Assignment 1

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1

设 Television Day Time Unit 为 x_1,Prime Time Unit 为 x_2,Radio Unit 为 x_3,Magazines Unit 为 x_4.

$$\max z = 400000x_1 + 900000x_2 + 500000x_3 + 200000x_4$$

$$\begin{cases} 4000x_1 + 75000x_2 + 3000x_3 + 15000x_4 \leqslant 800000 \\ 300000x_1 + 400000x_2 + 200000x_3 + 100000x_4 \geqslant 200000 \\ 4000x_1 + 75000x_2 \leq 500000 \\ x_1 \geqslant 3 \quad x_2 \geqslant 2 \\ 5 \leqslant x_3, x_4 \leqslant 10 \end{cases}$$

2

产销平衡的运输问题。

设 w_i 到 s_j 的运输量为 x_{ij}. 设 a_i 为 w_i 的容量,n_j 为 s_j 的需求量,c_{ij} 为 w_i 到 s_j 的 cost. 其中 $i = 1, 2, 3, \quad j = 1, \ldots 5$. 根据题意可知

$$a = [100, 200, 50] \quad n = [80, 90, 70, 60, 50]$$

$$c = \begin{bmatrix} 1 & 2 & 4 & 3 & 6 \\ 5 & 2 & 4 & 4 & 4 \\ 1 & 1 & 1 & 3 & 2 \end{bmatrix}$$

$$\min z = \sum_{i=1}^{3} \sum_{j=1}^{5} c_{ij} x_{ij}$$

$$\begin{cases} \sum_{j=1}^{5} x_{ij} = a_i & (i = 1, 2, 3) \\ \sum_{i=1}^{3} x_{ij} = n_j & (j = 1, 2, \cdots, 5) \\ x_{ij} \geqslant 0 & (i = 1, 2, 3; j = 1, 2, \cdots, 5) \end{cases}$$

3

(1)$\max z = x \quad s.t.\{1 \leqslant x \leqslant 2 \quad x < 0\}$, remove constraint 2 ,it is feasible.
(2)$\max z = x \quad s.t.\{x \geqslant 2 \quad x \leqslant 1\}$, remove constraint 2, it is unbounded.
(3) 同 (1)
(4)$\max z = x + y \quad s.t.\{x + y \leqslant 1 \quad x + y \geqslant 2 \quad x, y \geqslant 0\}$, remove constraint 2, it has an infinite number of optimal solutions.
(5)No Exist.
(6)$\max z = x \quad s.t.\{1 \leqslant x \leqslant 2\}$, add constraint $\{x < 0\}$, it is infeasible.
(7)$\max z = x + y \quad s.t.\{2x + y \leqslant 2 \quad x, y \geqslant 0\}$, add constraint $\{x + y \leqslant 1\}$, it has an infinite number of optimal solutions.
(8)$\max z = x + y \quad s.t.\{x + y \leqslant 1 \quad x, y \geqslant 0\}$, add constraint $\{x + 2y \leqslant 1\}$, it has exactly one optimal solution.
(9)No Exist.
(10)No Exist.
(11)$\min z = x \quad s.t.\{x \geqslant 1\}$, change objective function $\max z = x$, it is unbounded.
(12)$\max z = x \quad s.t.\{x \geqslant 1\}$, change objective function $\min z = x$, it has exactly one optimal solution.
(13)$\max z = x + y \quad s.t.\{x + y \geqslant 1 \quad x, y \geqslant 0\}$, change objective function $minz = x + y$, it has an infinite number of optimal solutions.

4

标准型为：

$$\max z = -x_1 - x_2 + 4x_3 + 0x_4 + 0x_5 + 0x_6$$

$$\begin{cases} x_1 + x_2 + 2x_3 + x_4 = 9 \\ x_1 + x_2 - x_3 + x_5 = 2 \\ -x_1 + x_2 + x_3 + x_6 = 4 \\ x_1, x_2 \cdots x_6 \geqslant 0 \end{cases}$$

simplex table 如下：

			-1	-1	4	0	0	0	
C_B	x_B	b	x_1	x_2	x_3	x_4	x_5	x_6	
0	x_4	9	1	1	2	1	0	0	$\frac{9}{2}$
0	x_5	2	1	1	-1	0	1	0	-
0	x_6	4	-1	1	[1]	0	0	1	4
$c_j - z_j$			-1	-1	4	0	0	0	
0	x_4	1	[3]	-1	0	1	0	-2	$\frac{1}{3}$
0	x_5	6	0	2	0	0	1	1	-
4	x_3	4	-1	1	1	0	0	1	-
$c_j - z_j$			3	-5	0	0	0	-4	
-1	x_1	$\frac{1}{3}$	1	$-\frac{1}{3}$	0	$\frac{1}{3}$	0	$-\frac{2}{3}$	
0	x_5	6	0	2	0	0	1	1	
4	x_3	$\frac{13}{3}$	0	$\frac{2}{3}$	1	$\frac{1}{3}$	0	$\frac{1}{3}$	
$c_j - z_j$			0	-4	0	-1	0	-2	

所以当 $X = (\frac{1}{3}, 0, \frac{13}{3}, 0, 0, 0)^T$ 时，z 取最小值为 -17.

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(1) 由题意可得 (1): $x_1 + x_3 - x_4 = 3 + 3\beta$, (2):$x_2 - x_3 = 1 - \beta$. 由此可得 simplex table with $X = (x_1, x_2)^T$ 如下：

			α	2	1	-4	
C_B	x_B	b	x_1	x_2	x_3	x_4	
α	x_1	$3 + 3\beta$	1	0	1	-1	
2	x_2	$1 - \beta$	0	1	-1	0	
$c_j - z_j$			0	0	$3 - \alpha$	$\alpha - 4$	

(2) 由$\begin{cases} 3-\alpha \leqslant 0 \\ \alpha - 4 \leqslant 0 \end{cases}$，$\alpha$ 的取值为 $3 \leqslant \alpha \leqslant 4$.

(3) 由$\begin{cases} 3+3\beta \geqslant 0 \\ 1 - \beta \geqslant 0 \end{cases}$，$\beta$ 的取值为 $-1 \leqslant \beta \leqslant 1$

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(1) 根据题意得 simplex table 如下:

		x_1	x_2	x_3	x_4	x_5	x_6	RHS
	1	-6	0	0	0	-1		-14
x_6	0	3	0	$-\frac{14}{3}$	0	1	1	7
x_2	0	6	1	2	0	$\frac{5}{2}$	0	5
x_4	0	0	0	$\frac{1}{3}$	1	0	0	0

得 $a=7, b=-6, c=0, d=1, e=0, f=\frac{1}{3}, g=0$.

(2) 由 simplex table 可知

$$\boldsymbol{B^{-1}} = \begin{bmatrix} 3 & 0 & -\frac{14}{3} \\ 6 & 1 & 2 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

(3) $\frac{\partial x_2}{\partial x_1} = -6 \quad \frac{\partial z}{\partial x_5} = -1 \quad \frac{\partial x_6}{\partial b_3} = 0$

(4) $\boldsymbol{a}_5 = \boldsymbol{a}_6 + \frac{5}{2}\boldsymbol{a}_2$

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(1) 对偶问题为:

$$\max 6y_1 + 3y_2$$

$$
\begin{cases}
2y_1 + y_2 \leqslant 3 \\
-y_1 + y_2 \leqslant 4 \\
y_1 + 2y_2 \leqslant 6 \\
6y_1 + y_2 \leqslant 7 \\
-5y_1 + 2y_2 \leqslant 1 \\
y_1, y_2 \geqslant 0
\end{cases}
$$

(2) 由 $w = (1, 1)$ 可知 dual problem constraint 1 和 4 为紧约束, 由对偶问题的性质可得:

$$
\begin{cases}
2x_1 + 6x_4 = 6 \\
x_1 + x_4 = 3
\end{cases}
$$

解得原问题最优解为 $X^\star = (3, 0, 0, 0, 0)^T, \min = 9.$

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(1)False

(2)False

(3)True

原问题和对偶问题解的关系如下表:

原问题	对偶问题
有可行解，且有最优解	有可行解，且有最优解
有可行解，但无最优解	无可行解
无可行解	无可行解
无可行解	有可行解，但无最优解

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(1)

$$
\boldsymbol{a}_2^{'} = \boldsymbol{B}^{-1}\boldsymbol{a}_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \end{bmatrix}
$$

$$\sigma_2 = C_2 - \boldsymbol{C}_B\boldsymbol{B}^{-1}\boldsymbol{a}_2 = -5 < 0$$

最优解不变，单纯型表如下:

	z	x_1	x_2	x_3	x_4	x_5	RHS
z	1	0	-5	-1	-2	0	-12
x_1	0	1	2	1	1	0	6
x_5	0	0	7	1	1	1	10

(2) 原问题变为 unbounded，simplex table 如下:

			2	-1	1	0	0	
C_B	x_B	b	x_1	x_2	x_3	x_4	x_5	
1	x_3	6	0	1	1	1	0	-
0	x_5	4	-1	2	0	0	1	-
$c_j - z_j$			2	-2	0	0	0	

(3)simplex table 如下:

			2	-1	1	0	0	
C_B	x_B	b	x_1	x_2	x_3	x_4	x_5	
1	x_3	6	3	1	1	1	0	
0	x_5	2	3	1	0	0	1	
$c_j - z_j$			-1	-2	0	-1	0	

最优解为 $\boldsymbol{X}^\star = (0,0,6,0,2)^T, \min = -6$

(4)simplex table 如下:

			2	-1	1	0	0	-1	
C_B	x_B	b	x_1	x_2	x_3	x_4	x_5	x_6	
0	x_4	6	[1]	1	1	1	0	-1	6
0	x_5	4	-1	2	0	0	1	2	-
$c_j - z_j$			-2	1	-1	0	0	-1	
2	x_1	6	1	1	1	1	0	-1	-
0	x_5	10	0	3	1	1	1	[1]	10
$c_j - z_j$			0	-3	-1	-2	0	1	
2	x_1	16	1	4	2	2	1	0	
-1	x_6	10	0	3	1	1	1	1	
$c_j - z_j$			0	-9	-3	-3	-1	0	

最优解为 $\boldsymbol{X}^{\star} = (16, 0, 0, 0, 0, 10)^T, \min = -22.$