Modelling Transportation Systems
ASSIGNMENT 1

- **Assignment due date: <u>May 21th, 2019</u>**

Question 1

The Reggio Advertising Company wishes to plan an advertising campaign in three different media—television, radio, and magazines. The purpose of the advertising program is to reach as many potential customers as possible. Results of a market study are given below:

	Television		Radio	Magazines
	Day Time	Prime Time		
Cost of an advertising unit	\$40,000	\$75,000	\$30,000	\$15,000
Number of potential customers reached per unit	400000	900000	500000	200000
Number of women customers reached per unit	300000	400000	200000	100000

The company does not want to spend more than \$800000 on advertising. It further requires that:
(1) at least 2 million exposures take place among women;
(2) advertising on television be limited to \$500000;
(3) at least 3 advertising units be bought on daytime television, and two units during prime time;
(4) the number of advertising units on radio and magazine should each be between 5 and 10. Formulate into a linear programming problem.

Question 2

Walmart has three warehouses (W1, W2 and W3) that store the same type of product and five supermarkets (S1 to S5) that need the products in a city. The number of products available at each warehouse, the number of products needed at each supermarket, and the transportation cost per unit product from each warehouse to each supermarket are shown below. Develop a linear optimization model to help Walmart make the decision of how to transport the products. (Just formulate this model, and you don't need to solve it.)

Warehouse	Number of products available
W1	100
W2	200
W3	50

Unit cost (\$)	Number of products available				
W1	1	2	4	3	6

W2	5	2	4	4	4
W3	5	1	1	3	2

Supermarket	**Number of products needed**
S1	80
S2	90
S3	70
S4	60
S5	50

Question 3

The following questions are on linear optimization models. For each question, you should either answer that such a linear optimization model does not exist (no need to provide the reason), or give an example of such a linear optimization model.

(i) A model is infeasible, and after removing one constraint, it is feasible.

(ii) A model is infeasible, and after removing one constraint, it is unbounded.

(iii) A model is infeasible, and after removing one constraint, it has an optimal solution.

(iv) A model is infeasible, and after removing one constraint, it has an infinite number of optimal solutions.

(v) A model has an optimal solution, and after adding one constraint, it is unbounded.

(vi) A model has an optimal solution, and after adding one constraint, it is infeasible.

(vii) A model has exactly one optimal solution, and after adding one constraint, it has an infinite number of optimal solutions.

(viii) A model has an infinite number of optimal solutions, and after adding one constraint, it has exactly one optimal solution.

(ix) A model is infeasible, and after changing its objective function, it has an optimal solution.

(x) A model has an optimal solution, and after changing its objective function, it is infeasible.

(xi) A model has an optimal solution, and after changing its objective function, it is unbounded.

(xii) A model is unbounded, and after changing its objective function, it has exactly one optimal solution.

(xiii) A model is unbounded, and after changing its objective function, it has an infinite number of optimal solutions.

Question 4

Solve the following Linear Programming problem by the simplex method and give the simplex table.

$$\text{Minimize} \quad z = x_1 + x_2 - 4x_3$$

$$\text{Subject to}\quad \begin{cases} x_1 + x_2 + 2x_3 \le 9 \\ x_1 + x_2 - x_3 \le 2 \\ -x_1 + x_2 + x_3 \le 4 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

Question 5

Consider the following linear programming problem with parameter α, β:

$$\text{Maximize}\quad z = \alpha x_1 + 2x_2 + x_3 - 4x_4$$

$$\text{Subject to}\quad \begin{cases} x_1 + x_2 - x_4 = 4 + 2\beta & (1) \\ 2x_1 - x_2 + 3x_3 - 2x_4 = 5 + 7\beta & (2) \\ x_i \ge 0,\ i = 1, \ldots, 4 \end{cases}$$

(i) Update the constraints as (1)=(1)+(2), (2)=(2)-2(1). Please give the simplex table with $\mathbf{X} = (x_1, x_2)^T$ as the BFS.

(ii) Assume that $\beta = 0$, $\mathbf{X} = (x_1, x_2)^T$ is the optimal BFS, then $\alpha = ?$

(iii) Assume that $\alpha = 3$, $\mathbf{X} = (x_1, x_2)^T$ is the optimal BFS, then $\beta = ?$

Question 6

The following is the current simplex tableau of a linear programming problem. The objective is to minimize $-28x_4 - x_5 - 2x_6$ and x_1, x_2 and x_3 are the slack variables.

	z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	1	b	c	0	0	-1	G	-14
x_6	0	3	0	-14/3	0	1	1	a
x_2	0	6	d	2	0	5/2	0	5
x_4	0	0	e	f	1	0	0	0

(i) Find the values of the unknowns *a* through *g* in the tableau.

(ii) Find $\mathbf{B}^{-1}$.

(iii) Find $\partial x_2 / \partial x_1, \partial z / \partial x_5, \partial x_6 / \partial b_3$.

(iv) Without explicitly finding the basic vectors $\mathbf{a}_6, \mathbf{a}_2, \mathbf{a}_4$ give the representation of the vector $\mathbf{a}_5$ in terms of these basic vectors.

Question 7

Consider the following LP problem:

$$\text{Minimize } 3x_1 + 4x_2 + 6x_3 + 7x_4 + x_5$$

$$\text{Subject to } \begin{cases} 2x_1 - x_2 + x_3 + 6x_4 - 5x_5 - x_6 = 6 \\ x_1 + x_2 + 2x_3 + x_4 + 2x_5 - x_7 = 3 \\ x_1, x_2, x_3 . x_4, x_5, x_6, x_7 \geq 0 \end{cases}$$

(i) Please give the dual problem of primal problem.

(ii) The optimal solution of the dual problem is $w = (w_1^*, w_2^*) = (1,1)$. Please give the optimal solution of the primal problem based on the characteristic of dual problem.

Question 8

Determine whether each of the following statements about LP is true or false, and show your reasons.

(i) If the primal problem exists the feasible solution, then its dual problem must exist the feasible solution;

(ii) If the dual problem does not have a feasible solution, then the primal problem must have no feasible solution;

(iii) If the primal problem and the dual problem both exist feasible solutions, then the LP must have finite optimal solutions.

Question 9

Consider the problem:

$$\text{Minimize } -2x_1 + x_2 - x_3$$

$$\text{Subject to } \begin{cases} x_1 + x_2 + x_3 \leq 6 \\ -x_1 + 2x_2 \leq 4 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

With the optimal tableau below:

	z	x_1	x_2	x_3	x_4	x_5	RHS
z	1	0	-3	-1	-2	0	-12
x_1	0	1	1	1	1	0	6
x_5	0	0	3	1	1	1	10

How would the optimal tableau change if:

(i) $\mathbf{a}_2$ in the constraint matrix $\mathbf{A}$ is changed from $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ to $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$.

(ii) $\mathbf{a}_1$ is changed from $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ to $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$.

(iii) $\mathbf{a}_1$ is changed from $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ to $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$.

(iv) A new activity $x_6 \geq 0$ with $c_6 = 1$ and $\mathbf{a}_6 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ is introduced?