Modelling Transportation Systems
ASSIGNMENT 2

- **Assignment due date: <u>May 28th, 2019</u>**

<u>Question 1</u>

Consider the following problem:

$$\text{Maximize} \quad z = f(x)$$

$$\text{Subject to} \quad a \le x \le b$$

(1) Suppose $f(x)$ is a convex function that has derivatives for all values of x. Show that $x = a$ or $x = b$ must be optimal for the nonlinear programming (please draw a picture).

(2) Suppose $f(x)$ is a convex function for which $f'(x)$ may not exist. Show that $x = a$ or $x = b$ must be optimal for the nonlinear programming (use the definition of a convex function).

<u>Question 2</u>

Locate the stationary points (values of the independent variables at which the slope of the function is zero) of $f(x)$ and determine whether they are local maxima, local minima or neither.

$$f(x) = x_1^3 - x_1x_2 + x_2^2 - 2x_1 + 3x_2 - 4$$

<u>Question 3</u>

Prove that the function $f(x) = 2x_1x_2x_3 - 4x_1x_3 - 2x_2x_3 + x_1^2 + x_2^2 + x_3^2 - 2x_1 - 4x_2 + 4x_3$ has stationary point $(0,3,1), (0,1,-1), (1,2,0), (2,1,1), (2,3,-1)$, and find the extreme point using sufficient conditions.

<u>Question 4</u>

Perform **four iterations** using steepest descent direction method on the following NLP problems:

- Maximize $f(x) = -x_1^2 - 2x_2^2 + 2x_1x_2 + 2x_2$ with $\mathbf{x}^0 = (0,0)^T$ as the initial point.

<u>Question 5</u>

Find an optimal solution for the following nonlinear programming problem by the KKT conditions, and show that the solution is unique.

$$\text{Minimize} \quad z(x_1, x_2) = 4(x_1 - 2)^2 + 3(x_2 - 4)^2$$

$$\text{Subject to} \quad \begin{aligned} x_1 + x_2 &\le 5 \\ x_1 &\ge 1 \\ x_2 &\ge 2 \end{aligned}$$

<u>Question 6</u>

Perform **two iterations** of the feasible-directions method on the following NLP. Begin at $\mathbf{X}^0 = (0, 0.75)^T$.

$$\text{Minimize} \quad f(x) = 2x_1^{\,2} - 2x_1x_2 + 2x_2^{\,2} - 4x_1 - 6x_2$$

$$\text{Subject to} \quad \begin{aligned} & x_1 + 5x_2 \le 5 \\ & 2x_1^{\,2} - x_2 \le 0 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Question 7

Consider the convex programming problem:

$$\text{Minimize} \quad z(x) = x_1^2 + x_2^2 - 3x_1 - 4x_2$$

$$\text{Subject to} \quad \begin{aligned} & x_1 + x_2 \le 1 \\ & x_1, x_2 \ge 0 \end{aligned}$$

(1) Starting from initial point $(x_1, x_2) = (\frac{1}{4}, \frac{1}{4})$, apply 3 iterations of the Frank-Wolfe algorithm.

(2) Check if the solution obtained in part (1) is the optimal one by using the KKT conditions.

Question 8

Find the user equilibrium flow patterns and the user equilibrium travel time for the network described in Figure 1. The link travel time functions are:

$$t(x_1) = 2 + x_1^{\,2}$$
$$t(x_2) = 1 + 3x_2$$
$$t(x_1) = 3 + x_3$$

where, $t_a(x_a)$ and x_a denote the travel time and flow on link a (a = 1,2,3) respectively. Travel demand between the OD pair (1-3) is q units, $q \ge 0$.

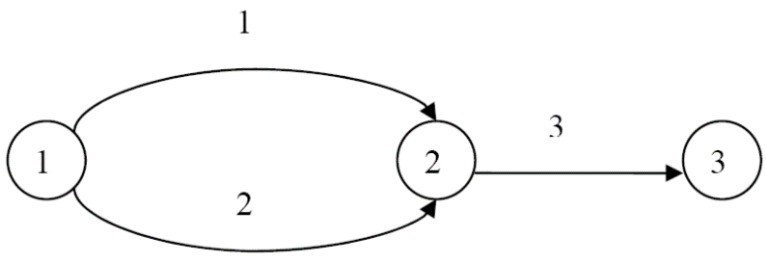

Figure 1. A simple network with one O-D pair

Question 9

A simple network with one OD pair (A-D) is depicted in Figure 2. Travel time functions are marked on each link. Total demand between the OD pair is 4 units.

(1) Find the user equilibrium flow patterns and the total travel cost of the network under the user equilibrium (UE) condition.

(2) How would the total travel cost of the whole network change if link B-C is converted into pavement (no vehicles)? Please state your opinion with clear reason.

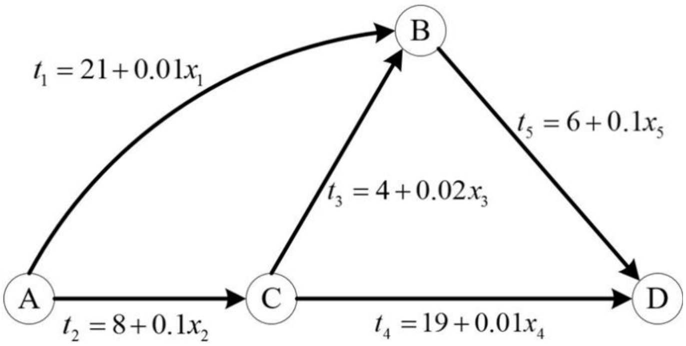

Figure 2. A simple network with one O-D pair