General Physics Study Guide

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1 Introduction

This study guide is for General Physics I & II. It covers basically all of the material in these courses. I can't guarantee it is all correct, but I think it's fairly comprehensive. I took that course during **Fall 2024 & Spring 2025** semesters, so that's when it was last updated. Material could have changed or been moved around since then. Also, just to flag, the material is not in the order that it was taught the year I took it. I grouped it based on concepts, rather than whatever the class does which I think is difficultly. As a result, you might encounter a really difficult topic out of nowhere. For example, Forces and Torques are similar concepts, but Forces is the second chapter and Torques is one of the last in the class. Despite that, I've decided to group them simply *because* they are similar topics. With that out of the way, I hope you find this guide helpful!

2 Prerequisite Math

Obviously physics needs a lot of math. This section covers is mostly about vector math you'll need for this course.

2.1 Special Angles

Here are very common angles that may be asked of you. It's best to memorize these angles and their values when trigonometric functions are used on them. These angles are reference angles, and because of that, they only represent magnitude. You need to add the sign after you are done computing the value based on what is physically happening in the scenario/question.

Operation	$0° = 0$	$30° = \frac{\pi}{6}$	$45° = \frac{\pi}{4}$	$60° = \frac{\pi}{3}$	$90° = \pi$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(\theta)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

2.2 Basic Vector Operation

A vector is a way to store numbers. In a physics sense, it's really just an arrow pointing from one place to another. Below are vector basics including how to add and multiply vectors. This covers all important vector operations done in this course. A normal number like 5 is called a scalar. In this class if something is not a vector, then it is a scalr.

2.2.1 Representing Vectors and Magnitude

Let $\vec{A}$ be a vector of 3 dimensions.

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} \tag{1}$$

There are other ways to represent vectors, but this is the way we'll do it in this class. Each $A_{something}$ literally just represents the x, y, or z coordinate of the arrow's tip.

A quick notational thing is the difference between $|\vec{A}|$ and $\vec{A}$. The first one represents the magnitude while the second one is the actual vector. If you think about our arrow representation, the magnitude of a vector is literally just how long it is. In a 2D space, if you think about a triangle, think about it as finding the hypotenuse from the length of the base and height. In fact, to find the magnitude of a vector from its normal form, you just use pythagorean's theorem. Below is that theorem in three dimensions.

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} \tag{2}$$

2.2.2 Vector Addition and Subtraction

$$\vec{A} \pm \vec{B} = \langle \vec{A}_x \pm \vec{B}_x,\ \vec{A}_y \pm \vec{B}_y,\ \vec{A}_z \pm \vec{B}_z \rangle = \vec{C} \tag{3}$$

2.2.3 Vector Multiplication

Vector multiplication comes in three flavors: multiplication by a scalar, the dot product, and the cross product. Scalar multiplication is multiplication between a scalar and a vector. The other two occur between two vectors. On a super high level, the dot product results in a scalar, whereas the cross product creates another vector.

Scalar Multiplication. Multiplying a vector by a scalar is by far the easiest vector multiplication. Suppose n is a scalar (that is, n is some number).

$$n\vec{A} = (A_x \times n)\hat{i} + (A_y \times n)\hat{j} + (A_z \times n)\hat{k} \tag{4}$$

You basically just distribute it over the vector.

The Dot Product. There are two ways to calculate the dot product.

$$\vec{A} \cdot \vec{B} = (\vec{A}_x \times \vec{B}_x) + (\vec{A}_y \times \vec{B}_y) + (\vec{A}_z \times \vec{B}_z) \tag{5}$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| \times |\vec{B}| \cos\theta \tag{6}$$

This is the second equation solved for theta, which often comes in handy.

$$\theta = \cos^{-1}\left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}| \times |\vec{B}|}\right) \tag{7}$$

The Cross Product. There are also two ways to calculate the cross product. A quick note: when you do the cross product of two 3D vectors, your resultant vector (the output) will be one dimensions smaller.

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta \tag{8}$$

$$\vec{A} \times \vec{B} = (A_y \times B_z - A_z \times B_y)\hat{i} + (A_z \times B_x - A_x \times B_z)\hat{j} + (A_x \times B_y - A_y \times B_x)\hat{k} \tag{9}$$

This formula will be used a lot less. In fact, you'll almost never use it.

3 Kinematics

Kinematics refers to the process of figuring out how something moves over time, including its velocity and acceleration.

3.1 Primer: Translational and Rotational Motion

Before discussing how to actually do kinematics, we'll start by talking about the two types of motion studied in this class: translational and rotational. **Translational** motion refers to things moving in a line or curve. For example, a car driving down the highway or a ball being thrown between two people. **Rotational** motion refers to the motion of something spinning such as someone on a merry go round or a pulley.

Something can have one and not the other, both, or neither. If I have a bowling ball in my hand and I'm just holding it, then it is stationary, so it has no motion. If I begin to spin the bowling ball on the floor and it stays in place, then it has rotational but not translational motion. If I slide it (that is, it's not rolling, the holes are always at the top for instance) then it has translational but not rotational motion. If I throw it down the alley and its rolling as it barrels towards the bowling pins, then it has both types of motion.

3.2 Primary Kinematic Equations

We start with $\vec{r}$ which represents position in a unit of distance (m). Before we begin, this variable is a little weird. In most of the equations given, you won't see a $\vec{r}$ because things will be generalized in one dimension. Just understand that $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$.

We can think about our kinematic equations in very general terms using derivatives. The core kinematic equations are as follow.

$$\vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} \tag{10}$$

$$\vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} \tag{11}$$

3.2.1 Special Case: Constant Acceleration

Then, we have the constant acceleration equations which is what you'll use more often.

$$\vec{v}_f = \vec{v}_i + \vec{a}t \tag{12}$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \tag{13}$$

$$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}(\vec{r}_f - \vec{r}_i) \tag{14}$$

3.2.2 Relative Kinematics Based on Postion

Since positions are relative, we can deduce the relative speed of an object A relative to a Point B if we know the speed of A relative to a position C and the speed of C relative to B's position. This applies to position, velocity, and acceleration. To solve for this, we can use the following formula.

$$v_{\frac{A}{B}} = v_{\frac{A}{C}} + v_{\frac{C}{B}} \tag{15}$$

3.3 Rotational Kinematics

Just a quick note before this section begins. Appendix **??** contains a useful chart that summarizes all of the translation and rotational counterparts. The section below provides a slightly more in-depth look at the topic, including useful formulas, but if you're just trying to remember what the rotational counterpart to displacement is, you might be better off checking the chart.

3.3.1 Introducing Rotational Kinematics

Rotational kinematics operate in practically the same way as translational kinematics, but deals with objects that are spinning or otherwise moving in a circular motion. Below, we detail how you can take tangential measurements and turn them into their rotational counterparts.

$$\vec{s} = r\theta$$

$$\vec{v} = r\omega$$

$$\vec{a} = r\alpha$$

$$t = t$$

In these equations, $\vec{r} = \vec{s}$ for clarity's sake because the r that is included actually refers to the path's radius. Both $\vec{r}$ and $\vec{s}$ mean position. You would refer to each counterpart as the "angular" version of the translational one (ex. displacement becomes angular displacement).

3.3.2 Primary Rotational Kinematics Equations

Since we have already defined θ as angular displacement (in radians) we can define the other kinematic values.

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} \tag{16}$$

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} \tag{17}$$

3.3.3 Special Case: Constant Rotational Acceleration

Now, we can combine the special case kinematic equations found in 3.2.1 with the definitions that relate the translational and angular values in 3.3.1. The process is simple, since the angular form of a kinematic value is the just the translational counterpart divided by r, we can just divide each of the 3.2.1 equations by r to create rotational counterparts.

$$\omega_f = \omega_i + \alpha t \tag{18}$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \tag{19}$$

$$\omega_f^2 = {\omega_i}^2 + 2\alpha(\theta_f - \theta_i) \tag{20}$$

3.3.4 Period and Frequency

As an object rotates around itself (or travels in a circular path), it's bound to come back to the same point eventually. Note that this only works for zero acceleration otherwise the time to finish one rotation would keep changing and that defeats the purpose of calculating such a measurement in the first place.The time it takes for one full rotation (usually called a cycle) is called the *period*, recorded in some unit of time.

$$T = \frac{2\pi}{\omega} \tag{21}$$

Frequency is the inverse of the period, defined as the number of rotations an object makes in one unit of time. Below are definitions of both in relation to angular velocity and each other.

$$f = \frac{1}{T} \tag{22}$$

An important thing to note is the unit of frequency. Since it's the inverse of T, its units are technically just "cycle per unit time", but if the unit of time is seconds, then we the unit hertz (hz). In other words $hz = \frac{1}{s}$.

3.3.5 Centripedal Acceleration

Centripetal acceleration, or rotational acceleration, is another important value that is unique to rotational motion. In effect, it is the inward acceleration that keeps an object that is moving in a circular path in that path. For example, you might be swinging a ball on a string in a circle; the reason it doesn't fly out is because the sting is applying a force (and therefore accelerating the ball) back into the circle as tangential velocity "tries to get it" to fly out. We can define centripetal acceleration using both tangential velocity and angular velocity.

$$\vec{a}_c = \frac{\vec{v}^{\,2}}{r} = \omega^2 r \tag{23}$$

4 Forces

4.1 Reference Frames

Reference frames, especially inertial reference frames, are an extremely important concept in the General Physics course. For the most part, homework and exam problems will exist within inertial reference frames, but it is still important to understand what a reference frame is and they relate to Newton's laws.

This subsection will discuss in the broadest terms what reference frames are. There is a lot more nuanced especially as you move into special relatively, but for the purposes of this study guide, I have boiled the concept down to the basic version we need to understand for this course.
Reference Frame: Refers to the parameters of observation, such as how the axes are situated or when $t = 0$.
Inertial Reference Frame: Refers to a reference frame in which Newton's laws hold true. That is, we can identify every force acting on an object and determine that motion is consistent with Newton's three laws. If you find yourself in a non-inertial reference frame, you may do one of two things.

1. Accept that you are in a non-inertial reference frame and calculate motion without Newton's laws.
2. Accept that you are in a non-inertial reference frame, but add a "fake" or "non-existent" force which allows you to use Newton's laws.

Example: Say a person is in a car that moves along a curved road. There are two possible observation points, otherwise known as reference frames. The first is an outside observer standing at the side of the road, and the second frame is of the person sitting inside of the car.

1. The observer on the side of the road *is* in an inertial reference frame:
   From the observer's point of view, all motion can be explained using Newton's laws. Put simply, the observer can determine that the car is moving, that it is being pulled into the curve by centripetal acceleration (friction on the tires).
2. The person inside the car *is not* in an inertial reference frame:
   The person inside the car has a different perspective. From their reference frame, the road is moving, not them. Due to this, they cannot determine that they have a tangential velocity to the curve of the road, and therefore, they cannot explain why they are being pushed off the road.
   (a) They can either accept they are in a non-inertial reference frame and make calculations accordingly (without Newton's laws), or
   (b) Accept they are in a non-inertial reference frame, but add a "fake" force pushing them outside the circle so that Newton's second law holds true. This allows them to use all of Newton's laws.

4.2 Newton's Laws

4.2.1 Newton's First Law: Inertia

Definition: In an inertial frame of reference, if there is *no* force on an object, then a stationary object remains at rest and ' an object in motion stays in motion with a constant velocity, $\vec{v}$.
Sloadism Definition: If there's no force on an object, then its movement doesn't change. If its stopped, it will stay stopped, if its moving, it'll keep moving at the same speed.

4.2.2 Newton's Second Law: Force

Formal Definition: In an inertial-reference frame, for an object of momentum $\vec{p}$, the net force is the change in momentum over time.
Sloadism Definition: The net force on an object is its impulse over time (or mass times acceleration).

$$F_{net} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = m\vec{a} \tag{24}$$

Where,
p = momentum of the object in newton-meters.
t = time in seconds.
m = the mass of the object in kilograms.
a = the acceleration of the object in meters per second.

4.2.3 Newton's Third Law: Action & Reaction

Definition: In an inertial reference frame, $F_{\text{A on B}} = -F_{\text{B on A}}$.
Sloadism Definition: Every action has an equal and opposite reaction.

Maybe add that thing about how the book and the table are not normal.

4.3 Hooke's Law

In physics, a lot of classic problems revolve around springs. Naturally, that means springs behave in a unique (but simple) way when it comes to forces. We'll only be considering ideal springs. That is massless springs with a fixed stretchiness. Hooke's law is the equation we use to determine the amount of force it takes to stretch or compress a string (and by Newton's Third Law, how much force the spring is exerting on whatever its pushing or pulling against). It is as follows:

$$\vec{F}_{spring} = k\Delta s \tag{25}$$

Where,

k is the elastic coefficient

Δs is the distance the spring has moved. You'll have to determine the direction just as you would any other force. You can think of the direction of pointing towards the center (so if its stretched and you have a typical x and y axis, then the direction is the negative one).

4.3.1 The Elastic Coefficient

The elastic coefficient k is a constant based on the spring/material that is being used. It describes how many newtons it takes to stretch or compress that material by one meter. It follows then that it's units are $\frac{N}{m}$.

5 Torques

Torques are the rotational equivalent to forces. If forces cause translational acceleration, motivating an object to move translationally, then torque motivates objects to move rotationally. Before we talk about torques, we'll first discuss the mass equivalent for rotational motion, then talk about Newton's Rotational Laws and a few important scenarios.

To law a few ground rules, we represent torques with τ in Nm. The counterclockwise direction is positive and the clockwise direction is negative.

5.1 Moments of Inertia

If mass is a measurement of how inert something is that means it measures how difficult something is to move (or otherwise to gain momentum). Likewise, the **moment of inertia** measures how difficult it is to rotate something. More intuitively, it measures how far apart mass is spread from something's pivot (the point it spins around). If two things have the same mass and one is packed closer, then it will have a smaller moment of inertia compared to the second object who has the same mass spread across a larger distance (for example, if you have two balls with the same mass, the smaller one has a smaller moment of inertia because the mass is spread out less). We represent the moment of inertia with the variable I and its units are kgm^2.

The moment of inertia depends on the shape of an object, so each one has a different formula (you use a different formula to calculate a disc's moment of inertia than for a ball). These will always be given, so I won't bother detailing them here.

5.1.1 The Parallel Axis Theorem

This theorem is used to find the moment of inertia for an object who's pivot axis has moved but stayed parallel to itself. For example, if spin a ball on my finger, then its axis is straight up (this would be $I_{original}$). If I instead decide to spin it by spinning in a circle with my arms out, then its axis is moved away from the center, but is still parallel (that is, straight up). To calculate the new moment of inertia, we can use the following equation.

$$I_{new} = I_{original} + md^2 \tag{26}$$

Where, m is the object's mass and
d is the distance the axis moved.

5.2 Newton's Rotational Laws

Just like with forces, we have three laws to consider. We'll look at them out of order since the second one is more involved (and arguably important) than the others. The same constraint that this must happen in an inertial reference frame applies.

5.2.1 Newton's First Rotational Law

In an inertial reference frame, if there is no torque acting on an object, then objects at rest stay at rest and objects in motion stay in motion with a constant velocity.

5.2.2 Newton's Third Rotational Law

In an inertial reference frame, torques are met with an equal and opposite reaction.

$$\tau_{AonB} = -\tau_{BonA} \tag{27}$$

5.2.3 Newton's Second Rotational Law

This is the same as Newton's Second Translation Law.

$$\tau_{net} = I\alpha \tag{28}$$