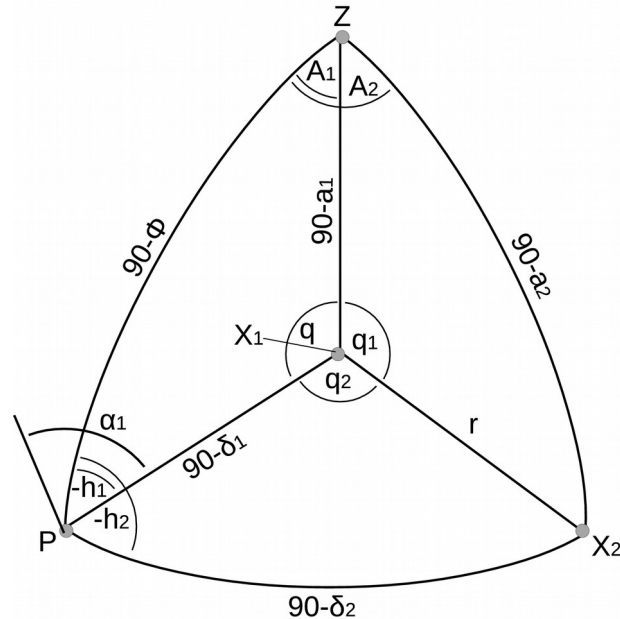


# Instantaneous location determination by Celestial Navigation.

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In the image above

- Z is the zenith of the observer
- P is the north pole
- $X_1$  and  $X_2$  are two selected stars with known equatorial coordinates.
- $\Phi$  is the latitude of the observer
- $\delta_1$  and  $\delta_2$  are the declinations from the equator of  $X_1$  and  $X_2$  respectively
- $a_1$  and  $a_2$  are the altitudes above the observer's horizon of  $X_1$  and  $X_2$  respectively
- $\alpha_1$  is the right ascension of  $X_1$ .  $\alpha_2$  is similar but not shown
- $A_1$  and  $A_2$  are the azimuths, measured east of north, of  $X_1$  and  $X_2$  respectively
- $h_1$  and  $h_2$  are the hour-angles to the observer's meridian for  $X_1$  and  $X_2$  respectively
- $q$  is the parallactic angle of  $X_1$

The object is to determine the location of the observer assuming the altitude and azimuth measurements ( $a_1$ ,  $a_2$ ,  $A_1$ ,  $A_2$ ) had zero error. By using this new location, subsequent conversions between horizontal and equatorial frames of reference can be done with increased accuracy.

It is also assumed that the declinations and right ascensions ( $\delta_1$ ,  $\delta_2$ ,  $RA_1$ ,  $RA_2$ ) are known with zero error.

Lastly it is assumed that the altitude and azimuth differences between  $X_1$  and  $X_2$  can be measured more accurately than their absolute angles (that the largest errors are common to both).

Using the cosine identity In  $PX_1X_2$  let

$$r_c = \cos(r)$$

$$r_c = \cos(90 - \delta_1) \cdot \cos(90 - \delta_2) + \sin(90 - \delta_1) \cdot \sin(90 - \delta_2) \cdot \cos(\alpha_2 - \alpha_1)$$

$$r_c = \sin(\delta_1) \cdot \sin(\delta_2) + \cos(\delta_1) \cdot \cos(\delta_2) \cdot (\alpha_2 - \alpha_1) \quad (1)$$

then let

$$r_s^2 = \sin^2(r) = 1 - \cos^2(r) = 1 - r_c^2 \quad (2)$$

In  $ZX_1X_2$  the sine rule gives

$$\frac{\sin(q_1)}{\sin(90 - a_2)} = \frac{\sin(A_2 - A_1)}{\sin(r)}$$

$$\frac{\sin(q_1)}{\cos(a_2)} = \frac{\sin(A_2 - A_1)}{\sin(r)}$$

then let

$$Q_1 = \sin(q_1) \cdot \sin(r) = \cos(a_2) \cdot \sin(A_2 - A_1) \quad (3)$$

similarly in  $PX_1X_2$ , let

$$Q_2 = \sin(q_2) \cdot \sin(r) = \cos(\delta_2) \cdot \sin(\alpha_2 - \alpha_1) \quad (4)$$

In  $ZX_1X_2$  the cosine rule gives

$$\cos(90 - a_2) = \cos(90 - a_1) \cdot \cos(r) + \sin(90 - a_1) \cdot \sin(r) \cdot \cos(q_1)$$

$$\sin(a_2) = \sin(a_1) \cdot \cos(r) + \cos(a_1) \cdot \sin(r) \cdot \cos(q_1)$$

then let

$$Q_1' = \cos(q_1) \cdot \sin(r) = \frac{\sin(a_2) - \sin(a_1) \cdot \cos(r)}{\cos(a_1)} \quad (5)$$

and similarly in  $PX_1X_2$ , let

$$Q_2' = \cos(q_2) \cdot \sin(r) = \frac{\sin(\delta_2) - \sin(\delta_1) \cdot \cos(r)}{\cos(\delta_1)} \quad (6)$$

From the angle sum identity

$$\cos(q_1 + q_2) = \cos(-q) = \cos(q) = \cos(q_1) \cdot \cos(q_2) - \sin(q_1) \cdot \sin(q_2)$$

or using the equations above

$$\cos(q) = \frac{Q_1' \cdot Q_2' - Q_1 \cdot Q_2}{r_s^2} \quad (7)$$

Using (7) and the cosine rule in  $PZX_1$ , it is now possible to determine  $\Phi$

$$\cos(90 - \Phi) = \cos(90 - \delta_1) \cdot \cos(90 - a_1) + \sin(90 - \delta_1) \cdot \sin(90 - a_1) \cdot \cos(q)$$

then let

$$\Phi_s = \sin(\Phi) = \sin(\delta_1) \cdot \sin(a_1) + \cos(\delta_1) \cdot \cos(a_1) \cdot \cos(q) \quad (8)$$

and because  $-90^\circ < \Phi < 90^\circ$

$$\Phi = \sin^{-1}(\Phi_s) \quad (9)$$

Applying the sine rule in  $PZX_1$  gives

$$\frac{\sin(-h_1)}{\sin(90 - a_1)} = \frac{\sin(q)}{\sin(90 - \Phi)}$$

then let

$$H = \sin(h_1) \cdot \cos(\Phi) = -\cos(a_1) \cdot \sin(q) \quad (10)$$

The cos rule in  $PZX_1$  gives

$$\cos(90 - a_1) = \cos(90 - \delta_1) \cdot \cos(90 - \Phi) + \sin(90 - \delta_1) \cdot \sin(90 - \Phi) \cdot \cos(-h_1)$$

$$\sin(a_1) = \sin(\delta_1) \cdot \sin(\Phi) + \cos(\delta_1) \cdot \cos(\Phi) \cdot \cos(h_1)$$

then let

$$H' = \cos(h) \cdot \cos(\Phi) = \frac{\sin(a_1) - \sin(\delta_1) \cdot \sin(\Phi)}{\cos(\delta_1)} \quad (11)$$

then, from (10) and (11)

$$h_1 = \tan^{-1} \left( \frac{H}{H'} \right)$$

the Local Sidereal Time ( $\theta_L$ ) is given by

$$\theta_L = \alpha_1 + h_1$$

if  $\theta_G$  is the Greenwich Sidereal Time, then the observer's longitude ( $\lambda_0$ ) is given by

$$\lambda_0 = \theta_L - \theta_G$$

By using these derived coordinates for conversion between horizontal and equatorial coordiante spaces, any errors in alignment between the observation platform and the horizon/zenith are eliminated in future calculations, enabling accurate redirection to any target in the celestial sky.

For reference, these are known equations not derived here. See Wikipedia

Conversion from Equatorial to Horizontal

$$A = \tan^{-1} \left( \frac{-\sin(h)}{-\cos(h) \cdot \sin(\Phi) + \tan(\delta) \cdot \cos(\Phi)} \right)$$

$$a = \sin^{-1}(\sin(\Phi) \cdot \sin(\delta) + \cos(\Phi) \cdot \cos(\delta) \cdot \cos(h))$$

Conversion from Horizontal to Equatorial

$$h = \tan^{-1} \left( \frac{-\sin(A)}{-\cos(A) \cdot \sin(\Phi) + \tan(a) \cdot \cos(\Phi)} \right)$$

$$\delta = \sin^{-1}(\sin(\Phi) \cdot \sin(a) + \cos(\Phi) \cdot \cos(a) \cdot \cos(A))$$

For a practical software implementation of the above equations see [github.com/mmoller2k/godob](https://github.com/mmoller2k/godob)