

Joint Optimization of Multicast Energy in Delay-constrained Mobile Wireless Networks

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Abstract—This paper studies the problem of optimizing multicast energy consumption in delay-constrained mobile wireless networks, where information from the source needs to be delivered to all the k destinations within an imposed delay constraint. Most existing works simply focus on deriving transmission schemes with the minimum transmitting energy, overlooking the energy consumption at the receiver side. Therefore, in this paper, we propose ConMap, a novel and general framework for efficient transmission scheme design that jointly optimizes both the transmitting and receiving energy. In doing so, we formulate our problem of designing minimum energy transmission scheme, called DeMEM, as a combinatorial optimization one, and prove that the approximation ratio of any polynomial time algorithm for DeMEM cannot be better than $\frac{1}{4} \ln k$. Aiming to provide more efficient approximation schemes, the proposed ConMap first converts DeMEM into an equivalent directed Steiner tree problem through creating auxiliary graph gadgets to capture energy consumption, then maps the computed tree back into a transmission scheme. The advantages of ConMap are threefolded: i) Generality—ConMap exhibits strong applicability to a wide range of energy models; ii) Flexibility—Any algorithm designed for the problem of directed Steiner tree can be embedded into our ConMap framework to achieve different performance guarantees and complexities; iii) Efficiency—ConMap preserves the approximation ratio of the embedded Steiner tree algorithm, to which only slight overhead will be incurred. The three features are then empirically validated, with ConMap also yielding near-optimal transmission schemes compared to a brute-force exact algorithm. To our best knowledge, this is the first work that jointly considers both the transmitting and receiving energy in the design of multicast transmission schemes in mobile wireless networks.

Index Terms—Mobile Wireless Networks, Energy Optimization, Multicast

I. INTRODUCTION

With the surge of mobile data traffic, energy consumption in communication is rising radically in recent years. As reported by a survey [1] where Telecom Italia is said to be the second largest consumer in Italy, energy consumption of mobile communication is already ranking the top. Hence, energy saving becomes a crucial issue that receives considerable attention in the design and implementation of mobile wireless networks.

As the majority of nodes are battery powered, a key problem of energy cost reduction in mobile wireless networks is to find

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a route with minimum total energy consumption for a given communication session [2]. Under such circumstance, multicast turns out to be more efficient due to the increasing demand of information sharing in those communication scenarios. It outperforms unicast by aggregating multiple flows from the same source so that network resources such as capacity and energy can be saved. However, as a traffic pattern that generalizes both unicast and broadcast, the issue of minimum-energy multicast is far more complicated compared to that of minimum-energy unicast, which is essentially a shortest path problem in static network.

Among existing works that indeed study the minimum-energy multicast problem, most of them [3], [4] are limited to static networks, which fail to well characterize many realistic scenarios where users are manifested to be moving. Consequently, the potential energy saving brought with mobility is also overlooked. Another concern lies in that the state of art mostly focuses on optimizing the energy consumed only at the transmitting side [5], [6], but does not take into consideration the receiving energy, which, as suggested by empirical studies [7], [8], is as significant as the transmission cost. Furthermore, due to the possible collisions and retransmissions in multicast/broadcast protocols, the energy consumption is not synonymous at transmitting and receiving sides. Interpreted in a more detailed way, the energy consumption of the transmitting side is usually determined by transmitting power, while at the receiving side, taking reliable MAC layer multicast as an example [9], the energy consumption may depend on the number of intended recipients as each transmission involves waiting for feedback from the receivers and resending the missing packets. Such inherent heterogeneity between transmitting and receiving energy can also be found in many other real applications such as wireless multihop networks [10] and full-duplex energy harvesting infrastructures [11]. Therefore, it is difficult to directly embed the receiving energy into existing algorithms. These motivate us to propose a general framework that jointly optimizes both the transmitting and receiving energy in mobile wireless networks.

In addition to the energy, our framework needs to incorporate the latency, another crucial performance metric, of which the demand varies in different scenarios. Real time applications such as video conferencing and traffic monitoring require the network latency to be fairly small, whereas in

other networks (e.g. satellites network), network delay is not so critical. Hence, it is reasonable to add to the network a delay constraint, meaning that all the packages must be delivered within a time length of D . Based on that, all the above cases can be well characterized by simply adjusting the size of D . Following this, there emerge a flurry of studies that quest for the relationship between delay constraint and other network performances. For instance, Small *et al.* show that relaxing delay constraint can bring about energy reduction or throughput gain [12]. Ra *et al.* propose an online algorithm for achieving near optimal energy-delay tradeoff in smartphone applications [13]. However, current literature on energy-delay tradeoff is limited to some specific applications, where the main goal is to empirically optimize the tradeoff without the corresponding theoretical exploration [13], [14]. Under such circumstance, two questions still remain open:

- How hard is it to achieve the minimum multicast energy under delay constraint in mobile wireless networks?
- How should we efficiently design such optimal multicast transmission schemes with minimum energy consumption in delay-constrained mobile wireless networks?

We therefore present a first look into the problem of multicast energy consumption optimization in delay-constrained wireless mobile networks, with both transmitting and receiving energy considered. Specifically, we consider a multicast session with k destinations selected in advance from all the n nodes in the network. With time divided into equal slots, we impose a delay constraint D on the multicast session in the sense that packets from the source must be delivered to all the k destinations within D time slots. We aim to design efficient multicast routing schemes that achieve the minimum transmitting and receiving energy. Unlike traditional multicast in wired network, which is essentially a minimum Steiner tree problem, the issue becomes more complicated when it comes to mobile wireless multicast. First, network topology is time-varying due to the mobility of nodes. This makes the task of designing an optimal transmission scheme rather challenging since the same tree may correspond to many different transmission schemes. Second, the “broadcast nature” in wireless communication, to a large extent, complicates the calculation of energy consumption since it cannot be obtained by simply taking the summation of edge weights in a minimum Steiner tree problem.

We approach the problem of multicast energy optimization in delay-constrained wireless mobile networks by formulating it as a combinatorial optimization problem called the DeMEM problem. Upon theoretical analysis on the approximation hardness of the DeMEM problem, we propose a novel energy optimizing framework called ConMap. Particularly, ConMap realizes multicast transmission in three major steps. First, it constructs an intermediate graph resulted from snapshots of graph states taken under D different slots and auxiliary gadgets for transmitting and receiving energy; Second, it builds a Steiner tree in that intermediate graph spanning all the destinations; Finally, it maps the obtained tree in that intermediate graph back into a multicast transmission scheme in the original network. The proposed ConMap flexibly allows us

to embed any directed Steiner tree algorithm into it, and turns out to be efficiently implementable with only a polynomial time complexity as well as preserving the approximation ratio of the underlying directed Steiner tree algorithms. Our main contributions are listed as follows:

- We prove that the approximation ratio of DeMEM problem in mobile wireless networks cannot be better than $\frac{1}{4} \ln k$ for any polynomial-time algorithm, which is an even stronger result than demonstration of its NP-hardness.
- We propose ConMap, a novel framework that jointly optimizes the transmitting and receiving energy in the multicast session and harnesses the results of existing directed Steiner tree algorithms to achieve a good approximation for DeMEM. Our framework does not rely on any specific characterization of energy consumption and is thus applicable to a broad range of applications.
- We empirically evaluate the performance of ConMap on real datasets, where ConMap exhibits good applicability to real network traces and yields near optimal transmission schemes compared to a brute-force exact algorithm. We also demonstrate the significance of jointly optimizing transmitting and receiving cost by showing that incorporating receiving energy into ConMap leads to transmission schemes of lower energy consumption.

The rest of this paper is organized as follows: Section II is contributed to literature review and we introduce our network and energy models in Section III. We prove the approximation hardness of DeMEM in Section IV. The approximation framework, ConMap, is proposed in Sections V and VI. We conduct simulations to evaluate the performance of our framework in Section VII and give concluding remarks as well as future directions in Section VIII.

II. RELATED WORK

A. Minimum Energy Multicast-related Problems

In the literature, the problem of transmitting energy consumption optimization is referred to as Minimum-Energy Multicast (MEM) problem, and has been extensively studied in *static* networks. Since both MEM problem and Minimum-Energy Broadcast (MEB) problem, a special case of MEM, are proven to be NP-hard [15], [16], [17], the community focuses on designing efficient heuristics. Wieselthier *et al.* study the MEM problem using the same approach as the MEB problem and propose three greedy heuristics [16], [3]. Wan *et al.* prove that both MEM problem and MEB problem can be approximated within constant ratio [4], [18]. Liang *et al.* propose approximation algorithms for MEB problem where the nodes have discrete transmission power levels [17]. Existing works also relate to the extensions of MEM and MEB such as MEM and MEB problem with reception cost [19], [9], duty-cycle-aware MEM [6] and delay-constrained MEM [5]. Furthermore, energy efficient multicast/broadcast algorithms for specific applications is also an active topic [20]. Gong *et al.* propose an efficient distributed algorithm for multicast tree construction in wireless Sensor Network [21]. However, as noted earlier, prior literature only focuses

on the minimization of transmitting energy, relies on the assumption that the receiving energy is directly proportional to the number of receivers [19] or considers the receiving energy in the context of network-wide broadcast [9]. In contrast, the proposed framework in our work jointly minimizes the transmitting and receiving energy, with applicability to more general scenarios.

In spite of a flurry of works addressing the issue of MEM in static networks, little is known about the energy consumption in mobile scenarios. As two exceptions, Wu et al. [22] study MEM in mobile *ad hoc* networks using network coding, but their focus is on energy gain from network coding rather than mobility. Guo et al. [2] propose an efficient Distributed Minimum Energy Multicast algorithm, but how delay constraint affects the energy consumption is not considered. In many cases where the timely delivery of messages is not of primary concern, relaxing delay constraint can bring gain in many aspects such as capacity and energy [12]. Considerable efforts have been made to investigate the theoretical delay-capacity relationship in large scale static and mobile wireless networks [23], [24], [25], [26].

Despite those dedications to multicast energy optimization, as far as we know, there is no prior work, other than ours, that considers the problem of jointly optimizing transmitting and receiving energy of multicast in mobile wireless networks.

B. Stochastic Network Control

Apart from the aforementioned works that mostly rely on graphical modeling of networks and deterministic packet arrival that are analogous to ours, there are a different line of research in the area of stochastic network control that deals with similar issues of network performance optimizations. The pioneering work of Tassiulas and Ephremides [27] studies the throughput optimal (unicast) routing policy of parallel-queue networks. The policy they proposed are subsequently referred to as back-pressure routing, and the Lyapunov-drift argument they applied set the basis for many works that follow. Neely et al. extend the throughput-optimal back pressure routing to time varying wireless networks [28]. Neely also proposes an energy-optimal unicast routing algorithm for time varying wireless networks [29]. Generalizing the unicast transmission pattern, Sinha et al. propose throughput optimal routing policy for multicast, broadcast and anycast transmission in wireless networks [30], [31]. Interested readers can refer to [32] for a complete survey and tutorial on this topic.

It is meaningful to distinguish our work from the aforementioned research in stochastic network control and analyze their pros and cons. Although the two kinds of works both focus on designing transmission/routing schemes with desirable properties, the feasibility of the schemes are defined in different fashions. In our work, a transmission scheme is feasible if all the designated destinations can successfully receive the packets within the delay constraint (as will be formally defined in Section III). While, in stochastic network control, each network node is associated with a queue for packets, and the feasibility condition of the transmitting policies are that the long term time average queue backlogs for all nodes must be

finite. The routing schemes designed in latter kind of works [27], [28], [29] make transmitting decisions in a distributed manner only based on nodes' queue backlogs. This gives them an advantage as the policies can achieve the properties without the knowledge of future packets arrival, future nodes mobility, etc [30], [31]. However, as the feasibility condition is defined using the time average instead of deterministic short term conditions, the algorithms in the latter works cannot be applied to derive delay-constrained transmission schemes. Indeed, the class of back pressure policies [32] often incur high packet delay that increases with the size of the networks. On the other hand, the algorithm designed in our work yields transmission schemes with constant delay (under a certain delay constraint threshold) and is approximately energy optimal.

III. MODELS AND ASSUMPTIONS

A. Mobile Wireless Network Model

We adopt the mobile wireless network model in [33] that can be reproduced as follows: time is divided into discrete time slots and the nodes remain static within a time slot. **And this is model permits a reasonable prediction of nodes' future movements. The reason is that the node mobility and the evolution of topology are heavily dependent on social and temporal characteristics of the network participants. And there are lots of examples such as the easy discovery of the topology for a DTN formed by public buses [41]. And more details of reason can be seen in [33].** With reasonable prediction of nodes' future movements, a mobile network can be modeled as a sequence of static graphs $\mathcal{G} = \{G_1, G_2, \dots\}$, with each static graph interpreted as a snapshot of nodes' positions in a time slot. Specifically, denoted as $G_t(V, E_t, w_t) \in \mathcal{G}$, each static graph is a weighted directed graph with $V = \{v_1, v_2, \dots, v_n\}$ corresponding to the n network nodes. $E_t \subseteq V \times V$ denotes the edge set and $w_t : E_t \mapsto \mathbb{R}^+$ assigns each edge a corresponding weight. An edge $e = (v_i, v_j) \in E_t$ means that v_i can send messages to v_j in time slot t with minimum transmission power $w_t(e)$.

B. Multicast Model

We consider a wireless multicast session with a source $s \in V$ and destination nodes set $T \subseteq V$ in mobile network \mathcal{G} . We impose an integral delay constraint D on the multicast session in the sense that the message must be delivered to all nodes in T within D slots. Therefore, for a mulicast session, we only need to consider the network snapshots within the delay constraint. Hence, in the sequel, we truncate the mobile network \mathcal{G} into D static graphs denoted as $\{G_1, G_2, \dots, G_D\}$.

Since we aim to design a minimum energy transmission scheme for a delay-constrained multicast session in a mobile wireless network, we next formally define the notion of transmission scheme. We refer to the process of some transmitter transmitting message to the intended receivers as a transmission. A transmission scheme specifies the transmitter, intended receivers, timing and transmission power of each transmission.

Definition 1: A transmission scheme $\pi \subseteq V \times 2^V \times \{1, \dots, D\} \times \mathbb{R}^+$ is a sequence of tuples $\{(u, V', t, p)\}$, where

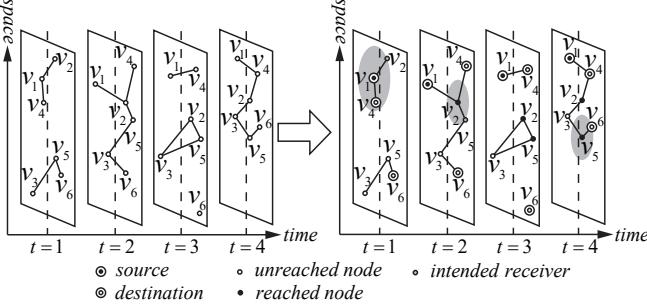


Fig. 1. A mobile wireless network and a feasible transmission scheme for a multicast session in the network; (a) a mobile wireless network (a sequence of snapshots); (b) a feasible transmission scheme for a multicast session in the network with v_1 being the source, v_4, v_6 being the destinations and a delay constraint of four time slots.

a tuple $\tau = (u_\tau, V_\tau, t_\tau, p_\tau) \in \pi$ denotes that node u_τ should transmit to the nodes in V_τ at time slot t_τ with power p_τ . Also, in a transmission scheme π , we define a node to be “reached in time slot t ” in a recursive way as follows: (i) the source s is reached in time slot 1, (ii) if a node is reached in time slot t , then it is reached in time slot t' for all $t' \geq t$, and (iii) if a node u is reached in time slot t and $(u, V, t, p) \in \pi$, then all nodes in V are reached after time slot t , or we say they have been reached in time slot $t+1$. Now, we give the definition of feasible transmission scheme and will restrict our consideration to the set of feasible transmission schemes in the sequel. Intuitively, a scheme is said to be feasible if it is without redundancy and qualified for the delay constrained multicast session.

Definition 2: A transmission scheme π is feasible if it observes the following conditions: (i) For all $\tau = (u, V', t, p) \in \pi$, it satisfies that u is reached in t and for all $v \in V'$, $e = (u, v) \in E_t$ and $p \geq \max_{e=(u,v) \in E_t, v \in V'} \{w_t(e)\}$. (ii) For all $\tau = (u, V', t, p) \in \pi$, no node in V' is reached before t . (iii) All nodes in T are reached in time slot D .

Figure 1 illustrates an example of a mobile wireless network, along with a feasible transmission scheme proposed under a multicast session in the network. The transmission scheme corresponding to the tuples $\{(v_1, \{v_2, v_4\}, 1, p_1), (v_2, \{v_5\}, 2, p_2), (v_5, \{v_6\}, 4, p_3)\}$.

C. Energy Model

For a transmission specified by tuple $\tau = (u_\tau, V_\tau, t_\tau, p_\tau)$, we model the energy consumption of this transmission as

$$\mathcal{E}(\tau) = p_\tau + f(|V_\tau|),$$

where the first part denotes the energy consumed by the transmitter side and f is a function of the number of intended receivers that can represent the receiving energy. Note that our framework does not restrict to any specific f and the choice of f may depend on practical settings [7], [9], which will be specified in Section VI. It follows that the energy consumption of a transmission scheme π is given by:

$$\mathcal{E}(\pi) = \sum_{\tau \in \pi} \mathcal{E}(\tau).$$

Now we formulate the DeMEM problem as follows.

Definition 3: (The DeMEM Problem) Given a mobile wireless network $\mathcal{G} = \{G_1, G_2, \dots, G_D\}$, a source node s , a set T of destination nodes, and a delay constraint D , the goal is to find a feasible transmission scheme with minimum energy consumption.

We note that in the definitions above, we make the following characterizations of communication in mobile wireless networks. First, we assume that the transmission rate is much faster than that of the nodes’ mobility. In other words, it is feasible to transmit multiple times to different receivers in a single time slot. This may not be desirable if we only consider transmission cost. However, it can bring possible gain when we take the receiving energy into account. Second, we can utilize the wireless broadcast nature during communications, i.e., different nodes can be reached within one transmission as long as the transmission power is large enough. Third, in the present work, we focus on the multicast of one data packet in the network, which alleviates us from the burden of interference and packet scheduling issues. We leave the minimum energy multicast with a streaming of packets as future work.

IV. APPROXIMATION HARDNESS

In this section we analyze the approximation hardness of DeMEM problem. We show that even without receiving energy (i.e., $f = 0$), the DeMEM problem cannot be approximated within a $\text{textrm{colorred}} \log k$ factor in polynomial time by reduction from acyclic directed Steiner tree problem.

Theorem 1: The DeMEM problem cannot be approximated in polynomial time within a factor better than $\frac{1}{4} \ln k$, where k is the number of destination.

Proof: Consider an instance of acyclic directed Steiner tree problem defined by an acyclic directed graph $G(V, E)$ and an edge cost function $d : E \rightarrow \mathbb{R}^+$, a root $r \in V$ and a set of vertices $S \subseteq V$ with $|S| = k$. The goal is to find a minimum cost Steiner tree (arborescence) covering all vertices in S . Based on these, we will construct an instance of the DeMEM problem by encoding the graph information into a mobile wireless network.

To begin with, we do a topological sort on the graph and get a sequence of nodes following the topological order. Without loss of generality, we only consider the nodes that do not lie before the root r in the sequence, i.e., a truncated sequence $\{r = v_0, v_1, \dots\}$. Then, we sort the relevant edges according to the order of their outgoing nodes in the truncated sequence into $\{e_0, e_1, \dots, e_l\}$. From this sequence, we will build a mobile wireless network \mathcal{G} .

First, we set V' as the set of nodes that appear in the truncated sequence. Then, for any generic edge $e_i = (u_i, v_i)$ in the edge sequence, we add (u_i, v_i) into $G_i(V', E_i, w_i)$ and set its cost as $w_i(e) = d(e)$. The constructed network \mathcal{G} consists of static graphs $\{G_1, G_2, \dots, G_l\}$ where each static graph has only one edge. Further, we designate r as the source, the nodes corresponding to the vertices in S as the destinations, and set the delay constraint $D = l$, the receiving energy function as zero. By above procedures, we have an instance of DeMEM problem and now we show that it is equivalent to the original acyclic directed Steiner tree instance.

For a Steiner tree in G , if it contains edges $\{e_{y_1}, e_{y_2}, \dots, e_{y_j}\}$, then obviously the corresponding transmission scheme with tuples $\{(u_{y_1}, \{v_{y_1}\}, y_1, d(e_{y_1})), \dots, (u_{y_j}, \{v_{y_j}\}, y_j, d(e_{y_j}))\}$ is feasible. Conversely, consider a feasible transmission scheme for the mobile wireless network. Since in each time slot there only exists one edge, each tuple in a feasible transmission scheme corresponds to some edge in the acyclic graph G . And from the requirements for the scheme to be feasible, it follows that those edges form a Steiner tree in G . Most importantly, the energy consumption of a feasible transmission scheme equals to the cost of its corresponding Steiner tree and all the reduction processes are implementable in polynomial time. Therefore, the above reduction is an approximation preserving reduction. Since for a fixed k there is no efficient $\frac{1}{4} \ln k$ approximation for acyclic directed Steiner tree problem unless $\text{NP} \subseteq \text{DTIME}(n^{\text{polylog} n})$ [34], we obtain the same approximation hardness for DeMEM problem. ■

V. THE APPROXIMATION FRAMEWORK: CONMAP

Since by Theorem 1, even approximating the DeMEM problem better than a logarithmic factor is NP-hard, it is unlikely that the DeMEM problem can be solved in polynomial time. Therefore, we need to design efficient approximation scheme to trade complexity for the optimality of the transmission scheme. In this section, we therefore propose a novel approximation framework – **ConMap**, which effectively achieves such tradeoff by transforming our DeMEM problem into a directed Steiner tree problem.

Given a specific DeMEM problem instance, **ConMap** first converts the mobile wireless network graph to an intermediate graph, then adopts established algorithms to compute an approximate minimum Steiner tree in the intermediate graph, and finally uses the computed Steiner tree to construct a near optimal transmission scheme for the original DeMEM problem. As we will disclose in the sequel, all the conversion processes are efficiently implementable, with a polynomial time complexity and the size of the intermediate graph being polynomial to that of the original network graph. Also, we can embed different algorithms for directed Steiner tree to obtain various performance guarantees.

Here for clarity and convenience, we first consider DeMEM problem without receiving energy and will demonstrate in Section VI how to integrate the receiving energy into the optimization.

A. Conversion into an Intermediate Graph

To derive an energy efficient transmission scheme, we firstly need to generate an intermediate graph so that the mobility of nodes and the broadcast nature of wireless environment can be captured. Note that the intermediate graph is a directed layered graph, with each layer corresponding to the network at a certain time slot.

Given a mobile wireless network $\mathcal{G} = \{G_1, G_2, \dots, G_D\}$, before we construct an intermediate graph, we adopt previous assumption [17] that there are p_v adjustable transmitting power levels for a generic node v . Note that even for nodes with

infinitely adjustable transmitting power, there are at most $(|V|-1)$ power levels that need to be taken into consideration. Upon the power level assignment, we can now construct an intermediate graph at layer t by applying to all nodes the procedures listed as follows, with the resulting layer denoted as L_t :

- 1) Create a vertex in L_t for each node in G_t and call these vertices “original vertices”.
- 2) For the p_u power levels of node u , add a set of vertices denoted as $\{w_{u1}, w_{u2}, \dots, w_{up_u}\}$ to the graph L_t . Hereby, without ambiguity, we will also refer to these vertices as “power levels” and their labels as their corresponding transmission powers.
- 3) Add a directed edge from u to each of its power levels and set the edge’s weight as the transmission power consumed by u when transmitting at that power level.
- 4) Add edges from w_{uj} to all the vertices that can be covered by u when transmitting at power level j and assign their weights as zero. Formally, a vertex v can be covered by u at power level j if and only if $\exists e = (u, v) \in E_t$ and $w_t(e) \leq w_{uj}$.

Upon the construction of D layers that correspond to D time slots, we connect them into the intermediate graph. Starting from L_1 , we establish the relation of adjacent layers by adding zero-weight edges between the original vertices in them that correspond to the same node in different time slots. These edges resemble the temporal links in the model of [33], with their directions following the chronological order. With all the operations above, we complete the construction of the intermediate graph denoted as \mathcal{L} , which, in the sequel, will be demonstrated to well capture both node mobility and the broadcast nature of wireless networks.

B. Computation of the Steiner Tree

We designate in L_1 the vertex that corresponds to the source node in the network as the root r and in L_D the original vertices that correspond to k destinations as terminals. Therefore, the DeMEM problem is equivalent to establishing a minimum Steiner tree rooted at r spanning all k terminals in the intermediate graph \mathcal{L} . Recall that in the intermediate graph, the weight of edges from original vertices to their power levels equals to the transmission power. And the weights of edges from power levels to the original vertices they cover are zero. It follows that the weight assignment ensures that the summation of the weights of edges in a Steiner tree characterizes the energy consumption of a wireless multicast session. In addition, the directed layering structure of the intermediate graph captures the nodes’ mobility and ensures that the Steiner tree in \mathcal{L} will be generated following the chronological order. Hence, the intermediate graph \mathcal{L} is an accurate abstraction of mobile wireless network.

Since our task now becomes computing the minimum Steiner tree in a directed graph, we can adopt existing efficient approximate algorithms [35], [36] to fulfill our purpose. **With considerable efforts searching a approximate algorithm that runs in polynomial time, the best algorithm that is available is still linear with n but exponential with k . And the approxima-**

tion factors of them are bad. However, for completeness, we present one as the following lemma.

Lemma 1: There exists an algorithm that provides an $l(l-1)k^{\frac{1}{l}}$ approximation to Steiner tree problem in directed graph in time $O(n^lk^l+n^2k+nm)$ where n is the number of vertices, k is the number of terminals and m is the number of edges. Specifically, set $l = \log k$, we get an $2\log^2 k$ approximation in time $O((nk)^{\log k})$ [35], [37].

Through implementation of the Steiner tree algorithm, we obtain an approximate Steiner tree in our intermediate graph, which will serve as the basis for the construction of transmission scheme.

C. Mapping into the Transmission Scheme

Based on the Steiner tree we computed, we proceed to design the corresponding transmission scheme. First, we present a crucial property of the conversion process.

Lemma 2: For an instance \mathcal{M} of DeMEM problem, let \mathcal{M}' be the directed Steiner tree instance converted from \mathcal{M} by ConMap framework. Each feasible transmission scheme in \mathcal{M} corresponds to a valid Steiner tree in \mathcal{M}' , and the energy consumption of the scheme equals to the cost of its corresponding Steiner tree.

Proof: We describe a procedure that maps a feasible transmission scheme π for \mathcal{M} to an edges set E_T that forms a Steiner tree for \mathcal{M}' . For each tuple $(u, V', t, p) \in \pi$, we add the edge from u to the power level corresponding to p in layer L_t together with the edges from the aforementioned power level to the original vertices of vertices in V' of L_t into E_T . Finally, we add the necessary inter-layer edges into the set. The edge set constructed by the above procedure will be a valid Steiner tree for \mathcal{M}' . In the sequel, we refer to the Steiner trees that have corresponding transmission schemes as being in *canonical form*. ■

From the proof of Lemma 2, we note that the energy consumption of any scheme is no less than the cost of the optimal Steiner tree in the constructed instance. Therefore, a transmission strategy mapped from an α -optimal Steiner tree is guaranteed to be an α -optimal transmission scheme. However, the Steiner tree computed in the intermediate graph may exhibit “aberrant phenomena” so that it has no corresponding feasible transmission scheme. Hence, in the final phase of ConMap, before we map our computed Steiner tree back into a transmission scheme, we need to prune it into canonical form.

The possible aberrant phenomena that can cause a Steiner tree E_T not to have its corresponding feasible transmission scheme is: there are edges from different power levels to multiple original vertices that correspond to the same network node, leading to redundant transmissions. To deal with this case we keep the edge with the original vertex in the earliest time slot and delete other redundant edges. An example of the pruning procedure is illustrated in Figure 2. Then, we add necessary inter-layer edges to reconnect the resulting edge set into a Steiner tree. Note that the above two procedures do not increase the cost of the resulting Steiner tree. Therefore, the pruned Steiner tree can only be closer to optimal.

After describing the three main stages of the framework, we summarize ConMap in **Algorithm 1**.

Algorithm 1 ConMap Steiner Framework for DeMEM

Input: An instance of the DeMEM problem \mathcal{M}

Output: A transmission scheme π for \mathcal{M}

- 1: Construct the corresponding intermediate graph \mathcal{L} of \mathcal{M} and form an instance of directed Steiner tree \mathcal{M}' .
 - 2: Compute a (approximate) minimum Steiner tree E_T in \mathcal{L} for \mathcal{M}' .
 - 3: Prune E_T into canonical form.
 - 4: Convert the pruned E_T to its corresponding transmission scheme π for \mathcal{M} .
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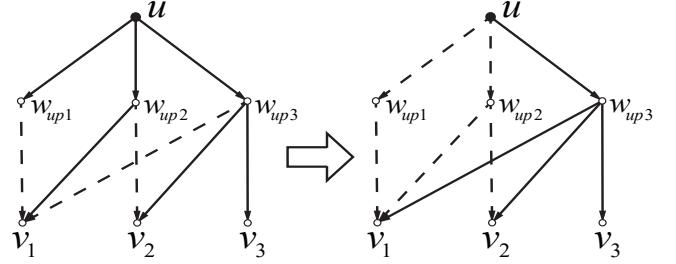


Fig. 2. **Illustration of the pruning procedure in ConMap without receiving energy.** The left part denotes a subgraph of a constructed Steiner tree that is not in canonical form. The right part represents the corresponding subgraph after the pruning procedure: absorb the original vertices that are connected to w_{up2} into w_{up3} . And the solid lines are the way that we have chosen to transmit and the dashed lines present the way that we have not chosen.

D. Illustration of ConMap

Now we illustrate our proposed framework, ConMap, using an example of a five-node mobile network, where multicast is between one source and two destinations with a delay constraint of three time slots. Figure 3 shows the intermediate graph of the network. Note that we omit some of the power levels due to space limitations. As shown in the figure, v_1 corresponds to the source, v_4 and v_5 correspond to the destinations. The solid lines denote the Steiner tree the algorithm computes in the intermediate graph. Here, the transmission scheme is $\{(v_1, \{v_2\}, 1, w_{11}), (v_2, \{v_3, v_5\}, 2, w_{22}), (v_3, \{v_4\}, 3, w_{32})\}$, which can be interpreted as follows: In the first time slot, the source transmits to v_2 using power w_{11} ; In the second time slot, v_2 transmits to v_3 and v_5 using power w_{22} ; In the last time slot, v_3 transmits to v_4 using power w_{32} .

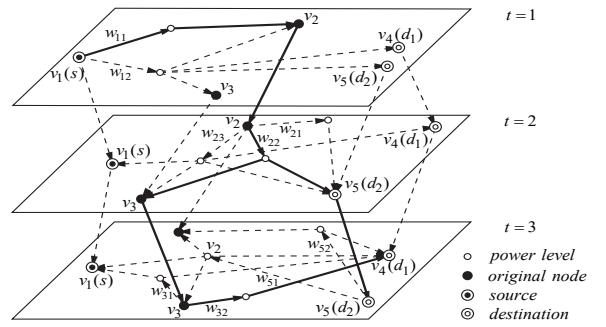


Fig. 3. **Illustration of ConMap on a five-node mobile network with a delay constraint of three time slots.**

E. Performance Analysis of ConMap

We proceed to provide analysis on the performance of the ConMap framework in terms of running time and approximation ratio. Without loss of generality, we assume the approximation guarantee of embedded algorithms for directed Steiner tree only depends on the number of terminals in the graph [34], [35].

Theorem 2: For a mobile network with n nodes, let k be the number of destinations and D be the delay constraint. Suppose the directed Steiner tree algorithm embedded in ConMap runs in $\mathcal{T}(|V|, |E|, k)$ time and achieves an approximation ratio of $g(k)$ on graph $G(V, E)$. If we only consider the transmitting energy, then ConMap returns a transmission scheme of which the energy cost is less than $g(k)$ times the optimal one in time $O(\mathcal{T}(Dn^2, Dn^3))$.

Proof: First, we focus on the time complexity of ConMap. In the first phase, since there are at most $(n - 1)$ power levels for each node and the intermediate graph contains D layers, it takes a time of $O(Dn^3)$ to construct such an intermediate graph. Note that the intermediate graph has at most Dn^2 vertices and Dn^3 edges. Hence, the time complexity of phase 2 is $O(\mathcal{T}(Dn^2, Dn^3))$. As for phase 3, the pruning and the converting process can both be done by traversing the tree, which takes $O(Dn^3)$ time. Hence, the total running time is $O(\mathcal{T}(Dn^2, Dn^3))$. Obviously the approximation ratio of the whole framework is determined by the second phase. By Lemma 2 and the fact that the number of terminals in the intermediate graph equals to the number of destinations we conclude that the approximation ratio of the proposed framework well preserves that obtained under the Steiner tree algorithm. ■

Now we give a concrete instantiation of our ConMap framework. If we embed the approximation algorithm for directed Steiner tree in [35], then we have a procedure that runs in $O((Dn^2k)^{\log k})$ time and returns a transmission scheme that is within a $2\log^2 k$ factor of the optimal one.

VI. INTEGRATION OF RECEIVING ENERGY

In this section, we illustrate how to integrate the optimization of receiving energy in our ConMap framework. Adopting the same assumption in [9], we consider the receiving energy of a multicast transmission grows sublinearly, linearly or superlinearly with respect to the number of intended receivers. The reasonability of this assumption is supported by Johnson et al. [9] through a theoretical abstraction of practical scenarios with different underlying protocols. Accordingly, we also divide the optimization technique into three regimes where the receiving energy function f is sublinear, linear or superlinear respectively. For completeness, we present the results in [9] regarding the modeling of receiving energy in the following proposition.

Proposition 1: [9] When the MAC layer of the wireless network has ACKs without repeated transmissions, the receiving energy is linear to the number of receivers; when the MAC layer model has repeated transmissions and no ACKs, the receiving energy is sublinear to the number of receivers; for certain reliable MAC layer multicast schemes with repeated

transmissions and ACKs, the receiving energy is superlinear to the number of receivers.

A. Linear Receiving Energy Functions

In this case, the receiving energy of a transmission is directly proportional to the number of receivers. Hence, we can suppose $f(k) = Ak$. The optimization combined with receiving energy can be easily integrated into the framework by setting the weights of all the edges from power levels to original vertices in the intermediate graph as A instead of zero and keeping the conversion procedures and the pruning process unchanged. We can easily verify that this modification does not influence the complexity and approximation ratio of the framework. Hence, straightforwardly we have the following theorem.

Theorem 3: For a mobile network with n, k, D be parameters defined as above, suppose the directed Steiner tree algorithm embedded in ConMap runs in $\mathcal{T}(|V|, |E|, k)$ time and achieves an approximation ratio of $g(k)$ on graph $G(V, E)$. Then ConMap returns a transmission scheme of which the energy cost is less than $g(k)$ times the optimal one in time $O(\mathcal{T}(Dn^2, Dn^3))$ when the receiving energy is linear to the number of intended receivers.

B. Sublinear and Superlinear Receiving Energy Functions

When the receiving energy is not linear with regard to the number of receivers, the straightforward modification made in the case of linearity is not applicable. We therefore present a new way to tackle this and merge the consideration of sublinear and superlinear receiving energy into our ConMap framework.

Construction of intermediate graph: The main idea lies in that instead of adding a simple edge from each power level of transmitters to their receivers, we create gadgets in the intermediate graph that charges the Steiner trees for the cost corresponding to receiving energy.

For a power level w_{up} (recall the definition in Section 5.1) of original vertex u that covers k receiving nodes v_1, v_2, \dots, v_k , the construction of the gadget is as follows. First, we create k rows of new vertices, with each row containing k vertices, which, in the sequel, will be referred to as “virtual vertices”. We denote the vertices in row i as $v_{i1}, v_{i2}, \dots, v_{ik}$. Second, we add a zero-weight edge from each virtual vertex to its corresponding receiving node (e.g., from $v_{11}, v_{21}, \dots, v_{k1}$ to v_1). Then, for the virtual vertices in two adjacent rows, we insert edges between them such that the subgraph formed by the nodes in each two adjacent rows is a complete bipartite graph. The direction of these edges is from the row with lower index to the row with higher index. And we set the weights of all the edges between row i and row $i + 1$ as $f(i + 1) - f(i)$. Finally, we add edges from v_p to all the virtual vertices in the first row and set their weights as $f(1)$. An example of the gadget is shown in Figure 4.

We justify the above construction by proving a similar argument as Lemma 2 in the following lemma.

Lemma 3: For an instance \mathcal{M} of DeMEM problem with receiving energy, let \mathcal{M}' be the directed Steiner tree instance

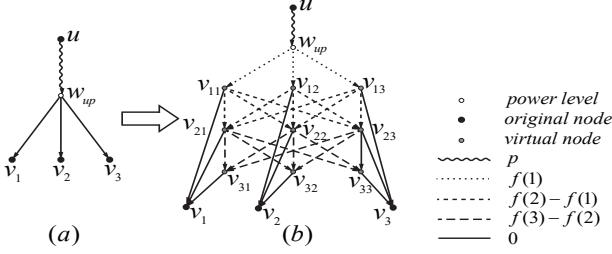


Fig. 4. Illustration of the gadgets for receiving energy: (a) the original gadget in ConMap without receiving energy for a power level of original vertex u and the nodes v_1, v_2, v_3 that u can cover transmitting at power p . (b) the corresponding gadget ConMap creates for this power level of u when considering the receiving energy.

converted from \mathcal{M} by ConMap framework considering the receiving energy. Each feasible transmission scheme in \mathcal{M} corresponds to a valid Steiner tree in \mathcal{M}' , and the energy consumption of the scheme equals to the cost of its corresponding Steiner tree.

Proof: Indeed, for a tuple $\tau = \{u_\tau, \{v_1, v_2, \dots, v_m\}, p_\tau, t_\tau\}$, it can be encoded as a subgraph consisting of an edge from the original vertex corresponding to u_τ to its power level p_τ in L_t , edges that form a path traversing one of each receiver node's corresponding virtual nodes once in the gadget on some row (e.g. $v_{11}, v_{22}, \dots, v_{mm}$) and edges from the traversed virtual vertices in the gadget to their corresponding receiver nodes (e.g. $(v_{11}, v_1), (v_{22}, v_2), \dots, (v_{mm}, v_m)$). It is easy to verify from the weight assignments of the edges that the total weight of this subgraph equals to the energy consumption of the transmission. And also as in the proof of Lemma 2, we add necessary edges between different layers, relating a feasible transmission scheme to a Steiner tree. ■

After the computation of a Steiner tree E_T in the intermediate graph, however, the existence of the gadgets complicates the pruning process for converting the constructed Steiner tree E_T into canonical form. In addition to the pruning procedures mentioned in the previous section, we also need to consider the aberrant phenomena caused by the edges in the gadgets. The pruning process is demonstrated in Figure 5 using two illustrative cases. Taking Figure 5(b) for example, the constructed Steiner tree entails two receivers v_1 and v_2 in the transmission but the edges that the algorithm chooses in the gadget are $(v_{11}, v_1), (v_{11}, v_{22}), (v_{22}, v_2), (v_{11}, v_{23}), (v_{23}, v_3)$ instead of the path $(v_{11}, v_{22}), (v_{22}, v_{33})$ and edges $(v_{11}, v_1), (v_{22}, v_2), (v_{33}, v_3)$, whereas the latter are the ones that correspond to a legitimate transmission scheme. Therefore, we incorporate searching such aberrant subgraphs in the gadgets and correct them to the legitimate ones in the pruning process of the framework. For the Steiner tree E_T that ConMap constructs for the intermediate graph, denote g_0 as a generic subgraph that lies in the intersection of E_T and some gadget. We formally present the pruning procedure for g_0 in **Algorithm 2**. We process all the subgraphs g_0 in E_T as in **Algorithm 2** and finally convert E_T to an feasible transmission scheme.

Performance analysis of ConMap with receiving energy: We summarize the performance guarantee of ConMap with receiving energy in the following theorem.

Theorem 4: For a mobile network with n nodes, let k be the

Algorithm 2 Pruning procedure for subgraph g_0

Input: An subgraph g_0 in the intersection of E_T and some gadget

Output: A pruned gadget that is eligible for a Steiner tree in canonical form

- 1: $G :=$ the gadget that g_0 lies in.
 - 2: $\{v_{r1}, v_{r2}, \dots, v_{rk}\} :=$ the set of receiving nodes in g_0 .
 - 3: $u, w_{up} :=$ the transmitting node and the transmitting power level in g_0 .
 - 4: Delete all the edges except (u, w_{up}) in g_0 .
 - 5: Add the edge (w_{up}, v_{r1}) to g_0 .
 - 6: Add the set of edges $\{(v_{r11}, v_{r22}), \dots, (v_{r_{k-1}(k-1)}, v_{rk})\}$ to g .
 - 7: Add the set of edges $\{(v_{r11}, v_{r1}), \dots, (v_{rk}, v_{rk})\}$ to g_0 .
 - 8: **return** g_0
-

number of destinations and D be the delay constraint. Suppose the directed Steiner tree algorithm embedded in ConMap runs in $\mathcal{T}(|V|, |E|, k)$ time and achieves an approximation ratio of $g(k)$ on graph $G(V, E)$. Then in time $O(\mathcal{T}(Dn^4, Dn^5))$, ConMap returns a transmission scheme of which the energy cost is less than $g(k)$ times the optimal when the receiving energy is sublinear to the number of receivers and less than $\frac{g(k)f(k)}{kf(1)}$ times the optimal when the receiving energy is superlinear to the number of receivers.

Proof: Based on Lemma 3, we only need to consider the change of cost of the tree brought by the pruning process. An important observation of the pruning procedure is that, the edges in the gadget can only move to upper level. Therefore, when f is sublinear, we have $f(i+2) - f(i+1) \leq f(i+1) - f(i)$, and it follows that the pruning procedure does not increase the cost of the resulting tree. When f is superlinear, we have $f(i+2) - f(i+1) \geq f(i+1) - f(i)$ for all i . And as we know $f(1)$ represents the least energy consumption. The consumption after the pruning procedure is $f(k)$. However, before pruning the least energy consumption is $kf(1)$. Hence, in the worst case the pruning procedure increases the cost to a factor of at most $\frac{f(k)}{kf(1)}$. For complexity issues, an m -receiver gadget contains at most m^2 vertices and m^3 edges. Hence, all the gadgets in the intermediate graph contain at most Dn^4 vertices and Dn^5 edges. Hence, if we use a Steiner tree algorithm with a time complexity of $\mathcal{T}(|V|, |E|, k)$ and an approximation ratio of $g(k)$, our Conmap framework will yield a $g(k)$ -approximation of the optimal transmission scheme for sublinear cost functions and a $\frac{g(k)f(k)}{kf(1)}$ -approximation for superlinear cost functions in a time complexity of $O(\mathcal{T}(Dn^4, Dn^5, k))$. ■

C. An Alternative Gadget Construction

As mentioned in the previous section, ConMap framework increases the approximation ratio of the embedding Steiner tree algorithm by a factor up to $\frac{f(k)}{kf(1)}$ when the receiving energy function is superlinear. Therefore, we propose a second method of constructing gadgets in ConMap to capture the receiving energy that can eliminate this factor for superlinear receiving energy functions. The method produces polynomial

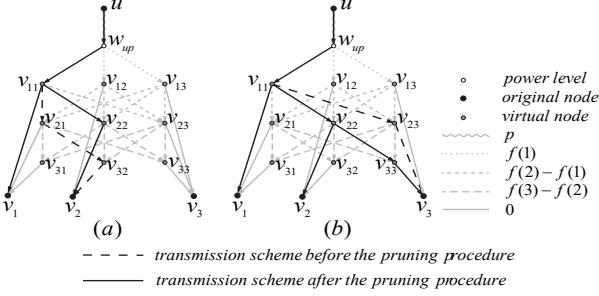


Fig. 5. Illustration of two pruning processes of converting the computed Steiner tree in the intermediate graph into canonical form. (a) and (b) are two cases of the conversion.

number of additional nodes when the maximum number of neighbors of a node in the network is logarithmic to the total number of nodes.

The construction works as follows. Starting from the original intermediate graph constructed in Section V, for each power level, we add a new vertex for each possible combination of receiving nodes and refer these vertices as receiving levels. Then, we add edges from each power level to all their corresponding receiving levels and from the receiving levels to all their corresponding receivers. The first set of edges are assigned with weights equal to the receiving energy, i.e., $f(k)$ for a receiving level with k receivers. And the second set of edges are zero-weighted ones.

Now, we justify the validity of the construction. In the gadgets constructed as above, a tuple (u, V', t, p) in a transmission scheme corresponds to a subgraph consisting of an edge from the original vertex associated with u to its power level corresponding to p in L_t , an edge from the aforementioned power level to the receiving level corresponding to V' as the intended receivers and edges from the receiving level to all the vertices associated with nodes in V' at time slot t . For the pruning procedure, apart from the first one mentioned in the previous section, we also need to guarantee that the receiving levels included in the computed Steiner tree is connected to all its receivers. If not, we can replace this receiving level with the receiving level that corresponds to the set of connected receivers only. This, again, will not increase the cost of the resulting Steiner tree, and thereby the approximation ratio can be preserved.

In terms of the time complexity of the framework, suppose the embedded Steiner tree algorithm runs in $\mathcal{T}(|V|, |E|, k)$ time. In this case, the number of additional vertices in a gadget is of $O(\Delta^2)$ (The Δ is the degree of original vertex u). Hence, the total time complexity of ConMap that applies this alternative gadget construction method becomes $O(\Gamma(\Delta^2 Dn^2, \Delta^2 Dn^3))$. Therefore, this method is suitable for implementation only when the maximum degree of a node is no larger than logarithmic to the total number of nodes in the network. An example of this alternative construction is illustrated in Figure 6.

D. Impact of Mobility Prediction

In this section, we briefly discuss the impact of the accuracy of future mobility prediction on the performance of ConMap from a theoretical perspective. We will characterize this impact empirically in Section VII-E. As assumed previously, the theoretical guarantees of ConMap are derived on the basis of perfect future mobility prediction. Now suppose that the mobility prediction we are able to make has an accuracy of $1 - \alpha$, i.e., for each edge $e = (u, v)$ in some G_t , it satisfies that $(1 - \alpha)w_e^* \leq w_e \leq (1 + \alpha)w_e^*$, where w_e^* is the true value of the weight of e . We subsequently prove that, based on this inaccurate prediction of future mobility, the degradation of ConMap will be no larger than a factor of $\frac{1}{1-\alpha}$, which demonstrates the robustness of ConMap against mobility prediction errors.

Let π_0 denote the optimal transmission scheme in the $(1 - \alpha)$ -accurate network and π^* denote the optimal transmission scheme in the true network. A straightforward observation is that the inaccuracy of the mobility prediction only has influence on transmitting energy optimization. Denote \mathcal{E} as the energy consumption function in the true network and \mathcal{E}' as the energy consumption function in the inaccurate network. Since $\frac{1}{1+\alpha}w_e \leq w_e^* \leq \frac{1}{1-\alpha}w_e$, we have $\mathcal{E}'(\pi^*) \leq \frac{1}{1-\alpha}\mathcal{E}(\pi^*)$. By the optimality of π^* in the inaccurate network we also have $\mathcal{E}'(\pi_0) \leq \mathcal{E}'(\pi^*)$. It follows that $\mathcal{E}'(\pi_0) \leq \frac{1}{1-\alpha}\mathcal{E}(\pi^*)$. Hence, if ConMap is guaranteed to find a transmission scheme that is less than β times the optimal one in the inaccurate network we possess, then the transmission scheme it finds is also within $\frac{\beta}{1-\alpha}$ of the optimal transmission scheme in the true network, where the energy consumption is calculated with respect to the true network.

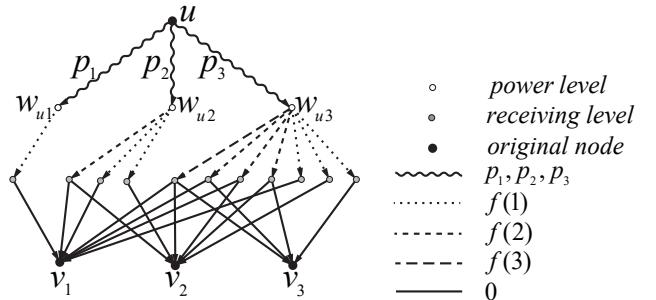


Fig. 6. An alternative construction of receiving gadgets.

VII. EXPERIMENTS

In this section, we empirically evaluate the performance of ConMap framework. We discuss our experimental settings in Section VII-A and present detailed empirical results in subsequent sections.

A. Experimental Setup

We validate the effectiveness of our ConMap framework based on three real datasets. The basic descriptions of the three data sets are presented as follows.

- **Brightkite Social Network [38]:** Brightkite is a location-based networking service provider where users share their locations by checking-in. We consider the users

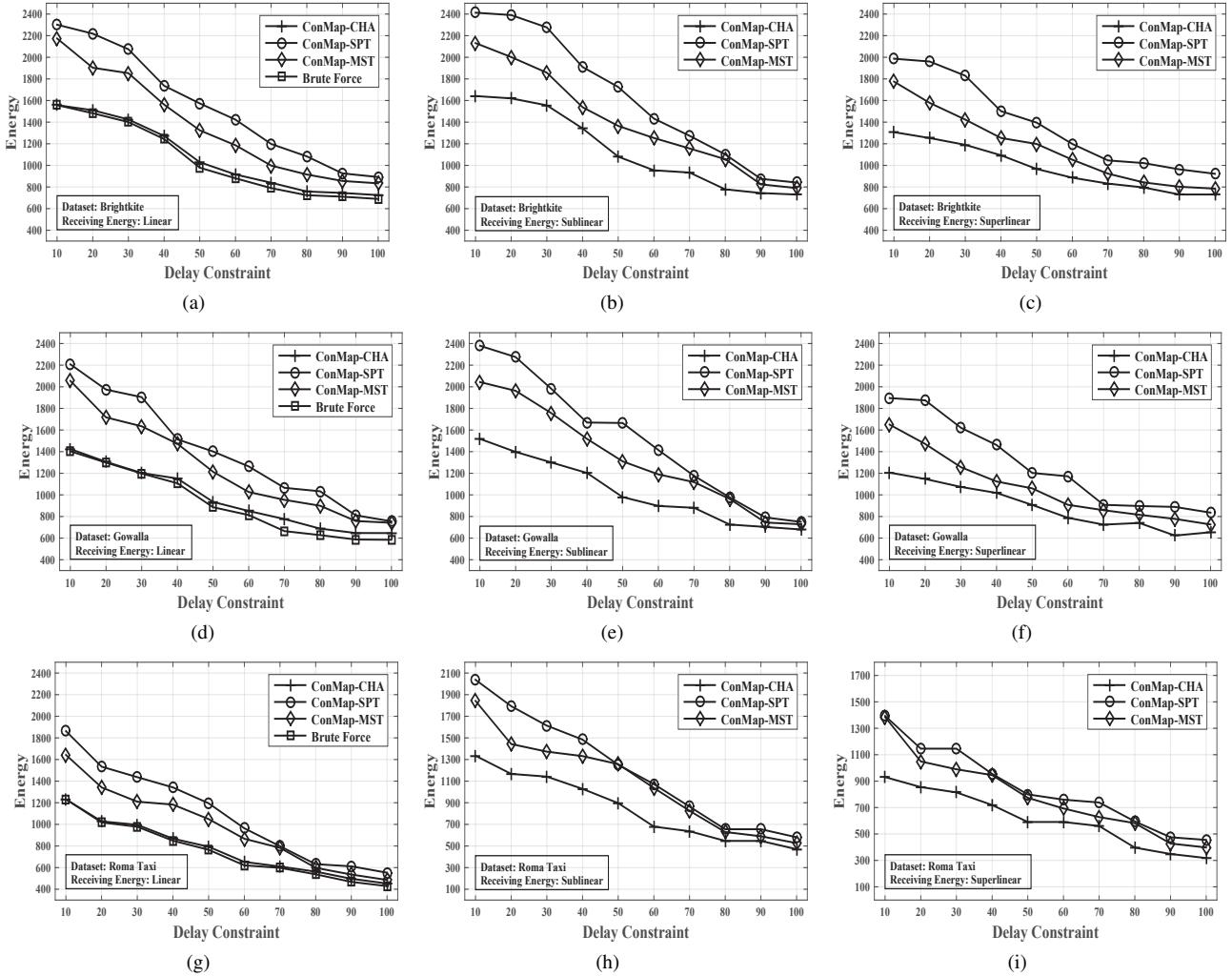


Fig. 7. The energy consumption of the transmission schemes yielded by the three/four algorithms under different scenarios with six destinations. The dataset and receiving energy function are labeled in the figures.

as network nodes and use their location information at different timestamps to create mobility trace.

- **Gowalla Social Network [38]:** Gowalla is a location-based social networking website where users share their locations by checking-in. Similarly, we consider users as network nodes and use their location information to create mobility trace.
- **Roma Taxi [39]:** This dataset contains mobility traces of taxi cabs in Rome, Italy. It contains GPS coordinates of approximately 320 taxis collected over 30 days.

Parameter setting: For each dataset, we extract ten groups data of 50 nodes in 100 time slots and normalize the distance between nodes into the range of 10 to 5000 for consistency. We designate one source and four/six destinations and set the delay constraint of the multicast session from 10 to 100 time slots. Due to space limitations, we defer all the graphical representation of results in the scenarios of four destinations to **Appendix A**. Furthermore, for the energy model, we adopt a widely used assumption that the transmission energy is α power of the transmission range [4], [3]. And for receiving energy, we select $f(k) = 50k$, $f(k) = 100k^{\frac{1}{2}}$ and $f(k) = 20k^2$ where k is the number of intended receivers in a transmission as the

representatives of linear, sublinear and superlinear functions of receiving energy, respectively. All the results demonstrated are the average of the ten groups. Note that we are interested in the relative quantity of energy consumption, and thus we do not relate the amount of energy consumption to specific physical quantity.

Algorithms used for performance comparison: To justify the performance and flexibility of our ConMap framework, we embed three different algorithms in ConMap for computing the Steiner tree in the intermediate graph. The first one is the **SPT heuristic** [3], [4]. It constructs the Steiner tree by finding the shortest path from the source to all the destinations. The second one is the **MST heuristic** [3], [4] that computes the Steiner tree by firstly establishing a minimum spanning arborescence and then pruning the unnecessary edges. And the last one is the **approximation algorithm** for directed Steiner tree problem proposed by Charikar et al. [35] and further improved in [37]. We also implement a **brute-force algorithm** to obtain the exact solutions for linear receiving energy function as the baseline. We omit the corresponding exact results for the cases of sublinear and suplinear settings primarily due to two reasons: (i) our algorithm adds a gadget

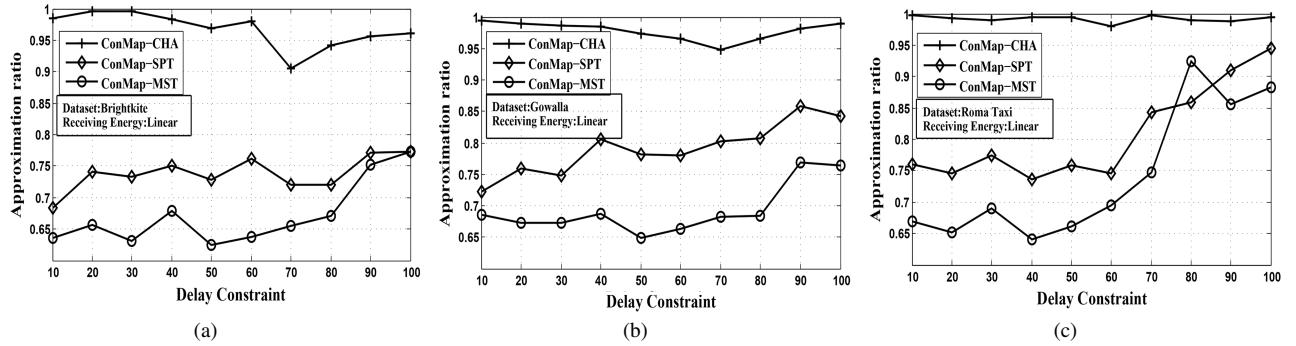


Fig. 8. The approximation ratio yielded by the energy consumption that has been shown above. The dataset, receiving energy function are labeled in the figures.

into the graph which adds the $O(n^2)$ nodes into the graph; (ii) the brute-force algorithm is an algorithm that searches all the possible combinations of the receivers for each node, thus exhibiting prohibitively large time complexity.

B. Evaluation of ConMap Framework

First, we investigate the performance of the proposed ConMap Framework. For each group of data under three different receiving energy functions, we apply the ConMap framework embedded with three directed Steiner tree algorithms mentioned above and name them **ConMap-SPT**, **ConMap-MST**, **ConMap-CHA** respectively. Imposing a delay constraint from 10 to 100 time slots, we compute the average energy consumption for the transmission schemes designed for the ten groups of data by each algorithm. Additionally, we run a brute-force algorithm on each group of data under linear receiving energy function to present the cost of the optimal transmission scheme for comparison. The results are plotted in Figure 7 (and Figure 10 in **Appendix A**).

As demonstrated in Figures 7(a), 7(d) and 7(g), when embedded with the Charikar's approximation algorithm [35] that has a logarithmic approximation ratio and the receiving energy function is linear, the energy consumption of the transmission schemes yielded by ConMap is only 9.1% and 2.6% higher than the optimal ones derived by the brute force algorithm in Brightkite, Gowalla and Roma Taxi datasets respectively. This verifies the conclusion that the ConMap framework indeed preserves the approximation ratio of the embedded directed Steiner tree algorithms, as we have proved theoretically in Theorem 2. We conjecture that the simulation results under superlinear and sublinear receiving energy functions still enjoy such property. However, as deriving the optimal transmission scheme in such cases is too computationally intensive, we leave the comparisons between ConMap and the brute force algorithm under superlinear and sublinear receiving energy functions as future work.

Apart from the Charikar's approximation algorithm which has relatively high time complexity, we can also embed faster heuristics into the framework to achieve the tradeoff between the optimality of the derived transmission schemes and the complexity of the algorithms. Figure 7 also shows this flexibility of ConMap, as we can more efficiently obtain transmission schemes that lead to more energy consumption by embedding SPT or MST heuristics in the framework.

Furthermore, it turns out that although the approximation ratio of MST and SPT heuristic can be proportional to the number of nodes [4] in worst cases, the performance of ConMap-MST and ConMap-SPT are significantly better when applied on real life datasets under different settings. As we can further see from the Figure 8, the simulation results indicate that in each scenario, the average approximation ratio is more than 0.68 for ConMap-MST and is more than 0.63 for ConMap-SPT. This demonstrates the generality of the ConMap framework. And more intuitively, we can see from Figures 8(a), 8(b), 8(c) that ConMap-CHA has the best approximation ratio which tends to be more than 0.9 and the approximation ratio tends to be larger as delay constraint becomes larger.

C. Significance of Receiving Energy

In order to capture the importance of jointly optimizing the transmitting energy and receiving energy for multicast in mobile wireless networks, we embed the Charikar's approximation algorithm into the original ConMap framework that only considers the transmitting energy (as depicted in Section V) and apply it to the aforementioned scenarios. We compare it with the ConMap framework integrated with receiving energy (ConMap-CHA) and summarize in Figure 9 (and Figure 12 in **Appendix A**) the average energy consumption of the schemes they derive under different delay constraints. For the ConMap without receiving energy consideration, the optimization is completed with respect to transmitting energy only, but the energy consumption of the schemes derived by ConMap is calculated for the sum of transmitting and receiving energy.

From Figure 9, we can see that in the three datasets, when the energy receiving function is superlinear or sublinear, the consideration of receiving energy can bring a gain of more than 15% in terms of energy consumption of the derived schemes. The gain for linear energy model is comparatively smaller as linear receiving energy function is more homogeneous to transmitting energy.

D. Characterization of Delay-Energy Tradeoff

Our empirical results also delineate the delay-energy tradeoff existing in mobile wireless networks. As shown in Figure 7, the achievable energy consumption of transmission schemes decreases with the relaxation of delay constraint on the multicast session. More specifically, by relaxing the delay

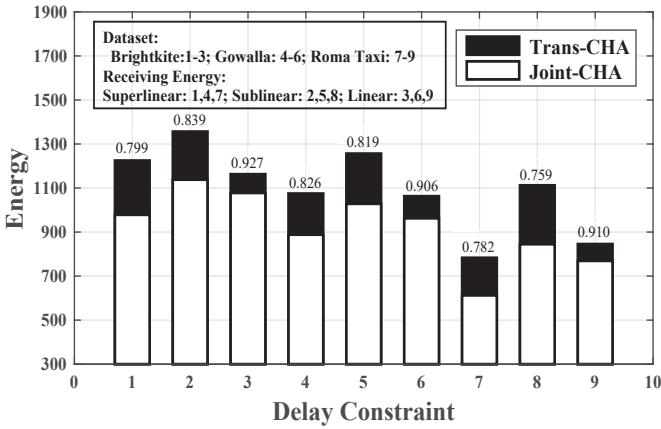


Fig. 9. Comparisons of the performance of ConMap with (white) and without (black) consideration for receiving energy in different scenarios with six destinations. The ratios between the average energy achieved by ConMap with receiving energy and that by ConMap without receiving energy is labeled on the figure.

constraint from 10 time slots to 100 time slots, the minimum achievable energy decreases by about 50%. Interpreted from the perspective of mobile wireless networks, this is attributed to the fact that as the delay constraint becomes looser, nodes can store and carry the message until there are better chances to forward it, therefore it leads to the feasibility of transmission schemes with smaller energy consumption. The experimental results also conform to the theoretical scaling laws derived in previous works [25], [26] for mobile wireless networks. However, by relaxing the delay constraint from 10 time slots to 100 time slots the time complexity also becomes larger. Since the ConMap-CHA has relatively high time complexity among all the three algorithms and the ConMap-MST can also suit many kinds of other scenarios, we choose to run the ConMap-MST algorithm in the Brightkite dataset and record the corresponding time complexity. We summarize the result in Table II, where the unit of the time is set to be one second in the table. The result also verify our theoretical form of the time complexity, which is derived to be $O(T(Dn^4, Dn^5))$. As a result, in practice we need to choose a best Delay Constraint to balance between the Energy consumption and time complexity.

E. Robustness of ConMap Against Mobility Prediction Errors

In order to empirically evaluate the robustness of ConMap against inaccurate mobility prediction, we add perturbations of 0.05, 0.10 and 0.15 to the original network data respectively, i.e., randomly perturb the edge weights of the original network into values in the range of $(1 - \alpha, 1 + \alpha)$ ($\alpha = 0.05, 0.10, 0.15, 0.20, 0.25$) of the original ones. We then calculate the relative increase in the cost of the transmission schemes derived by **ConMap-CHA** when there are six destinations and the delay constraint equals to 60. The results are shown in Table I. Again, the results are the average of the ten groups of destinations.

From the table, we can see that even when the mobility prediction has an error of 0.25, the impact on ConMap is still no larger than 0.12 for all data sets and all receiving energy regimes. Also sometimes the inaccuracy of the mobility

prediction accidentally improves the transmission schemes (as indicated by the negative entries in the table). Therefore, reasonable future mobility prediction is sufficient for ConMap to work well in practice, and ConMap can tolerate certain amount of prediction error.

VIII. CONCLUSION AND FUTURE WORK

In this paper, we studied the problem of jointly optimizing multicast energy consumption in delay-constrained mobile wireless networks. We first formulated our problem, called DeMEM, as a combinatorial optimization problem, and then proved that the approximation ratio of any polynomial time algorithms for DeMEM cannot be better than $\frac{1}{4} \ln k$. As the approximation hardness of DeMEM implies its NP-hardness, aiming to provide efficient approximation schemes, we proposed a novel approximation framework – ConMap. The proposed ConMap first converts DeMEM into an equivalent directed Steiner tree problem through creating auxiliary graph gadgets to capture energy consumption, then maps the computed tree back into a transmission scheme. We have demonstrated the generality, flexibility and efficiency of ConMap framework by theoretical analysis and extensive simulations on real life network datasets. Our experiments also reveal useful insights as the significance of the consideration of receiving energy and the delay-energy tradeoff in mobile wireless networks.

There are several prospective directions that are worthy of future exploration based on the present work. First, it will be interesting to take into account the streaming of packets that need to be sent. In this way, it can be foreseen that during transmission one also needs to consider scheduling later packets in the stream; Second, the proposed algorithm is centralized, based on the assumption that node mobility is known exactly a priori. Hence, it will be nice to relax the assumption a little bit, and design a way to implement ConMap in a distributed fashion which is more apt for practical situations; Last but not least, the incorporation of interference into the model is also likely to bring about different results, especially when we consider multiple transmissions at each time slot. We believe that multiple transmission has the potential of improving both network performance and feasibility, given that power levels are finite.

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Receiving Energy	Data sets	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.15$	$\alpha = 0.20$	$\alpha = 0.25$
Linear	Brightkite	-0.012	0.024	0.036	0.068	0.082
	Gowalla	0.005	0.013	0.012	0.025	0.048
	Roma Taxi	0.026	0.047	0.065	0.078	0.093
Sublinear	Brightkite	-0.010	0.025	0.034	0.069	0.083
	Gowalla	-0.015	-0.002	0.014	0.023	0.033
	Roma Taxi	0.021	0.044	0.069	0.079	0.102
Superlinear	Brightkite	0.004	0.027	0.038	0.067	0.099
	Gowalla	0.009	0.015	0.019	0.032	0.055
	Roma Taxi	0.029	0.051	0.073	0.091	0.112

TABLE I
Relative increase of ConMap-CHA when the mobility prediction is inaccurate.

Receiving Energy	D=10	D =20	D=30	D=40	D=50	D=60	D=70	D=80	D=90	D=100
Linear	0.98271	1.7478	2.6718	3.4284	4.3861	5.0947	6.2312	7.0891	7.9021	8.6539
Sublinear	18.1364	34.6246	53.5352	70.2988	88.1364	105.6275	122.8926	141.1611	158.5848	177.0769
Superlinear	16.9433	31.1417	45.0861	62.0332	77.1518	93.669	110.6978	121.2471	137.8163	151.8032

TABLE II
Time complexity of the ConMap-CHA runs in Brightkite dataset with $n = 50$.

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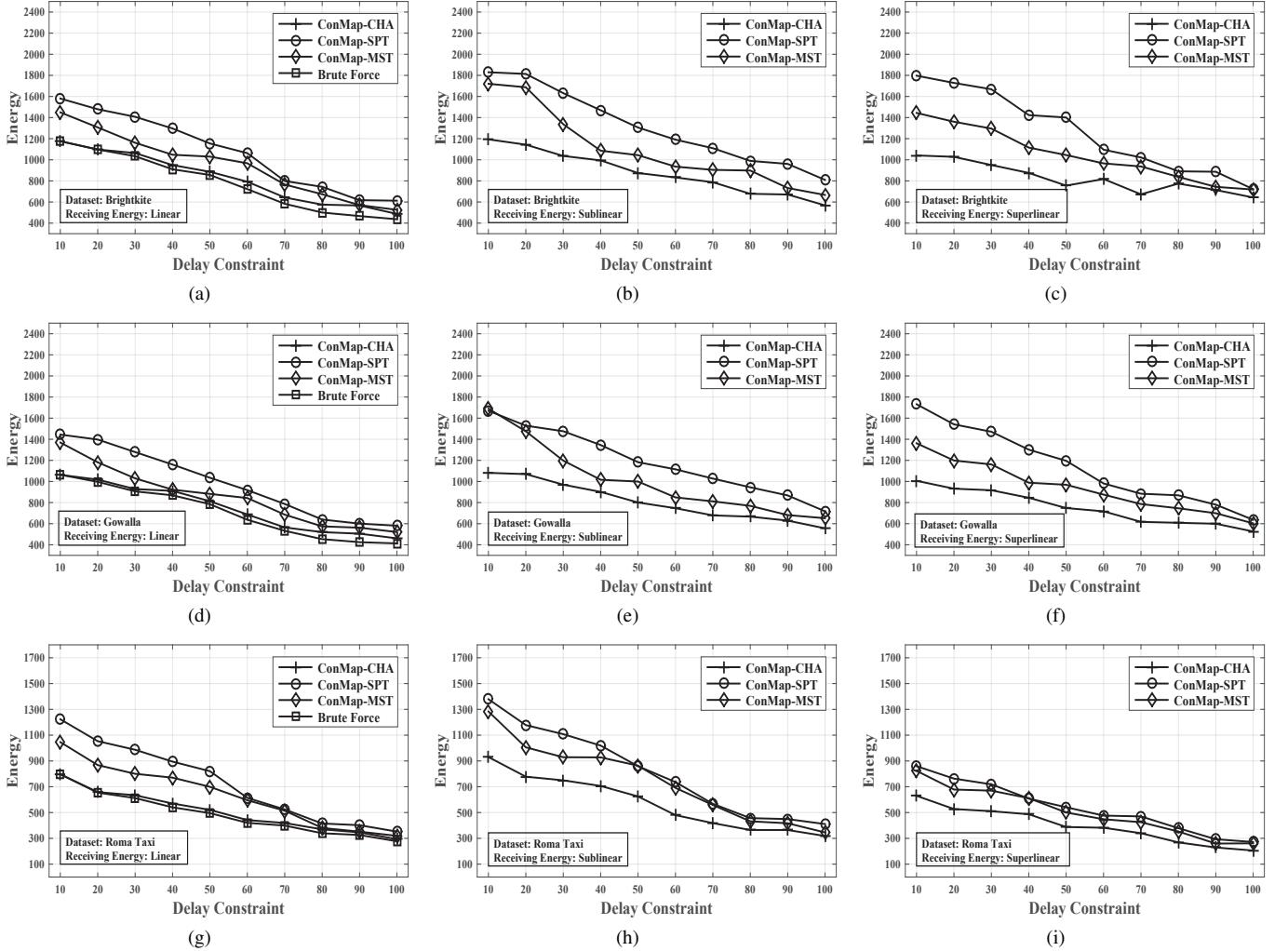


Fig. 10. The energy consumption of the transmission schemes yielded by the three/four algorithms under different scenarios with four destinations. The dataset, receiving energy function and approximation ratio are labeled in the figures.

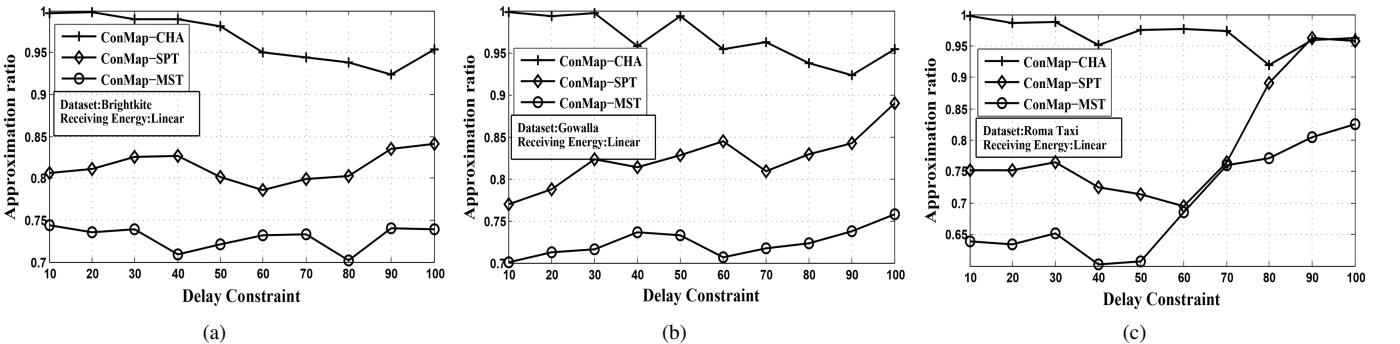


Fig. 11. The approximation ratio yielded by the energy consumption that has been shown above. The dataset, receiving energy function are labeled in the figures.

APPENDIX A GRAPHICAL RESULTS OF SCENARIOS WITH FOUR DESTINATIONS

In this section, we present the figures (Figures 10, 11 and 12) that demonstrate our simulation results on the settings with four multicast destinations.

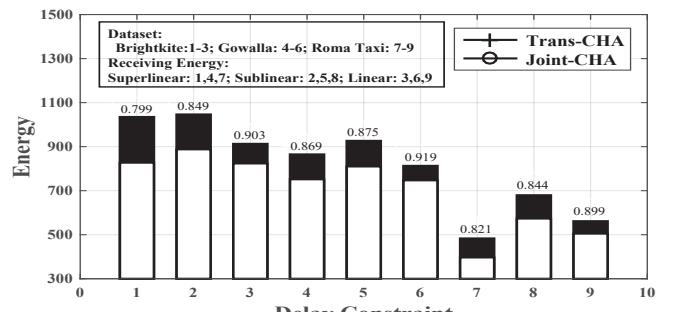


Fig. 12. Comparisons of the performance of ConMap with (white) and without (black) consideration for receiving energy in different scenarios with four destinations. The ratios between the average energy achieved by ConMap with receiving energy and that by ConMap without receiving energy is labeled on the figure.