EE605

Chapter 1: End of Chapter Problems

2

A

$$[6,8] \cup [2,7) = [2,8]$$

B

$$[6,8] \cap [2,7) = [6,7)$$

C

$$[0.1]^c = (-\infty, 0) \cup (1, \infty)$$

D

$$[6,8] - (2,7] = (7,8]$$

13

$$S = \{a, b, d\}$$

$$\begin{array}{lllll} (1)P(a) & = & P(\{a\}) & = & 0.5 \\ (2)P(d) & = & P(\{d\}) & = & 0.25 \end{array}$$

a Find the probability that B wins.

$$P(a) + P(b) + P(d) = 1$$

$$P(b) = 0.25$$

b Find the probability that B wins, or a draw occurs.

$$P(b \cup d) = p(b) + p(d) = 0.5$$

15

a

$$P(x_2 = 4) = \frac{1}{6}$$

b

$$X_1 + X_2 = 7$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \end{bmatrix}$$

$$p(x_1 + x_2) = \frac{1}{6} \times \frac{1}{6} \times 2 + \frac{1}{6} \times \frac{1}{6} \times 2 + \frac{1}{6} \times \frac{1}{6} \times 2 = \frac{1}{6}$$

$$p(x_1 + x_2) = \frac{1}{6}$$

C

$$X_1 \neq 2, X_2 \geq 4$$

$$p(X_1 \neq 2, X_2 \geq 4) = \frac{5}{6} \times \frac{3}{6} = \frac{5}{12}$$

$$p(X_1 \neq 2, X_2 \geq 4) = \frac{5}{12}$$

16

$$S = \{1,2,3, \cdots\}$$

$$P(k) = P(\{k\}) = \frac{c}{3^k} \text{ , for } k = 1,2, \cdots,$$

a. Find c.

$$p(1) + p(2) + \ldots + p(k) = 1$$

$$S_n = a_1 \frac{1 - q^n}{1 - q}$$

$$1 = \frac{1}{3} \frac{1 - \frac{1}{3}^n}{1 - \frac{1}{3}} c$$

$$c = 2$$

$$P(k) = \frac{2}{3^k}$$

b. Find P (2,4,6).

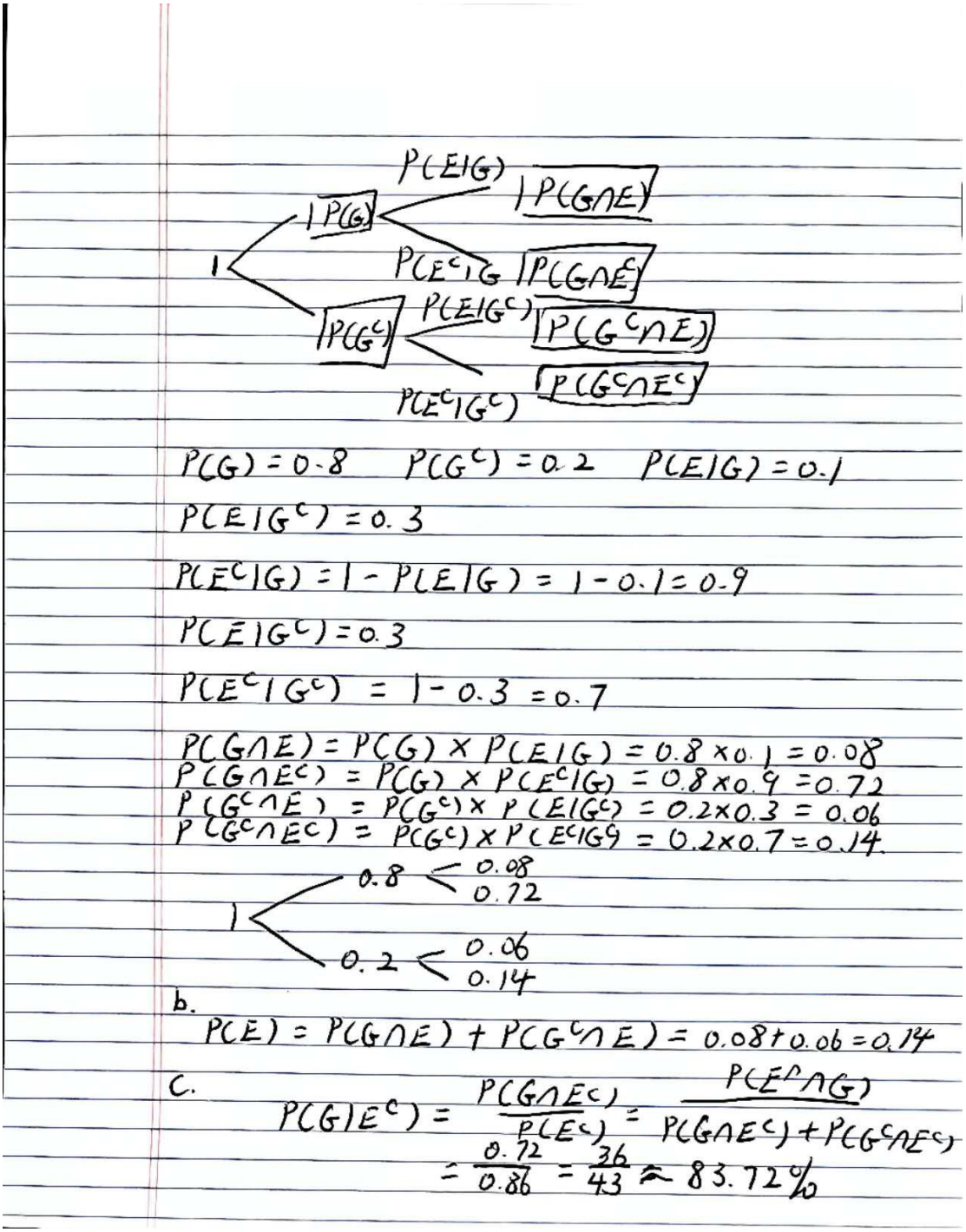
$$P(2,4,6) = \frac{2}{3^2} + \frac{2}{3^4} + \frac{2}{3^6} = \frac{182}{729}$$

c. Find P (3,4,5, ⋯}).

$$P(3,4,5,\cdots) = 1 - P(1,2) = 1 - \frac{2}{3} - \frac{2}{9} = \frac{1}{9}$$

$$P(3,4,5,\cdots) = \frac{1}{9}$$

27



$P(G) = 0.8 \quad P(G^c) = 0.2 \quad P(E|G) = 0.1$

$P(E|G^c) = 0.3$

$P(E^c|G) = 1 - P(E|G) = 1 - 0.1 = 0.9$

$P(E|G^c) = 0.3$

$P(E^c|G^c) = 1 - 0.3 = 0.7$

$P(G \cap E) = P(G) \times P(E|G) = 0.8 \times 0.1 = 0.08$
$P(G \cap E^c) = P(G) \times P(E^c|G) = 0.8 \times 0.9 = 0.72$
$P(G^c \cap E) = P(G^c) \times P(E|G^c) = 0.2 \times 0.3 = 0.06$
$P(G^c \cap E^c) = P(G^c) \times P(E^c|G^c) = 0.2 \times 0.7 = 0.14$

b.

$P(E) = P(G \cap E) + P(G^c \cap E) = 0.08 + 0.06 = 0.14$

c.

$$P(G|E^c) = \frac{P(G \cap E^c)}{P(E^c)} = \frac{P(E^c \cap G)}{P(G \cap E^c) + P(G^c \cap E^c)}$$
$$= \frac{0.72}{0.86} = \frac{36}{43} \approx 83.72\%$$

31

Let A = "The email is spam", A^c = "It is not spam".

Let B = "The word refinance appears."

emails are spam

$$P(A) = 50\%$$

1% of spam emails contain the word "refinance".

$$P(B \mid A) = 1\%$$

0.001% of non-spam emails contain the word "refinance".

$$P(B \mid A^C) = 0.001\%$$

$$P(A^C) = 1 - P(A) = 50\%$$

$$\begin{aligned} P(B) &= P(B \mid A)P(A) + P(B \mid A^C)P(A^C) \\ &= 50\% \times 0.001\% + 1\% \times 50\% \\ &= 0.5005\% \end{aligned}$$

$$P(B) = 0.5005\%$$

$$\begin{aligned} P(A \mid B) &= \frac{P(B \mid A)P(A)}{P(B)} \\ &= \frac{1\% \times 50\%}{0.5005\%} \\ &= 99.9000999001\% \\ &\approx 99.90\% \end{aligned}$$

$$P(A \mid B) = 99.90\%$$