Unraveling Impact of Critical Sensing Range on Mobile Camera Sensor Networks

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Abstract—In camera sensor networks (CSNs), full view coverage, meaning that any direction of any point in the operational region is covered by at least one camera sensor, plays a significant role in object identification. While prior work is dedicated to static CSNs for the seek of critical condition to achieve full view coverage, such performance still remains unknown in mobile CSNs. In this paper we take the initiative to address this issue, where a centralized parameter, i.e., equivalent sensing radius (ESR), is defined to unravel the critical requirement for asymptotic full view coverage in mobile heterogeneous CSNs in the sense that camera sensors of different sensing capabilities are moving around in target area. Specifically, we derive ESR under three different mobilities, i.e., 1-dimensional and 2-dimensional random walks and random rotating model, and then explore respectively the corresponding critical conditions to achieve almost surely coverage[1]. The static network is introduced as a baseline in order to gain a clear understanding of how mobility affects coverage performance differently. Interestingly, we find that both 1 dimensional and 2 dimensional random walks exhibit a smaller ESR than static one whereas ESR is even larger in random rotating mobility than that in static CSNs. Moreover, the almost surely coverage is found to be around 1.225 times of the critical condition to achieve coverage with high probability[2], and therefore turns out to be a stronger result compared to the traditional coverage with high probability. We then turn to the impact of various mobility patterns on sensing energy consumption, a metric that is closely related to ESR, and show that it can be decreased by random walks under certain delay tolerance. The relationship between ESR and percentage of full view coverage is also discussed and the results unify those under homogeneous CSNs.

1 Introduction

Coverage, as a crucial performance metric, is commonly used in Wireless Sensor Networks (WSNs) in measurement of how well a target field is monitored by sensors. Intuitively, a better guarantee of coverage can lead to higher network controllability, and therefore manifests its importance in a wide range of control-aware applications such as security surveillance, traffic control, environmental monitoring, intrusion detection, industrial process control [1] and etc. As a kind of derivative of WSNs, Camera Sensor Networks (CSNs) have recently attracted an increasing amount of attention due to the significant ability of visual information collection, and consequently can provide more comprehensive and accurate information about real-time situation. Different from traditional sensors that possess omnidirectional sensing ability, a camera sensor is only capable of sensing within a certain angle of view, beyond which it fails to capture any information.

Such phenomenon can be briefly attributed to the viewing direction, which, as a property that exclusively belongs to camera sensors, distinguishes the coverage issue of CSNs from the one that has been intensively studied in conventional WSNs [2]- [13]. The main reason is that the model suggested by those works characterizes coverage through simply assuming that an object is considered to be covered if it is within the sensor's sensing range, which is usually supposed as a disk. However, when it comes to a CSN, such simplification falls short of well reflecting the features of camera sensors in the sense that the model fails to embody viewing direction. To solve this, Wang *et al.* [14] took a pioneer step ahead by proposing a novel concept called **full view coverage** in judgement of coverage performance in CSNs. An object is said to be *full view covered* if its viewed direction is always sufficiently close to its facing direction, regardless of wherever it actually faces. The advantage of full view coverage lies in incorporating the object recognition [15], and meanwhile guarantees that every perspective of an object at any point is under the view of some camera sensor if the target area is full-view covered.

A key step to construct a full-view covered CSN is to find out under which conditions such full view coverage can be achieved. Nevertheless, in contrast to the huge efforts made in traditional WSNs, the issue of coverage in CSNs still remains underexplored. With their proposed coverage metric as stated above, Wang *et al.* [14] considered two types of deployment, i.e., random and uniform deployment and the lattice based one in CSNs, and provided a sufficient condition for full view coverage in the former one as well as a critical (i.e. both necessary and sufficient) condition under the latter. Following that, Wu *et al.* [16] introduced heterogeneity into the CSN, where they also analyzed the necessary and sufficient conditions to achieve full view coverage, respectively. Another line of existing works are con-

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1. Let A_n be a countable collection of sets, and $\limsup_{n\to\infty} A_n = \bigcap_{n=1}^{\infty}\bigcup_{m\geq n} A_m$, which means that for every element in the $\limsup$, for every N, there exists an A_n with $n \geq N$ that has the element. For event A_n, if $\mathbb{P}(\limsup_{n\to\infty} A_n) = 1$, we say the event A_n will almost surely happen or happen infinitely often. Let $\liminf_{n\to\infty} A_n = \bigcup_{n=1}^{\infty}\bigcap_{m\geq n} A_m$, which means that for any element in the $\liminf$, there is an N such that the element is in every A_n for any $n \geq N$. For event A_n, if $\mathbb{P}(\liminf_{n\to\infty} A_n) = 1$, we say the event A_n will happen eventually.

2. If event A_n satisfies $lim_{n\to\infty} P(A_n) = 1$, then event A_n will happen with high probability.

cerned with full view barrier coverage in CSNs [17] [18].

All these works are commonly based on static networks for the seek of coverage condition. With recent development in electronic technology and image sensors, it is possible to deploy mobile CSNs with camera sensors moving in the area of interest and takeing pictures or live videos simultaneously. The ability of mobility greatly expands CSN's application range [19], while the use of mobile camera sensors also brings about benefit of enlarging the monitored area as mobile cameras are able to move toward any corner of the area of interest. Specifically, as demonstrated by Liu *et al.* [20] and Saipulla *et al.* [21], mobility can lead to improvement of barrier coverage performance since it may reduce the detection time of intruders. However, a question remains unknown: **how could mobility potentially enhance coverage in CSNs? And to what extent?**

To address this issue, we present a first look into coverage problem in mobile CSNs. Leveraging the conception of full view coverage in [14], we focus on the critical coverage condition in three different mobility models, i.e, 1-dimensional and 2-dimensional random walks as well as random rotating mobility. Moreover, we use static network as a baseline to get a clear understanding of the benefit brought by mobility. For the sake of tractability, here we consider asymptotic coverage in the sense that the total number of cameras approaches to infinity. Specifically we focus on a metric Equivalent Sensing Range (ESR), which, as pointed out by previous study [13], plays a vital role in determining the full coverage of the whole sensor network, regardless of the total number of sensors or the sensing radius of a single senor under random mobility patterns, and is thus a much easier and more general way to operate coverage control. However, unlike the sensing range of traditional sensors, here it relies heavily on several key factors such as the angle of view (or in other words, viewing direction), sensing radius, deployment density of camera sensors, and etc. To quantify this element, we introduce the conception of Equivalent Sensing Range (ESR), of which rigorous definition will be provided in Section II.

Here it is worthwhile noting that in our definition of ESR, we also take into consideration the heterogeneity of camera sensors. The introduction of heterogeneity coincides well with the fact that camera sensors may come from different manufacturers and thus have different sensing parameters, or the sensing capability of cameras will decline with the elapse of time or vary under different obstruction of terrains. Specifically, we deal with camera heterogeneity through dividing them into different groups according to their sensing parameters as is similarly conducted in [22] [23]. Then we define in all the four scenarios the corresponding ESRs, which incorporate the combined effects of viewing direction, camera heterogeneity and mobility patterns. Based on those, we derive critical ESRs under four mobility cases[3] with uniform sensor deployment [4]. The merit of critical ESR lies in facilitating the evaluation of the overhead for a CSN to achieve full view coverage, and both the advantages and drawbacks incurred by mobility are disclosed through the results.

However, there are several significant works that seem to be similar with our work. The work [29] is one of them. We have to admit that our work does share some similarity with Kumar's literature in terms of the technical structure. Despite of that, there still exist may differences between the two works.First of all, our work assumes that the sensor just have partial view, which is much closer to the reality. And the Φ considered is not simply a constant, as we shown in Section 2.1 that "All sensors in group G_y have identical sensing radius r_y and angle Φ_y ,but either $r_y \neq r_z$ or $\Phi_y \neq \Phi_z$ will hold if $y \neq z$. So our Φ will change as n varies. In addition, though our analysis looks similar to that in Kumar's work in terms of the structure, our main contribution is on the CRITICAL ESR, which is $\sqrt{\frac{3}{2}}$ ESR in the literature. With the introduction of critical ESR, we are able to find the tighter condition than that discovered in Kumar's work. Last but not least, our work addresses the first of several future directions that Kumar pointed out in Section 4 in his work. Overall, all those factors suffice to differ our work from Kumar's.

Also there was a book [28] giving out a really general and fantastic analysis of the coverage process. And someone may argue that our work can be generated from [28]. However, there are still some major differences between our work and the book. First of all, as Kumar's work [29] shows "According to Hall(1988)(Theorem 3.11) that the corresponding lower bound on $r(n)$ is $\pi r^2(n) = 4\frac{\log n+\log\log n+c(n)}{n}$ for $c(n) \to \infty$." And our result is that once $\pi r^2(n) = 2\frac{\log n+\log\log n+c(n)}{n}$ can satisfies the whole coverage of the area. Therefore, our result is stronger. Further, as we notice from Hall(1988)(Chapter 1.6)that the expected amount of the target destroyed by salvo of size n as:

$$e_n = \pi - \int_{|y|<1} [1-(2\pi\sigma^2)^{-1} \int_{|x-y|\leq r \& \theta \leq \frac{\phi}{2}} \exp\{-(2\sigma^2)^{-1}(x_1^2+x_2^2)\} dx_1 dx_2]^n dy_1 dy_2$$

After simplicification we have:$e_n \to e_\infty(f) \equiv \pi - \int_{|y|\leq 1} \exp\{-\lambda\frac{\phi}{\pi}f(y)\}dy$. We could not get the exact expression of λ here to get the whole area tends to be distract. However, our result gives out the expression.

Our main contributions are highlighted as follows.

1. We provide the critical conditions (critical ESR) of full view coverage or coverage with high probability under four different mobile situations. Specifically, our results disclose that the critical ESRs derived under random walks can be reduced by approximately an

3. We can treat static network as a special case of mobility.

4. As the major concern in the present work lies in the effect of mobility, we leave it a future work for exploration of other deployments.

order of $\Theta\left(\sqrt{\frac{\log n+\log\log n}{n\theta}}\right)$, where n and θ respectively represent the number of camera sensors and viewing angle, compared to that under static CSNs, whereas the random rotating mobility leads to a critical ESR twice than that of static CSNs.

2. We also derive the critical condition to achieve almost surely coverage, which is shown to be approximately 1.225 times of that to achieve coverage with high probability. Therefore, the almost surely coverage result turns out to be stronger compared to the traditional coverage with high probability. More delicate relationship between the two types of coverage is also discussed.

3. We present an extra look into sensing energy consumption, a metric that is closely related to ESR. Comparing with static networks, we demonstrate that both 1-dimensional and 2-dimensional random walks can reduce the sensing energy consumption by an order of $\Theta(\frac{\log n+\log\log n}{n\theta})$, at the expense of $\Theta(1)$ delay under uniform deployment. In contrast, random rotating mobility incurs no change on energy consumption, but with the same delay cost.

The rest of the paper is structured as follows. The basic models and definitions are described in Section 2. We show the geometric analysis and preliminaries in Section 3. In Section 4, we study the static model and derive the ESR to achieve full view coverage. The corresponding analysis in mobile CSNs are available in Section 6.2. Section 7 is dedicated to detailed discussion of theoretical results while Section VII presents simulations. In Section 8, we give the concluding remarks.

2 Models and Definitions

2.1 Deployment Scheme and Sensing Model

In this paper, the operational region of the sensor network is assumed to be a square of unit area. Similar to the previous literature, we ignore the boundary effect by considering a torus topology to simplify the analysis[5]. n sensors are randomly and uniformly deployed in the operational region, independently of each other. The random strategy is favored in the situations where the operational region is inimical and hostile, or it is expensive and difficult to place sensors by human or programmed robots. Under such circumstance, wireless sensors may be sprinkled from aircrafts, delivered by artillery shell, rocket, missile or thrown from a ship, instead of manual placement by human beings or programmed robots.

A camera sensor S can sense perfectly in a sector of radius r and angle ϕ, but has no sensing capability outside that sector. Without confusion, S also represents the location of the sensor. The angular bisector of ϕ is recognized as *orientation* of S, denoted by $\vec{f}$. This model is commonly used in literature [24] and [25], called *binary sector model*. Further, since the quality of information provided by a camera is sensitive to its viewpoint, there are other two essential directions to be considered. The direction towards which a point P faces is called its *facing direction*, denoted by $\vec{p}$. The vector $\overrightarrow{PS}$ is called *viewed direction* of the object, which reflects the viewpoint of sensor S. Figure 1 illustrates these directions which will be considered in subsequent discussion.

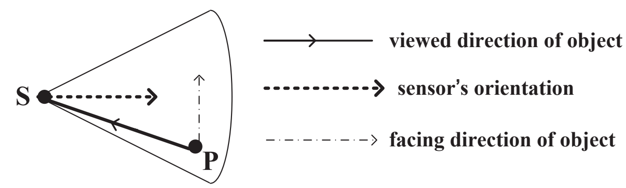

Fig. 1. For sensor S and point P, the orientation, viewed direction and facing direction are depicted respectively.

We consider heterogeneous sensors, of which the different qualities are described by partitioning sensors into u groups $G_1, G_2, \cdots, G_u$. As the total number of sensors is n, each group G_y $(y = 1, 2, \cdots, u)$ has $n_y = c_y n$ sensors, where c_y is a constant invariant to n. Clearly, c_y satisfies $0 < c_y < 1$ and $\sum_{y=1}^{u} c_y = 1$. All sensors in group G_y have identical sensing radius r_y and angle ϕ_y, but either $r_y \neq r_z$ or $\phi_y \neq \phi_z$ will hold if $y \neq z$ $(y, z = 1, 2, \cdots, u)$. We mainly study the asymptotic coverage here, implying that n is a variable approaching to infinity, whereas r_y and ϕ_y, which is sometimes denoted by $r_y(n)$ and $\phi_y(n)$, are dependent variables of n. Hence, the requirements for $r_y(n)$ and $\phi_y(n)$ change as n varies.

2.2 Static and Mobility Patterns

For mobility patterns, we divide the sensing process into time slots with unit length, and sensors can move according to certain mobility patterns in each time slot. When assuming the network works in a large amount of time slots, a single time slot can also be viewed as an instant.

Static Model: Wherever a sensor is located, its orientation $\vec{f}$ faces towards all possible directions with equal probability. And once a sensor is deployed, neither its orientation $\vec{f}$ nor its location will change, which means that the camera will not steer its lens during the operation.

2-Dimensional Random Walk Mobility Model: At the very beginning of each time slot, each sensor uniformly chooses a random direction $\sigma \in [0, 2\pi)$, and then it rotates its sensor's orientation to the chosen one and moves along the direction with a constant velocity v in each time slot on a 2-dimensional surface and the velocity is $\Theta(1)$.

1-Dimensional Random Walk Mobility Model: Sensors are classified into two types of equal quantity, i.e., H-nodes and V-nodes. And sensors of each type move horizontally and vertically, respectively. At the very beginning of each time slot, each sensor randomly and uniformly chooses a direction along its moving dimension and travels in the selected direction for a certain distance

5. Actually, coverage problem near the boundaries differs significantly from general situations. However, it is beyond the scope of this paper.

D, a random variable uniformly distributed from 0 to 1.[6] The velocities of the sensors are not considered, as long as the sensors could reach the destination within the time slot, and remain stationary until the next slot.

Random Rotating Mobility Model: Cameras can rotate and change their orientation in a clockwise/counterclockwise manner. At the very beginning of each time slot, each sensor randomly chooses a rotating direction, i.e. a clockwise or counterclockwise one, and then rotates an angle Ψ, a random variable uniformly distributed between 0 and 2π. Note that the results can be easily expanded to more general cases where Ψ follows a certain distribution function $f_\Psi(\psi)$. We omit it here for the sake of brevity. Similarly, the velocity of sensors is also ignored.

The static model has been widely adopted due to its favorable property of characterizing lower and upper bounds of the performance. Note that in some previous literatures, it is also called I.I.D. mobility pattern. Since I.I.D mobility model does not change the coverage area of sensors, we can simply treat it as a quasi-static model, or view static model as I.I.D. mobility model with an infinity period. Comparatively, the 2-dimensional random walk mobility model can highly exploit the randomness of the motion of the nodes and is closer to realistic situations where the statistics of the moving habit is unknown. The 1-dimensional random walk mobility model is motivated by certain networks where nodes move along determined tracks such as the networks employed in streets, systems consisted of satellites moving in fixed orbits and etc. In the random rotation mobility that we propose, camera sensors are allowed to rotate their orientation to broaden the viewing angle.

2.3 Performance Metrics

To assess the full view coverage performance in CSNs, we give the following five definitions.

2.3.1 Definition of θ-view coverage

For a specific facing direction $\vec{p}$ of point P, it achieves θ-view coverage if it is covered by at least one sensor and the angle between $\vec{p}$ and its viewed direction is no more than θ. Here, $\theta \in (0, \pi]$ is a predefined constant parameter called effective angle.

2.3.2 Definition of full θ-view coverage

For a point P, it is said to be full θ-view covered if every possible facing direction $\vec{p}$ is θ-view covered. The operational region achieves full θ-view coverage if and only if (iff) every point in this region achieve full θ-view coverage. For the sake of simplicity, we also call it **full view coverage** without incurring too much ambiguity throughout the rest of the paper.

6. Long distance travel is energy-consuming. And if the sensor can travel beyond the dimension of the operational region (i.e., $D > 1$), it can always cover the area along its moving dimension which is meaningless.

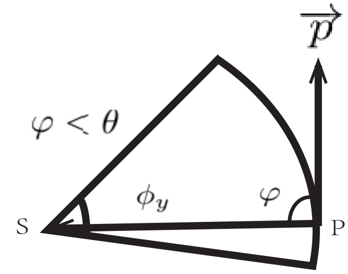

Fig. 2. The sensor's angle is ϕ_y and the angle φ between the viewed direction and the facing direction needs to be less than the effective angle θ.

2.3.3 Definition of full view coverage in a period T

If during a time period T (T time slots), the network is in the state of full view coverage for at least one time slot, we say the network achieves full view coverage in period T.

2.3.4 Definition of Equivalent Sensing Radius

For heterogeneous camera sensor networks, we define the equivalent sensing radius (ESR) for each static and mobility pattern to analyze the asymptotic full view-coverage. Specifically, the ESR is $r = \sqrt{\sum_{y=1}^{u} c_y \frac{\phi_y}{2\pi} r_y^2}$ for static model, and is $r = \sum_{y=1}^{u} c_y \frac{\phi_y}{2\pi} r_y$ for both 1-dimensional and 2-dimensional random walks and the ESR and is $r = \sqrt{\sum_{y=1}^{u} c_y (\frac{17}{8} - \frac{1}{2}(\frac{3}{2} - \frac{\phi_y}{2\pi})^2) r_y^2}$ for random rotating mobility model.

In this part, $\frac{\phi_y}{2\pi}$ is viewed as the weight of each sensor's radius. When $\phi = 2\pi$, it is equivalent to a sensor whose sensing range is a circle, and ESR in this case is reduced to that of omnidirectional sensors [22].

As we mentioned in the above passage, we would like to discuss three moving states of sensors, so that the motivation of ESR is to unify them as well as present the combination of r_y and ϕ_y of the camera sensor. And our goal is to find a suitable characteristic of sensors to achieve the full view coverage of a given area with a given number of sensors. With the critical ESR we can find the suitable characteristic of sensors. It is undeniable that there are many kinds of indexes that can be used. However, the reason that we choose ESR is that we can easily get the sensing radius (r_y) and viewing angle (ϕ_y) when we buy a camera sensor. As a result we can easily confirm our result in the reality.

Intuitively, the coverage of the network is positively correlated with ESR. The ESR needed when the network exactly achieves asymptotic coverage is called critical ESR, which is defined as follows.

2.3.5 Definition of Critical ESR

Let $\mathcal{H}$ denotes the event that the operational region is full view covered. Then

$$\lim_{n\to\infty} P(\mathcal{H}) = 1, \text{if } r_i \geq cR_i(n) \text{ for any } c > 1;$$
$$\lim_{n\to\infty} P(\mathcal{H}) < 1, \text{if } r_i \leq \hat{c}R_i(n) \text{ for any } 0 < \hat{c} < 1,$$

where $R_i(n)$ is the critical ESR under four different static and mobile patterns, with $i = stat$, $r.r.$, $2.r.w.$, $1.r.w.$, representing the abbreviations of "static model", "random rotating mobility model", "2-dimensional random walk mobility model" and"1-dimensional random walk mobility model", respectively.

When ESR exceeds the critical one, the operational region will be full covered with probability one when n is sufficiently large, and guarantees the sufficiency of critical ESR. In contrast, when ESR is below the critical value, even though n is large enough, the operational region still cannot be full covered with probability one, which reflects the necessity of critical ESR.

3 Overview of the Geometric Analysis

It has been shown in Wang *et al.* [22] that a dense grid $\mathbb{M}$ with $\sqrt{m} \times \sqrt{m}$ is almost always covered when $m = n \log n$. Based on which we can also prove that the θ-view coverage of a facing direction set $\mathbb{K}$ formed by k directions of a point can guarantee its full view coverage when $k = n \log n$. Technically, a key factor behind such result lies in the facing direction set of a point. Figure 3 (b) illustrates an example of a facing direction set $\mathbb{K}$ of point P. We use $k = 8$ facing directions to uniformly distribute the angle of circumference into 8 parts. The correlation between full view coverage of P and k directions is presented in Lemma 1.

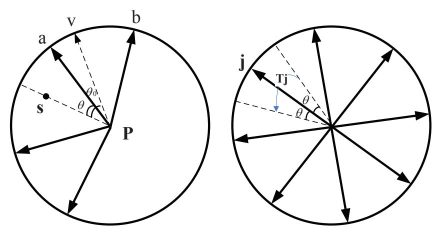

Fig. 3. (a) A set of four nearest directions including a and b in the direction set $\mathbb{K}$; (b) The possible area T_j to θ-view cover orientation O_j.

Lemma 1: Assume θ, θ_0, k are constants and $\theta_0 = \theta + \frac{2\pi}{k}$. $\mathbb{K}$ is what we shown above. If these k directions can all achieve θ-view coverage, then point P can achieve full view coverage with effective angle θ_0.

Proof: Let v be an arbitrary facing direction of point P. Without loss of generality, we assume it is inside the sector formed by virtual orientation a and b in $\mathbb{K}$ and is closest to orientation a, as shown in Figure 3 (a). By assumption, there exists at least one sensor that can cover a, with effective angle θ. Suppose one of them locates at point s (in Figure 3 (a)), and $\angle(s, a) < \theta$. s also represents the viewed direction without confusion. Besides $\angle(a, v) < \frac{2\pi}{k}$, then

$$\angle(s, v) = \angle(s, a) + \angle(a, v) < \theta + \frac{2\pi}{k} = \theta_0. \qquad (1)$$

□

Obviously Eq. (1) still holds when the sensor locates between a and b. Also $\lim_{k\to\infty} \theta_0 = \theta$, which means θ_0 is only slightly larger than θ, when k is large enough. With Theorem 4.1 in [3], the following theorem is derived.

Theorem 1: For point P, if a k facing direction set $\mathbb{K}$ satisfies $k = n \log n$, the θ-view coverage of set $\mathbb{K}$ can promise the full view coverage of P with effective angle θ when n is large enough.

Thus we can focus on the θ-view coverage of orientation set $\mathbb{K}$ for the dense gird $\mathbb{M}$ to estimate full view coverage performance of the operational region.

4 Critical Sensing Range in Static CSNs

We start with the analysis of full view coverage for static camera sensor networks, and obtain the critical ESR of heterogeneous cameras for coverage with high probability. We also derive the critical equivalent sensing range for almost surely coverage. We first have the following theorem.

Theorem 2: Under the uniform deployment with static model, the critical ESR for static heterogeneous CSNs to achieve asymptotic full view coverage is

$$R_{stat}(n) = \sqrt{\frac{2(\log n + \log\log n)}{n\theta}}.$$

Let $\mathbb{P}_{i,j,S_y}$ denote the probability that orientation O_j of point P_i is θ-viewed covered by Sensor S in group G_y. To make O_j of set $\mathbb{K}$ θ-viewed covered, at least one sensor should locate in sector T_j, as shown in Figure 3(b). For sector T_j, the angular bisector is orientation j, with an angle 2θ. Then

$$\begin{aligned}\mathbb{P}_{i,j,S_y} &= \mathbb{P}(S \text{ falls in } T_j) \times \mathbb{P}(S \text{ has proper orientation}) \\ &= \frac{2\theta}{2\pi} \times \pi r_y^2(n) \times \frac{\phi_y}{2\pi} = \frac{r_y^2(n)\phi_y\theta}{2\pi}\end{aligned}$$

4.1 Necessary Condition of Theorem 2

Let $\mathcal{G}_{stat}(n, u)$ denote the network that each point in $\mathbb{M}$ achieves full view coverage when the category of sensors is u. And we use $P_{f-stat}(n, u)$ to represent the probability that $\mathcal{G}_{stat}(n, u)$ has at least one point that is not full view covered. Then we derive the following proposition. For simplicity, we say a direction uncovered and not θ-view covered equivalently, and a point uncovered and not full view covered interchangeably. To simplify the proof, we define a variable $\omega(n)$ to combine the $r_{stat}(n)$ with the $R_{stat}(n)$.

Proposition 1: In the static heterogeneous CSN, if

$$r_{stat}(n) = \sqrt{\frac{2(\log n + \log\log n + \omega(n))}{n\theta}},$$

$m = n \log n$ and $k = n \log n$, then

$$\liminf_{n\to\infty} P_{f-stat}(n, u) \geq e^{-2\omega} - \frac{\theta}{\pi} e^{-3\omega},$$

where $\omega = \lim_{n\to\infty} \omega(n)$.

Proof: To simplify the complexity of the proof, we provide the following lemma first.

Lemma 2: Given a variable $x = x(n)$ satisfies $0 < x(n) < \frac{1}{2}$, and a variable $y = y(n) > 0$, then $(1 - x)^y \sim e^{-xy}$ if $x^2 y$ approaches to zero as $n \to \infty$.

Using the similar method in the proof of Lemma 1 [16], the proof of Lemma 2 can easily follow. Then we study the case that $r(n) = \sqrt{\frac{2(\log n + \log\log n + \omega(n))}{n\theta}}$ for a fixed ω. Referring to Bonferroni inequalities, we get

$$
\begin{aligned}
&P_{f-stat}(n, r(n)) \\
&\geq \sum_{P_i \in \mathbb{M}} \mathbb{P}(\{\text{some point } P_i \text{ is not full view covered}\}) \\
&\geq \sum_{P_i \in \mathbb{M}} \mathbb{P}(\{P_i \text{ is the only uncovered point}\}) \\
&\geq \sum_{P_i \in \mathbb{M}} \sum_{O_j \in \mathbb{K}} \mathbb{P}(\{\text{only } O_j \text{ of } P_i \text{ is uncovered}\}) \\
&\overset{1}{\geq} \sum_{P_i \in \mathbb{M}} \sum_{O_j \in \mathbb{K}} \mathbb{P}(\{O_j \text{ of } P_i \text{ is uncovered}\}) \\
&- \sum_{P_i \in \mathbb{M}} \sum_{O_j, O_h \in \mathbb{K}}^{O_j \neq O_h} \mathbb{P}(\{\ O_j \text{ and } O_h \text{ of } P_i \text{ are uncovered}\}).
\end{aligned} \tag{2}
$$

The $\overset{1}{\geq}$ is where Bonferroni inequalities applied. For the first term of the R.H.S. of Eq. (2)

$$
\begin{aligned}
&\mathbb{P}(\{O_j \text{ of } P_i \text{ is uncovered}\}) \\
&\geq \prod_{y=1}^{u} \mathbb{P}(\{O_j \text{ is uncovered by sensors in } G_y\}) \\
&= \prod_{y=1}^{u} \left(1 - \frac{r_y^2(n)\phi_y\theta}{2\pi}\right)^{c_y n},
\end{aligned} \tag{3}
$$

where $\frac{r_y^2(n)\phi_y\theta}{2\pi}$ represents the probability that orientation O_j of point P_i is θ-viewed covered by Sensor S in group G_y, while $c_y n$ represents the number of sensors in group G_y. Then with Lemma 2 and Eq. (3), we obtain that

$$
\begin{aligned}
&\sum_{P_i \in \mathbb{M}} \sum_{O_j \in \mathbb{K}} \mathbb{P}(\{O_j \text{ of } P_i \text{ are uncovered}) \\
&\quad \geq mk \prod_{y=1}^{u} \left(1 - \frac{r_y^2(n)\phi_y\theta}{2\pi}\right)^{c_y n} \\
&\quad \sim mke^{-n\theta \sum_{y=1}^{u} c_y \frac{\phi_y}{2\pi} r_y^2(n)} = mke^{-n\theta r_{stat}^2(n)} \\
&\quad = (n\log n)^2 e^{-2(\log n + \log\log n + \omega)} = e^{-2\omega}.
\end{aligned} \tag{4}
$$

For the second term of the R.H.S. of Eq. (2)

$$
\begin{aligned}
&\mathbb{P}(\{O_j \text{ and } O_h \text{ of } P_i \text{ are uncovered}\}) \\
&\leq \frac{2\theta}{2\pi} \prod_{y=1}^{u} \left(1 - \frac{3}{2}\frac{r_y^2(n)\phi_y\theta}{2\pi}\right)^{c_y n} + \\
&\left(1 - \frac{2\theta}{2\pi}\right) \prod_{y=1}^{u} \left(1 - 2\frac{r_y^2(n)\phi_y\theta}{2\pi}\right)^{c_y n},
\end{aligned} \tag{5}
$$

where the two terms on the right side correspond to the cases where $\angle(O_j, O_h) \leq 2\theta$ and $\angle(O_j, O_h) > 2\theta$, respectively. For the first term, $\frac{3}{2}\frac{r_y^2(n)\phi_y\theta}{2\pi}$ is the average area sensors may locate to θ-view cover O_j or O_h. Since the overlapping area between O_j and O_h is a random variable uniformly distributed between 0 and $\theta r^2(n)$, the corresponding possible area is also a random variable, uniformly distributed between $\frac{r_y^2(n)\phi_y\theta}{2\pi}$ and $2\frac{r_y^2(n)\phi_y\theta}{2\pi}$, so that its expectation is $\frac{3}{2}\frac{r_y^2(n)\phi_y\theta}{2\pi}$. The second term can be analyzed in a similar manner.

Then according to Lemma 2 and Eqn. (5), we obtain

$$
\begin{aligned}
&\sum_{P_i \in \mathbb{M}} \sum_{O_j, O_h \in \mathbb{K}}^{O_j \neq O_h} \mathbb{P}(\{O_j \text{ and } O_h \text{ of } P_i \text{ are uncovered}) \\
&\quad \leq mk^2 \frac{2\theta}{2\pi} \prod_{y=1}^{u} \left(\frac{2\pi - \frac{3}{2}r_y^2(n)\phi_y\theta}{2\pi}\right)^{c_y n} \\
&\quad\quad + mk^2 \left(1 - \frac{2\theta}{2\pi}\right) \prod_{y=1}^{u} \left(\frac{2\pi - 2r_y^2(n)\phi_y\theta}{2\pi}\right)^{c_y n} \\
&\quad \sim mk^2 \frac{\theta}{\pi} e^{-\frac{3n\theta}{2} r_{stat}^2} + mk^2 \left(1 - \frac{\theta}{\pi}\right) e^{-2n\theta r_{stat}^2} \\
&\quad = \frac{\theta}{\pi} e^{-3\omega} + \left(1 - \frac{\theta}{\pi}\right) e^{-4\omega} \frac{1}{n\log n}.
\end{aligned}
$$

Since we consider the asymptotic coverage problem where the total number of cameras n approaches to infinity, for any fixed ω, we can obtain $\liminf_{n\to\infty} P_{f-stat}(n, u) \geq e^{-2\omega} - \frac{\theta}{\pi}e^{-3\omega}$. Now we consider the $\omega = \liminf_{n\to\infty} \omega(n)$,which indicates that $\omega(n) < \omega + \delta$ for any $\delta > 0$, for all $n > N_\delta$. Since $P_{f-stat}(n, u)$ is monotonically decreasing in r_{stat} and thus in ω, we have $\liminf_{n\to\infty} P_{f-stat}(n, u) \geq e^{-2(\omega+\delta)} - \frac{\theta}{\pi}e^{-3(\omega+\delta)}$, for all $n > N_\delta$. □

It has been known that $P_{f-stat}(n, u)$ is bounded away from zero. Combined with the definition of ESR for static model, we know that $r_{stat} \geq R_{stat} = \sqrt{\frac{2(\log n + \log\log n)}{n\theta}}$ is necessary to achieve the full view coverage of $\mathbb{M}$. Moreover, if sensing range r_{stat} is smaller than $\sqrt{\frac{3}{2}}$ of critical ESR of static model, the result can be extended as stated in the following Theorem.

Theorem 3: Under the uniform deployment with static model, if the CSN satisfies $r_{stat}(n) < \sqrt{\frac{3}{2}}R_{stat}$, there still will be an nonegligible probability that the network is uncovered. So we can say that the $r_{stat}(n) > \sqrt{\frac{3}{2}}R_{stat}$ is the necessary condition of Theorem 2

Proof: We denote the event that the operational region with n camera sensors has at least one point that is not full view covered as $\widehat{\mathcal{H}_n}$, and use $\mathbb{P}(\widehat{\mathcal{H}_n})$ to represent the corresponding probability.

Assuming $r_{stat}(n) = c\sqrt{\frac{3}{2}}R_{stat}$, and referring to Bonferroni inequalities, we get

$$
\begin{aligned}
\mathbb{P}(\widehat{\mathcal{H}_n}) \geq &\sum_{P_i \in \mathbb{M}} \sum_{O_j \in \mathbb{K}} \mathbb{P}(\{O_j \text{ of } P_i \text{ is uncovered}\}) \\
&- \sum_{P_i \in \mathbb{M}} \sum_{O_j, O_h \in \mathbb{K}}^{O_j \neq O_h} \mathbb{P}(\{\ O_j \text{ and } O_h \text{ of } P_i \text{ are uncovered}\})
\end{aligned}
$$

$$= mk\prod_{y=1}^{u}\left(1-\frac{r_y^2(n)\phi_y\theta}{2\pi}\right)^{c_yn} - mk^2\frac{2\theta}{2\pi}\prod_{y=1}^{u}\left(1-\frac{3}{2}\frac{r_y^2(n)\phi_y\theta}{2\pi}\right)^{c_yn}$$
$$- mk^2\left(1-\frac{2\theta}{2\pi}\right)\prod_{y=1}^{u}\left(1-2\frac{r_y^2(n)\phi_y\theta}{2\pi}\right)^{c_yn}$$
$$\sim mke^{-n\theta\sum_{y=1}^{u}c_y\frac{\phi_y}{2\pi}r_y^2(n)} - mk^2\frac{\theta}{\pi}e^{-\frac{3n\theta}{2}\sum_{y=1}^{u}c_y\frac{\phi_y}{2\pi}r_y^2(n)}$$
$$- mk^2\left(1-\frac{\theta}{\pi}\right)e^{-2n\theta\sum_{y=1}^{u}c_y\frac{\phi_y}{2\pi}r_y^2(n)}$$
$$= \frac{1}{(n\log n)^{3c^2-2}} - \frac{\theta}{\pi}\frac{1}{(n\log n)^{\frac{9}{2}c^2-3}} - \left(1-\frac{\theta}{\pi}\right)\frac{1}{(n\log n)^{6c^2-3}}$$

If $r_{stat}(n)$ is smaller than $\sqrt{\frac{3}{2}}R_{stat}$, namely, $c<1$, then according to the characteristic of P-series, we know that

$$\sum_{n=1}^{\infty}\mathbb{P}(\widehat{\mathcal{H}_n}) > \sum_{n=1}^{\infty}\frac{1}{(n\log n)^{3c^2-2}} - \sum_{n=1}^{\infty}\frac{\theta}{\pi}\frac{1}{(n\log n)^{\frac{9}{2}c^2-3}} - \sum_{n=1}^{\infty}\left(1-\frac{\theta}{\pi}\right)\frac{1}{(n\log n)^{6c^2-3}} > \infty.$$

And $\{\widehat{\mathcal{H}_n}\}$ is a sequence of independence events. Then using *Borel–Cantelli Lemma* in [26], we know that

$$\mathbb{P}(\limsup_{n\to\infty}\widehat{\mathcal{H}_n}) = 1,$$

which means the event $\widehat{\mathcal{H}_n}$ will infinitely often happen under an asymptotic network. Namely, when $r_{stat}(n)\le\sqrt{\frac{3}{2}}R_{stat}$, for any N, there is always an n which is larger than N, that event $\widehat{\mathcal{H}_n}$ will happen. Shortly, the network is almost surely uncovered when $r_{stat}(n)$ is smaller than $\sqrt{\frac{3}{2}}R_{stat}$. □

4.2 Sufficient Condition of Theorem 2

Now we turn to explore the sufficient condition. First, we obtain the following proposition.

Proposition 2: In CSN, if n sensors are randomly and uniformly deployed in a unit square, and $r_{stat}(n) = cR_{stat}$ where $c>1$, then

$$\liminf_{n\to\infty}\mathbb{P}(\widehat{\mathcal{H}}) = 0. \quad (6)$$

where $\widehat{\mathcal{H}}$ denotes the event that the operational region is not full view covered as defined in Section 2.

The proof is easy to complete so we skip it here due to space limitations. Then from Proposition 2 and the definition of critical ESR for static model, we know that $r_{stat}\ge R_{stat}=\sqrt{\frac{2(\log n+\log\log n)}{n\theta}}$ is sufficient to achieve the full view coverage of $\mathbb{M}$. Based on that, we can further obtain the result where sensing range is larger than critical ESR in static network, as is stated in Theorem 4.

Theorem 4: Under the uniform deployment with static model, if the CSN satisfies $r_{stat}(n) > cR_{stat}$, $c>1$, then it is sufficient for the network to achieve full coverage.

4.3 Critical ESR for Full View Coverage of the Operational Range

So far we have already proved that $R_{stat} = \sqrt{\frac{2(\log n+\log\log n)}{n\theta}}$ is the sufficient condition to achieve full view coverage for dense grid $\mathbb{M}$. Referring to LEMMA 3.1 in [3], as well as Lemma 1 and Theorem 1 in this paper and using similar approach as THEOREM 4.1 in [3], the density of the dense grid $m = n\log n$ and the density of the orientation set $k = n\log n$ are sufficiently large to evaluate the full view coverage of the whole area. Moreover, referring to Theorems 3 and 4, we conclude that $R_{a.s.c.} = \sqrt{\frac{3}{2}}R_{stat}$ is the critical condition to achieve almost surely coverage for static model.

5 The Critical Sensing Range for Mobile CSNs

Now we proceed to investigate full view coverage problem for CSNs under uniform deployment mobile scenarios. Recall that we particularly consider three different mobile patterns, namely, 2-dimensional random walk mobility model, 1-dimensional random walk mobility model and random rotating mobility model.

5.1 Critical ESR Under 2-Dimensional Random Walk

We investigate full view coverage in one time slot under 2-Dimensional Random Walk Mobility Model, and Figure 4 illustrates the effect of random walk mobility of the sensor on area coverage. We will first analyze full view coverage for dense grid $\mathbb{M}$, and then expand it to the whole area.

Theorem 5: Under the uniform deployment with 2-dimensional random walk mobility model, the critical ESR for mobile heterogeneous CSNs to achieve asymptotic full view coverage is

$$R_{2.r.w}(n) = \begin{cases}\frac{\log n+\log\log n}{2nTv\sin\theta} & \text{if } \theta<\frac{\pi}{2}\\ \frac{\log n+\log\log n}{2nTv} & \text{if } \theta\ge\frac{\pi}{2}\end{cases}.$$

We will focus on the proof of the case where $\theta<\frac{\pi}{2}$, and the proof is similar when $\theta\ge\frac{\pi}{2}$.

5.1.1 Failure Probability of an Orientation in $\mathbb{K}$

Let $\mathcal{F}_{i,j}$ denote the event that orientation O_j of point P_i is not θ-viewed covered during the time slot τ, and $\mathbb{P}(\mathcal{F}_{i,j})$ denote the corresponding probability. We use $\mathbb{P}_{i,j,S_y}$ to represent that O_j of point P_i is θ-viewed covered by Sensor S in group G_y. Then we obtain

$$\mathbb{P}_{i,j,S_y} = \left((\theta+\alpha)r_y^2(n) + 2vTr_y(n)\sin\theta\right)\frac{\phi_y}{2\pi}. \quad (7)$$

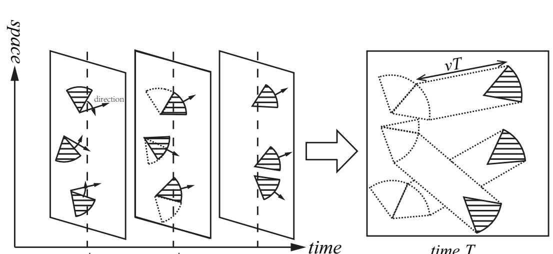

Fig. 4. $T/3$, $2T/3$ and T; the right one illustrates the trace of sensor mobility during the whole interval [0, T). The shadowed disks constitute the area being covered at the given time instant, and the union of the region inside the dotted line and the shadowed disks represents the area being covered during the time interval.

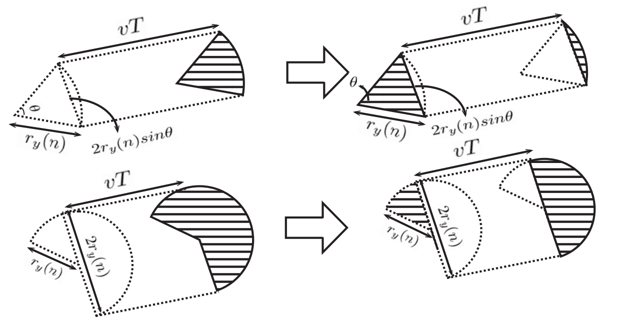

Fig. 5. Illustration of calculating the mobility area. It is difficult to calculate the mobility area directly, so we rearrange the area into a square and then can easily obtain the result.

In Eq. (7), $(\theta+\alpha)r_y^2(n)+2vTr_y(n)\sin\theta$ represents the possible area the sensor may locate in order to θ-view cover O_j during T slots, if it does not change its direction during the process. (Similarly as we can see from the Fig. 5 that the Eq. (7) can change into $(\theta+\alpha)r_y^2(n)+2vTr_y(n)$.) In this formula $\theta r_y^2(n)$ represents the possible area where sensors in group $G_y, y=1,2,..,u$ might locate if it is stationary,like sector T_j in Figure 3. $\alpha r_y^2(n)$ represents the additional area due to rotation, which is caused by the sensor's initial orientation and its chosen direction δ. Considering its mobility character, the possible area can be $2vTr_y(n)\sin\theta$ more, like the region inside the dotted line in Figure 4. If the sensor changes its direction during this period, the sensing area will overlap, making it no larger than $2vTr_y(n)\sin\theta$. For formula $\frac{\phi_y}{2\pi}$, it represents the probability that the sensor in group G_y has proper orientation to sense the point. With all those factors determined, $\mathbb{P}(\mathcal{F}_{i,j})$ can then be calculated.

5.1.2 Necessary ESR for Full View Coverage

Here, we use $\widehat{\mathcal{H}^\tau}$ to denote the event that the dense grid $\mathbb{M}$ is not fully full view covered in the time slot τ, and present the following proposition regarding the necessary condition. We will slightly abuse the notation and use the ω which represent the same meaning as that in Proposition 1

Proposition 3: In the mobile heterogeneous CSN with 2-dimensional random walk mobility model, if $r_{2.r.w.}=\frac{\log n+\log\log n+\omega(n)}{2nTv\sin\theta}$ and the density of the dense grid $\mathbb{M}$ is $m=n\log n$, the density of the orientation set $\mathbb{K}$ is $k=n\log n$, then

$$\liminf_{n\to\infty}\mathbb{P}_\tau(\widehat{\mathcal{H}^\tau})\geq e^{-2\omega}-\frac{\theta}{\pi}e^{-3\omega},$$

where $\omega=\lim_{n\to\infty}\omega(n)$.

Proof: Similar to the proof of Proposition 1, we first study the case where $r_{2.r.w.}=\frac{\log n+\log\log n+\omega}{2nTv\sin\theta}$, for a fix ω.

$$\mathbb{P}_\tau(\widehat{\mathcal{H}^\tau})\geq\sum_{P_i\in\mathbb{M}}\sum_{O_j\in\mathbb{K}}\mathbb{P}_\tau(\{O_j \text{ of } P_i \text{ is uncovered}\})$$
$$-\sum_{P_i\in\mathbb{M}}\sum_{O_j,O_h\in\mathbb{K}}^{O_j\neq O_h}\mathbb{P}_\tau(\{O_j \text{ and } O_h \text{ of } P_i \text{ are uncovered}). \tag{8}$$

And we calculate that

$$\begin{aligned}&\mathbb{P}_\tau(\{O_j\ of\ P_i \text{ is uncovered}\})\\ &=\prod_{y=1}^{u}\left(1-\left((\theta+\alpha)r_y^2(n)+2vTr_y(n)\sin\theta\right)\frac{\phi_y}{2\pi}\right)^{c_yn}\\ &=\prod_{y=1}^{u}\left(1-(1+\lambda_y)\frac{\phi_yvTr_y(n)\sin\theta}{\pi}\right)^{c_yn},\end{aligned}$$

where $\lambda_y=\frac{(\theta+\alpha)r_y(n)}{2vT\sin\theta}=\Theta(r_y(n))=o(1)$, since the asymptotic coverage problem is considered.

Then we can bound the first term of R.H.S of Eq. (8),

$$\sum_{P_i\in\mathbb{M}}\sum_{O_j\in\mathbb{K}}\mathbb{P}_\tau(\{O_j\ of\ P_i \text{ is uncovered}\}) \tag{9}$$

$$\geq mke^{-4vT\sin\theta n}\sum_{y=1}^{u}c_y\frac{\phi_y}{2\pi}r_y$$

$$=mke^{-4vT\sin\theta nr_{2.r.w.}}$$
$$=e^{-2\omega}.$$

Similarly, we bound the second term

$$\sum_{P_i\in\mathbb{M}}\sum_{O_j,O_h\in\mathbb{K}}^{O_j\neq O_h}\mathbb{P}_\tau(\{O_j \text{ and } O_h \text{ of } P_i \text{ are uncovered})\sim\frac{\theta}{\pi}e^{-3\omega}.$$

Then we have $\liminf_{n\to\infty}\mathbb{P}_\tau(\widehat{\mathcal{H}^\tau})\geq e^{-2\omega}-\frac{\theta}{\pi}e^{-3\omega}$. Since ω is a function of n, the conclusion holds. □

According to Proposition 3, we know that $R_{2.r.w.}\geq\frac{\log n+\log\log n+\omega(n)}{2nTv\sin\theta}$ is necessary to achieve the full view coverage of $\mathbb{M}$. Moreover, if sensing range is smaller than $\sqrt{\frac{3}{2}}$ of critical ESR of the 2-dimensional random walk mobility model, we can extend our result to the following theorem. We omit the proof here since it shares a similar technique adopted in the proof of Theorem 3.

Theorem 6: Under the uniform deployment with the 2-dimensional random walk mobility model, if the CSN satisfies $r_{2.r.w.}(n)<\sqrt{\frac{3}{2}}R_{2.r.w.}$, there still will be an nonegligible probability that the network is uncovered. So we can say that the $r_{2.r.w.}>\sqrt{\frac{3}{2}}R_{2.r.w.}$ is the necessary condition for the network to achieve full view coverage.

5.1.3 Sufficient ESR for Full View Coverage

Before we proceed, we first present the following proposition.

Proposition 4: In CSN, if n sensors are randomly and uniformly deployed in a unit square, and $r_{2.r.w.}(n)=cR_{2.r.w.}(n)$ where $c>1$, then

$$\liminf_{n\to\infty}\mathbb{P}_\tau(\widehat{\mathcal{H}_\tau})=0. \tag{10}$$

The proof can be completed using a similar approach as in Proposition 2 following that fact that Eq. (10) can be further written as

$$\begin{aligned}
&\mathbb{P}_\tau(\widehat{\mathcal{H}_\tau}) = \mathbb{P}_\tau(\bigcup_{i=1}^{m} \mathcal{F}_i) \\
&\quad \leq mk \prod_{y=1}^{u} \left(1 - \left((\theta+\alpha) r_y^2(n) + 2vTr_y(n)\sin\theta\right) \frac{\phi_y}{2\pi}\right)^{c_y n} \\
&\quad \sim (n\log n)^2 e^{-4vT\sin\theta n r_{2.r.w.}} \\
&\quad = \frac{1}{(n\log n)^{2c^2-2}} \to 0,
\end{aligned} \tag{11}$$

for any $c > 1$.

From Proposition 4 and the definition of critical ESR for 2-dimensional random walk mobility model, we know that $r_{2.r.w.} \geq cR_{2.r.w.} = \frac{\log n + \log\log n + \omega(n)}{2nTv\sin\theta}$,namely $c > 1$, is sufficient to achieve the full view coverage of $\mathbb{M}$. Moreover, Theorem 7 states an extended result if sensing range is more than critical ESR of the 2-dimensional random walk.

Theorem 7: Under the uniform deployment with 2-dimensional random walk mobility model, if the CSN satisfies $r_{2.r.w.}(n) > cR_{2.r.w.}$,$c > 1$, then it is sufficient for the network to achieve full coverage.

5.1.4 Critical ESR for Full View Coverage of the Operational Range

Similar to the analysis in the static model, Theorem 5 follows. Namely, $R_{2.r.w.} = \frac{\log n + \log\log n}{2nTv\sin\theta}$ is the critical condition to achieve coverage with high probability. Moreover, referring to Theorems 6 and 7, we conclude that $R_{a.s.c.} = \sqrt{\frac{3}{2}} R_{2.r.w.}$ is the critical condition to achieve almost surely coverage for 2-dimensional random walk mobility model. Apparently, there is a decrease in critical ESR under 2-dimensional random walk compared to static case. Further more, we proceed to analyze how it will be affected in 1-dimensional random walk.

5.2 Critical ESR Under 1-Dimensional Random Walk

5.2.1 Failure Probability of an Orientation in $\mathbb{K}$

Similarly, let $\mathcal{F}_{i,j}$ denote the event that orientation O_j of point P_i is not θ-viewed covered during the time slot τ, and $\mathbb{P}(\mathcal{F}_{i,j})$ denote the corresponding probability. We use $\mathbb{P}_{i,j,S_y}$ to represent that O_j of point P_i is θ-view covered by Sensor S in group G_y.

It can be derived from Wang *et al.* [22] that under 1-dimensional random walk mobility model, the probability that S falls in the circle around of P_i, with radius r_y is $\mathbb{P}_{i,S} = \frac{4}{3} r_y$. And it is clear that $\mathbb{P}$(S falls in cirle around P_i)=$\mathbb{P}_{i,S}$. Then we obtain

$$\begin{aligned}
\mathbb{P}_{i,j,S_y} &= \mathbb{P}(S \text{ falls in } T_j) \times \mathbb{P}(S \text{ has a proper orientation}) \\
&= \mathbb{P}(S \text{ falls in the cirle around } P_i) \times \frac{2\theta}{2\pi} \times \frac{\phi_y}{2\pi} \\
&= \frac{\theta\phi_y}{2\pi^2} \mathbb{P}_{i,S} = \frac{2\theta\phi_y r_y(n)}{3\pi^2},
\end{aligned} \tag{12}$$

based on which $\mathbb{P}(\mathcal{F}_{i,j})$ can be easily calculated.

5.2.2 Necessary ESR for Full View Coverage

Here, we use $\widehat{\mathcal{H}^\tau}$ denote the event that the dense grid $\mathbb{M}$ is not fully full view covered in time slot τ. And we now present the following proposition regarding the necessary condition.We will slightly abuse the notation and use the ω which represent the same meaning as that in Proposition 1.

Proposition 5: In the mobile heterogeneous CSN with 1-dimensional random walk mobility model, if $r_{1.r.w.} = \frac{3\pi(\log n + \log\log n + \omega(n)}{)} 2\theta n$ and the density of the dense grid $\mathbb{M}$ is $m = n\log n$, the density of the orientation set $\mathbb{K}$ is $k = n\log n$, then

$$\liminf_{n\to\infty} \mathbb{P}_\tau(\widehat{\mathcal{H}^\tau}) \geq e^{-2\omega} - \frac{\theta}{\pi} e^{-3\omega},$$

where $\omega = \lim_{n\to\infty} \omega(n)$.

Proof: Similar to the proof of Proposition 1, we first study the case where $r_{1.r.w.} = \frac{3\pi(\log n + \log\log n + \xi(n))}{2\theta n}$, for a fix ξ.

$$\begin{aligned}
\mathbb{P}_\tau(\widehat{\mathcal{H}^\tau}) \geq &\sum_{P_i\in\mathbb{M}} \sum_{O_j\in\mathbb{K}} \mathbb{P}_\tau(\{O_j \text{ of } P_i \text{ is uncovered}\}) \\
&- \sum_{P_i\in\mathbb{M}} \sum_{O_j,O_h\in\mathbb{K}}^{O_j\neq O_h} \mathbb{P}_\tau(\{O_j \text{ and } O_h \text{ of } P_i \text{ are uncovered}).
\end{aligned} \tag{13}$$

And we calculate that $\mathbb{P}_\tau(\{O_j \text{ of } P_i \text{ is uncovered}\}) = \prod_{y=1}^{u} \left(1 - \frac{2\theta\phi_y r_y(n)}{3\pi^2}\right)^{c_y n}$.

Then we can bound the first term of R.H.S of Eq. (13),

$$\sum_{P_i\in\mathbb{M}} \sum_{O_j\in\mathbb{K}} \mathbb{P}_\tau(\{O_j \text{ of } P_i \text{ is uncovered}\}) \geq e^{-2\omega}, \tag{14}$$

for any $\gamma > 1$ and all $n > N_\xi$.

Then we bound the second term

$$\begin{aligned}
&\sum_{P_i\in\mathbb{M}} \sum_{O_j,O_h\in\mathbb{K}}^{O_j\neq O_h} \mathbb{P}_\tau(\{O_j \text{ and } O_h \text{ of } P_i \text{ are uncovered}) \\
&\leq mk^2 \frac{2\theta}{2\pi} \prod_{y=1}^{u} \left(1 - \frac{r_y(n)\phi_y\theta}{\pi^2}\right)^{c_y n} \\
&+ mk^2 \left(1 - \frac{2\theta}{2\pi}\right) \prod_{y=1}^{u} \left(1 - \frac{4}{3}\frac{r_y(n)\phi_y\theta}{\pi^2}\right)^{c_y n} \\
&\sim mk^2 \frac{\theta}{\pi} e^{-\frac{2n\theta}{\pi}\sum_{y=1}^{u} c_y \frac{\phi_y}{2\pi} r_y(n)} \\
&+ mk^2 \left(1 - \frac{\theta}{\pi}\right) e^{-\frac{8n\theta}{3\pi}\sum_{y=1}^{u} c_y \frac{\phi_y}{2\pi} r_y(n)} \\
&= mk^2 \frac{\theta}{\pi} e^{-\frac{2n\theta}{\pi} r_{1.r.w.}} + mk^2 \left(1 - \frac{\theta}{\pi}\right) e^{-\frac{8n\theta}{3\pi} r_{1.r.w.}} \\
&= \frac{\theta}{\pi} e^{-3\omega} + \left(1 - \frac{\theta}{\pi}\right) e^{-4\omega} \frac{1}{n\log n}.
\end{aligned}$$

Since we consider the asymptotic coverage problem, which means that the total number of cameras n approaches to infinity, then we have

$$\mathbb{P}_\tau(\widehat{\mathcal{H}^\tau}) \geq e^{-2\omega} - \frac{\theta}{\pi} e^{-3\omega}. \tag{15}$$

Taking into account that ω is a function of n, the conclusion still holds. □

According to Proposition 5, we know that $r_{1.r.w.}(n) \geq R_{1.r.w.} = \frac{3\pi(\log n+\log\log n)}{2\theta n}$ can achieve the full view coverage of $\mathbb{M}$. Following that, the result can be naturally extended to the case where sensing range is smaller than $\sqrt{\frac{3}{2}}$ of critical ESR of the 1-dimensional random walk, as is stated in Theorem 8. And we omit the corresponding proof since it follows a similar manner to that of Theorem 3.

Theorem 8: Under the uniform deployment with the 1-dimensional random walk mobility model, if the CSN satisfies $r_{1.r.w.}(n) < \sqrt{\frac{3}{2}}R_{1.r.w.}$, there still will be an nonegligible probability that the network is uncovered. So we can say that the $r_{1.r.w.}(n) > \sqrt{\frac{3}{2}}R_{1.r.w.}$ is the necessary condition for the network to achieve full view coverage.

5.2.3 Sufficient ESR for Full View Coverage

First, we obtain the following proposition.

Proposition 6: In CSN, if n sensors are randomly and uniformly deployed in a unit square, and $r_{1.r.w.} = cR_{1.r.w.}(n)$ where $c > 1$, then

$$\liminf_{n\to\infty} \mathbb{P}_\tau(\widehat{\mathcal{H}_\tau}) = 0. \tag{16}$$

The proposition can be proved using a similar approach as in Proposition 2, given the fact that

$$\mathbb{P}_\tau(\widehat{\mathcal{H}_\tau}) = \mathbb{P}_\tau(\bigcup_{i=1}^{m}\mathcal{F}_i) \leq mk\prod_{y=1}^{u}\left(1-\frac{2\theta\phi_y r_y(n)}{3\pi^2}\right)^{c_y n}$$
$$\sim (n\log n)^2 e^{-\frac{4\theta}{3\pi}nr_{1.r.w.}} = \frac{1}{(n\log n)^{2c^2-2}} \to 0,$$

for any $c > 1$.

From Proposition 5 and the definition of critical ESR for 1-dimensional random walk mobility model, we know that $r_{1.r.w.} \geq R_{1.r.w.} = \frac{3\pi(\log n+\log\log n)}{2\theta n}$ is sufficient to achieve the full view coverage of $\mathbb{M}$. Based on this we can make our conclusion in the following theorem.

Theorem 9: Under the uniform deployment with 1-dimensional random walk mobility model, if the CSN satisfies $r_{1.r.w.}(n) > cR_{1.r.w.}$,$c > 1$, then it is sufficient for the network to achieve full coverage.

The proof again can be done by adopting a similar method ysed in the proof of Theorem 4, and we skip it for the sake of concision.

5.2.4 Critical ESR for Full View Coverage of the Operational Range

Similar to the analysis in static model, we can reach the following theorem regarding the relation between critical ESR and full view coverage.

Under the uniform deployment with 1-dimensional random walk mobility model, $R_{1.r.w.}(n) = \frac{3\pi(\log n+\log\log n)}{2\theta n}$ is the critical condition to achieve coverage with high probability. Moreover, referring to Theorems 8 and 9, we draw the conclusion that $R_{a.s.c.} = \sqrt{\frac{3}{2}}R_{1.r.w.}$ is the critical condition to achieve almost surely coverage for 1-dimensional random walk mobility model. Again, we see a decrease in critical ESR compared to that in static networks. This implies that even restricted mobility such as 1-dimensional random walk can lead to an improved coverage condition. The next part subsequently discloses the effect of random rotating mobility.

5.3 Critical ESR Under Random Rotating Mobility

We investigate the situation in one time slot under the random rotating mobility pattern, the effect of which on area coverage is illustrated in Figure 6. To proceed, we will still firstly analyze the full view coverage for dense grid $\mathbb{M}$, and then extend it to the whole area.

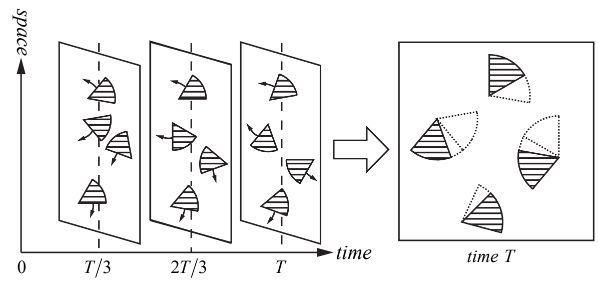

Fig. 6. Full view coverage of CSNs under random rotating walk: the left figure depicts a sequence of snapshots showing camera sensors' position change in time slots $T/3$, $2T/3$ and T; the right one illustrates the trace of sensor mobility during the whole interval [0, T). The shadowed disks constitute the area being covered at the given time instant, and the union of the region inside the dotted line and the shadowed disks represents the area being covered during the time interval.

5.3.1 Failure Probability of an Orientation in $\mathbb{K}$

Similarly, $\mathcal{F}_{i,j}$ denotes the event that orientation O_j of point P_i is not θ-viewed covered, and $\mathbb{P}(\mathcal{F}_{i,j})$ denotes the corresponding probability. With $\mathbb{P}_{i,j,S_y}$ representing the same meaning as that under 2-dimensional random walk, we have

$$\begin{aligned}\mathbb{P}_{i,j,S_y} &= \mathbb{P}(\text{S falls in } T_j) \times \mathbb{P}(\text{S has proper orientation})\\ &= \pi r_y^2(n) \times \frac{2\theta}{2\pi} \times \mathbb{P}(\text{S has proper orientation}).\end{aligned}$$

Here the event that the sensor has proper orientation means that the supposed viewed direction $\overrightarrow{PS}$ locates in the sensing region of the sensor. We will first calculate the probability that sensor S has proper orientation, which is denoted as $\mathbb{P}(S)$ in the following.

The initial angle from the bisector of the sensor to $\overrightarrow{PS}$ is denoted as G, a variable random uniformly distributed from 0 to 2π according to the deployment pattern. And the angle the sensor moves in a time slot is denoted as H, which is also a random variable distributed uniformly from 0 to 2π. Still, the sensor can rotate clockwise or counterclockwise, and the results are the same.

As shown in Figure 7, we set orientation $\overrightarrow{PS}$ as angle 0. When the bisector of the sensor initially locates in the sectorial area between $\overrightarrow{P_1S}$ and $\overrightarrow{P_2S}$, it can surely have a proper orientation. Otherwise when it moves counterclockwise, the bisector should go through $\overrightarrow{P_1S}$,

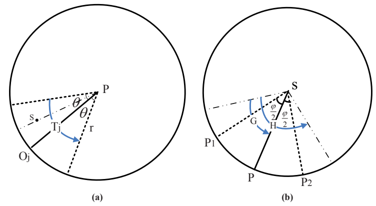

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Fig. 7. The figure on the left shows the sensor that locates in T_j with the supposed viewed direction, and the one on the right illustrates the rotating process of the sensor.

and when clockwise, the bisector should come cross $\overrightarrow{P_2S}$, which can be formulated as

$$\begin{cases} G - H < \frac{\phi_y}{2}, & \text{if } S \text{ moves counterclockwisely,} \\ G + H > 2\pi - \frac{\phi_y}{2}, & \text{if } S \text{ moves clockwisely.} \end{cases} \tag{17}$$

When the sensor moves clockwise, we obtain

$$\begin{aligned} \mathbb{P}(S) &= \frac{\phi_y}{2\pi} + \left(1 - \frac{\phi_y}{2\pi}\right) \mathbb{P}\left(G + H > 2\pi - \frac{\phi_y}{2}\right) \\ &= \frac{\phi_y}{2\pi} + \left(1 - \frac{\phi_y}{2\pi}\right)\left(\frac{1}{2} + \frac{\phi_y}{8\pi}\right) \\ &= \frac{1}{2}\left(1 + \frac{3\phi_y}{4\pi} - \frac{\phi_y^2}{8\pi^2}\right). \end{aligned} \tag{18}$$

When the sensor moves counterclockwise, similarly,

$$\mathbb{P}(S) = \frac{1}{2}\left(1 + \frac{3\phi_y}{4\pi} - \frac{\phi_y^2}{8\pi^2}\right).$$

Then we have

$$\begin{aligned} \mathbb{P}_{i,j,S_y} &= \pi r_y(n)^2 \times \frac{2\theta}{2\pi} \times \mathbb{P}(S) \\ &= \frac{\theta r_y^2(n)}{2}\left(1 + \frac{3\phi_y}{4\pi} - \frac{\phi_y^2}{8\pi^2}\right). \end{aligned} \tag{19}$$

Then, $\mathbb{P}(\mathcal{F}_{i,j})$ can be easily calculated.

5.3.2 Necessary ESR for Full View Coverage

Here, we use $\widehat{\mathcal{H}^\tau}$ to denote the event that the dense grid $\mathbb{M}$ is not fully full view covered in time slot τ. The following lemma states the correlation between network density m, the number of directions k as well as the sensing range.We will slightly abuse the notation and use the ω which represent the same meaning as that in Proposition 1. We now present the following proposition regarding the necessary condition.

Proposition 7: In the mobile heterogeneous CSN with random rotating mobility model, if $r_{r.r.} = \sqrt{\frac{4(\log n + \log\log n + \omega(n))}{n\theta}}$ and the density of the dense grid $\mathbb{M}$ is $m = n\log n$, the density of the orientation set $\mathbb{K}$ is $k = n\log n$, then

$$\liminf_{n\to\infty} \mathbb{P}_\tau(\widehat{\mathcal{H}^\tau}) \geq e^{-2\omega} - e^{-3\omega},$$

where $\omega = \lim_{n\to\infty}\omega(n)$.

Proof: We use a similar apporach as in Proposition 1 and first study the case where $r_{r.r.} = \sqrt{\frac{4(\log n + \log\log n + \omega)}{n\theta}}$, for a fix ω.

$$\begin{aligned} \mathbb{P}_\tau(\widehat{\mathcal{H}^\tau}) \geq & \sum_{P_i\in\mathbb{M}} \sum_{O_j\in\mathbb{K}} \mathbb{P}_\tau(\{O_j \text{ of } P_i \text{ is uncovered}\}) \\ & - \sum_{P_i\in\mathbb{M}} \sum_{O_j,O_h\in\mathbb{K}}^{O_j\neq O_h} \mathbb{P}_\tau(\{O_j \text{ and } O_h \text{ of } P_i \text{ are uncovered}). \end{aligned} \tag{20}$$

And we calculate that

$$\begin{aligned} & \mathbb{P}_\tau(\{O_j \text{ of } P_i \text{ is uncovered}\}) \\ & = \prod_{y=1}^{u}\left(1 - \frac{\theta}{2}\left(1 + \frac{\phi}{\pi} - \frac{\phi^2}{4\pi^2}\right) r_y^2(n)\right)^{c_y n}. \end{aligned}$$

Then the first term of R.H.S of Eq. (20) can be bounded as

$$\sum_{P_i\in\mathbb{M}} \sum_{O_j\in\mathbb{K}} \mathbb{P}_\tau(\{O_j \text{ of } P_i \text{ is uncovered}\}) \geq e^{-2\omega}, \tag{21}$$

for all $n > N_\omega$.

Similarly, the second term of R.H.S of Eq. (20) has

$$\sum_{P_i\in\mathbb{M}} \sum_{O_j,O_h\in\mathbb{K}}^{O_j\neq O_h} \mathbb{P}_\tau(\{O_j \text{ and } O_h \text{ of } P_i \text{ are uncovered}) \sim e^{-3\omega}. \tag{22}$$

Since we consider the coverage problem in an asymptotic sense where the total number of cameras n approaches to infinity, we have

$$\mathbb{P}_\tau(\widehat{\mathcal{H}^\tau}) \geq e^{-2\omega} - e^{-3\omega}. \tag{23}$$

Taking into account that ω is a function of n, the conclusion still holds. □

According to Proposition 7, we know that $R_{r.r.}(n) = \sqrt{\frac{4(\log n + \log\log n)}{n\theta}}$ is necessary to achieve the full view coverage of $\mathbb{M}$. The result also holds for the case where ω changes, and we thus finish the necessary part. Theorem 10 presents the result where the sensing range is smaller than $\sqrt{\frac{3}{2}}$ of critical ESR of random rotating mobility model.

Theorem 10: Under the uniform deployment where sensors move according to random rotating mobility model, if the CSN satisfies $r_{r.r}(n) < \sqrt{\frac{3}{2}}R_{r.r.}$, there still will be an nonegligible probability that the network is uncovered. So we can say that the $r_{r.r}(n) > \sqrt{\frac{3}{2}}R_{r.r.}$ is the necessary condition for the network to achieve full view coverage.

5.3.3 Sufficient ESR for Full View Coverage

The following proposition provides the relation between ESR and critical ESR under random rotating mobility model.

Proposition 8: In CSN, if n sensors are randomly and uniformly deployed in a unit square, and $r_{r.r.}(n) = cR_{r.r.}(n)$ where $c > 1$, then

$$\liminf_{n\to\infty} \mathbb{P}_\tau(\widehat{\mathcal{H}_\tau}) = 0. \tag{24}$$

Here it again can be derived that

$$\begin{aligned}\mathbb{P}_\tau(\widehat{\mathcal{H}_\tau}) &= \mathbb{P}_\tau(\bigcup_{i=1}^{m} \mathcal{F}_i) \\ &\le mk \prod_{y=1}^{u} \left[1 - \frac{\theta}{2}\left(1 + \frac{3\phi}{4\pi} - \frac{\phi^2}{8\pi^2}\right) r_y^2(n)\right]^{c_y n} \\ &\sim (n \log n)^2 e^{-\frac{n\theta}{2} r_{r.r.}^2} = \frac{1}{(n \log n)^{2c^2-2}} \to 0,\end{aligned} \tag{25}$$

for any $c > 1$.

According to Proposition 8, along with the definition of critical ESR for random rotating mobility model, we know that $r_{r.r.}(n) \ge R_{r.r.} = \sqrt{\frac{4(\log n + \log\log n)}{n\theta}}$ is sufficient to achieve the full view coverage of $\mathbb{M}$. And the result when sensing range is larger than $\sqrt{\frac{3}{2}}$ of critical ESR of the random rotating mobility model also follows, and is presented in Theorem 11, which can be proved by adopting the similar technique in the proof of Theorem 4.

Theorem 11: Under the uniform deployment with random rotating mobility model, if the CSN satisfies $r_{r.r.}(n) > cR_{r.r.}$,$c > 1$, then it is sufficient for the network to achieve full coverage.

5.3.4 Critical ESR for full view coverage of the operational range

Similar to the analysis in the static model, we can reach the following theorem.

Under uniform deployment with sensors moving according to random rotating mobility model, $R_{r.r.} = \sqrt{\frac{4(\log n + \log\log n)}{n\theta}}$ is the sufficient condition to achieve coverage with high probability. Moreover, referring to Theorems 10 and 11, we conclude that $R_{a.s.c.} = \sqrt{\frac{3}{2}} R_{r.r.}$ is the critical condition to achieve almost surely coverage for random rotating mobility model.

Surprisingly, the critical ESR is manifested to be larger under random rotating mobility than that obtained under static CSNs. The result discloses that mobility does not always lead to performance improvement. All the insights behind the theoretical results will be further discussed in the next section.

6 Discussion of Theoretical Findings

6.1 Relationship between Coverage with High Probability and Almost Surely Coverage

According to Sections 4 and 5, we find that the critical ESR ($R_{a.s.c}$) to achieve almost surely coverage is $\sqrt{\frac{3}{2}}$ times of that to achieve coverage with high probability for both static and mobile situations considered. In this section, we take the static model as an example to disclose the relationship between coverage with high probability and almost surely coverage.

According to Theorem 4, we know that when $r_{stat}(n)$ is larger than R_{stat}, it is sufficient for the operational region to achieve full coverage with high probability. Recall that when $r_{stat}(n) \ge R_{stat}$, $\mathbb{P}(\liminf_{n\to\infty} \mathcal{H}_n) = 1$. Technically, as defined in Section 2, this means that the event $\mathcal{H}_n$ will eventually almost surely happen or that ultimately all of the event $\mathcal{H}_n$ will occur almost surely under asymptotic network. Namely, there exists an N so that the operational region will be almost surely covered for all n which is larger than N.

According to Theorem 3 and Proposition 1, we know that when $r(n)$ is between R_{stat} and $\sqrt{\frac{3}{2}} R_{stat}$, the operational region is being covered with probability one. However, when $r(n)$ is smaller than $\sqrt{\frac{3}{2}} R_{stat}$, $\mathbb{P}(\limsup_{n\to\infty} \widehat{\mathcal{H}_n}) = 1$. As defined in Section 2, this means that the event $\widehat{\mathcal{H}_n}$ will happen under asymptotic network. In other words, for any number N, there exists an n which is larger than N so that the operational region is not being covered.

According to Theorem 2, we know that when $r(n)$ is below R_{stat}, the operational region is being covered with probability less than one. And the probability of coverage or coverage percentage can be calculated according to the sensing radius.

6.2 Critical Condition for Homogeneous CSNs

Previous sections mainly focus on heterogeneous static and mobile CSNs in the sense that sensors may have different sensing parameters such as sensing radius and sensing angles. Based on those, we can further expand our results to homogeneous case, where all sensors have identical sensing parameters. We only provide the main results here since the analysis shares a similar idea to that in heterogeneous cases.

- With static model, the critical sensing radius (CSR) is $R(n) = \sqrt{\frac{4\pi(\log n + \log\log n)}{n\theta\phi}}$.
- With 2-dimensional random walk mobility model, the CSR is
$$R(n) = \begin{cases} \frac{\pi(\log n + \log\log n)}{nTv\phi \sin\theta} & \text{if } \theta < \frac{\pi}{2} \\ \frac{\pi(\log n + \log\log n)}{nTv\phi} & \text{if } \theta \ge \frac{\pi}{2} \end{cases}$$
- With 1-dimensional random walk mobility model, the CSR is $R(n) = \frac{3\pi^2(\log n + \log\log n)}{\theta n\phi}$.
- With random rotating mobility model, the CSR is $R(n) = \sqrt{\frac{4(\log n + \log\log n)}{n\theta(1 + \frac{3\phi}{4\pi} - \frac{\phi^2}{8\pi^2})}}$.

In COROLLARY 5.1 of [3], Kumar presented that in a static and homogeneous network under uniform deployment, $c(n) \ge 1 + \frac{\phi(np) + k\log\log(np)}{\log(np)}$ is sufficient for a unit square to be asymptotically k-covered, where $c(n) = \frac{np\pi r^2}{\log(np)}$, $\phi(np) = o(\log\log(np))$ and p is the probability that a sensor is currently operating. By assuming that $p = 1$, $k = 1$, and ignoring $\phi(np)$ as $n \to \infty$, we translate this landmark result to our model, and obtain

$$r \ge \sqrt{\frac{(\log n + \log\log n)}{n\theta}},$$

which matches our result under static model when taking $\phi = 2\pi$ to represent omni-directional sensors. This result verifies the generality of our model.

To have an overview of all the derived results in the present work, Table 1 summarizes the results obtained in both homogeneous and heterogeneous networks.

6.3 Impact of Mobility on Sensing Energy Consumption

We consider the impact of mobility here. Sensors are considered to have critical ESR, with radius $r_y = r_i, i = stat, 2.r.w., 1.r.w., r.r.$, under static, 2-dimensional random walk, 1-dimensional random walk, and random rotating correspondingly. As we just convert the value of the angle of each sensor to the weight of its radius when we derive the critical ESR, the sensors can be viewed as omnidirectional traditional sensors and we here use the area the sensor covers to represent the sensing energy consumption of it.

We have the following results:

(a) Under Static Model:

$$\overline{E}_{stat} = \Theta\left(\frac{\log n + \log\log n}{n}\right). \tag{26}$$

(b) Under 2-Dimensional Random Walk Mobility Model:

$$\overline{E}_{2.r.w.} = \Theta\left(\left(\frac{\log n + \log\log n}{n}\right)^2\right). \tag{27}$$

(c) Under 1-Dimensional Random Walk Mobility Model:

$$\overline{E}_{1.r.w.} = \Theta\left(\left(\frac{\log n + \log\log n}{n}\right)^2\right). \tag{28}$$

(d) Under Random Rotating Mobility Model:

$$\overline{E}_{r.r.} = \Theta\left(\frac{\log n + \log\log n}{n}\right). \tag{29}$$

Therefore, taking static model as a baseline, we have

$$\overline{E}_{2.r.w.} = \overline{E}_{1.r.w.} = \Theta\left(\frac{\log n + \log\log n}{n}\right) \times \overline{E}_{stat},$$

$$\overline{E}_{r.r.} = \overline{E}_{stat},$$

which indicates that compared with static model, both the 2-dimensional random walk mobility model and 1-dimensional random walk mobility model can decrease the energy consumption in CSNs. And this improvement sacrifices the delay upper bounded by $\Theta(1)$ as the movement is divided into time slots. This is actually a tradeoff between energy consumption and the delay.

However, for random rotating mobility, the energy consumption is the same as when sensors are stationary, but it still causes a delay upper bounded by $\Theta(1)$, due to the division of the time slots. Furthermore, this results in much more energy consumption for movement. Thus, the movements like random rotating should be avoided for full view coverage.

More importantly, from previous theoretical analysis we could conclude that when we consider the critical sensing range under 2-dimensional random walk mobility model and 1-dimensional random walk mobility model, the rectangular area the sensor covers when it moves contributes most for coverage performance rather than the sectorial area it covers when it is static. For instance, under 2-dimensional random walk mobility model, the area $\theta r_y^2(n)$ in Eq. (7) does not affect the final result.

7 Simulation Results

In this part, we analyze the numerical results to validate the theoretical results on critical ESR to achieve full view coverage. Moreover, we investigate the relationship between ESR and the percentage of full view coverage.

7.1 Simulation Setup

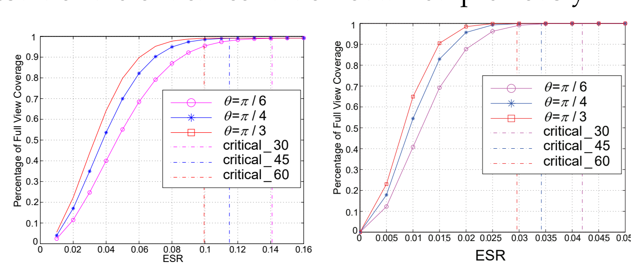
We again take the static model as an instance. The simulation can be easily extended to the other three mobile models. The target area is a unit square and we use two settings for sensor density, i.e., $n = 25 * 25$ and $n = 100 * 100$. For simplicity, we consider the homogeneous case, namely all the sensors have the same sensing parameter (sensing radius and angle). The effective angle is fixed, and we use three values for the fixed effective angle, i.e., $\theta = \pi/6, \pi/4, \pi/3$ (or 30, 45, 60 in degree) respectively.

We vary the ESR in simulations from 0 to 0.16 for $n = 625$, and from 0 to 0.05 for $n = 10000$, to observe the percentage of full view coverage, which is defined as the percentage of points that are full view covered.

7.2 Analysis of Simulation Results

7.2.1 Impact of ESRs on Full View Coverage

Figures 6(a) and 6(b) report the results of the percentage of full view coverage under different ESRs. We let the x-axis denote the percentage of full view coverage and the y-axis denote the ESR. The results in the two figures are obtained in the cases where $n = 625$ and $n = 10000$, respectively. In both cases, the ESRs needed for full view coverage increase as the required probability increases, although the ESR for $n = 10000$ is much lower than that for $n = 625$. According to the formulation derive in Section 4, we calculate the critical ESR for full view coverage when $\theta = \pi/6, \pi/4, \pi/3$ (or 30, 45, 60 in degree), respectively, and use dotted lines to indicate the critical ESR on Figures 6(a) and 6(b). It is clear that the network is able to achieve full view coverage with probability one when ESR is larger than the critical ESR. This verifies our result of the critical condition obtained previously.



(a) (b)

Fig. 8. Relationship between equivalent sensing range $R(n)$ and percentage of full view coverage under different n.

Moreover, we observe from Figures 6(a) and 6(b) that although the ESR needed to achieve full view coverage for the whole area may be high whereas the ESR

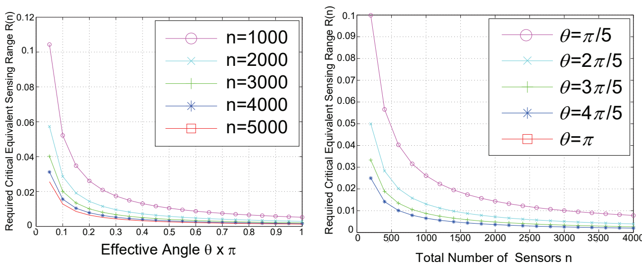
TABLE 1
Comparison of ESR and CSR

Network Type	ESR for heterogeneous network	CSR for homogeneous network	Energy Consumption
Static	$R(n)=\sqrt{\frac{2(\log n+\log\log n)}{n\theta}}$	$R(n)=\sqrt{\frac{4\pi(\log n+\log\log n)}{n\theta\phi}}$	$\Theta(\frac{\log n+\log\log n}{n})$
2.r.w.	$R(n)=\begin{cases}\frac{\log n+\log\log n}{2nTv\sin\theta} & \text{if } \theta<\frac{\pi}{2}\\ \frac{\log n+\log\log n}{2nTv} & \text{if } \theta\geq\frac{\pi}{2}\end{cases}$	$R(n)=\begin{cases}\frac{\pi(\log n+\log\log n)}{nTv\phi\sin\theta} & \text{if } \theta<\frac{\pi}{2}\\ \frac{\pi(\log n+\log\log n)}{nTv\phi} & \text{if } \theta\geq\frac{\pi}{2}\end{cases}$	$\Theta((\frac{\log n+\log\log n}{n})^2)$
1.r.w.	$R(n)=\frac{3\pi(\log n+\log\log n)}{2\theta n}$	$R(n)=\frac{3\pi^2(\log n+\log\log n)}{\theta n\phi}$	$\Theta((\frac{\log n+\log\log n}{n})^2)$
r.r.	$R(n)=\sqrt{\frac{4(\log n+\log\log n)}{n\theta}}$	$R(n)=\sqrt{\frac{4(\log n+\log\log n)}{n\theta(1+\frac{3\phi}{4\pi}-\frac{\phi^2}{8\pi^2})}}$	$\Theta(\frac{\log n+\log\log n}{n})$

needed for a high percentage (but not 100%) of full view coverage is much lower. For example, given $\theta=\pi/3$ and sensor density $n=10000$, 90% of the field is full view covered when ESR $r(n)=0.015$, which is only around half of the required critical ESR to achieve 100% full view coverage with ESR $r(n)=0.0296$. Hence, our results can provide useful guidelines in CSN design by balancing coverage performance and ESR according to certain engineering requirements.

7.2.2 Impact of Parameters n and θ on CSR

Here we continue to analyze the influence of the number of camera sensors n and sensing angle θ on the critical ESR denoted by $R(n)$. Figure 7(a) plots the relationship between $R(n)$ and θ, when n changes accordingly under 1-dimensional random walk mobility model. As a counterpart, Figure 7(b) illustrates the relationship between $R(n)$ and n, when θ changes accordingly under 1-dimensional random walk. When n is fixed, $R(n)$



(a) (b)

Fig. 9. Relationship between (a) $R(n)$ and θ, (b) $R(n)$ and n, when n changes accordingly under 1-dimensional random walk mobility model

becomes larger, as θ decreases for all the static and mobility patterns we have discussed. Hence, we need sensors of larger sensing range when a better view of object's face is required. It is obvious since larger sensing region render more sensors to cover a certain object, making it more likely to catch its frontal image. When it is sufficiently large (such as the case of n=4000, 5000 shown in Figure 7(a), n incurs no further influence on network performance. This fact coincides with the instinct that when there are plenty of sensors in the network, adding more sensors will not further reduce the critical equivalent sensing range. Furthermore, as can be seen in Figure 7(a), changing n will lead to a more apparent change of $R(n)$ for smaller effective angle θ, whereas n will have little influence on $R(n)$ when θ goes to π. Similar analysis also holds for Figure 7(b).

8 Conclusion

This paper studied the coverage problem in both static and mobile CSNs. In heterogeneous scenarios, we defined a metric named ESR for the corresponding modeling, and derived the critical sensing range for full view coverage under static model, 2-dimensional random walk mobility, 1-dimensional random walk and random rotating model. The results indicate that random walk mobility model can decrease the sensing energy consumption under certain delay tolerance. Furthermore, we derived the critical condition to achieve almost surely coverage, which is a much stronger result compared with the traditional coverage with probability one. We find that the critical condition to achieve almost surely coverage is around 1.225 times of that to achieve coverage with high probability. We also extended our result from heterogeneous networks to homogeneous ones for the corresponding ESR and CSR.

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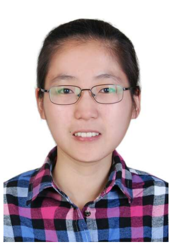

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