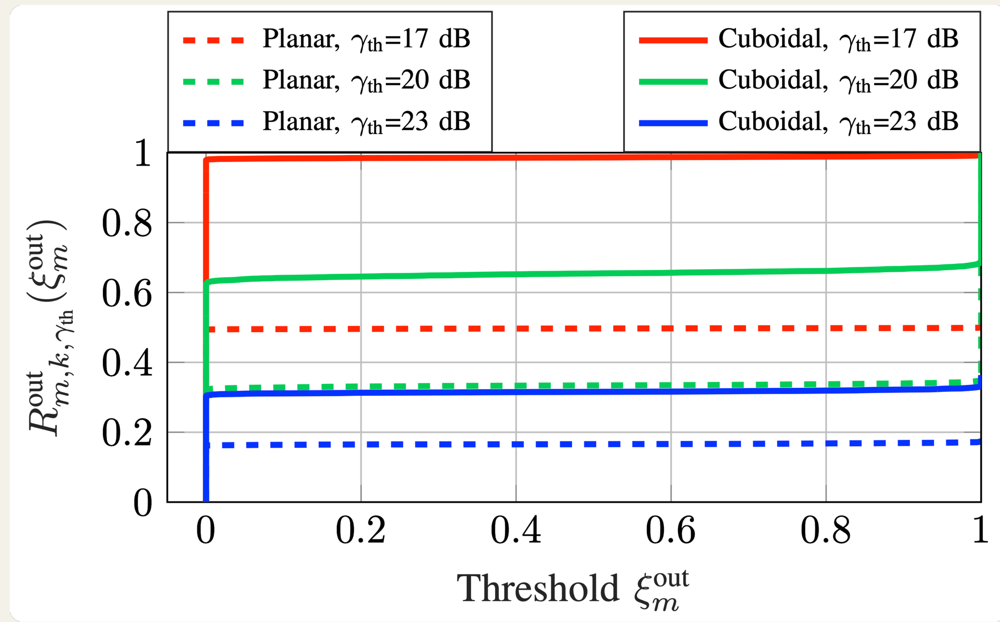

Fig. 5: Outage probability coverage of the two types of the array under different instantaneous SNR thresholds.

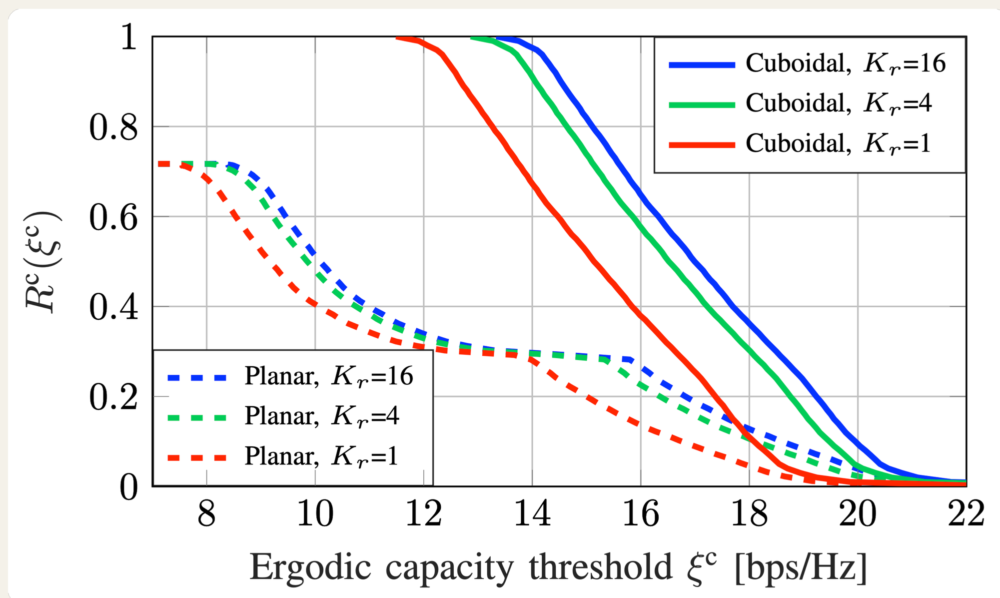

Fig. 6: Ergodic capacity coverage of the two types of the array under different values of Rician factor.

Conclusion

- This work considered a downlink, far-field THz band MIMO wireless system with multiple BSs and one UE equipped with a 3D array over the Rician fading channel.
- By deriving the localization error bound in terms of PEB and OEB, and communication KPIs of instantaneous SNR, outage probability, and ergodic capacity, we analyzed and compared the planar and 3D array configuration w.r.t. the coverage of these metrics
- The numerical results revealed a higher coverage for 3D array in both localization and communication KPIs given a suitable threshold, and minor performance loss in certain areas compared with the planar array.

References

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