Discrete Math Study Guide

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Contents

1 Logical Symbols and Deductive Reasoning

1.1 Variables and Statements

A **variable** is a symbol that stands in for some specific value, be it a person, number, etc.

A **statement** is a something that may evaluate to true or false. It is usually either in the form P if it does not depend on a variable or $P(x)$ if the statement's truth depends on what the input is.

1.2 Connective Symbols

1.3 Logical Laws

1.3.1 Associative Law

$$(P \wedge Q) \wedge R = P \wedge (Q \wedge R) \tag{1}$$
$$(P \vee Q) \vee R = P \vee (Q \vee R) \tag{2}$$

1.3.2 Communative Law

$$P \wedge Q = Q \wedge P \tag{3}$$
$$P \vee Q = Q \vee P \tag{4}$$

1.3.3 Distributive Law

$$P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R) \tag{5}$$
$$P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R) \tag{6}$$

1.3.4 Double Negation Law

$$\neg\neg P = P \tag{7}$$

1.3.5 De Morgan's Law

$$\neg(P \wedge Q) = (\neg P \vee \neg Q) \tag{8}$$
$$\neg(P \vee Q) = (\neg P \wedge \neg Q) \tag{9}$$

1.3.6 Idempotent Law

$$P \wedge P = P \tag{10}$$
$$P \vee P = P \tag{11}$$

1.3.7 Absorption Law

$$P \wedge (P \vee Q) = P \quad (12)$$
$$P \wedge (P \vee Q) = P \quad (13)$$

1.4 Truth Tables

Truth tables are are relative straight forward concept. The aim is to evaluate the truth of statement by breaking it down into its smallest parts, then seeing if the final statement is true or false based on the truth of the sub-statements. Here is a simple example,

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

1.5 The Conditional

1.5.1 Definition

The conditional can be thought of as an "if, then" statement. It primarily demonstrates some relationship between two statements. In symbols, it is represented as,

$$P \to Q$$

This statement can be read several ways in English:

1. P implies Q
2. P only if Q
3. P is a sufficient condition for Q
4. Q, if P
5. Q is a necessary condition for P

1.5.2 The Truth of a Conditional

We can demonstrate the truth of the conditional via a truth table.

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

To put the truth table into plain words, the conditional is true only if both Q is true or if P and Q are both false. In other words, the conditional is only false if only Q is false.

1.5.3 The Conditonal in Logical Connectives

We can write the conditional in terms of basic logical connectives. The definition of conditional in these terms is as follows.

$$P \to Q \quad \equiv \quad \neg P \lor Q \quad \equiv \quad P \land \neg Q$$

Note that the rightmost statement is the same as the middle statement, but De Morgan's Law was applied.

We can verify that these statements are equivalent via another truth table.

P	Q	$P \to Q$	$\neg P \lor Q$	$P \land \neg Q$
T	T	T	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

1.5.4 The Converse

The converse of a conditional is simply the conditional, but the statements have been swapped around.

$$P \to Q \quad \not\equiv \quad Q \to P$$

We could write a truth table to demonstrate that these statements are **NOT** equivalent, but we will use the definition of the conditional (the logical symbols version) to demonstrate intuitively that these are not the same.

$$\begin{aligned} P \to Q &\equiv \neg P \lor Q \\ Q \to P &\equiv \neg Q \lor P \qquad (14) \\ \neg P \lor Q &\not\equiv \neg Q \lor P \qquad (15) \end{aligned}$$

1.5.5 The Contrapositive

The contrapositive of a conditional is a negated version of the original statement. Unlike the converse of conditional, the contrapositive is equivalent to the original statement.

$$P \to Q \quad \equiv \quad \neg Q \to P$$

We could use a truth table to show that these statements are equivalent, but we can also use the logical forms of the conditionals achieve the same end.

$$\begin{aligned} P \to Q &\equiv \neg P \lor Q \\ \neg Q \to P &\equiv \neg\neg Q \lor \neg P \\ \neg P \lor Q &\equiv \neg\neg Q \lor P \end{aligned}$$

1.6 The Biconditional

1.6.1 Definition

The biconditional is often read as "if an only if". It can be written in terms of the conditional or logical connectors. It is written as follows.

$$P \leftrightarrow Q$$

In terms of the conditional,

$$P \leftrightarrow Q \quad \equiv \quad (P \to Q) \land (Q \to P) \tag{16}$$

The final definition is in terms of logical connectors.

$$P \leftrightarrow \quad \equiv \quad (\neg P \lor Q) \land (\neg Q \lor P) \tag{17}$$

Any statements that are equivalent to those above are valid definition of the biconditional.

1.6.2 The Truth of a Biconditional

We can determine the truth of a biconditional via a truth table.

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

As we can see, the biconditional only evaluates to true if (and only if) both statements involved are true.