

# Discrete Math Study Guide

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# 1 Logical Symbols and Deductive Reasoning

## 1.1 Variables and Statements

A **variable** is a symbol that stands in for some specific value, be it a person, number, etc.

A **statement** is a something that may evaluate to true or false. It is usually either in the form  $P$  if it does not depend on a variable or  $P(x)$  if the statement's truth depends on what the input is.

## 1.2 Connective Symbols

### 1.3 Logical Laws

#### 1.3.1 Associative Law

$$(P \wedge Q) \wedge R = P \wedge (Q \wedge R) \quad (1)$$

$$(P \vee Q) \vee R = P \vee (Q \vee R) \quad (2)$$

#### 1.3.2 Communative Law

$$P \wedge Q = Q \wedge P \quad (3)$$

$$P \vee Q = Q \vee P \quad (4)$$

#### 1.3.3 Distributive Law

$$P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R) \quad (5)$$

$$P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R) \quad (6)$$

#### 1.3.4 Double Negation Law

$$\neg\neg P = P \quad (7)$$

#### 1.3.5 De Morgan's Law

$$\neg(P \wedge Q) = (\neg P \vee \neg Q) \quad (8)$$

$$\neg(P \vee Q) = (\neg P \wedge \neg Q) \quad (9)$$

#### 1.3.6 Idempotent Law

$$P \wedge P = P \quad (10)$$

$$P \vee P = P \quad (11)$$

### 1.3.7 Absorption Law

$$P \wedge (P \vee Q) = P \quad (12)$$

$$P \wedge (P \vee Q) = P \quad (13)$$

## 1.4 Truth Tables

Truth tables are a relatively straightforward concept. The aim is to evaluate the truth of a statement by breaking it down into its smallest parts, then seeing if the final statement is true or false based on the truth of the sub-statements. Here is a simple example,

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

## 1.5 The Conditional

### 1.5.1 Definition

The conditional can be thought of as an "if, then" statement. It primarily demonstrates some relationship between two statements. In symbols, it is represented as,

$$P \rightarrow Q$$

This statement can be read several ways in English:

1.  $P$  implies  $Q$
2.  $P$  only if  $Q$
3.  $P$  is a sufficient condition for  $Q$
4.  $Q$ , if  $P$
5.  $Q$  is a necessary condition for  $P$

### 1.5.2 The Truth of a Conditional

We can demonstrate the truth of the conditional via a truth table.

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

To put the truth table into plain words, the conditional is true only if both  $Q$  is true or if  $P$  and  $Q$  are both false. In other words, the conditional is only false if only  $Q$  is false.

### 1.5.3 The Conditional in Logical Connectives

We can write the conditional in terms of basic logical connectives. The definition of conditional in these terms is as follows.

$$P \rightarrow Q \equiv \neg P \vee Q \equiv P \wedge \neg Q$$

Note that the rightmost statement is the same as the middle statement, but De Morgan's Law was applied.

We can verify that these statements are equivalent via another truth table.

P	Q	$P \rightarrow Q$	$\neg P \vee Q$	$P \wedge \neg Q$
T	T	T	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

### 1.5.4 The Converse

The converse of a conditional is simply the conditional, but the statements have been swapped around.

$$P \rightarrow Q \not\equiv Q \rightarrow P$$

We could write a truth table to demonstrate that these statements are **NOT** equivalent, but we will use the definition of the conditional (the logical symbols version) to demonstrate intuitively that these are not the same.

$$P \rightarrow Q \equiv \neg P \vee Q$$

$$Q \rightarrow P \equiv \neg Q \vee P \tag{14}$$

$$\neg P \vee Q \not\equiv \neg Q \vee P \tag{15}$$

### 1.5.5 The Contrapositive

The contrapositive of a conditional is a negated version of the original statement. Unlike the converse of conditional, the contrapositive is equivalent to the original statement.

$$P \rightarrow Q \equiv \neg Q \rightarrow \neg P$$

We could use a truth table to show that these statements are equivalent, but we can also use the logical forms of the conditionals achieve the same end.

$$P \rightarrow Q \equiv \neg P \vee Q$$

$$\neg Q \rightarrow \neg P \equiv \neg \neg Q \vee \neg P$$

$$\neg P \vee Q \equiv \neg \neg Q \vee P$$

## 1.6 The Biconditional

### 1.6.1 Definition

The biconditional is often read as "if and only if". It can be written in terms of the conditional or logical connectors. It is written as follows.

$$P \leftrightarrow Q$$

In terms of the conditional,

$$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P) \quad (16)$$

The final definition is in terms of logical connectors.

$$P \leftrightarrow Q \equiv (\neg P \vee Q) \wedge (\neg Q \vee P) \quad (17)$$

Any statements that are equivalent to those above are valid definitions of the biconditional.

### 1.6.2 The Truth of a Biconditional

We can determine the truth of a biconditional via a truth table.

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

As we can see, the biconditional only evaluates to true if (and only if) both statements involved are true.

### 1.7 Arguments

## 2 Quantifiers

### 2.1 Motivating Quantifiers

### 2.2 The Universe of Discourse

### 2.3 The Universal Quantifier

### 2.4 The Existential Quantifier

#### 2.4.1 Uniqueness

### 2.5 Bound Variables

### 2.6 Quantifier Negation

## 3 Set Theory

### 3.1 Defining Sets

#### 3.1.1 Important Sets

#### 3.1.2 Truth Sets

### 3.2 Basic Set Operations

### 3.3 Index Sets

### 3.4 Families of Sets

#### 3.4.1 The Power Set

### 3.5 Operations on Families of Sets

## 4 Introductory Proof Strategies

### 4.1 Theorems, Propositions, and Lemmas

### 4.2 Proof Writing Basics

### 4.3 Direct Proofs

### 4.4 Proof by Contrapositive