# Problem Set 3

Data Structures and Algorithms, Fall 2019

**Due: September 26, in class.**

## From CLRS

Exercise 6.1-7, 6.2-5, 6.3-3, 6.4-3, 6.4-5, 6.5-8. (For Exercise 6.5-8, also justify your `Heap-Delete` implementation is correct.) Problem 6-3.

## Additional Problem One

*[This is NOT a bonus problem, and you ARE required to solve it.]*

Professor F. Lake tells his class that it is asymptotically faster to square an $n$-bit integer than to multiply two $n$-bit integers. Should you believe him? Justify your answer.

## Additional Problem Two

*[This is NOT a bonus problem, and you ARE required to solve it.]*

The following cruel and unusual sorting algorithm was proposed by Gary Miller. Assume for this problem that the input size $n$ is always a power of 2.

```
Cruel(A[1···n])
1: if (n > 1) then
2:     Cruel(A[1···(n/2)]).
3:     Cruel(A[(n/2+1)···n]).
4:     Unusal(A[1···n]).
```

```
Unusal(A[1···n])
1: if (n == 2) then
2:     if (A[1] > A[2]) then
3:         Swap(A[1], A[2]).
4: else
5:     for (i = 1 to n/4) do                    ▷ Swap 2nd and 3rd quarters.
6:         Swap(A[i + n/4], A[i + n/2]).
7:     Unusal(A[1···(n/2)]).
8:     Unusal(A[(n/2+1)···n]).
9:     Unusal(A[(n/4+1)···(3n/4)]).             ▷ Recurse on middle half.
```

**(a).** Prove by induction that `Cruel` correctly sorts any input array.
**(b).** Prove that `Cruel` would *not* correctly sort if we remove the **for** loop from `Unusual`.
**(c).** Prove that `Cruel` would *not* correctly sort if we swap the last two lines of `Unusual`.
**(d).** What is the running time of `Unusual` and `Cruel`? Justify your answers.