Research Statement

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According to my mother, my answer to, "what do you want to be when you grow up?" was, from the time I was old enough to comprehend the question, "a scientist." Before that it was, "a lizard." In a very real sense I have always been interested in discovery.

As an undergraduate, I started out as a computer science major, only picking up mathematics as a second major in my sophomore year. I was fascinated with data structures, particularly trees and graphs, which can have remarkable properties when used wisely. As I progressed in my mathematics classes, I discovered that mathematical structures are also fascinating, and eventually developed an interest in algebraic structures in particular.

1 Undergraduate research

In my undergraduate studies I primarily concentrated on computer science research. I worked as an undergraduate research assistant at Jacksonville State University in fast encoding of multi-reference frame video encoding under Dr. Monica Trifas and Dr. Ming Yang. Briefly, modern video streaming is done by only occasionally transmitting a full video frame, called a reference frame. The rest of the frames are divided into small square blocks and estimated motion vectors from these small blocks to corresponding blocks in the nearest reference frame are transmitted. A multi-reference frame video encoder uses multiple reference frames to estimate these motion vectors, which results in higher-quality video, but encoding that is potentially orders of magnitude slower.

In the last summer at Jacksonville State University, I traveled to Utah State University to work on a summer research project with Dr. Xiaojun Qi in computer vision, specifically image classification. We used a support vector machine to train the computer to classify images based on content, e.g. contains flags vs. contains birds.

2 Lie theory

My dissertation work and current research are in the area of Lie theory, particularly in semisimple Lie groups.

A *Lie group* is a group that also has the structure of a smooth manifold such that both multiplication and inversion of group elements are smooth maps. A smooth manifold itself is a topological space that is locally homeomorphic to a Euclidean space $\mathbb{R}^n$, and for any two local homeomorphisms φ_1 and φ_2 whose domains intersect, $\varphi_1 \circ \varphi_2^{-1} : \mathbb{R}^n \to \mathbb{R}^n$ must have partial derivatives of all orders. In my research I have restricted my attention to finite-dimensional Lie groups.

To every Lie group G there is an associated *Lie algebra* $\mathfrak{g}$, isomorphic as a vector space to the tangent space at the identity of G, and having an additional nonassociative algebra structure given by the Lie bracket. Every Lie algebra is isomorphic to a matrix algebra, whwere $[X, Y] = XY - YX$ where juxtaposition is ordinary matrix multiplication.

A Lie algebra is *simple* if it is nonabelian and its only ideals are itself and the trivial Lie algebra. A Lie algebra is *semisimple* if it is the direct sum of simple Lie algebras. A Lie group is semisimple if its associated Lie algebra is semisimple.

A semisimple Lie group admits a preorder, called Kostant's preorder, and inequalities with Kostant's preorder are primarily where my interest lies. Kostant's preorder has a very lengthy and technical definition involving the Iwasawa decomposition, complete multiplicative Jordan decomposition, and vector log-majorization, and I will omit the definition here. It is, however, worth note that many inequalities involving matrices and vector majorization have been generalized to semisimple Lie groups, and many more seem to be good candidates for doing so.

3 Matrix theory

Here's some specific stuff about Matrix theory, specifically future work in matrix means and generalizing audenaert's result

4 Topological groups

Here's some specific stuff about topological groups, and my interest in integration.

5 Multilinear algebra

Here's some specific stuff about multilinear algebra, and my interest in symmetry classes and tensor rank, as well as obstacles to studying tensor rank. (I.e. algebraic geometry)

6 Scholarship of education

IBL stuff probably.

7 Other interests

1. Category theory
2. Computability
3. Quantum computation
4. Programming languages