Problem Set 13

Data Structures and Algorithms, Fall 2019

Due: December 19, in class.

From CLRS

Exercise 15.4-5, 15.5-4, 26.1-4, 26.1-7, 26.2-3, 26.2-11, 26.3-3 (a constant is not the right answer), 26.3-5, 34.1-4, 34.1-6.

Additional Problem One

Suppose you are given a sequence of integers separated by $+$ and $\times$ signs; for example:

$$1 + 3 \times 2 \times 0 + 1 \times 6 + 7$$

You can change the value of this expression by adding parentheses in different places. For example:

$$(1 + (3 \times 2)) \times 0 + (1 \times 6) + 7 = 13$$
$$((1 + (3 \times 2 \times 0) + 1) \times 6) + 7 = 19$$
$$(1 + 3) \times 2 \times (0 + 1) \times (6 + 7) = 104$$

Assume any arithmetic operation takes $O(1)$ time, answer the following questions.

(a) Describe and analyze an algorithm to compute the maximum possible value the given expression can take by adding parentheses, assuming all integers in the input are non-negative.

(b) Describe and analyze an algorithm to compute the maximum possible value the given expression can take by adding parentheses, with no restrictions on the input numbers.

Additional Problem Two

You are helping arranging the work schedules of doctors in a hospital. In particular, you need to ensure there is at least one doctor covering each vacation day.

More specifically, there are k vacation periods (e.g., the week of Christmas, the July 4^{th} weekend, the Thanksgiving weekend), each spanning several contiguous days. Let D_j be the set of days included in the j^{th} vacation period. We refer to the union of all these days, $\bigcup_j D_j$, as the set of all *vacation days*.

There are n doctors at the hospital, and doctor i has a set of vacation days S_i when he or she is available to work. (This may include certain days from a given vacation period but not others; so, for example, a doctor may be able to work the Friday, Saturday, or Sunday of Thanksgiving weekend, but not the Thursday.)

Design and analyze an algorithm that takes this information and determines whether it is possible to select a single doctor to work on each vacation day, subject to the following two constraints:

- For a given parameter c, each doctor should be assigned to work at most c vacation days total, and only days when he or she is available.

- For each vacation period j, each doctor should be assigned to work at most one of the days in the set D_j. (In other words, although a particular doctor may work on several vacation days over the course of a year, he or she should not be assigned to work two or more days of the Thanksgiving weekend, or two or more days of the July 4^{th} weekend, etc.)

Additional Problem Three

Suppose you are given a flow network G with integer edge capacities and an integer-valued maximum flow f in G. Describe and analyze algorithms for each of the following two operations:

(a) `Increment`(e): Increase the capacity of edge e by 1 and update the maximum flow.

(b) `Decrement`(e): Decrease the capacity of edge e by 1 and update the maximum flow.

Both algorithms should modify f so that it is still a maximum flow, more quickly than recomputing a maximum flow from scratch.