Executive Summary

ComfortHealth, a Canadian home care agency that provides medical care to patients in their homes, has experienced a rise in demand due to the growing aging population and government incentives. To cope with this expansion, the agency hired a data consultant to create a comprehensive three-year annual service plan that includes hiring and allocating the workforce and medical facilities. The consultant used a linear regression model to forecast demand for the next three years and an optimization model to find the optimal solution for facility allocation and staff planning. Assuming the company has flexibility over the number of patients they accept or the amount of demand they satisfy. The optimal solution suggested that ComfortHealth open centers E and F and allocate all existing staff (total of 560) to these two centers, with no new hires for all three years (284 HPs to center E and 276 HPs to center F). This solution would generate a profit of 6.8 million dollars due to government reimbursement of $42 for every hour of service, which is more than enough to cover the hourly pay for healthcare professionals ($37.85 per hour of service) and the fact that the model would try to minimize travelling expenses. Specifically, the model follows the "focus-on-own-region" strategy, in which ComfortHealth would allocate all or most resources to first serve people in the center's region (i.e., the distance between the center and region equals zero). By looking at the distance table, centers E and F would first serve people from North Toronto and Mississauga East, respectively. This strategy would also apply to centers A, B, and D. Still; the model chose E and F because the operating costs for E and F were much lower than other existing centers, even after incurring a one-time opening cost for the first year.

If the model changes the constraint such that it has to accept all patients, ComfortHealth would have to operate all centers except C and hire around 1,419 new healthcare professionals in the first year, costing the company around 21 million dollars. This is not viable for ComfortHealth, and to make it viable, the model must weaken the demand constraint. After trial and error, setting the model to ensure that at least 44% of the demand for each region is satisfied would yield a viable solution. This solution would generate a total cost of around zero, which means there is no profit or loss for ComfortHealth over the three years. As a healthcare agent, ComfortHealth's purpose should be to provide services to as many people as possible while maintaining service quality and a sustainable operation.

Nevertheless, the company would satisfy more demand (around 44% than the full accept/reject flexibility case (only 27% of demand was fulfilled). ComfortHealth would only have to operate four centers (A, B, E, and F) and hire around 400 new healthcare professionals.

The plan's success depends on the accuracy of the forecasted demand, which the model assumed to have a steady, quadratic growth rate across all regions. However, the demand could change due to various reasons, such as changes in the aging population or government policies. Therefore, ComfortHealth must continuously monitor and update its demand forecast to adjust its staffing and facility allocation accordingly. The company cannot adjust the allocation of NPs from one center to the other after the first year. Still, it could change the capacity allocation of each center to its dedicated regions. ComfortHealth could reference the optimal solution from the 44% demand satisfaction model.

Introduction

ComfortHealth is a reputable home care agency based in Canada, dedicated to providing quality medical care and support to patients within the comfort of their homes. The agency has witnessed a significant increase in demand for its services due to a growing ageing population and extensive government incentives, expanding the company's reach.

The importance of home care in Canada cannot be overstated, as it has proven to be a strategic area that contributes to the overall improvement of the quality of care provided, postpones institutionalization, and significantly reduces the operational costs of hospitals.

As a data consultant hired by the firm, my primary objective is to create a comprehensive three-year annual service plan for ComfortHealth, detailing the hiring and allocation of the workforce and medical facilities. This plan will also outline how the agency intends to serve the various geographical areas covered by ComfortHealth during the next three years.

To ensure the effective execution of the annual service plan, a thorough analysis of the current and projected demand for home care services must be conducted. This analysis will serve as a guide to determine the number of healthcare professionals needed and the optimal allocation of medical facilities.

The following analysis will show the process of obtaining the model and optimal solution.

Analysis

Demand Forecast:

Before building the model, we would need to forecast of the demand for 2023~2025. **Figure 1** (Left) below was constructed using historical demand from 2013~2022. Based on this graph, the demand growth is quadratic and steady (a curvy line) with respect to time; moreover, all regions are growing at the same rate. Assuming the demand for the future would grow at a similar rate as the previous years. Demand for 2023~2025 could be forecasted using a linear regression model with past data; the prediction is plotted below (**Figure 1 - Right**).

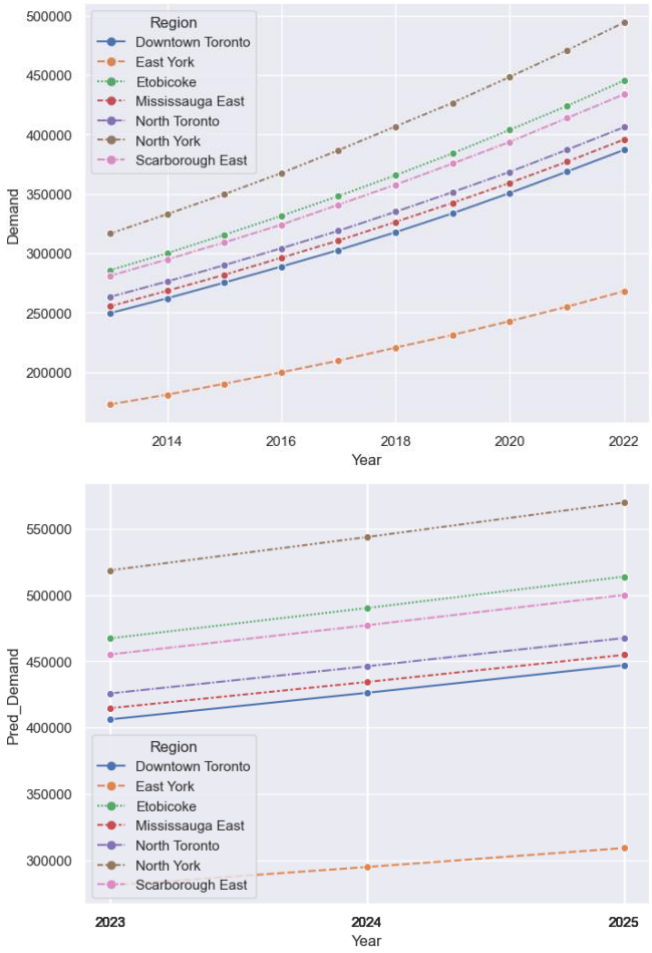

***Figure 1**: (Left) Historical Demand 2013~2022; (Right) Forecasted Demand 2023~2025*

Facility & Staff Planning:

Serval models were built using forecasted demand to help us answer managerial questions. The following analysis assumes that the realized demand would be the same as the forecasted demand calculated previously using a linear regression model. Please refer to **Appendix A** for the mathematical representation of the model and assumption used.

Case 1: Full accept/decline flexibility (model output: Appendix B)

The first model assumes that ComfortHealth does not have to accept all service requests and is free to choose who to serve. In this case, the optimal solution from the model has a total cost of -6,803,096.22 dollars; in other words, ComfortHealth made a profit of round 6.8 million dollars. Profit would exist because of the government reimbursement ($42 for every hour of service), which would be more than enough to cover the hourly pay for HPs ($37.85

per hour of service, growing at 2.5% yearly) if there is no travelling expense. Note that the total service expense for every hour of service equals to:

$$c_{ijt} = 37.85 * 1.025^{t-1} + 0.25 * d_{ij} - 42$$

where d represents the distance between center i and region j. If d_{ij}= 0, and t = 1, this equation would yield a negative number: -4.15 (=37.85 – 42) which means government reimbursement would cover the pay for NPs. This means ComfortHealth would be able to profit for every hour of service provided if the distance between a center and a region is less than 16.6 km (= 4.15/0.25). To clarify, we would define "center i locates in region j" if the distance between center i and region j equals zero (dij = 0).

Thus, to minimize the expense or maximize the profit (from government reimbursement), the model would try to allocate all or most of the capacity to only serve people in the region where the center is located (i.e. dij = 0). Based on this assumption, the model suggests ComfortHealth open centers E and F and allocate all existing staff (total of 560) to these two centers (allocate 284 HPs to center E and, 276 HPs to center F).

The optimal solution reflects this situation, where center E only serves people from North Toronto (since center E is located in North Toronto), similar to center F, which only serves Mississauga East. Note that center E also served Downtown Toronto (the nearest region after North Toronto); because center E has satisfied all the demand from North Toronto, it allocates leftover capacity to Downtown Toronto. (Please reference Figure 2 and Table 1 in the Appendix)

This "focus on own region first" strategy could also be applied to centers A, B and D (Note center C is not included because it lives outside of all regions). Why did the model choose only to open and operate E and F? If we break down the total cost of the optimal solution (figure 5), other than total service revenue and cost, the model will also have to consider operating cost and opening cost. Note that if we open centers E and F, we will incur a one-time opening cost for the first year which would boost total cost for the first year, but the operating cost for E and F is much lower compared to existing centers. Thus, over the three years term, the total cost from centers E and F would be lower compared to any combination of other existing centers (A, B, and D).

Finally, the optimal solution for this case did not plan to hire any new NPs over the three years. This is true because hiring a new employee costs more than the profit they could generate over three years.

Case 2: Accepts all patients (model output: Appendix C)

Rejecting most of the demand and focusing only on two regions might damage the company's reputation. ComfortHealth would try to fulfill all demands/requests for this case.

Recall that the previous model suggests not hiring new employees and restricting providing service to only regions where centers are located. If we change the constraint such that the model would have to satisfy all demand from all regions, the optimal solution will have a huge change. To meet all service requests, ComfortHealth would have to operate all centers except C since C does not locate in any region (distance > 0 between any region and center C), and almost every center would have to hire around 300 people in the first year (total 1419 new hires, hiring cost = $21,285,000).

The allocation of the initial 560 works changed from focusing on centers E and F to A and E (233 for A, 323 for E, and 4 for C). The model chose this because both A and E have their dedicated regions (distance = 0), and the average distance between centers A or E to other regions is much lower than other centers. (Appendix)

Similar to the solution of flexible demand, the model will first try to allocate all/majority capacity to fulfill demand in the region which the center locates, then the next closest region and so on. For example, center E tried to fulfill demands from North Toronto (distance = 0 km) and Downtown Toronto (distance = 6.37 km) first, then it would get to Etobicoke (distance = 23.2).

To fulfill all demand, the cost becomes positive this time, which means ComfortHealth would lose around 10 million dollars over the three years (Appendix). If we break down the total cost for this optimal solution, we can see that the service revenue is high enough to cover service, operating, and opening costs but not the hiring cost. As mentioned above, hiring 1419 new NPs would cost around 21 million dollars in the first year, which is certainly not viable because the service revenue is more than the service cost indicating that the model can still profit from government reimbursement by making sure that most resources for each center would be allocated to the nearest region first. Thus, if we extend the year of

operations, the effect of this one-time payment for hiring in year one would be smaller, which means the longer the new NPs work, the more profit they could bring to the company.

However, generating profit from government reimbursement would only be possible if 1. the payment of the NPs does not grow over time and 2. the demand would increases as expected. First, by assumption, the hourly pay for NPs increases over time by 2.5% annually; in year five, this payment would increase to $42.82 [= 37.85 * (1.025)^5]. Thus, starting from year five and forward, government reimbursement would not be able to cover any cost. Second, the demand forecast could be more stable, which means the realized demand may be lower than expected. Therefore, it is not recommended for ComfortHealth to invest 21 million dollars at be beginning of year one since the cost would always be higher than the profit from both short-term and long-term perspective. To make this viable we could change the constraint for the demand in the model to make sure that at least some percentage of the demand for each region is satisfied. By setting the constraint such that more than 44% of demand for each region should be satisfied, the total cost is now closer to 0 (check **Appendix D** for model output detail). This means that over the three-year period ComfortHealth would not make nor loss any money, however, satisfying much more demand compared to full accept/reject flexibility case (~ 27% of demand was fulfilled). For this particular optimal solution, ComfortHealth would only have to operate 4 centers (A, B, E, F) and only hire around 400 new NPs.

Sensitivity Analysis on Demand Forecasted

Everything that we have analyzed so far depends on the forecast of the demand, which was assumed to be the linear (quadratic) with respect to time. NPs would not be reallocated after the first year, but the capacity from center i to region j is flexible. In some cases, if the realized demand differs from the expected demand for some regions, ComfortHealth could adjust accordingly to reduce cost. For example, in the accept-all-patients case, center E is located in North Toronto (with a distance = 0); other than fulfilling North Toronto, it also has to provide capacity for Downtown Toronto and Etobicoke. Center E and Downtown Toronto are 6.37 km and 23.2 km between E and Etobicoke. Based on our previous calculation, if the distance between a center and a region is less than 16 km, then ComfortHealth could profit from government reimbursement; thus, if the realized demand for Etobicoke decreases while realized demand for Downtown Toronto and North Toronto increase, ComfortHealth could allocate more capacity in regions like Downtown Toronto and North Toronto to reduce cost

(by increasing profit). However, this would be the best case for ComfortHealth. The realized demand may be lower than expected for regions with a distance of fewer than 16 km but higher for regions that are hard to reach. To accommodate these uncertainties, it would be better to use the optimal solution from the third model in which at least 44% of the demand is satisfied in each region.

Conclusion

The optimal solutions for cases one and two are not viable. As a healthcare agent, ComfortHealth's purpose should be to provide services to as many people as possible while maintaining service quality and a sustainable operation. ComfortHealth should not try to maximize its profit which is reflected in the optimal solution of case one, and it should not try to make itself bankrupt, which happens if it follows the optimal solution from case two. Nevertheless, the company would satisfy more demand (around 44% than the full accept/reject flexibility case (only 27% of demand was fulfilled).

Appendix

***Figure 2**: Approximated Map*

	Center A	Center B	Center C	Center D	Center E	Center F
Downtown Toronto	25.77	29.88	19.1	29.88	6.37	18.83
East York	2.24	16.04	22.45	16.04	20.93	30.7
Etobicoke	25.72	31.11	26.93	31.11	23.2	25.64
Mississauga East	28.65	33.93	21.58	0	20.65	0
North Toronto	11.64	25.05	22.99	25.05	0	20.65
North York	0	22.98	20.52	22.98	11.64	28.65
Scarborough East	19.76	0	14.86	26.78	25.89	17.01
Average Distance	16.25	22.71	21.20	21.69	15.53	20.21

***Table 1:** Distance between regions and centers*

Appendix A

Model:

Parameters:

f_i: $Operating\ expense\ for\ center\ i$

O_i: $Opening\ expense\ for\ center\ i$

K_i: $Center\ capacity\ for\ center\ i$

d_{ij}: $Distance\ between\ center\ i\ and\ region\ j$

Variables:

y_{it}: $whether\ to\ open\ center\ i\ at\ time\ t$

x_{ijt}: $number\ of\ hours\ allocated\ from\ center\ i\ to\ region\ j\ at\ time\ t$

h_{it}: $number\ of\ new\ hires\ for\ center\ i\ at\ time\ t$

w_{it}: $number\ of\ existing\ workers\ for\ center\ i\ at\ time\ t$

Objective Function:

$$Max \sum_{t=1}^{T}\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ijt}x_{ijt} + \sum_{t=1}^{T}\sum_{i=1}^{n} f_i y_{it} + \sum_{i=1}^{n} O_i y_{i1} + \sum_{t=1}^{T}\sum_{i=1}^{n} h_{it} * 15{,}000$$

$$where: c_{ijt} = 37.85 * 1.025^{t-1} + 0.25 * d_{ij} - 42$$

Constraints:

- Constriant for the variables bounds:

$$x_{ijt}, h_{it}, w_{it} \geq 0 \quad for\ i = 1, \dots, n; \quad for\ j = 1, \dots, m; \quad for\ t = 1, \dots, T$$
$$h_{it} \leq 300 \quad for\ i = 1, \dots, n; \quad for\ t = 1, \dots, T$$
$$y_{it} \in \{0,1\} \quad for\ i = 1, \dots, n; \quad for\ t = 1, \dots, T$$

- Constraint for Center A, B, C, D must be open:

$$y_{i1} = 1 \quad for\ i = 1, \dots, n$$

- Constraint for opened centers in period 1 must remain open:

$$y_{it} = y_{i(t-1)} \quad for\ i = 1, \dots, n; \quad for\ t = 2, \dots, T$$

- Constriant for Demand - all demand must be satisifed:

$$\sum_{i=1}^{n} x_{ijt} = D_{jt} \qquad for\ j = 1, \dots, m\ ; \quad for\ t = 1, \dots, T$$

- Constraint for Center Capacity – all centers would not pass it's maximum capacity

$$\sum_{j=1}^{m} x_{ijt} \leq K_i y_{it} \qquad for\ j = 1, \dots, m\ ; \quad for\ t = 1, \dots, T$$

- Constraint for HP's (workers) Capacity – all centers should not surpass their maximum working hour

$$\sum_{j=1}^{m} x_{ijt} \leq 6 * 250 * (h_{it} + w_{it}) \qquad for\ i = 1, \dots, n\ ; \quad for\ t = 1, \dots, T$$

- Constraint for total number of worker that can be allcated in the first year:

$$\sum_{i=1}^{n} w_{i1} = 560 \qquad for\ i = 1, \dots, n$$

- Constraint for the flow of workers (workers can not reallocated to other places after year 1)

$$w_{it} = w_{i(t-1)} + h_{i(t-1)} \quad for\ i = 1, \dots, n; \quad for\ t = 2, \dots, T$$

Appendix B

	In dollar $
total_operating_cost	-900000.00
total_opening	-350000.00
total_service_cost	-97786903.78
total_hiring_cost	-0.00
total_service_revenue	105840000.00
total_Profit_/_Cost	6803096.22

	Center	Open_Y	Year	num_HPs	num_new_hire	East_York	Etobicoke	Mississauga_East	North_Toronto	North_York	Scarborough_East	Downtown_Toronto
0	Center A	0.00	2023	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	Center B	0.00	2023	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	Center C	0.00	2023	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	Center D	0.00	2023	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	Center E	1.00	2023	284	0	0.00	0.00	0.00	425697.00	0.00	0.00	303.00
5	Center F	1.00	2023	276	0	0.00	0.00	414000.00	0.00	0.00	0.00	0.00
6	Center A	0.00	2024	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	Center B	0.00	2024	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	Center C	0.00	2024	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	Center D	0.00	2024	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	Center E	1.00	2024	284	0	0.00	0.00	0.00	426000.00	0.00	0.00	0.00
11	Center F	1.00	2024	276	0	0.00	0.00	414000.00	0.00	0.00	0.00	0.00
12	Center A	0.00	2025	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
13	Center B	0.00	2025	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
14	Center C	0.00	2025	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15	Center D	0.00	2025	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
16	Center E	1.00	2025	284	0	0.00	0.00	0.00	426000.00	0.00	0.00	0.00
17	Center F	1.00	2025	276	0	0.00	0.00	414000.00	0.00	0.00	0.00	0.00

Appendix C

	In dollar $
total_operating_cost	-3405000.00
total_opening	-350000.00
total_service_cost	-374430814.60
total_hiring_cost	-24225000.00
total_service_revenue	392361984.00
total_Profit_/_Cost	-10048830.60

	Center	Open_Y	Year	num_HPs	num_new_hire	East_York	Etobicoke	Mississauga_East	North_Toronto	North_York	Scarborough_East	Downtown_Toronto
0	Center A	1.00	2023	233	300	281010.00	0.00	0.00	0.00	518490.00	0.00	0.00
1	Center B	1.00	2023	4	300	67.00	606.00	0.00	0.00	0.00	455085.00	0.00
2	Center C	0.00	2023	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	Center D	1.00	2023	0	221	0.00	0.00	331500.00	0.00	0.00	0.00	0.00
4	Center E	1.00	2023	323	300	0.00	102676.00	0.00	425697.00	0.00	0.00	406127.00
5	Center F	1.00	2023	0	298	0.00	363936.00	83064.00	0.00	0.00	0.00	0.00
6	Center A	1.00	2024	533	26	294832.00	0.00	0.00	0.00	543579.00	0.00	0.00
7	Center B	1.00	2024	304	29	0.00	0.00	0.00	0.00	0.00	477079.00	0.00
8	Center C	0.00	2024	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	Center D	1.00	2024	221	0	0.00	0.00	244198.00	0.00	0.00	0.00	0.00
10	Center E	1.00	2024	623	114	0.00	233174.00	0.00	446171.00	0.00	0.00	426155.00
11	Center F	1.00	2024	298	0	0.00	256882.00	190118.00	0.00	0.00	0.00	0.00
12	Center A	1.00	2025	559	27	309116.00	0.00	0.00	0.00	569558.00	0.00	0.00
13	Center B	1.00	2025	333	0	0.00	0.00	0.00	0.00	0.00	499500.00	0.00
14	Center C	0.00	2025	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15	Center D	1.00	2025	221	0	0.00	0.00	330832.00	0.00	0.00	0.00	0.00
16	Center E	1.00	2025	737	0	0.00	191095.00	0.00	467419.00	0.00	0.00	446986.00
17	Center F	1.00	2025	298	0	0.00	322654.00	123937.00	0.00	0.00	409.00	0.00

Appendix D

	In dollar $
total_operating_cost	-2550000.00
total_opening	-350000.00
total_service_cost	-173408656.74
total_hiring_cost	-6075000.00
total_service_revenue	182385000.42
total_Profit_/_Cost	1343.68

	Center	Open_Y	Year	num_HPs	num_new_hire	East_York	Etobicoke	Mississauga_East	North_Toronto	North_York	Scarborough_East	Downtown_Toronto
0	Center A	1.00	2023	5	255	124517.11	0.00	0.00	0.00	265482.89	0.00	0.00
1	Center B	1.00	2023	0	148	0.00	0.00	0.00	0.00	0.00	222000.00	0.00
2	Center C	0.00	2023	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	Center D	0.00	2023	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	Center E	1.00	2023	420	2	0.00	206977.57	0.00	246108.17	0.00	0.00	179914.26
5	Center F	1.00	2023	135	0	0.00	0.00	202500.00	0.00	0.00	0.00	0.00
6	Center A	1.00	2024	260	0	130610.58	0.00	0.00	0.00	259389.42	0.00	0.00
7	Center B	1.00	2024	148	0	0.00	0.00	0.00	0.00	0.00	222000.00	0.00
8	Center C	0.00	2024	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	Center D	0.00	2024	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	Center E	1.00	2024	422	0	0.00	217094.81	0.00	227118.53	0.00	0.00	188786.66
11	Center F	1.00	2024	135	0	0.00	0.00	202500.00	0.00	0.00	0.00	0.00
12	Center A	1.00	2025	260	0	136938.39	0.00	0.00	0.00	253061.61	0.00	0.00
13	Center B	1.00	2025	148	0	0.00	0.00	0.00	0.00	0.00	222000.00	0.00
14	Center C	0.00	2025	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15	Center D	0.00	2025	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
16	Center E	1.00	2025	422	0	0.00	227590.81	0.00	207394.40	0.00	0.00	198014.80
17	Center F	1.00	2025	135	0	0.00	0.00	202500.00	0.00	0.00	0.00	0.00