

Discrete Math Study Guide

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Contents

1	Logical Symbols	2
1.1	Logical "And", "Or", and "Not"	2
1.2	The Conditional	2
1.2.1	Alternate Language	2
1.2.2	Truth Table	2
1.3	The Biconditional	2
2	Logical Laws	2
2.1	Associative Law	2
2.2	Communicative Law	2
2.3	Idempotent Law	3
2.4	Absorption Law	3
2.5	Distributive Law	3
2.6	De Morgan's Law	3
2.7	Double Negation Law	3
2.8	Tautologies and Contradictions	3
2.8.1	Tautologies	3
2.8.2	Contradictions	3
3	Introducing Proofs	4
3.1	Notation	4
3.2	Writing Proofs	4
3.3	General Strategies for Proofs	4
3.4	Proving Negated Statements	4

1 Logical Symbols

1.1 Logical "And", "Or", and "Not"

These are the absolute basic logical symbols. Basically anything can be written in terms of these.

$$\wedge = \text{And}$$

$$\vee = \text{Or}$$

$$\neg = \text{Not}$$

1.2 The Conditional

Definition: $P \rightarrow Q$ means " P implies Q ". It can also be written with basic logical operators as $\neg P \vee Q$.

1.2.1 Alternate Language

Here are a few other ways that $P \rightarrow Q$ can be written in English.

1. Q , if P
2. P only if Q
3. P is a sufficient condition for Q
4. Q is a necessary condition for P

1.2.2 Truth Table

Hello there.

1.3 The Biconditional

2 Logical Laws

2.1 Associative Law

$$(P \vee Q) \vee R = P \vee (Q \vee R)$$

$$(P \wedge Q) \wedge R = P \wedge (Q \wedge R)$$

2.2 Commutative Law

$$P \vee Q = Q \vee P$$

$$P \wedge Q = Q \wedge P$$

2.3 Idempotent Law

$$P \vee P = P$$

$$P \wedge P = P$$

2.4 Absorption Law

$$P \vee (P \wedge Q) = P$$

$$P \wedge (P \vee Q) = P$$

2.5 Distributive Law

$$P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$$

$$P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$$

2.6 De Morgan's Law

$$\neg(P \vee Q) = (\neg P \wedge \neg Q)$$

$$\neg(P \wedge Q) = (\neg P \vee \neg Q)$$

2.7 Double Negation Law

$$\neg\neg P = P$$

2.8 Tautologies and Contradictions

2.8.1 Tautologies

Tautology: A formula that is always *true* independent of the truth values of its statements.

$$P \wedge \text{Tautology} = P$$

$$P \vee \text{Tautology} = \text{Tautology}$$

$$\neg\text{Tautology} = \text{Contradiction}$$

2.8.2 Contradictions

Contradiction: A formula that is always *false* independent of the truth values of its statements.

$$P \wedge \text{Contradiction} = \text{Contradiction}$$

$$P \vee \text{Contradiction} = P$$

$$\neg\text{Contradiction} = \text{Tautology}$$

3 Introducing Proofs

3.1 Notation

3.2 Writing Proofs

3.3 General Strategies for Proofs

3.4 Proving Negated Statements