Modelling Transportation Systems
ASSIGNMENT 3

◆ **Assignment due date: <u>June 1th, 2019</u>**

Question 1

Please convert the following nonlinear 0-1 programming problem into a linear 0-1 programming problem:

$$\text{Maximize} \quad z = x_1^2 + x_2 x_3 - x_3^3$$

$$\text{Subject to} \quad \begin{cases} -2x_1 + 3x_2 + x_3 \le 3 \\ x_i = 0 \ or \ 1 \ (i = 1,2,3) \end{cases}$$

Question 2

Solve the following Integer Programming problem by Branch-and-Bound algorithm:

$$\textbf{(a)} \ \text{Maximize} \quad P = 2x_1 + 3x_2 + x_3 + 2x_4$$

$$\text{Subject to} \quad \begin{cases} 5x_1 + 2x_2 + x_3 + x_4 \le 15 \\ 2x_1 + 6x_2 + 10x_3 + 8x_4 \le 60 \\ x_1 + x_2 + x_3 + x_4 \le 8 \\ 2x_1 + 2x_2 + 3x_3 + 3x_4 \le 16 \\ x_1 \le 3, x_2 \le 7, x_3 \le 5, x_4 \le 5 \\ x_1, x_2, x_3, x_4 \ \text{are integers} \end{cases}$$

$$\textbf{(b)} \ \text{Maximize} \quad z = x_1 + x_2$$

$$\text{Subject to} \quad \begin{cases} 2x_1 + 5x_2 \le 16 \\ 6x_1 + 5x_2 \le 30 \\ x_1, x_2 \ge 0 \\ x_1, x_2 \ \text{are integers} \end{cases}$$

Question 3

Solve the following Integer Programming problem by cut plane algorithm:

$$\text{Minimize} \quad z = 3x_1 + 4x_2$$

$$\text{Subject to} \quad \begin{cases} 3x_1 + x_2 \ge 4 \\ x_1 + 2x_2 \ge 4 \\ x_1, x_2 \ge 0 \\ x_1, x_2 \ \text{are integers} \end{cases}$$

Question 4

Solve the following 0-1 Integer Programming problems:

$$\text{Minimize} \quad z = 4x_1 + 3x_2 + 2x_3$$

$$\text{Subject to} \quad \begin{cases} 2x_1 - 5x_2 + 3x_3 \le 4 \\ 4x_1 + x_2 + 3x_3 \ge 3 \\ x_2 + x_3 \ge 1 \\ x_i = 0 \text{ } or \text{ } 1 \ (i = 1, 2, 3) \end{cases}$$

Question 5

There are six cities (city 1–6) in Kilroy County. The county must determine the locations of fire stations. The county wants to build the minimum number of fire stations needed to ensure that at least one fire station is within 15 minutes (driving time) of each city. The times (in minutes) required to drive between the cities in Kilroy County are shown in the following table. Formulate an IP that will tell Kilroy how many fire stations should be built and where they should be located.

From	To					
	City 1	**City 2**	**City 3**	**City 4**	**City 5**	**City 6**
City 1	0	10	20	30	30	20
City 2	10	0	25	35	20	10
City 3	20	25	0	15	30	20
City 4	30	35	15	0	15	25
City 5	30	20	30	15	0	14
City 6	20	10	20	25	14	0

Question 6

A manufacturing engineer facing the problem of reducing the costs of production in his department, decided to see whether he can reduce the number of work stations involved in the production of a certain product. The final product consists of 6 components each of which is fabricated within a single work station and then added to the product as it is passed through these stations. Even though more than one of these components may be fabricated in any one station, there is a limit of 16 minutes on how long the product can stay at any given station. In addition, due to the geometry of the parts, some components cannot be added unless others have been previously attached (This type of restriction is sometimes referred to as a precedence relation).

Assembly Component (i)	Time Required for Assembly (Min) (t)	Predecessors

1	9	-
2	3	-
3	5	2
4	7	-
5	3	1,3
6	8	4

The objective is to formulate this problem as an integer programming problem, the solution of which will correlate the assembly of the components with the minimum number of work stations. (Let i denote a component, j denote a work station)

Question 7

(i) A railway management runs different types of trains (shown in the table below).

Type	**Number of first-class**	**Number of second-class**
1	4	4
2	6	3
3	2	5

And the income from these types of trains is in the proportion 5:6:4. The management has altogether 22 first-class and 25 second-class carriages at its disposal. How many trains of each type should be run?

(ii) Solve the same problem with types in the following table:

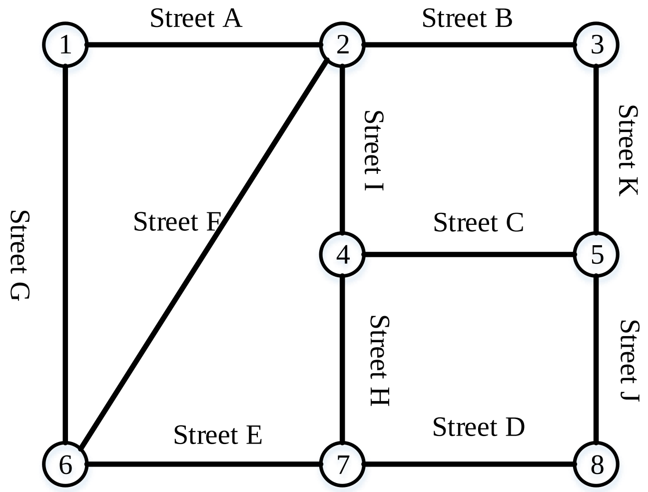
Type	**Number of first-class**	**Number of second-class**
1	6	4
2	6	6
3	5	7

When the income is in the proportions 26:30:19, and the management has 33 first-class and 7 second-class carriage at its disposal. Solve the problem by simplex method.

Question 8

To promote on-campus safety, the U of A Security Department is in the process of installing emergency telephones at selected locations. The department wants to install the minimum number of telephones,

provided that each of the campus main streets is served by at least one telephone. The following figure maps the principal streets (A to K) on campus.
It is logical to place the telephones at street intersections so that each telephone will serve at least two streets. The following figure shows that the layout of the streets requires a maximum of eight telephone locations.



Question 9

Consider the assignment problem. There are $N = 10$ workers and N jobs to do. Each worker must do a job. The cost of letting worker i do job j is c_{ij}. How to assign the jobs to the workers in order to minimize the total costs?