Misspecified Cramér-Rao Bound of RIS-aided Localization under Geometry Mismatch

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1 Motivation

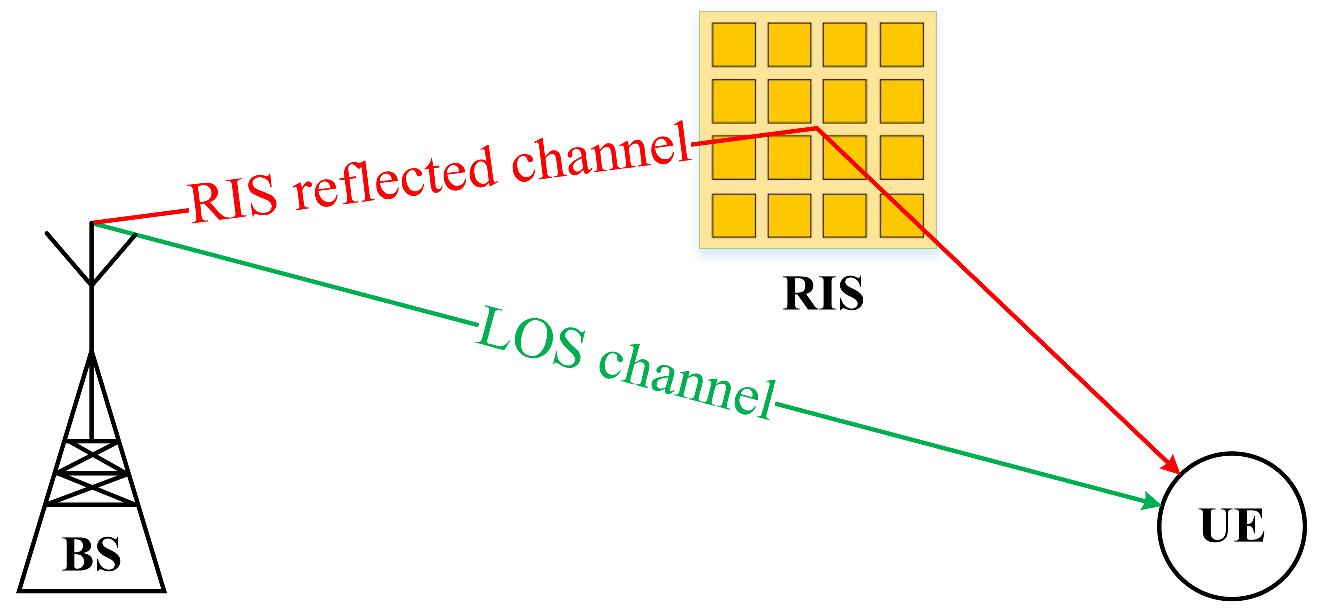

Figure 1: Illustration of a RIS-aided localization system.

- In 5G/6G wireless systems, reconfigurable intelligent surface (RIS) can play a role as a passive anchor to enable and enhance localization in various scenarios.
- However, most existing RIS-aided localization works assume that the geometry of the RIS is perfectly known, which is not realistic in practice.
- This work aims to characterize the effect of the model mismatch caused by RIS geometry error on the localization performance.
- Specifically, we derive the **misspecified Cramér-Rao bound (MCRB)** for a single-input-single-output (SISO) RIS-aided localization system with RIS geometry mismatch.
- The main contribution is that we derive a **closed-form** solution to the pseudo-true parameter determination problem (which is a core step to derive the MCRB).

2 Channel Model

- A downlink SISO wireless system with a BS, a UE, and a RIS.
- Transmission of L OFDM pilot symbols with K subcarriers.

The received baseband signal in the OFDM block with index ℓ is given by

$$\mathbf{y}_\ell = \underbrace{\overbrace{g_{\mathrm{b}}\mathbf{d}(\tau_{\mathrm{b}}) \odot \mathbf{x}}^{\text{LOS channel}} + \overbrace{g_{\mathrm{r}}\mathbf{b}(\boldsymbol{\phi})^{\mathsf{T}}\boldsymbol{\gamma}_\ell(\mathbf{d}(\tau_{\mathrm{r}}) \odot \mathbf{x})}^{\text{RIS reflected channel}}}_{\boldsymbol{\mu}_\ell} + \mathbf{n}, \tag{1}$$

3 Bound Derivation

The accessible geometry information of the RIS can be modeled as $\tilde{\mathbf{p}}_{\mathrm{r}} = \mathbf{p}_{\mathrm{r}} + \mathbf{u}$, $\tilde{\mathbf{R}}_{\mathrm{r}} = \mathbf{R}_{\mathrm{p}}(\mathbf{v})\mathbf{R}_{\mathrm{r}}$, where $\mathbf{u}$ and $\mathbf{v}$ are the calibration errors. Define channel parameters and localization parameters:

$$\boldsymbol{\eta}_{\mathrm{ch}} \triangleq [\phi_{\mathrm{az}}, \phi_{\mathrm{el}}, \tau_{\mathrm{b}}, \tau_{\mathrm{r}}, \Re(g_{\mathrm{b}}), \Im(g_{\mathrm{b}}), \Re(g_{\mathrm{r}}), \Im(g_{\mathrm{r}})]^{\mathsf{T}}, \tag{2}$$

$$\boldsymbol{\eta} \triangleq [\phi_{\mathrm{az}}, \phi_{\mathrm{el}}, \tau_{\mathrm{b}}, \tau_{\mathrm{r}}]^{\mathsf{T}}, \tag{3}$$

$$\mathbf{r} \triangleq [\mathbf{p}^{\mathsf{T}}, \Delta]^{\mathsf{T}} \in \mathbb{R}^4. \tag{4}$$

Two-stage localization: $\mathbf{y} \longrightarrow \boldsymbol{\eta} \longrightarrow \mathbf{r}$

The **true** likelihood function (f_{T}) and **mismatched** likelihood function (f_{M}):

$$\ln f_{\mathrm{T}} = -\frac{1}{2}(\hat{\boldsymbol{\eta}} - \mathbf{g}(\mathbf{r}|\mathbf{p}_{\mathrm{r}}, \mathbf{R}_{\mathrm{r}}))^{\mathsf{T}}\boldsymbol{\Sigma}^{-1}(\hat{\boldsymbol{\eta}} - \mathbf{g}(\mathbf{r}|\mathbf{p}_{\mathrm{r}}, \mathbf{R}_{\mathrm{r}})),$$

$$\ln f_{\mathrm{M}} = -\frac{1}{2}(\hat{\boldsymbol{\eta}} - \mathbf{g}(\mathbf{r}|\tilde{\mathbf{p}}_{\mathrm{r}}, \tilde{\mathbf{R}}_{\mathrm{r}}))^{\mathsf{T}}\boldsymbol{\Sigma}^{-1}(\hat{\boldsymbol{\eta}} - \mathbf{g}(\mathbf{r}|\tilde{\mathbf{p}}_{\mathrm{r}}, \tilde{\mathbf{R}}_{\mathrm{r}})).$$

The lower bound matrix of the estimation MSE based on f_{M}:

$$\mathrm{LBM}(\hat{\mathbf{r}}, \bar{\mathbf{r}}) = \underbrace{\mathbf{A}_{\mathbf{r}_0}^{-1}\mathbf{B}_{\mathbf{r}_0}\mathbf{A}_{\mathbf{r}_0}^{-1}}_{\mathrm{MCRB}(\mathbf{r}_0)} + \underbrace{(\bar{\mathbf{r}} - \mathbf{r}_0)(\bar{\mathbf{r}} - \mathbf{r}_0)^{\mathsf{T}}}_{\mathrm{Bias}(\mathbf{r}_0)}, \tag{5}$$

User PEB: $\sqrt{\mathbb{E}\{\|\mathbf{p} - \hat{\mathbf{p}}\|^2\}} \geq \sqrt{\mathrm{tr}\big([\mathrm{LBM}(\hat{\mathbf{r}}, \bar{\mathbf{r}})]_{1:3,1:3}\big)} \triangleq$ LB. Here, $\mathbf{r}_0$ is the pseudo-true parameter vector that minimizes the KLD between f_{T} and f_{M}:

$$\mathbf{r}_0 = \arg\min_{\mathbf{r}} D(f_{\mathrm{T}}(\hat{\boldsymbol{\eta}}|\bar{\mathbf{r}}) \| f_{\mathrm{M}}(\hat{\boldsymbol{\eta}}|\mathbf{r})), \tag{6}$$

$$D(f_{\mathrm{T}}(\hat{\boldsymbol{\eta}}|\bar{\mathbf{r}}) \| f_{\mathrm{M}}(\hat{\boldsymbol{\eta}}|\mathbf{r})) = \mathbb{E}_{f_{\mathrm{T}}}\{\ln f_{\mathrm{T}}(\hat{\boldsymbol{\eta}}|\bar{\mathbf{r}}) - \ln f_{\mathrm{M}}(\hat{\boldsymbol{\eta}}|\mathbf{r})\},$$

$$= \ \ldots = \frac{1}{2}\mathbf{h}(\mathbf{r})^{\mathsf{T}}\boldsymbol{\Sigma}^{-1}\mathbf{h}(\mathbf{r}). \tag{7}$$

Let $\mathbf{r}_0 = [\mathbf{p}_0^{\mathsf{T}}, \Delta_0]^{\mathsf{T}}$ be the pseudo-true parameters and $\bar{\mathbf{r}} = [\bar{\mathbf{p}}^{\mathsf{T}}, \bar{\Delta}]^{\mathsf{T}}$ be the true parameters, then $\mathbf{h}(\mathbf{r}_0) = \mathbf{0}$ gives

$$(\alpha + \|\mathbf{p}_0 - \tilde{\mathbf{p}}_{\mathrm{r}}\| - \|\mathbf{p}_0 - \mathbf{p}_{\mathrm{b}}\|)\frac{\mathbf{p}_0 - \tilde{\mathbf{p}}_{\mathrm{r}}}{\|\mathbf{p}_0 - \tilde{\mathbf{p}}_{\mathrm{r}}\|} = \tilde{\mathbf{R}}_{\mathrm{r}}\mathbf{R}_{\mathrm{r}}^{\mathsf{T}}(\bar{\mathbf{p}} - \mathbf{p}_{\mathrm{r}}), \tag{8}$$

It can be inferred that a pseudo-true UE position $\mathbf{p}_0$ can be obtained as the intersection of a line s_l and a hyperboloid s_h given by

$$s_l: \quad \mathbf{p} = x\tilde{\mathbf{R}}_{\mathrm{r}}\mathbf{R}_{\mathrm{r}}^{\mathsf{T}}(\bar{\mathbf{p}} - \mathbf{p}_{\mathrm{r}}) + \tilde{\mathbf{p}}_{\mathrm{r}}, \tag{9}$$

$$s_h: \quad \|\mathbf{p} - \tilde{\mathbf{p}}_{\mathrm{r}}\| - \|\mathbf{p} - \mathbf{p}_{\mathrm{b}}\| = \beta. \tag{10}$$

4 Results Validation

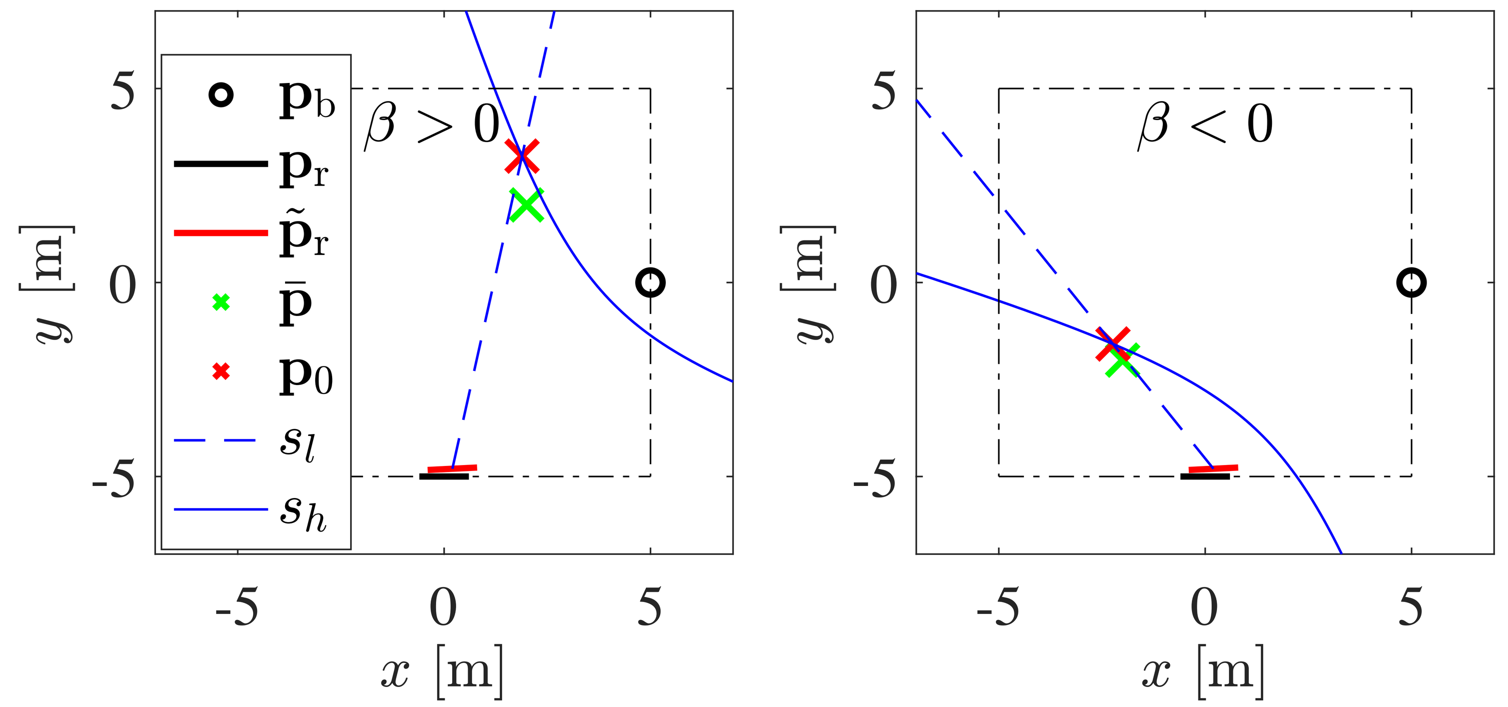

Figure 2: The geometric relationship between the pseudo-true UE position $\mathbf{p}_0$ and true UE position $\bar{\mathbf{p}}$.

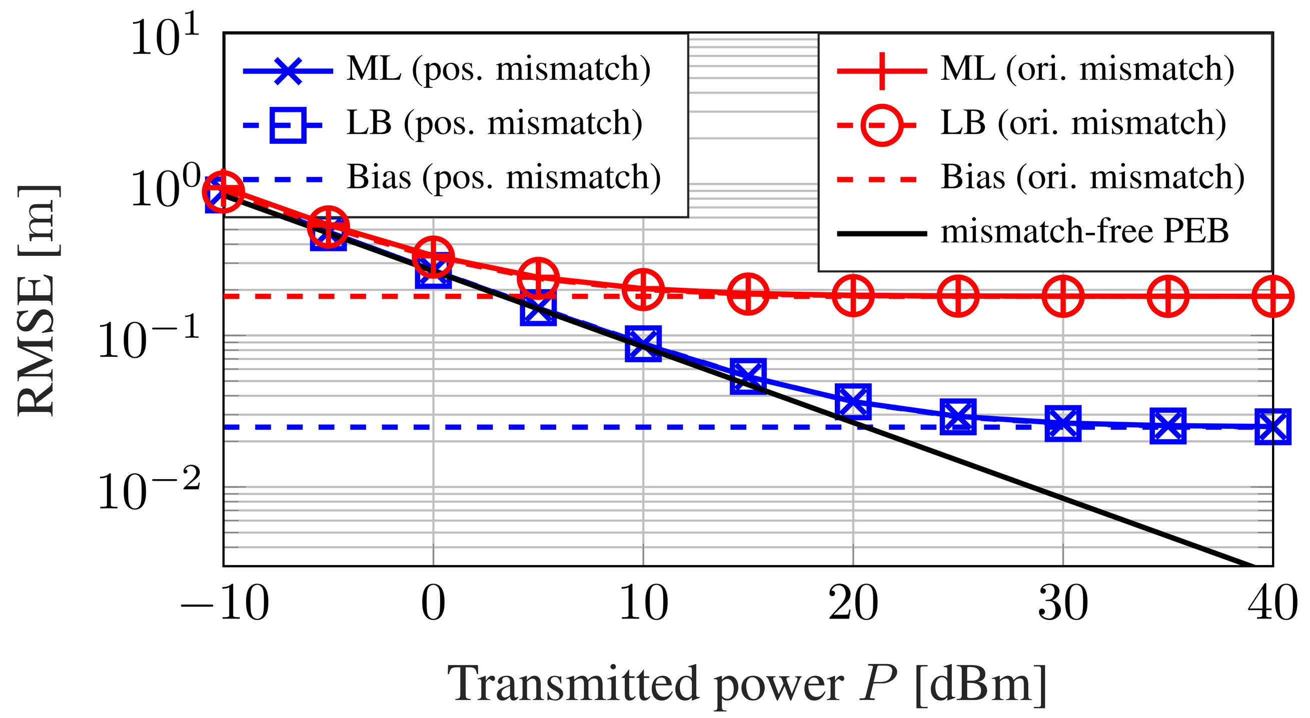

Figure 3: ML-RMSE, LB, and bias term versus transmitted power (SNR) for RIS position mismatch ($\mathbf{u} = 0.01 \times \mathbf{1}$ m) and orientation mismatch ($\mathbf{v} = 0.5 \times \mathbf{1}$ deg).

1. The RMSE of the ML estimator closely follows the LB, which demonstrates the validity of our derivation.
2. At low SNR levels, LB and mismatch-free PEB coincide → The RIS geometry mismatch is not the main source of error.
3. At higher SNR, LB deviates from the mismatch-free PEB and saturates → The positioning performance is more severely affected by RIS geometry mismatch.

P. Zheng, H. Chen, T. Ballal, H. Wymeersch, and T. Y. Al-Naffouri, "Misspecified Cramér-Rao bound of RIS-aided localization under geometry mismatch," preprint arXiv:2211.06990, 2022.