How can we apply the Fourier transform to a 2D signal f(x,y) by using 1D FT?

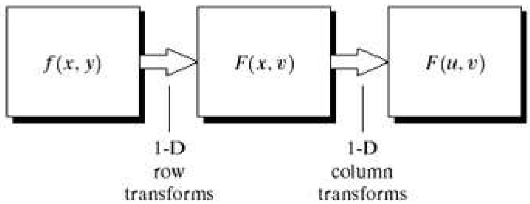


FIGURE 4.35 Computation of the 2-D Fourier transform as a series of 1-D transforms.

Separability

The discrete Fourier transform in Eq. (4.2-16) can be expressed in the separable form

$$F(u, v) = \frac{1}{M}\sum_{x=0}^{M-1} e^{-j2\pi ux/M} \frac{1}{N}\sum_{y=0}^{N-1} f(x, y)e^{-j2\pi vy/N} = \frac{1}{M}\sum_{x=0}^{M-1} F(x, v)e^{-j2\pi ux/M} \qquad (4.6\text{-}14)$$

where

$$F(x, v) = \frac{1}{N}\sum_{y=0}^{N-1} f(x, y)e^{-j2\pi vy/N}. \qquad (4.6\text{-}15)$$

Basics of filtering in the frequency domain

Filtering in the frequency domain is straightforward. It consists of the following steps:

1. Multiply the input image by $(-1)^{x+y}$ to center the transform, as indicated in Eq. (4.2-21).
2. Compute $F(u, v)$, the DFT of the image from (1).
3. Multiply $F(u, v)$ by a *filter* function $H(u, v)$.
4. Compute the inverse DFT of the result in (3).
5. Obtain the real part of the result in (4).
6. Multiply the result in (5) by $(-1)^{x+y}$.

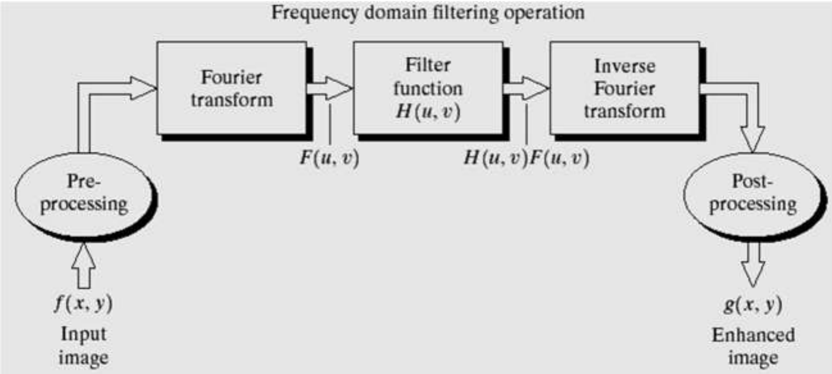


FIGURE 4.5 Basic steps for filtering in the frequency domain.

Some basic filters and their properties

$$H(u, v) = \begin{cases} 0 & \text{if } (u, v) = (M/2, N/2) \\ 1 & \text{otherwise.} \end{cases} \quad (4.2\text{-}29)$$

All this filter would do is set $F(0, 0)$ to zero and leave all other frequency components of the Fourier transform untouched, as desired. The processed image

The filter just discussed is called a *notch filter* because it is a constant function with a hole (notch) at the origin. The result of processing the image in

Low and high pass filters:

gories. A filter that attenuates high frequencies while "passing" low frequencies is called a *lowpass filter*. A filter that has the opposite characteristic is appropriately called a *highpass filter*. We would expect a lowpass-filtered image to have less sharp detail than the original because the high frequencies have been attenuated. Similarly, a highpass-filtered image would have less gray level variations in smooth areas and emphasized transitional (e.g., edge) gray-level detail. Such an image will appear sharper.

Low frequencies in the Fourier transform are responsible for the general gray-level appearance of an image over smooth areas, while high frequencies are responsible for detail, such as edges and noise. These ideas are discussed in more

FIGURE 4.6 Result of filtering the image in Fig. 4.4(a) with a notch filter that set to 0 the $F(0, 0)$ term in the Fourier transform.

how is this image displayed if the average value is 0?!

the $F(0, 0)$ term has been set to zero. This is typical of highpassed results, and a procedure often followed is to add a constant to the filter so that it will not completely eliminate $F(0, 0)$. The result of using this procedure is shown in read it from book

Guassian filters:

cerned with complex numbers. In addition, Gaussian curves are intuitive and easy to manipulate. (2) These functions behave reciprocally with respect to one another. In other words, when $H(u)$ has a broad profile (large value of σ), $h(x)$ has a narrow profile, and vice versa. In fact, when σ approaches infinity, $H(u)$ tends toward a constant function and $h(x)$ tends toward an impulse. This is ex-

Smoothing frequency-domain filters:

1-ideal lowpass filters1:

Ideal low pass is a filter which passes all the low frequencies within a bound while attenuating the high frequencies out of that bound.

The simplest lowpass filter we can envision is a filter that "cuts off" all high-frequency components of the Fourier transform that are at a distance greater than a specified distance D_0 from the origin of the (centered) transform. Such a filter is called a two-dimensional (2-D) *ideal lowpass filter* (ILPF) and has the transfer function

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \leq D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases} \tag{4.3-2}$$

where D_0 is a specified nonnegative quantity, and $D(u,v)$ is the distance from point (u,v) to the center of the frequency rectangle. If the image in question is

$$D(u,v) = \left[(u - M/2)^2 + (v - N/2)^2\right]^{1/2}.$$

Note that the narrower the filter in the frequency domain is the more severe are the blurring and ringing!

The cross section of ILPF in frequency domain looks like a box filter, a cross section of the corresponding spatial filter has the shape of a since function. Filtering in spatial domain is done by convolving h(x,y) with the image. convolving a sinc function with an impulse copies the sinc at the location of the impulse. the centre lobe of the sinc is responsible for blurring and the outer smaller lobes are responsible for ringing. (for below).

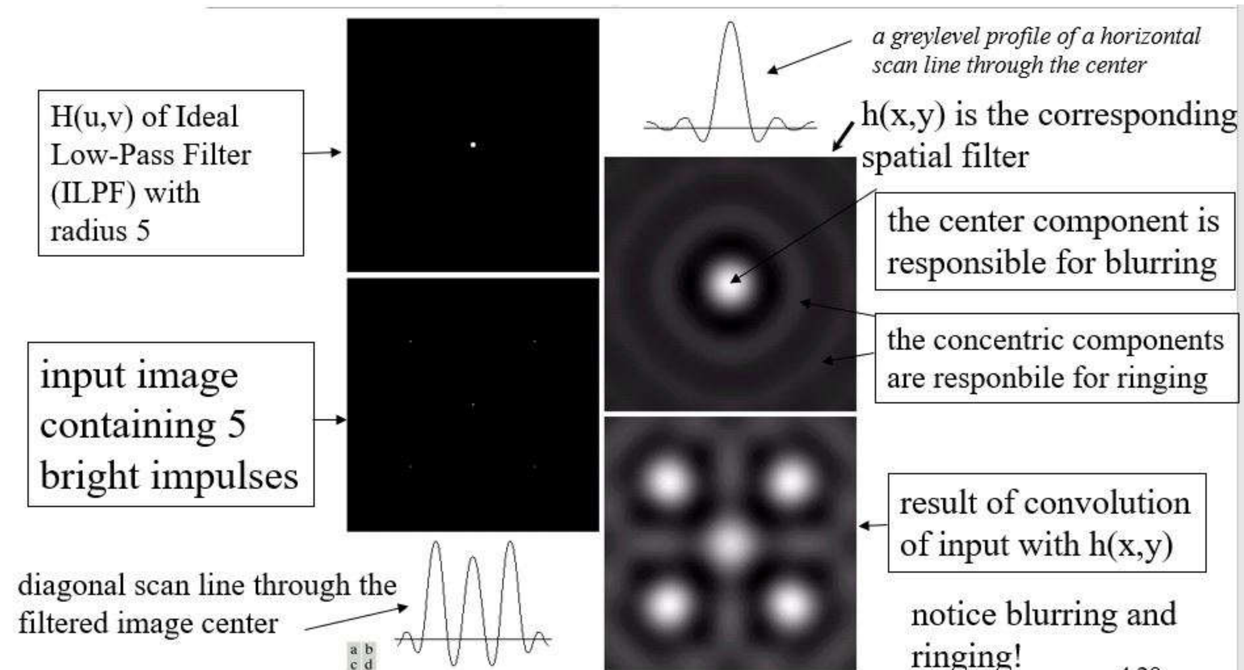


FIGURE 4.13 (a) A frequency-domain ILPF of radius 5. (b) Corresponding spatial filter (note the ringing). (c) Five impulses in the spatial domain, simulating the values of five pixels. (d) Convolution of (b) and (c) in the spatial domain.

2-butterworth lowpass filter

How to achieve blurring with little or no ringing? Butterworth LPF is one technique

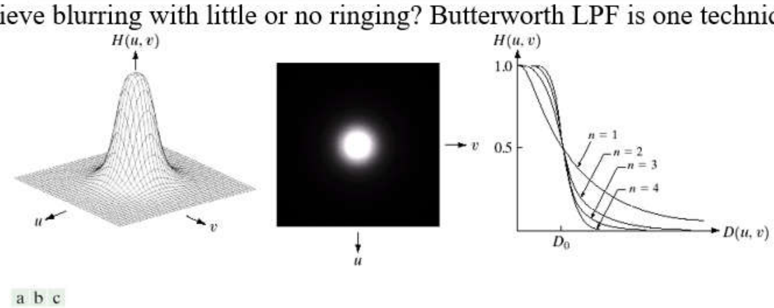


FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

Transfer function of a BLPF of order n and cut-off frequency at distance D_0 (at which H(u,v) is at ½ its max value) from the origin:

$$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}} \quad \text{where} \quad D(u,v) = [(u-M/2)^2 + (v-N/2)^2]^{1/2}$$

D(u,v) is just the distance from point *(u,v)* to the center of the FT 4.29

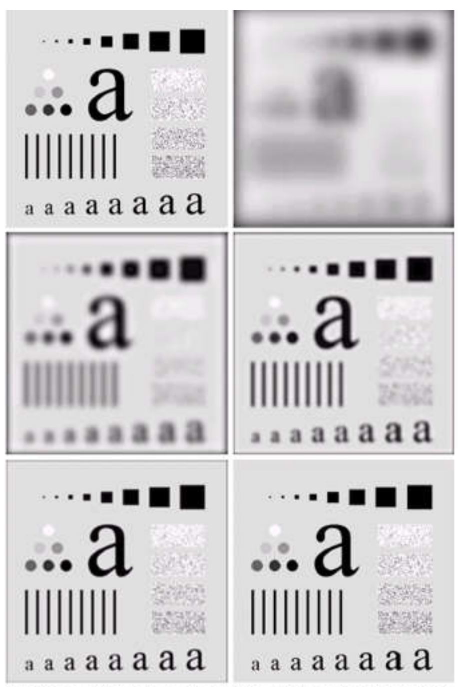
BLPF represents a transition between the sharpness of ideal filter and smoothness of the gaussian filter.

Unlike the ILPF, the BLPF transfer function does not have a sharp discontinuity that establishes a clear cutoff between passed and filtered frequencies.

A Butterworth filter of order 1 has no ringing. Ringing generally is imperceptible in filters of order 2, but can become a significant factor in filters of higher order. Figure 4.16 shows an interesting comparison between the *spatial*

ters are identical. In general, BLPFs of order 2 are a good compromise between effective lowpass filtering and acceptable ringing characteristics.

Filtering with BLPF with **n=2** and increasing cut-off as was done with the Ideal LPF



a b
c d
e f

FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

Note the smooth transition in blurring achieved as a function of increasing cutoff but no ringing is present in any of the filtered images with this particular BLPF (with n=2)

this is attributed to the smooth transition bet low and high frequencies

3-Gaussian lowpass filter:

The 2-D Gaussian low-pass filter (GLPF) has this form:

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2} \qquad D_0 = \sigma$$

σ is a measure of the spread of the Gaussian curve
recall that the inverse FT of the GLPF is also Gaussian, i.e. it has no ringing!
at the cutoff frequency D_0, H(u,v) decreases to 0.607 of its max value.

4.32

Results of GLPFs

Remarks:

1. Note the smooth transition in blurring achieved as a function of increasing cutoff frequency.

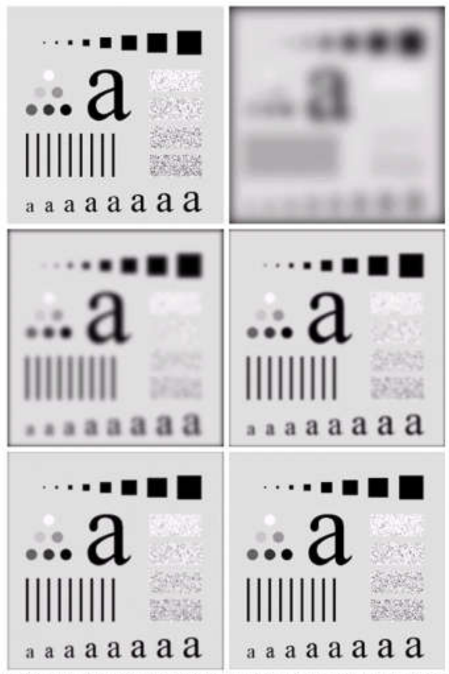


FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

a b
c d
e f

2. Less smoothing than BLPFs since the latter have tighter control over the transitions bet low and high frequencies.

The price paid for tighter control by using BLP is possible ringing.

3. No ringing!

4.33

Sharpening Frequency Domain Filters

the high-frequency components of its Fourier transform. Because edges and other abrupt changes in gray levels are associated with high-frequency components, image sharpening can be achieved in the frequency domain by a *high-pass filtering* process, which attenuates the low-frequency components without disturbing high-frequency information in the Fourier transform. As in Section

4.4.1 Ideal Highpass Filters

A 2-D ideal highpass filter (IHPF) is defined as

$$H(u, v) = \begin{cases} 0 & \text{if } D(u, v) \leq D_0 \\ 1 & \text{if } D(u, v) > D_0 \end{cases}$$

Eqs. (4.4-1) and (4.4-2). As intended, this filter is the opposite of the ideal lowpass filter in the sense that it sets to zero all frequencies inside a circle of radius D_0 while passing, without attenuation, all frequencies outside the circle. As in

Ideal high-pass filters enhance edges but suffer from ringing artefacts, just like Ideal LPF.

Ideal HPFs are expected to suffer from the same ringing effects as Ideal LPF

2-Butterworth highpass filter:

$$H(u, v) = \frac{1}{1 + \left[D_0 / D(u, v)\right]^{2n}}$$

1.BHPF to behave smoother than IHPS

2.

ing the smaller objects is comparable. The transition into higher values of cutoff frequencies is much smoother with the BHPF.

3.gaussian highpass filter:

$$H(u, v) = 1 - e^{-D^2(u, v)/2D_0^2}$$

BHPF, we show in Fig. 4.26 comparable results using GHPFs. As expected, the results obtained are smoother than with the previous two filters. Even the filtering of the smaller objects and thin bars is cleaner with the Gaussian filter.

4. laplacian

$$H(u,v) = -\left[(u - M/2)^2 + (v - N/2)^2\right]$$

High Frequency Emphasis Filtering

- How to emphasise more the contribution to enhancement of high-frequency components of an image and still maintain the zero frequency?

$$H_{hfe}(u,v) = a + bH_{hp}(u,v) \quad where \quad a \geq 0 \; and \; b > a$$

- Typical values of a range in 0.25 to 0.5 and b between 1.5 and 2.0.
- When $a=A-1$ and $b=1$ it reduces to high-boost filtering
- When $b>1$, high frequencies are emphasized.

Butterworth high-pass and histogram equalization

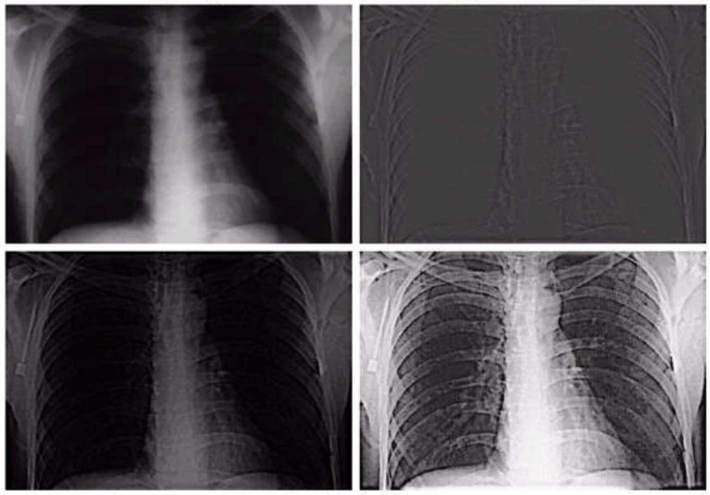


FIGURE 4.30 (a) A chest X-ray image. (b) Result of Butterworth highpass filtering. (c) Result of high-frequency emphasis filtering. (d) Result of performing histogram equalization on (c). (Original image courtesy Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)

X-ray images cannot be focused in the same manner as a lens, so they tend to produce slightly blurred images with biased (towards black) greylevels → complement freq dom filtering with spatial dom filtering!

4.16

Question in first exam

Recall that the image is formed through the multiplicative illumination-reflectance process:

$$f(x,y) = i(x,y)\,r(x,y)$$

where *i(x,y)* is the illumination and *r(x,y)* is the reflectance component

Question: how can we operate on the frequency components of illumination and reflectance?

Recall that: $FT[f(x,y)] \neq FT[i(x,y)]\,FT[r(x,y)]$

Let's make this transformation:

$$z(x,y) = \ln(f(x,y)) = \ln(i(x,y)) + \ln(r(x,y))$$

Then $FT[z(x,y)] = FT[\ln(f(x,y))] = FT[\ln(i(x,y))] + FT[\ln(r(x,y))]$ *or*

$$Z(u,v) = F_i(u,v) + F_r(u,v)$$

Z(u,v) can then be filtered by a *H(u,v),* i.e.

$$S(u,v) = H(u,v)Z(u,v) = H(u,v)F_i(u,v) + H(u,v)F_r(u,v)$$

4.17

$$
\begin{aligned}
s(x, y) &= \Im^{-1}\{S(u, v)\} \\
&= \Im^{-1}\{H(u, v)F_i(u, v)\} + \Im^{-1}\{H(u, v)F_r(u, v)\}.
\end{aligned} \tag{4.5-6}
$$

By letting

$$
i'(x, y) = \Im^{-1}\{H(u, v)F_i(u, v)\} \tag{4.5-7}
$$

and

$$
r'(x, y) = \Im^{-1}\{H(u, v)F_r(u, v)\}, \tag{4.5-8}
$$

Eq. (4.5-6) can be expressed in the form

$$
s(x, y) = i'(x, y) + r'(x, y). \tag{4.5-9}
$$

Finally, as $z(x, y)$ was formed by taking the logarithm of the original image $f(x, y)$, the inverse (exponential) operation yields the desired enhanced image, denoted by $g(x, y)$; that is,

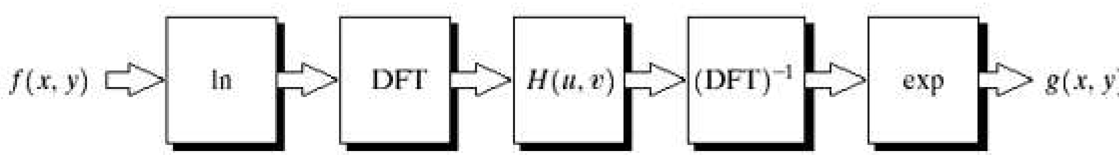
$$
\begin{aligned}
g(x, y) &= e^{s(x, y)} \\
&= e^{i'(x, y)} \cdot e^{r'(x, y)} \\
&= i_0(x, y)r_0(x, y)
\end{aligned} \tag{4.5-10}
$$

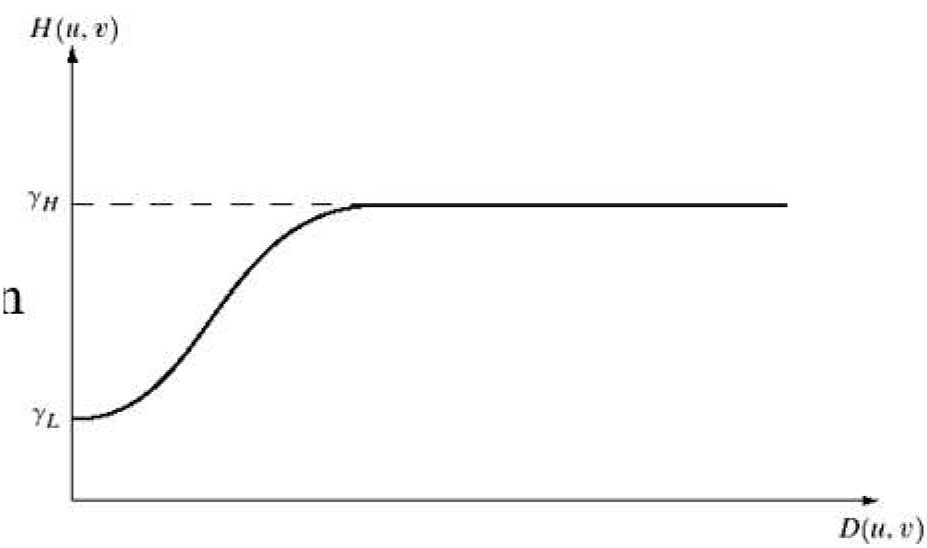
where

$$
r_0(x, y) = e^{r'(x, y)} \qquad i_0(x, y) = e^{i'(x, y)} \tag{4.5-11}
$$

4.18

Homomorphic filtering:





The illumination component of an image generally is characterized by slow spatial variations, while the reflectance component tends to vary abruptly, particularly at the junctions of dissimilar objects. These characteristics lead to associating the low frequencies of the Fourier transform of the logarithm of an image with illumination and the high frequencies with reflectance. Although these associations

A good deal of control can be gained over the illumination and reflectance components with a homomorphic filter. This control requires specification of a filter function $H(u, v)$ that affects the low- and high-frequency components of the Fourier transform in different ways. Figure 4.32 shows a cross section of such a filter. If the parameters γ_L and γ_H are chosen so that $\gamma_L < 1$ and $\gamma_H > 1$, the filter function shown in Fig. 4.32 tends to decrease the contribution made by the low frequencies (illumination) and amplify the contribution made by high frequencies (reflectance). The net result is simultaneous dynamic range compression and contrast enhancement.

In which cases zero padding necessary when applying DFT?

Zero padding must be used to compute linear convolution and avoid mixing in the case of circular convolution zero padding is also necessary for implementing fast DFT (FFT) in which the input signal is padded with zero to have a length that is a power of 2.