第四章 隐变量模型

Latent Variable Models

- This chapter presents tools for illuminating structure in data in the presence of measurement difficulties, endogeneity, and unobservable or latent variables.
- There are several approaches to uncovering data structure:
  - Principal components analysis is widely used as an exploratory method for revealing structure in data.
  - Factor analysis, a close relative of principal components analysis, is a statistical approach for examining the underlying structure in multivariate data.

Principal Components Analysis

- Principal components analysis has two primary objectives: to reduce a relatively large multivariate data set, and to interpret data.
- Principal components analysis "explains" the variance–covariance structure using a few linear combinations of the originally measured variables.
- Through this process a more parsimonious description of the data is provided——reducing or explaining the variance of many variables with fewer, well-chosen combinations of variables.

Principal Components Analysis

- If a large proportion (70 to 90%) of the total population variance is attributed to a few uncorrelated principal components, then these components can replace the original variables without much loss of information and also describe different dimensions in the data.
- Principal components analysis relies on the correlation matrix of variables, so the method is suitable for variables measured on the interval and ratio scales.

Principal Components Analysis

- If the original variables are uncorrelated, then principal components analysis accomplishes nothing.
- Observational data containing a large number of correlated variables
- Experimental data with randomized treatments

- If it is found that the variance in 20 or 30 original variables is described adequately with four or five principal components (dimensions), then principal components analysis will have succeeded.

Principal Components Analysis

- Principal components analysis begins by noting that *n* observations, each with *p* variables or measurements upon them, is expressed in an $n \times p$ matrix ***X***:

$$X_{n \times p} = \begin{bmatrix} x_{11} & \cdots & x_{1p} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{n1} \end{bmatrix} \qquad (4.1)$$

- Principal components analysis is not a statistical model, and there is no distinction between dependent and independent variables.

Principal Components Analysis

- If the principal components analysis is useful, there are $K < n$ principal components, with the first principal component:

$$Z_1 = a_{11}x_1 + a_{12}x_2 + \ldots + a_{1p}x_p \tag{4.2}$$

- which maximizes the variability across individuals, subject to the constraint

$$a_{11}^2 + a_{12}^2 + \ldots + a_{1p}^2 = 1 \tag{4.3}$$

- VAR[Z_1] is maximized given the constraint in Equation 4.3, with the constraint imposed simply because the solution is indeterminate, otherwise one could simply increase one or more of the a_{ij} values to increase the variance.

Principal Components Analysis

- A second principal component, Z_2, is then sought that maximizes the variability across individuals subject to the constraints that $a_{21}^2 + a_{22}^2 + \ldots + a_{2p}^2 = 1$ and $COR[Z_1, Z_2] = 0$.
- A third principal component is added subject to the same constraint on the a_{ij} values, with the additional constraint that $COR[Z_1, Z_2, Z_3] = 0$.
- Additional principal components are added up to p, the number of variables in the original data set.

Principal Components Analysis

- The eigenvalues of the sample variance–covariance matrix $\boldsymbol{X}$ are the variances of the principal components. The corresponding eigenvector provides the parameters to satisfy Equation 4.3.
- The symmetric $p \times p$ sample variance–covariance matrix is given as

$$s^2[X] = \begin{bmatrix} s^2(x_1) & s(x_1, x_2) & \cdots & s(x_1, x_p) \\ s(x_2, x_1) & s^2(x_2) & \cdots & s(x_2, x_p) \\ \vdots & \vdots & \ddots & \vdots \\ s(x_p, x_1) & s(x_p, x_2) & \cdots & s^2(x_p) \end{bmatrix} \tag{4.4}$$

Principal Components Analysis

- The diagonal elements of this matrix represent the estimated variances of random variables 1 through *p*, and the off-diagonal elements represent the estimated covariances between variables.
- The sum of the eigenvalues λ_p of the sample variance–covariance matrix is equal to the sum of the diagonal elements in $s^2[X]$, or the sum of the variances of the *p* variables in matrix ***X***; that is,

$$\lambda_1 + \lambda_2 + \ldots + \lambda_p = VAR(x_1) + VAR(x_2) + \ldots + VAR(x_p) \qquad (4.5)$$

Principal Components Analysis

- Because the sum of the diagonal elements represents the total sample variance, and the sum of the eigenvalues is equal to the trace of $s^2[X]$, then the variance in the principal components accounts for all of the variation in the original data.
- There are p eigenvalues, and the proportion of total variance explained by the jth principal component is given by

$$VAR_j = \frac{\lambda_j}{\lambda_1 + \lambda_2 + \ldots + \lambda_p}, \; j = 1, 2, \ldots, p \qquad (4.6)$$

Principal Components Analysis

- To avoid excessive influence of measurement units, the principal components analysis is carried out using a standardized variance–covariance matrix, or the correlation matrix.
- The correlation matrix is simply the variance– covariance matrix as obtained by using the standardized variables instead of the original variables, such that

$$Z_{ij} = \frac{X_{ij} - \bar{X}_j}{\sigma_j}, i = 1, 2, \ldots, n; \ j = 1, 2, \ldots, p$$

replace the original X_{ij} terms.

Principal Components Analysis

- Because the correlation matrix is often used, variables used in principal components analysis are restricted to interval and ratio scales unless corrections are made.
- Using the correlation matrix, the sum of the diagonal terms, and the sum of eigenvalues, is equal to p (the number of variables).

Principal Components Analysis

- With the basic statistical mechanics in place, the basic steps in principal components analysis are as follows:
  1. Standardize all observed variables in the ***X*** matrix;
  2. Calculate the variance–covariance matrix, which is the correlation matrix after standardization;
  3. Determine the eigenvalues and corresponding eigenvectors of the correlation matrix (the parameters of the *i*th principal component are given by the eigenvector, whereas the variance is given by the eigenvalue);
  4. Discard any components that account for a relatively small proportion of the variation in the data.

Principal Components Analysis

- The intent of principal components analysis is to illuminate underlying commonality, or structure, in the data.
- In exploratory studies relying on observational data, it is often the case that some variables measure the same underlying construct, and this underlying construct is what is being sought in principal components analysis.
- The goal is to reduce the number of potential explanatory variables and gain insight regarding what underlying dimensions have been captured in the data.

Principal Components Analysis

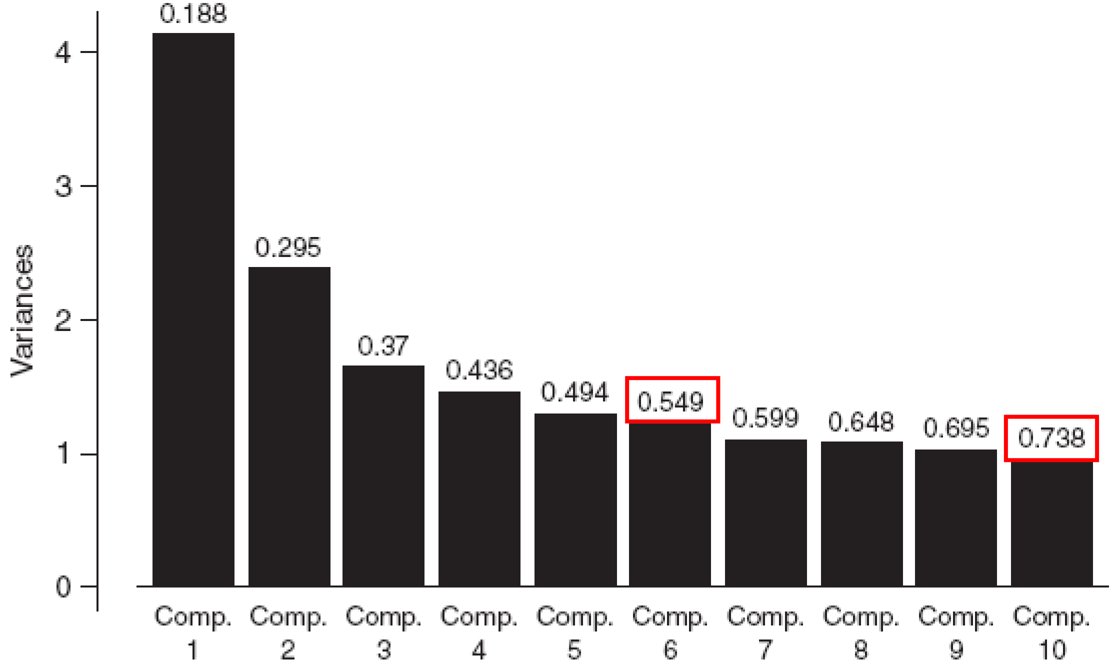
A survey of 281 commuters was conducted in the Seattle metropolitan area. The intent of the survey was to gather information on commuters' opinions or attitudes regarding high-occupancy vehicle (HOV) lanes (lanes that are restricted for use by vehicles with two or more occupants). Commuters were asked a series of attitudinal questions regarding HOV lanes and their use, in addition to a number of sociodemographic and commuting behavior questions. A description of the variables is provided in Table 8.1.

It is useful to assess whether a large number of variables, including attitudinal, sociodemographic, and travel behavior, can be distilled into a smaller set of variables. To do this, a principal components analysis is performed on the correlation matrix (not the variance–covariance matrix).

Variable Abbreviation	Variable Description
MODE	Usual mode of travel: 0 if drive alone, 1 if two person carpool, 2 if three or more person carpool, 3 if van pool, 4 if bus, 5 if bicycle or walk, 6 if motorcycle, 7 if other
HOVUSE	Have used HOV lanes: 1 if yes, 0 if no
HOVMODE	If used HOV lanes, what mode is most often used: 0 in a bus, 1 in two person carpool, 2 in three or more person carpool, 3 in van pool, 4 alone in vehicle, 5 on motorcycle
HOVDECLINE	Sometimes eligible for HOV lane use but do not use: 1 if yes, 0 if no
HOVDECREAS	Reason for not using HOV lanes when eligible: 0 if slower than regular lanes, 1 if too much trouble to change lanes, 2 if HOV lanes are not safe, 3 if traffic moves fast enough, 4 if forget to use HOV lanes, 5 if other
MODE1YR	Usual mode of travel 1 year ago: 0 if drive alone, 1 if two person carpool, 2 if three or more person carpool, 3 if van pool, 4 if bus, 5 if bicycle or walk, 6 if motorcycle, 7 if other
COM1YR	Commuted to work in Seattle a year ago: 1 if yes, 0 if no
FLEXSTAR	Have flexible work start times: 1 if yes, 0 if no
CHNGDEPTM	Changed departure times to work in the last year: 1 if yes, 0 if no
MINERLYWRK	On average, number of minutes leaving earlier for work relative to last year
MINLTWRK	On average, number of minutes leaving later for work relative to last year
DEPCHNGREAS	If changed departure times to work in the last year, reason: 0 if change in travel mode, 1 if increasing traffic congestion, 2 if change in work start time, 3 if presence of HOV lanes, 4 if change in residence, 5 if change in lifestyle, 6 if other
CHNGRTE	Changed route to work in the last year: 1 if yes, 0 if no
CHNGRTEREAS	If changed route to work in the last year, reason: 0 if change in travel mode, 1 if increasing traffic congestion, 2 if change in work start time, 3 if presence of HOV lanes, 4 if change in residence, 5 if change in lifestyle, 6 if other
I90CM	Usually commute to or from work on Interstate 90: 1 if yes, 0 if no
I90CMT1YR	Usually commuted to or from work on Interstate 90 last year: 1 if yes, 0 if no
HOVPST5	On your past five commutes to work, how often have you used HOV lanes
DAPST5	On your past five commutes to work, how often did you drive alone
CRPPST5	On your past five commutes to work, how often did you carpool with one other person
CRPPST52MR	On your past five commutes to work, how often did you carpool with two or more people
VNPPST5	On your past five commutes to work, how often did you take a van pool
BUSPST5	On your past five commutes to work, how often did you take a bus
NONMOTPST5	On your past five commutes to work, how often did you bicycle or walk
MOTPST5	On your past five commutes to work, how often did you take a motorcycle
OTHPST5	On your past five commutes to work, how often did you take a mode other than those listed in variables 18 through 24
	(continued)
CHGRTEPST5	On your past five commutes to work, how often have you changed route or departure time
HOVSAVTIME	HOV lanes save all commuters time: 0 if strongly disagree, 1 if disagree, 2 if neutral, 3 if agree, 4 if agree strongly
HOVADUSE	Existing HOV lanes are being adequately used: 0 if strongly disagree, 1 if disagree, 2 if neutral, 3 if agree, 4 if agree strongly
HOVOPN	HOV lanes should be open to all traffic: 0 if strongly disagree, 1 if disagree, 2 if neutral, 3 if agree, 4 if agree strongly
GPTOHOV	Converting some regular lanes to HOV lanes is a good idea: 0 if strongly disagree, 1 if disagree, 2 if neutral, 3 if agree, 4 if agree strongly
GTTOHOV2	Converting some regular lanes to HOV lanes is a good idea only if it is done before traffic congestion becomes serious: 0 if strongly disagree, 1 if disagree, 2 if neutral, 3 if agree, 4 if agree strongly
GEND	Gender: 1 if male, 0 if female
AGE	Age in years: 0 if under 21, 1 if 22 to 30, 2 if 31 to 40, 3 if 41 to 50, 4 if 51 to 64, 5 if 65 or older
HHINCM	Annual household income (U.S. dollars): 0 if no income, 1 if 1 to 9,999, 2 if 10,000 to 19,999, 3 if 20,000 to 29,999, 4 if 30,000 to 39,999, 5 if 40,000 to 49,999, 6 if 50,000 to 74,999, 7 if 75,000 to 100,000, 8 if over 100,000
EDUC	Highest level of education: 0 if did not finish high school, 1 if high school, 2 if community college or trade school, 3 if college/university, 4 if post college graduate degree
FAMSIZ	Number of household members
NUMADLT	Number of adults in household (aged 16 or older)
NUMWRKS	Number of household members working outside the home
NUMCARS	Number of licensed motor vehicles in the household
ZIPWRK	Postal zip code of workplace
ZIPHM	Postal zip code of home
HOVCMNT	Type of survey comment left by respondent regarding opinions on HOV lanes: 0 if no comment on HOV lanes, 1 if comment not in favor of HOV lanes, 2 if comment positive toward HOV lanes but critical of HOV lane policies, 3 if comment positive toward HOV lanes, 4 if neutral HOV lane comment

Principal Components Analysis

Figure 8.1 shows a graph of the first ten principal components. The graph shows that the first principal component represents about 19% of the total variance, the second principal component an additional 10%, the third principal component about 8%, the fourth about 7%, and the remaining principal components about 5% each. Ten principal components account for about 74% of the variance, and six principal components account for about 55% of the variance contained in the 23 variables that were used in the principal components analysis. Thus, there is some evidence that some variables, at least, are explaining similar dimensions of the underlying phenomenon.



Principal Components Analysis

Table 8.2 shows the variable parameters for the six principal components. For example, the first principal component is given by

$$Z_1 = -0.380(HOVPST5) + 0.396(DAPST5) - 0.303(CRPPST5)$$

$$-\ 0.109(CRPPST52MR) - 0.161(BUSPST5) - 0.325(HOVSAVTIME)\ .$$

$$+\ 0.364(HOVOPN) - 0.339(GTTOHOV2) + 0.117(GEND)$$

Factor Loadings of Principal Components Analysis: HOV Lane Survey Data

Variable	Comp. 1	Comp. 2	Comp. 3	Comp. 4	Comp. 5	Comp. 6
			Travel Behavior Variables			
HOVPST5	−0.380		−0.284	0.236		
DAPST5	0.396		0.274	−0.283		0.128
CRPPST5	−0.303		−0.223	0.240	0.282	0.221
CRPPST52MR	−0.109			0.167	0.196	−0.107
VNPPST5				−0.146		
BUSPST5	−0.161	−0.140	−0.227	0.112	−0.514	−0.395
NONMOTPST5						0.471
MOTPST5			0.104		0.381	−0.418
CHGRTEPST5					0.525	−0.302
			HOV Attitude Variables			
HOVSAVTIME	−0.325		0.301	−0.140		
HOVADUSE	−0.321		0.227	−0.133		
HOVOPN	0.364		−0.216	0.210		
GPTOHOV	−0.339	0.125	0.230	−0.115		
GTTOHOV2	−0.260		0.245	−0.153		
			Sociodemographic Variables			
GEND	0.117		0.388	0.180		−0.199
AGE			0.268	0.341	−0.363	−0.270
HHINCM		0.304	0.131	0.489		0.101
EDUC		0.188	0.247	0.443		0.247
FAMSIZ		0.429	−0.122			
NUMADLT		0.516	−0.188	−0.128	−0.133	
NUMWRKS		0.451	−0.242	−0.137		
NUMCARS		0.372	−0.106		0.107	−0.268

Note: Loadings < 0.10 shown as blanks.

Principal Components Analysis

All of the variables had estimated parameters (or loadings). However, parameters less than 0.1 were omitted from Table 8.2 because of their relatively small magnitude. The first principal component loaded strongly on travel behavior variables and HOV attitude variables. In addition, Z_1 increases with decreases in any non-drive-alone travel variables (HOV, Car Pool, Bus), increases with decreases in pro-HOV attitudes, and increases for males. By analyzing the principal components in this way, some of the relationships between variables are better understood.

$$Z_1 = -0.380(HOVPST5) + 0.396(DAPST5) - 0.303(CRPPST5)$$

$$-\,0.109(CRPPST52MR) - 0.161(BUSPST5) - 0.325(HOVSAVTIME)\,.$$

$$+\,0.364(HOVOPN) - 0.339(GTTOHOV2) + 0.117(GEND)$$

or Loadings of Principal Components Analysis: HOV Lane Su

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DAPST5	0.396		0.274
CRPPST5	–0.303		–0.223
CRPPST52MR	–0.109		
VNPPST5			
BUSPST5	–0.161	–0.140	–0.227
NONMOTPST5			
MOTPST5			0.104
CHGRTEPST5			
			HOV Attitude Variables
HOVSAVTIME	–0.325		0.301
HOVADUSE	–0.321		0.227
HOVOPN	0.364		–0.216
GPTOHOV	–0.339	0.125	0.230
GTTOHOV2	–0.260		0.245
			Sociodemographic Variables
GEND	0.117		0.388
AGE			0.268
HHINCM		0.304	0.131
EDUC		0.188	0.247
FAMSIZ		0.429	–0.122
NUMADLT		0.516	–0.188
NUMWRKS		0.451	–0.242
NUMCARS		0.372	–0.106

Note: Loadings < 0.10 shown as blanks.

Factor Analysis

- Factor analysis is a close relative of principal components analysis. It was developed to gain insight into psychometric measurements, specifically the directly unobservable variable intelligence.
- The aim of the analysis is to reduce the number of p variables to a smaller set of parsimonious $K < p$ variables.
- The objective is to describe the covariance among many variables in terms of a few unobservable factors.

Factor Analysis

- Between Principal Components and Factor Analysis :
  - The difference: Factor analysis is based on a specific statistical model, whereas principal components analysis is not;
  - The similarities: ① Relying on the correlation matrix;
    ② Being suitable for variables measured on interval and ratio scales.
- Just as for other statistical models, there should be a theoretical rationale for conducting a factor analysis. There should be a theoretically motivated reason to suspect that some variables may be measuring the same underlying phenomenon, with the expectation of examining whether the data support this expected underlying measurement model or process.

Factor Analysis

- The factor analysis model is formulated by expressing the X_i terms as linear functions, such that

$$\begin{aligned}
X_1-\mu_1 &= \ell_{11}F_1+\ell_{12}F_2+\ \dots\ +\ell_{1m}F_m+\varepsilon_1\\
X_2-\mu_2 &= \ell_{21}F_1+\ell_{22}F_2+\ \dots\ +\ell_{2m}F_m+\varepsilon_2\\
\vdots\quad & \qquad\ \vdots \qquad\qquad\quad \ddots \qquad\quad \vdots\\
X_p-\mu_p &= \ell_{p1}F_1+\ell_{p2}F_2+\ \dots\ +\ell_{pm}F_m+\varepsilon_p
\end{aligned} \tag{4.7}$$

where in matrix notation the factor analysis model is given as:

$$(\mathbf{X}-\boldsymbol{\mu})_{p\times 1}=\mathbf{L}_{p\times m}\mathbf{F}_{m\times 1}+\boldsymbol{\varepsilon}_{p\times 1'} \tag{4.8}$$

where the F are factors and the ℓ_{ij} are the factor loadings. The ε_i are associated only with the X_i, and the p random errors and m factor loadings are unobservable or latent.

Factor Analysis

- With p equations and $p+m$ unknowns, the unknowns cannot be directly solved without additional information.
- To solve for the unknown factor loadings and errors, restrictions are imposed. The types of restrictions determine the type of factor analysis model.
- Factor loadings that are either close to 1 or close to 0 are sought.
  - A factor loading close to 1 suggests that a variable X_i is largely influenced by F_j;
  - A factor loading close to 0 suggests that a variable X_i is not substantively influenced by F_j.
- A collection of factor loadings that is as diverse as possible is sought, lending itself to easy interpretation.

Factor Analysis

- The factor rotation method used determines the type of factor analysis model, orthogonal or oblique.
- The orthogonal factor analysis model satisfies the following conditions:

$$\begin{aligned}
&F, \varepsilon \text{ are independent} \\
&E[\mathbf{F}] = \mathbf{0} \\
&COV[\mathbf{F}] = \mathbf{I} \\
&E[\boldsymbol{\epsilon}] = \mathbf{0} \\
&COV[\boldsymbol{\epsilon}] = \boldsymbol{\upsilon}, \text{ where } \boldsymbol{\upsilon} \text{ is a diagonal matrix}
\end{aligned} \tag{4.9}$$

- Varimax rotation: maximizing the sum of the variances of the factor loadings, a common method for conducting an orthogonal rotation.

Factor Analysis

- The oblique factor analysis model relaxes the restriction of uncorrelated factor loadings, resulting in factors that are non-orthogonal.
- Oblique factor analysis is conducted with the intent to achieve a structure that is more interpretable. Specifically, computational strategies have been developed to rotate factors such that clusters of variables are best represented, without the constraint of orthogonality.
- However, the oblique factors produced by such rotations are often not easily interpreted, sometimes resulting in factors with less-than-obvious meaning (that is, with many cross loadings).

Factor Analysis

- Interpretation of factor analysis is straightforward. Variables that have high factor loadings are thought to be highly influential in describing the factor, whereas variables with low factor loadings are less influential in describing the factor.
- Inspection of the variables with high factor loadings on a specific factor is used to uncover structure or commonality among the variables. One must then determine the underlying constructs that are common to variables that load highly on specific factors.

Factor Analysis

Continuing from the previous example, a factor analysis on continuous variables is conducted to determine which variables might be explaining similar underlying phenomenon. The same set of variables used for the principal components analysis is again used in an exploratory factor analysis with orthogonal varimax rotation. Because six principal components explained roughly 55% of the data variance, the number of factors estimated was six.

Table 8.3 shows the factor loadings resulting from this factor analysis. As was done in the principal components analysis, factor loadings less than 0.1 are blanks in the table. Table 8.3, in general, is sparser than Table 8.2, simply because factors are orthogonal to one another (correlations between factors are zero), and because a factor set solution was sought that maximizes the variances of the factor loadings.

TABLE 8.3

Factor Loadings of Factor Analysis: HOV Lane Survey Data

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
			Travel Behavior Variables			
HOVPST5	0.306	0.658	0.323	0.147		0.147
DAPST5	–0.278	–0.846	–0.358	–0.206		–0.167
CRPPST5	0.221	0.930	–0.217	–0.166	–0.109	
CRPPST52MR	0.112			0.987		
VNPPST5						0.675
BUSPST5	0.113	0.108	0.983			
NONMOTPST5						0.298
MOTPST5					0.992	
CHGRTEPST5	–0.125		–0.113		0.158	
			HOV Attitude Variables			
HOVSAVTIME	0.734	0.142				
HOVADUSE	0.642	0.110				0.135
HOVOPN	–0.814	–0.117				
GPTOHOV	0.681	0.156				
GTTOHOV2	0.515					
			Sociodemographic Variables			
AGE		–0.128				
HHINCM			–0.167			
EDUC						
FAMSIZ						–0.129
NUMWRKS	0.105					
NUMCARS						

Note: Loadings <0.10 shown as blanks.

TABLE 8.2

Factor Loadings of Principal Components Analysis: HOV Lane Survey Data

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NUMCARS		0.372	–0.106		0.107	–0.268

Note: Loadings < 0.10 shown as blanks.

Factor Analysis

Inspection of Table 8.3 shows that the equation for X_1 is given as $HOVPST5 = 0.306F_1 + 0.658F_2 + 0.323F_3 + 0.147F_4 + 0.147F_6$, where factors 1 through 5 are unobserved or latent. As in the principal components analysis, travel behavior variables and HOV attitudinal variables seem to load heavily on factor 1. That many of the factors include travel behavior variables suggests that many dimensions of travel behavior are reflected in these variables. There appear to be two factors that reflect dimensions in the data related to attitudes toward HOV lanes. Sociodemographic variables do not seem to load on any particular factor, and so probably represent unique dimensions in the data.

Factor Loadings of Factor Analysis: HOV Lane Survey Data

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
			Travel Behavior Variables			
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