Southeast University
School of Transportation

Urban Transportation Planning

Chinese-English course (2019)

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9:50AM, Friday, 31th May

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Lecture review

- Uncertainty analysis:
  - to quantify the uncertainty around the mean estimate of one or more outcomes.
- Sensitivity analysis:
  - to identify the most significant parameters in the model and to quantify how the input uncertainty influences the outputs.
    - Local approach
      - one-at-a-time sensitivity analysis measure
    - Global approach
      - the improved Sobol' method

Lecture schedule

Lecture	Week	Date/Time	Topic
1	9	28 April 9：50-12：15	Transportation planning & demand and supply & trip-based model
2	10	5 May 9：50-12：15	ABM：data process
3	11	10 May 9：50-12：15	ABM：scheduling
4	12	17 May 9：50-12：15	ABM：uncertainty analysis
5	13	24 May 9：50-12：15	ABM：sensitivity analysis
6	14	31 May 9：50-12：15	Project Evaluation I
7	15	7 June 9：50-12：15	Festival
8	16	14 June 9：50-12：15	Project Evaluation II

Contents

- Overview of engineering economic
  - Discount rate and time value of money
  - Net present value (NPV)
  - Internal rate of return(IRR)
  - Incremental rate of return(ΔROR)
  - Payback period
- Discounting
  - Discounting methods
  - NPV=PV (B) – PV(C) & NPV=PV(NB)
  - Timing of benefits and costs
  - Long-lived projects
  - Comparing projects with different time duration

Overview of engineering economic

- Discount rate (折扣率) and time value of money（货币时间价值)
  - The purchasing power of money normally decreases over any given period of time due to inflation and uncertainty.
  - A discount rate adjusts the value of money for time, expressing expected future monetary quantities in terms of their worth today. (以今天的价值表示预期的未来货币数量)
  - There are two different kinds of interest rates:
    - *Real interest: rate exclusive of inflation*
    - *Nominal interest: rate inclusive of inflation*

Note: The selection of interest rate depends on the type of decision-making. Normally, the nominal interest will be used, as planning requires using future revenues and costs.

Example of interest calculation

Example of Simple Compound Interest
If you have $100 today and invest it at 10% simple annual compound interest rate per year for 2 years, you will have the following: Interest earned in year 1: $100 x %10 = $10 Interest earned in Year 2: $110 x %10 = $11 Total interest earned in 2 Years = $21 Present value = $100 Future value at the end of Year 2 = $121

$$FV = 100 \times (1 + 10\%)^2 = 121$$

Example of interest calculation

Example of Interest Calculation

Suppose a planned project is suddenly delayed for 2 years. Construction, labor, and materials costs are expected to increase 2% annually during the delay, but the unused funds could meanwhile accrue 1.75% interest in other investments.

Current construction cost: $10,000,000

Therefore, in 2 years, the money not spent on construction could earn $353,063 from interest.

Rising materials and labor costs would increase construction costs $404,000 in two years. Considering both interest and inflation, a 2-year delay would cost overall $50,937.

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Therefore, in 2 years, the money not spent on construction could earn $353,063 from interest.

Rising materials and labor costs would increase construction costs $404,000 in two years. Considering both interest and inflation, a 2-year delay would cost overall $50,937.

$$Benefit: B_2 = \$10{,}000{,}000 \times (1 + 1.75\%)^2 - \$10{,}000{,}000 = \$353{,}063$$

$$Cost: C_2 = \$10{,}000{,}000 \times (1 + 2\%)^2 - \$10{,}000{,}000 = \$404{,}000$$

Net present value (NPV)

– A net present value (NPV，净现值) calculation is used to state a project's worth or cost for its entire life cycle in today's dollars or at some specific point in time.

$$NPV = -C_i + SV\left(\frac{1}{1+DR}\right)^{proj\ life} - IPC\left(\frac{1}{1+DR}\right)^{year} + \sum_{y=1}^{proj\ life} (B_y - C_y)\left(\frac{1}{1+DR}\right)^{y}$$

– where, C_i is the initial project cost, SV is the salvage value, IPC is interim project costs (generally involving highway capacity additions and upgrades), DR is the discount rate (as a proportion, rather than as a percentage, e.g., 0.05 instead of 5%), B_y and C_y denote the monetized benefits and costs realized in year y.

Note: the benefit assessment do not monetize benefits such as crash and air emissions reductions. The NPV of transportation projects only covers expected costs at a specific point. (e.g., toll revenue)

Internal rate of return (IRR)

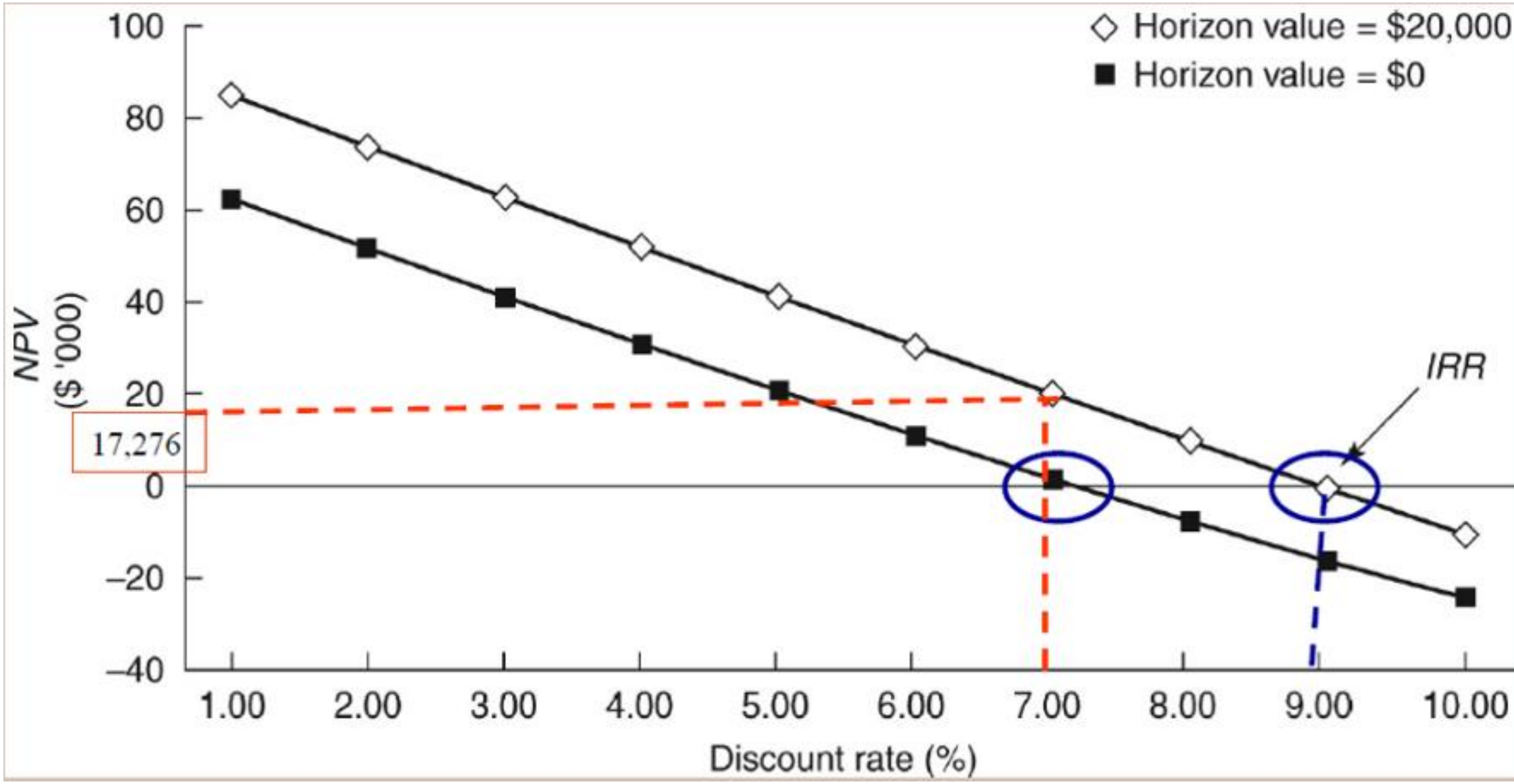
– The project's Internal Rate of Return (IRR，内部收益率) determines the discount rate at which the sum of discounted costs equals the sum of discounted benefits (at their present-year worth).

– The IRR indicates the annual rate of return that would be derived from an equivalent project of similar size and similar duration.

$$C_i + \sum_{y=1}^{proj\ life}(C_y)\left(\frac{1}{1+IRR}\right)^y - SV\left(\frac{1}{1+IRR}\right)^{proj\ life} + IPC\left(\frac{1}{1+DR}\right)^{year} = \sum_{y=1}^{proj\ life}(B_y)\left(\frac{1}{1+IRR}\right)^y$$

Note: IRR is the effective interest rate used to measure the value of an investment. IRR can be used only when the project will generate income. To evaluate alternatives using IRR, the alternatives' IRRs should be greater than the minimum accepted rate of return(MARR), which is the lowest interest rate that investors would accept, given the risk of the investment and the opportunity cost of foregoing other projects.

Internal rate of return (IRR)



– The Internal rate of return (IRR) of a project equals the discount rate at which the project's NPV=0

Example of IRR

Example of IRR

TxDOT considers building a new toll or managed lane highway with the following cash flow for first 5 years. What is the IRR for this period?

Year	Cash Flow	Year	Cash Flow
0	-$10,000,000	3	$2,345,000
1	$1,340,000	4	$2,680,000
2	$2,010,000	5	$2,847,500

Example of IRR

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Year	Cash Flow	Year	Cash Flow
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1	$1,340,000	4	$2,680,000
2	$2,010,000	5	$2,847,500

IRR: NPV=0 $\Longrightarrow$ *NB – NC = 0*

$Cost: C = \$10{,}000{,}000$

$$Benefit: B = \frac{\$1{,}340{,}000}{(1+DR)} + \frac{\$2{,}010{,}000}{(1+DR)^2} + \cdots + + \frac{\$2{,}847{,}500}{(1+DR)^5}$$

IRR=3.6%

Incremental rate of return (ΔROR)

– The NPV of one alternative can be greater than its competing alternative but require greater investment. In this situation, incremental rate of return (ΔROR，递增收益率) can be used. ΔROR is the interest rate earned on the extra cost of a higher cost alternative. If the alternative's ΔROR is greater than the MARR, the alternative would be beneficial.

Example of ΔROR

TxDOT considers two alternatives for a project, A and B. The table shows the required investment and returned benefit of each alternative. If TxDOT's MARR is 8%, which alternative should be built?

Year	Alternative A	Alternative B
0	-$2,500,000	-$6,000,000
1	$746,000	$1,664,000
2	$746,000	$1,664,000
3	$746,000	$1,664,000
4	$746,000	$1,664,000
5	$746,000	$1,664,000
IRR	15.01%	11.99%

Example of ΔROR

– The IRR for both alternatives is greater than the MARR. So, both projects are acceptable investments. Now the question is whether alternative B is worth the extra $3,500,000 in initial investment.

Example of ΔROR

Year	Alternative A	Alternative B	Δ(A,B)
0	- $2,500,000	-$6,000,000	-$3,500,000
1	$746,000	$1,664,000	$918,000
2	$746,000	$1,664,000	$918,000
3	$746,000	$1,664,000	$918,000
4	$746,000	$1,664,000	$918,000
5	$746,000	$1,664,000	$918,000
IRR	15.01%	11.99%	9.78%

– The extra $3,500,000 investment in alternative B yields 9.78% IRR. As the incremental rate of return is till higher than MARR, alternative B should be selected.

Payback period

– Payback period (投资回收期) is the period of time required before the project's benefits are equal to the project's cost.

Example of Payback Period

Year	Alternative A	Alternative B
0	-$1000	-$1000
1	$200	$300
2	$500	$300
3	$800	$300
4	$1100	$300
5	$1400	$300

Payback periods for the alternatives shown in the table at a 10% discount rate are as follows:

Alternative A $$1000 = 200 + 500 + (800 * 0.375)$$

The payback period is equal to 2.375 years.

Alternative B
Uniform annual benefits $$\frac{1000}{300} = 3.33$$

The payback period is equal to 3.33 years.

Discounting

- Project with life of one year:
  - Discounting takes place over periods not years
  - However, for simplicity, we assume that each period is a year. In other words, we default the project lasts for one year.
  - There are three possible methods to evaluate potential projects. Each gives the same answer:
    - future value analysis;
    - present value analysis;
    - net present value analysis.

Discounting methods

- **Future value analysis:**
  - Choose the project with the largest future value, ***FV***, where the future value in one year of an amount ***X*** invested at interest rate ***i*** is:

$$FV = X(1 + i)$$

Where, ***X*** denotes the present value;

Discounting methods

- Present value analysis:
  - Choose the project with the largest present value, ***PV***, where the present value of an amount ***Y*** received in one year is:

$$FV = X(1+i)$$

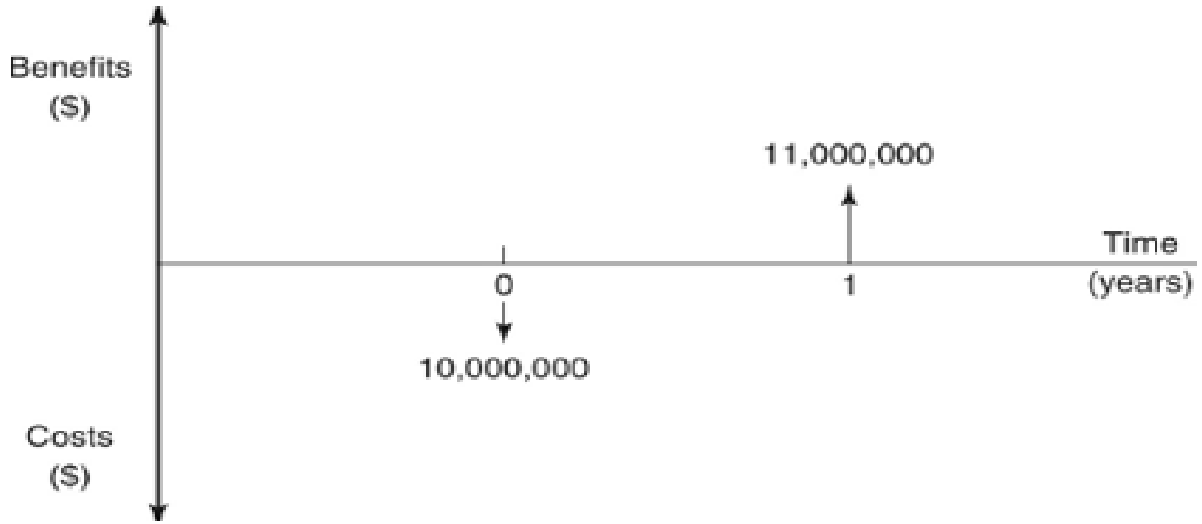
$$PV = Y/(1+i)$$

$$\Longrightarrow \quad PV = \frac{FV}{1+i}$$

Where, ***X*** is the present value; ***Y*** is the future value.

Note: It implies that discounting (the process of calculating the present value of future amounts) is the opposite of compounding (the process of calculating future values).

Discounting methods



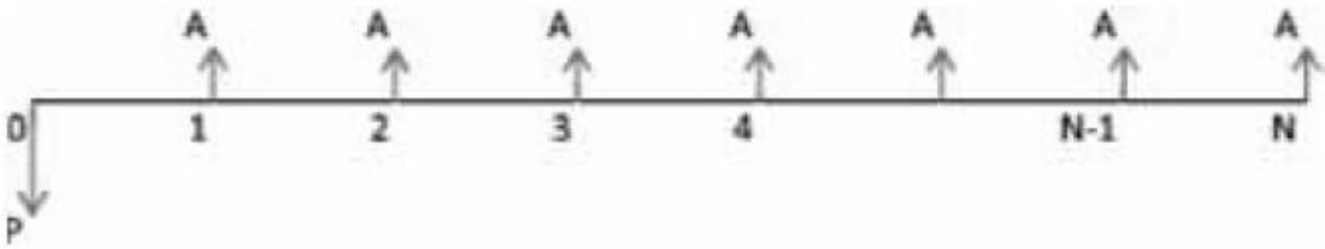
- Net present value analysis:
  - Choose the project with the largest **net** present value, ***NPV***, which calculates the sum of the present values of all the benefits and costs of a project (including the initial investment):

$$NPV = PV(benefits) - PV(costs)$$

PV-Single payment 单笔支付

$$PV = \frac{FV}{(1+i)^N} = FV(1+i)^{-N}$$

PV-Equal payment series 等值支付



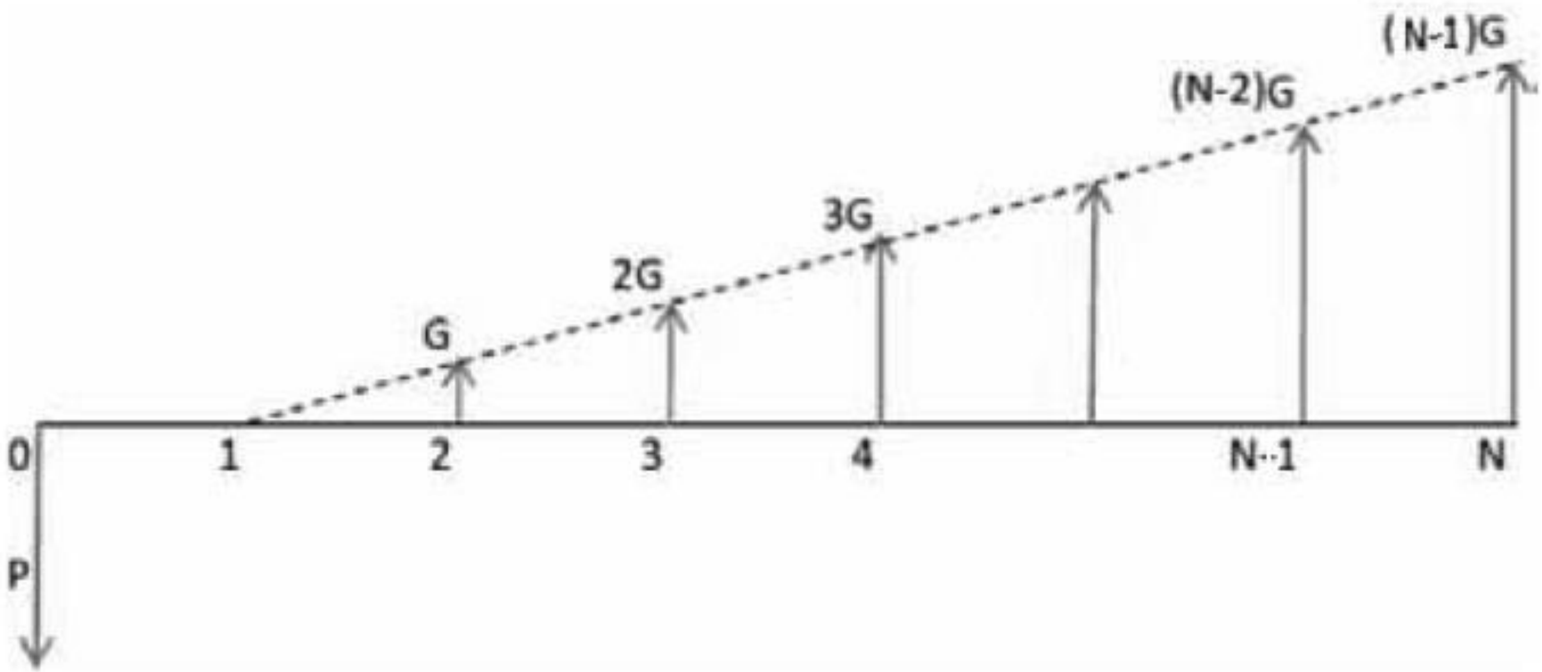
$$To\ find\ P, given\ A\ (P|A,i,N): P = \frac{A}{i}[1-(1+i)^{-N}]$$

$$P = \frac{A}{i}\ (if\ N = \infty)$$

Example of Equal Payment Series

$$P = \frac{1{,}000{,}000}{0.05}[1 - 1.05^{20}] = \$33{,}065{,}954$$

PV-Linear gradient series 线性梯度变化



$$To\ find\ P, given\ G\ (P|G,i,N): P = G\left[\frac{(1+i)^N - iN - 1}{i^2(1+i)^N}\right]$$

Example of Linear Gradient Series

TxDOT considers benefits of a project as shown in this table. Toll revenue is assumed to increase by a rate of $80,000 each year. What is the NPV of the project? Assume the discount rate is 10%.

Year	End-of-Year Payment
1	$1,000,000
2	$1,080,000
3	$1,160,000
4	$1,240,000
5	$1,320,000

The total project cash flow consists of two cash flows:

1. Annual Cash Flow A = $1,000,000/period

2. Linear Gradient Cash Flow G = $80,000/period

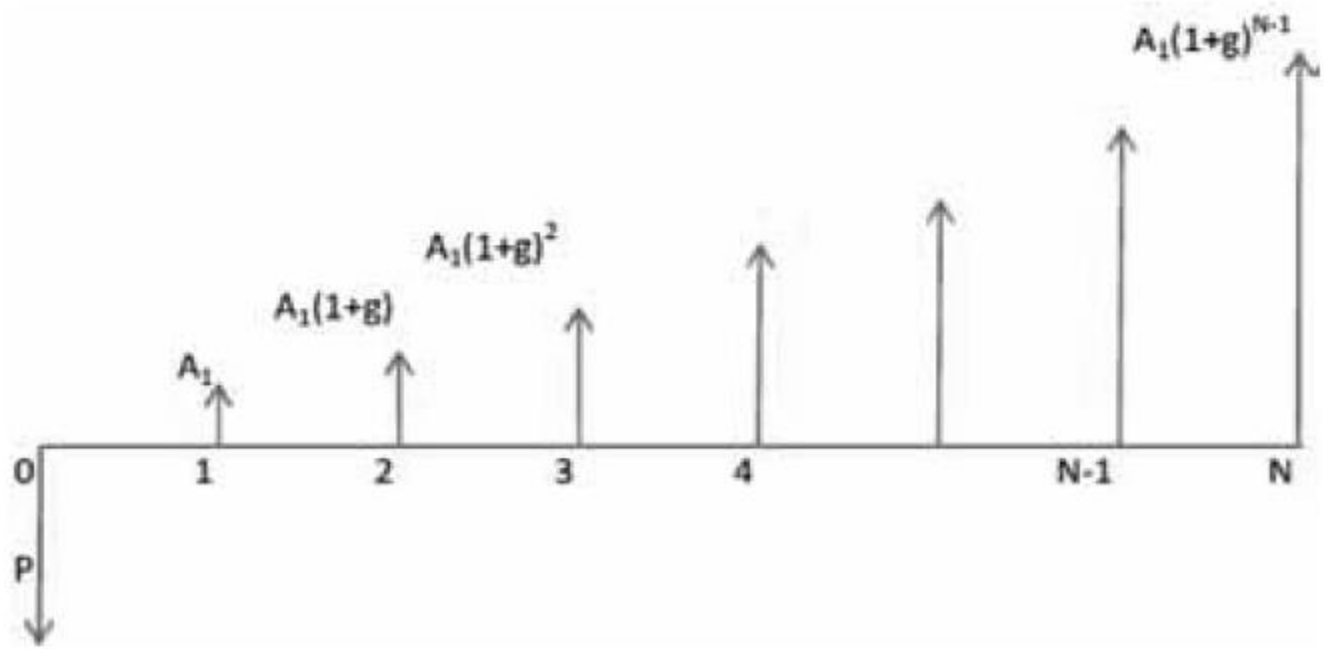
So, the NPV of this series of payments at a 10% discount rate is

$$NPV = \frac{1{,}000{,}000}{0.10}[1 - 1.1^{-5}] + 80\left[\frac{(1+0.1)^5 - 0.1 \times 5 - 1}{0.1^2 \times (1+0.1)^5}\right]$$
$$= \$4{,}339{,}731$$

PV-Geometric gradient series 几何梯度支付

Geometric Gradient Series

To calculate the present value of a cash flow that changes by a fixed percentage (g) each time period:



$$To\ find\ P, given\ A_1\ and\ g: (P|A, g, i, N): P = A_1\left[\frac{1-(1+g)^N(1+i)^{-N}}{i-g}\right]$$

$$(if\ i \neq g)$$

$$P = A_1\left(\frac{N}{1+i}\right)\ (if\ i = g)$$

Discounting methods

- Usually projects are evaluated relative to the current status.
  - If there is ***<u>only one</u>*** new potential project and its impacts are calculated relative to the status quo.
    - it should be selected if its NPV > 0,
    - and should not be selected if its NPV < 0.
  - If the impacts of ***<u>multiple</u>*** alternative projects are calculated relative to the status,
    - we should choose the project with the highest NPV (NPV>0).
    - if NPV<0 for all the alternatives, we should maintain the projects.

Compounding & Discounting over multiple years

FV, PV, over multiple years

- Interest is compounded when an amount is invested for a number of years and interest earned each period is reinvested.
- Interest on reinvested interest is called compound interest. The future value , ***FV***, the present value ***PV*** for ***n*** years with compounded interest rate ***i*** is:

$$FV = X(1+i)^n$$

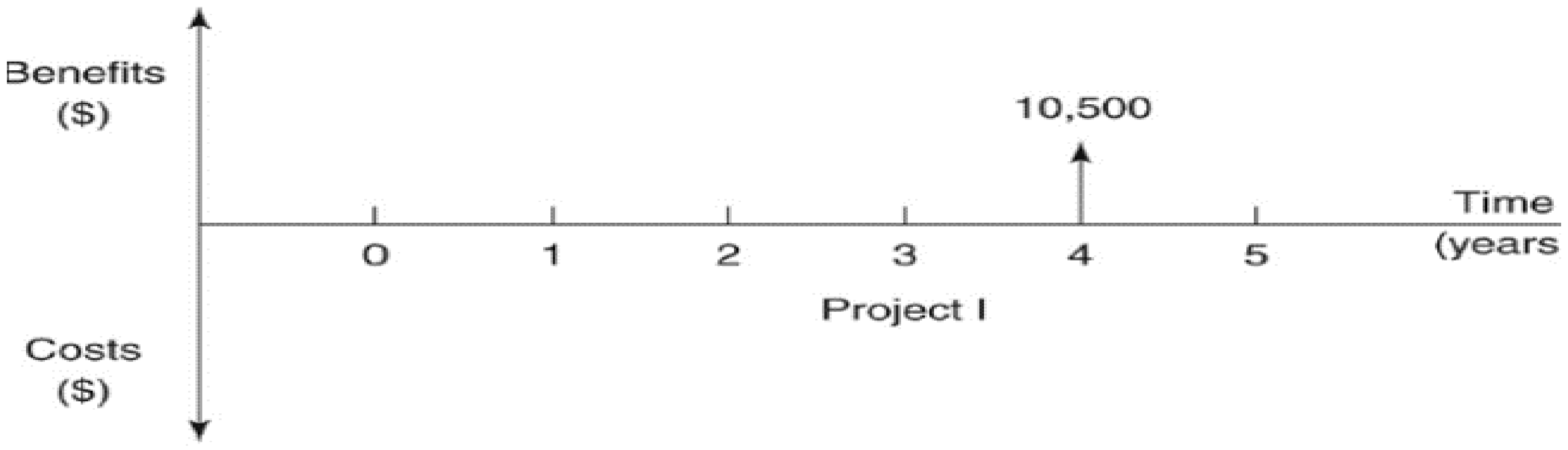
$$PV = \frac{Y}{(1+i)^n}$$

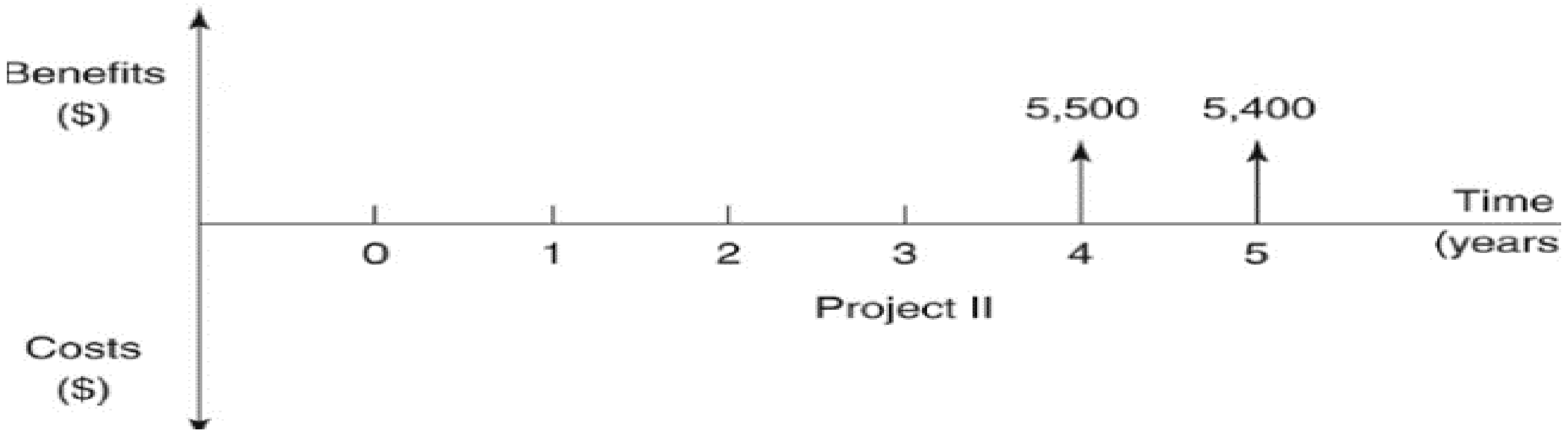
- The present value of benefits or costs over ***n*** years with compounded interest rate ***i*** is :

$$PV(B) = \sum_{t=0}^{n} \frac{B_t}{(1+i)^t}$$

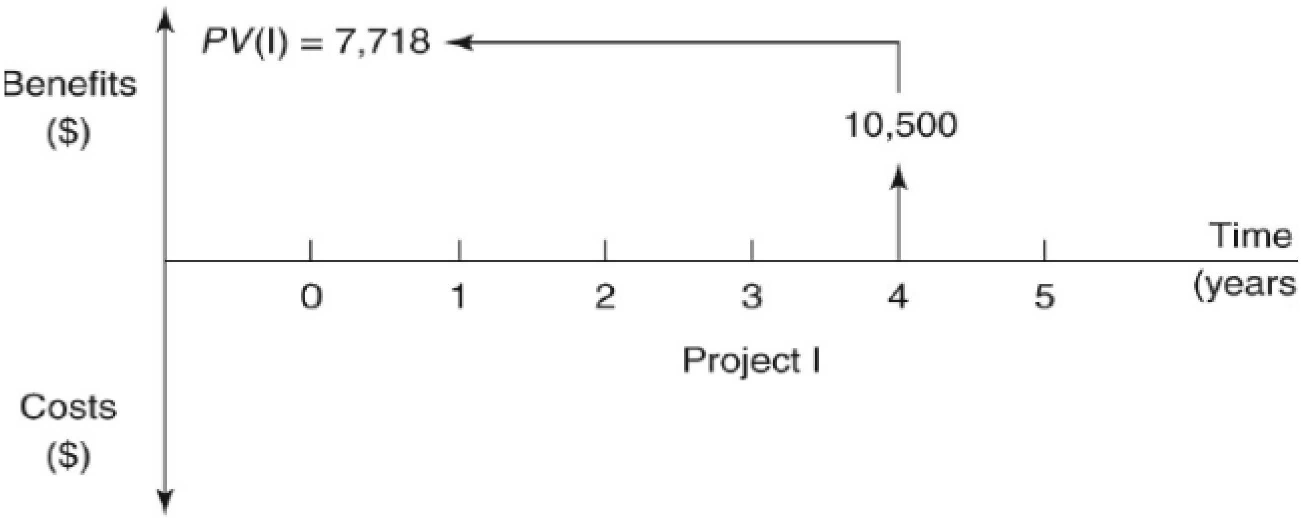
$$PV(C) = \sum_{t=0}^{n} \frac{C_t}{(1+i)^t}$$

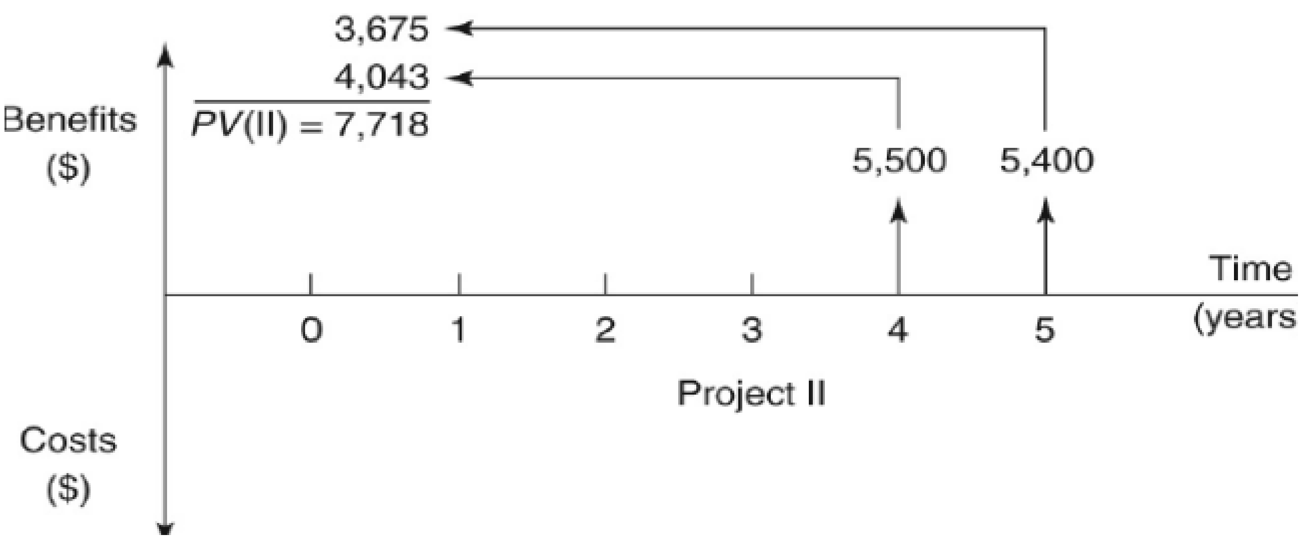
Time line of benefits and costs





Time line of benefits and costs





Project I:

The 4th year FV = 10500

Discount rate = 8%

Project II:

The 4th year FV = 5500

The 5th year FV = 5400

Discount rate = 8%

The NPV of the library information system

Yr		Annual benefits	Annual Costs	PV Annual Benefits	PV Annual Costs	PV Annual Net benefits
0	Purchase & Install	0	325.000	0	325.000	-325.000
1	Annual Benefits & Costs	100.000	20.000	93.458	18.692	74.766
2	Annual Benefits & Costs	100.000	20.000	87.344	17.469	69.875
3	Annual Benefits & Costs	100.000	20.000	81.630	16.326	65.304
4	Annual Benefits & Costs	100.000	20.000	76.290	15.258	61.032
5	Annual Benefits & Costs	100.000	20.000	71.299	14.260	57.039
5	Liquidation	20.000		14.260	0	14.260

Compounded rate= 7%

∑PB = 424.281

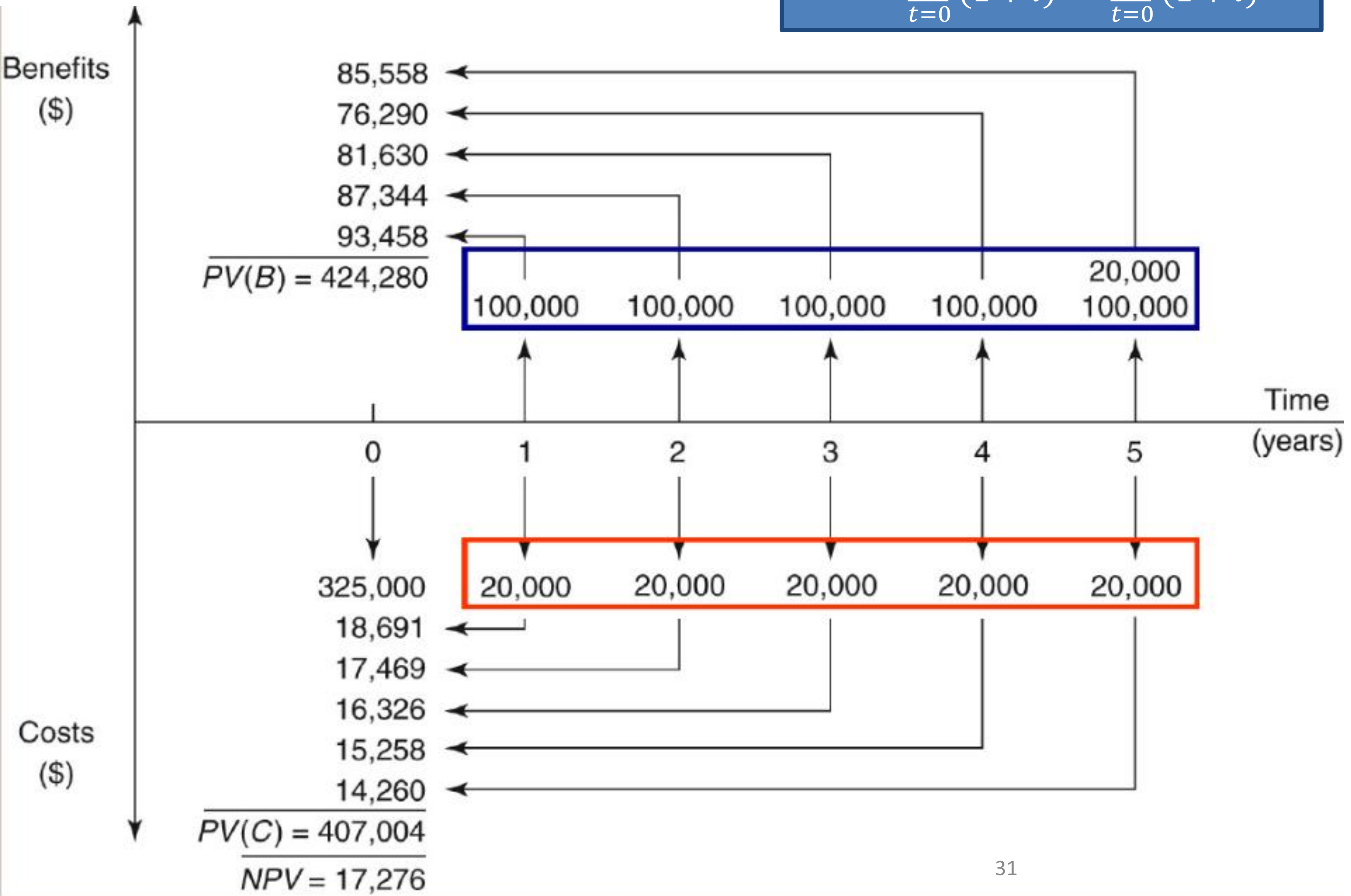
∑ PC = 407.005

∑ NB = 17.276

$$NPV = \sum_{t=0}^{n} \frac{B_t}{(1+i)^t} - \sum_{t=0}^{n} \frac{C_t}{(1+i)^t}$$

Time line of the benefits and cost information system

$$NPV = \sum_{t=0}^{n} \frac{B_t}{(1+i)^t} - \sum_{t=0}^{n} \frac{C_t}{(1+i)^t}$$



The NPV of the library information system

Yr		Annual benefits	Annual Costs	Annual Net benefits	PV Annual Net benefits
0	Purchase & Install	0	325.000	-325.000	-325.000
1	Annual Benefits & Costs	100.000	20.000	80.000	74.766
2	Annual Benefits & Costs	100.000	20.000	80.000	69.875
3	Annual Benefits & Costs	100.000	20.000	80.000	65.304
4	Annual Benefits & Costs	100.000	20.000	80.000	61.032
5	Annual Benefits & Costs	100.000	20.000	80.000	57.039
5	Liquidation	20.000		20.000	14.260
					17.276

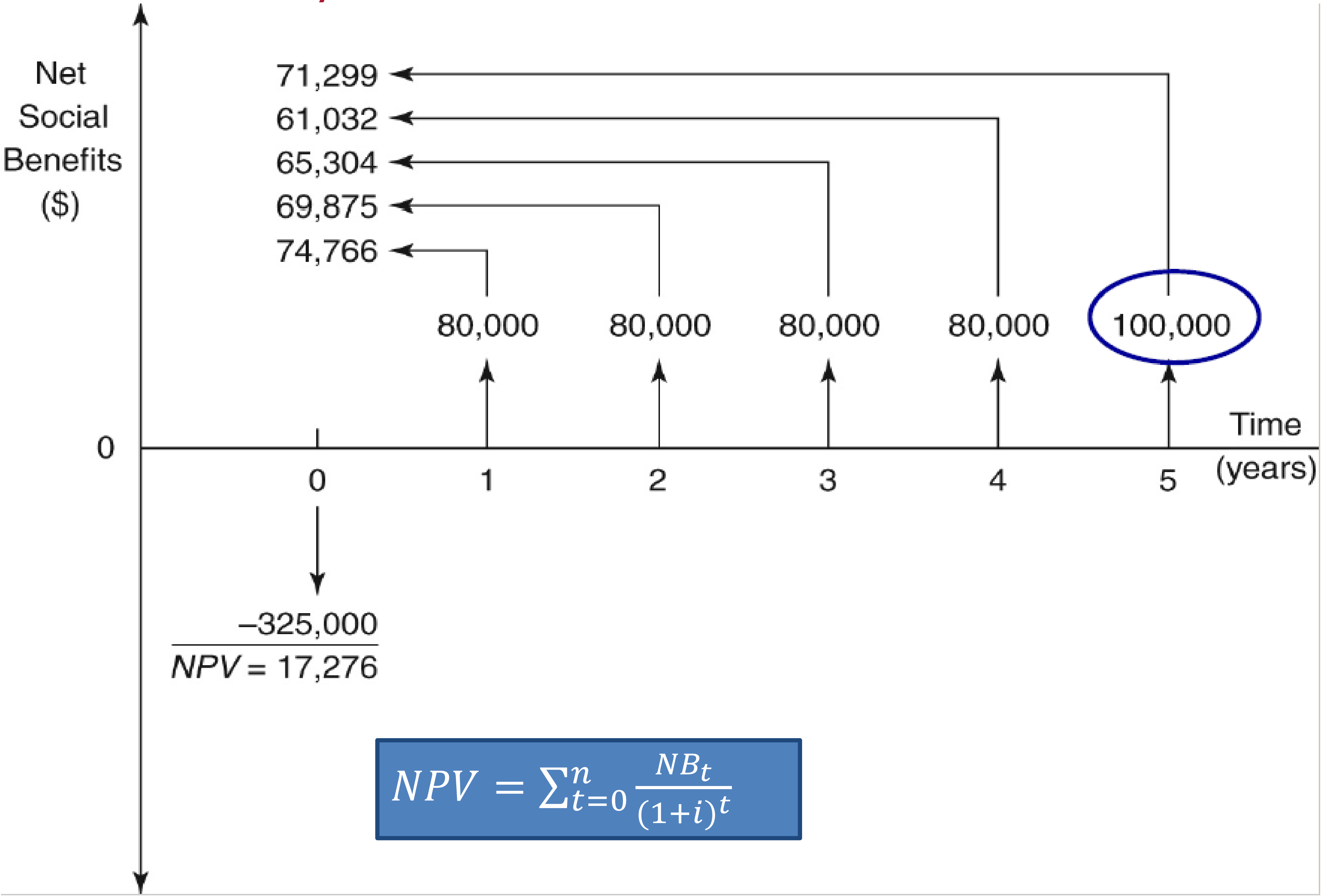
Net Benefit = Benefit-Cost

NPV=PV(B)-PV(C)

Compounded rate= 7%

$$NPV = \sum_{t=0}^{n} \frac{B_t}{(1+i)^t} - \sum_{t=0}^{n} \frac{C_t}{(1+i)^t} = \sum_{t=0}^{n} \frac{NB_t}{(1+i)^t}$$

Time line of the benefits and costs of the library information system



$$NPV = \sum_{t=0}^{n} \frac{NB_t}{(1+i)^t}$$

Compounding & Discounting over multiple years

NPV over multiple years

- The net present value ***NPV*** for ***n*** years with compounded interest rate ***i*** is:

$$NPV = PV(benefits) - PV(costs)$$

$$NPV = \sum_{t=0}^{n} \frac{B_t}{(1+i)^t} - \sum_{t=0}^{n} \frac{C_t}{(1+i)^t}$$

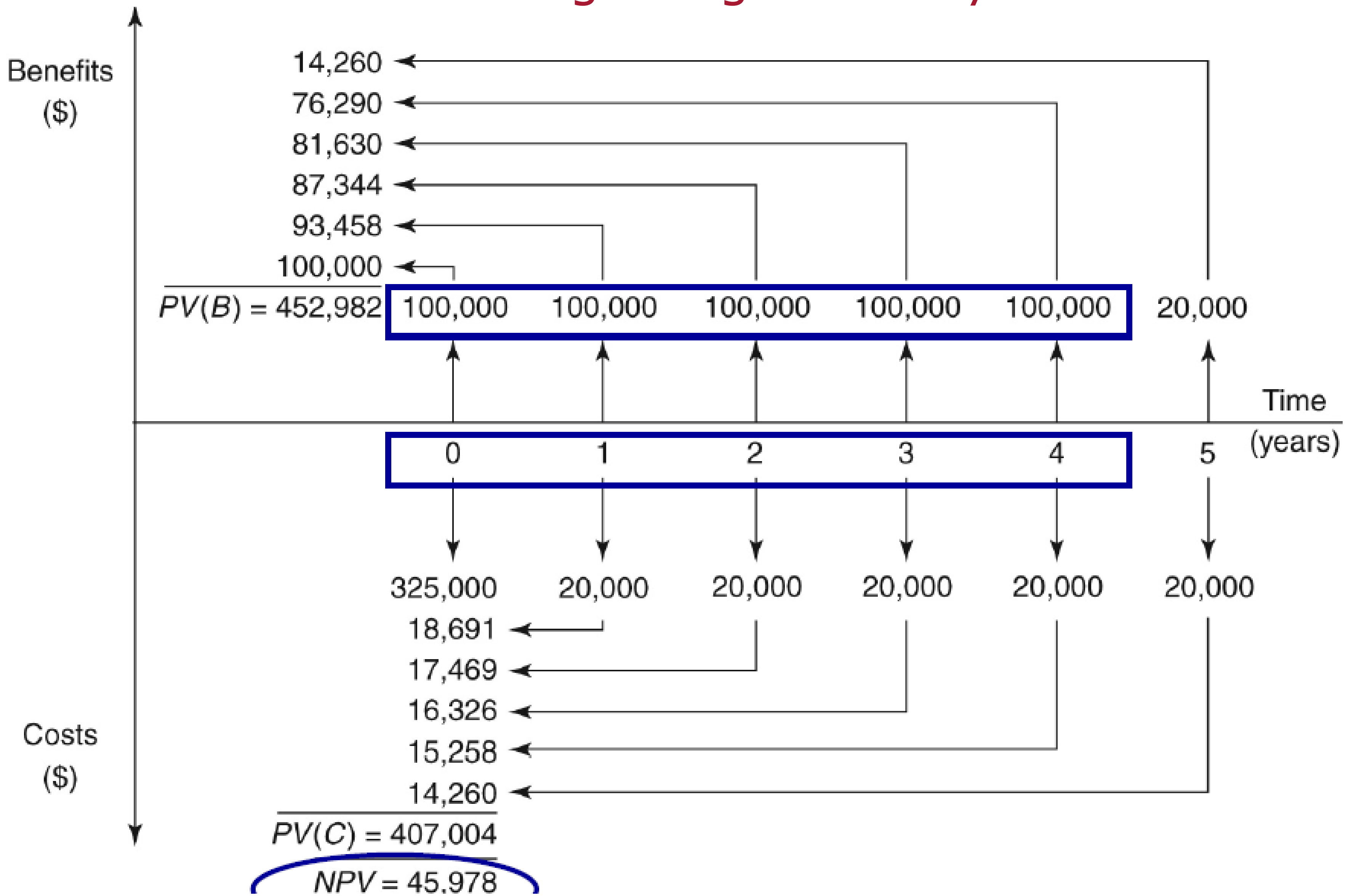
- Or, equivalently, the NPV of a project equals the present value of the net benefits (*NBi*=*Bi*-*Ci*):

$$NPV = \sum_{t=0}^{n} \frac{NB_t}{(1+i)^t}$$

Timing of benefits and costs

- We have assumed that impacts occur immediately, or at the end of the first year, or at the end of the second year, and so on.
- If most benefits occur in the beginning of the year while costs occur at the end of the year, the NPV will be higher than if both occurred at the end.
- If most costs are incurred during the first few years of a project and most benefits arise later, this assumption is conservative in the sense that the NPV is lower than if it were computed under an alternative assumption.

Time line of the benefits and costs assuming user benefits occur at the beginning of each year



Exercise

- A highway department is considering building a temporary bridge to cut travel time during the three years it will take to build a permanent bridge. The temporary bridge can be put up in a few weeks at a cost of $740,000. At the end of three years, it would be removed and the steel would be sold for scrap. The real net cost of this would be $81,000.
- Based on estimated time savings and wage rates, fuel savings, and reductions in risks of accidents, department analysts predict that the benefits in real dollars would be $275,000 during the first year, $295,000 during the second year, and $315,000 during the third year. Departmental regulations require use of a real discount rate of 5%.

Exercise

- a. Calculate the present value of net benefits assuming that the benefits are realized at the end of each of the three years.
- b. Calculate the present value of net benefits assuming that the benefits are realized at the beginning of each of the three years.
- c. Calculate the present value of net benefits assuming that the benefits are realized in the middle of each of the three year.
- d. Calculate the present value of net benefits assuming that half of each year's benefits are realized at the beginning of the year and the other half at the end of the year.
- e. Does the temporary bridge pass the net benefits test?

Exercise

Temporary bridge evaluation		
Cost	initial cost	\$740,000
	the 3rd year	\$81,000
Benefit	1st year	\$275,000
	2nd year	\$295,000
	3rd year	\$315,000

- Begin by calculating the present value of the costs, which are realized at the beginning of year 1 and at the end of year 3:

$$PV(cost) = \$740{,}000 + \frac{\$81{,}000}{(1+0.05)^3}$$

$$= \$740{,}000 + \$69{,}971$$

$$= \$809{,}971$$

Exercise

- Begin by calculating the present value of the costs, which are realized at the beginning of year 1 and at the end of year 3:

$$PV(cost) = \$740{,}000 + \frac{\$81{,}000}{(1+0.05)^3} = \$809{,}971$$

- a. Calculate the present value of net benefits assuming that the benefits are realized at the end of each of the three years.

$$PV(benefits) = \frac{\$275{,}000}{(1+0.05)^1} + \frac{\$295{,}000}{(1+0.05)^2} + \frac{\$315{,}000}{(1+0.05)^3}$$
$$= \$261{,}905 + \$267{,}574 + \$272{,}109$$
$$= \$801{,}588$$

$$NPV = PV(benefits) - PV(costs) = \$801{,}588 - \$809{,}971 = -\$8{,}383$$

Exercise

- Begin by calculating the present value of the costs, which are realized at the beginning of year 1 and at the end of year 3:

$$PV(costs) = \$740{,}000 + \frac{\$81{,}000}{(1+0.05)^3} = \$809{,}971$$

- b. Calculate the present value of net benefits assuming that the benefits are realized at the beginning of each of the three years.

$$PV(benefits) = \$275{,}000 + \frac{\$295{,}000}{(1+0.05)^1} + \frac{\$315{,}000}{(1+0.05)^2} = \$841{,}666$$

$$NPV = PV(benefits) - PV(costs) = \$841{,}666 - \$809{,}971 = \$31{,}695$$

Exercise

- Begin by calculating the present value of the costs, which are realized at the beginning of year 1 and at the end of year 3:

$$PV(costs) = \$740{,}000 + \frac{\$81{,}000}{(1+0.05)^3} = \$809{,}971$$

- c. Calculate the present value of net benefits assuming that the benefits are realized in the middle of each of the three years.

$$PV(benefits) = \frac{\$275{,}000}{(1+0.05)^{0.5}} + \frac{\$295{,}000}{(1+0.05)^{1.5}} + \frac{\$315{,}000}{(1+0.05)^{2.5}} = \$821{,}383$$

$$NPV = PV(benefits) - PV(costs) = \$821{,}383 - \$809{,}971 = \$11{,}412$$

Exercise

- Begin by calculating the present value of the costs, which are realized at the beginning of year 1 and at the end of year 3:

$$PV(costs) = \$740{,}000 + \frac{\$81{,}000}{(1+0.05)^3} = \$809{,}971$$

- d. Calculate the present value of net benefits assuming that half of each year's benefits are realized at the beginning of the year and the other half at the end of the year.

$$PV(benefits) = (\$841{,}666\ +\ \$801{,}588)/2 = \$821{,}627$$

$$NPV\ = PV(benefits) - PV(costs) = \$821{,}627 - \$809{,}971 = \$11{,}656$$

Alternatively, *average the NPV of answer a and b*:
$NPV = (-\$8{,}383 + \$31{,}695)/2 = \$11{,}656$

Exercise

- e. Does the temporary bridge pass the net benefits test?

	The benefits are realized			
When the benefits are happened	a. at the end of each year	b. at the beginning of each year	c. in the middle of each year	d. ½ at the beginning and ½ at the end of each year
NPV	−$8,383	$31,695	$11,412	$11,656

- The present value of net benefits depends on when the benefits are assumed to happen.
- In this case, either method (c) or (d) is most appropriate, and the temporary bridge passes the net benefits test.

Long-lived projects

NPV over multiple years

- The net present value ***NPV*** for ***n*** years with compounded interest rate ***i*** is:

$$NPV = \sum_{t=0}^{n} \frac{B_t}{(1+i)^t} - \sum_{t=0}^{n} \frac{C_t}{(1+i)^t}$$

- Or, the NPV of a project equals the present value of the net benefits:

$$NPV = \sum_{t=0}^{n} \frac{NB_t}{(1+i)^t}$$

- Some projects may have benefits (and costs) that occur far in the future. We can use a generalized version of equation with infinity ∞ replacing n:

$$NPV = \sum_{t=0}^{\infty} \frac{NB_t}{(1+i)^t}$$

Long-lived projects

NPV over multiple years

- Some projects can be reasonably divided into two periods:

1. a "near future" (the discounting period), which pertains to the first k periods,

2. and a "far future", which pertains to the subsequent periods and is captured by the horizon value, H_K.

$$NPV = \sum_{t=0}^{k} \frac{NB_t}{(1+i)^t} + PV(H_k)$$

- Where, $PV(H_k)$ is the present value of the horizon value (i.e. the PV of all benefits and costs arise after the first K periods).
- Usually, there is a natural choice for k, the "useful" life of the project, such as when the asset are sold.

Comparing projects with different time duration

- If the time spans are different, such projects are not directly comparable.
- Two methods to evaluate projects with different life spans:
  1. Rolling over the shorter project (滚动短期项目)
  2. Equivalent annual benefit method（等效年度收益法)

Comparing projects with different time duration

1. Rolling over the shorter project

   – If project A spans n times the number of years as project B, then assume that project B is repeated n times and compare the NPV of n repeated project Bs to the NPV of project A.

   – For example, if project A lasts 30 years and project B lasts 15 years, compare the NPV of project A to the NPV of 2 Project B's, where the latter is computed:

$$NPV = x + \frac{x}{(1+i)^{15}}$$

   – Where, x =NPV of project B (15-year)

Comparing projects with different time duration

2. Equivalent annual net benefit method

- The EANB is the amount received each year for the life of the project that has the same NPV as the project itself.
- The EANB of a project is computed by dividing the NPV by the appropriate annuity factor, a_i^n

$$EANB = \frac{NPV}{a_i^n}$$

- The annuity factor a_i^n is the present value of an annuity of $1 for the life of the project (n years), where i=interest rate used to compute the NPV.
- Obviously, we would choose the project with the highest EANB.

(等效年度收益)

Present value of an Annuity

- The present value of an annuity of \$A per year (with payments received at the end of each year) for n years with interest at i percent is given by:

$$PV = \sum_{t=1}^{n} \frac{A}{(1+i)^t}$$

- This is the sum of n terms of a geometric series with the common ratio equal to 1/(1+i). Consequently,

$$PV = Aa_i^n, \qquad where\ a_i^n = \frac{1-(1+i)^{-n}}{i}, \qquad a_i^n = \sum_{t=1}^{n} \frac{1}{(1+i)^t}$$

- The term a_i^n, called an annuity factor, which equals to the present value of an annuity of \$1 per year for n years when the interest rate is i percent.

Example of equivalent annual net benefit method

水电站大坝

$$EANB = \frac{NPV}{a_i^n}, \mathrm{i} = 8\%$$

热电联厂

Hydroelectric dam (HED)	Cogeneration plant (CGP)
n= 75 years	n= 15 years
NVP = \$ 30 mln	NVP = \$ 24 mln
$a_{0,08}^{75} = 12.461$	$a_{0,08}^{15} = 8.559$
EANB = \$ 30 mln / 12.461	EANB = \$ 24 mln / 8.559
= \$ 2,407 mln	= \$ 2,804 mln

- Shorter projects also have an additional benefit (not included in EANB) because one does not necessarily have to roll-over the shorter project with identical installation when it's finished.
- A better option might be available at that time.

Questions

1. Discounting
2. Net present value (NPV)
3. Internal rate of return(IRR)
4. Timing of benefits and costs
5. Comparing projects with different time duration

Thanks for your attention!

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