SGN-12006 Basic Course in Image and Video Processing

EXERCISE 7

19.10.2015-21.10.2015

This exercise consists of both lab exercises and homework. Complete the lab exercises and present your results for the TA. Prerequisite for submitting the homework is attendance in an exercise session. Homework should be submitted only online using Moodle2.

Follow the naming format 'ExN_surname_ID.pdf' (N is the number of exercise). Also please clearly write down your full name and student number in the document. The homework report should be no more than 1 page long and it should be done individually (no pairs allowed). Questions on this exercise should be addressed to TA's email address: (firstname.surname@tut.fi).

Lab exercises

Based on the experiences from the previous years, Task 1 may be time consuming. The code required is rather short, but you should concentrate on the given information to understand it and to be able to implement it. Therefore, you may consider finishing the other tasks first to not run out of time.

1) Haar Transform (2 points)

The Haar transform can be expressed in the matrix form as $\mathbf{T} = \mathbf{HIH}^\mathbf{T}$ (inverse transform $\mathbf{H}^\mathbf{T}\mathbf{TH}$) where **I** is an *NxN* image matrix, **H** is an *NxN* transformation matrix and **T** is the resulting *NxN* transform.

For Haar transform, the transformation matrix **H** contains the Haar basis functions $h_k(x)$. They are defined on a continuous, closed interval, $x \in [0, 1]$ for $k = 0,1,2 \ldots, N-1$ where $N = 2^n$.

The integer k is defined as

$$k = 2^p + q - 1$$

where p, q are integers: $0 \le p \le n-1$; $q = 0$ or 1 for $p = 0$ and $1 \le q \le 2^p$ for $p \neq 0$.

The Haar functions are defined as

$$h_0(x) = h_{00}(x) = \frac{1}{\sqrt{N}}, x \in [0,1]$$

$$h_k(x) = h_{pq}(x) = \frac{1}{\sqrt{N}} \begin{cases} 2^{p/2}, & \frac{q-1}{2^p} \le x < \frac{q-1/2}{2^p} \\ -2^{p/2}, & \frac{q-1/2}{2^p} \le x < \frac{q}{2^p} \\ 0, & \text{otherwise for } x \in [0,1] \end{cases}$$

The i^{th} row of an *NxN* Haar transform matrix contains the elements of $h_i(x)$ for $x = 0/N, 1/N, 2/N, \ldots, (N-1)/N$. In other words, $H(i, j) = h_i(j/N)$.

Your task is to write a Matlab function `haarmtx(N)` that produces the Haar transform matrix **H** for the given *N* and to show the 16*x*16 transform matrix to the TA.

Hint: Start with creating vectors *k*, *p* and *q*, where *p* = floor(log2(*k*)), *p*(0) = 0.
For N=4, vectors *k*, *p* and *q* along with the transformation matrix $\mathbf{H}_4$ are given below to check your answers.

k	*p*	*q*
0	0	0
1	0	1
2	1	1
3	1	2

$$\mathbf{H}_4 = \frac{1}{\sqrt{4}}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ \sqrt{2} & -\sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & -\sqrt{2} \end{bmatrix}$$

2) Walsh-Hadamard Transform (2 points)

For Hadamard transform, $\mathbf{H}=\mathbf{H}^T$ so the transform can be expressed in the matrix form as **T = HIH** (inverse transform **HTH**) where **I** is an *NxN* image matrix, **H** is an *NxN* transformation matrix and **T** is the resulting *NxN* transform. The same is true with Walsh-Hadamard transform matrix **W**.

For Hadamard transform, the transformation matrix **H** is an *NxN* matrix where $N = 2^n$ and $N = 1,2,\ldots$ defined as

$$\mathbf{H}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \mathbf{H}_n = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{H}_{n-1} & \mathbf{H}_{n-1} \\ \mathbf{H}_{n-1} & -\mathbf{H}_{n-1} \end{bmatrix}$$

Walsh transform is equal to Hadamard transform, but the rows of the transform matrix **H** are reordered so that on the top of the transform matrix W is the row of the Hadamard transform matrix with the smallest number of sign changes (from *i* to -*i* and from -*i* to *i*) and below rows are ordered in increasing order of sign changes.

Write a MATLAB function `walsh_hadamard(N)` that produces the Hadamard transform matrix **H** for the given *N* (do not use MATLAB's `hadamard` function) and then reorders the rows to a Walsh-Hadamard transform matrix **W**. Show the 16*x*16 transform matrix to the TA.

Hint: You may use the following code to fine out the number of sign changes on each line

```
p = sum([zeros( size(H,1),1) H] .* [H zeros( size(H,1),1) ] < 0,2);
```

3) Compression with DFT, DCT, Haar and Walsh-Hadamard Transforms (3 points)

a) Load the image *cameraman.tif* and perform on it the DFT, DCT, Haar and Walsh-Hadamard transforms (each transform is applied on the original image). Display each transform and try to make the characteristics of the transforms visible in the images. Note that different display methods are necessary in different cases.

b) Out of each transform select only a quarter of the coefficients ($N^2/4$ out of N^2, where $N \times N$ is the dimension of the image) so that they contain most of the image energy. Do not select individual coefficients, but a continuous square section. Put the rest of the coefficients to zero. Display.

c) Reconstruct the images using the inverse transform. Visually compare the reconstructed images. Compute also the mean square error between the reconstructed images and the original.

Hints: DFT: exercise7, DCT: `dct2, idct2`, Haar: taks1, Walsh-Hadamard: task2

Homework:

4) Singular-Value-Decomposition (SVD) (1 points)

Describe the relation between the rank of a matrix, A, and its singular values? How Singular Value Decomposition (SVD) can be used for image compression or noise reduction?

5) Karhunen-Loeve Transform (KLT) (2 points)

Briefly describe how SVD can be used to find the eigenvalues of a matrix, A? How KLT is used to map data into a lower dimension?